The Minimal Massive Majoron

Xavier Ponce Díaz

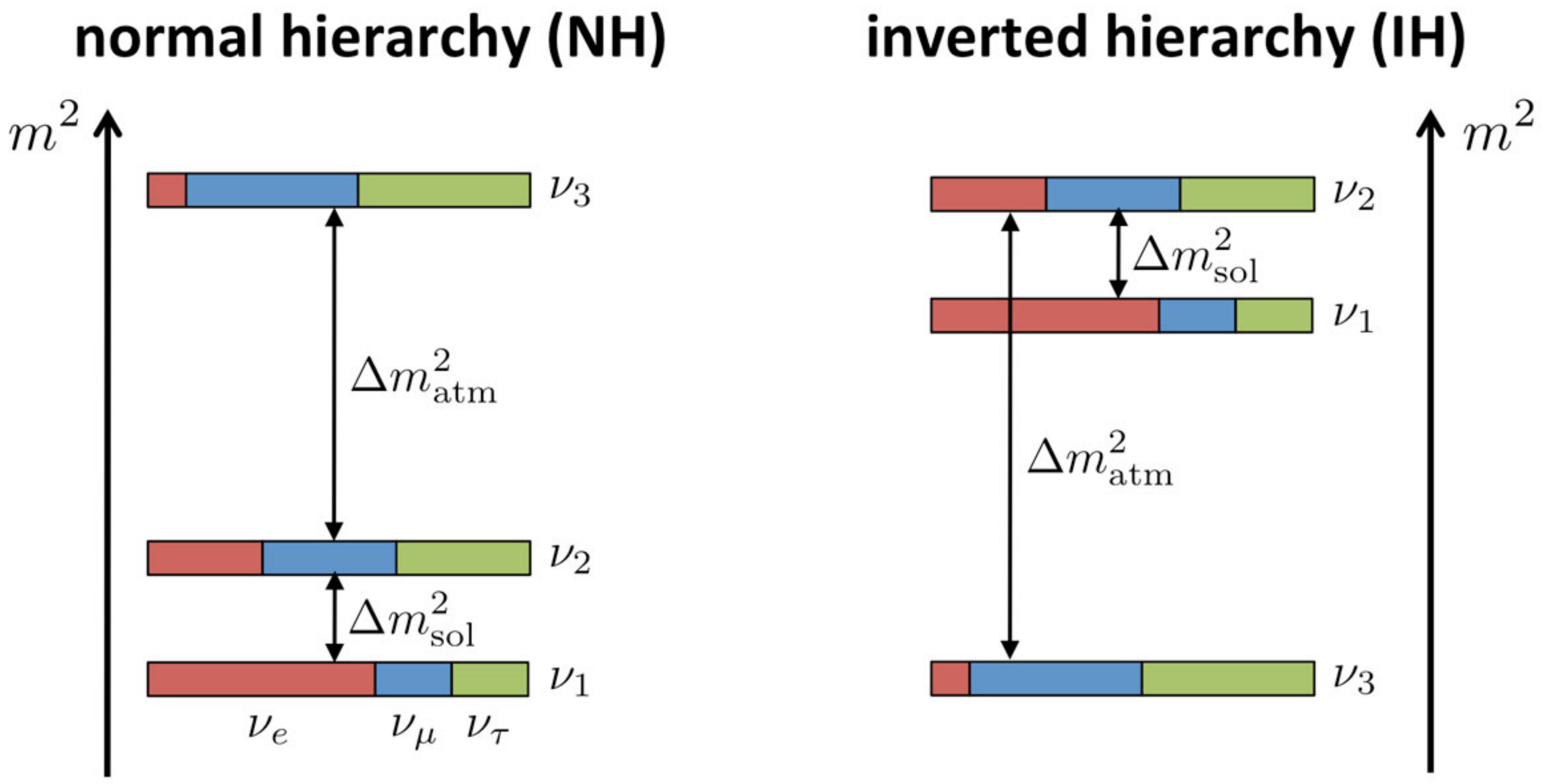
based on: arXiv:2312.13417,
with Arturo de Giorgi, Luca Merlo and Stefano Rigolin

INFN

HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Università degli Studi di Padova

The Majoron



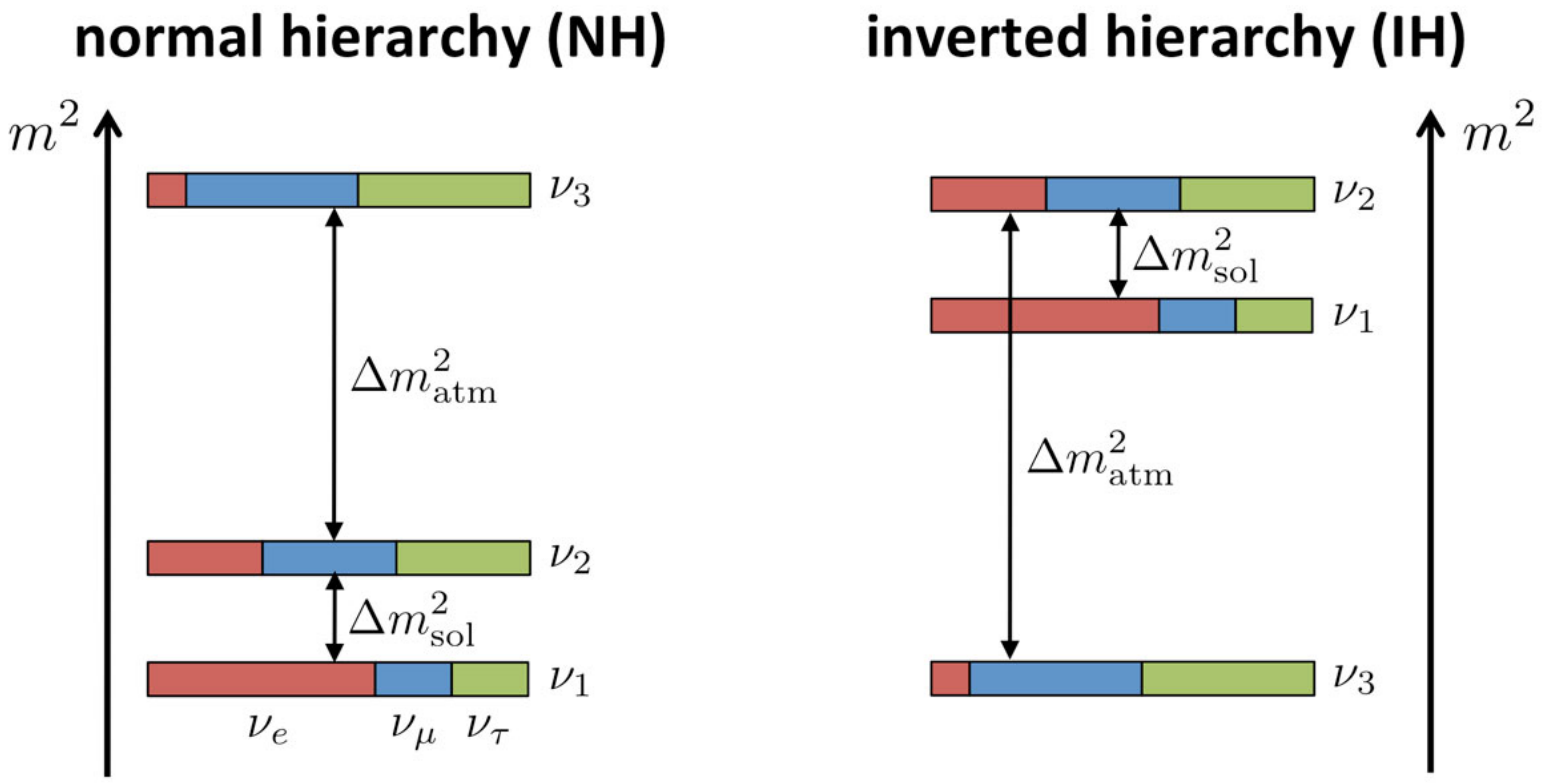
normal hierarchy (NH)
inverted hierarchy (IH)
m^2
ν_3
ν_2
ν_1
$\Delta m^2_{\mathrm{atm}}$
$\Delta m^2_{\mathrm{sol}}$
ν_e
ν_μ
ν_τ

The Majoron

Add RH neutrinos, singlets under SM gauge group:

$$\mathcal{L}_{\text{type-I}} = \overline{\ell}\,\hat{Y}_N N_R H + \frac{1}{2}\overline{N_R^c}\hat{\Lambda}N_R + \text{h.c.}$$

$$\hat{m}_\nu \simeq \frac{v^2}{2}\hat{Y}_N\hat{\Lambda}^{-1}\hat{Y}_N^T$$



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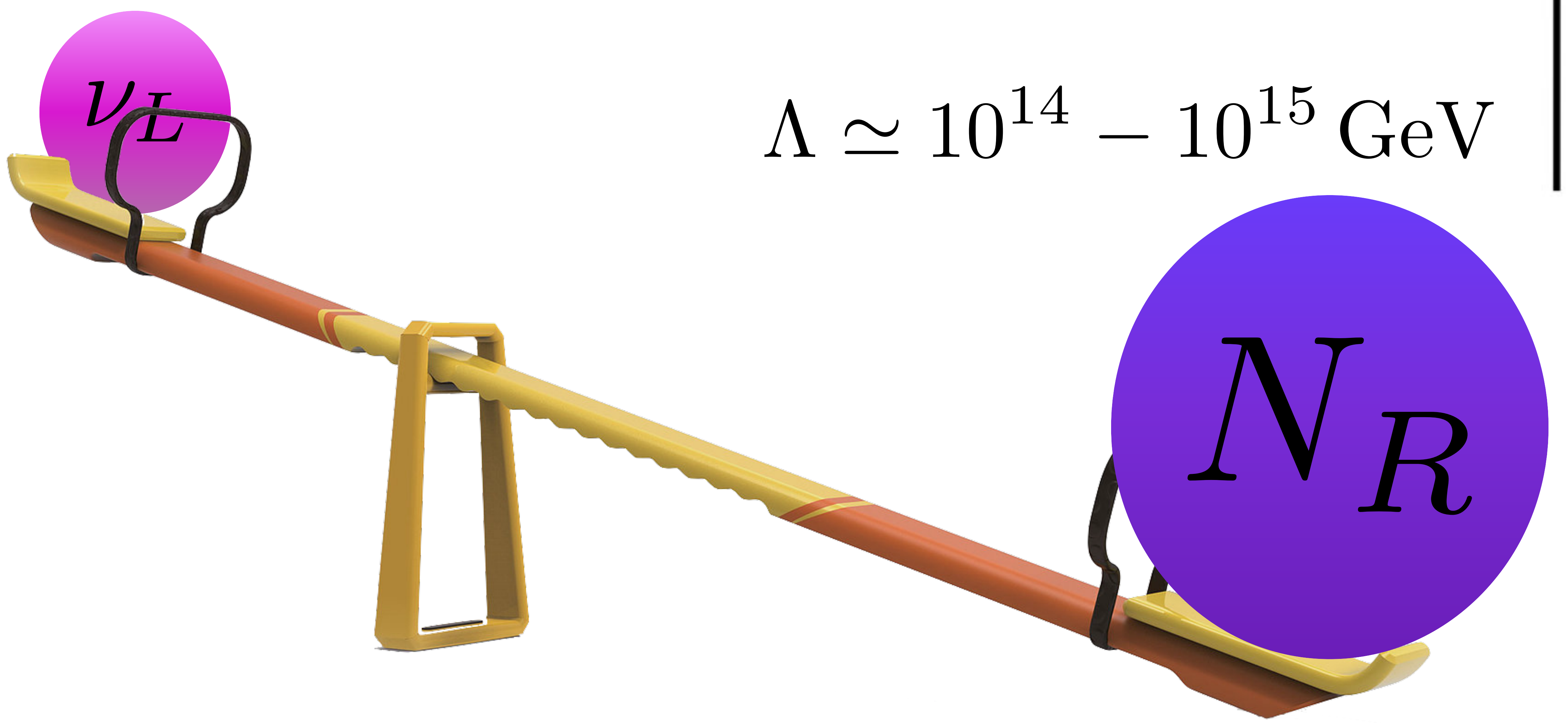
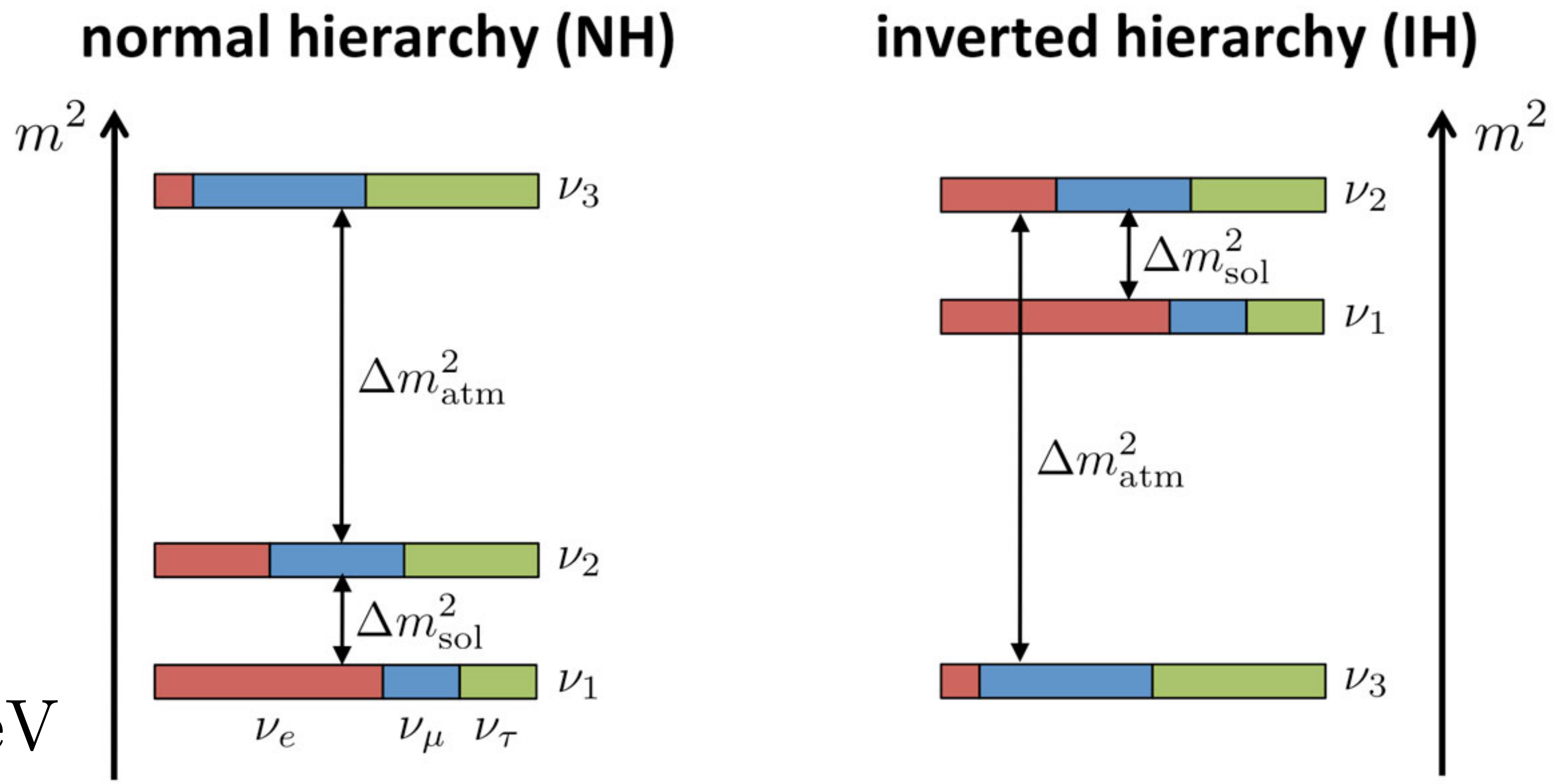
$m_\nu \lesssim 1\,\text{eV}$

$\Lambda \simeq 10^{14} - 10^{15}\,\text{GeV}$





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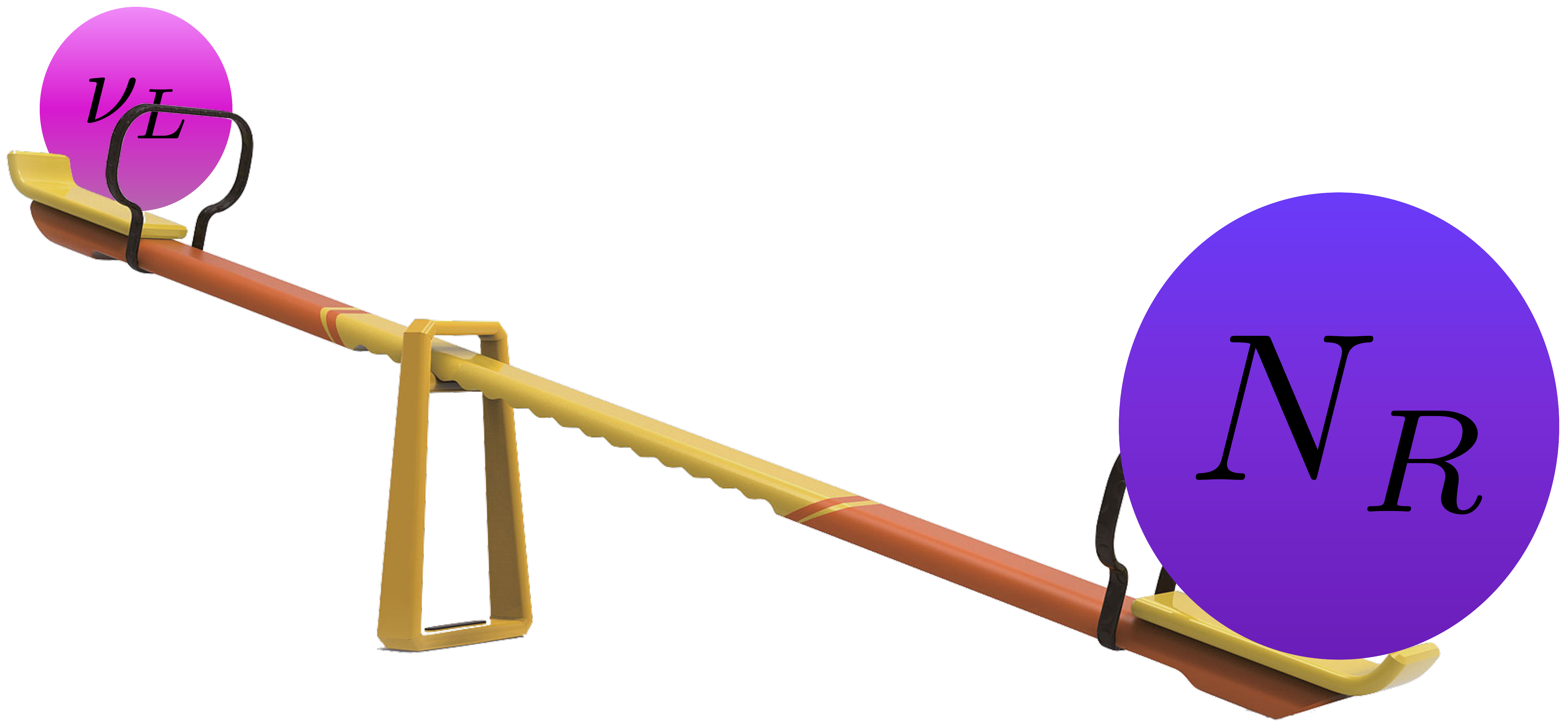

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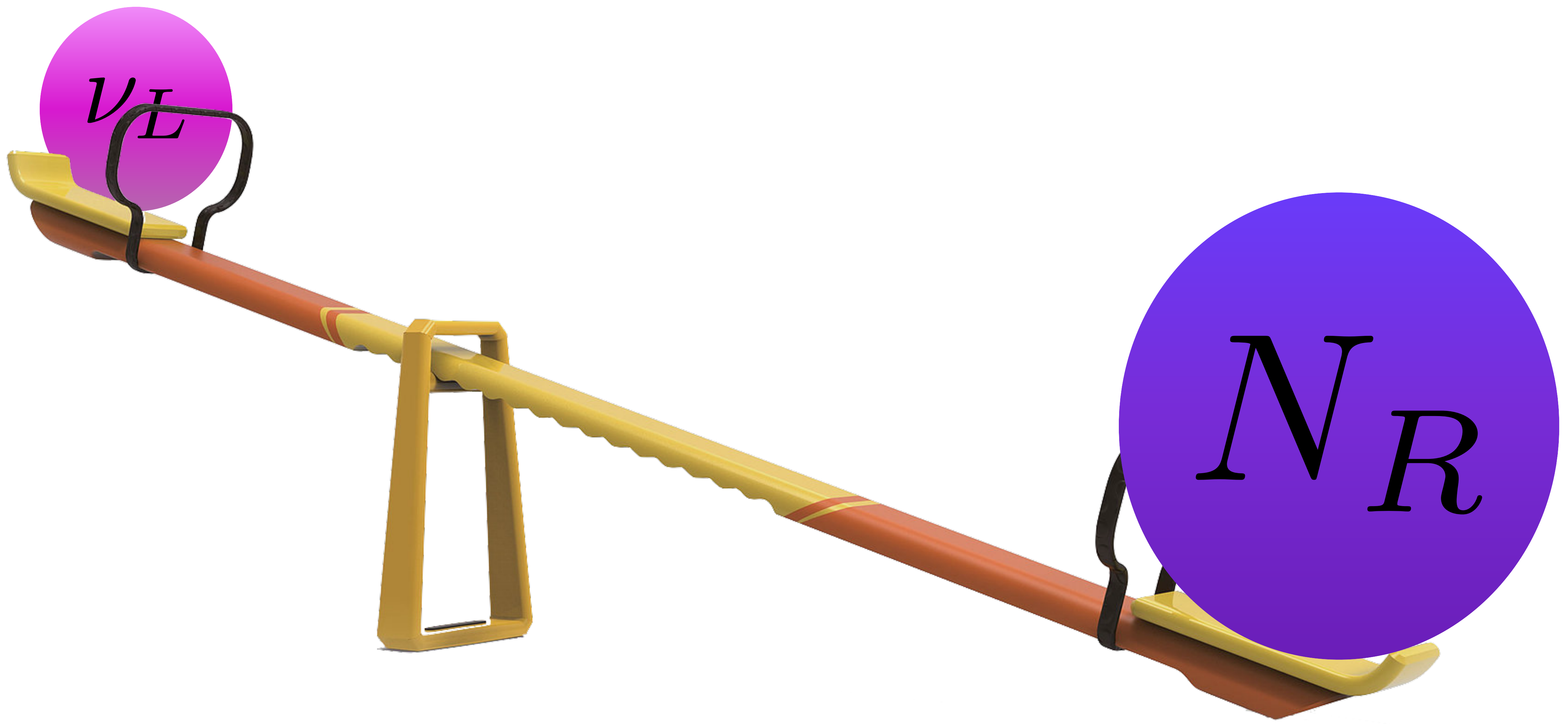
$$V(\phi) = -\mu_\phi^2|\phi|^2 + \frac{1}{2}\lambda_\phi|\phi|^4$$



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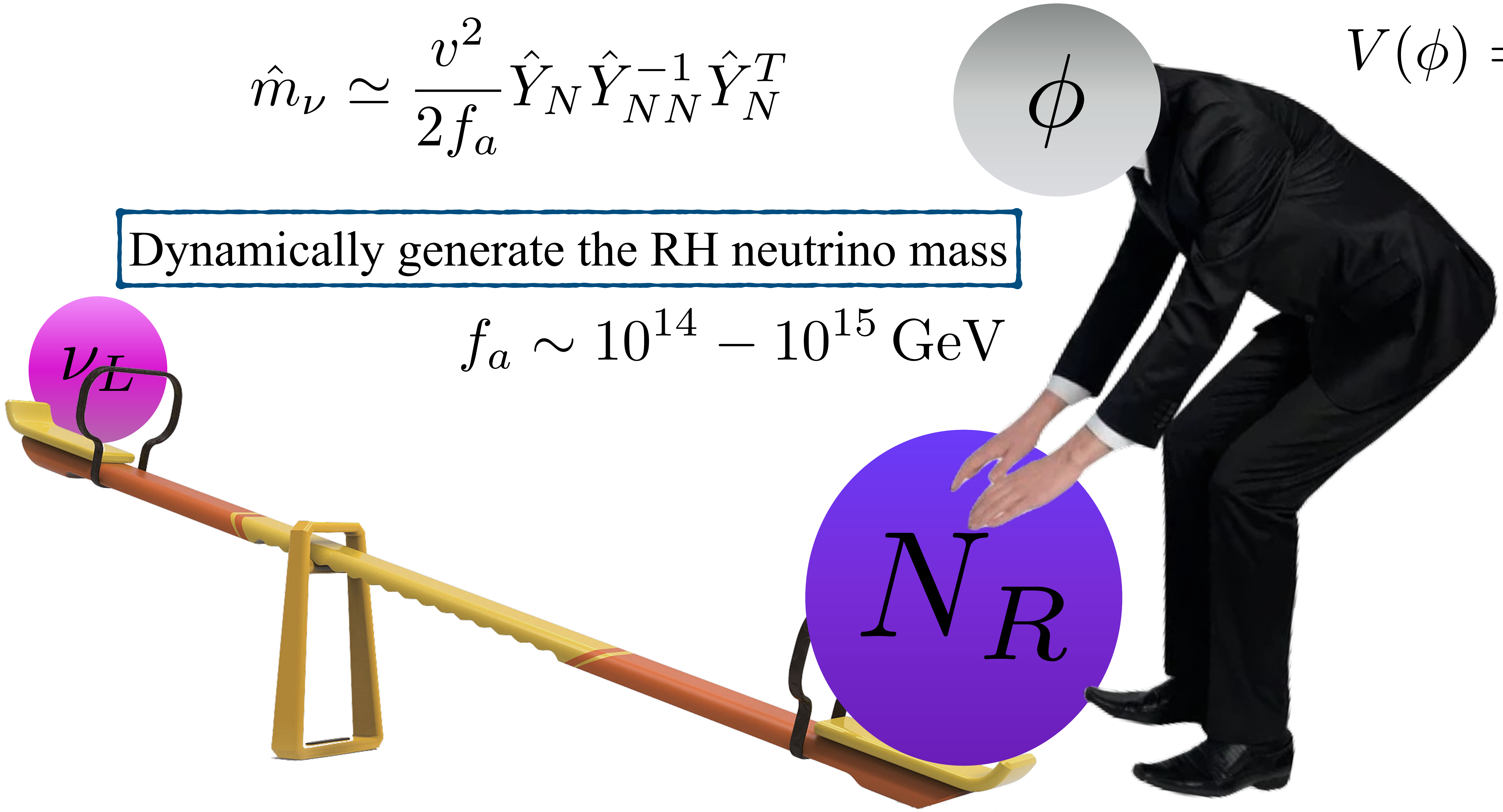
Dynamically generate the RH neutrino mass

$f_a \sim 10^{14} - 10^{15}$ GeV



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The Majoron is the angular mode associated with the breaking of a $U(1)_{B-L}$

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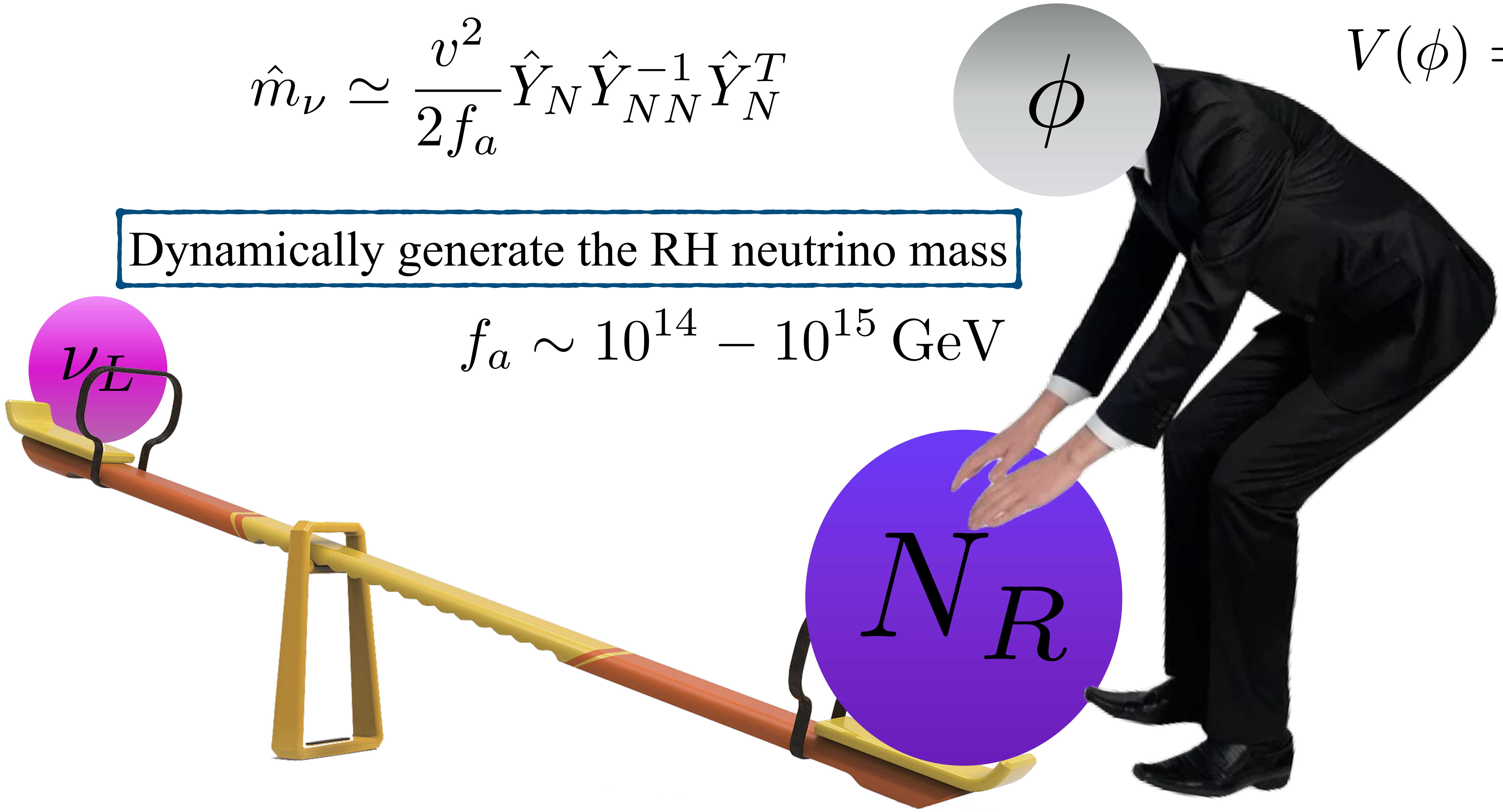

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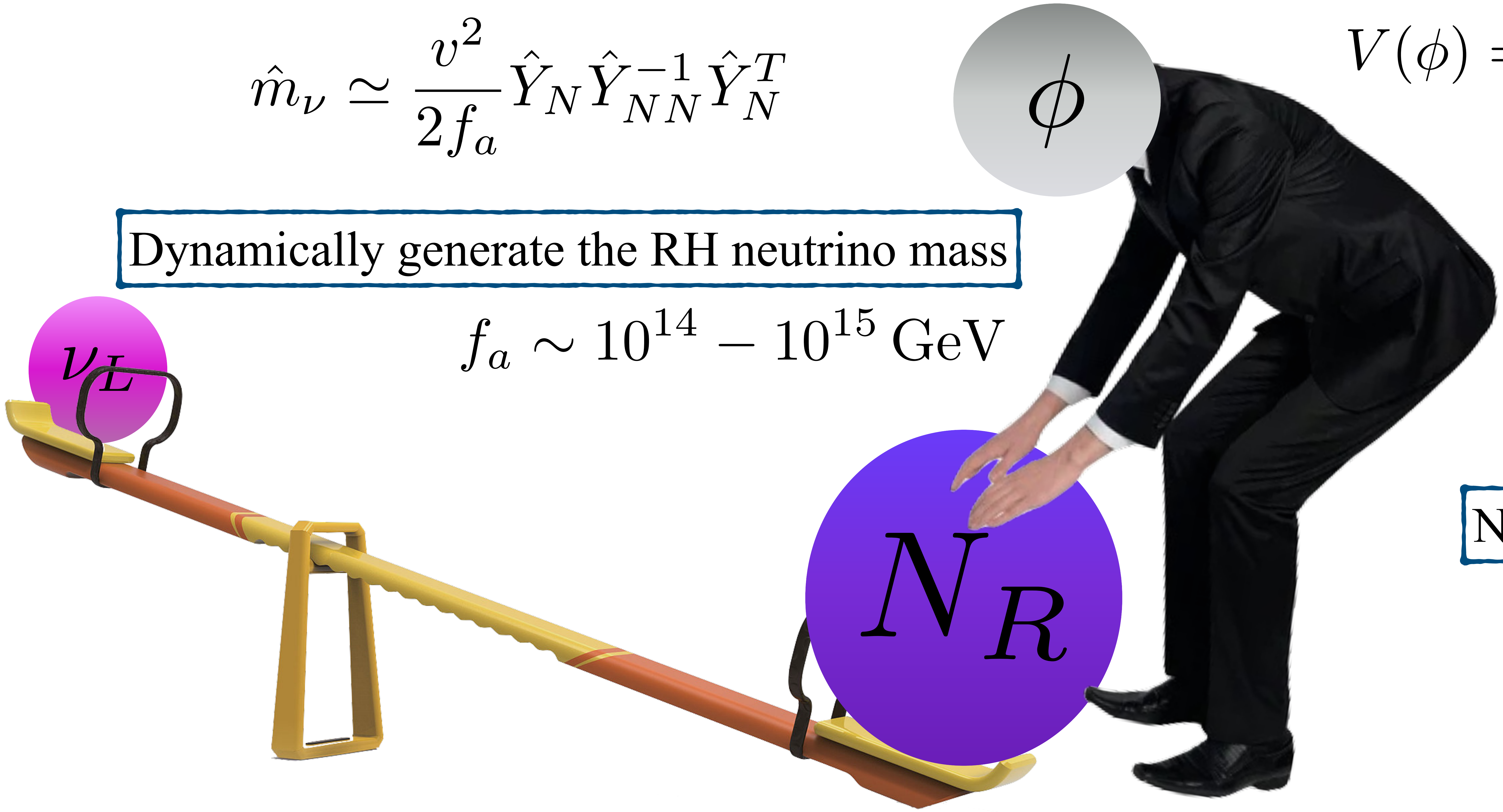

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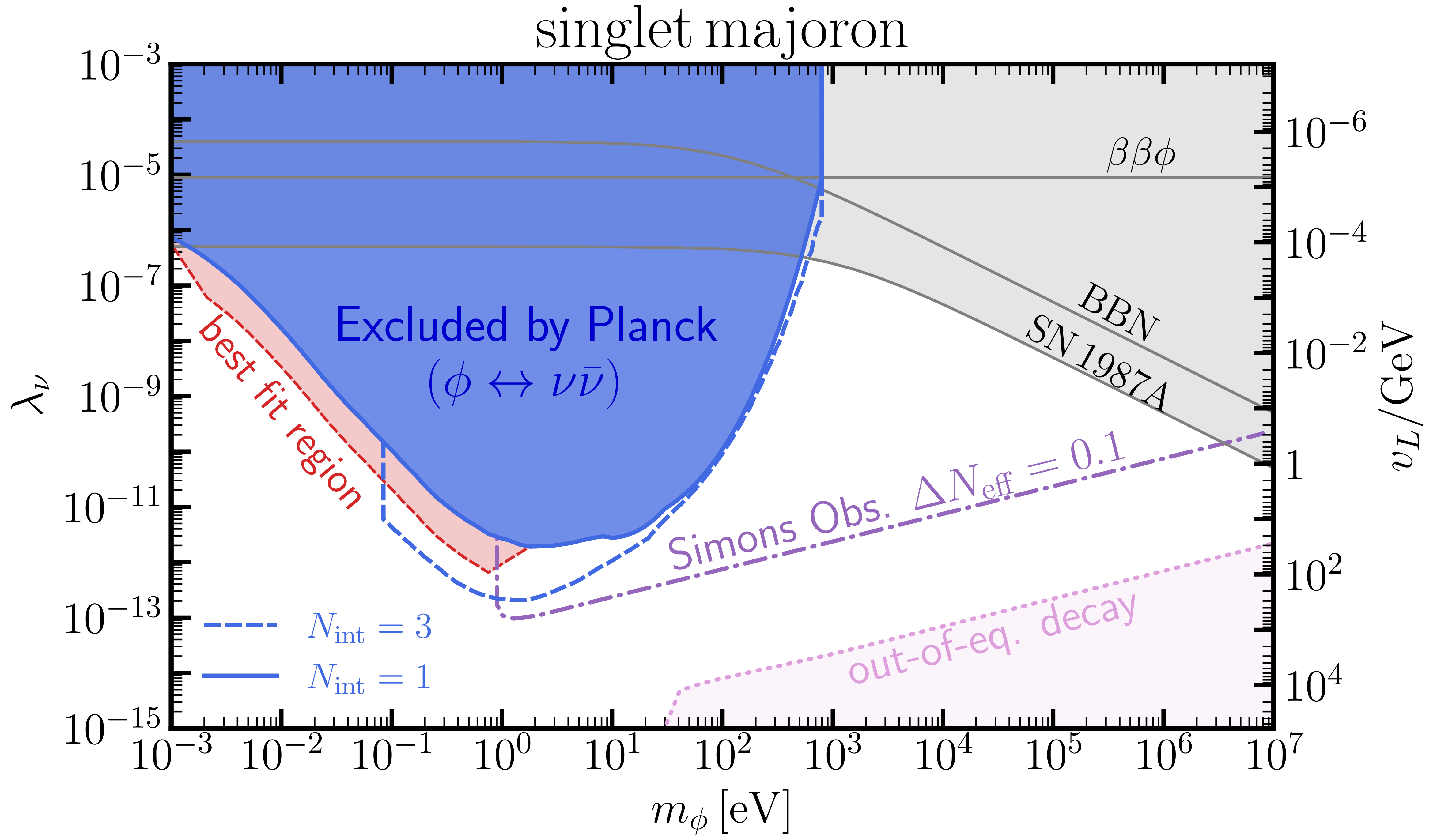
Natural light Dark Matter candidate

Motivation

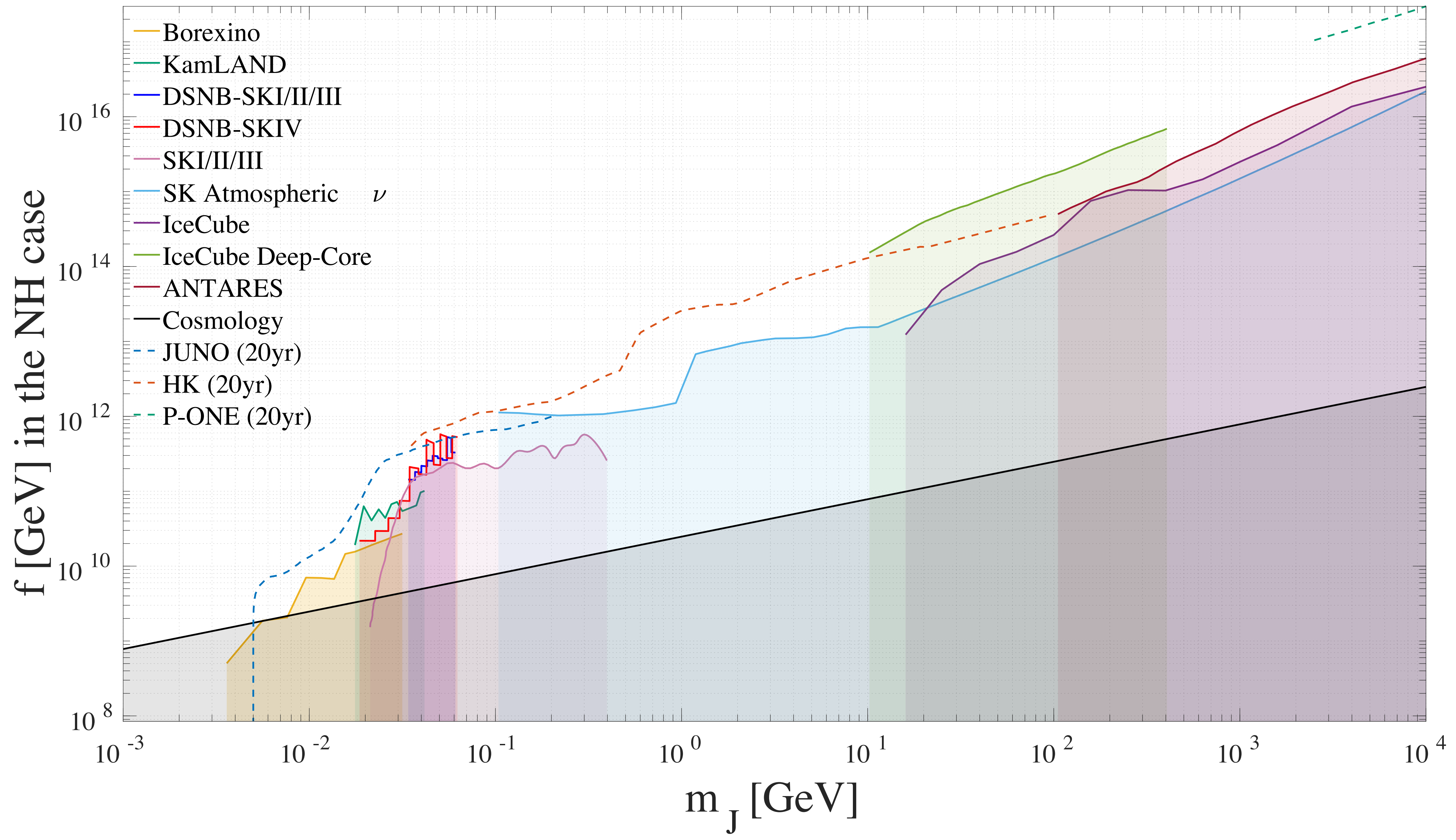
$$m_a^2 = ?^2$$

Cosmological probes



Sandner, Escudero, Witte, [2305.01692]

Terrestrial Experiments



Akita, Niibo, [2304.04430]

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$$V_{\text{grav.}} = g_{2m+n} \frac{|\phi|^{2m} \phi^n}{M_{\text{Pl}}^{2m+n-4}} + \text{h.c.}$$

Alonso, Urbano, [1706.07415]

Akhmedov, Berezhiani, Mohapatra, Senjanović [hep-ph/9209285]

Babu, Rothstein, Seckel [hep-ph/9301213]

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  - Explicit breaking terms in the fermion sector:

$$m_a^2 \sim \frac{1}{8\pi^2} \frac{\Lambda_{11}\Lambda_{22}\Lambda_{12}\Lambda_{21}}{f_a^2} \log \frac{f_a^2}{\mu^2}$$

$$m_a^2 \sim \frac{v^2}{8\pi^2} \frac{\Lambda_{ij}\Lambda_{kl}(\hat{Y}_N)_{\alpha 1}(\hat{Y}_N)_{\alpha 2}}{f_a^2} \log \frac{f_a^2}{\mu^2}$$

$$m_a^2 \sim \frac{v^4}{8\pi^2} \frac{(\hat{Y}_N)_{\alpha 1}(\hat{Y}_N)_{\alpha 2}(\hat{Y}_N)_{\beta 1}(\hat{Y}_N)_{\beta 2}}{f_a^2} \log \frac{f_a^2}{\mu^2}$$

Frigerio, Hambye, Massó [1107.4564]

Neutrino Mass Models 101

$$-\mathcal{L}_{\nu\mathrm{M}} \supset \frac{1}{2}\,\overline{\chi}_L\,\mathcal{M}_\chi\,\chi_L^c \qquad \chi_L = \begin{pmatrix} \nu_L \\ N_R^c \\ S_R^c \end{pmatrix}$$

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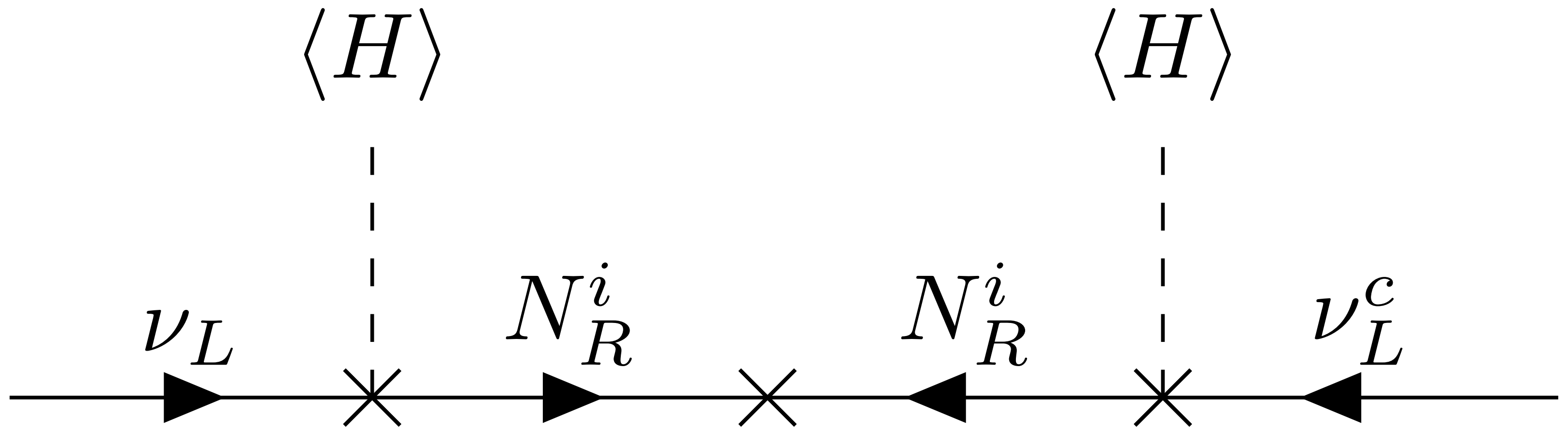

Type-I Seesaw

$$\mathcal{M}_\chi^{\text{Type-I}} = \begin{pmatrix} 0 & m_N & m_S \\ m_N^T & \Lambda_{NN} & \Lambda_{NS} \\ m_S^T & \Lambda_{NS} & \Lambda_{SS} \end{pmatrix}$$

$$m_\nu^{\text{type-I}} \simeq -m_i \hat{\Lambda}_{ij}^{-1} m_j^T$$

$$\Lambda \sim 10^{14}\ \mathrm{GeV}$$

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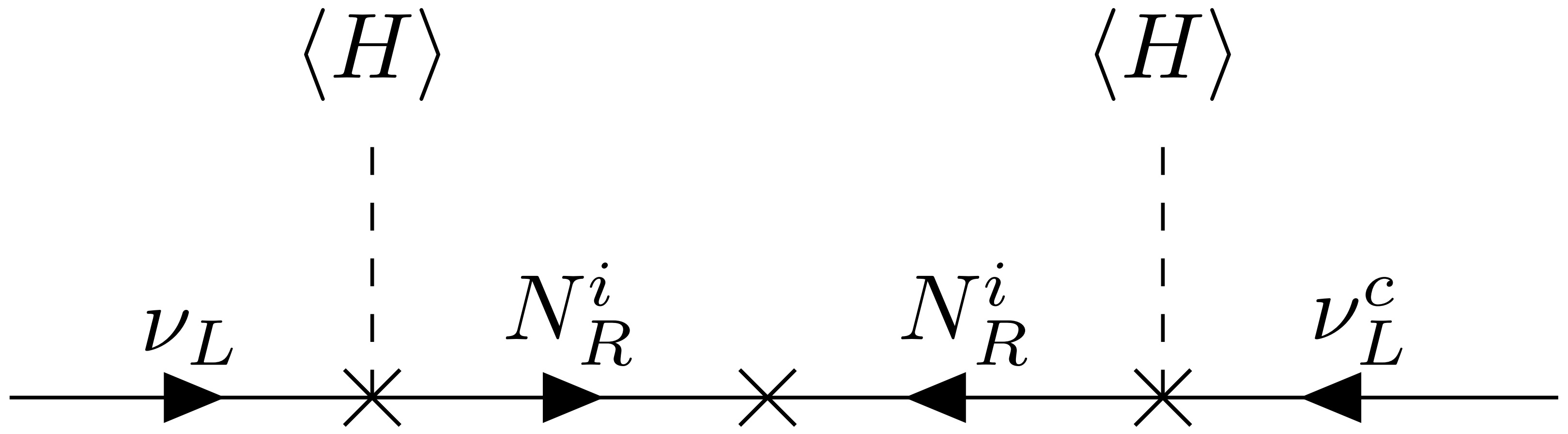

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$m_S,\, \Lambda_{NN},\, \Lambda_{SS}$
break L

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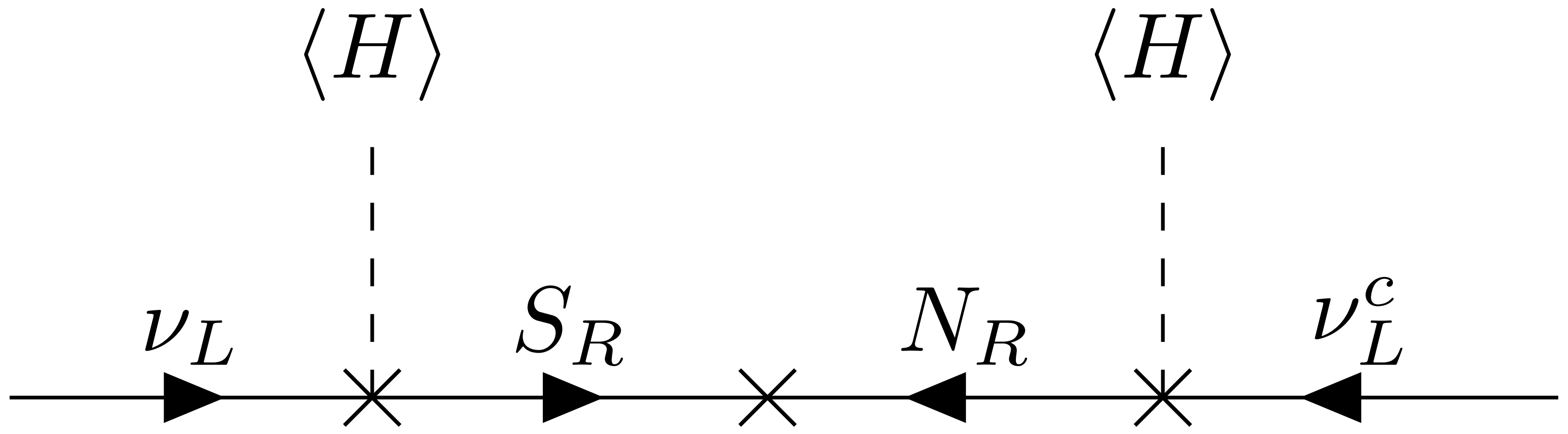

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Linear Seesaw

$$\mathcal{M}_\chi^{\text{LSS}} = \begin{pmatrix} 0 & m_N & \epsilon m_S \\ m_N^T & 0 & \Lambda_{NS} \\ \epsilon m_S^T & \Lambda_{NS} & 0 \end{pmatrix}$$

$$L(\ell_L) = L(N_R) = -L(S_R)$$

$$m_\nu^{\text{LSS}} \simeq -\epsilon\frac{m_S\, m_N^T + m_N\, m_S^T}{\Lambda_{NS}}$$

$$\epsilon m_S \sim 10\,\text{eV}, \quad \Lambda_{NS} \sim 1\,\text{TeV}$$

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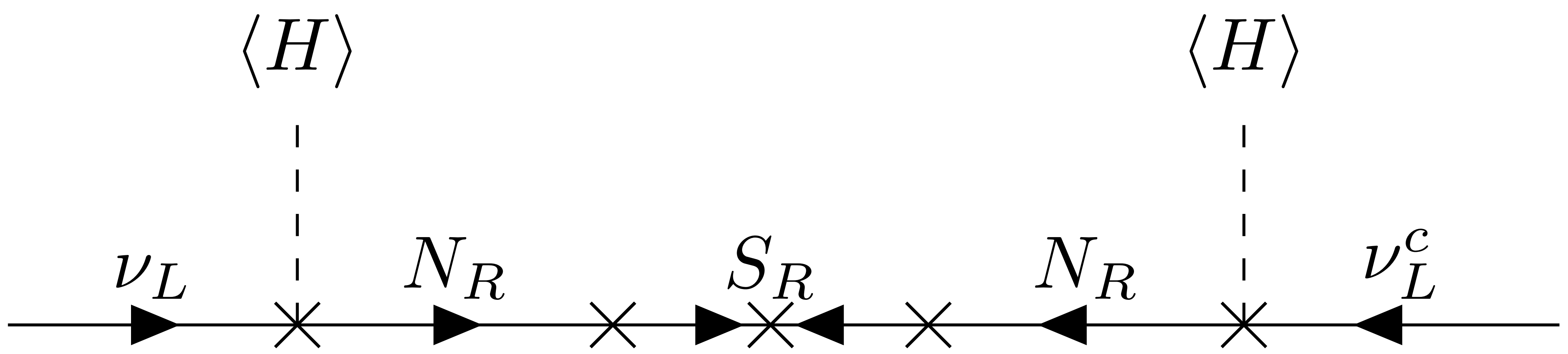

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$$\epsilon m_S \sim 10\,\mathrm{eV}, \quad \Lambda_{NS} \sim 1\,\mathrm{TeV}$$

Inverse Seesaw

$$\mathcal{M}_\chi^{\text{ISS}} = \begin{pmatrix} 0 & m_N & 0 \\ m_N^T & 0 & \Lambda_{NS} \\ 0 & \Lambda_{NS} & \mu \end{pmatrix}$$

$$L(\ell_L) = L(N_R) = -L(S_R)$$

$$m_\nu^{\text{ISS}} \simeq -\mu \frac{m_N m_N^T}{\Lambda_{NS}^2}$$

$$\mu \sim 1\,\mathrm{keV}, \quad \Lambda_{NS} \sim 1\,\mathrm{TeV}$$

Neutrino Mass Models 101

$$-\mathcal{L}_{\nu\mathrm{M}} \supset \frac{1}{2}\,\overline{\chi}_L\,\mathcal{M}_\chi\,\chi_L^c \qquad \chi_L = \begin{pmatrix} \nu_L \\ N_R^c \\ S_R^c \end{pmatrix} \qquad \mathcal{M}_\chi = \begin{pmatrix} 0 & \hat{m}(H) \\ \hat{m}^T(H) & \hat{\Lambda}(\phi) \end{pmatrix}$$

Remember
Need an explicit breaking to generate a Majoron mass

Type-I Seesaw + ϕ

$$\mathcal{M}_\chi^{\text{Type-I}} = \begin{pmatrix} 0 & m_N & m_S \\ m_N^T & Y_{NN}\phi & Y_{NS}\phi \\ m_S^T & Y_{NS}\phi & Y_{SS}\phi \end{pmatrix}$$

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$$PQ(\ell_L) = PQ(N_R) = PQ(S_R) = -\frac{1}{2}PQ(\phi)$$

ϵm_S

$\mu \equiv \epsilon Y_{SS}\langle\phi\rangle$ No longer breaking terms!

Majoron Neutrino Mass Models

The list of models: We identify all the possible non-redundant PQ symmetries

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Up to trivial exchanges: $\phi \longleftrightarrow \phi^* \quad N_R \longleftrightarrow S_R$

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Two different scales: $f_a,\ \Lambda$

Extra requirements to fix the two scales

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Two different scales: f_a, Λ

Extra requirements to fix the two scales

$$m_\nu \simeq -\mu \frac{m_N m_N^T}{\Lambda_{NS}^2} \sim Y_{SS} \frac{m_N m_N^T}{Y_{NS}^2 f_a}$$

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$$m_\nu \simeq -\mu \frac{m_N m_N^T}{\Lambda_{NS}^2} \sim Y_{SS} \frac{m_N m_N^T}{Y_{NS}^2 f_a}$$

$m_N m_N^T$ is a rank-1 matrix, it can only produce 1 massive light neutrino

Up to trivial exchanges: $\phi \longleftrightarrow \phi^*$ $N_R \longleftrightarrow S_R$

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$$-\mathcal{L}_{\rm PQ}^{\rm mmM} = \overline{\ell_L}\,\tilde{H}\,Y_N\,N_R + \frac{Y_{NS}}{2}\,\phi^*\,\left(\overline{N_R^c}\,S_R + \overline{S_R^c}\,N_R\right) + \frac{Y_{NN}}{2}\,\phi\,\overline{N_R^c}\,N_R + \text{h.c.}$$

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$$-\mathcal{L}_{\epsilon\rm PQ}^{\rm mmM} = \epsilon\overline{\ell_L}\,\tilde{H}\,Y_S\,S_R + \text{h.c.}\,,$$

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$$-\mathcal{L}_{\epsilon\rm PQ}^{\rm mmM} = \epsilon\overline{\ell_L}\,\tilde{H}\,Y_S\,S_R + \text{h.c.}\,,$$

$$\mathcal{M}_\chi = \begin{pmatrix} 0 & m_N & \epsilon m_S \\ m_N^T & Y_{NN}\phi & Y_{NS}\phi^* \\ \epsilon m_S^T & Y_{NS}\phi^* & 0 \end{pmatrix}$$

Minimality:

i) Minimal number of fields to account for the correct neutrino spectrum
$N_R,\, S_R\, +\, \phi$

→ Light neutrino masses can be correctly generated

$$m_\nu^{\rm TL} = -\epsilon\frac{m_S\,m_N^T + m_N\,m_S^T}{\Lambda_{NS}} + \epsilon^2\,\frac{\Lambda_{NN}}{\Lambda_{NS}}\,\frac{m_S\,m_S^T}{\Lambda_{NS}}\,,$$

The Minimal Massive Majoron

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ii) One explicit breaking links neutrino and Majoron mass. Other breaking-effects are assumed to be subleading

$$\epsilon \ll Y_i \sim \left[10^{-2}\,, 1\right]$$

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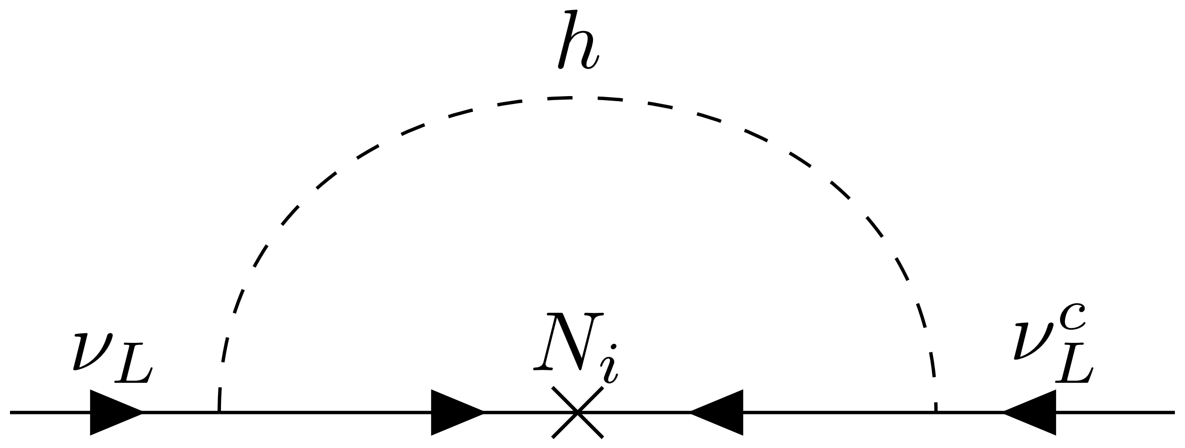
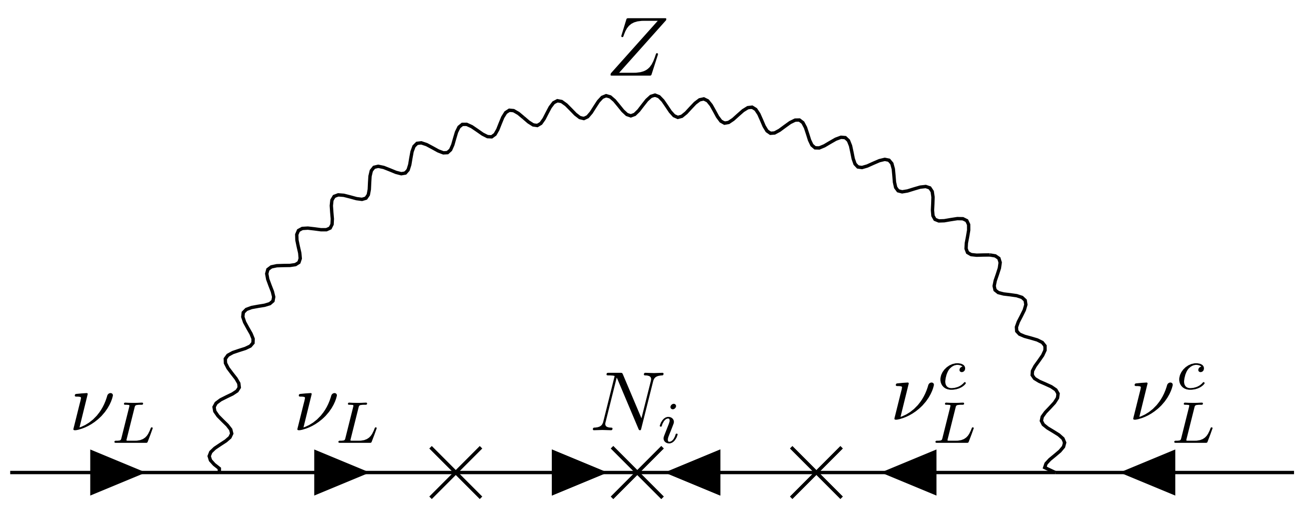
iii) Only one scale $\Lambda \sim f_a = \langle\phi\rangle$

$$\Lambda_{NN} \equiv \frac{f_a}{\sqrt{2}}Y_{NN}\,,\quad \Lambda_{NS} \equiv \frac{f_a}{\sqrt{2}}Y_{NS}$$

Radiative Neutrino Masses

López-Pavón, Pascoli, Wong [1209.5342]





Two massive RH neutrinos

$$M_{N,S} = \frac{\Lambda_{NS}}{2}\left[\sqrt{4 + \left(\frac{\Lambda_{NN}}{\Lambda_{NS}}\right)^2} \mp \left(\frac{\Lambda_{NN}}{\Lambda_{NS}}\right)\right]$$

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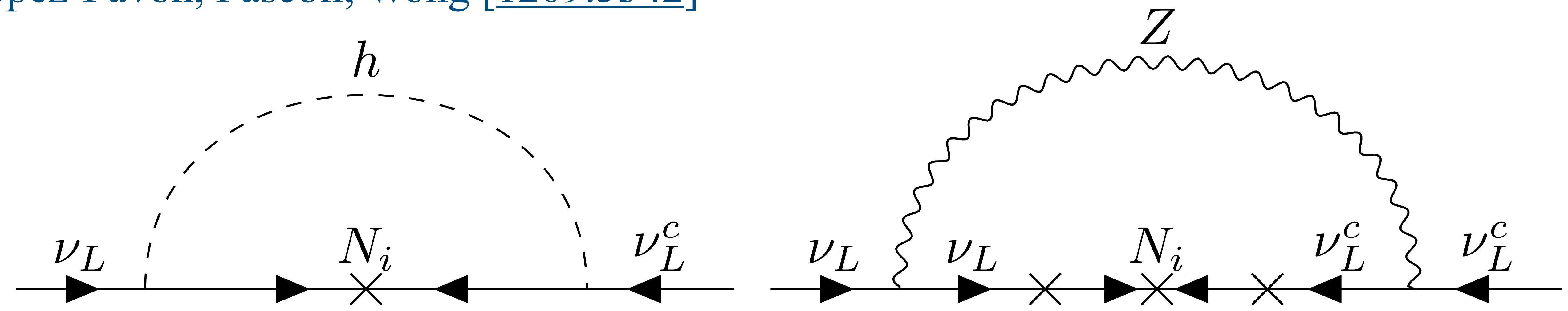

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1-loop contribution

Need to tame the loop contribution!

Radiative Neutrino Masses

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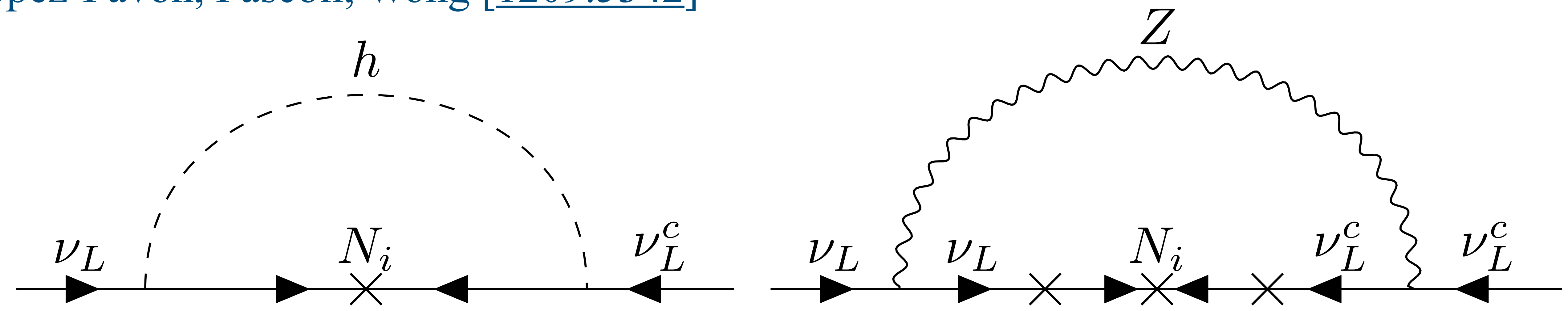

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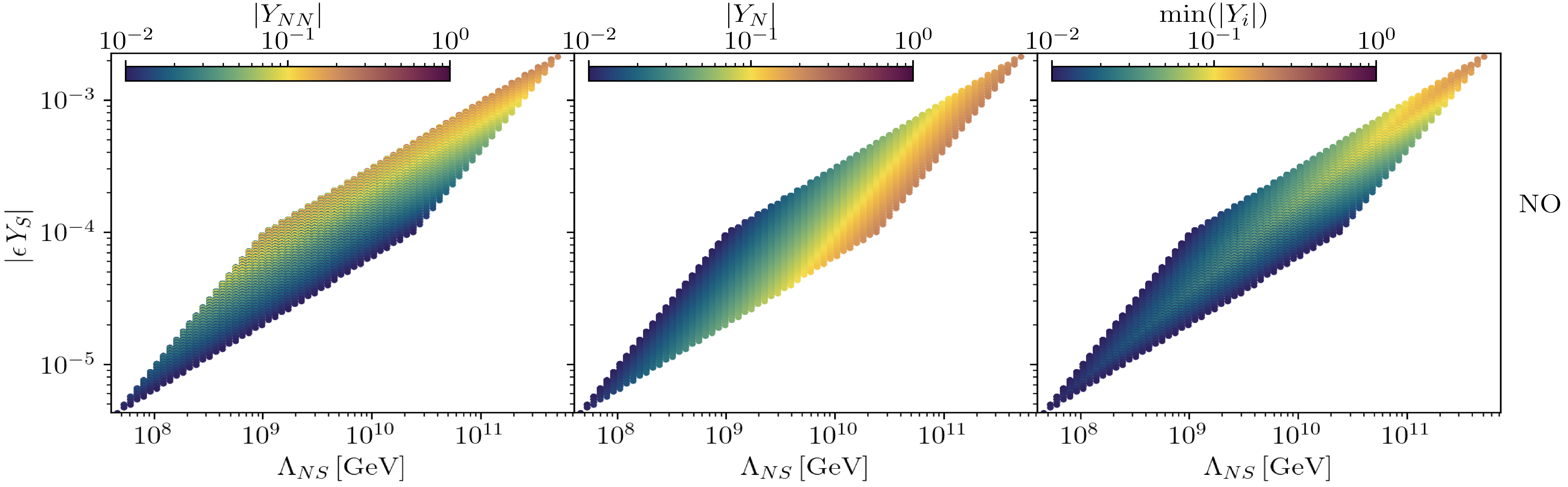

The Minimal Massive Majoron

Using the Coleman-Weinberg potential:

$$V_{\mathrm{CW}} = -\frac{1}{2} \times \frac{1}{16\pi^2} \mathrm{Tr} \left[\left(\mathcal{M}_\chi \mathcal{M}_\chi^\dagger \right)^2 \left(\log \left(\frac{\mathcal{M}_\chi \mathcal{M}_\chi^\dagger}{\mu_R^2} \right) - \frac{3}{2} \right) \right] \quad \text{with } \mathcal{M}_\chi = \mathcal{M}_\chi(\phi,\, H)$$

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$$V_{\rm CW} = -\,\epsilon \frac{Y_{NS}\, Y_{NN}\, |Y_N \cdot Y_S|\, v^2\, f_a^2}{16\pi^2} \cos\left(\vartheta_\eta + \frac{2a}{f_a}\right) \times \qquad \vartheta_\eta \equiv \arg(Y_N^\dagger Y_S)$$

$$\times \left[\frac{(Y_{NN}^2 + 2Y_{NS}^2)}{Y_{NN}\sqrt{Y_{NN}^2 + 4Y_{NS}^2}} \mathrm{Arcoth}\left(\frac{(Y_{NN}^2 + 2Y_{NS}^2)}{Y_{NN}\sqrt{Y_{NN}^2 + 4Y_{NS}^2}}\right) + \left(\log\left(\frac{Y_{NS}^2 f_a^2}{2\mu_R^2}\right) - 1\right)\right]$$

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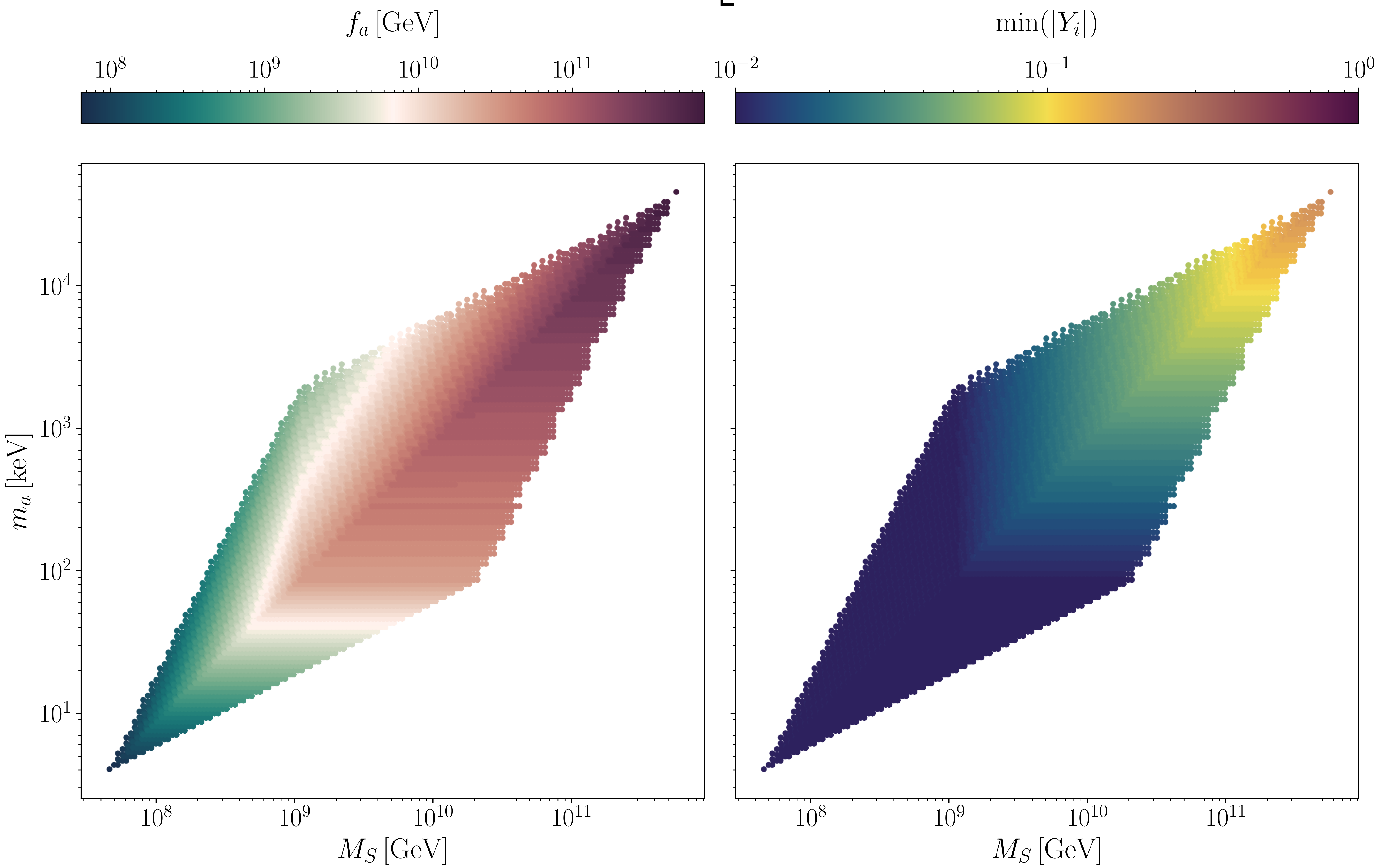
$$m_a^2 = \frac{|\epsilon m_S \cdot m_N|}{\pi^2} \frac{\sqrt{M_N M_S}}{M_N + M_S} \left[\frac{(M_S^2 + M_N^2)}{f_a^2} \log\left(\frac{M_S}{M_N} \right) + \frac{(M_S^2 - M_N^2)}{f_a^2} \left(\log\left(\frac{M_N M_S}{\mu_R^2} \right) - 1 \right) \right]$$

$$m_a^2 \sim \epsilon \frac{v^2}{\pi^2} \ll f_a^2$$

The Minimal Massive Majoron

$$m_a^2 \simeq \frac{\sqrt{|\eta||\Delta m_{32}^2|}}{2\,\pi^2}\,\frac{M_N M_S}{M_N + M_S}\left[\frac{(M_S^2 + M_N^2)}{f_a^2}\log\left(\frac{M_S}{M_N}\right) + \frac{(M_S^2 - M_N^2)}{f_a^2}\left(\log\left(\frac{M_N M_S}{\mu_R^2}\right) - 1\right)\right]$$



$$|\eta| \equiv \frac{1 - \sqrt{r}}{1 + \sqrt{r}}\,, \quad \text{for the NO}$$

$$r \equiv \frac{|\Delta m_{\text{sol.}}^2|}{|\Delta m_{\text{atm.}}^2|}$$

Gavela, Hambie, Hernández, Hernández [0906.2461]

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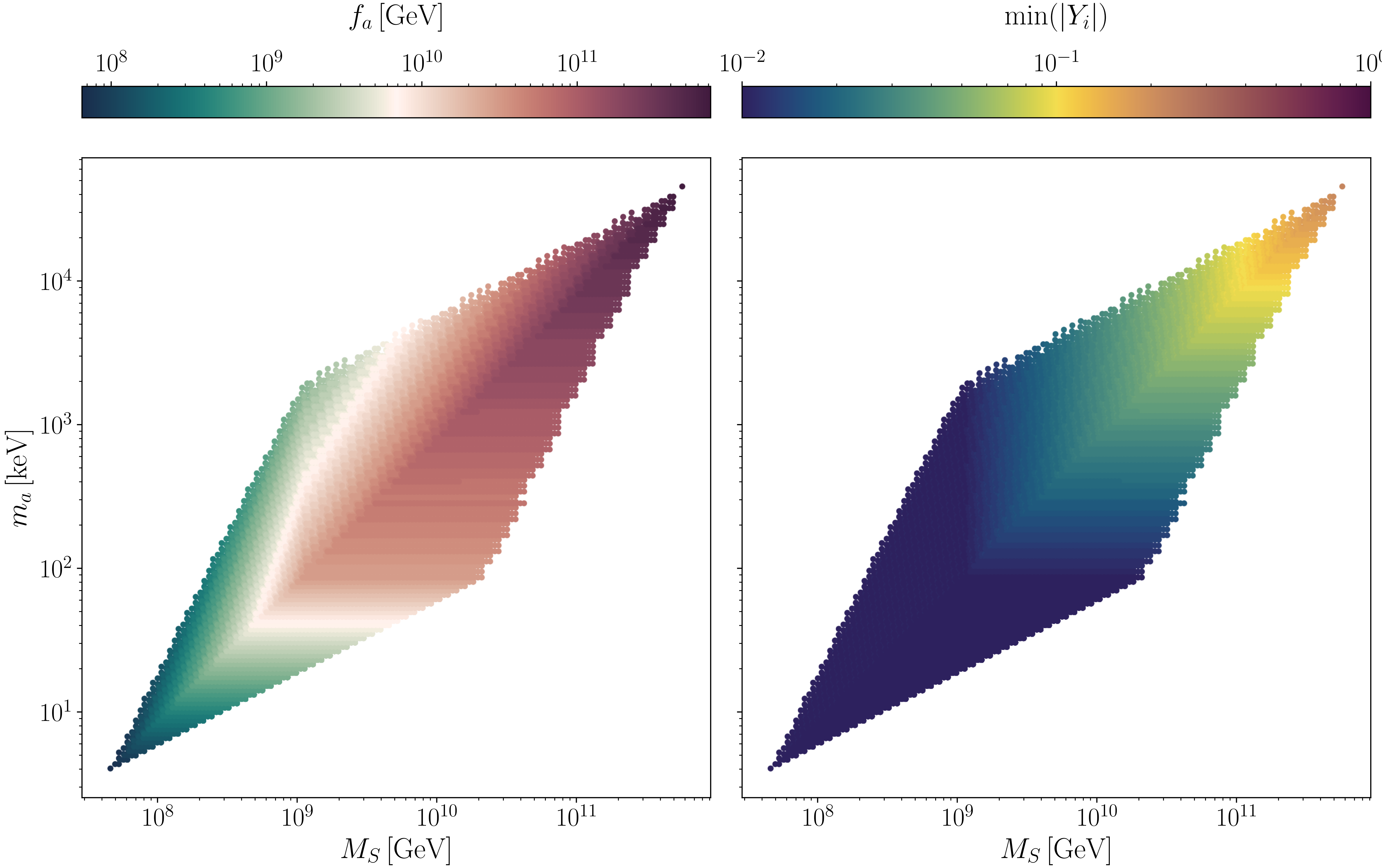

The Minimal Massive Majoron can be found

$$f_a \sim \left[10^8\,,\,10^{11}\right]\,\text{GeV}$$

$$m_a \sim [1\,\text{keV},\,10\,\text{MeV}]$$

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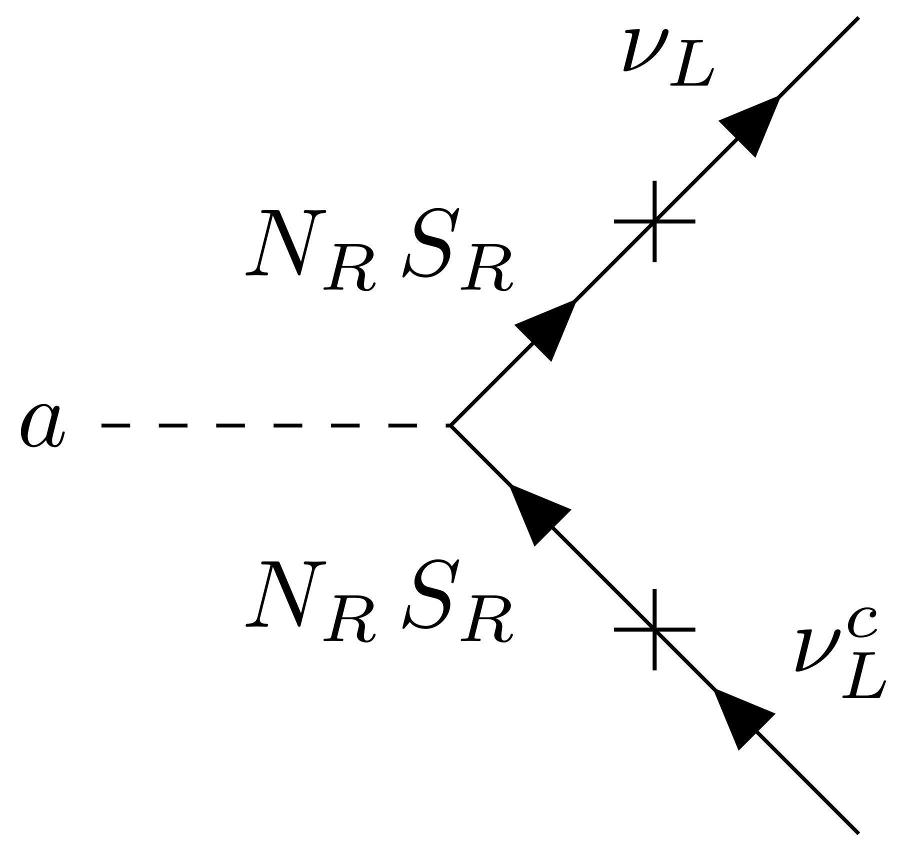
Gavela, Hambie, Hernández, Hernández [0906.2461]

Phenomenology of the Majoron

$$\mathcal{L}_{a\nu\nu} \simeq -\frac{i\,a}{2f_a}\overline{\nu_L}\, m_\nu\, \gamma_5\, \nu_L^c$$



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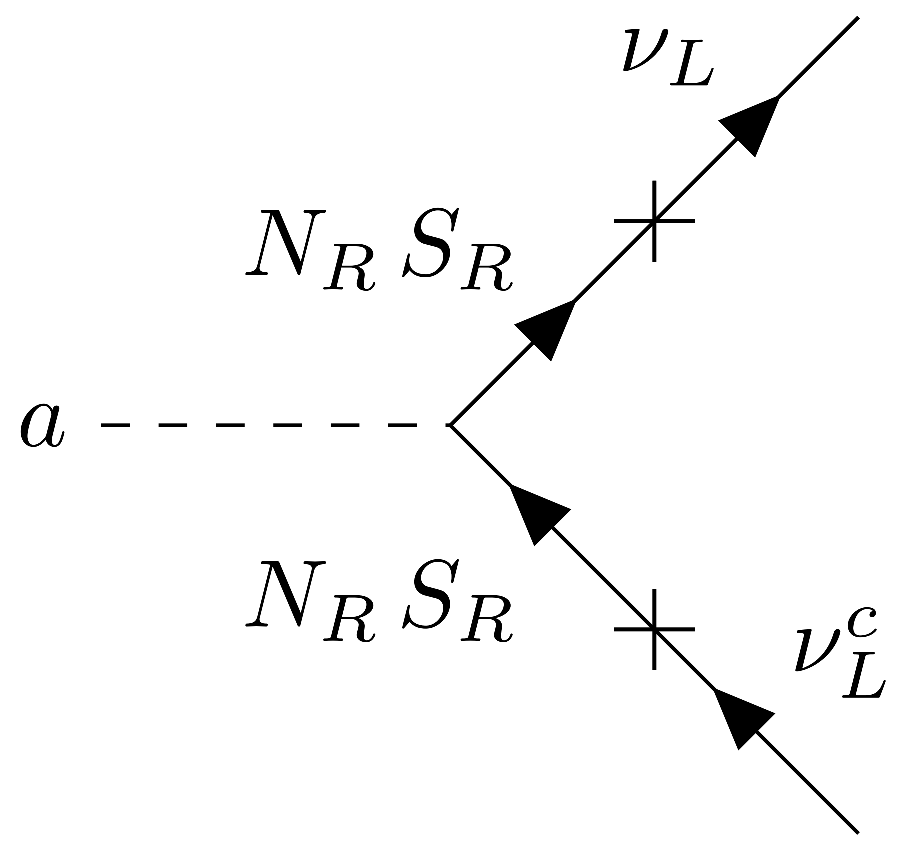

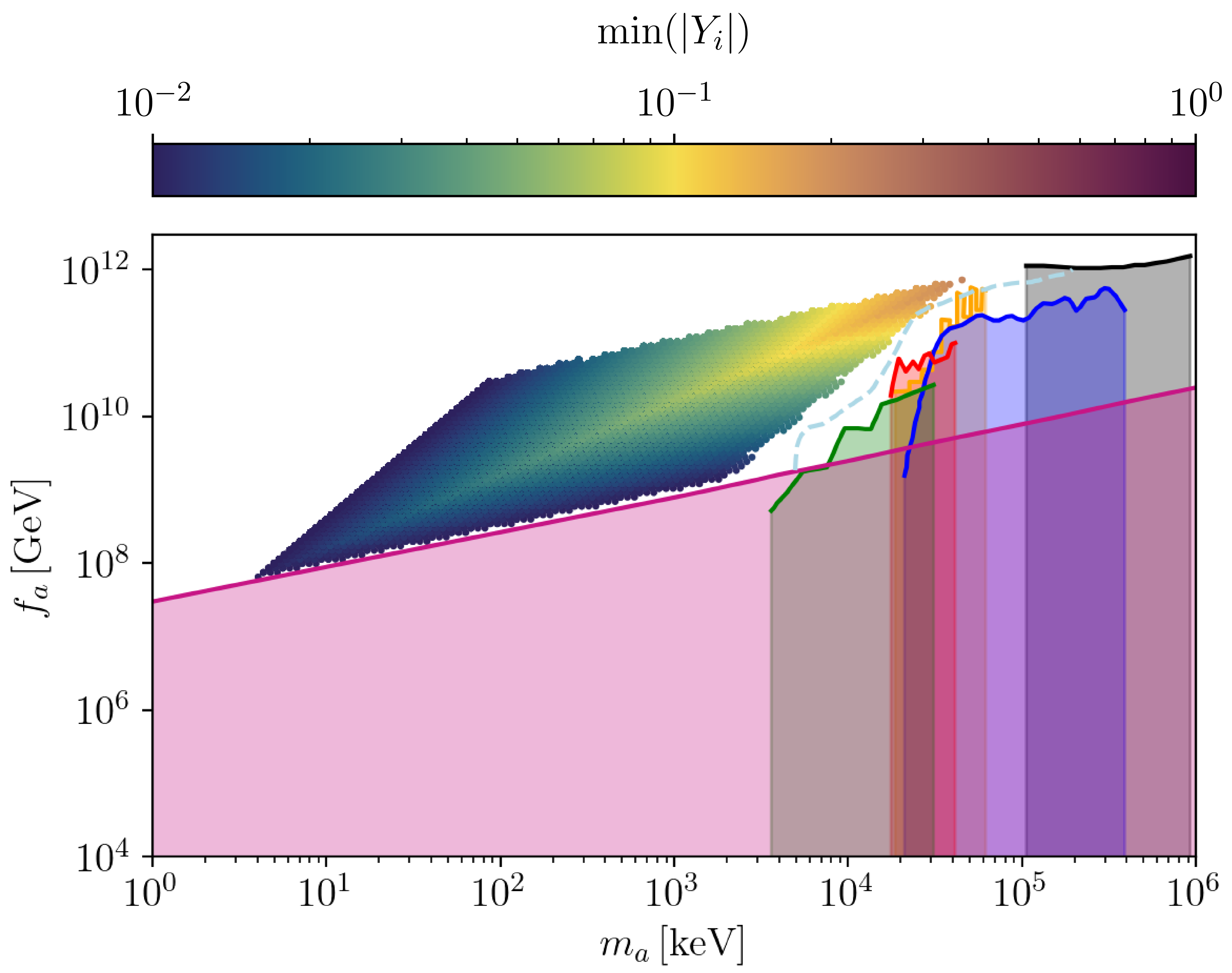

Akita, Niibo, [2304.04430]

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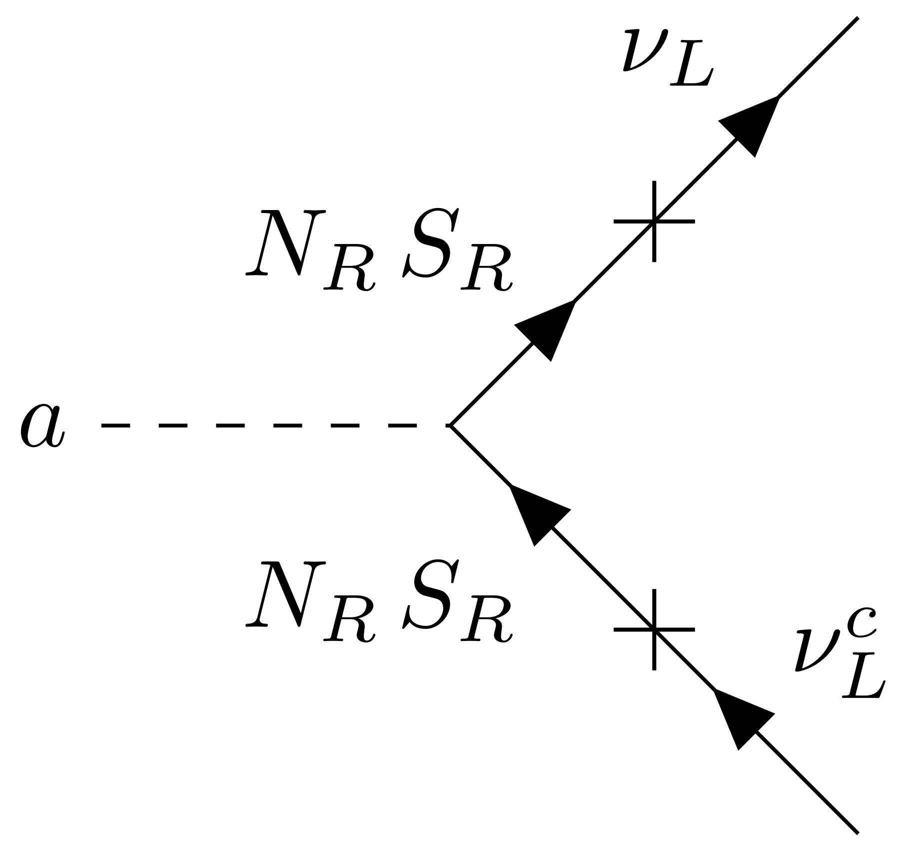

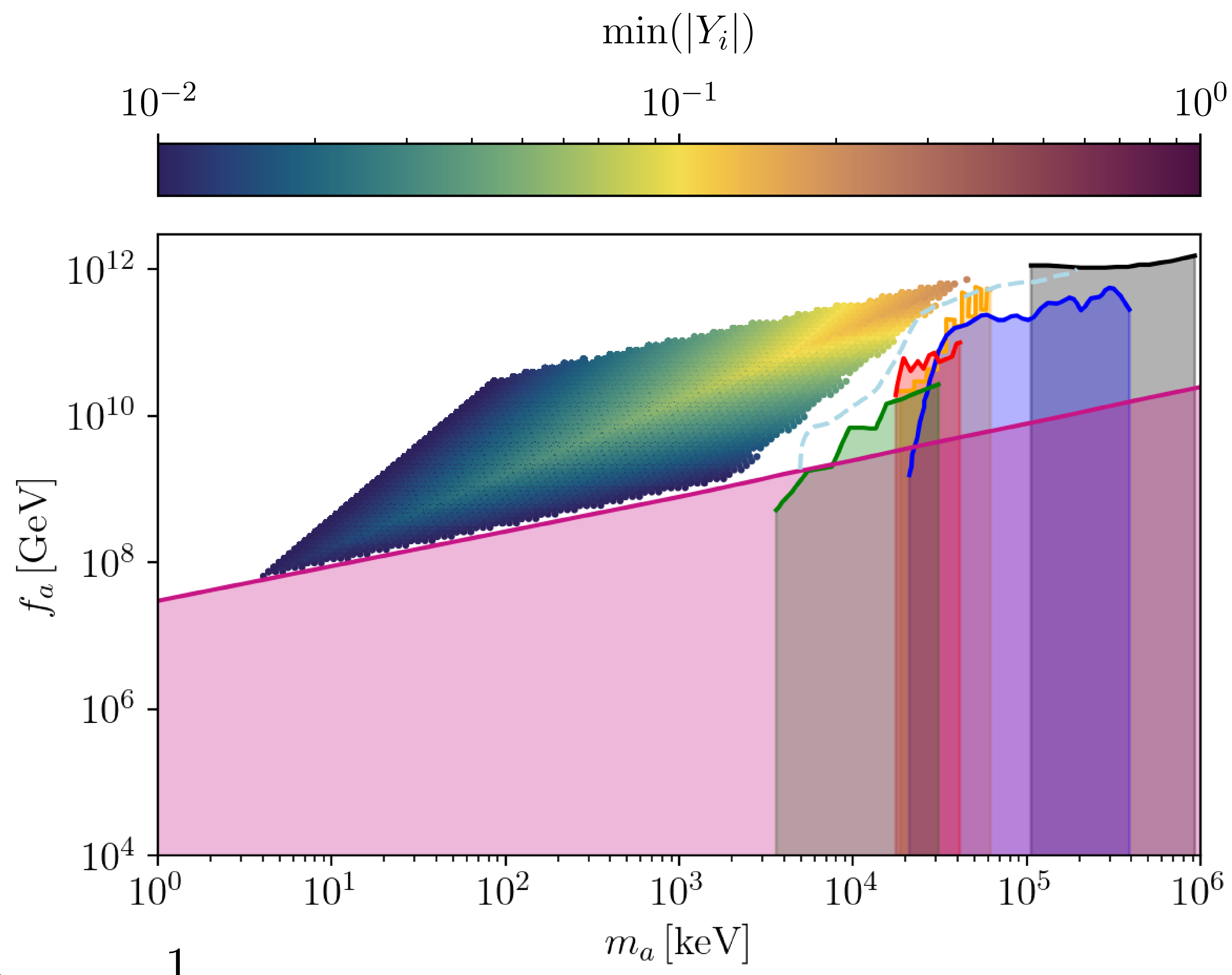

$$\Gamma(a \to 2\nu) \simeq \frac{m_a}{16\pi f_a^2}\sum_{i=1}^{3} m_i^2 \sim$$

$$\frac{1}{3\times 10^{19}\,\mathrm{sec}}\left(\frac{m_a}{1\,\mathrm{MeV}}\right)\left(\frac{10^9\,\mathrm{GeV}}{f_a}\right)^2\left(\frac{\sum m_i^2}{10^{-3}\mathrm{eV}^2}\right) \lesssim \frac{1}{10^{19}\mathrm{sec}}$$

From CMB +BAO

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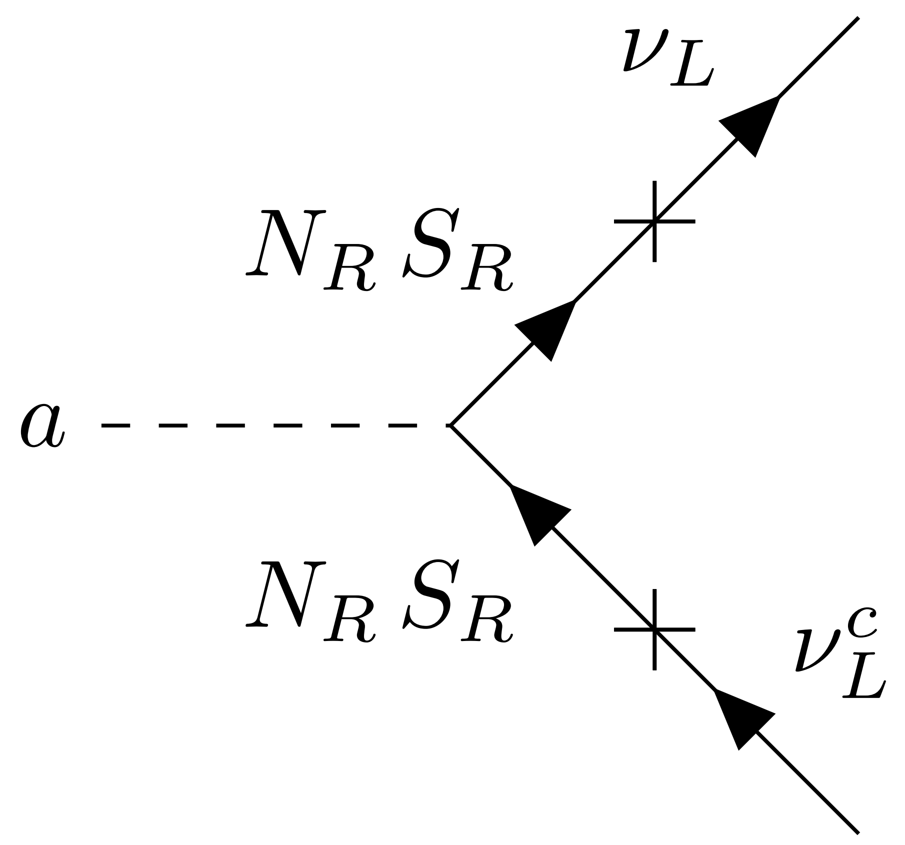

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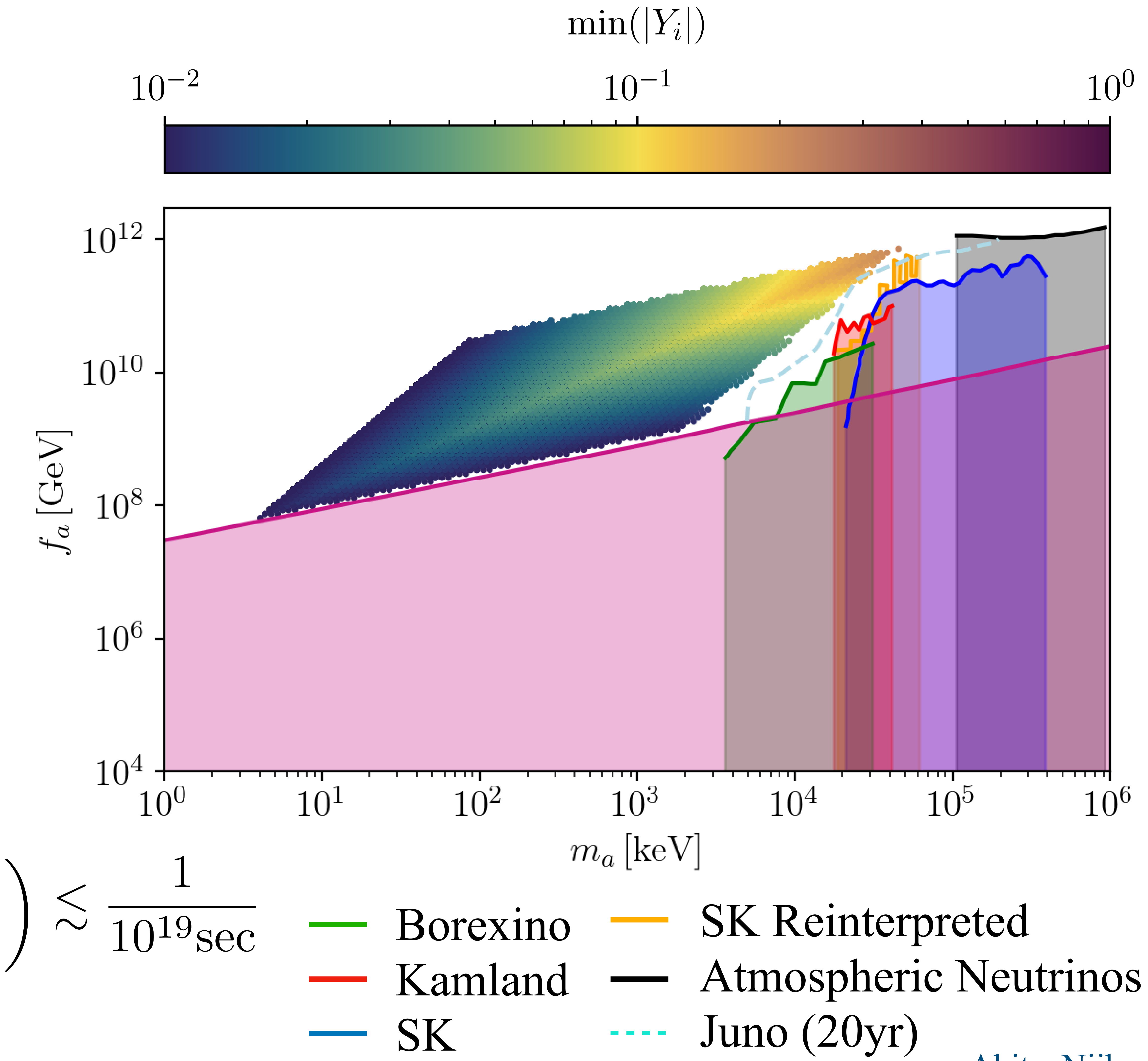
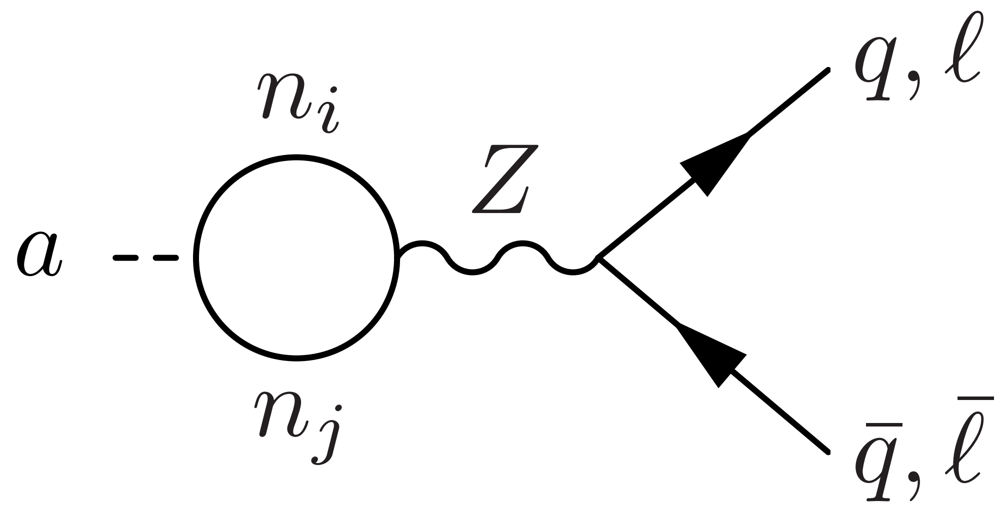


Akita, Niibo, [2304.04430]

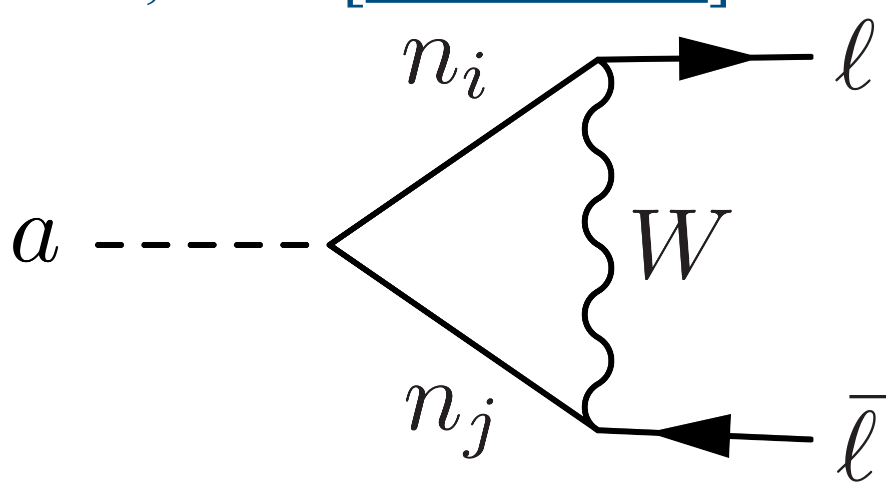
Phenomenology of the Majoron

$$\mathcal{L}_{a\ell\ell} \simeq \frac{i\,a}{16\pi^2 f_a}\bar{\ell}\left(M_\ell \,\mathrm{tr}\,[K]\,\gamma_5 + 2\,M_\ell\,K\,P_L - 2\,K\,M_\ell\,P_R\right)\ell$$

$$K \equiv \frac{\hat{m}\hat{m}^\dagger}{v^2} = \frac{m_N m_N^\dagger + \epsilon^2 m_S m_S^\dagger}{v^2}$$



Heeck, Patel [1909.02029]



Phenomenology of the Majoron

$$\mathcal{L}_{aee} \simeq i \frac{m_e}{16\pi^2 f_a} \left(\mathrm{tr}K - 2(K)_{ee}\right) a\, \bar{e}\gamma_5 e \equiv i\, g_{ae}\, a\, \bar{e}\gamma_5 e$$

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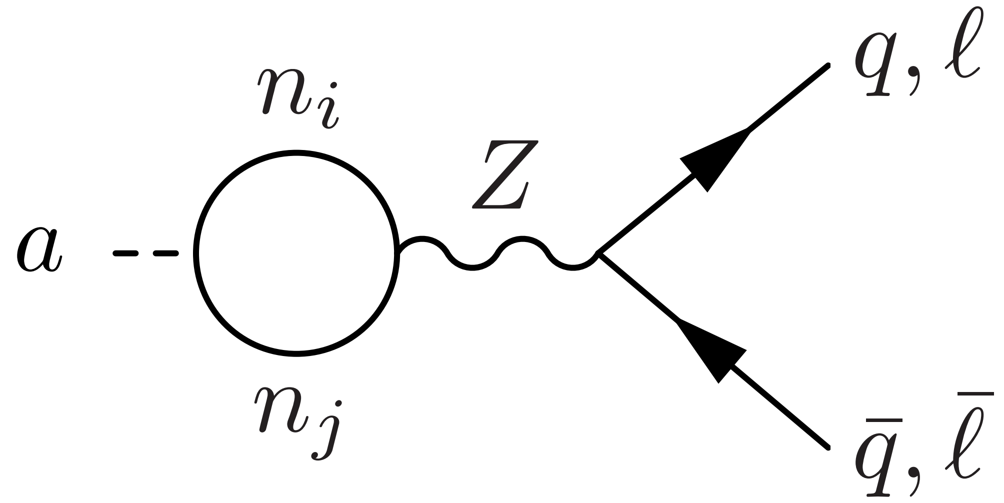
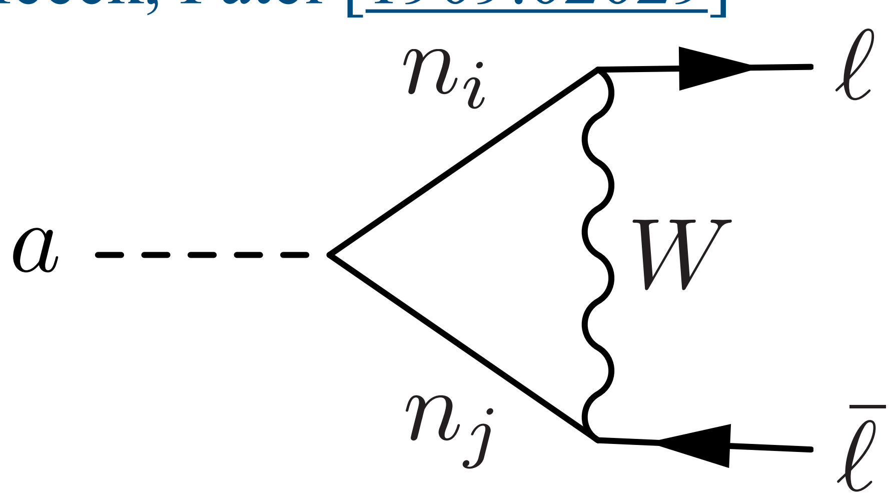
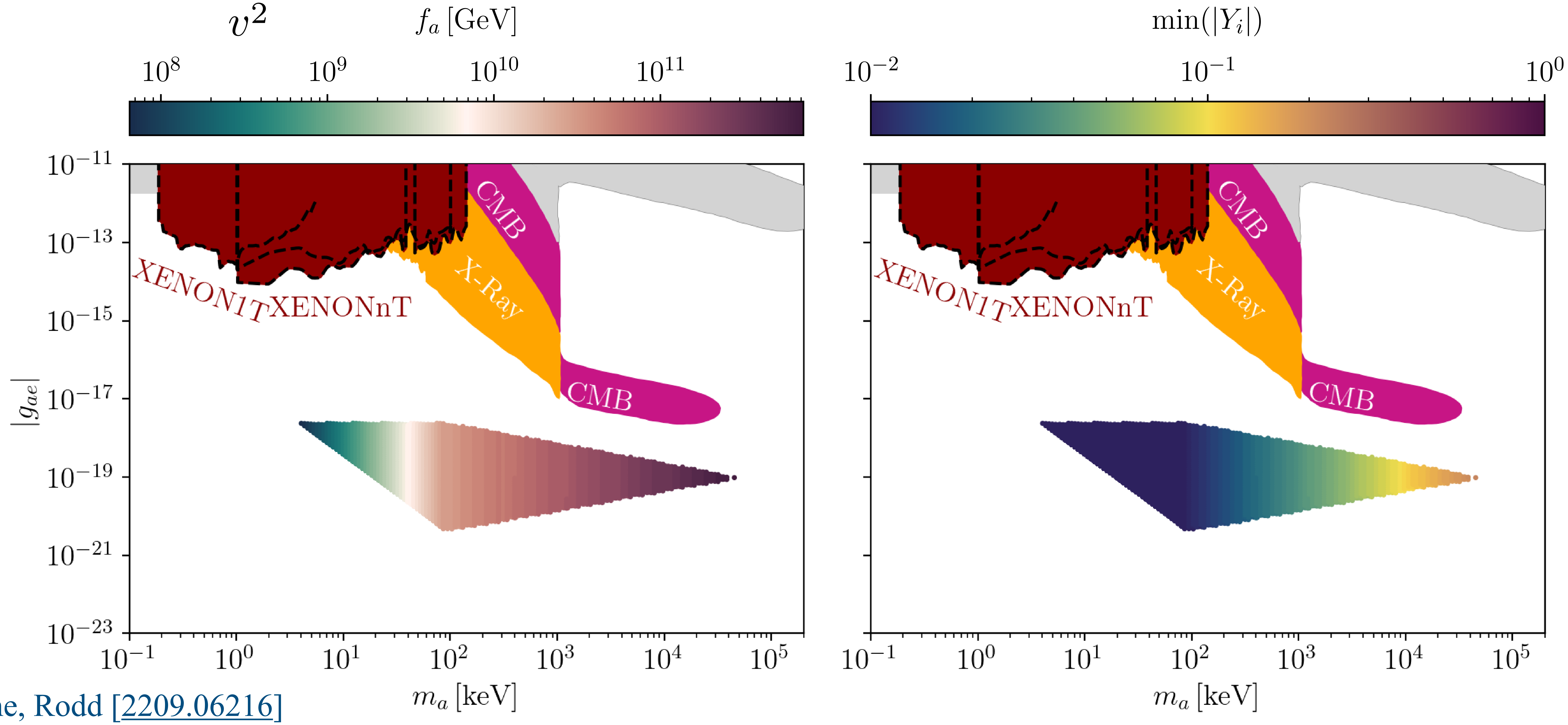
Heeck, Patel [1909.02029]







Langhoff, Outmezguine, Rodd [2209.06216]

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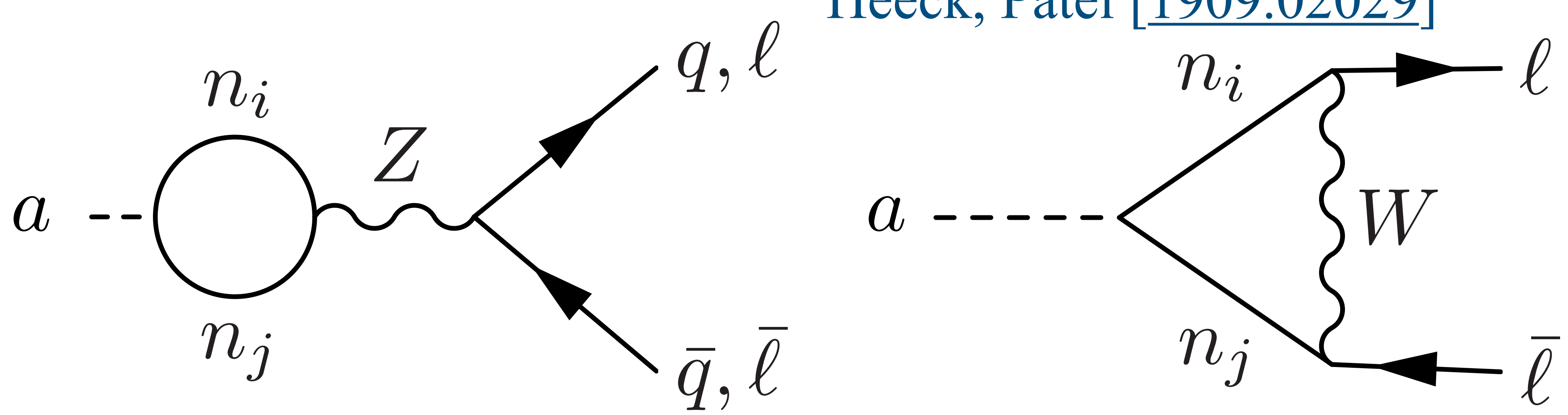
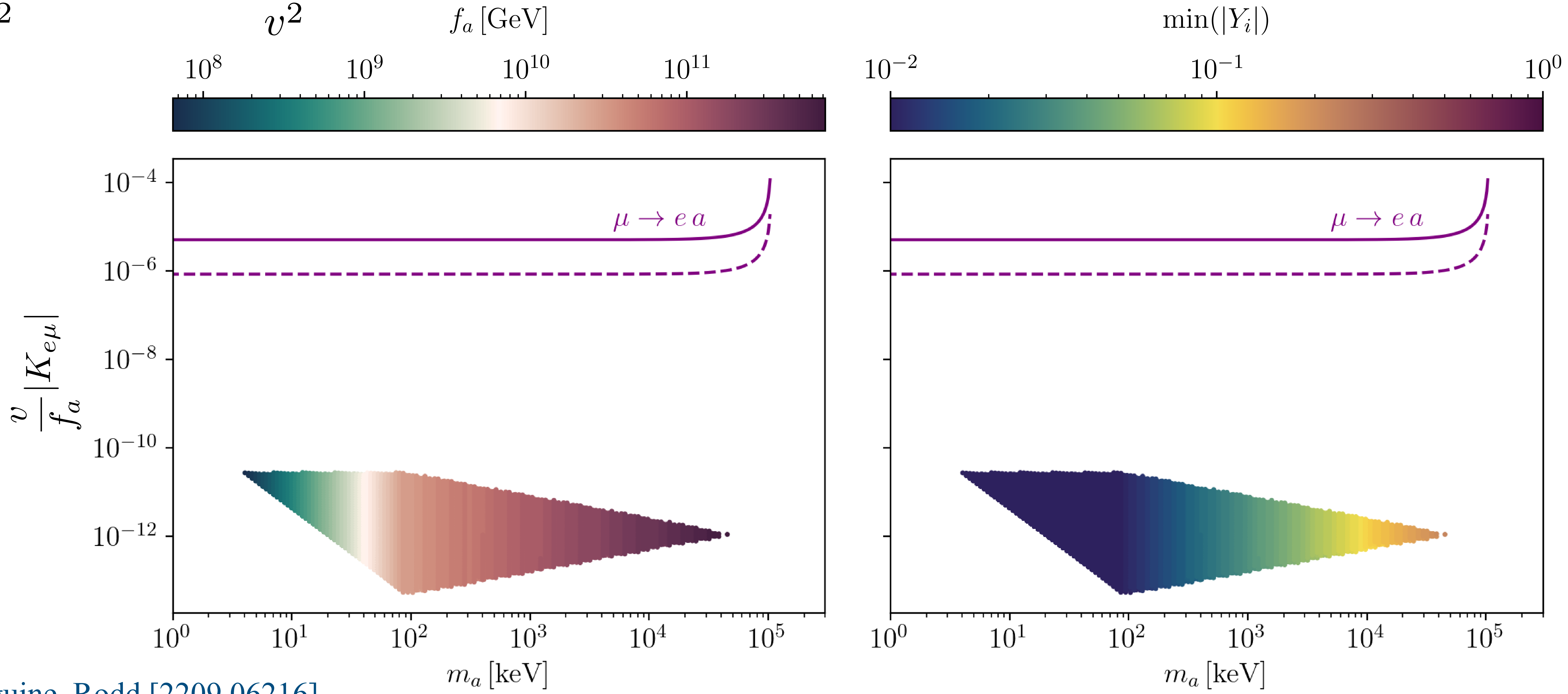


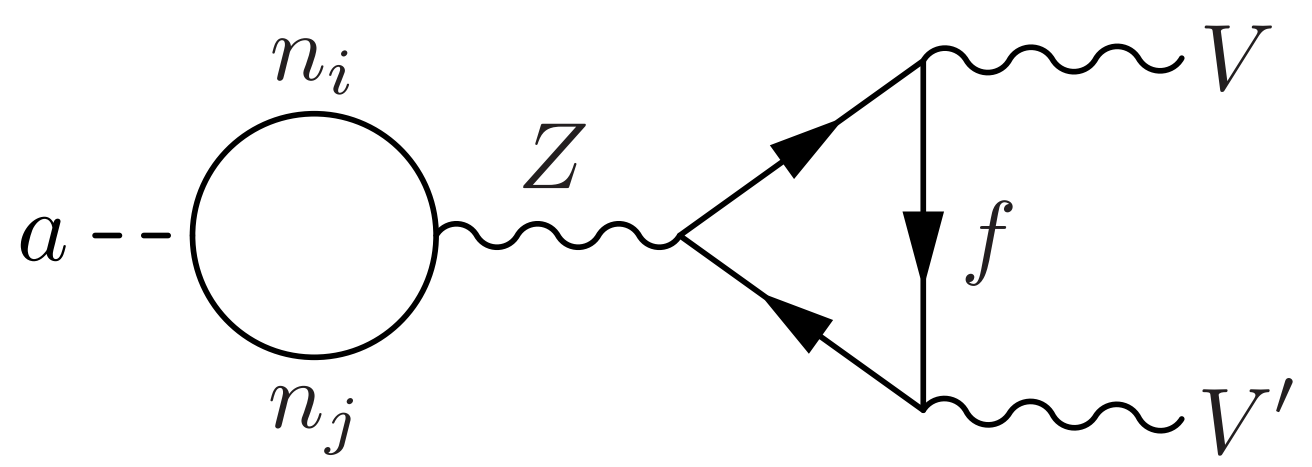


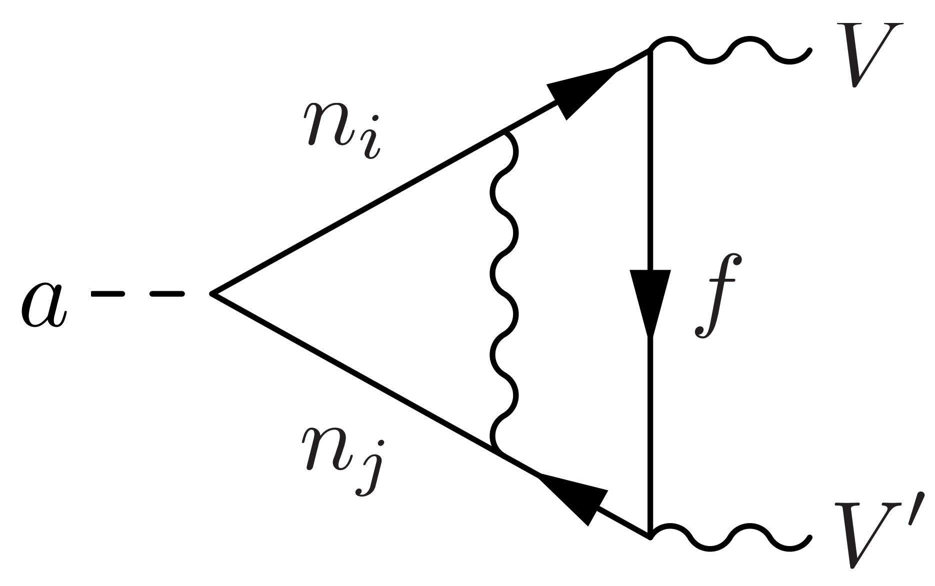
Langhoff, Outmezguine, Rodd [2209.06216]

Phenomenology of the Majoron

Heeck, Patel [1909.02029]

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\widetilde{F}^{\mu\nu} \qquad g_{a\gamma\gamma} \simeq -\frac{\alpha_{\rm em}}{8\pi^3 f_a}\left[\mathrm{tr}K\sum_f N_c^f Q_f^2 T_3^f\, h\left(\frac{m_a^2}{4m_f^2}\right) + \sum_{\ell=e,\mu,\tau} K_{\ell\ell}\, h\left(\frac{m_a^2}{4m_\ell^2}\right)\right]$$





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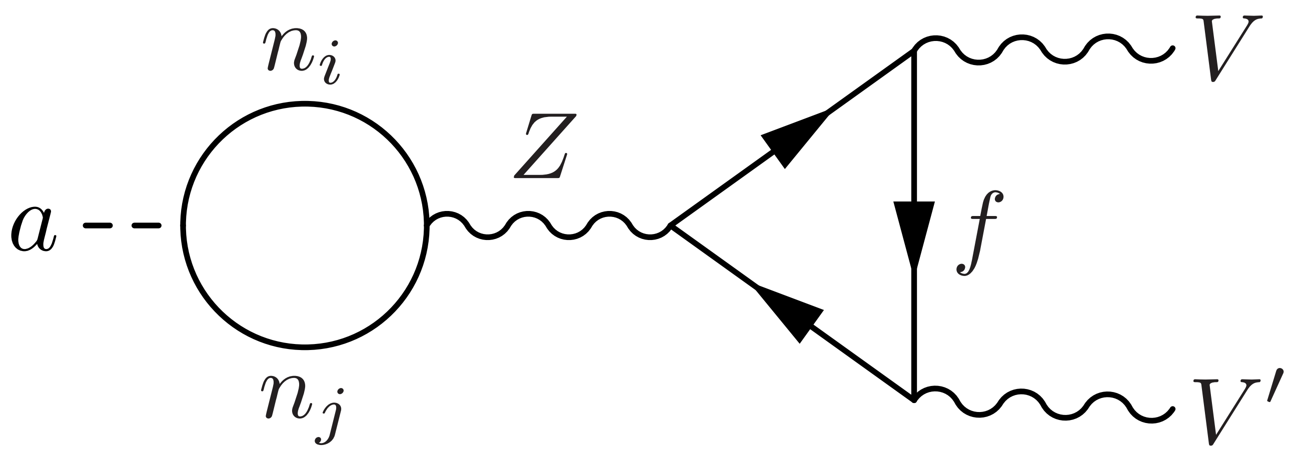
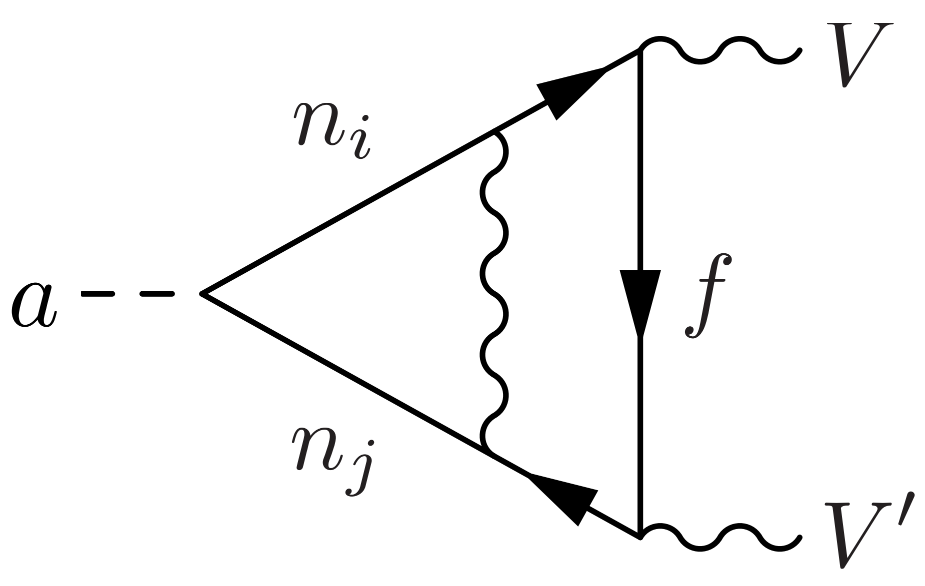
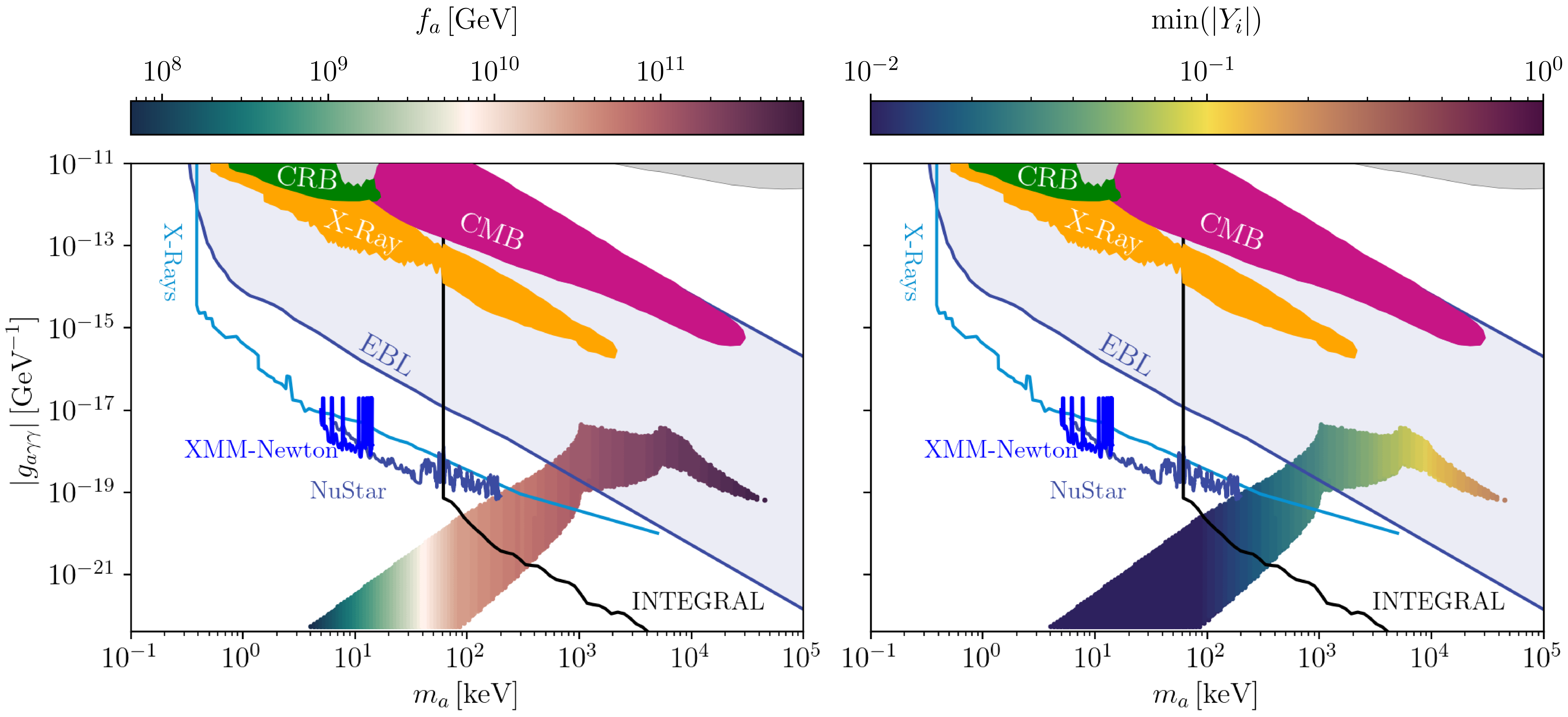






Cadamuro, Redondo [1110.2895]

Langhoff, Outmezguine, Rodd [2209.06216]

Conclusions

- We investigate all possible textures of non-redundant models
- Imposing a set of minimal conditions we are able to link together the masses of the Majoron and light neutrinos.
- Obtaining a prediction for the parameter space!

$$f_a \sim \left[10^{8}\,,\; 10^{11}\right]\,\mathrm{GeV}$$
$$m_a \sim [1\,\mathrm{keV},\; 10\,\mathrm{MeV}]$$

- This model can be tested via Cosmology + neutrino detectors, if this particle is Dark Matter!

Thanks for your attention!

Acknowledgements

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 860881.

HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Back up

Light Neutrino Masses

One can factorise the mass-scale contributions from the flavour structure:

When $m_{T_1} \gg m_{T_2}, \delta m_\nu^{1L}$
Is fixed by the PMNS

$$m_\nu = m_\nu^{\rm TL} + \delta m_\nu^{\rm 1L} \equiv m_{T_1}(\mathbf{u}\mathbf{v}^T + \mathbf{v}\mathbf{u}^T) + m_{T_2}\mathbf{v}\mathbf{v}^T + m_L \mathbf{u}\mathbf{u}^T$$

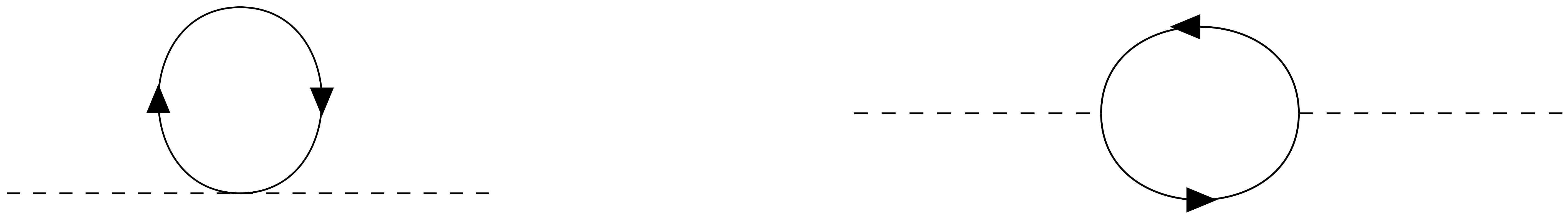
$$m_{T_1} = -\epsilon \frac{|Y_N||Y_S| v_{\rm EW}^2}{2\Lambda_{NS}}\,, \qquad m_{T_2} = \epsilon^2 \frac{\Lambda_{NN}}{\Lambda_{NS}} \frac{|Y_S|^2 v_{\rm EW}^2}{2\Lambda_{NS}}\,, \qquad m_L = \frac{|Y_N|^2}{16\pi^2} \frac{M_H^2 + 3M_Z^2}{M_N + M_S} \log\left(\frac{M_S}{M_N}\right).$$

$$|m_\pm|^2 = \frac{1}{2}\left[m_C^2 - \tau^2(2m_{T_1}^2 - m_{T_2}^2 - m_L^2) \pm \sqrt{\left(m_C^2 - \tau^2(2m_{T_1}^2 - m_{T_2}^2 - m_L^2)\right)^2 - 4\tau^4(m_{T_1}^2 - m_L m_{T_2})^2}\,\right],$$

$$m_C \equiv |2m_{T_1} + \eta m_L + \eta^* m_{T_2}|\,, \qquad \eta \equiv \mathbf{u}^\dagger \mathbf{v} \equiv |\eta| e^{i\vartheta_\eta}\,, \qquad \tau^2 \equiv 1 - |\eta|^2\,.$$

NO:	$\lvert m_1\rvert^2 = 0\,,$	$\lvert m_2\rvert^2 = \lvert m_-\rvert^2\,,$	$\lvert m_3\rvert^2 = \lvert m_+\rvert^2\,,$
IO:	$\lvert m_1\rvert^2 = \lvert m_-\rvert^2\,,$	$\lvert m_2\rvert^2 = \lvert m_+\rvert^2\,,$	$\lvert m_3\rvert^2 = 0\,.$

Majoron Mass: Loop Computation



$$\mathcal{L}_{\text{Yuk}} = \overline{\ell_L}\, H\, Y_e\, e_R + \overline{\ell_L}\, \tilde{H}\, Y_N\, N_R + \epsilon\, \overline{\ell_L}\, \tilde{H}\, Y_S\, S_R + \frac{Y_{NS}}{2}\, \phi^* \left(\overline{N_R^c}\, S_R + \overline{S_R^c}\, N_R\right) + \frac{Y_{NN}}{2}\, \phi\, \overline{N_R^c}\, N_R$$

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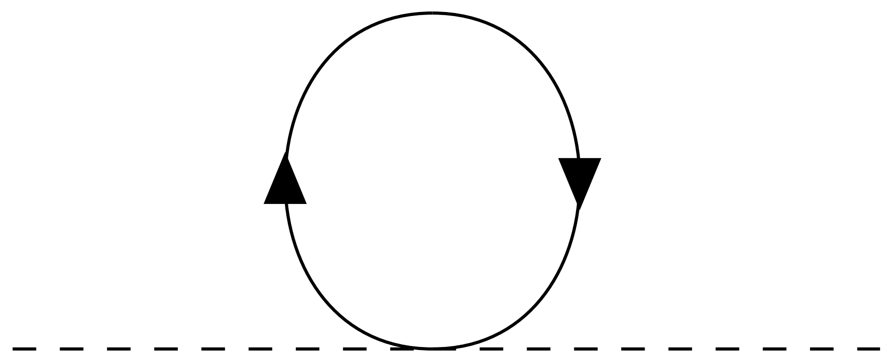
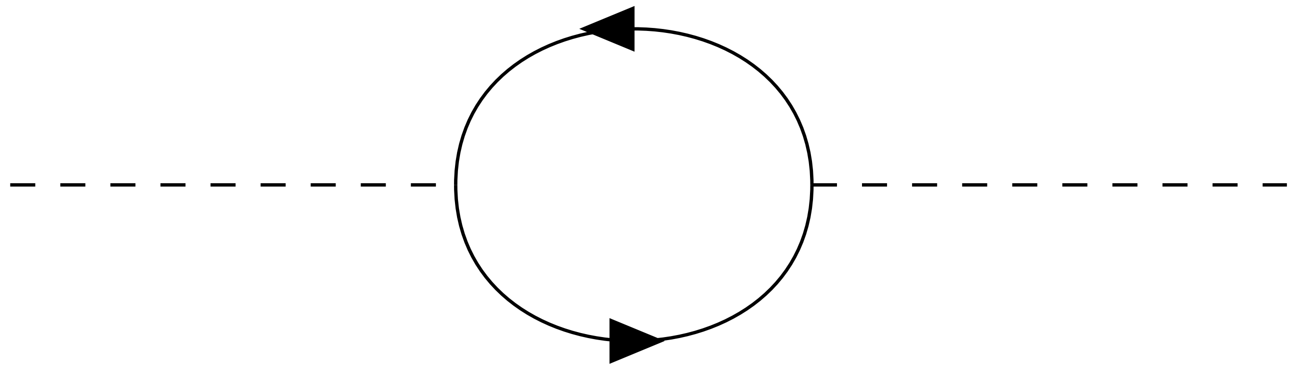
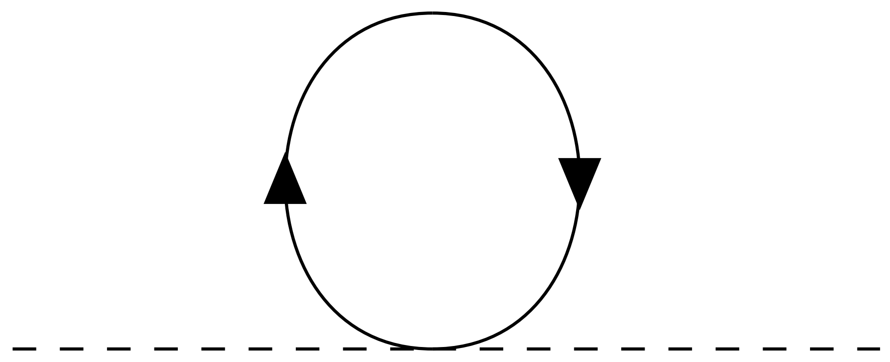


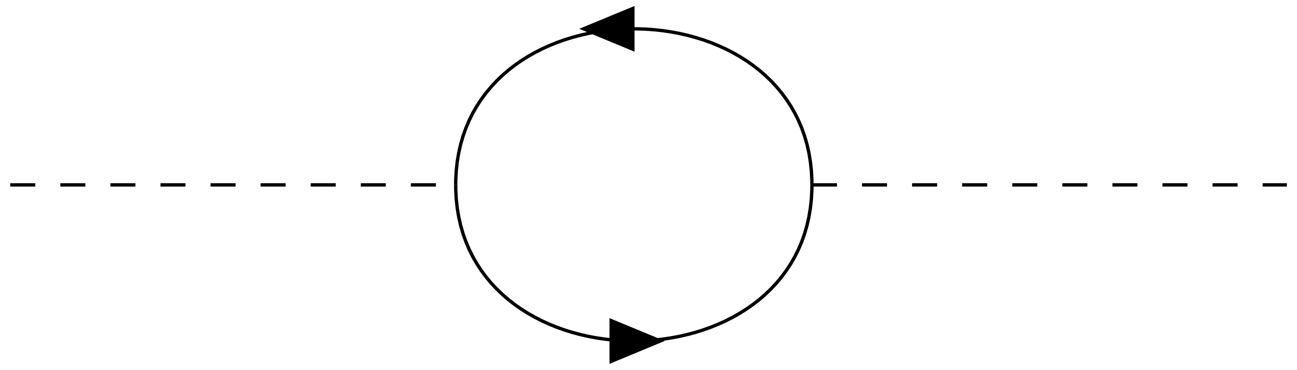


$$\mathcal{L}_{\text{Yuk}} = \overline{\ell_L}\, H\, Y_e\, e_R + \overline{\ell_L}\, \tilde{H}\, Y_N\, N_R + \epsilon\, \overline{\ell_L}\, \tilde{H}\, Y_S\, S_R + \frac{Y_{NS}}{2}\, \phi^* \left(\overline{N_R^c}\, S_R + \overline{S_R^c}\, N_R\right) + \frac{Y_{NN}}{2}\, \phi\, \overline{N_R^c}\, N_R$$

Perform the field redefinition: $\{N_R,\, \ell_L,\, e_R\} \to \{N_R,\, \ell_L,\, e_R\}\, e^{-ia/(2f_a)}\,, \quad S_R \to S_R\, e^{3ia/(2f_a)}$

Majoron Mass: Loop Computation





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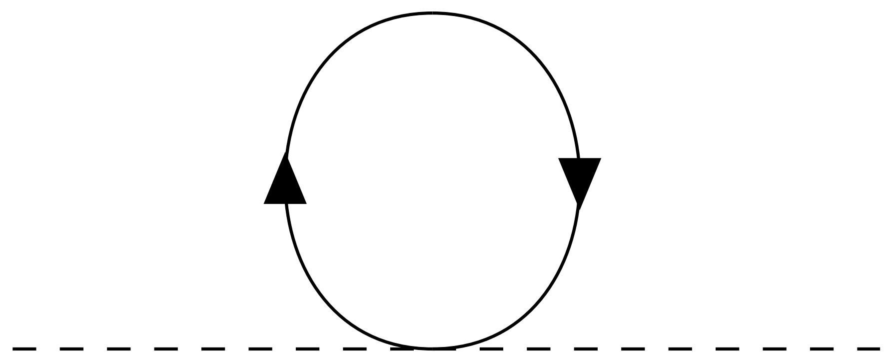
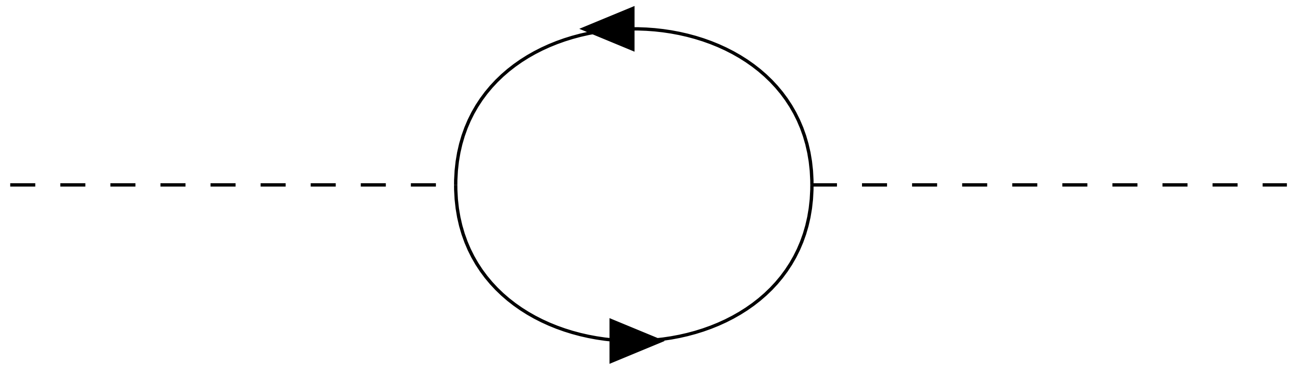
$$\mathcal{L}_a = \frac{\partial_\mu a}{2f_a} \left(\overline{\nu}_L \gamma^\mu \nu_L + \overline{N}_R \gamma^\mu N_R - 3\, \overline{S}_R \gamma^\mu S_R\right) + \left(\epsilon\, \overline{\ell}_L\, \tilde{H}\, Y_S\, S_R\, e^{2ia/f_a} + \text{h.c.}\right)$$

Majoron Mass: Loop Computation





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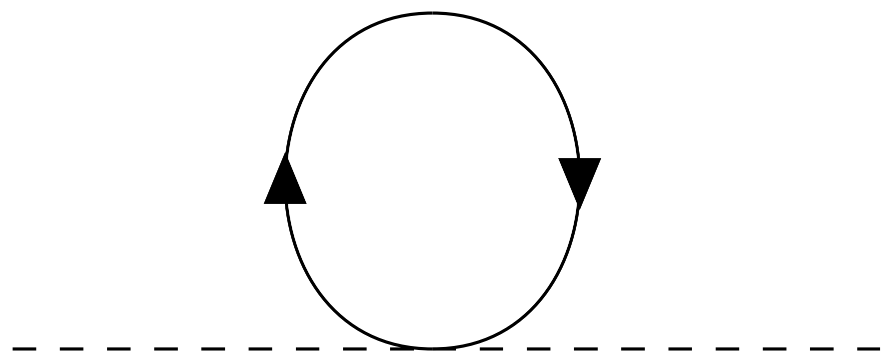
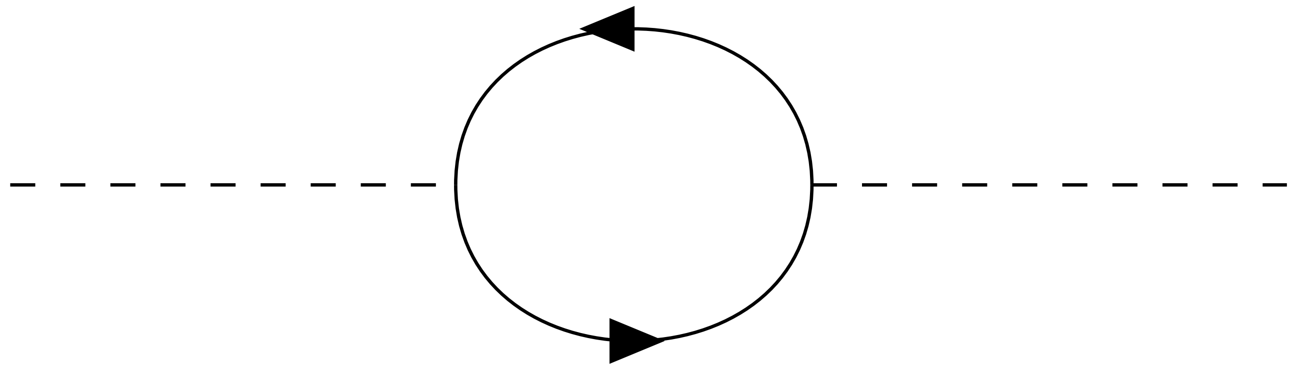
Majoron Mass: Loop Computation





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Perform the field redefinition: $\{N_R,\, \ell_L,\, e_R\} \to \{N_R,\, \ell_L,\, e_R\}\, e^{-ia/(2f_a)}\,, \quad S_R \to S_R\, e^{3ia/(2f_a)}$

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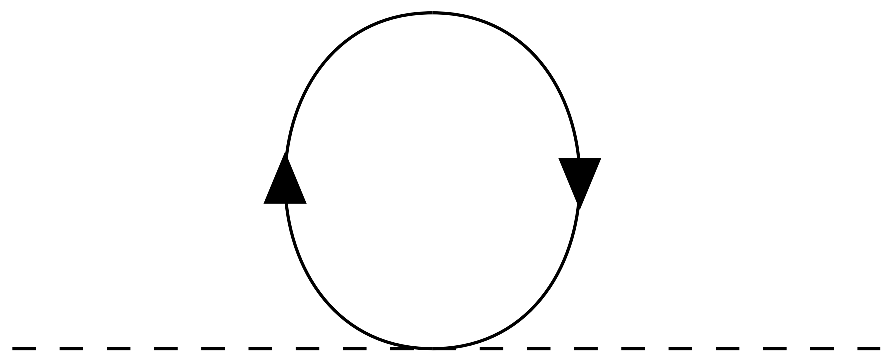
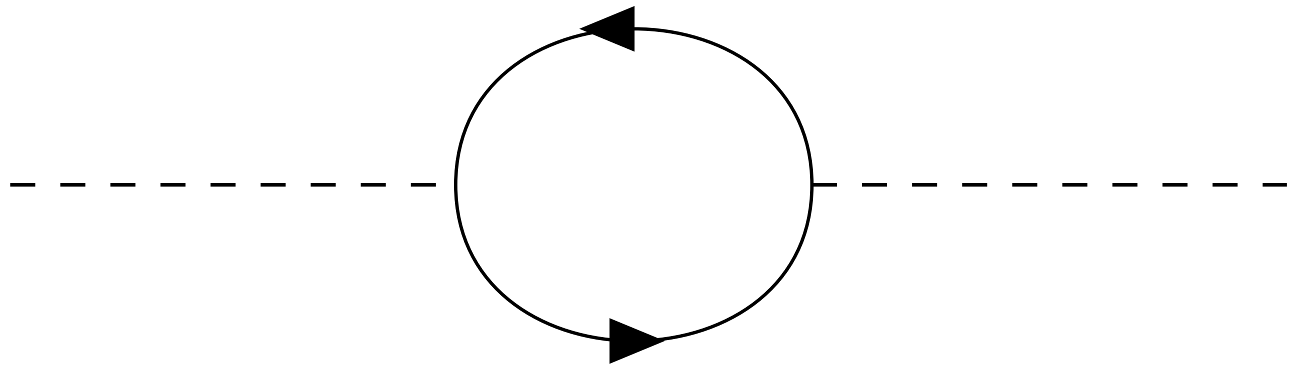
Majoron Mass: Loop Computation





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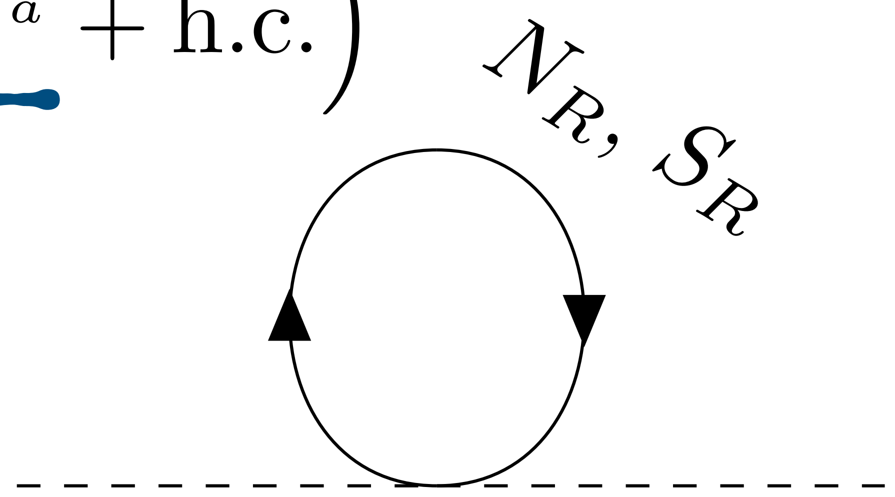

After diagonalization

$$\mathcal{L}_a \supset \frac{|\epsilon m_S \cdot m_N|}{2\sqrt{M_N\, M_S}(M_N + M_S)}\left(M_N \overline{S_R^c} S_R - M_S \overline{N_R^c} N_R\right)\frac{a^2}{f_a^2} + \text{h.c.}$$

Problem reduced to only two loops!