The Chiral Lagrangian for CP-violating Axion-like particles

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Axion-Like Particles: Motivations

Axion-Like Particles (ALPs)

Axion-Like Particles (ALPs) are the pseudo Nambu-Goldstone bosons (pNGBs) that emerge from the **spontaneous breaking of an anomalous symmetry in the UV**

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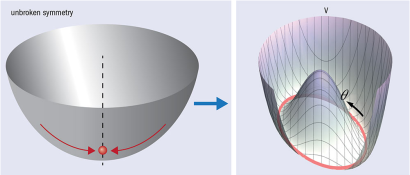

Differently from the QCD axion, ALPs:

- Need not solving the strong CP problem
- Have arbitrary masses and couplings ($f_\phi m_\phi \not\sim f_\pi m_\pi$)

Axion-Like Particles: Motivations

ALPs can address several open problems in particle physics:

- Strong CP problem (**QCD axion**)
- Hierarchy problem (**relaxion**)
- Flavour problem (**axiflavon/flaxion**)
- The observed **dark matter** abundance
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ALPs can be **probed experimentally** via:

- **Higgs and Z boson decay processes** ($h \to Z\phi$, $Z \to \gamma\phi$)
- **Flavour-changing neutral current** processes ($K^\pm \to \pi^\pm\phi$)
- **Electric Dipole Moments (EDMs)** of particles, nucleons, atoms, molecules (iff the ALP has CP-violating interactions)

Probing the CP violating ALP

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Our idea: **probe CP-violating ALPs** at low energies. We started from the most general $SU(3)_c \times U(1)_{em}$ invariant

EFT for a CP-violating ALP ϕ at the EW scale ($\Lambda \gg M_W$)

$$\mathcal{L}_{\text{ALP}}^{\text{dim-5}} \supset +e^2 \frac{C_\gamma}{\Lambda} \phi F^{\mu\nu} F_{\mu\nu} + e^2 \frac{\tilde{C}_\gamma}{\Lambda} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} + g_s^2 \frac{C_g}{\Lambda} \phi G_a^{\mu\nu} G_{\mu\nu}^a + g_s^2 \frac{\tilde{C}_g}{\Lambda} \phi G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a + \frac{v}{\Lambda} y_S^{ij} \phi \bar{f}_i f_j + i \frac{v}{\Lambda} y_P^{ij} \phi \bar{f}_i \gamma_5 f_j + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$

[Di Luzio, Gröber, Paradisi,'20]

Jarlskog invariants: $C_a \tilde{C}_b$, $y_S^{ii} \tilde{C}_a$, $y_P^{ii} C_a$, $y_S^{ii} y_P^{jj}$, $y_S^{ik} y_{SM}^{kk} y_P^{ki}$

Chiral Perturbation theory: a quick recap - I

χPT is an effective field theory describing strong interactions at low energies (see, e.g. [Pich,'95]).

- Symmetries: $\mathrm{G}^0_{\mathrm{QCD}} \supset SU(3)_L \times SU(3)_R \longrightarrow SU(3)_V$

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Leading order chiral Lagrangian

$$\mathcal{L}^0_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i\gamma^\mu(\bar{q}_L D_\mu q_L + \bar{q}_R D_\mu q_R)$$

$$\mathcal{L}^{\mathcal{O}(\mathrm{p}^2)}_{\chi\mathrm{PT}} = \frac{f^2}{4} \mathrm{Tr}(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) \qquad \text{with} \quad q^T = (u, d, s)$$

Chiral Perturbation theory: a quick recap - II

External gauge and scalar **fields** enter as sources in $\mathcal{L}_{\mathrm{QCD}}$:

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}^0_{\mathrm{QCD}} + \bar{q}\gamma^\mu(2r_\mu P_R + 2\ell_\mu P_L)q - \bar{q}(s - i\gamma_5 p)q$$

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These enter $\mathcal{L}_{\chi\mathrm{pt}}$ via

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{f^2}{4}\mathrm{Tr}\left[D_\mu\Sigma^\dagger D^\mu\Sigma + \Sigma^\dagger\chi + \chi^\dagger\Sigma\right]$$

$$D_\mu\Sigma = \partial_\mu\Sigma + i\Sigma\ell_\mu - ir_\mu\Sigma, \qquad \chi = 2B_0(s + ip)$$

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Chiral counterparts to quark-containing operators are found exploiting the low-energy path-integral **duality**

$$\int \mathcal{D}q\,\mathcal{D}\bar{q}\,\mathcal{D}G_\mu\, \exp\left(i\int d^4x\,\mathcal{L}^{\text{ext}}_{\text{QCD}}\right) = \int \mathcal{D}\Sigma\, \exp\left(i\int d^4x\,\mathcal{L}^{\text{ext}}_{\chi\text{pt}}\right)\ (*)$$

From quarks to mesons

We want to find the **chiral counterpart** to our Lagrangian

EFT for a CP-violating ALP ϕ at the QCD scale at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\text{QCD scale}} = e^2 \frac{C_\gamma}{\Lambda} \phi F F + e^2 \frac{\tilde{C}'_\gamma}{\Lambda} \phi F \tilde{F} + g_s^2 \frac{C_g}{\Lambda} \phi G G + g_s^2 \frac{\tilde{C}'_g}{\Lambda} \phi G \tilde{G} + \frac{\partial_\mu \phi}{\Lambda} \bar{q} \gamma^\mu (Y_S + Y_P \gamma_5) q + \frac{v}{\Lambda} \phi \bar{q} y_{q,S} q + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Chiral counterparts to quark-containing operators are found exploiting the low-energy path-integral **duality** (*). For instance:

Example

$$\bar{q}_i y_{ij}^S q_j = -y_{ij}^S \frac{\partial \mathcal{L}_{\text{QCD}}}{\partial y_{ij}^S} \longrightarrow -y_{ij}^S \frac{\partial \mathcal{L}_{\chi\text{pt}}}{\partial y_{ij}^S} = -\frac{f_\pi^2}{2} B_0 \text{Tr} \left[y^S (\Sigma + \Sigma^\dagger) \right]$$

Getting rid of gluons

- Eliminate ϕGG thanks to the **trace anomaly** equation [Leutwyler, Shifman,'89]:

$$T^{\mu}{}_{\mu} = \sum_q m_q \bar{q} q - \frac{\alpha_s}{8\pi} \beta^0_{\text{QCD}} \, G_a^{\mu\nu} G^a_{\mu\nu} - \frac{\alpha_{\text{em}}}{8\pi} \beta^0_{\text{QED}} \, F^{\mu\nu} F_{\mu\nu}$$

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- Eliminate $\phi G \tilde{G}$ via an **ALP-dependent quark field redefinition**[Georgi, Kaplan, Randall,'86]:

$$q \to q = \exp\left[i \frac{\phi}{\Lambda} \left(Q_V + \lambda_g^* Q_A \gamma_5 \right) \right] q'$$

with Q_V and Q_A are arbitrary hermitian 3×3 matrices (Q_V is diagonal, Tr(Q_A)$= 1/2$, $\lambda_g^* = 32\pi^2 \tilde{C}_g'$).

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- Other **couplings** are **modified** (currents, masses, ...)!

Chiral Lagrangian for the CPV ALP

All of the previous modifications lead to the following

EFT for a CP-violating ALP ϕ at the QCD scale at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\text{QCD scale}} = e^2 \frac{c_\gamma}{\Lambda} \phi F F + e^2 \frac{\tilde{c}_\gamma}{\Lambda} \phi F \tilde{F} + \frac{\partial_\mu \phi}{\Lambda} \bar{q} \gamma^\mu \left(Y_V + Y_A \gamma_5 \right) q$$
$$- \kappa \frac{\phi}{\Lambda} T^\mu{}_\mu + \frac{v}{\Lambda} \phi \bar{q} \mathcal{Z} q + \bar{q}_L M_q^\phi q_R + \text{h.c.} + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Its counterpart is found by using the **duality** in (*)

Mesonic Chiral Lagrangian for a CP-violating ALP ϕ at $\mathcal{O}(\Lambda^{-2})$

$$\mathcal{L}_{\text{ALP}}^{\chi\text{pt}} = \frac{\partial_\mu \phi}{\Lambda} \left[2\,\text{Tr}(Y_V T_a) j_V^{\mu,a} + 2\,\text{Tr}(Y_A T_a) j_A^{\mu,a} \right] + \frac{f_\pi^2}{2} B_0 \text{Tr} \left[M_\phi \Sigma^\dagger + \Sigma M_\phi{}^\dagger \right]$$
$$+ \kappa \frac{f_\pi^2}{2} \frac{\phi}{\Lambda} \left[\text{Tr}(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger) + 4 B_0 \text{Tr} \left[M_q (\Sigma + \Sigma^\dagger) \right] \right]$$
$$- \frac{f_\pi^2}{2} \frac{v}{\Lambda} B_0 \phi \,\text{Tr} \left[\mathcal{Z} (\Sigma + \Sigma^\dagger) \right] + e^2 \frac{c_\gamma}{\Lambda} \phi F F + e^2 \frac{\tilde{c}_\gamma}{\Lambda} \phi F \tilde{F} + \mathcal{L}_{\text{ALP, leptons}}^{\text{QCD scale}}$$

Matching onto the low-energy Lagrangian ($n_f = 2$)

The $\mathcal{O}(\Lambda^{-2})$ low-energy Lagrangian $\mathcal{L}_{\phi\chi}$ valid for $E < 1{-}2$ GeV is:

low-energy CP-violating ALP Lagrangian

$$\mathcal{L}_{\phi\chi} = -\frac{1}{3}\frac{m_\pi^2}{m_\pi^2 - M_\phi^2}\frac{\Delta_{ud}}{f_\pi \Lambda}\Big[-2\partial\phi\left(2\pi^+\pi^-\partial\pi_0 + \pi_0\pi^+\partial\pi^- + \pi_0\pi^-\partial\pi^+\right)$$
$$+ M_\phi^2\phi\left(\pi_0^3 + 2\pi^+\pi^-\pi_0\right)\Big] + 2\kappa\frac{\phi}{\Lambda}[\partial_\mu\pi^+\partial^\mu\pi^- + \frac{1}{2}\partial_\mu\pi^0\partial^\mu\pi^0]$$
$$- m_\pi^2\omega\frac{\phi}{\Lambda}[\pi^+\pi^- + \frac{1}{2}\pi_0^2] + C_N^S\frac{\phi}{\Lambda}\bar{N}_v N_v + C_N^A\frac{\partial_\mu\phi}{\Lambda}\bar{N}_v\gamma^\mu\gamma_5 N_v$$
$$+ e^2\widetilde{C}'_\gamma\frac{\phi}{\Lambda}F\widetilde{F} + e^2 C'_\gamma\frac{\phi}{\Lambda}FF + +i\frac{v}{\Lambda}y_{P,\ell}^{ij}\,\phi\bar{\ell}_i\gamma_5\ell_j + \frac{v}{\Lambda}y_{S,\ell}^{ij}\,\phi\bar{\ell}_i\ell_j$$

All the couplings in $\mathcal{L}_{\phi\chi}$ can be expressed in terms of those in $\mathcal{L}_{\text{ALP}}^{\text{dim-5}}$ or at most of **measurable/computable** quantities.

Example: $Y_A^{ij} = -y_{q,P}^{ij}\frac{v}{m_i + m_j} - 32\pi^2\, Q_A^{ij}\tilde{C}_g$

CPV Jarlskog invariants ($n_f = 2$)

The **low-energy Jarlskog invariants** are found from $\mathcal{L}_{\phi\chi}$ by multiplying the Wilson coefficients of operators possessing **opposite CP** transformation properties

Example

$$c_\gamma FF \xrightarrow{CP} c_\gamma FF \qquad\qquad \tilde{c}_\gamma F\widetilde{F} \xrightarrow{CP} -\tilde{c}_\gamma F\widetilde{F} \quad \longrightarrow \quad c_\gamma \tilde{c}_\gamma \text{ is a Jarlskog invariant!}$$

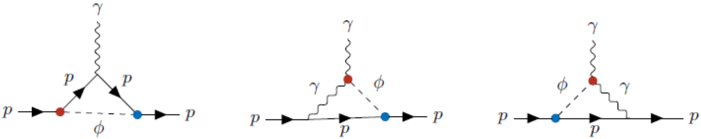
	c_γ	$y_{\ell,S}$	κ	$\mathcal{Z}$	$C_{\phi NN}$
$\tilde{c}_\gamma$	$\tilde{c}_\gamma\, c_\gamma$	$\tilde{c}_\gamma\, y_{\ell,S}$	$\tilde{c}_\gamma\, \kappa$	$\tilde{c}_\gamma\, \mathcal{Z}$	$\tilde{c}_\gamma\, C_{\phi NN}$
$y_{\ell,P}$	$y_{\ell,P}\, c_\gamma$	$y_{\ell,P}\, y_{\ell,S}$	$y_{\ell,P}\, \kappa$	$y_{\ell,P}\, \mathcal{Z}$	$y_{\ell,P}\, C_{\phi NN}$
Δ^A_{ud}	$\Delta^A_{ud}\, c_\gamma$	$\Delta^A_{ud}\, y_{\ell,S}$	$\Delta^A_{ud}\, \kappa$	$\Delta^A_{ud}\, \mathcal{Z}$	$\Delta^A_{ud}\, C_{\phi NN}$
$\tilde{C}_{\phi N}$	$\tilde{C}_{\phi N}\, c_\gamma$	$\tilde{C}_{\phi N}\, y_{\ell,S}$	$\tilde{C}_{\phi N}\, \kappa$	$\tilde{C}_{\phi N}\, \mathcal{Z}$	$\tilde{C}_{\phi N}\, C_{\phi NN}$

Table: Jarlskog invariants of the low-energy chiral Lagrangian $\mathcal{L}_{\phi\chi}$

Phenomenological applications

Phenomenological applications we have studied include:

- **EDMs** of protons, neutrons, atoms, molecules ...



$$d_p \simeq -\frac{e\,Q_p}{4\pi^2\Lambda^2}\left[C_{\phi \mathrm{pp}}\,\tilde{C}_{\phi \mathrm{p}} + 6e^2 m_p c_\gamma\,\tilde{C}_{\phi \mathrm{p}} + 2e^2\tilde{c}_\gamma\,C_{\phi \mathrm{pp}}\right]$$

$$\longrightarrow C_g\,\tilde{C}_g < 4.4\times 10^{-8}$$

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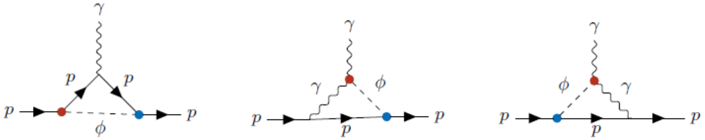

$$d_p \simeq -\frac{e\,Q_p}{4\pi^2\Lambda^2}\left[C_{\phi pp}\tilde{C}_{\phi p} + 6e^2 m_p c_\gamma \tilde{C}_{\phi p} + 2e^2\tilde{c}_\gamma C_{\phi pp}\right]$$

$$\longrightarrow C_g\tilde{C}_g < 4.4\times 10^{-8}$$

- **Ratio of the BRs** for $\phi \to 2\pi$ over $\phi \to 3\pi$

Summary

We have:

- Constructed the most general **Chiral Lagrangian** for a **CPV ALP** both in a **2-flavors** and in a **3-flavors** setting
- Provided the **matching dictionary** relating the IR couplings in the chiral Lagrangian to the UV couplings at the EW scale
- Classified the **low-energy Jarlskog invariants** of the theory.
- Written a `FeynRules` **model** for both the 2- and the 3-flavors setting $\rightarrow$ extensive, automatized pheno analyses

Thanks for your attention!

Backup slides

Kinetic and Mass mixing in a 2-flavor setting - I

From the coupling to the **axial current** and from the **mass** term we have both **kinetic and mass mixing** between ϕ and π_0:

$$\mathcal{L}_{\chi\text{pt}}^{\text{ALP mixing}} = \frac{1}{2}\partial^\mu \varphi^T \, \mathbf{Z} \, \partial_\mu \varphi - \frac{1}{2}\varphi^T \, \mathbf{M} \, \varphi \qquad \text{with} \qquad \varphi = \begin{pmatrix} \phi \\ \pi_0 \end{pmatrix}$$

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$$\mathbf{Z} = \begin{bmatrix} 1 & \epsilon \\ \epsilon & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{M} = \begin{bmatrix} m_\phi^2 & -\epsilon\,\alpha \\ -\epsilon\,\alpha & m_\pi^2 \end{bmatrix} \quad \epsilon = (Y_A^u - Y_A^d)\frac{f_\pi}{\Lambda},$$

$$\alpha = 2\frac{m_\pi^2}{(Y_A^u - Y_A^d)} \lambda_g^* \frac{m_u q_u - m_d q_d}{m_u + m_d},$$

$$\phi_{\text{ph}} = \phi + \epsilon \frac{m_\pi^2 + \alpha}{m_\pi^2 - m_\phi^2}\pi_0 \quad \pi_{0,\text{ph}} = \pi_0 - \epsilon \frac{m_\phi^2 + \alpha}{m_\pi^2 - m_\phi^2}\phi$$

Kinetic and Mass mixing in a 2-flavor setting - II

On the choice of α

$\alpha = \alpha(Q_A)$ can be tuned at will by choosing proper values of q_A^i.

The standard choice is $\alpha = 0$, but setting $\alpha = -m_\phi^2$ [Bauer, Neubert, Renner, Schnubel, Thamm,'21] yields much simpler expressions !

$$q_{u/d} = \frac{1}{2}\frac{m_{d/u}}{m_u + m_d}$$

$$q_{u/d} = \frac{1}{2}\frac{m_{d/u}}{m_u + m_d} \mp \frac{m_\phi^2}{m_\pi^2 - m_\phi^2}\frac{\Delta_{ud}^A}{2\lambda_g^*}, \quad \Delta_{ud}^A = \frac{m_\pi^2 - m_\phi^2}{m_\pi^2}(Y_A^u - Y_A^d)$$

Some comments

- In a **3-flavors** case, the ALP will mix with all the neutral mesons. Due to $\mathrm{Tr}(Q_A) = 1/2$ we can choose the q_A^i in order to avoid the mixing of the ALP with η or π_0 (**not both!**)

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- **Baryons** can be included as well via the Lagrangian pieces

$$\mathcal{L}_{\text{HN}} = i\bar{N}_v\gamma^\mu D_\mu N_v - g_A\bar{N}_v\gamma^\mu\gamma_5\mathcal{A}_\mu N_v$$

where $D_\mu = \partial_\mu + \mathcal{V}_\mu$

$$\mathcal{A}^\mu = \frac{i}{2}(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi) = \frac{\partial^\mu\pi}{2f_\pi} + \ldots \qquad \xi(x) = \exp\left[i\frac{\pi(x)}{2f_\pi}\right]$$

$$\mathcal{V}^\mu = \frac{1}{2}(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi) = \frac{1}{8}\frac{[\pi, \partial^\mu\pi]}{f_\pi^2} + \ldots$$

Getting rid of ϕGG

The coupling of the ALP with the **scalar** gluonic density can be eliminated thanks to the **trace anomaly** equation [Leutwyler, Shifman,'89]:

$$T^{\mu}{}_{\mu} = \sum_q m_q \bar{q} q - \frac{\alpha_s}{8\pi} \beta^0_{\text{QCD}} \, G_a^{\mu\nu} G^a_{\mu\nu} - \frac{\alpha_{\text{em}}}{8\pi} \beta^0_{\text{QED}} \, F^{\mu\nu} F_{\mu\nu}$$

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This:

- Introduces the operator $\phi \, \theta^{\mu}_{\mu}$
- Modifies the coupling of the operator $\phi \bar{f} f$ $(y^S \to \mathcal{Z})$
- Modifies the coupling of the operator ϕFF $(C_\gamma \to C'_\gamma)$

Getting rid of $\phi\tilde{G}G$

The coupling of the ALP with the **pseudoscalar** gluonic density is eliminated via an **ALP-dependent quark field redefinition**:

$$q \to q = \exp\left[i\frac{\phi}{\Lambda}\left(Q_V + \lambda_g^* Q_A\gamma_5\right)\right] q'$$

with Q_V and Q_A are arbitrary hermitian 3×3 matrices (Q_V is diagonal).

Getting rid of $\phi\tilde{G}G$

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with Q_V and Q_A are arbitrary hermitian 3×3 matrices (Q_V is diagonal).

- Eliminates $\phi\tilde{G}G$ if $\mathrm{Tr}(Q_A) = 1/2$, $\lambda_g^* = 32\pi^2 \tilde{C}'_g$
- Modifies the coupling of the operator $\phi\tilde{F}F$ ($\tilde{C}'_\gamma \to \tilde{C}''_\gamma$)
- Modifies the coupling of the operators $\partial_\mu\phi\,\bar{f}\gamma_\mu(\gamma_5)f$ (via the kinetic term for fermions)
- Modifies the mass term for quarks as $\bar{q}_L M_q q_R \quad \to \quad \bar{q}'_L e^{i\frac{\phi}{\Lambda}\lambda_g^* Q_A} M_q e^{i\frac{\phi}{\Lambda}\lambda_g^* Q_A} q'_R = \bar{q}'_L M_q^\phi q'_R$

Chiral Perturbation theory for Baryons - I

The baryon octet $B(x)$ is described by the 3×3 matrix

$$B = \begin{bmatrix} \frac{\Sigma_0}{\sqrt{2}} + \frac{\Lambda_0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma_0}{\sqrt{2}} + \frac{\Lambda_0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda_0 \end{bmatrix}$$

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The expansion in terms of p/Λ_{QCD} does **not converge** because $p \sim m_B \sim \Lambda_{\mathrm{QCD}}$.
By parametrizing the momentum as $p = m_B v + k$ (v is the velocity of the baryon) we can define the **definite-velocity** baryon field B_v:

$$B_v(x) = \frac{1 + \not{v}}{2} e^{i m_B v_\mu x^\mu} B(x)$$

Its derivatives produce powers of k, allowing for a meaningful perturbative expansion.

Chiral Perturbation theory for Baryons - II

Introducing the quantities

$$\xi = \exp\left[i\frac{\pi}{2f_\pi}\right]$$

$$\mathcal{A}^\mu = \frac{i}{2}(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi) = \frac{\partial^\mu\pi}{2f_\pi} + \dots$$

$$\mathcal{V}^\mu = \frac{1}{2}(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi) = \frac{1}{8}\frac{[\pi, \partial^\mu\pi]}{f_\pi^2} + \dots$$

one can build the leading-order heavy baryon Lagrangian :

$$\mathcal{L}_{\mathrm{HB}} = i\mathrm{Tr}(\bar{B}_v\gamma^\mu D_\mu B_v) - D\,\mathrm{Tr}(\bar{B}_v\gamma^\mu\gamma_5\{\mathcal{A}_\mu, B_v\}) - F\,\mathrm{Tr}(\bar{B}_v\gamma^\mu\gamma_5[\mathcal{A}_\mu, B_v])$$

where $D_\mu = \partial_\mu + [\mathcal{V}_\mu, \cdot]$

Chiral Perturbation theory for Baryons - III

Introducing the quantities

$$N_v = \begin{pmatrix} p_v \\ n_v \end{pmatrix}$$

$$\xi = \exp\left[i\frac{\pi}{2f_\pi}\right]$$

$$\mathcal{A}^\mu = \frac{i}{2}(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi) = \frac{\partial^\mu\pi}{2f_\pi} + \dots$$

$$\mathcal{V}^\mu = \frac{1}{2}(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi) = \frac{1}{8}\frac{[\pi, \partial^\mu\pi]}{f_\pi^2} + \dots$$

one can build the leading-order heavy baryon Lagrangian :

$$\mathcal{L}_{\mathrm{HN}} = i\bar{N}_v\gamma^\mu D_\mu N_v - g_A\bar{N}_v\gamma^\mu\gamma_5\mathcal{A}_\mu N_v$$

where $D_\mu = \partial_\mu + \mathcal{V}_\mu$

FeynRules model

- Available both for the **2-flavors** and the **3-flavors** case
- **Customizable** in the choice of the mixing coefficients (**choice of** Q_A)
- Allows for the extraction of the **Feynman rules** for the low-energy chiral Lagrangian and for extensive phenomenological analyses
- Interface to `FeynArts` and `FeynCalc` easy to build