Describing inclusive semileptonic $b \to c\ell\bar{\nu}$ decays to higher precision for inclusive V_{cb} determinations

La Thuile 2024 - Les Rencontres de Physique de la Vallée d'Aoste

Ilija S. Milutin[1]

in collaboration with
Thomas Mannel[1] and K. Keri Vos[2]

Based on: JHEP 02 (2024) 226 [ArXiv: 2311.12002]

[1]CPPS, Universität Siegen, Germany

[2]GWFP, Maastricht University and Nikhef, The Netherlands

Universität Siegen

TP1 Theoretical Particle Physics
CPPS Center for Particle Physics Siegen

- $b \to c(u)\ell\bar{\nu}$ decays allow for studies of physics beyond the Standard Model and extraction of CKM parameters
- V_{cb} is an important input for many Standard Model predictions
- Long-standing puzzle between different determinations
  - exclusive: known-final state, e.g. $B \to D\ell\bar{\nu}$
  - inclusive: summed over all possible final states, e.g. $B \to X_c\ell\bar{\nu}$
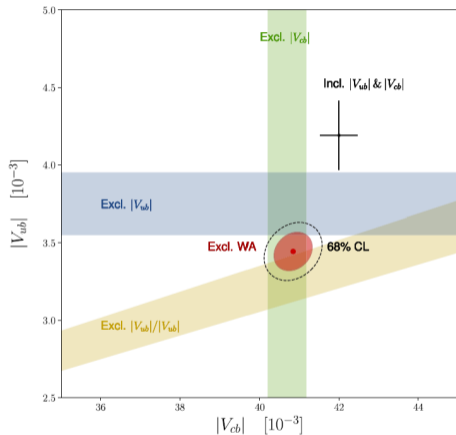
Bernlochner, Prim, Vos (*Eur. Phys. J. Spec. Top.* (2024))

Setting up the Heavy Quark Expansion

- m_b is large compared to $\Lambda_{\mathrm{QCD}} \to$ power series in $\Lambda_{\mathrm{QCD}}/m_b$
  Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel,...

- Split the momentum of the heavy quark as $p_b = m_b v + k$ where $v^2 = 1$

- Expand in residual momentum $k \sim iD$

- Perform Operator Product Expansion (OPE)

  Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel,...

$$d\Gamma = d\Gamma^{(3)} + \frac{1}{m_b^2}d\Gamma^{(5)} + \frac{1}{m_b^3}d\Gamma^{(6)} + \frac{1}{m_b^4}d\Gamma^{(7)} + ... \,, \qquad d\Gamma^{(n)} = \sum_i \mathcal{C}_i^{(n)}\langle B|\mathcal{O}_i^{(n)}|B\rangle$$

  - $d\Gamma^{(n)}$ are power series in $\mathcal{O}(\alpha_s)$
  - $\mathcal{C}_i^{(n)}$ perturbative Wilson coefficients
  - $\mathcal{O}^{(n)}$ local operators of dimension n
  - $\langle B|\mathcal{O}_i^{(n)}|B\rangle$ non-perturbative matrix elements $\rightarrow$ need to be extracted from data

Matrix elements

- The number of matrix elements increases very fast with the dimension:

$$d\Gamma = d\Gamma^{(3)} + \frac{1}{m_b^2}d\Gamma^{(5)} + \frac{1}{m_b^3}d\Gamma^{(6)} + \frac{1}{m_b^4}d\Gamma^{(7)} + \frac{1}{m_b^5}d\Gamma^{(8)} + ...$$

- $d\Gamma^{(3)}$: Partonic result ($d\Gamma^{(4)} = 0$ due to Heavy Quark Symmetries)
- $d\Gamma^{(5)}$: 2 parameters

$$2m_B\mu_\pi^2 = -\langle B|\bar{b}_v(iD)^2 b_v|B\rangle$$
$$2m_B\mu_G^2 = \langle B|\bar{b}_v(-i\sigma^{\mu\nu})(iD_\mu)(iD_\nu)b_v|B\rangle$$

- $d\Gamma^{(6)}$: 2 parameters

$$2m_B\rho_D^3 = \frac{1}{2}\langle B|\bar{b}_v[iD_\mu, [ivD, iD^\mu]]b_v|B\rangle$$
$$2m_B\rho_{LS}^3 = \frac{1}{2}\langle B|\bar{b}_v\{iD_\mu, [ivD, iD_\nu]\}(-i\sigma^{\mu\nu})b_v|B\rangle$$

- $d\Gamma^{(7)}$: 9 parameters (at tree level) Mannel, Turczyk, Uraltsev [1009.4622]; Kobach, Pal [1704.00008]
- $d\Gamma^{(8)}$: 18 parameters (at tree level) Mannel, Turczyk, Uraltsev [1009.4622]; Kobach, Pal [1704.00008]

Reparametrization Invariance

- In HQE, choice of v_μ is not unique Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen,...
- Lorentz invariance of QCD $\to$ Reparametrization Invariance (RPI) imposed under infinitesimal change $v_\mu \to v_\mu + \delta v_\mu$, leading to

$$\delta_{\mathrm{RP}} v_\mu = \delta v_\mu \ , \qquad \delta_{\mathrm{RP}} i D_\mu = -m_b \delta v_\mu \ , \qquad \delta_{\mathrm{RP}} b_v(x) = i m_b (x \cdot \delta v) b_v(x)$$

Reparametrization Invariance

- In HQE, choice of v_μ is not unique Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen,...
- Lorentz invariance of QCD $\rightarrow$ Reparametrization Invariance (RPI) imposed under infinitesimal change $v_\mu \rightarrow v_\mu + \delta v_\mu$, leading to
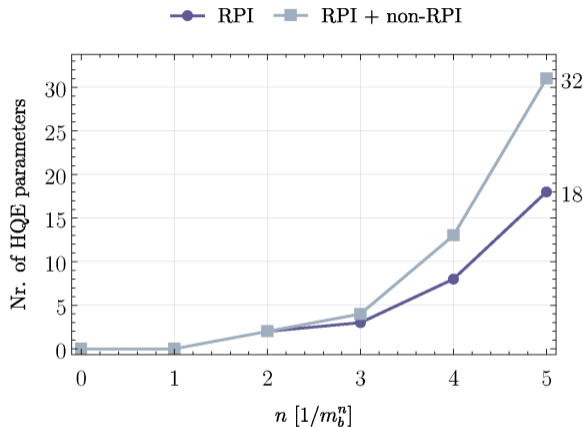
$$\delta_{\mathrm{RP}} v_\mu = \delta v_\mu \ , \qquad \delta_{\mathrm{RP}} iD_\mu = -m_b \delta v_\mu \ , \qquad \delta_{\mathrm{RP}} b_v(x) = im_b(x \cdot \delta v) b_v(x)$$

- RPI relates different orders n in $1/m_b$ expansion Mannel, Vos [1802.09409]

$$\delta_{\mathrm{RP}} \mathcal{C}^{(n)} \propto m_b \mathcal{C}^{(n+1)}$$

- This allows us to find combinations of operators which are RPI
- Lorentz invariant quantities depend on reduced set of RPI parameters

Counting RPI operators

- Up to $1/m_b^4$: total of 8 independent parameters <small>Mannel, Vos [1802.09409]</small>
- At $1/m_b^5$, we find only 10 RPI operators and 8 non-RPI operators <small>Mannel, ISM, Vos [2311.12002]</small>

New inclusive V_{cb} determination

- Need an RPI-observable $\rightarrow$ only depend on reduced set of RPI operators

New inclusive V_{cb} determination

- Need an RPI-observable → only depend on reduced set of RPI operators
- Dilepton invariant mass (q^2) moments are RPI and can be used to extract $|V_{cb}^{\text{incl}}|$

  (q^2-cut needed due to experimental setup)

  Bernlochner, Welsch, Fael, Olschewsky, Persson, von Tonder, Vos [2205.10274]

$$\langle (q^2)^n \rangle_{\text{cut}} = \frac{\int_{q^2 > q_{\text{cut}}^2} \mathrm{d}q^2 \, (q^2)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}}{\int_{q^2 > q_{\text{cut}}^2} \mathrm{d}q^2 \, \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}}$$

New inclusive V_{cb} determination

- Need an RPI-observable $\rightarrow$ only depend on reduced set of RPI operators
- Dilepton invariant mass (q^2) moments are RPI and can be used to extract $|V_{cb}^{\text{incl}}|$

  (q^2-cut needed due to experimental setup)

  Bernlochner, Welsch, Fael, Olschewsky, Persson, von Tonder, Vos [2205.10274]

$$\langle (q^2)^n \rangle_{\text{cut}} = \frac{\int_{q^2 > q_{\text{cut}}^2} \mathrm{d}q^2 \; (q^2)^n \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}}{\int_{q^2 > q_{\text{cut}}^2} \mathrm{d}q^2 \; \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}}$$

- Data $\rightarrow$ values for reduced set of RPI parameters up to $1/m_b^4 \rightarrow Br(\bar{B} \rightarrow X_c \ell \bar{\nu}) \rightarrow$

$$|V_{cb}^{\text{incl}}| = (41.69 \pm 0.63) \times 10^{-3} \quad \text{Bernlochner, Vos, et al. [2205.10274]}$$

New inclusive V_{cb} determination

- Need an RPI-observable $\rightarrow$ only depend on reduced set of RPI operators
- Dilepton invariant mass (q^2) moments are RPI and can be used to extract $|V_{cb}^{\text{incl}}|$

  (q^2-cut needed due to experimental setup)

  Bernlochner, Welsch, Fael, Olschewsky, Persson, von Tonder, Vos [2205.10274]

$$\langle (q^2)^n \rangle_{\text{cut}} = \frac{\int_{q^2 > q_{\text{cut}}^2} dq^2 \, (q^2)^n \frac{d\Gamma}{dq^2}}{\int_{q^2 > q_{\text{cut}}^2} dq^2 \, \frac{d\Gamma}{dq^2}}$$

- Data $\rightarrow$ values for reduced set of RPI parameters up to $1/m_b^4 \rightarrow Br(\bar{B} \rightarrow X_c \ell \bar{\nu}) \rightarrow$

$$|V_{cb}^{\text{incl}}| = (41.69 \pm 0.63) \times 10^{-3} \quad \text{Bernlochner, Vos, et al. [2205.10274]}$$

- First determination of V_{cb} up to $\mathcal{O}(1/m_b^4)$ and first extraction of $1/m_b^4$ matrix elements from data
- Agreement at $1 - 2\sigma$ level with previous $\mathcal{O}(1/m_b^3)$ determinations

  Finauri, Gambino [2310.20324]; Bordone, Capdevila, Gambino [2107.00604]; Alberti, Gambino, Healey, Nandi[1411.6560]; Gambino, Schwanda [1307.4551]

Where do we currently stand?

Γ	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	✓
$1/m_b^2$	✓	✓		
$1/m_b^3$	✓	✓		
$1/m_b^4$	✓			
$m_b^{\mathrm{kin}}/\bar{m}_c$		✓	✓	✓

$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
$1/m_b^2$	✓	✓		
$1/m_b^3$	✓	✓		
$1/m_b^4$	✓			

- Green: known perturbative corrections Jezabek, Kuhn (1989); Melnikov (2008); Pak, Czarnecki (2008); Becher, Boos, Lunghi (2007); Alberti, Gambino, Nandi (2014); Mannel, Pivovarov, Rosenthal (2015); Gambino, Healey, Turczyk (2016); Mannel, Pivovarov (2020); Fael, Schonwald, Steinhauser (2020, 2021)

- Next step in HQE is dimension 8: $1/m_b^5$

- Dimension 8 contains: Bigi, Mannel, Turczyk, Uraltsev [0911.3322]

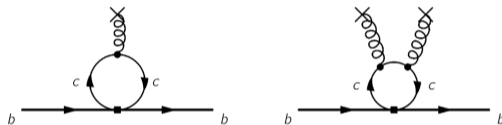"genuine" $\dfrac{1}{m_b^5}$ &   Intrinsic Charm $\dfrac{1}{m_b^3 m_c^2}$

Going higher in the $1/m_b$ expansion

- Dimension 8 contains: Bigi, Mannel, Turczyk, Uraltsev [0911.3322]

<div align="center">

"genuine" $\quad \dfrac{1}{m_b^5} \quad$ & $\quad$ Intrinsic Charm $\quad \dfrac{1}{m_b^3 m_c^2}$

</div>
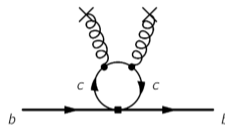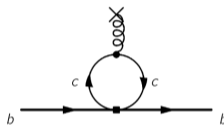
- Numerically: $m_c^2 \sim m_b \Lambda_{\mathrm{QCD}}$
- Dimension 8 contains enhanced terms which contribute like $1/m_b^4$ terms

Going higher in the $1/m_b$ expansion

- Dimension 8 contains: Bigi, Mannel, Turczyk, Uraltsev [0911.3322]

$$\text{"genuine"} \quad \frac{1}{m_b^5} \quad \& \quad \text{Intrinsic Charm} \quad \frac{1}{m_b^3 m_c^2}$$
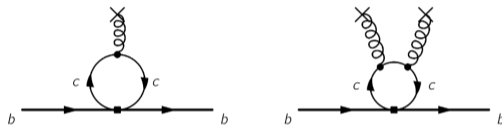
- Numerically: $m_c^2 \sim m_b \Lambda_{\text{QCD}}$
- Dimension 8 contains enhanced terms which contribute like $1/m_b^4$ terms

- We need $1/m_b^3 \times 1/m_c^2$ contributions to complete the calculation at $1/m_b^4$
  - We calculate the full dimension 8 contributions
  - We extract the Intrinsic Charm contribution
  - We find that only 1 combination of parameters describes the IC in the q^2-moments
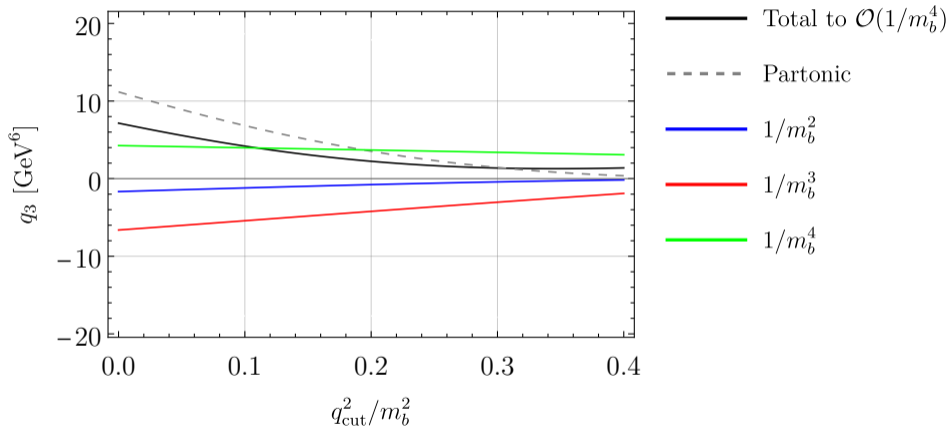   Mannel, ISM, Vos [2311.12002]

How to get the matrix elements...

- At $1/m_b^5 \to$ too many HQE parameters to fit to data
- LLSA: Lowest-Lying-State Approximation to estimate through known parameters μ_π^2, μ_G^2, ρ_D^3 and ρ_{LS}^3
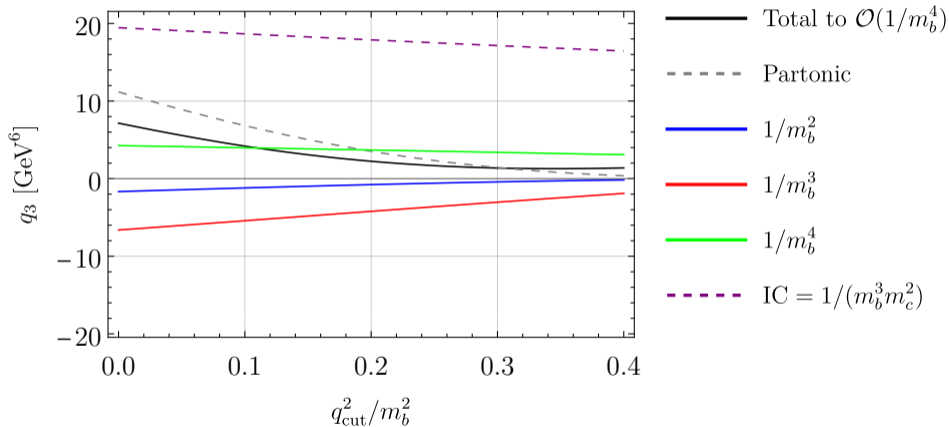 Mannel, Turczyk, Uraltsev [1009.4622]

$$\langle B|\bar{b} \, AC\Gamma \, b(0)|B\rangle = \frac{1}{2m_B} \sum_n \langle B|\bar{b} \, A \, b(0)|n\rangle \cdot \langle n|\bar{b} \, C\Gamma \, b(0)|B\rangle$$
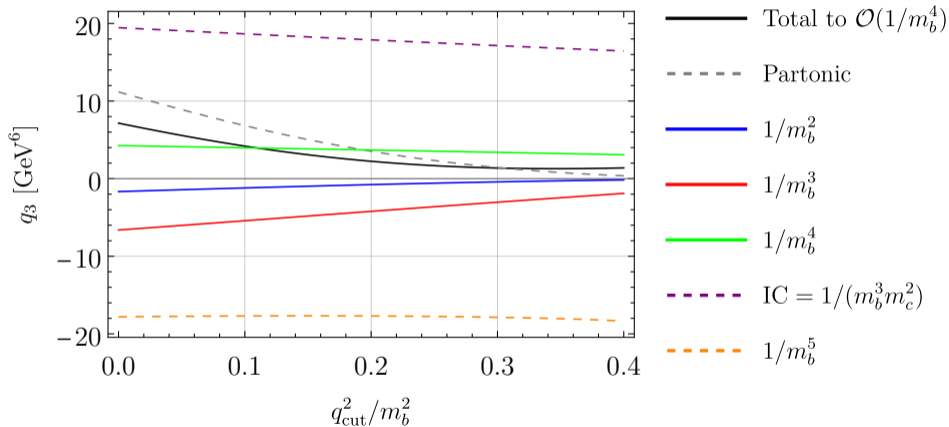
  - $A = iD_{\mu_1}...iD_{\mu_k}$, $C = iD_{\mu_{k+1}}...iD_{\mu_n}$

Mannel, ISM, Vos [2311.12002]

q^2-moments
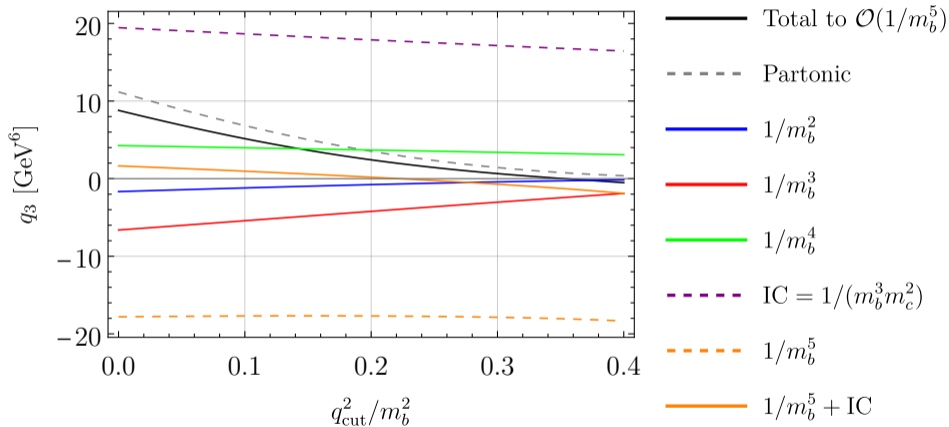

Mannel, ISM, Vos [2311.12002]

q^2-moments

Mannel, ISM, Vos [2311.12002]

Mannel, ISM, Vos [2311.12002]

- The $1/m_b^3 \times 1/m_c^2$ contributions are (partially) cancelled by the strict $1/m_b^5$ contributions

Conclusions and outlook

- The $1/m_b^3 \times 1/m_c^2$ contributions are (partially) cancelled by the strict $1/m_b^5$ contributions
- We find an unexpectedly small overall contribution of the dimension-8 operators

Conclusions and outlook

- The $1/m_b^3 \times 1/m_c^2$ contributions are (partially) cancelled by the strict $1/m_b^5$ contributions
- We find an unexpectedly small overall contribution of the dimension-8 operators
- q^2 moments to $1/m_b^5$ available in open-source library `kolya`

Conclusions and outlook

- The $1/m_b^3 \times 1/m_c^2$ contributions are (partially) cancelled by the strict $1/m_b^5$ contributions
- We find an unexpectedly small overall contribution of the dimension-8 operators
- q^2 moments to $1/m_b^5$ available in open-source library `kolya`
- We provide easy calculation of any kinematic moment to $1/m_b^5$ for $B \to X_c \ell \bar{\nu}$

  Mannel, ISM, Vos [2311.12002]

Conclusions and outlook

- The $1/m_b^3 \times 1/m_c^2$ contributions are (partially) cancelled by the strict $1/m_b^5$ contributions
- We find an unexpectedly small overall contribution of the dimension-8 operators
- q^2 moments to $1/m_b^5$ available in open-source library `kolya`
- We provide easy calculation of any kinematic moment to $1/m_b^5$ for $B \to X_c \ell \bar{\nu}$
  Mannel, ISM, Vos [2311.12002]
- Work in progress:
  - Full $|V_{cb}|^{\text{incl}}$ fit to $1/m_b^5$
  - Further understand theory uncertainties through the study of
    Quark-Hadron Duality Violation

- Assume for all dimension 8 operators that $X_i^5 \sim \Lambda_{\text{QCD}}^5$ and vary signs:

Combinations of signs of X_i^5

Legend:
- $*$ m_b^5
- $*$ IC
- $*$ Sum
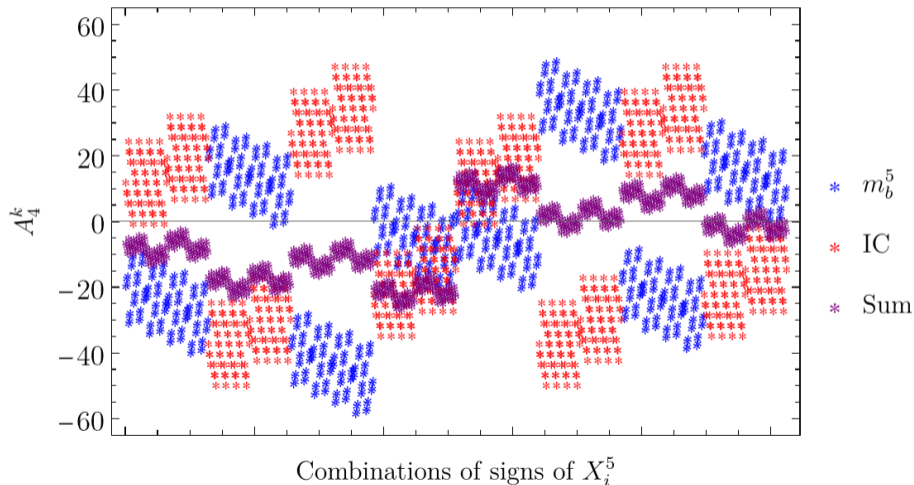
- **Step 1:** expand charm propagator ($Q^\mu = m_b v^\mu$)

$$
-iS_{\mathrm{BGF}} = \frac{1}{\slashed{Q} + i\slashed{D} - m_c}
$$

$$
= \frac{1}{\slashed{Q} - m_c} - \frac{1}{\slashed{Q} - m_c}(i\slashed{D})\frac{1}{\slashed{Q} - m_c}
$$

$$
+ \frac{1}{\slashed{Q} - m_c}(i\slashed{D})\frac{1}{\slashed{Q} - m_c}(i\slashed{D})\frac{1}{\slashed{Q} - m_c} + ...
$$

- **Step 2:** insert in forward matrix element

$$T = \left[\Gamma \frac{1}{\not{Q} - m_c} \Gamma^\dagger \right]_{\alpha\beta} \langle \bar{b}_\alpha b_\beta \rangle$$

$$- \left[\Gamma \frac{1}{\not{Q} - m_c} \gamma^\mu \frac{1}{\not{Q} - m_c} \Gamma^\dagger \right]_{\alpha\beta} \langle \bar{b}_\alpha (iD_\mu) b_\beta \rangle$$

$$+ \left[\Gamma \frac{1}{\not{Q} - m_c} \gamma^\mu \frac{1}{\not{Q} - m_c} \gamma^\nu \frac{1}{\not{Q} - m_c} \Gamma^\dagger \right]_{\alpha\beta} \langle \bar{b}_\alpha (iD_\mu)(iD_\nu) b_\beta \rangle$$

$$+ \ldots$$

Back-up: Calculation of forward matrix element

- **Step 3:** Determine Trace formula
  - Start at dimension 8 → lengthy, but systemically calculable
  - Compute dim-7, including $1/m_b$ correction through e.o.m.

$$(ivD)b_v = -\frac{1}{2m_b}(i\not{D})(i\not{D})b_v$$

  - ...
  - Compute dim-3, including corrections up to $1/m_b^5$

$$\langle \bar{b}_\alpha b_\beta \rangle = 2m_B \left(\frac{1+\not{v}}{4} + \frac{1}{8m_b^2}(\mu_G^2 - \mu_\pi^2) + \mathcal{O}(1/m_b^6) \right)_{\beta\alpha}$$

    - Need full (non-RPI) set of basic parameters up to $1/m_b^5$
- **Step 4:** Compute the trace with the geometric series