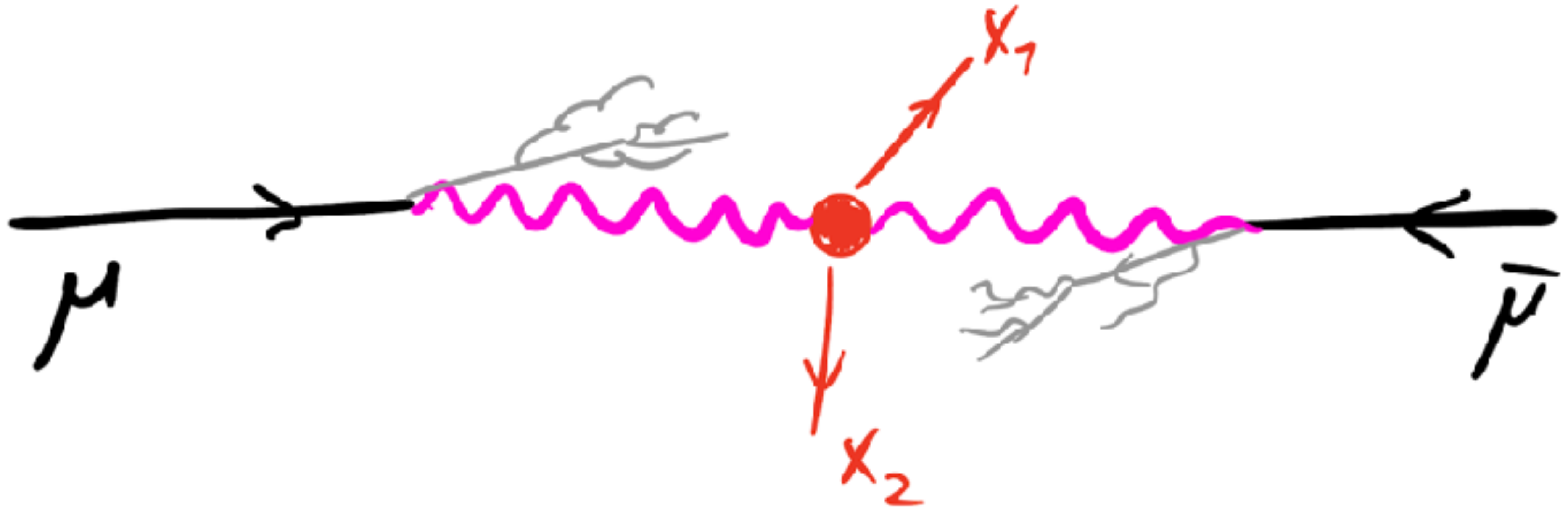
EW PDFs for Muon Colliders and applications



LePDF

Francesco Garosi, D.M., Sokratis Trifinopoulos
JHEP 09 (2023) 107 [**2303.16964**]

Source + Downloads available at
https://github.com/DavidMarzocca/LePDF

David Marzocca

+ ongoing work with A. Stanzione, F. Garosi, R. Capdevilla, B. Stechauner

INFN Trieste

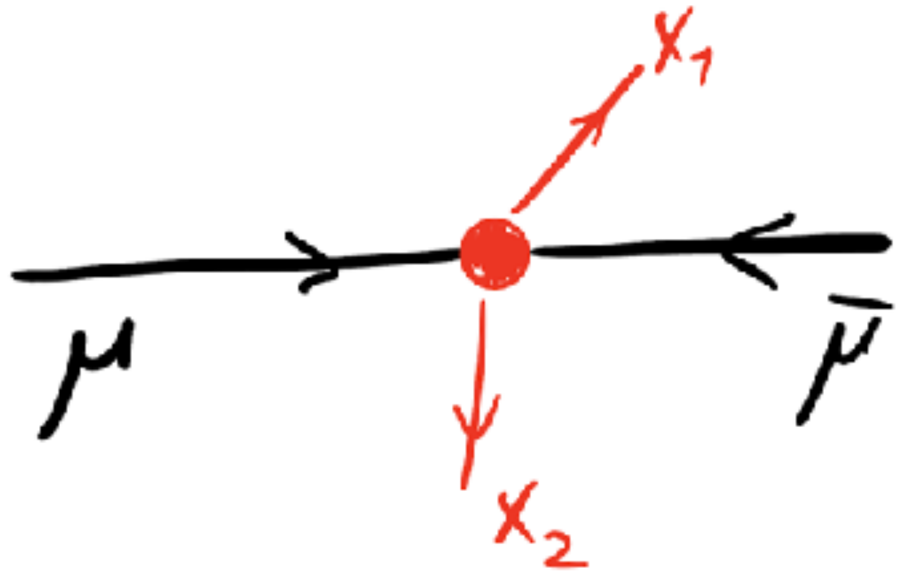
La Thuile 2024 - 08/03/2024

PDFs of a muon

The muon (or electron) is an elementary particle.
At zeroth order in perturbation theory it carries all the momentum of the beam.

The production of **heavy states** with $M_X \sim \sqrt{s}$ is dominated by the **annihilation** $\mu^+ \mu^- \to X$

(e.g. QED pair production of heavy particles)



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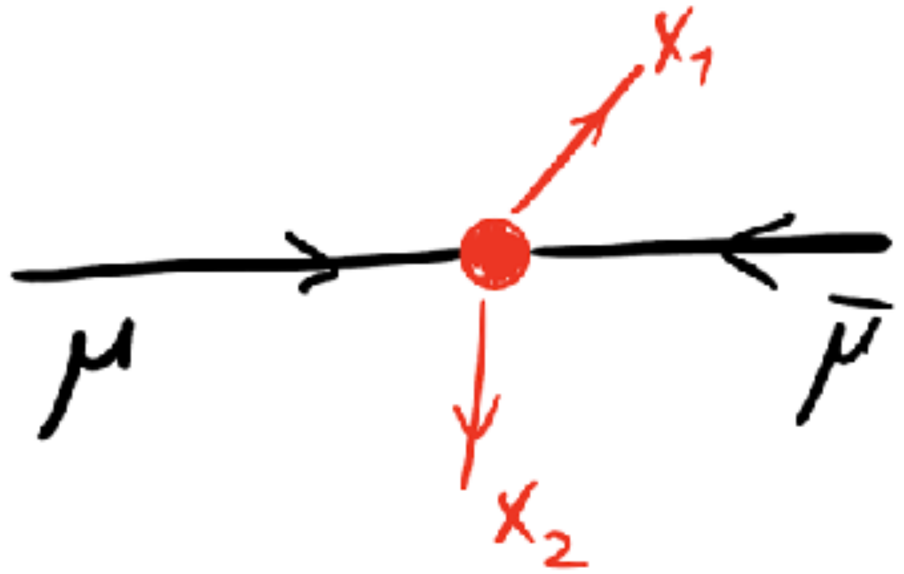

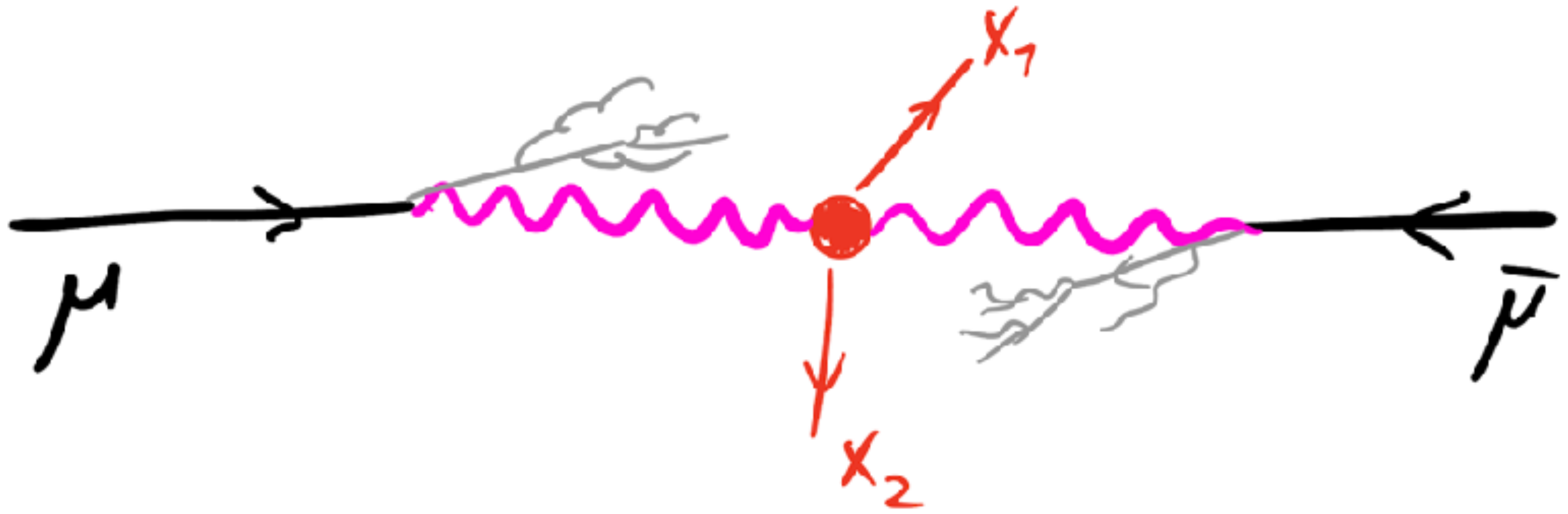

For processes well above threshold, the **contribution from collinear virtual bosons** emitted from the muons can become **dominant**.

"The muon collider is a weak boson collider"

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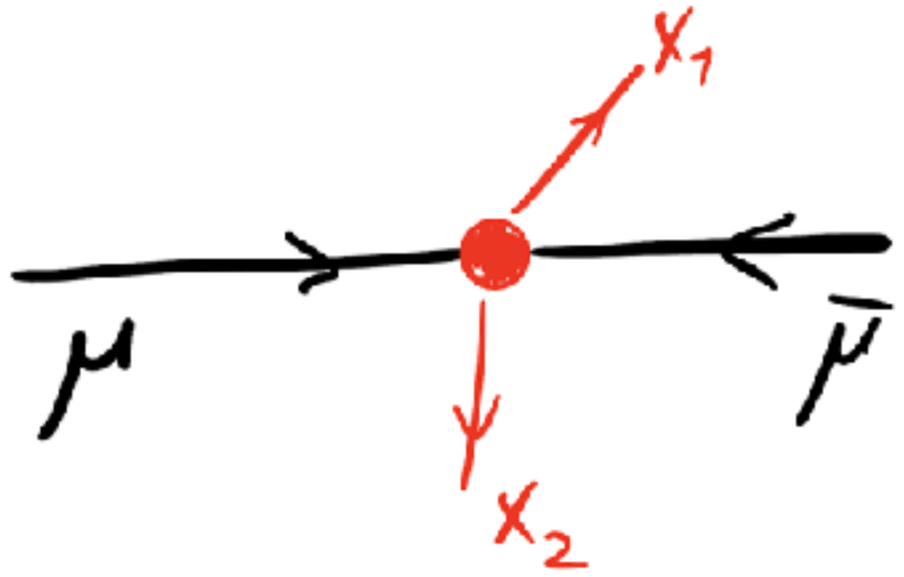

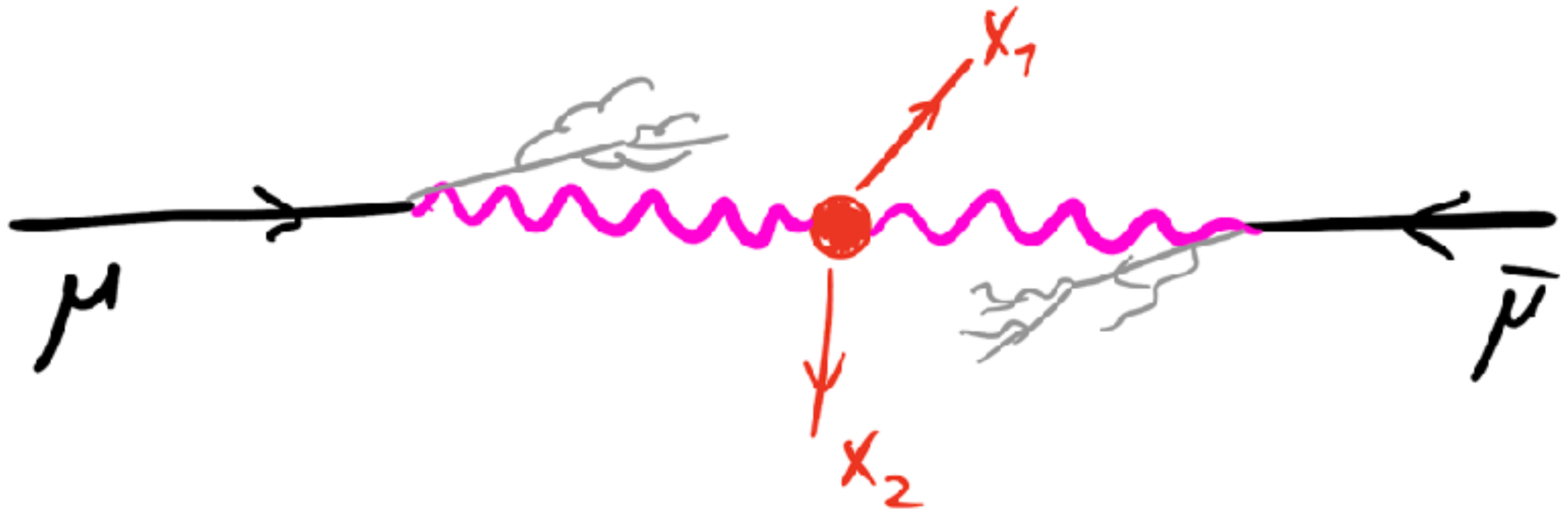

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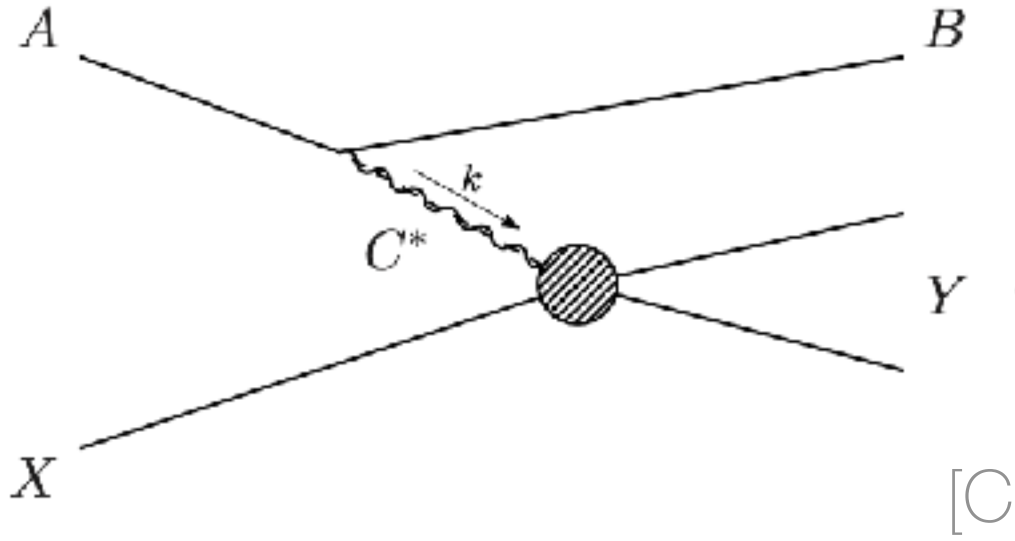
Collinear Factorization



The amplitudes for collinear splitting and hard scattering can be factorised

$$i\mathcal{M}(AX \to BY) = \sum_{C} i\mathcal{M}^{\text{hard}}(CX \to Y)\frac{i}{Q^2} i\mathcal{M}^{\text{split}}(A \to BC^*)\,[1+\mathcal{O}(\delta)] \quad \text{if}$$

$$\delta_m \equiv m/E \ll 1$$

$$\delta_\perp \equiv |\mathbf{k}_\perp|/E \ll 1$$

$(m = m_C)$

on-shell

$Q^2 = k^2 - m_C^2$

[Cuomo, Vecchi, Wulzer 1911.12366, …]

PDFs of a muon

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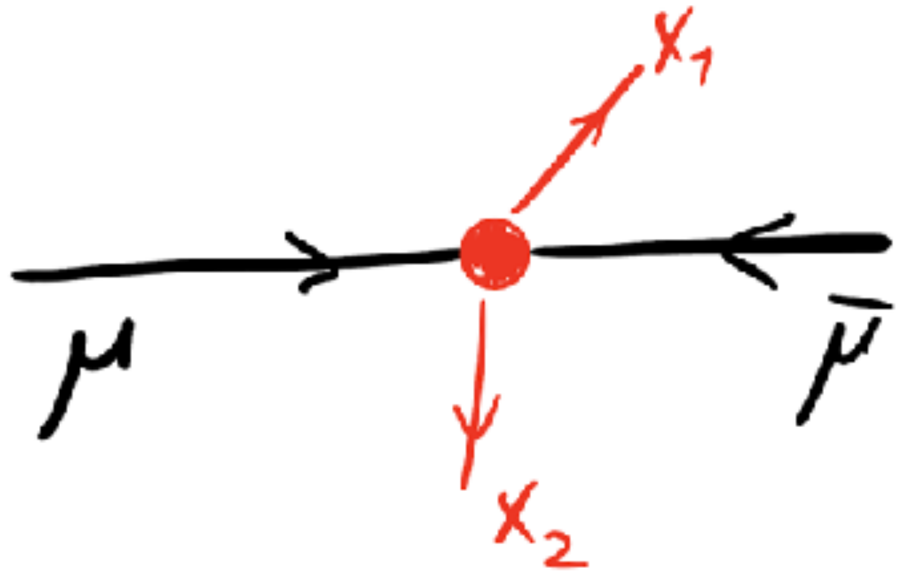

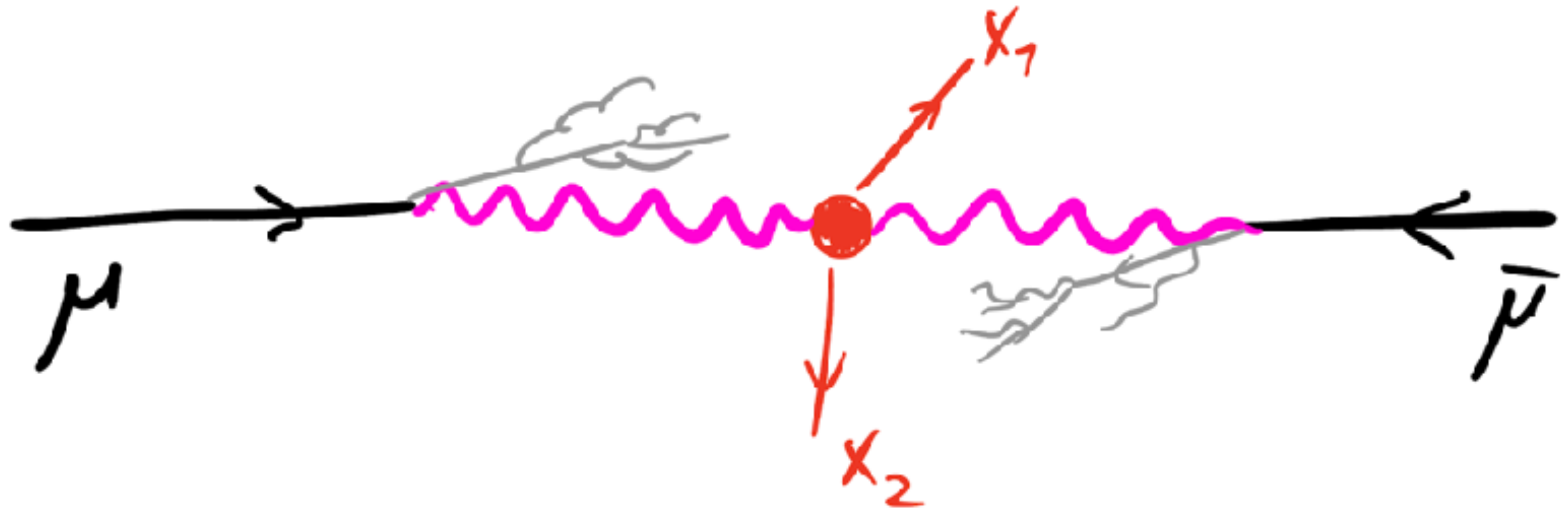

For processes well above threshold, the **contribution from collinear virtual bosons** emitted from the muons can become **dominant**.

"The muon collider is a weak boson collider"

This can be described in terms of **generalised Parton Distribution Functions**, like for proton colliders:

$$\sigma(\mu\bar{\mu} \to C + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{ij} f_i(x_1,\mu)\, f_j(x_2,\mu)\, \hat{\sigma}(ij \to C)(\hat{s})$$

PDFs of a muon

Unlike for protons, since the muon is elementary this can be done **from first principles**.

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The **boundary condition** is set by $f_\mu(x,\ m_\mu) = \delta(1\text{-}x) + O(\alpha), \quad f_{i\neq\mu}(x,\ m_\mu) = 0 + O(\alpha)$

NLO corrections in Frixione [1909.03886]

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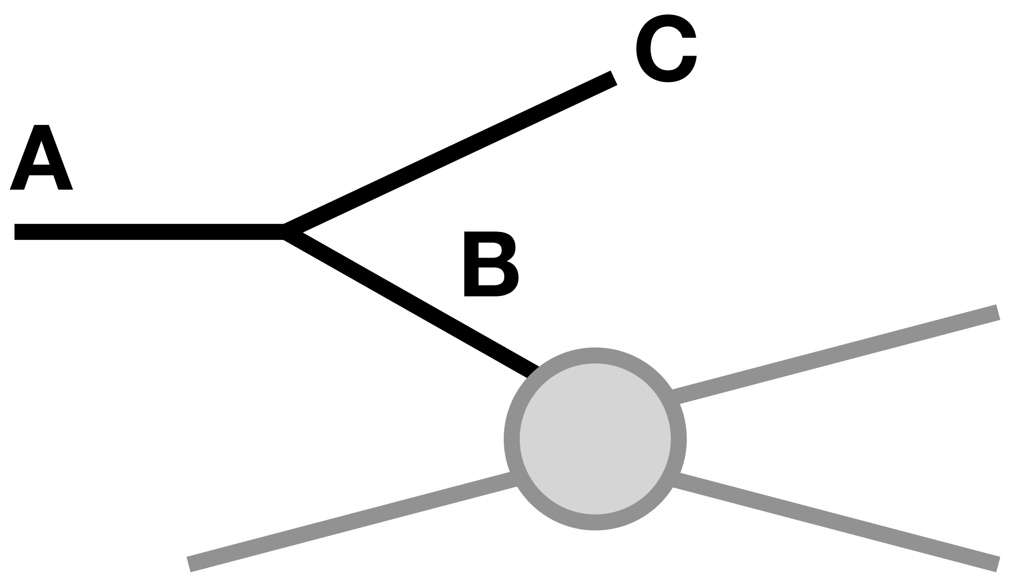
NLO corrections in Frixione [1909.03886]

The **SM DGLAP equations** describe the evolution of the PDFs M. Ciafaloni, P. Ciafaloni, D. Comelli hep-ph/0111109, hep-ph/0505047]



$$Q^2 \frac{d f_B(x,Q^2)}{dQ^2} = \underbrace{P_B^v\, f_B(x,Q^2)}_{\text{Virtual corrections}} + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \underbrace{\widetilde{P}^C_{BA}}_{\text{Real emission}} \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \underbrace{\widetilde{U}^C_{BA}}_{\text{ultra-collinear terms (EWSB)}} \otimes f_A$$

Chen, Han, Tweedie [1611.00788]

For scales **below *m*$_W$ we can use QED+QCD** interactions. **Above**, the **complete SM** is needed.

Above the EW scale

All SM interactions and fields must be considered and

several new effects must be taken into account:

- **PDFs become polarised**, since EW interactions are chiral. Bauer, Webber [1808.08831]
- At high energies **EW Sudakov double logarithms** are generated. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0001142, hep-ph/0505047], Bauer, Webber [1703.08562, 1808.08831], Chen, Han, Tweedie [1611.00788], Han, Ma, Xie [2103.09844], F. Garosi, D.M., S. Trifinopoulos [2303.16964]
- Neutral bosons interfere with each other: **Z/γ and h/Z_L PDFs mix**. P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047] Chen, Han, Tweedie [1611.00788]
- **Mass effects** of partons with EW masses (W, Z, h, t) become relevant and remain so even at multi-TeV scale.
- EW symmetry is broken. Another set of splitting functions, proportional to v^2 instead of $p_T{}^2$, arise: **ultra-collinear splitting functions**. Chen, Han, Tweedie [1611.00788]

LePDF - implementation

We work in the **mass eigenstate basis** and **solve the DGLAP numerically** in **x-space**, discretising the [10^{-6}, -1] interval

After identifying PDFs which are identical because of flavour symmetry, we remain with **42 independent PDFs**:

$$
\begin{aligned}
&f_{e_L} = f_{\tau_L}\,, \quad f_{\bar{\ell}_L} = f_{\bar{e}_L} = f_{\bar{\mu}_L} = f_{\bar{\tau}_L}\,,\\
&f_{e_R} = f_{\tau_R}\,, \quad f_{\bar{\ell}_R} = f_{\bar{e}_R} = f_{\bar{\mu}_R} = f_{\bar{\tau}_R}\,,\\
&f_{\nu_e} = f_{\nu_\tau}\,, \quad f_{\bar{\nu}_\ell} = f_{\bar{\nu}_e} = f_{\bar{\nu}_\mu} = f_{\bar{\nu}_\tau}\,,\\
&f_{u_L} = f_{c_L}\,, \quad f_{\bar{u}_L} = f_{\bar{c}_L}\,, \quad f_{u_R} = f_{c_R}\,, \quad f_{\bar{u}_R} = f_{\bar{c}_R}\,,\\
&f_{d_L} = f_{s_L}\,, \quad f_{\bar{d}_L} = f_{\bar{s}_L}\,, \quad f_{d_R} = f_{s_R}\,, \quad f_{\bar{d}_R} = f_{\bar{s}_R}\,.
\end{aligned}
$$

Leptons	μ_L	μ_R	e_L	e_R	ν_μ	ν_e	$\bar{\ell}_L$	$\bar{\ell}_R$	$\bar{\nu}_\ell$
Quarks	u_L	d_L	u_R	d_R	t_L	t_R	b_L	b_R	$+$ h.c.
Gauge Bosons	$\gamma_\pm$	$Z_\pm$	$Z\gamma_\pm$	$W^\pm_\pm$	$G_\pm$				
Scalars	h	Z_L	hZ_L	$W^\pm_L$					

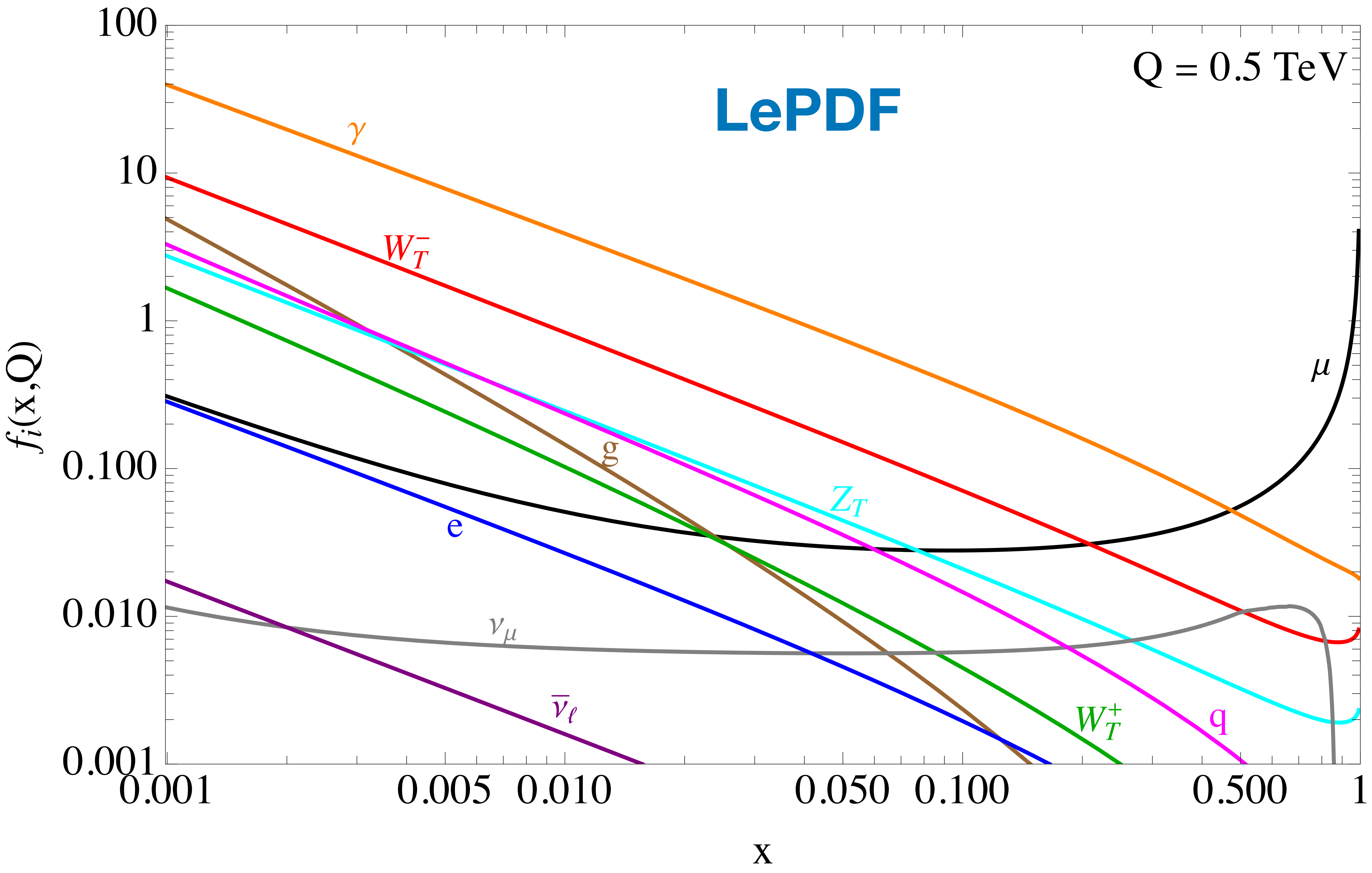
Starting from $Q_{\text{EW}} = m_W$, heavy states are added at the corresponding mass threshold.

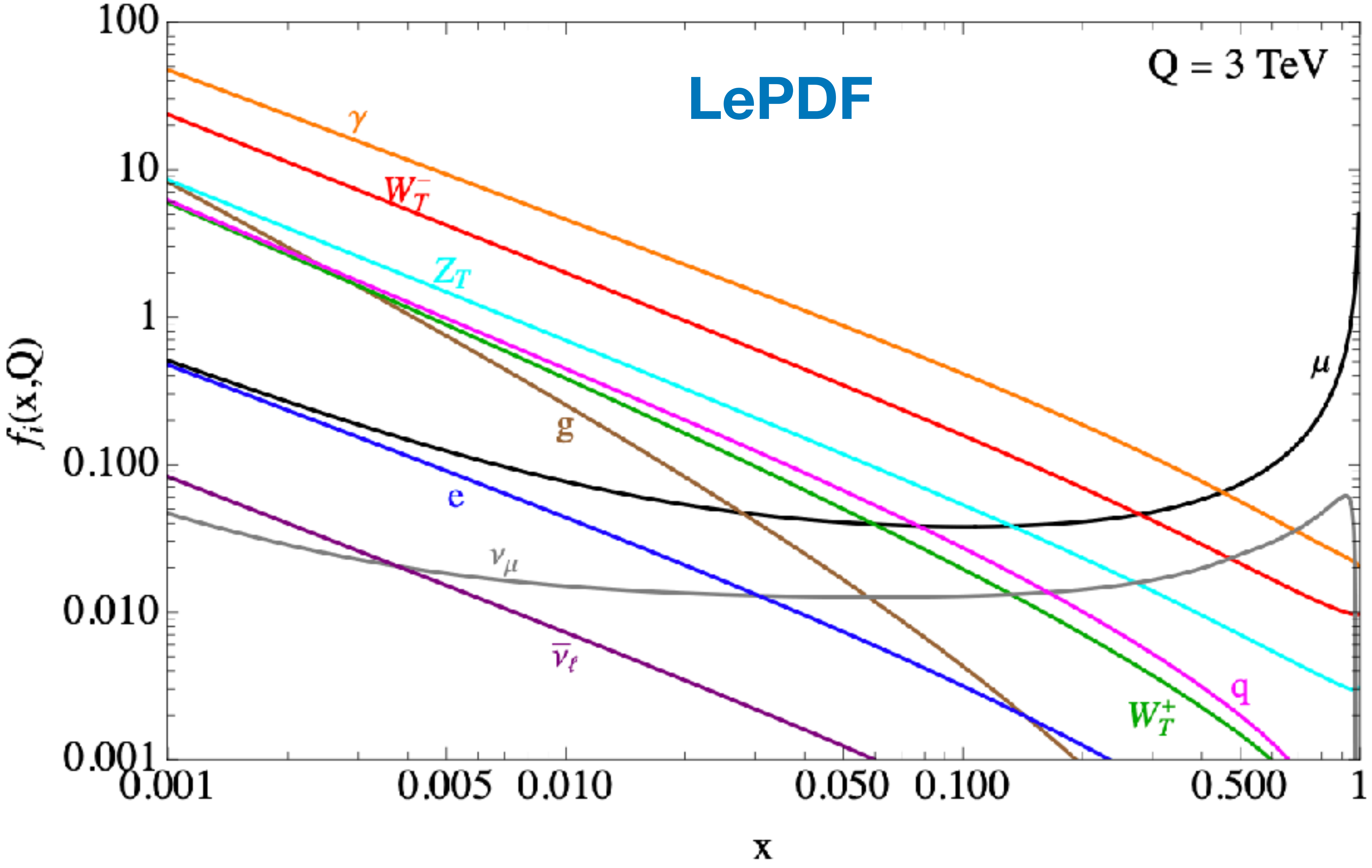
The uncertainties due to x and t discretisation are estimated to be of ~1% and ~0.1%, respectively.

All EW & SM interactions are implemented, including all features listed in the previous slide.

LePDF

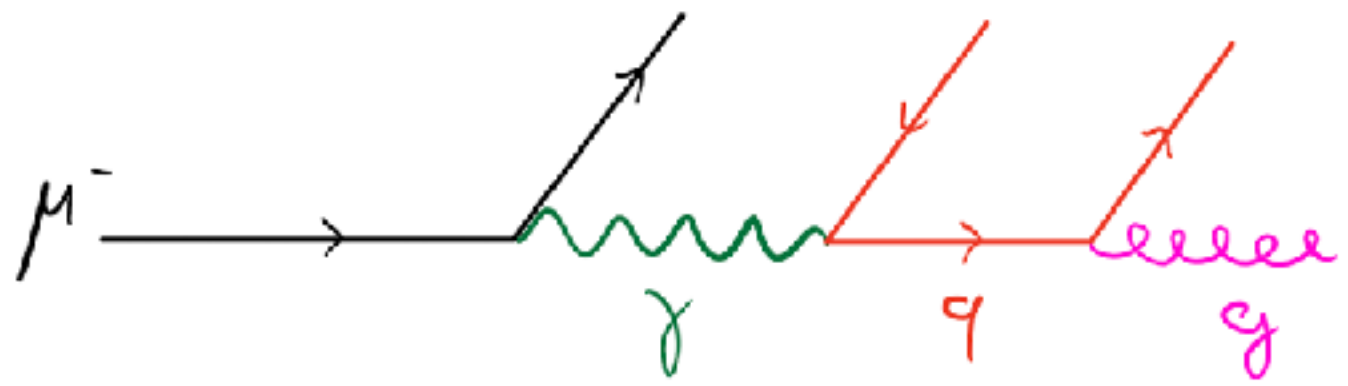
Summing over polarisations:





- **Large** **EW boson** **PDFs**, above EW scale and small x
- Non negligible **gluon** and **quark** content.

Han, Ma, Xie [2007.14300, 2103.09844]



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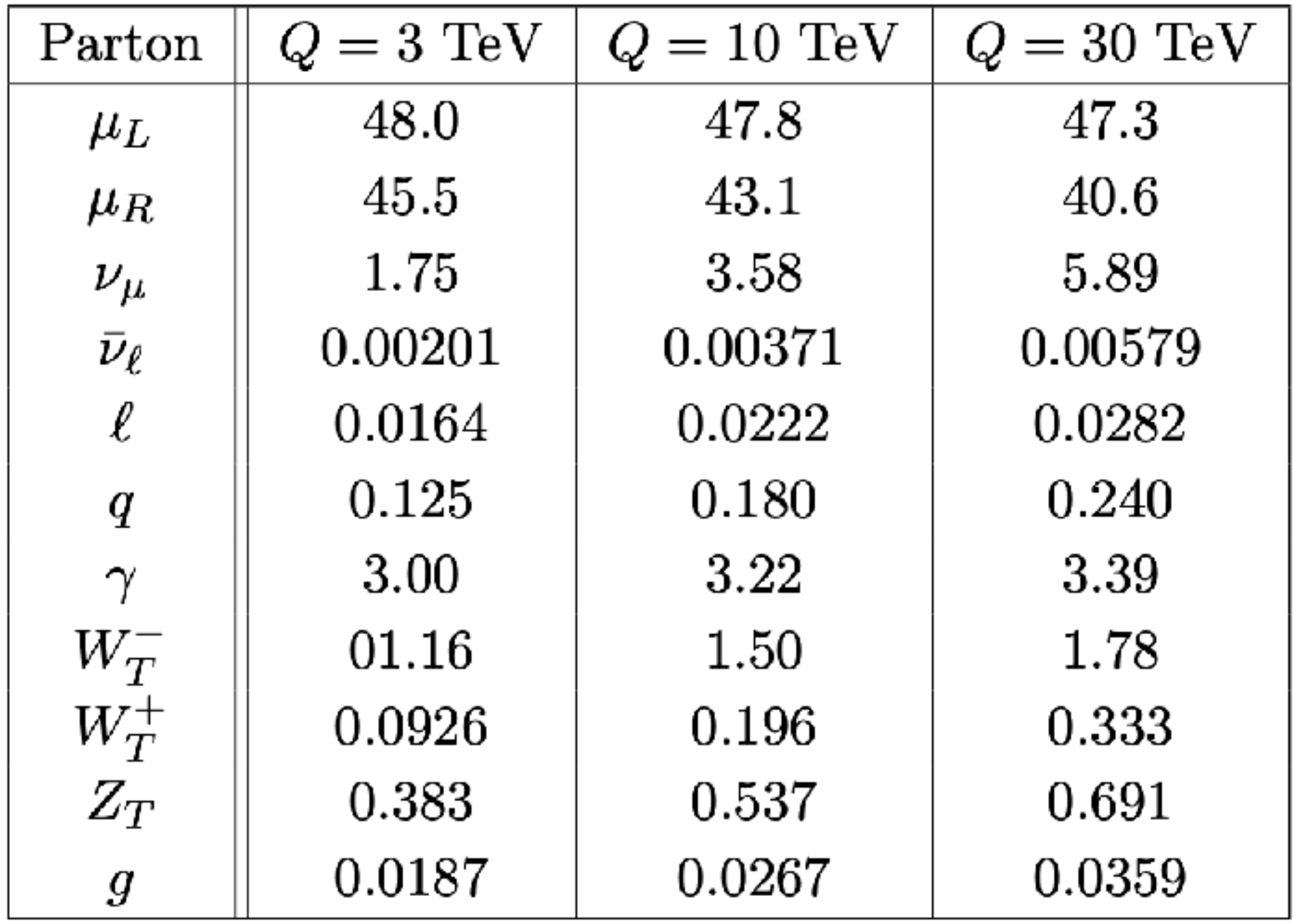
Momentum fractions



Parton	$Q = 3$ TeV	$Q = 10$ TeV	$Q = 30$ TeV
μ_L	48.0	47.8	47.3
μ_R	45.5	43.1	40.6
ν_μ	1.75	3.58	5.89
$\bar{\nu}_\ell$	0.00201	0.00371	0.00579
ℓ	0.0164	0.0222	0.0282
q	0.125	0.180	0.240
γ	3.00	3.22	3.39
W_T^-	01.16	1.50	1.78
W_T^+	0.0926	0.196	0.333
Z_T	0.383	0.537	0.691
g	0.0187	0.0267	0.0359

Table 4. Fraction of the momentum carried by each parton at $Q = 3, 10, 30$ TeV.

Momentum conservation

$$\sum_i \int dx \; x \, f_i(x, Q^2) = 1{,}0037$$

Fermion number conservation

Q = 3 TeV

$$\int dx \left(f_{\ell_L} + f_{\nu_\ell} + f_{\ell_R} - f_{\bar{\ell}_L} - f_{\bar{\nu}_\ell} - f_{\bar{\ell}_R} \right)$$

e: 6×10^{-7}

μ: 1.0018

τ: 6×10^{-7}

$$\int dx \left(f_{u^i_L} + f_{d^i_L} + f_{u^i_R} + f_{d^i_R} - f_{u^i_L} - f_{d^i_L} - f_{u^i_R} - f_{d^i_R} \right)$$

u,d: 1.6×10^{-7}

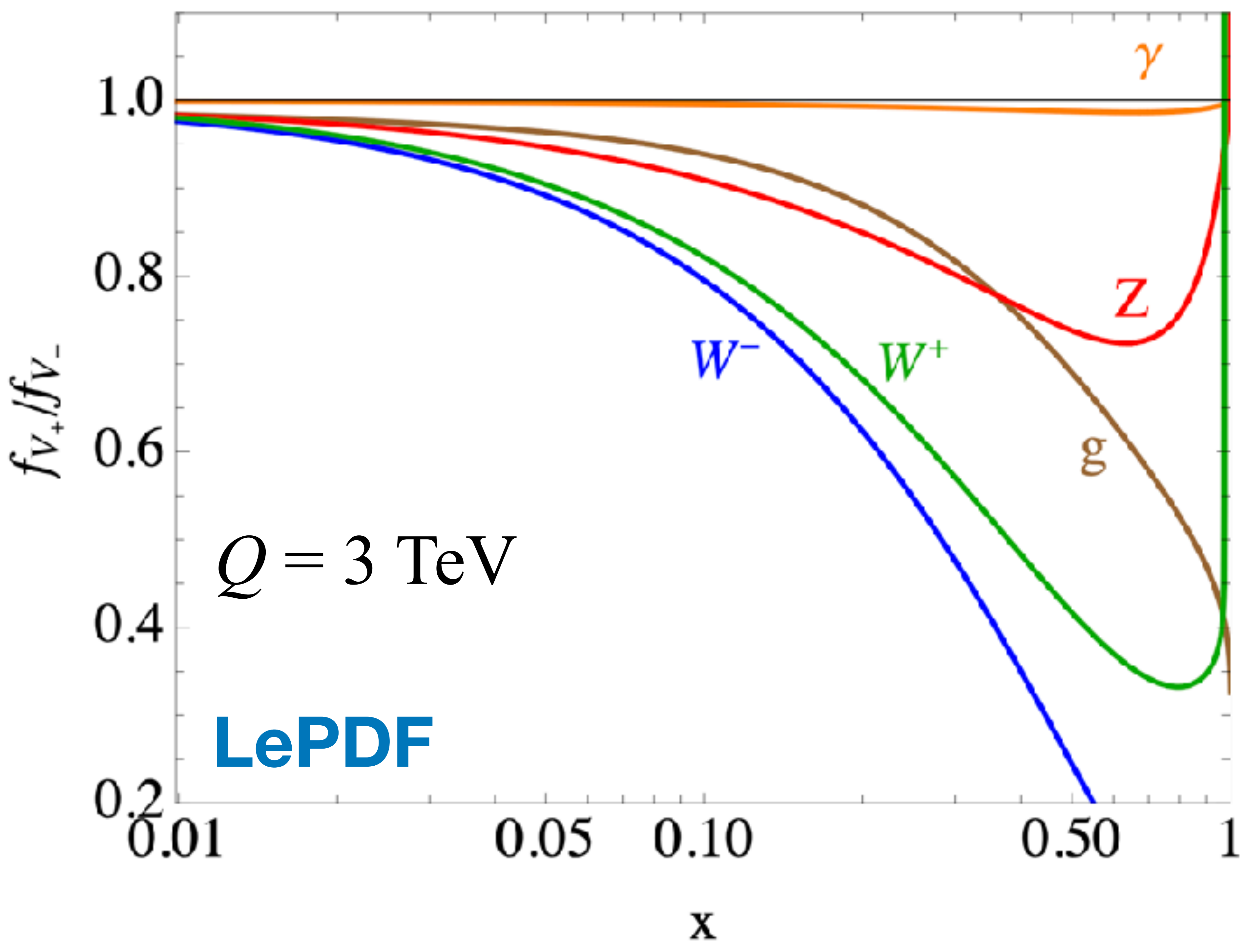
c,s: 1.6×10^{-7}

t,b: 4×10^{-5}

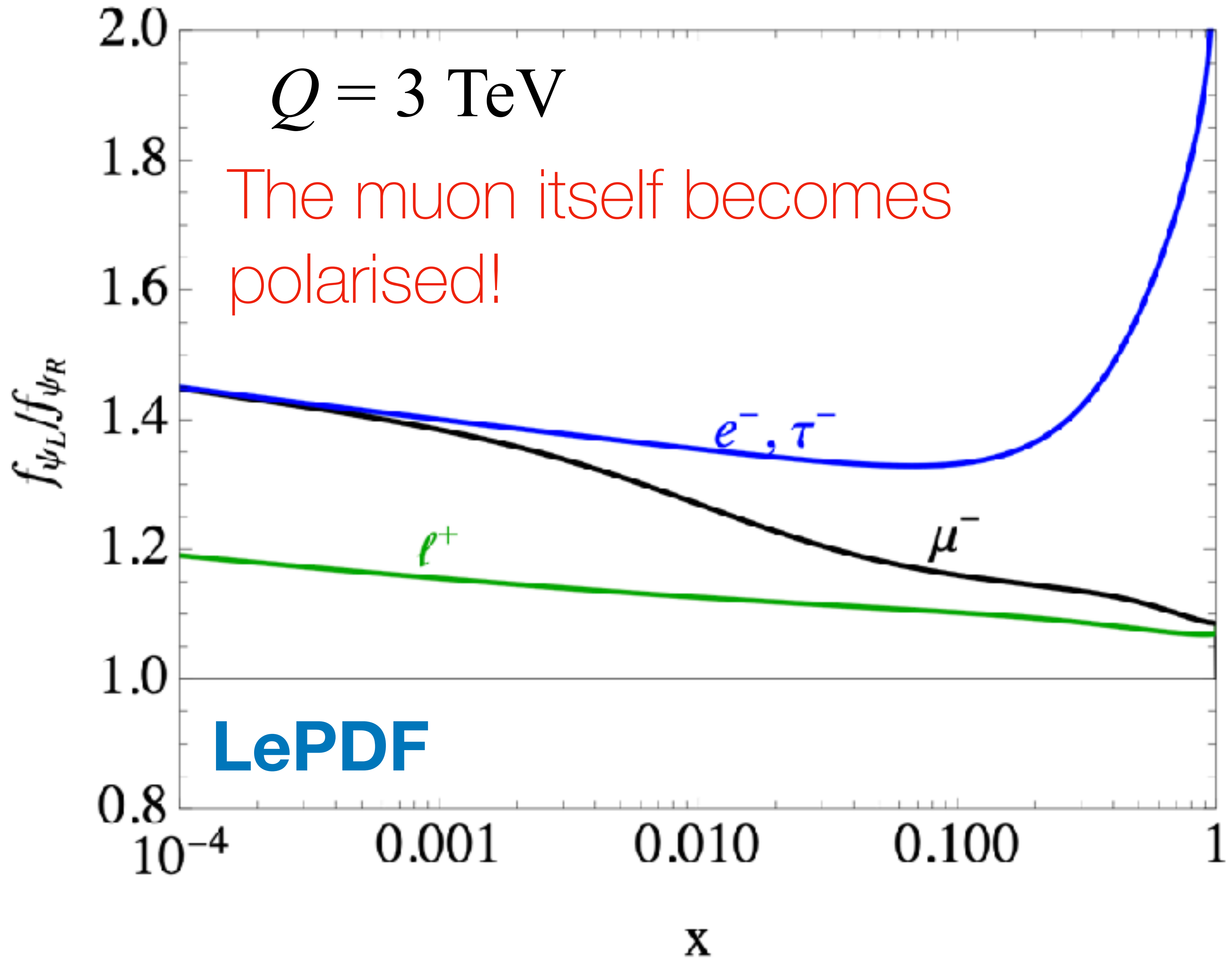
Polarisation

Since EW interactions are chiral, PDFs become polarised. Bauer, Webber [1808.08831]

Vectors polarisation: V_+ / V_-



Fermions polarisation: ψ_L / ψ_R



The muon itself becomes polarised!

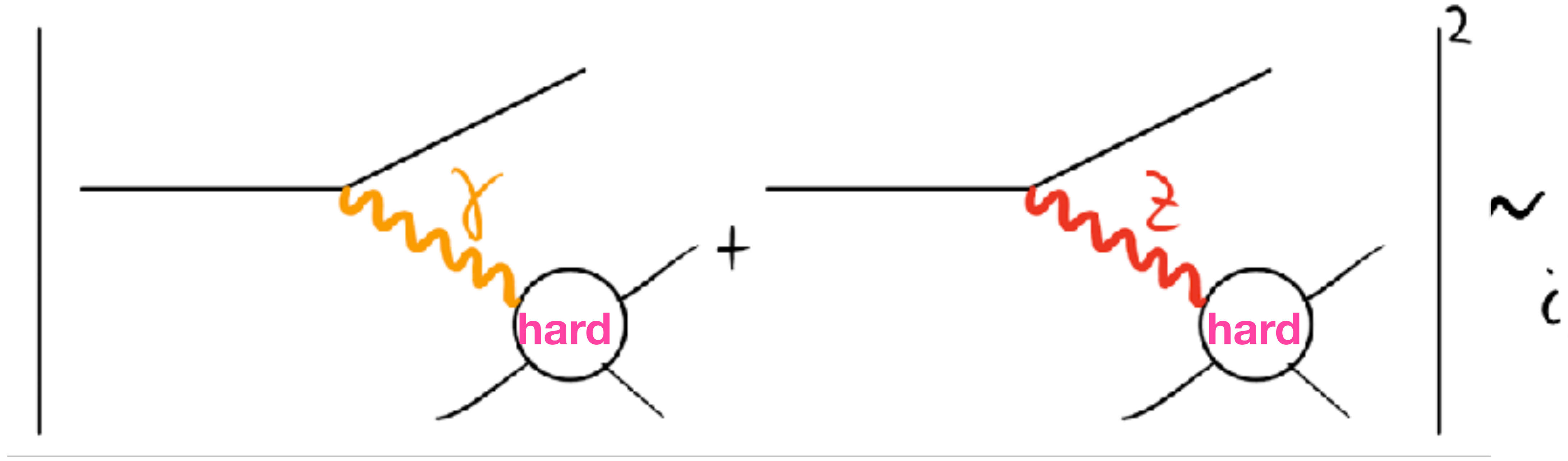
O(1) polarisation effects! (except for photon PDF)

E.g. in case of W^- PDF, coupled to μ_L, the PDF for RH W's goes to zero for $x \to 1$ faster than LH W's, since $P_{V+f_L}(z) = (1-z)/z$ while $P_{V-f_L}(z) = 1/z$.

Photon - Z mixing PDF

Factorization takes place at amplitude level.

$$i\mathcal{M}(AX \to BY) = \sum_{C} i\mathcal{M}^{\mathrm{hard}}(CX \to Y)\frac{i}{Q^2} i\mathcal{M}^{\mathrm{split}}(A \to BC^*)\left[1 + \mathcal{O}(\delta)\right]$$



$$\sim \sum_{i,j=\gamma,Z} \mathcal{M}_i^{\mathrm{split}} \mathcal{M}_i^{\mathrm{hard}} \mathcal{M}_j^{\mathrm{split}\,*} \mathcal{M}_j^{\mathrm{hard}\,*}$$

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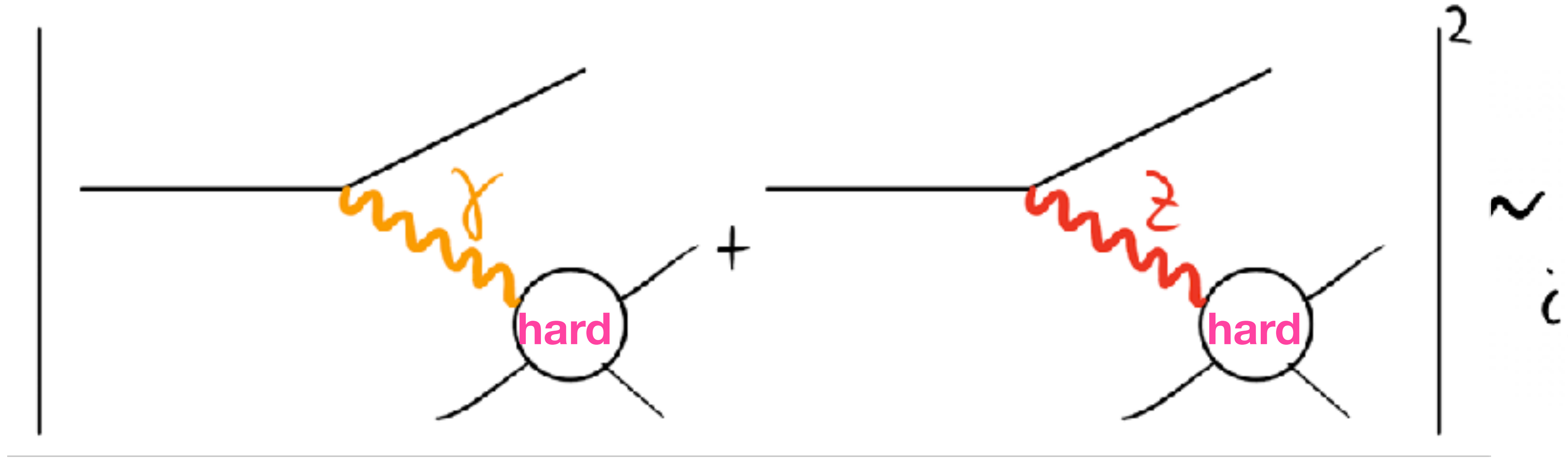

$$\sim \sum_{i,j=\gamma,Z} \mathcal{M}_i^{split}\,\mathcal{M}_i^{hard}\,\mathcal{M}_j^{split\,*}\,\mathcal{M}_j^{hard\,*} \sim \sum_{i,j=\gamma,Z} d\rho_{ij}^{split}\,d\rho_{ji}^{hard}$$

The splitting function is generalised to a **splitting density matrix**, traced with the **density matrix for the hard scattering**:

$$d\sigma \sim \mathrm{Tr}\left[\underbrace{\begin{pmatrix} f_\gamma & f_{Z\gamma} \\ f_{Z\gamma}^* & f_Z \end{pmatrix}}_{d\rho^{split}} \cdot \underbrace{\begin{pmatrix} |\mathcal{M}_\gamma^h|^2 & \mathcal{M}_Z^h \mathcal{M}_\gamma^{h*} \\ \mathcal{M}_\gamma^h \mathcal{M}_Z^{h*} & |\mathcal{M}_Z^h|^2 \end{pmatrix}}_{d\rho^{hard}}\right]$$ up to $O(k_T^2/E^2, m^2/E^2)$

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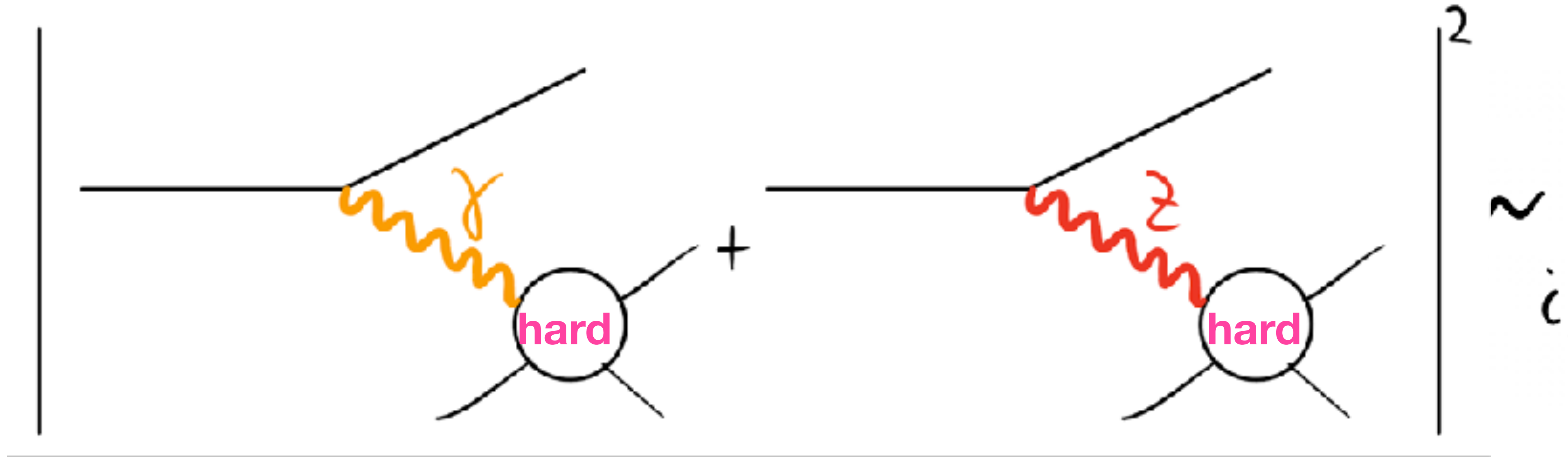

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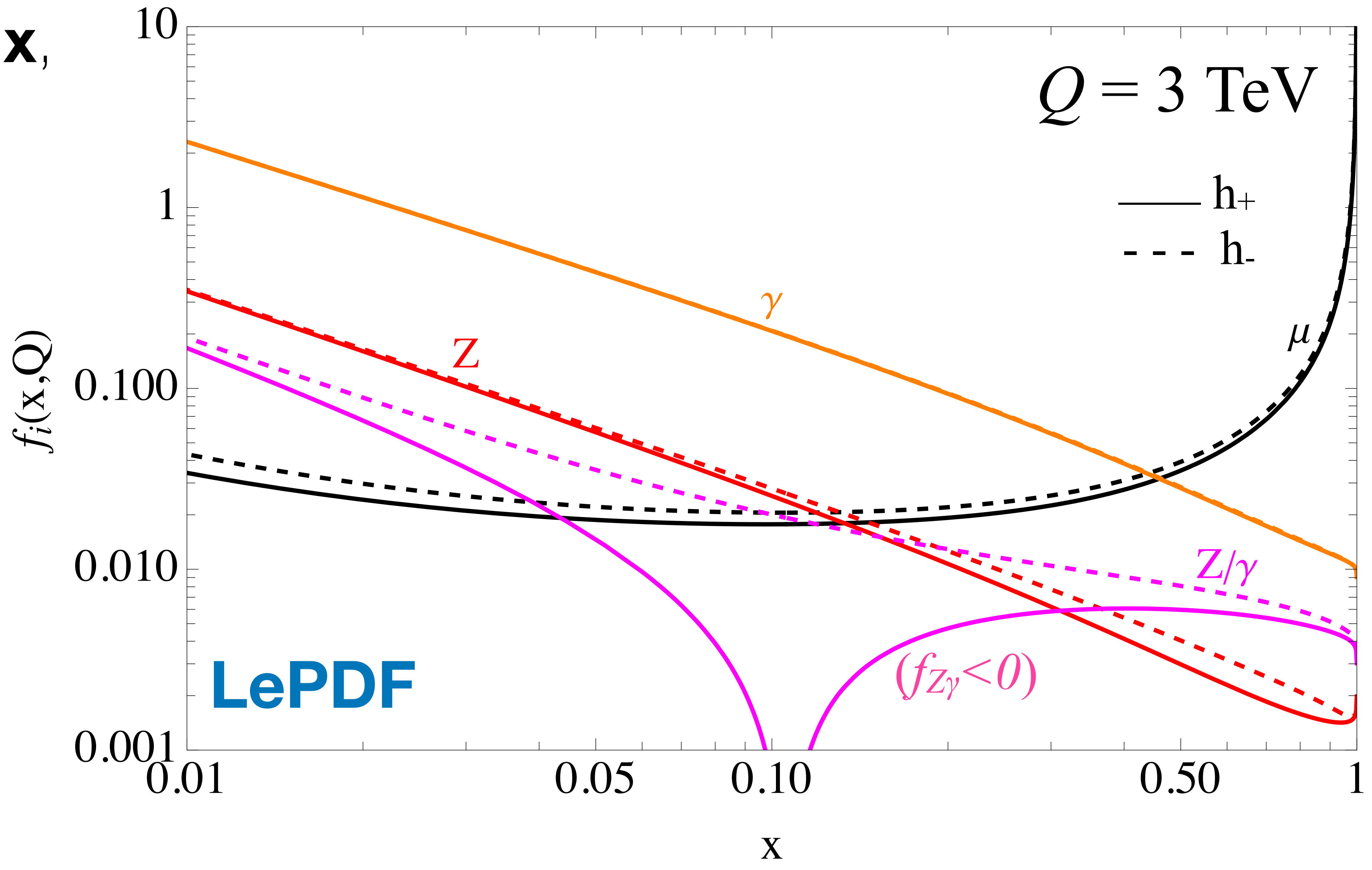
up to $O(k_T^2/E^2, m^2/E^2)$

In the collinear limit this can be described by a **mixed Zγ PDF**. (Similarly also for Z_L and H)

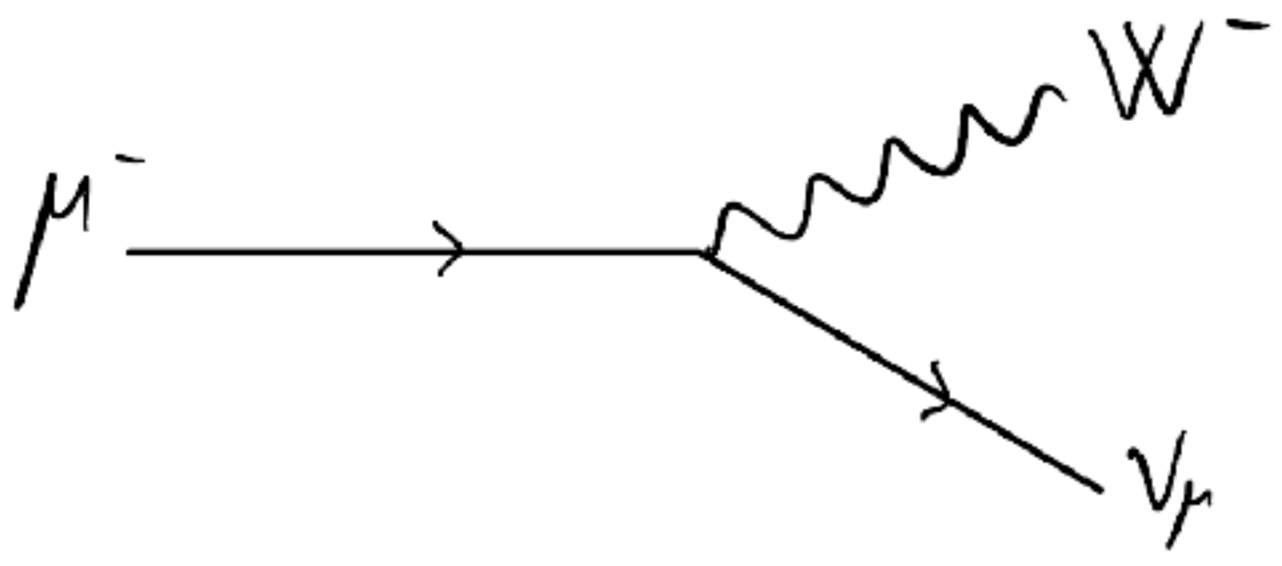
P. Ciafaloni, Comelli [hep-ph/0007096, hep-ph/0505047]
Chen, Han, Tweedie [1611.00788]



Effective Vector Boson Approximation

At energies **above the EW scale**, **collinear emission of EW gauge bosons** can be described at LO with the **Effective Vector Boson Approximation**

Fermi ('24) Weizsacker, Williams ('34) Landau, Lifschitz ('34) Kane, Repko, Rolnik; Dawson; Chanowitz, Gaillard '84, See also Borel et al. [1202.1904], Costantini et al. [2005.10289] Ruiz et al. [2111.02442], etc…



Including W-mass effects:

$$f_{W^-_\pm}^{(\alpha)}(x,Q^2) = \frac{\alpha_2}{8\pi}P^f_{V_\pm f_L}(x)\left(\log\frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2}\right)$$

$$f_{W^-_L}^{(\alpha)}(x,Q^2) = \frac{\alpha_2}{4\pi}\frac{1-x}{x}\frac{Q^2}{Q^2+(1-x)m_W^2}$$

(similar expressions also for Z_T, Z_L, Z/γ)

For $Q \gg m_W$:

$$f_{W^-_\pm}^{(\alpha)}(x,Q^2) \approx \frac{\alpha_2}{8\pi}P^f_{V_\pm f_L}(x)\log\frac{Q^2}{m_W^2}$$

← This one is now implemented in `MadGraph5_aMC@NLO` [Ruiz, Costantini, Maltoni, Mattelaer 2111.02442]

NOTE: mass effects remain of O(1) also at TeV scale! Chen, Han, Tweedie [1611.00788]

Do we need SM/EW PDFs?

Collinear factorisation works if $p_T,\ m_W \ll E_{hard}$, so it can be **viable for a 3 TeV MuC**.

Particularly **useful for processes well below threshold** $E_{hard} \ll E_{collider}$ (e.g. production of EW final states).

The $W,\ Z$ PDFs are suppressed compared to the photon one by a factor $\sim \log m_W{}^2/m_\mu{}^2 \sim \mathrm{O}(10)$.
Nevertheless, they induce the **dominant contribution in a large class of processes** (vector boson collider).

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Why not just EVA?

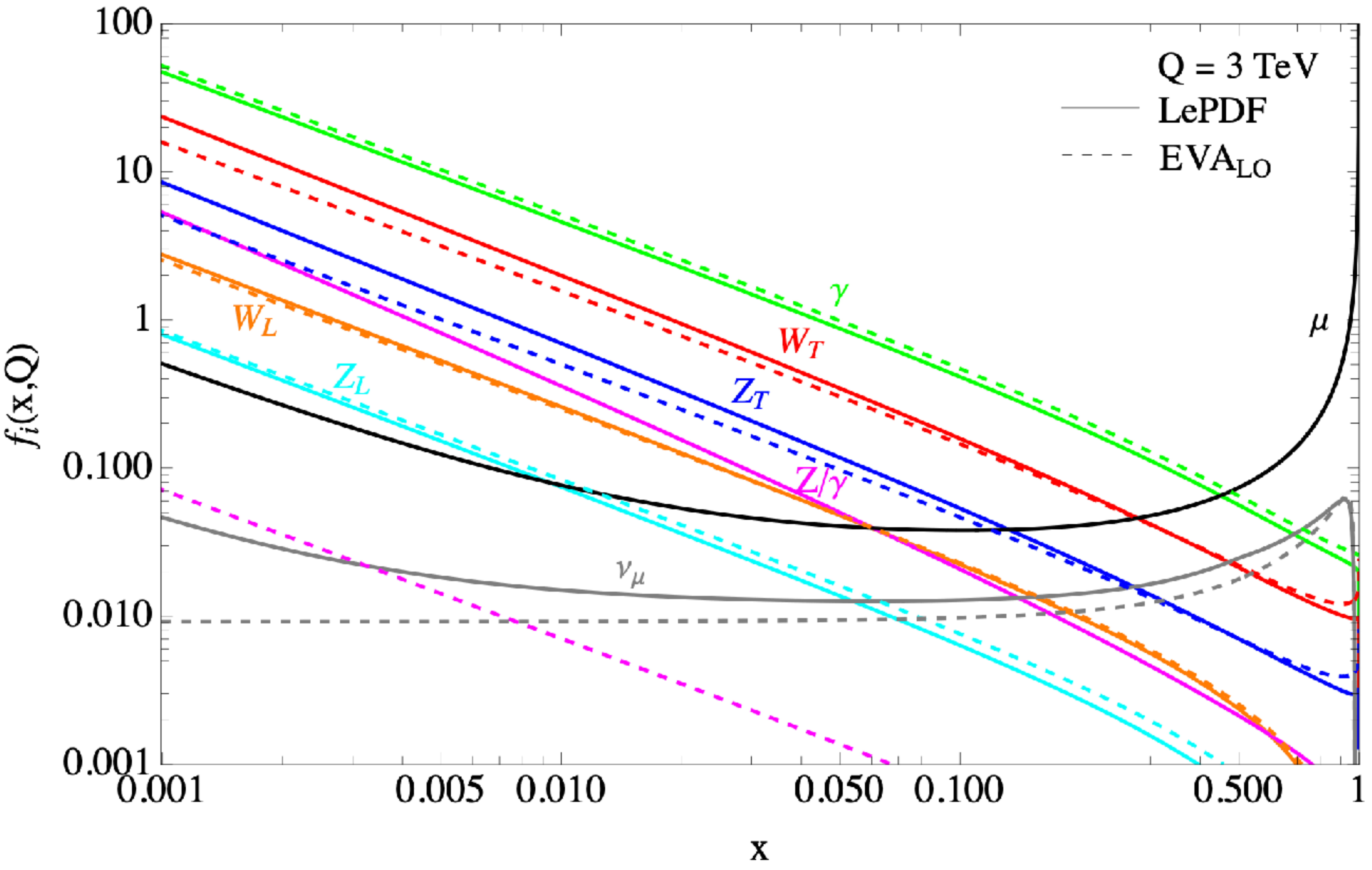
- For **QCD (gluon and quarks) DGLAP resummation is required** since α_s is large at small scales.
- The expected **relative corrections to the LO EVA** result are proportional to (*Sudakov double logs*) $\alpha_2 \left(\log \frac{Q^2}{M_W^2}\right)^2 \sim 1$ for $Q \sim 1.5\ \mathrm{TeV}$. still sizeable at lower Q.

For precise vector boson PDFs at the TeV scale it is important to re-sum the EW double logs.

→ **PDF approach**

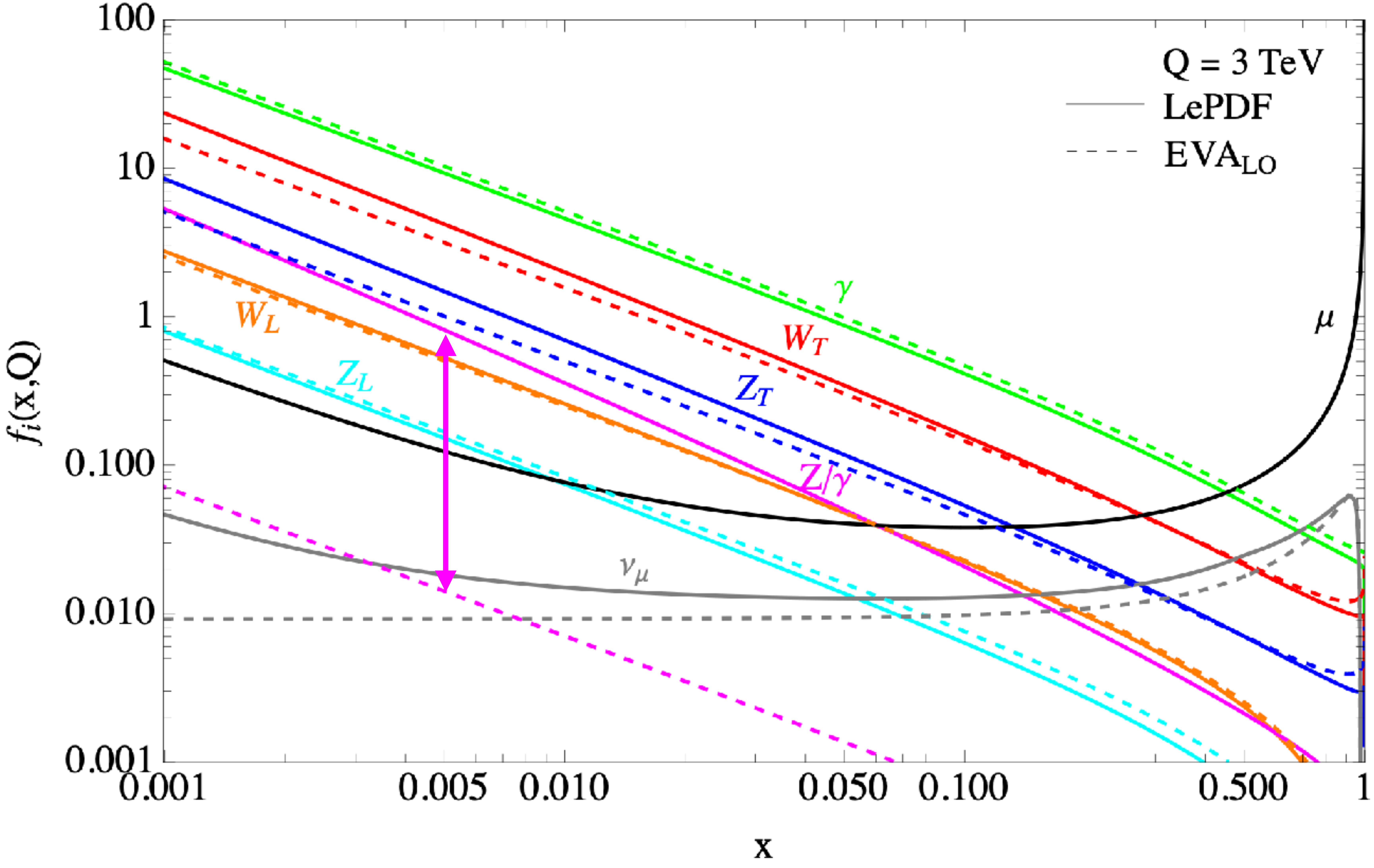
M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]

LePDF vs. EVA


Q = 3 TeV
LePDF
EVA$_{LO}$
γ
W_T
Z_T
W_L
Z_L
Z/γ
ν_μ
μ
f_i(x,Q)
x
100
10
1
0.100
0.010
0.001
0.001
0.005
0.010
0.050
0.100
0.500
1

LePDF vs. EVA

$$f^{(\alpha)}_{Z/\gamma\pm}(x,Q^2) = -\frac{\sqrt{\alpha_\gamma\alpha_2}}{2\pi c_W}\left(P^f_{V_\pm f_L}(x)Q^Z_{\mu_L} + P^f_{V_\pm f_R}(x)Q^Z_{\mu_R}\right)\log\frac{Q^2+(1-x)m_Z^2}{m_\mu^2+(1-x)m_Z^2}$$



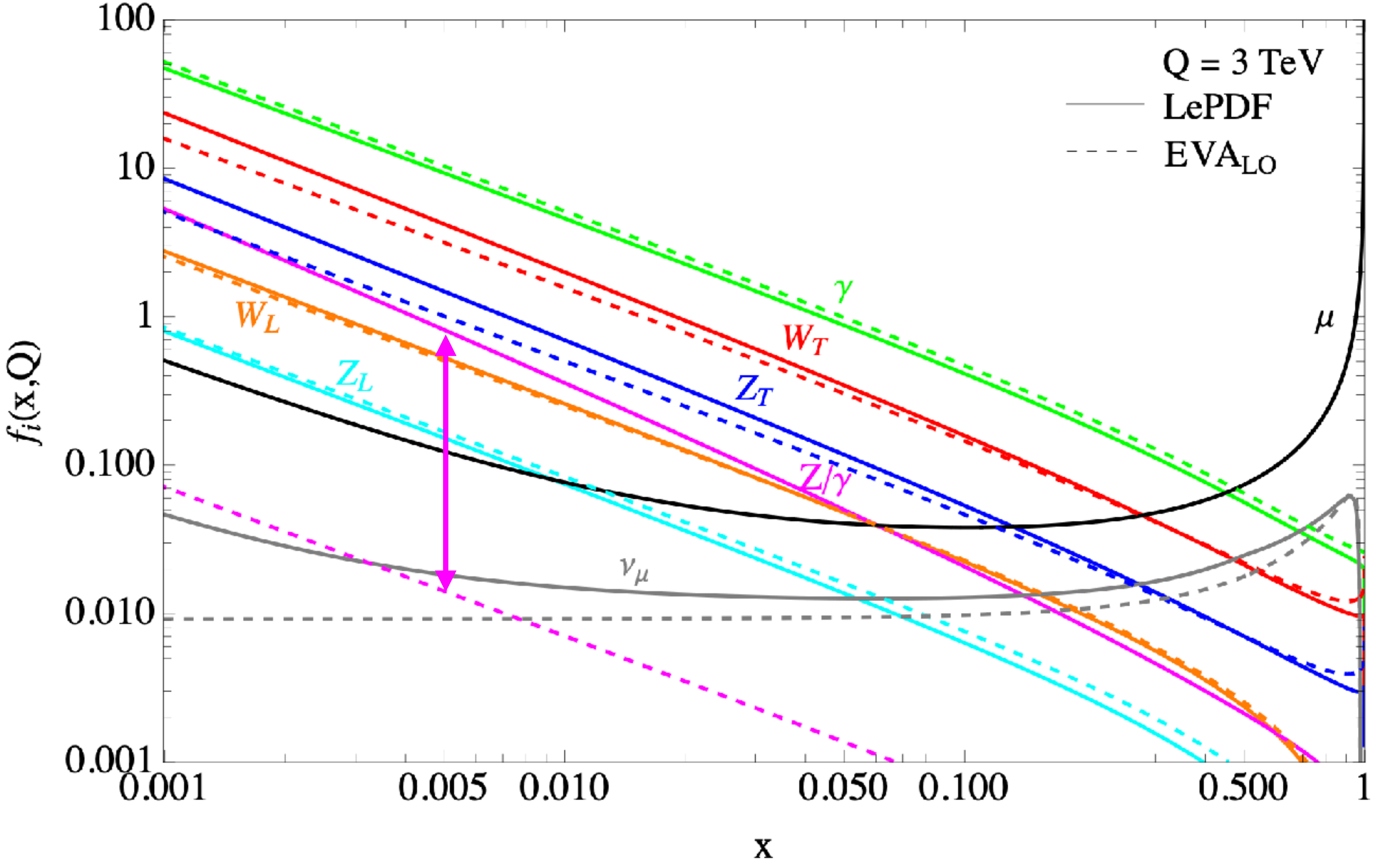
- The **EVA Z/γ PDF is off by ~10²**, due to the fact that in EVA the muon is taken unpolarised and

$$Q^Z_{\mu_L} + Q^Z_{\mu_R} = -\tfrac{1}{2} + 2s^2_W \ll 1$$

Instead, the muon gains a O(1) polarisation, so the actual Z/γ PDF is much larger.

LePDF vs. EVA

$$f^{(\alpha)}_{Z/\gamma\pm}(x,Q^2) = -\frac{\sqrt{\alpha_\gamma\alpha_2}}{2\pi c_W}\left(P^f_{V_\pm f_L}(x)Q^Z_{\mu_L} + P^f_{V_\pm f_R}(x)Q^Z_{\mu_R}\right)\log\frac{Q^2+(1-x)m_Z^2}{m_\mu^2+(1-x)m_Z^2}$$



◆ The **EVA Z/γ PDF is off by ~10^2**, due to the fact that in EVA the muon is taken unpolarised and

$$Q^Z_{\mu_L} + Q^Z_{\mu_R} = -\frac{1}{2} + 2s_W^2 \ll 1$$

Instead, the muon gains a O(1) polarisation, so the actual Z/γ PDF is much larger.

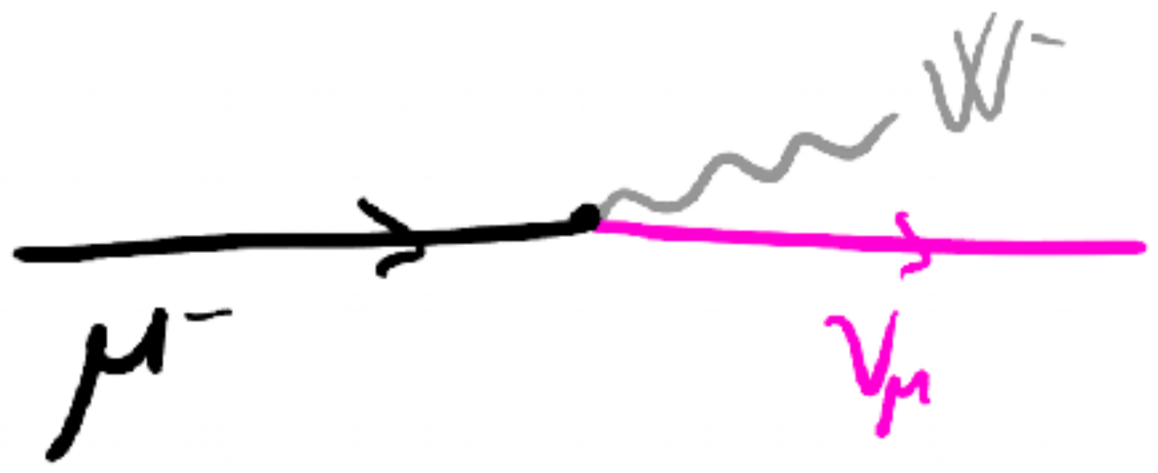
We can also see a **sizeable deviation** (in this log-log plot) for the **W_T** and **Z_T** PDF.

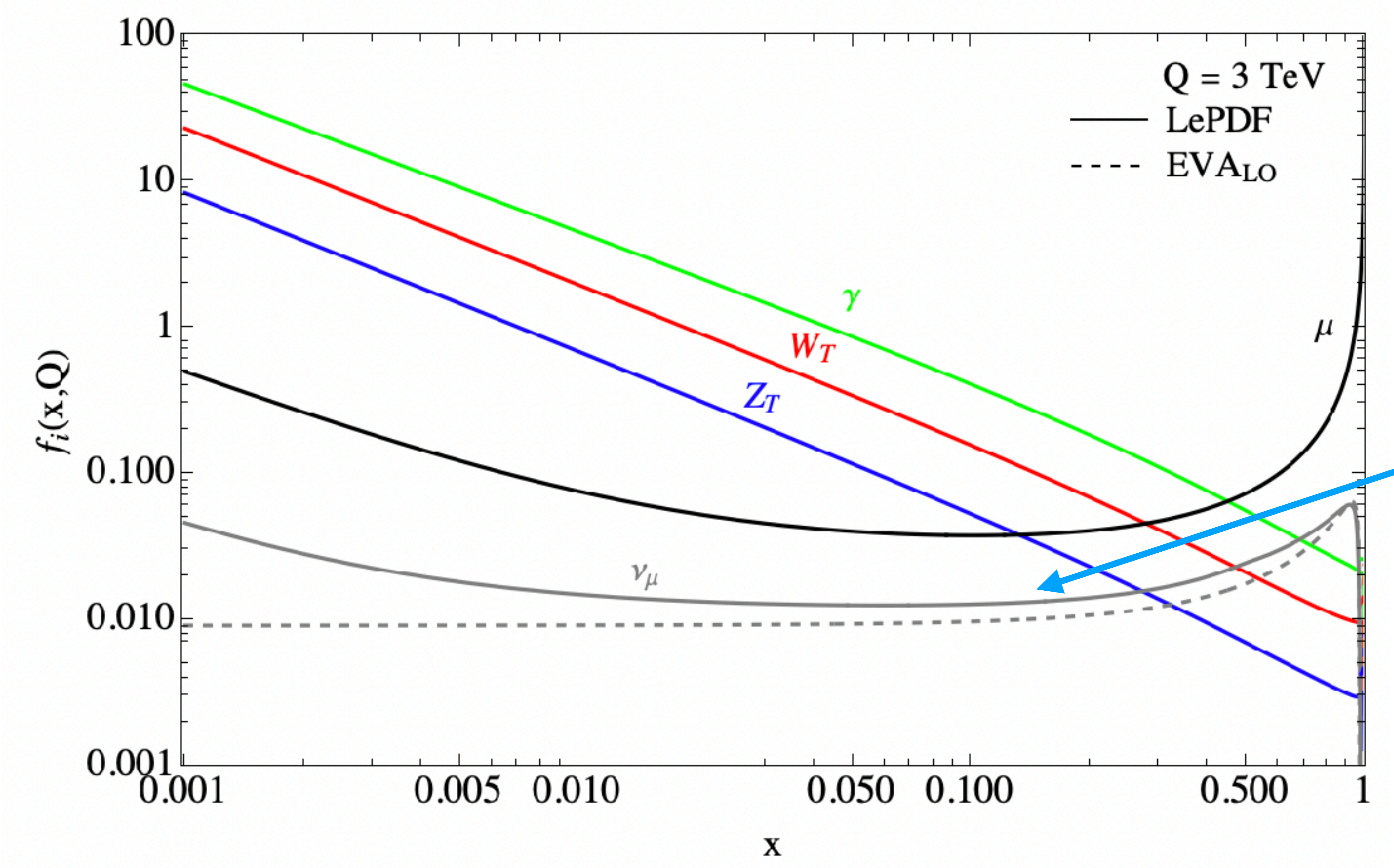
Mostly due to the double-log arising at O(α^2) from VV interactions.

Applications of LePDF … beyond EVA

Muon Neutrino PDF

Emission of **collinear W^- from the muon** generates a **l** **muon neutrino content inside of the muon**.





Particularly **large at x ≳ 0.3** due to the IR divergence of the $\mu \to W\, \nu_\mu$ splitting

Muon Neutrino PDF from LePDF

Muon Neutrino PDF

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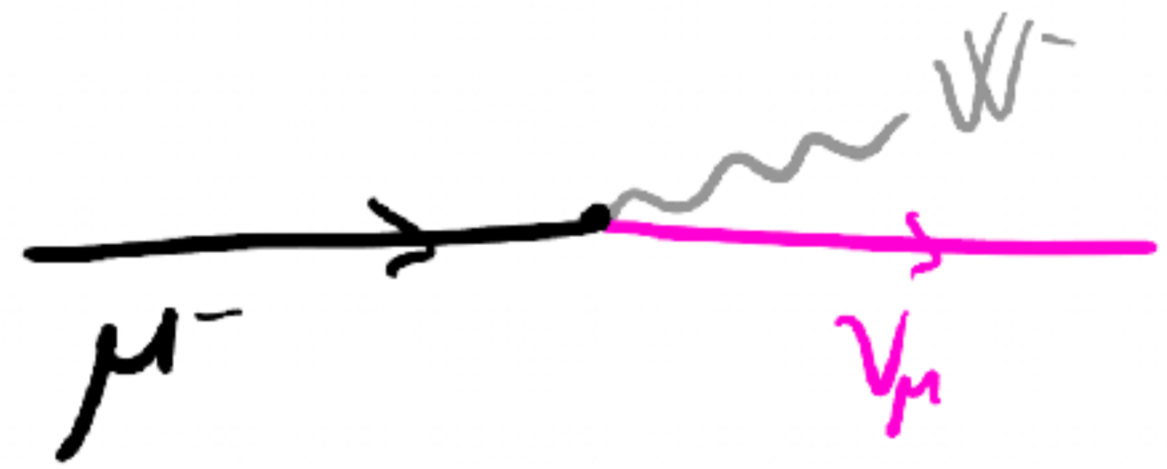
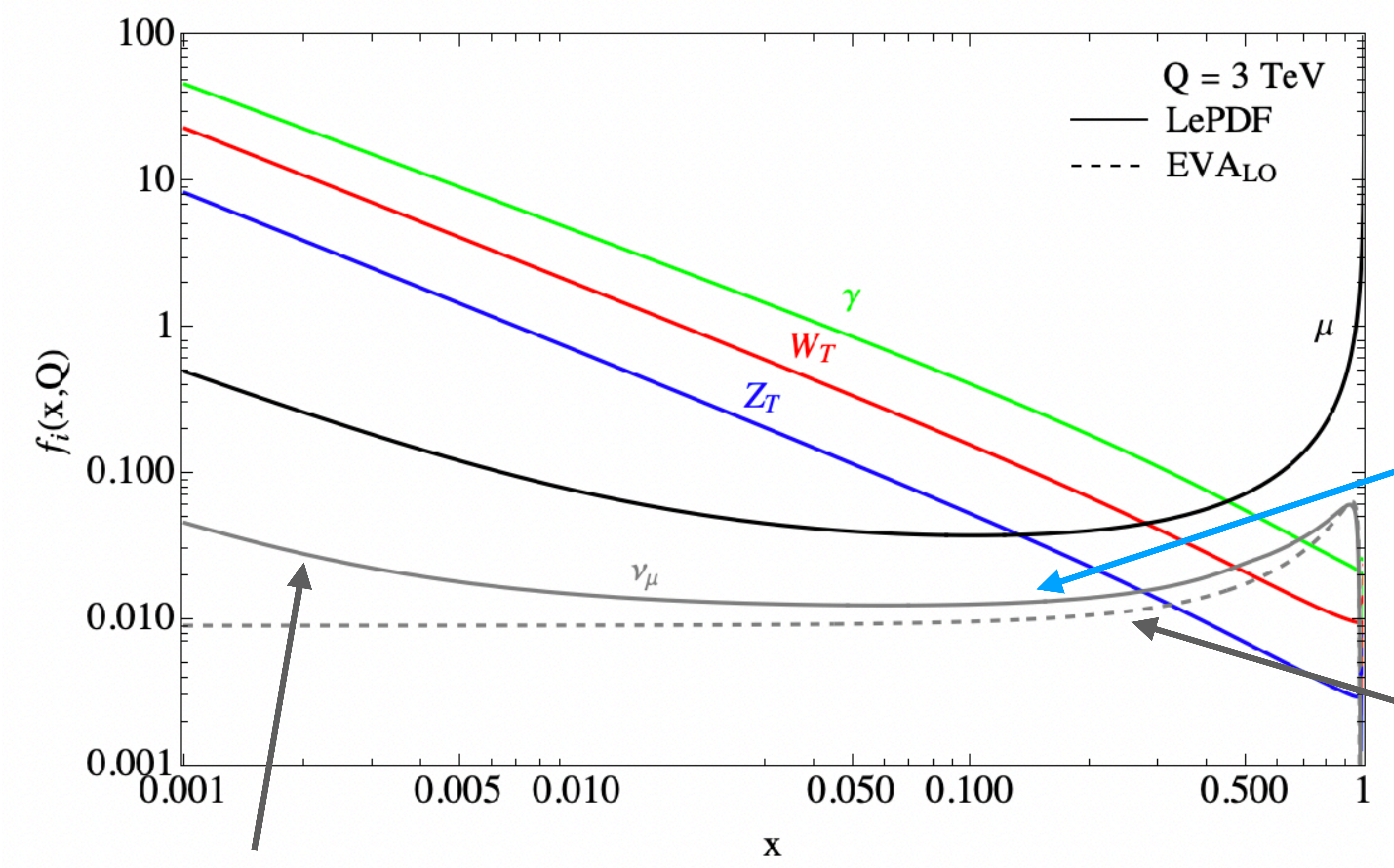




Particularly **large at x ≳ 0.3** due to the IR divergence of the $\mu \longrightarrow W\, \nu_\mu$ splitting

Muon Neutrino PDF from LePDF

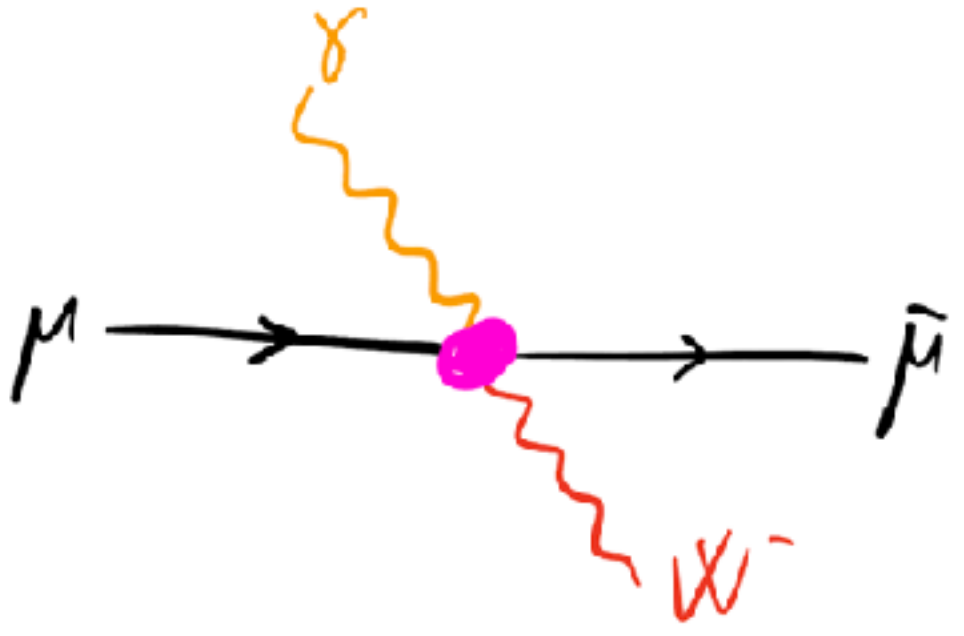
We can compute the **ν_μ PDF at $O(\alpha)$** (as for EVA)

$$f^{(\alpha)}_{\nu_\mu}(x,Q^2) = \frac{\alpha_2}{8\pi}\,\theta\left(Q^2 - \frac{m_W^2}{(1-x)^2}\right) P^V_{ff}(x)\left(\log\frac{Q^2 + x m_W^2}{m_W^2} + \right.$$
$$\left. + \log\frac{(1-x)^2}{1+x(1-x)^2} + \frac{x m_W^2}{Q^2 + x m_W^2} + \frac{1}{1+x(1-x)^2} - 1\right)$$

Here $Z \longrightarrow \overline{\nu}_\mu\, \nu_\mu$ dominates $O(\alpha^2)$

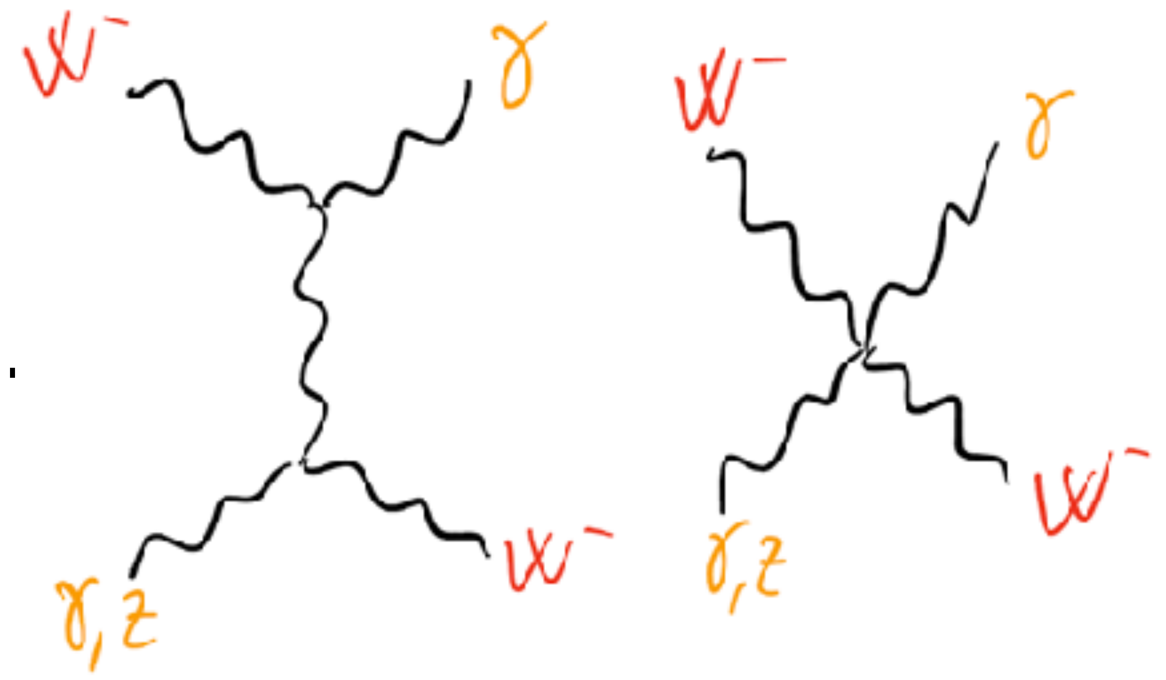
f_{ν_μ} in Wγ production

[work in progress with F. Garsosi, R. Capdevilla, B. Stechauner]



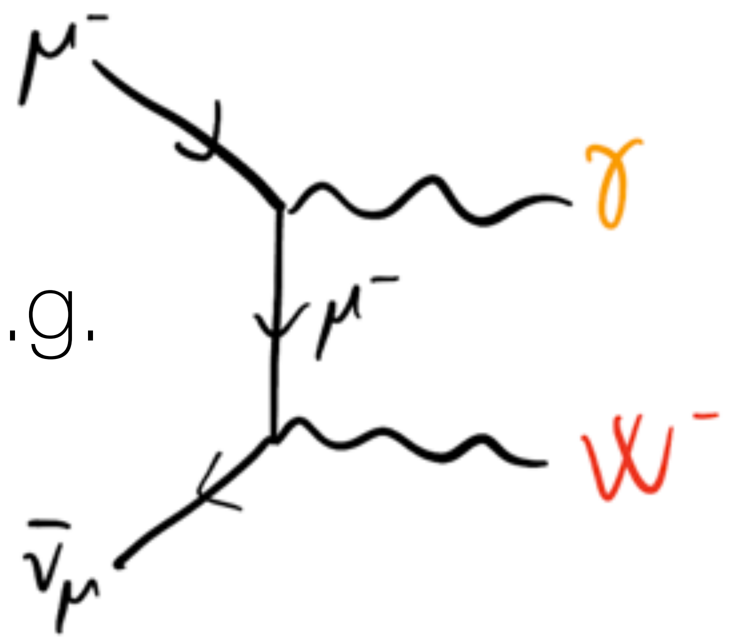
Dominant contributions from VBF

e.g.



But also contribution from the neutrino PDF

e.g.



$$\sigma(\mu\bar{\mu} \to C + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{ij} f_i(x_1,\mu)\, f_j(x_2,\mu)\, \hat{\sigma}(ij \to C)(\hat{s})$$

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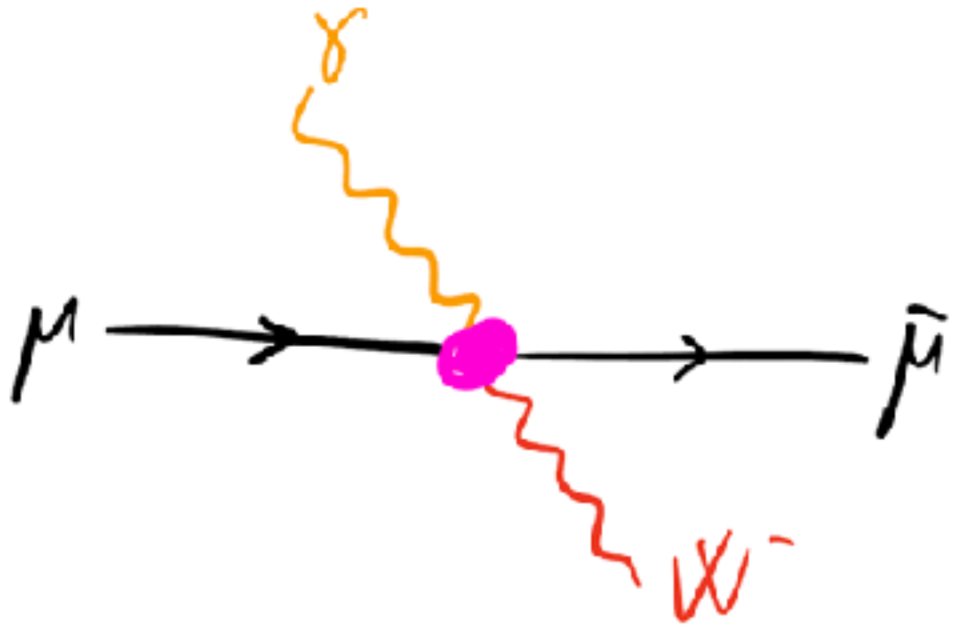

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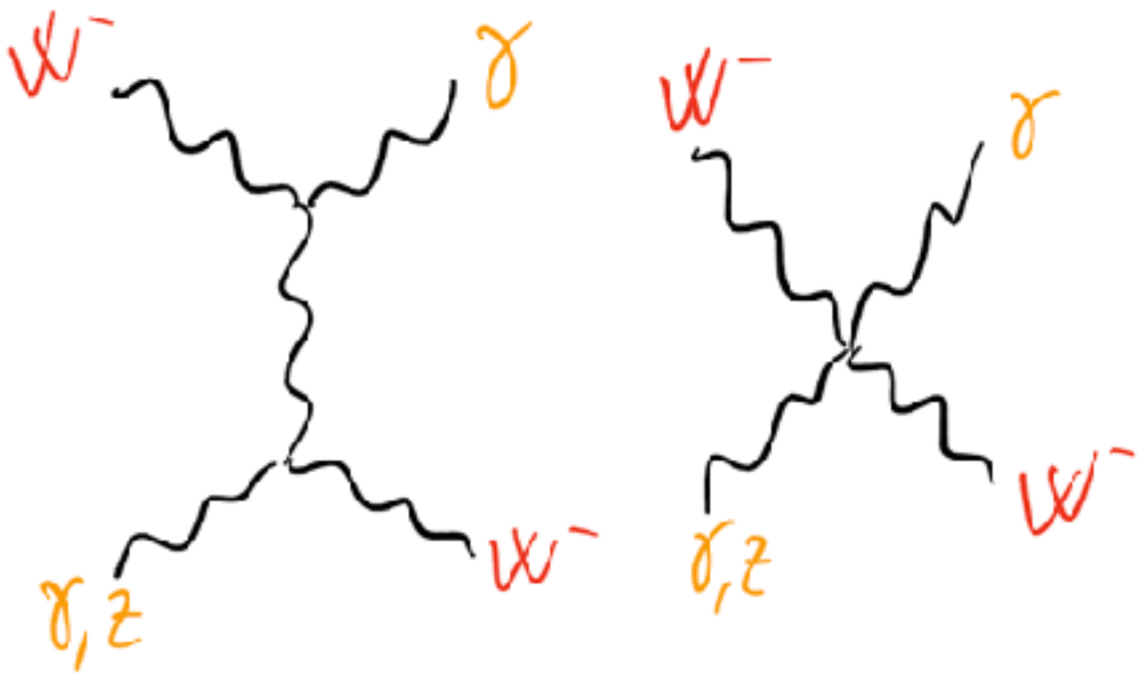

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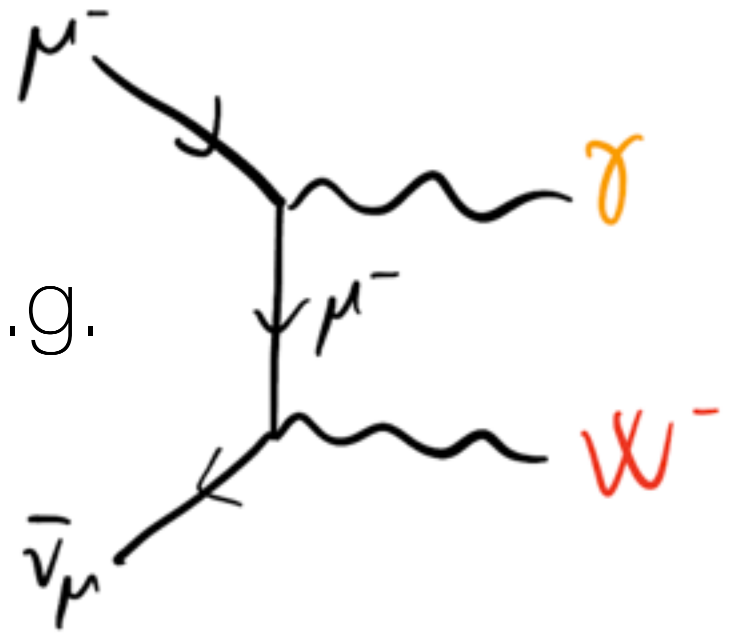

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@ 10 TeV Muon Collider

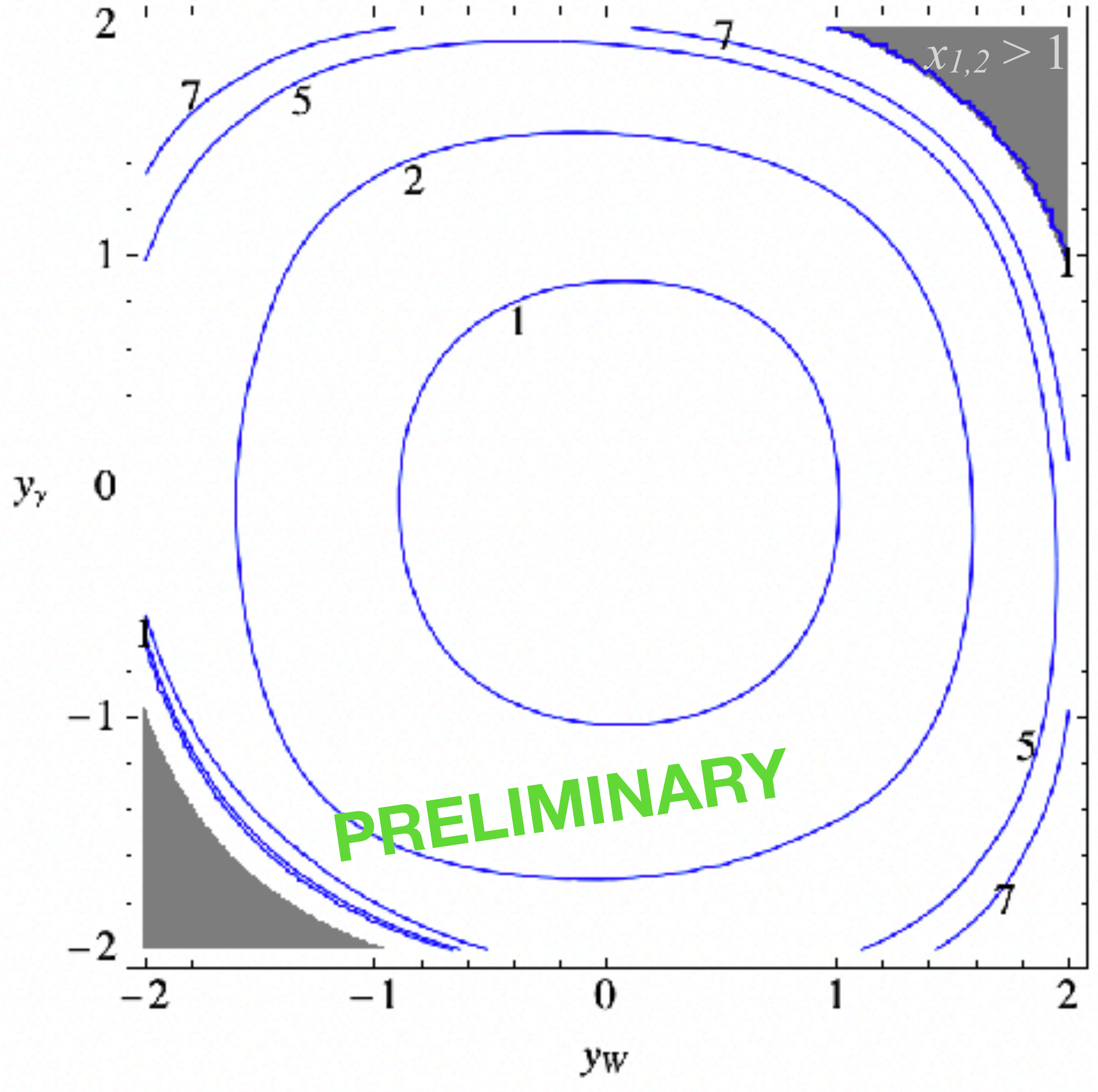
Differential cross section in:

y_γ, y_W, $p_T{}^\gamma$

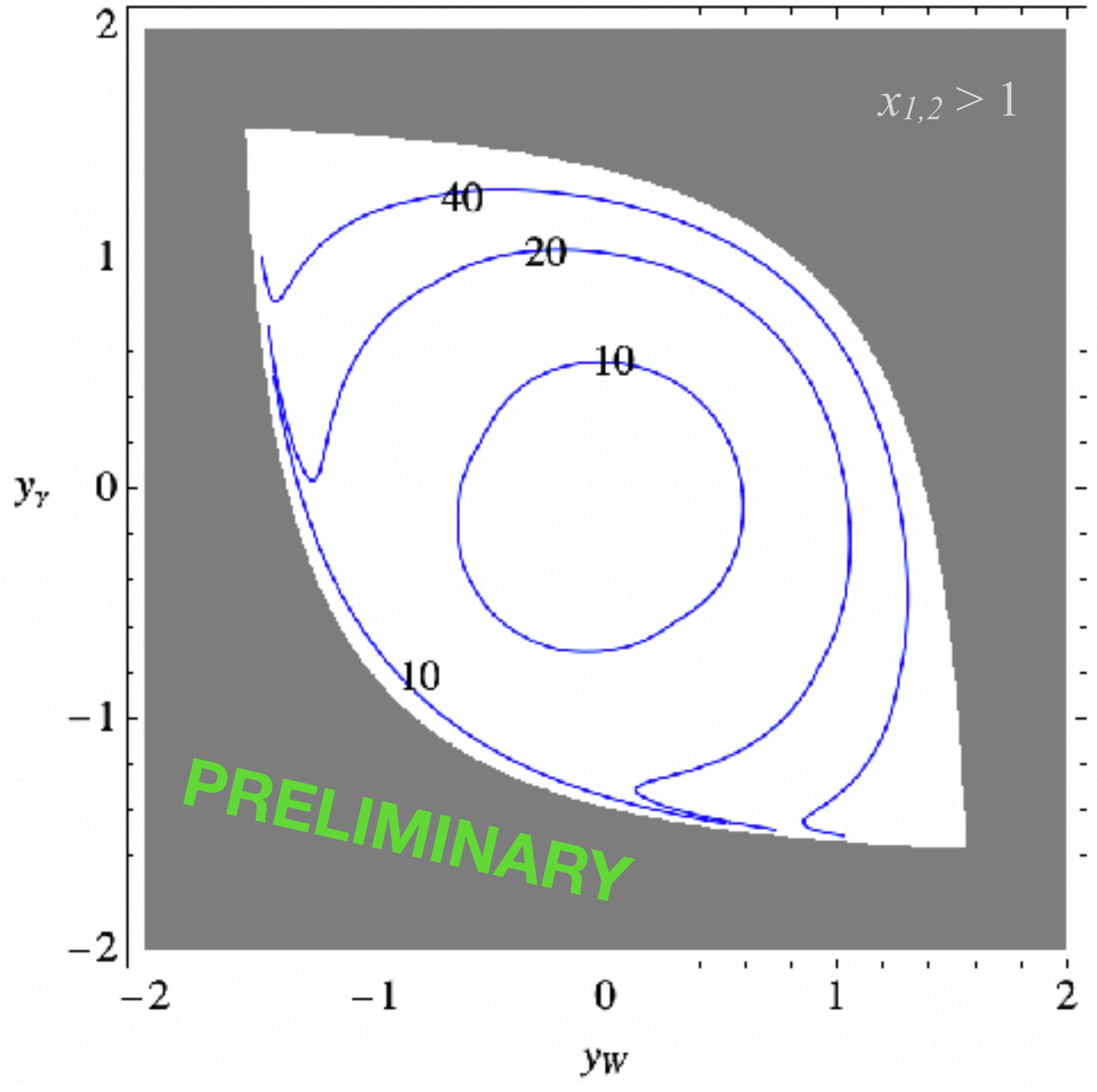
We plot:

$$R_\mu \equiv \frac{\sigma_{\mu\nu}}{\sigma_{FULL}} \quad [\%]$$

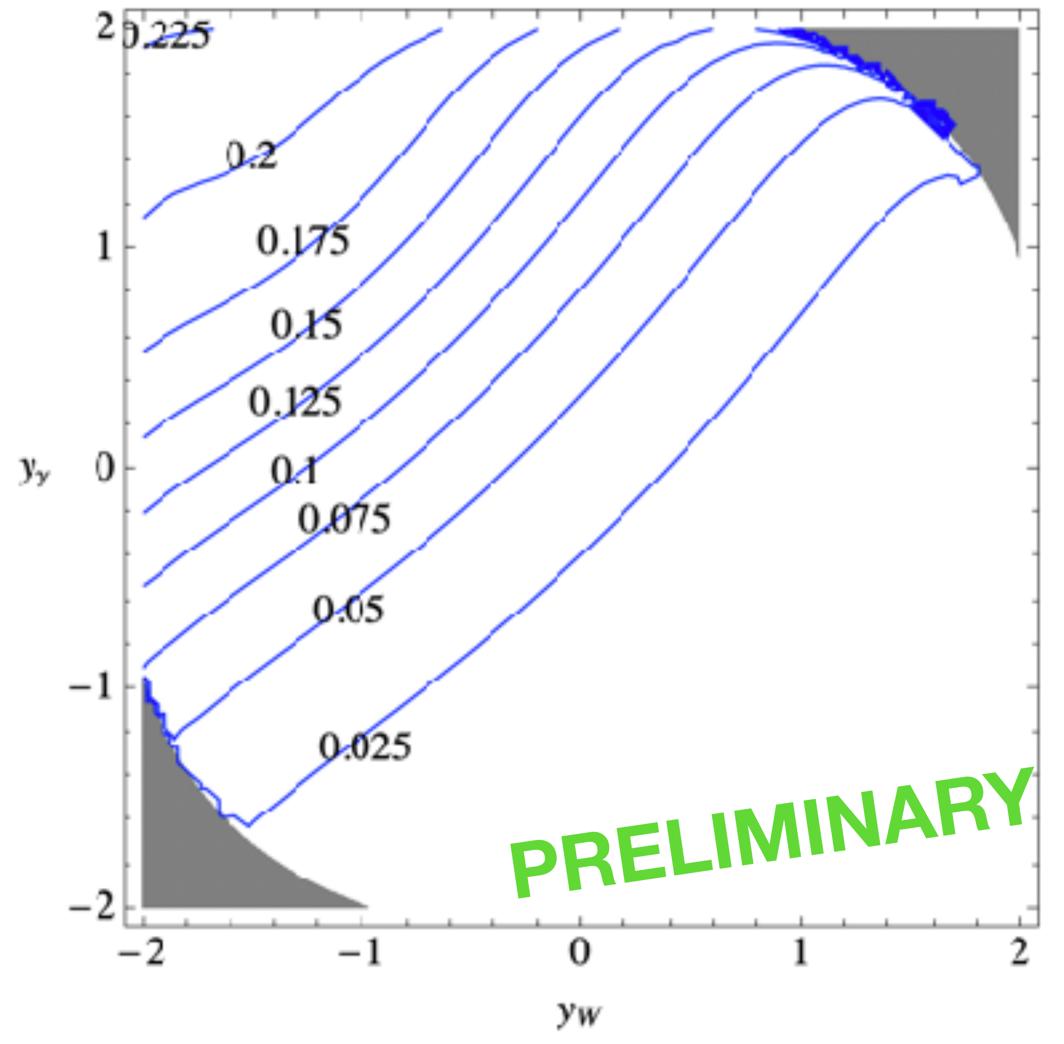
$p_T{}^\gamma = 1\text{TeV}$



$p_T{}^\gamma = 2\text{TeV}$



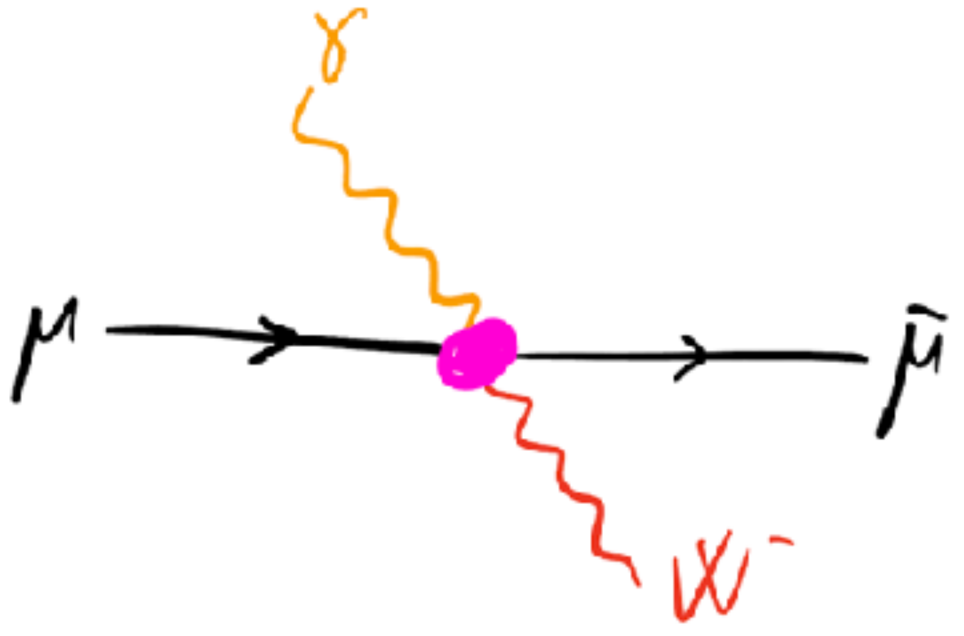
Differential xsec [ab/GeV]



Thanks to F. Garosi for the plots!

f_{ν_μ} in Wγ production

[work in progress with F. Garsosi, R. Capdevilla, B. Stechauner]



Dominant contributions from VBF

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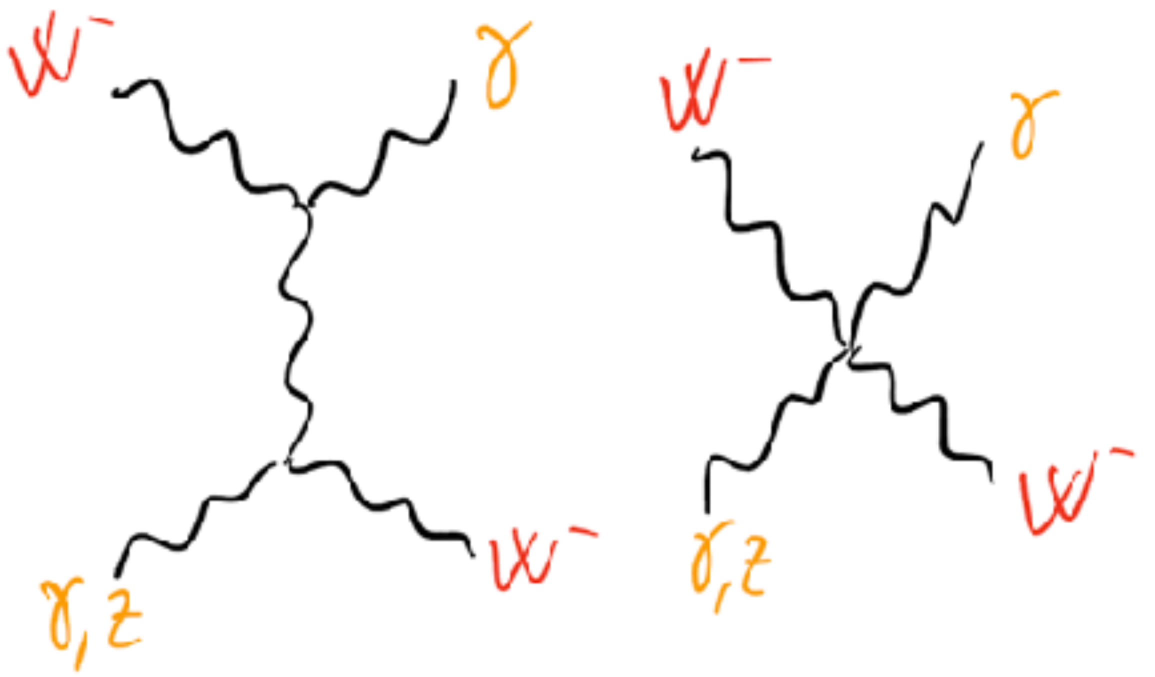

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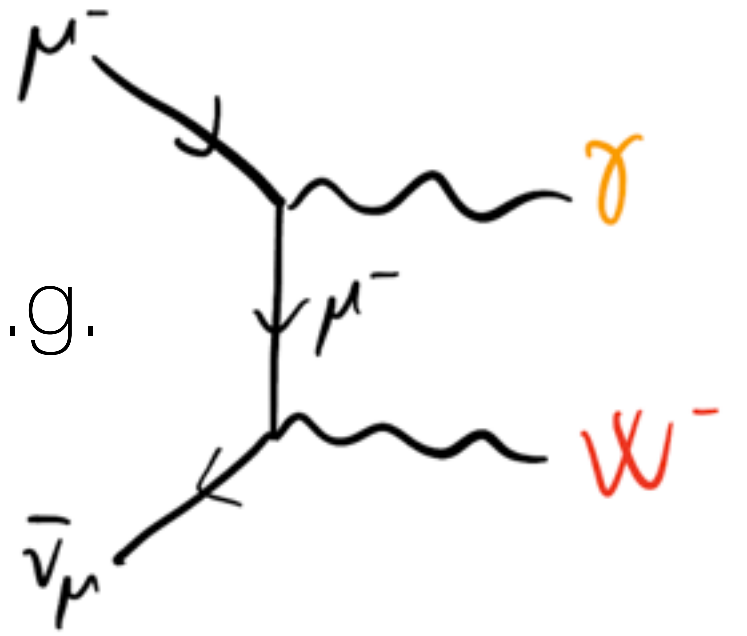

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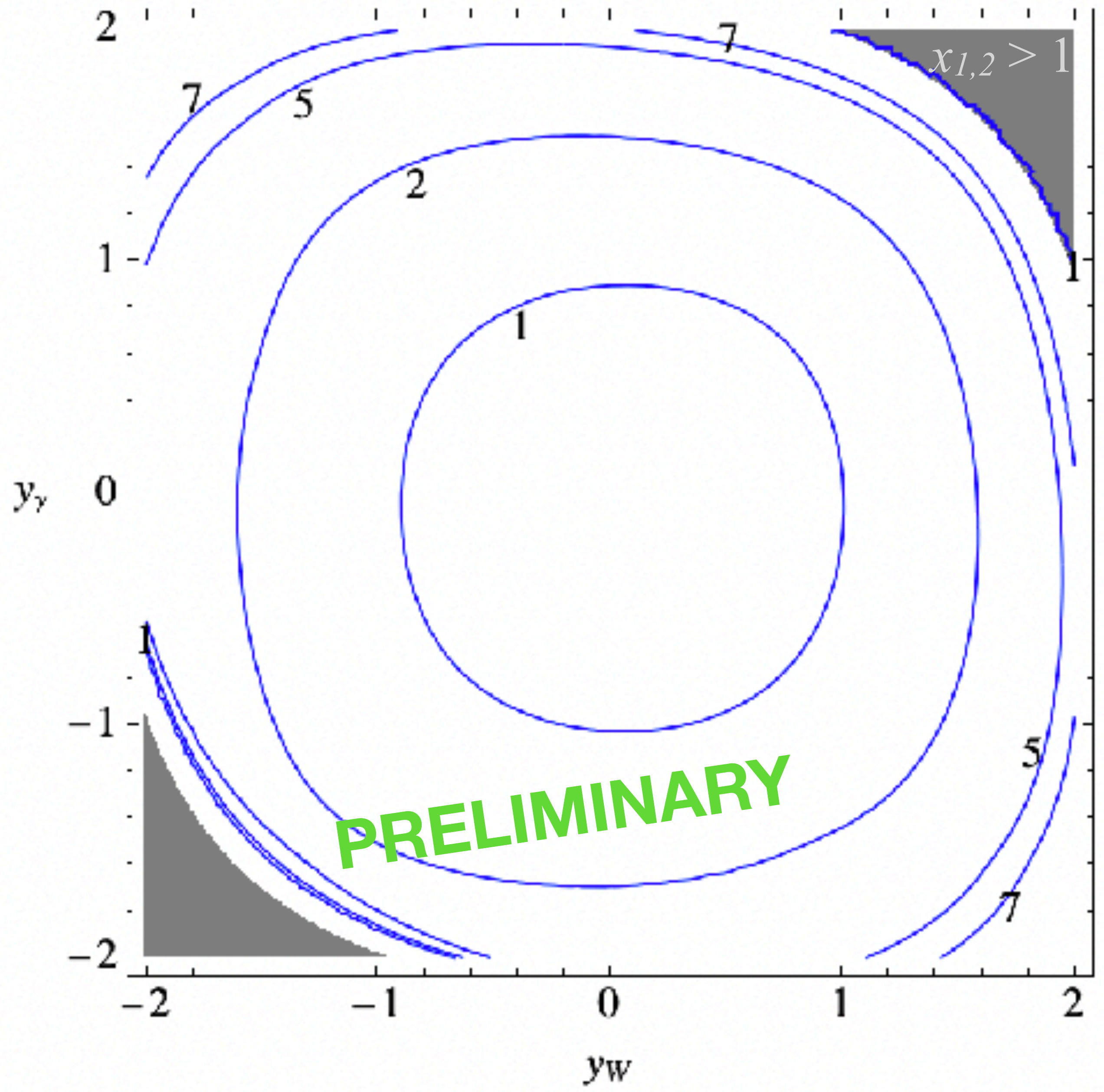
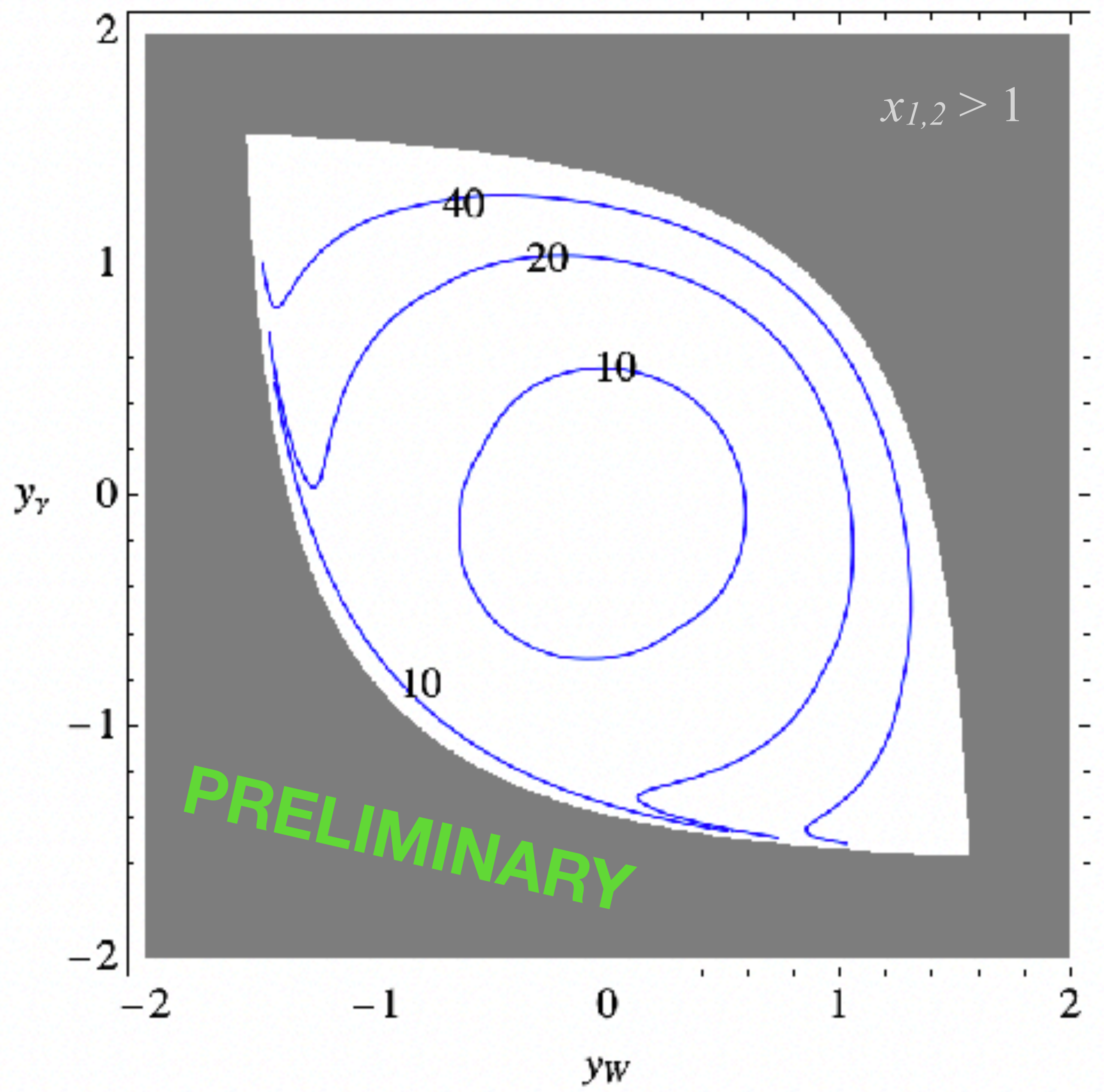
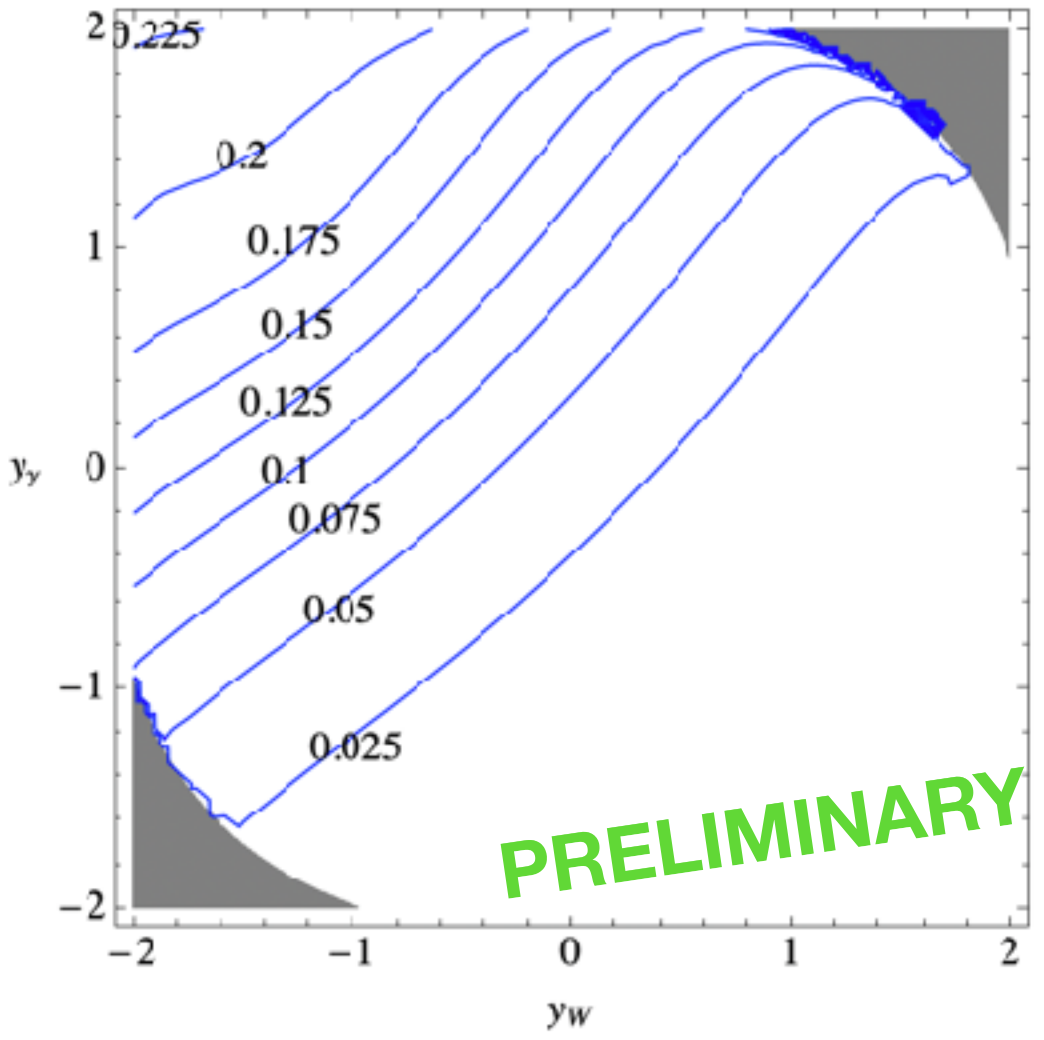


$p_T^\gamma = 2\text{TeV}$



Differential xsec [ab/GeV]

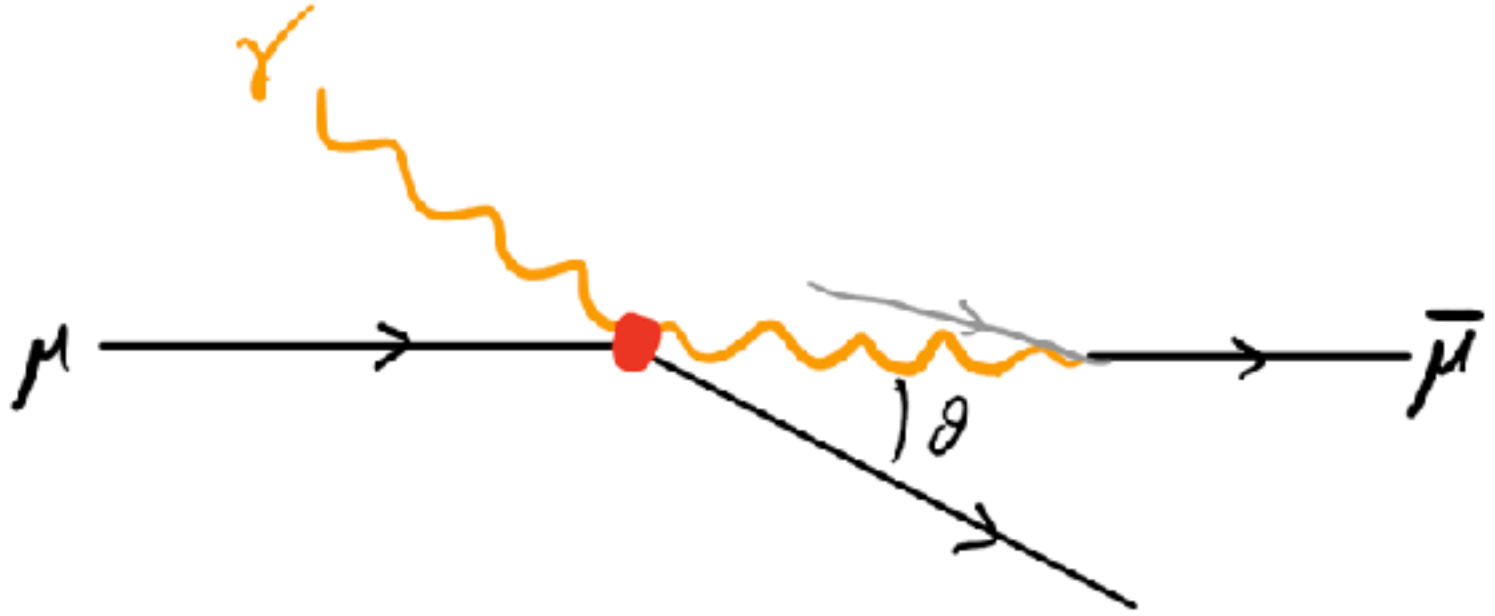


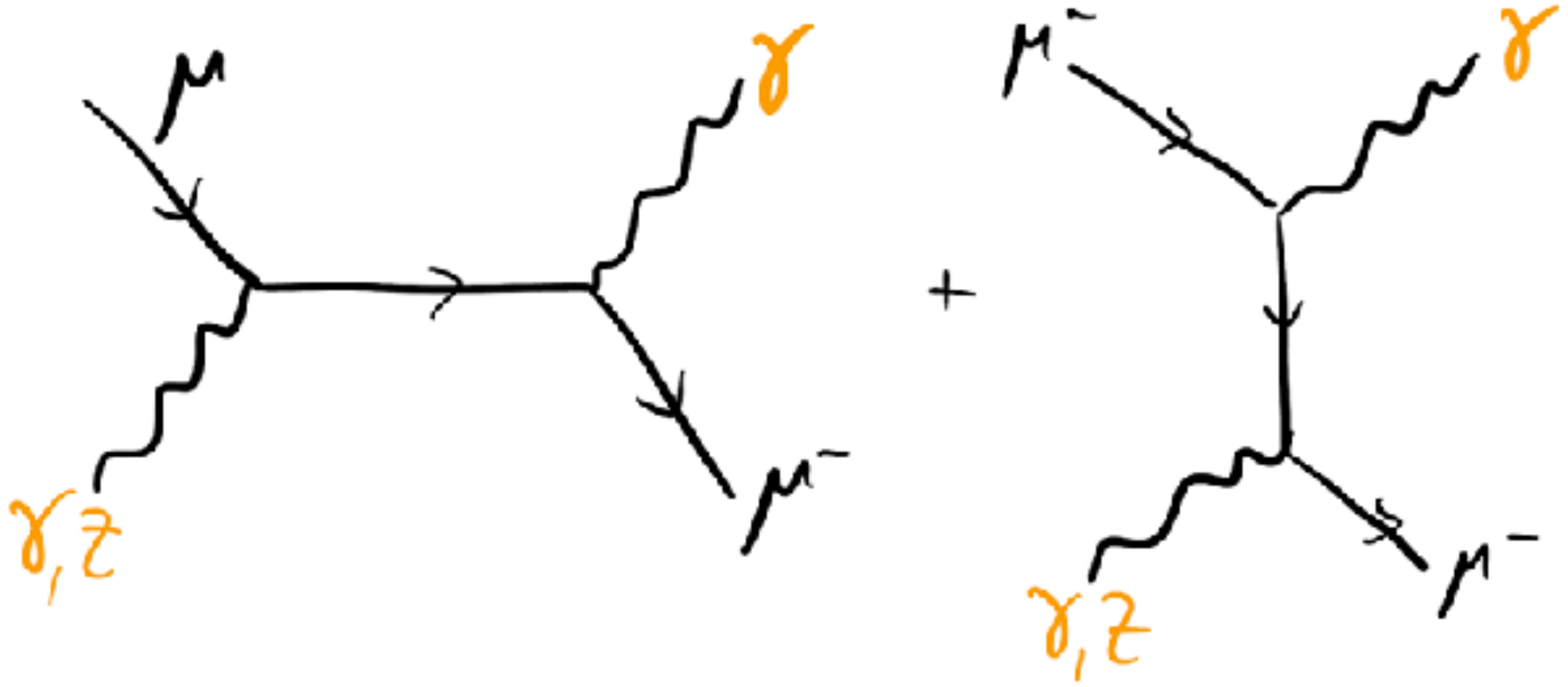
Thanks to F. Garosi for the plots!

The **muon neutrino PDF** can **contribute from few % up to ~ 40%.**

Compton Scattering @ MuC

[work in progress with A. Stanzione]

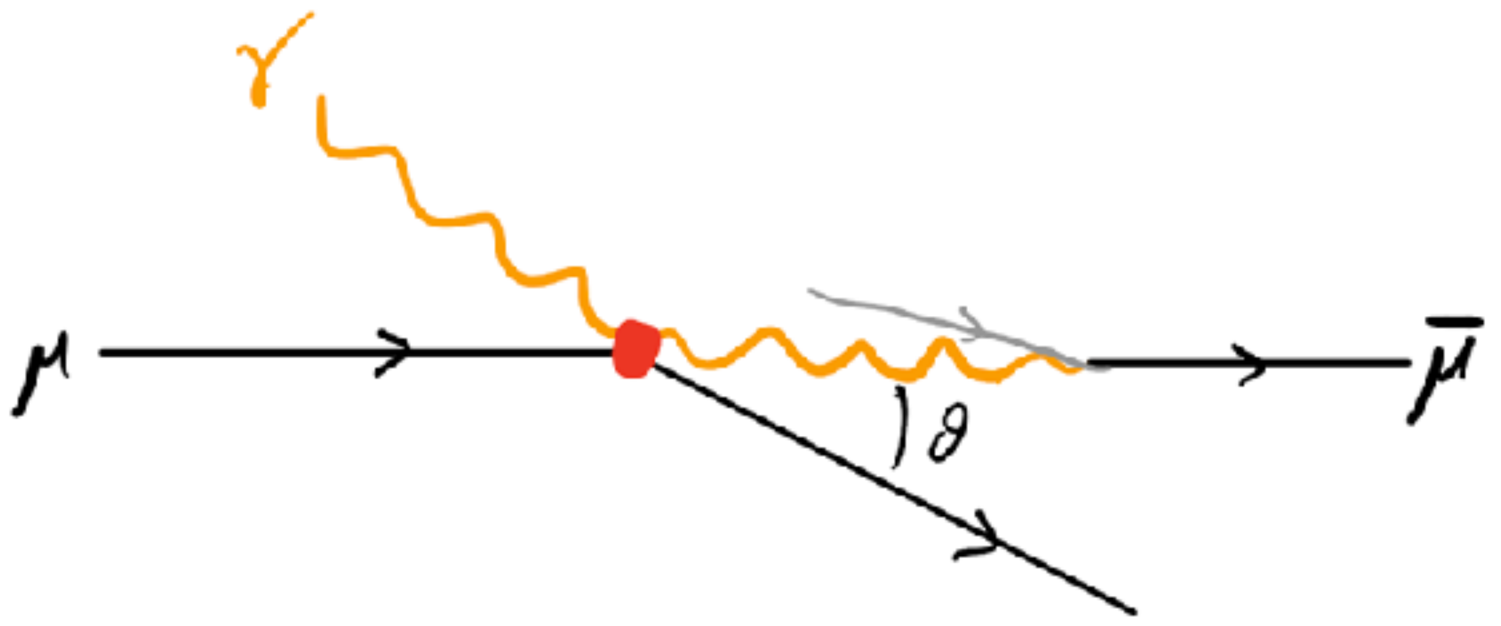




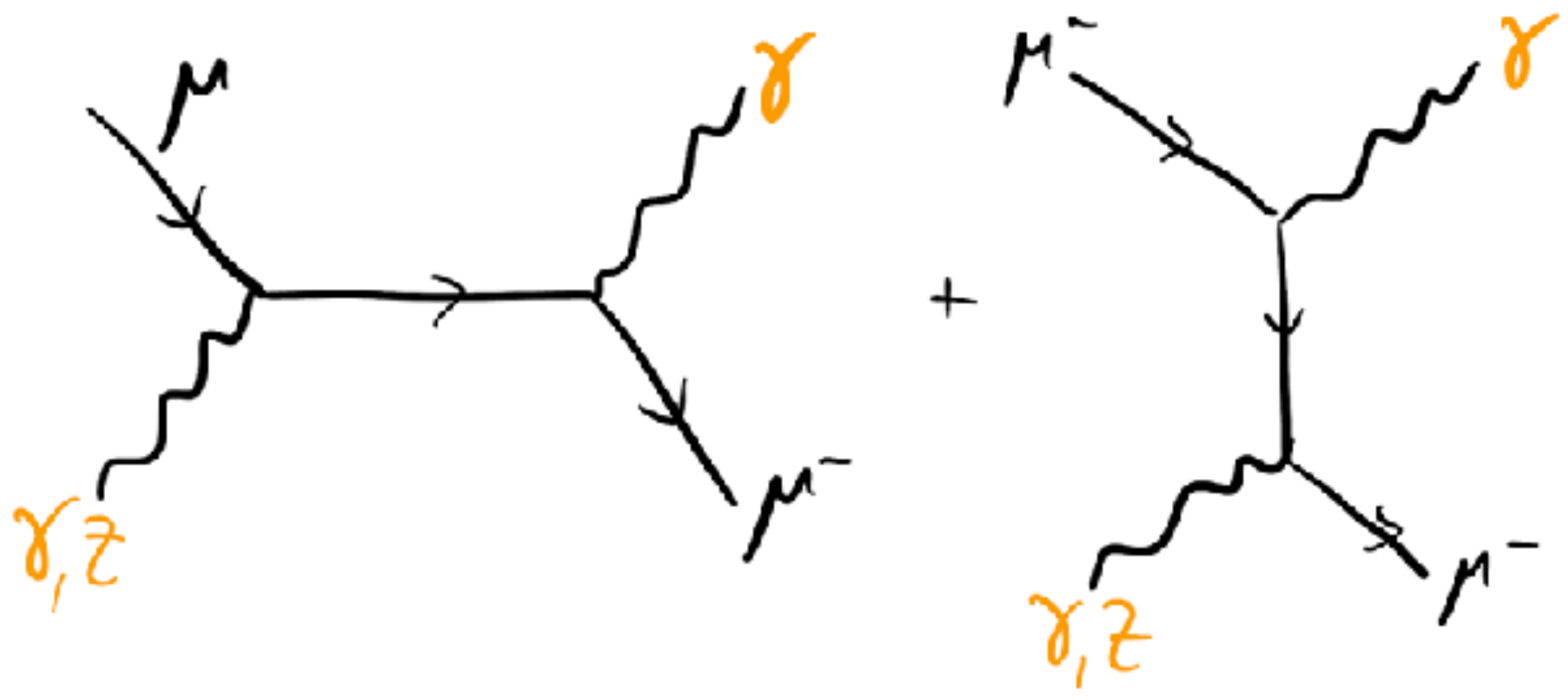
What is the impact of the mixed $Z\gamma$ PDF?

Compton Scattering @ MuC

[work in progress with A. Stanzione]



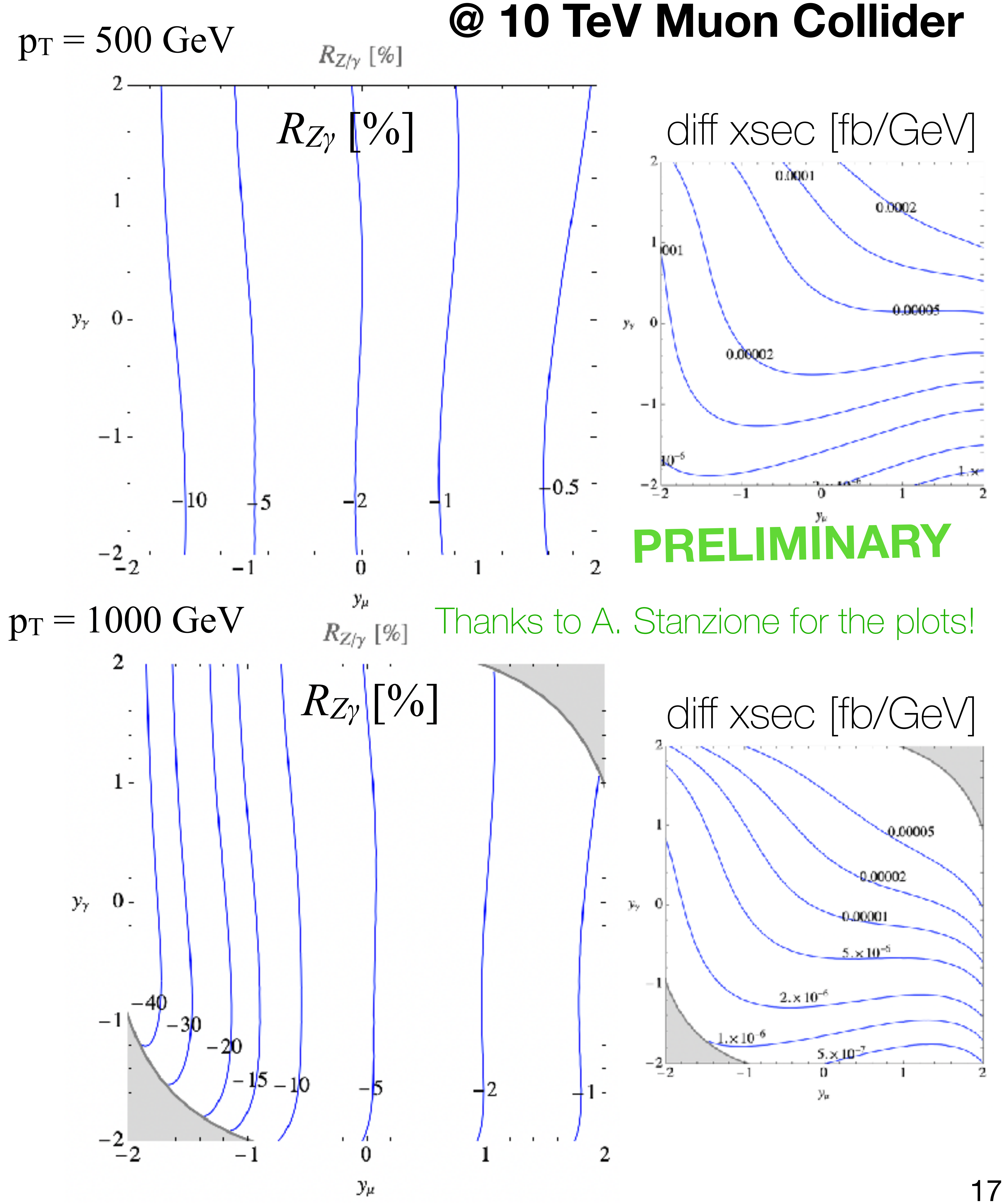
$$R_{Z/\gamma} \equiv \frac{\sigma_{Z\gamma}}{\sigma_{FULL}}$$



$$\frac{d^3\sigma}{dy_\mu dy_\gamma dp_T}$$

What is the impact of the mixed $Z\gamma$ PDF?

@ 10 TeV Muon Collider

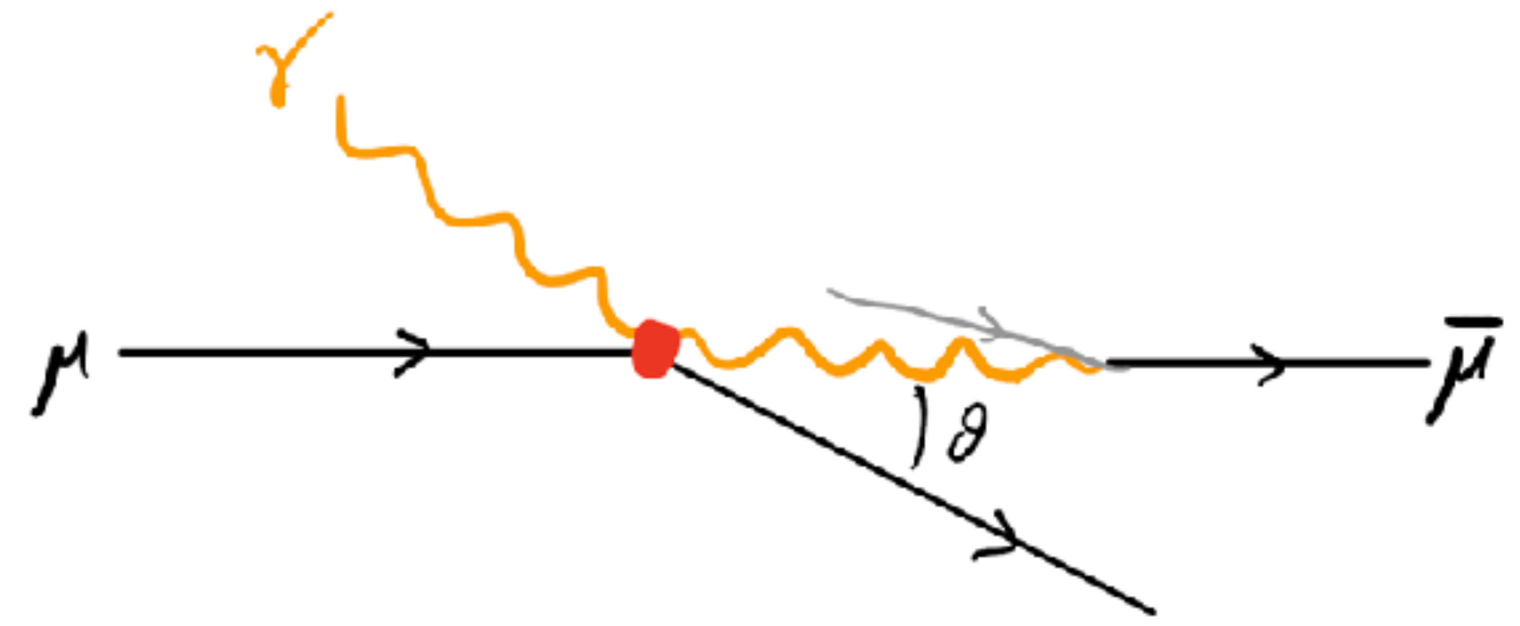


PRELIMINARY

Thanks to A. Stanzione for the plots!

Compton Scattering @ MuC

[work in progress with A. Stanzione]



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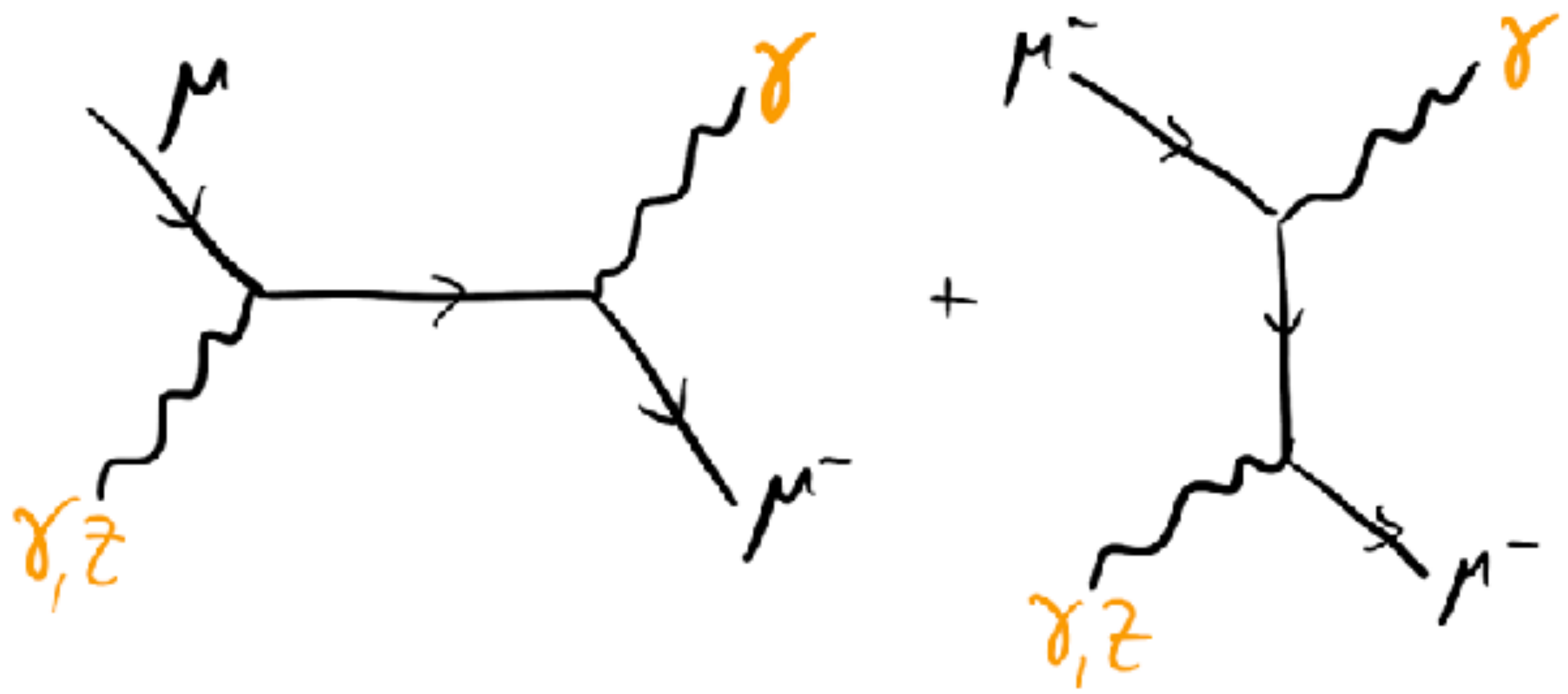


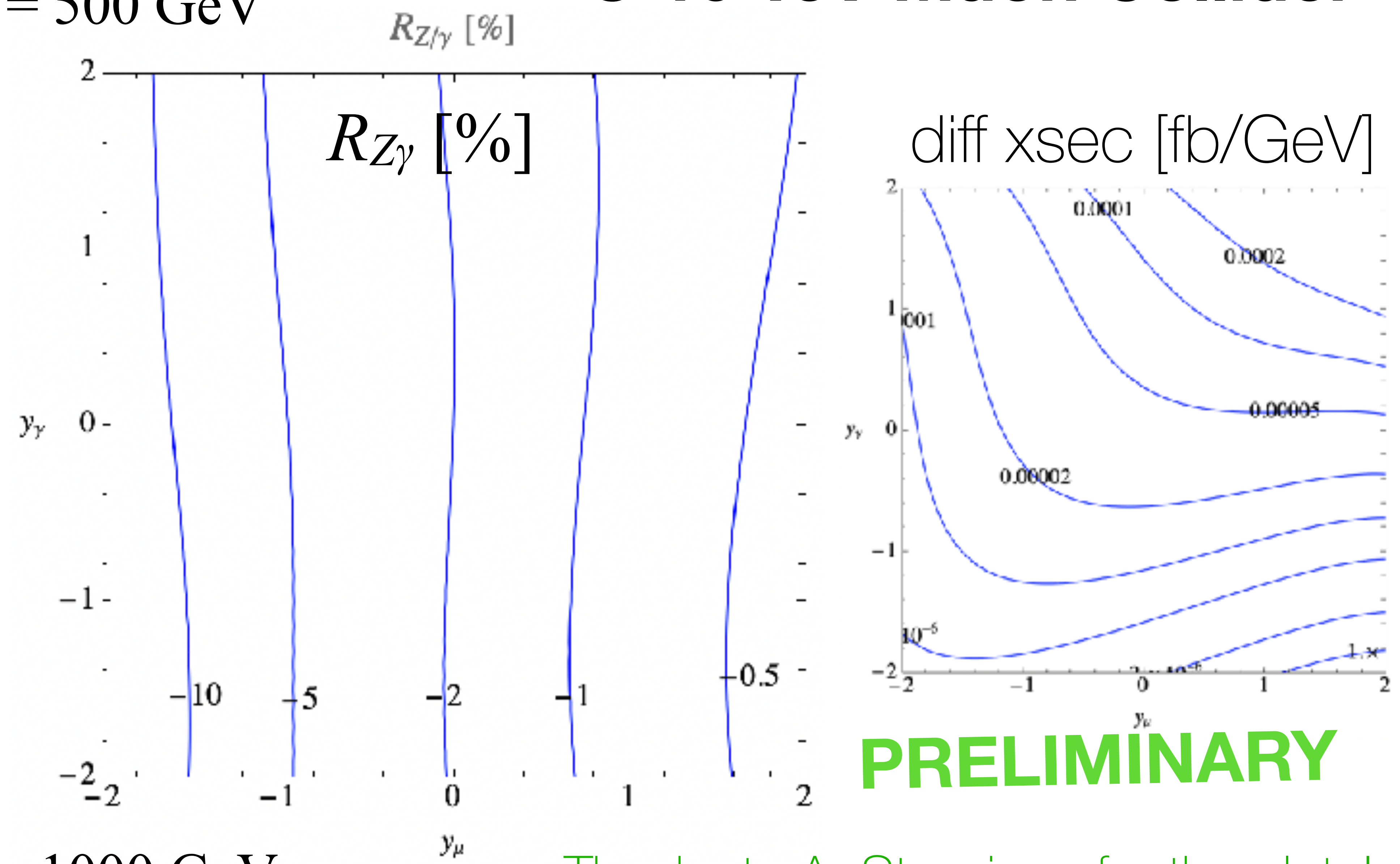
$$\frac{d^3\sigma}{dy_\mu dy_\gamma dp_T}$$

What is the impact of the mixed $Z\gamma$ PDF?

The **mixed $Z\gamma$ PDF** can **contribute from few % up to ~ 40%,** depending on the phase space region.

@ 10 TeV Muon Collider

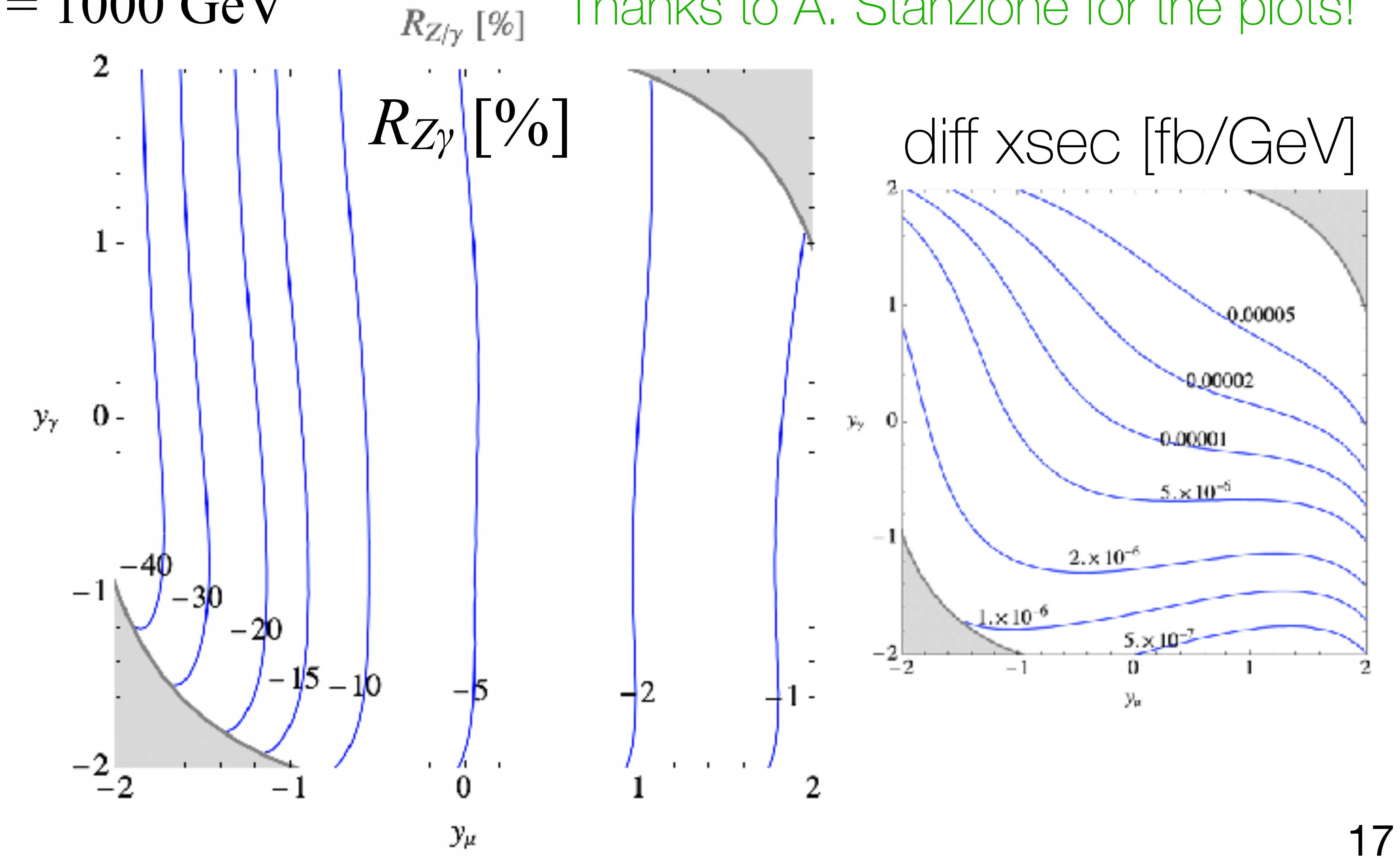
$p_T = 500$ GeV



PRELIMINARY

Thanks to A. Stanzione for the plots!

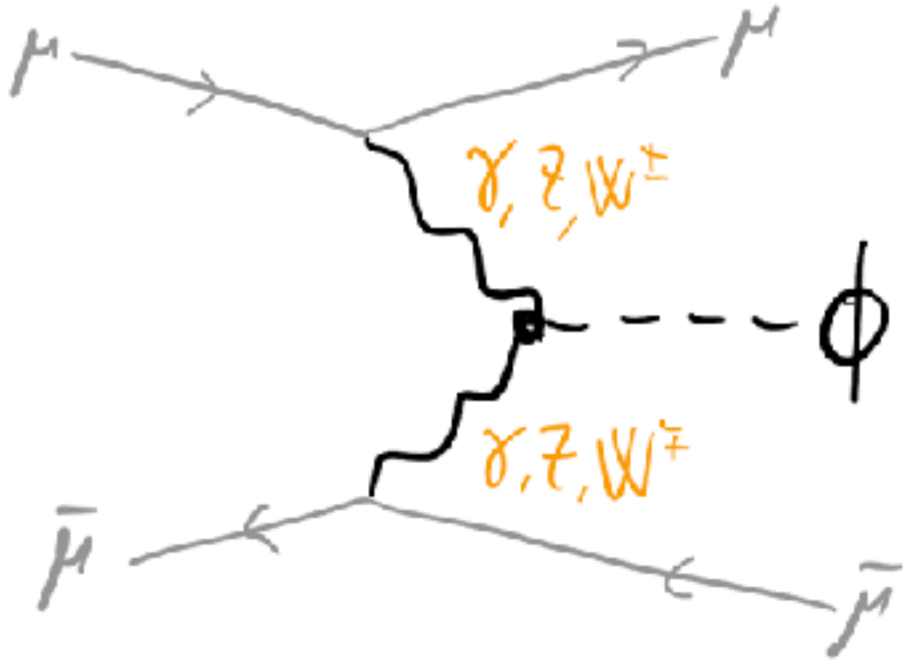
$p_T = 1000$ GeV



Singlet (pseudo-)scalar production

[work in progress with A. Stanzione]

$$\mathcal{L} = \frac{C_B}{\Lambda} B_{\mu\nu} B^{\mu\nu} \phi + \frac{C_W}{\Lambda} W^a_{\mu\nu} W^{a\mu\nu} \phi$$



This singlet scalar can be produced at muon colliders by (transverse) vector boson fusion.
What is the **impact** of the **mixed $Z\gamma$ PDF**?

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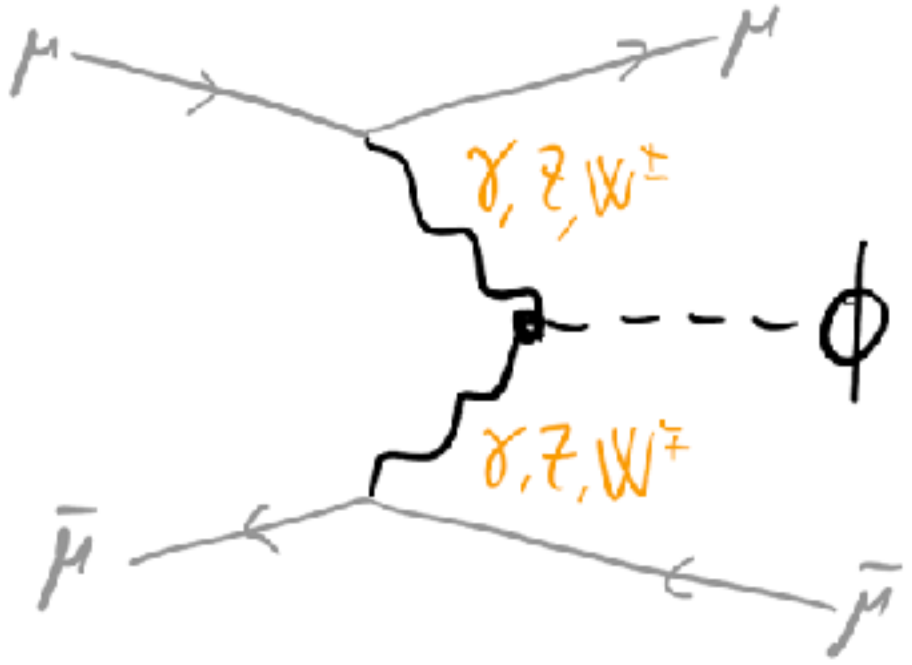

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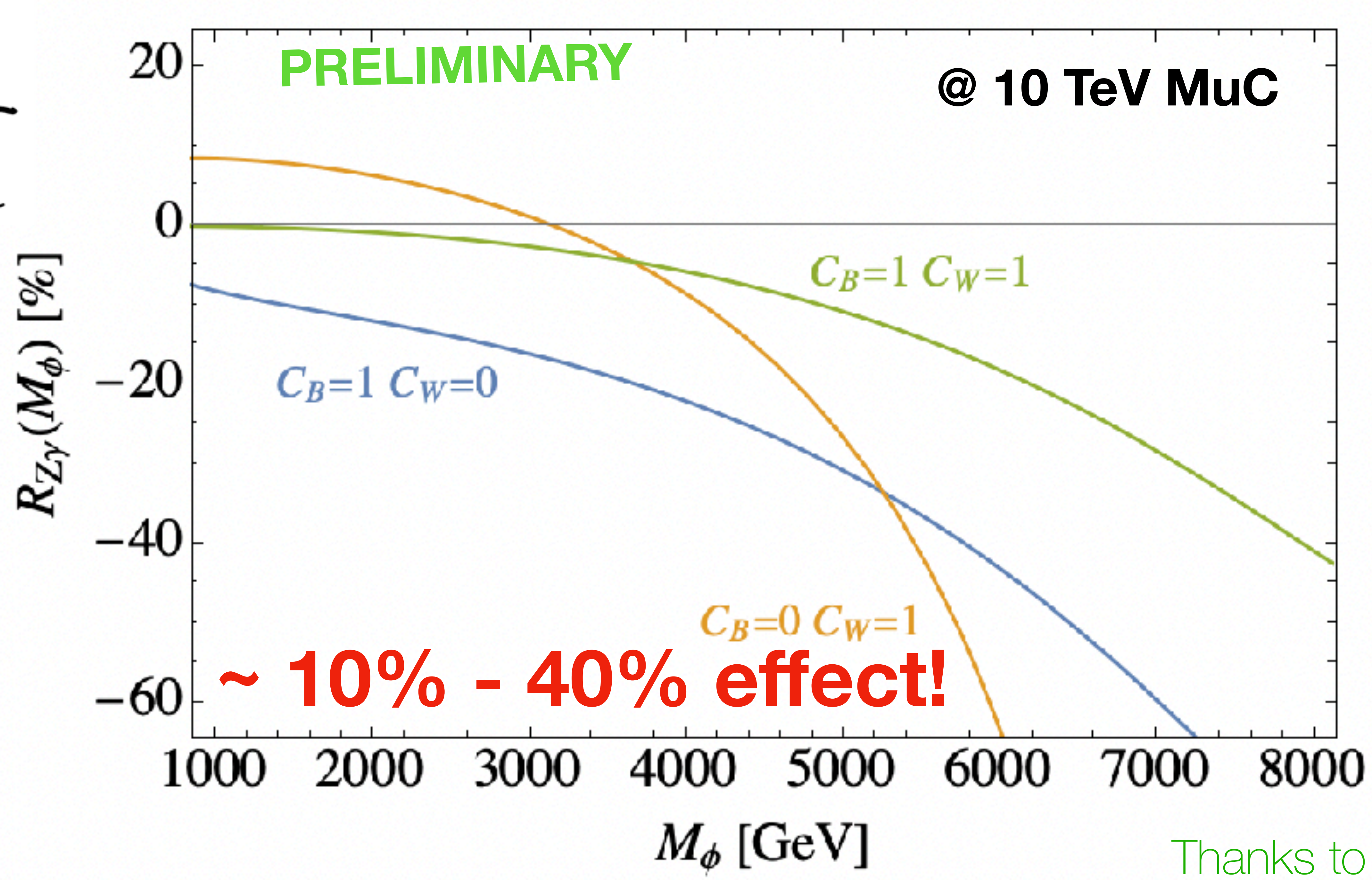
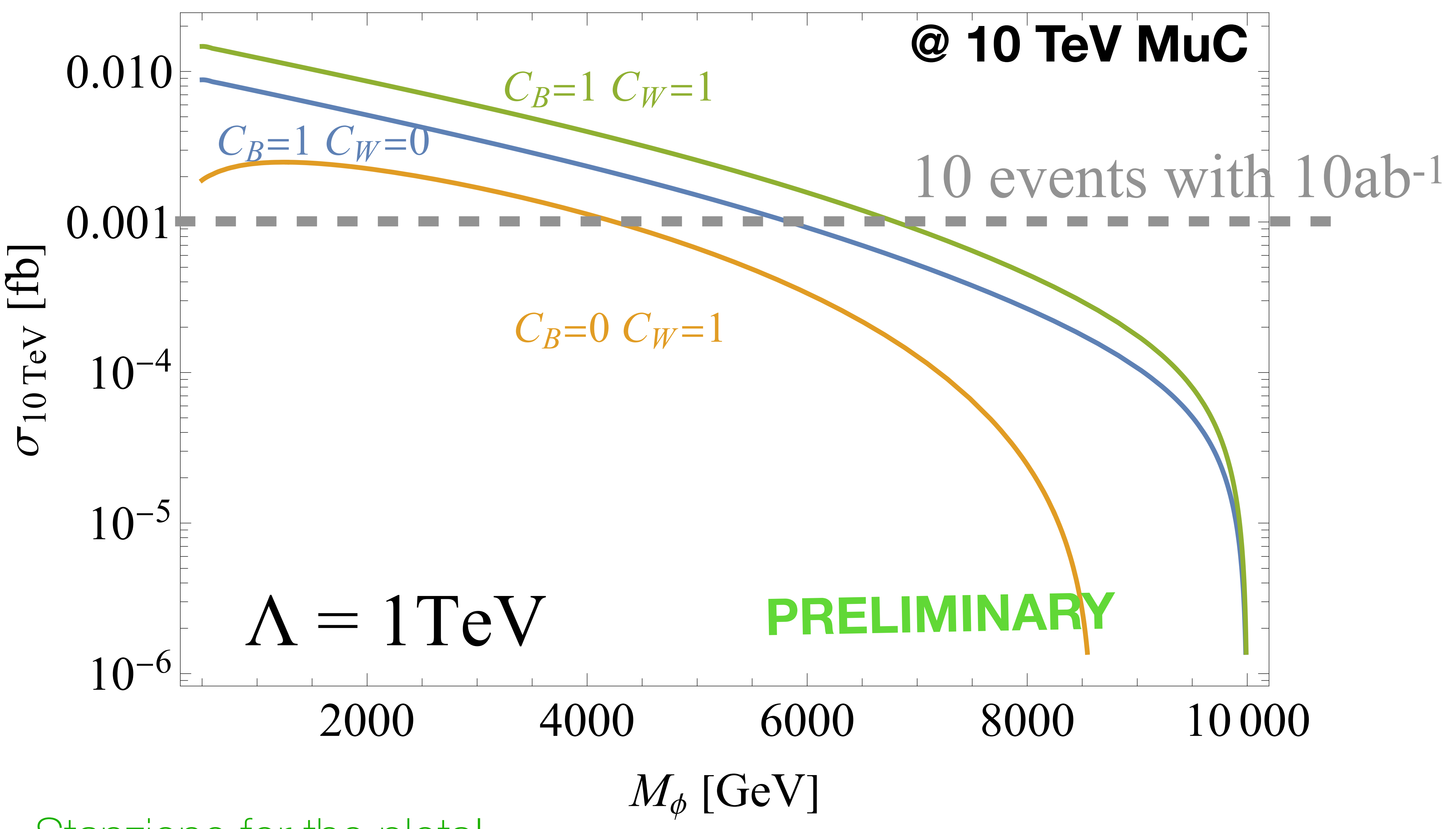
$$R_{Z\gamma} \equiv \frac{\sigma_{Z\gamma}}{\sigma_{FULL}}$$



~ 10% - 40% effect!



Thanks to A. Stanzione for the plots!

Conclusions

We derived resummed **SM PDFs for lepton colliders** at the leading-log level: **LePDF**.

The results are made public in a **LHAPDF6**-*type* format: **extended to include helicity dependence**.
https://github.com/DavidMarzocca/LePDF

We show that the implementation of EVA with the $Q \gg m_W$ approximation is not sufficient, even at TeV scales.
When mass terms are included, **EVA @ LO deviates by**:

- **up to O(30-40%) for Z_T and W_T** at small x and large Q (few TeV) ,
- **~10^2 for the Z/γ** PDF.

The **muon neutrino PDF** inside a muon can impact physics studies: from few % **up to ~40%** effect!

The **mixed $Z\gamma$ PDF** can impact the xsec for several final states (SM or BSM) by **up to ~40%**.

We hope this tool can allow **more comprehensive studies of physics potential at Muon Colliders**!

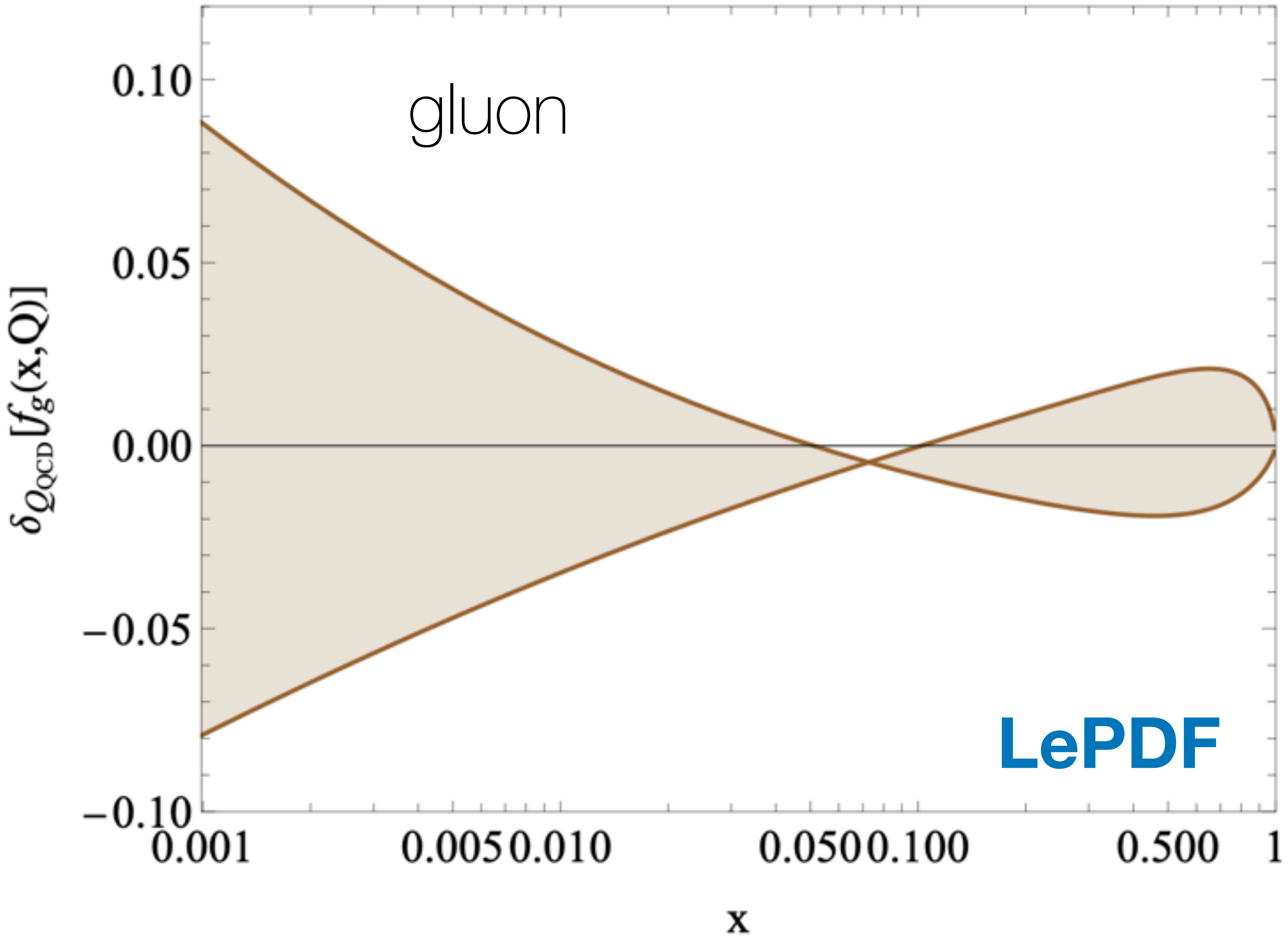
Thank you!

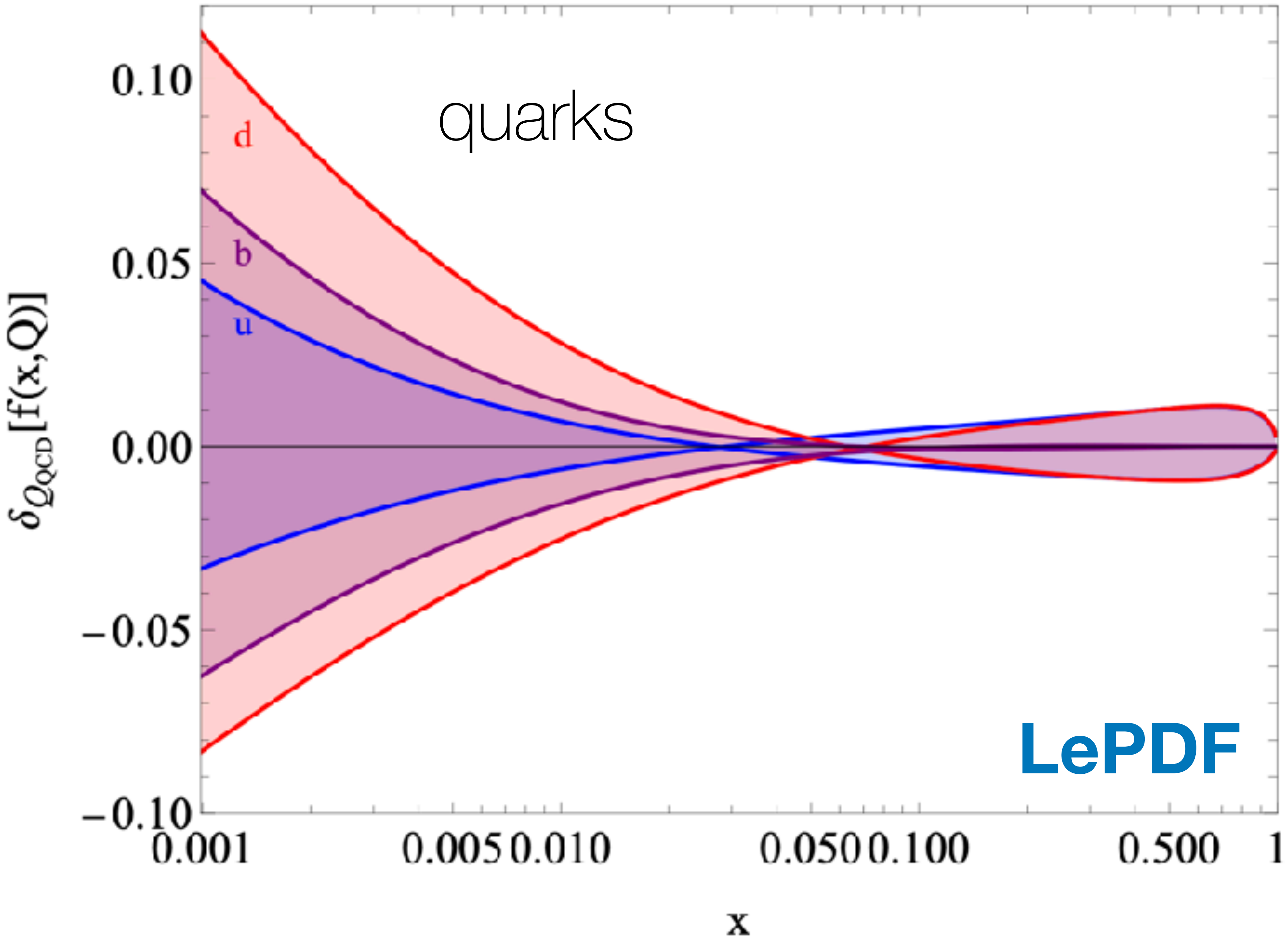
Backup

Uncertainty due to choice of Q_{QCD}

Changing the scale in the interval $Q_{QCD} =$ [0.5 - 1] GeV

Relative variation in the PDFs, evaluated at the $\boldsymbol{m_W}$ **scale**.





For **leptons** and the **photon**, relative variations are **smaller than 10^{-5}**.

EW Sudakov double logs from ISR

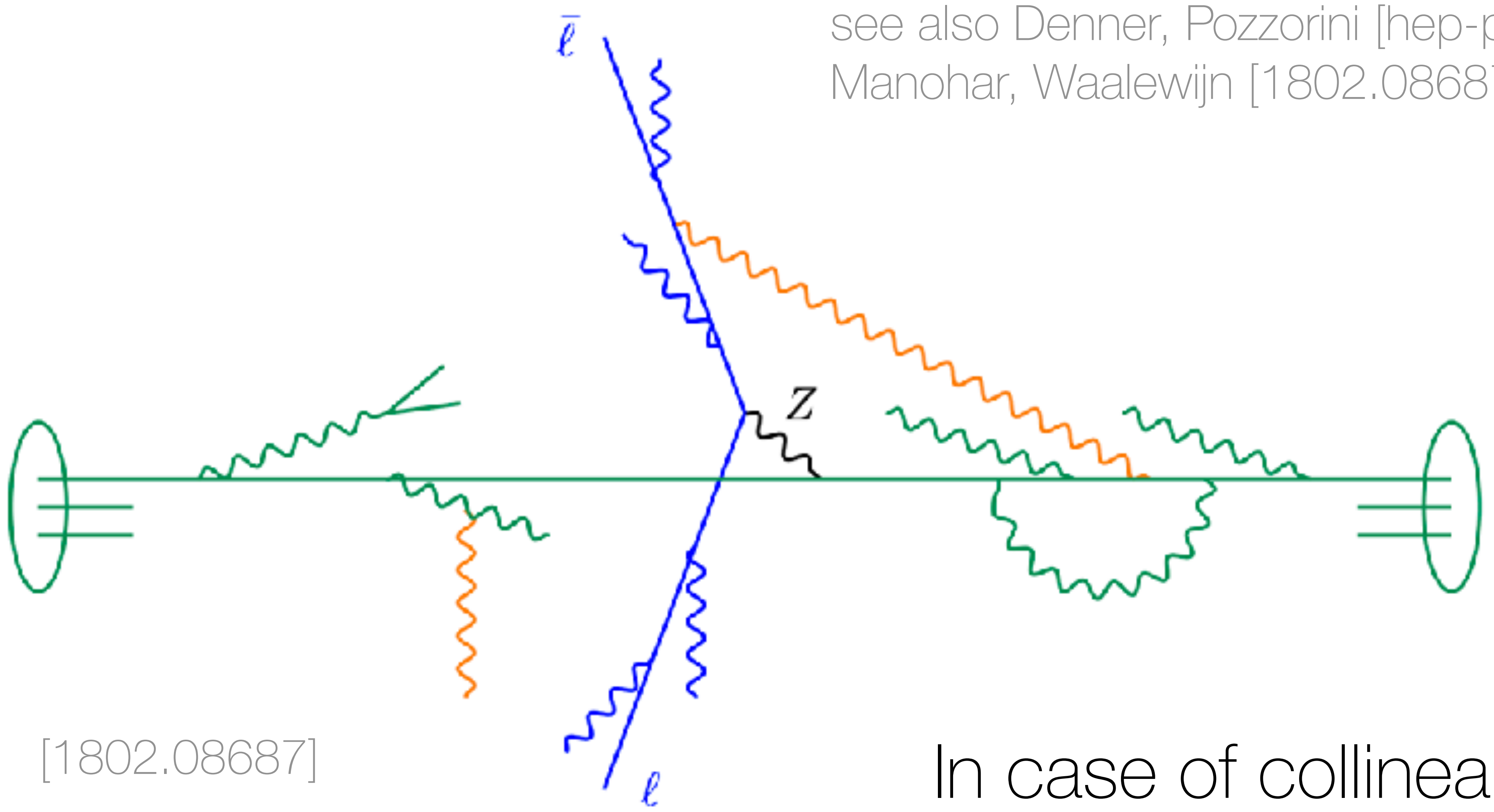
The Bloch-Nordsieck theorem is violated for non-abelian gauge theories

→ IR divergencies are not cancelled in inclusive processes, since the initial state is EW non-singlet

→ We are often interested in exclusive processes, since we measure the SU(2) charge (W vs Z, t vs b, etc…)

The **EW Sudakov double logs** arises as a non-cancellation of the IR soft divergences $(z \to 1)$ between real emission and virtual corrections.

P. Ciafaloni, Comelli [hep-ph/9809321], Fadin et al. [hep-ph/9910338], M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0001142, hep-ph/0103315]
see also Denner, Pozzorini [hep-ph/0010201], Pozzorini [hep-ph/0201077], Manohar [1409.1918], Pagani, Zaro [2110.03714], …
Manohar, Waalewijn [1802.08687], Chen, Glioti, Rattazzi, Ricci, Wulzer [2202.10509]



[1802.08687]

Here I am interested in **resumming the EW double logs** related to the **initial-state radiation**.
At the leading-log level we can neglect soft radiation

Manohar, Waalewijn [1802.08687]

In case of collinear W emission they can be implemented (and resummed) at the **Leading Log** level by putting an **explicit IR cutoff** $z_{max} = 1 - Q_{EW}/Q$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]; Bauer, Ferland, Webber [1703.08562]; Manohar, Waalewijn [1802.08687]

EW Sudakov double logs from ISR

In case of collinear W emission they can be implemented (and resumed) at he **Double Log** level equations by putting an **explicit IR cutoff $z_{max} = 1 - Q_{EW} / Q$** $(Q_{EW} = m_W)$

M. Ciafaloni, P. Ciafaloni, Comelli [hep-ph/0111109]
Bauer, Ferland, Webber [1703.08562]
see Manohar, Waalewijn [1802.08687] for a different approach

$$\frac{\alpha_{ABC}(Q)}{2\pi}\int_x^1 \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right) \quad \rightarrow \quad \frac{\alpha_{ABC}(Q)}{2\pi}\int_x^{\boxed{z_{\max}^{ABC}(Q)}} \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right)$$

This modifies also the **virtual corrections** as:

$$P_A^v(Q) \supset -\sum_{B,C} \frac{\alpha_{ABC}(Q)}{2\pi} \int_0^{z_{\max}^{ABC}(Q)} dz\, z\, P_{BA}^C(z)$$

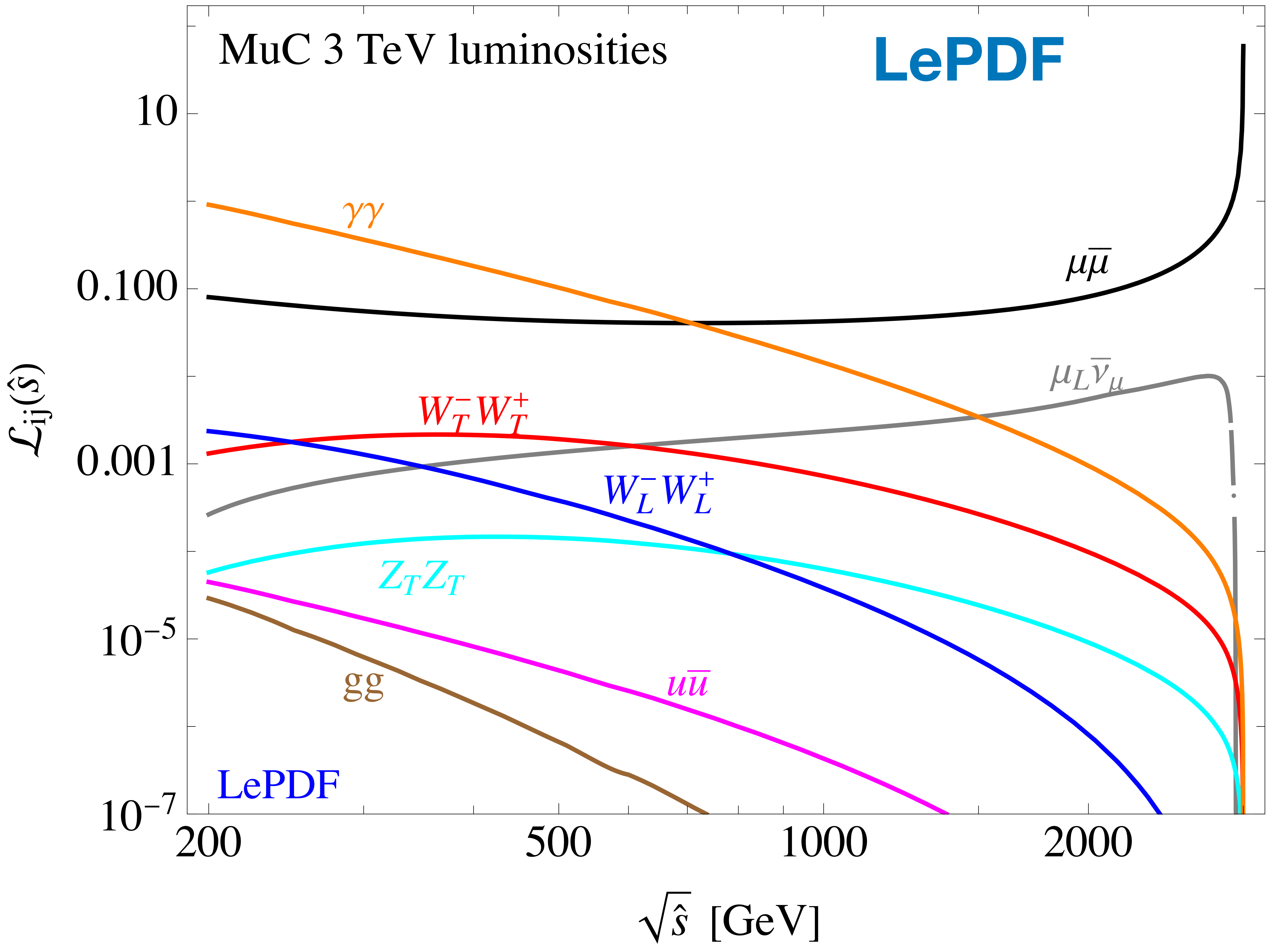
The non-cancellation of the z_{max} dependence between emission and virtual corrections generates the double logs.

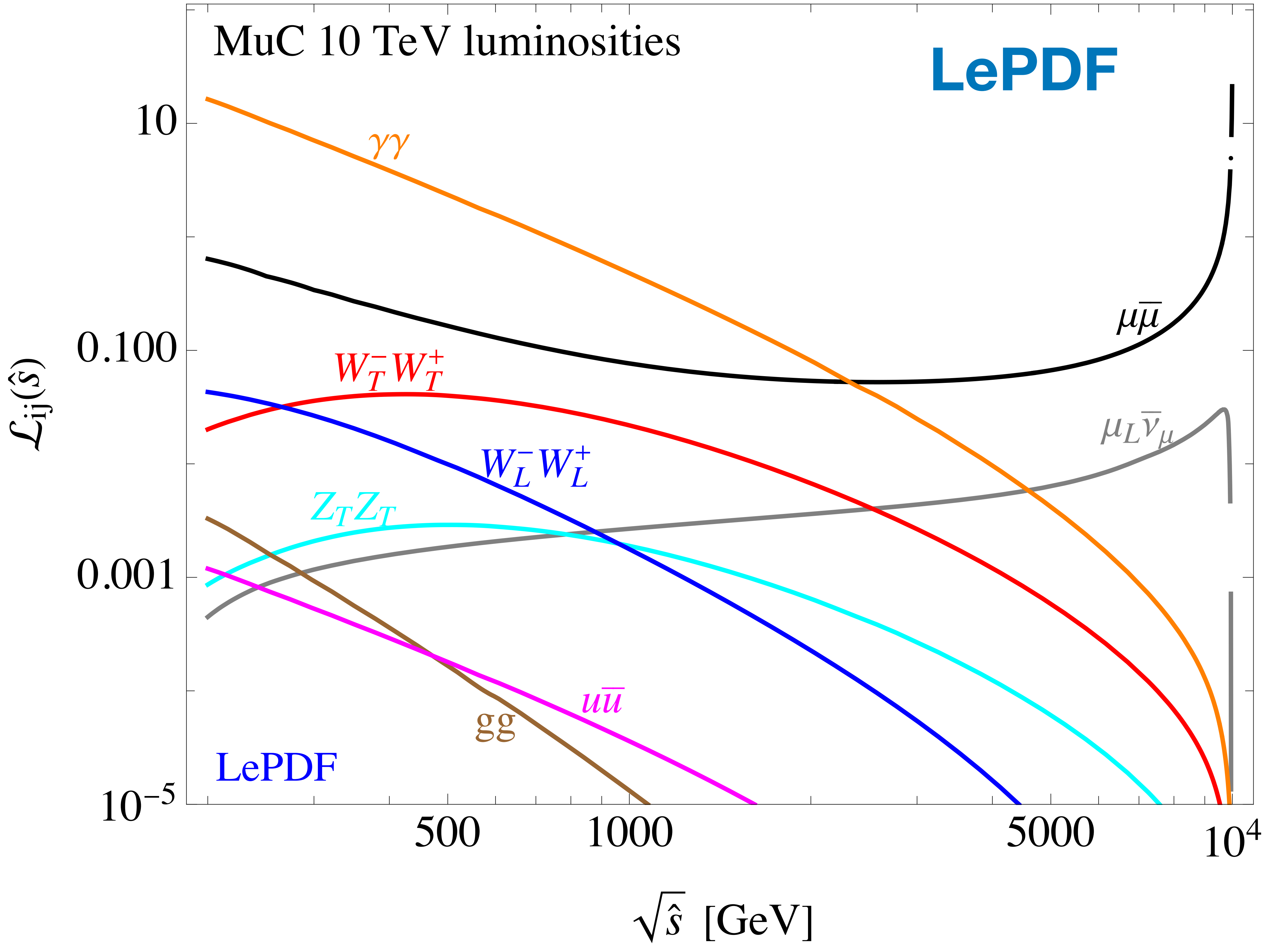
This happens if $P_{BA}^C,\ U_{BA}^C \propto \frac{1}{1-z}$ and $A \neq B$ otherwise we set z_{max}=1 and use the +-distribution.

PDFs of a muon

Some examples of **parton luminosities** for muon colliders.

$$\mathcal{L}_{ij}(\hat{s}) = \int_{\hat{s}/s_0}^{1} dx \frac{1}{x} f_i^{(\mu)}\left(x, \frac{\sqrt{\hat{s}}}{2}\right) f_j^{(\bar{\mu})}\left(\frac{\hat{s}}{x s_0}, \frac{\sqrt{\hat{s}}}{2}\right)$$

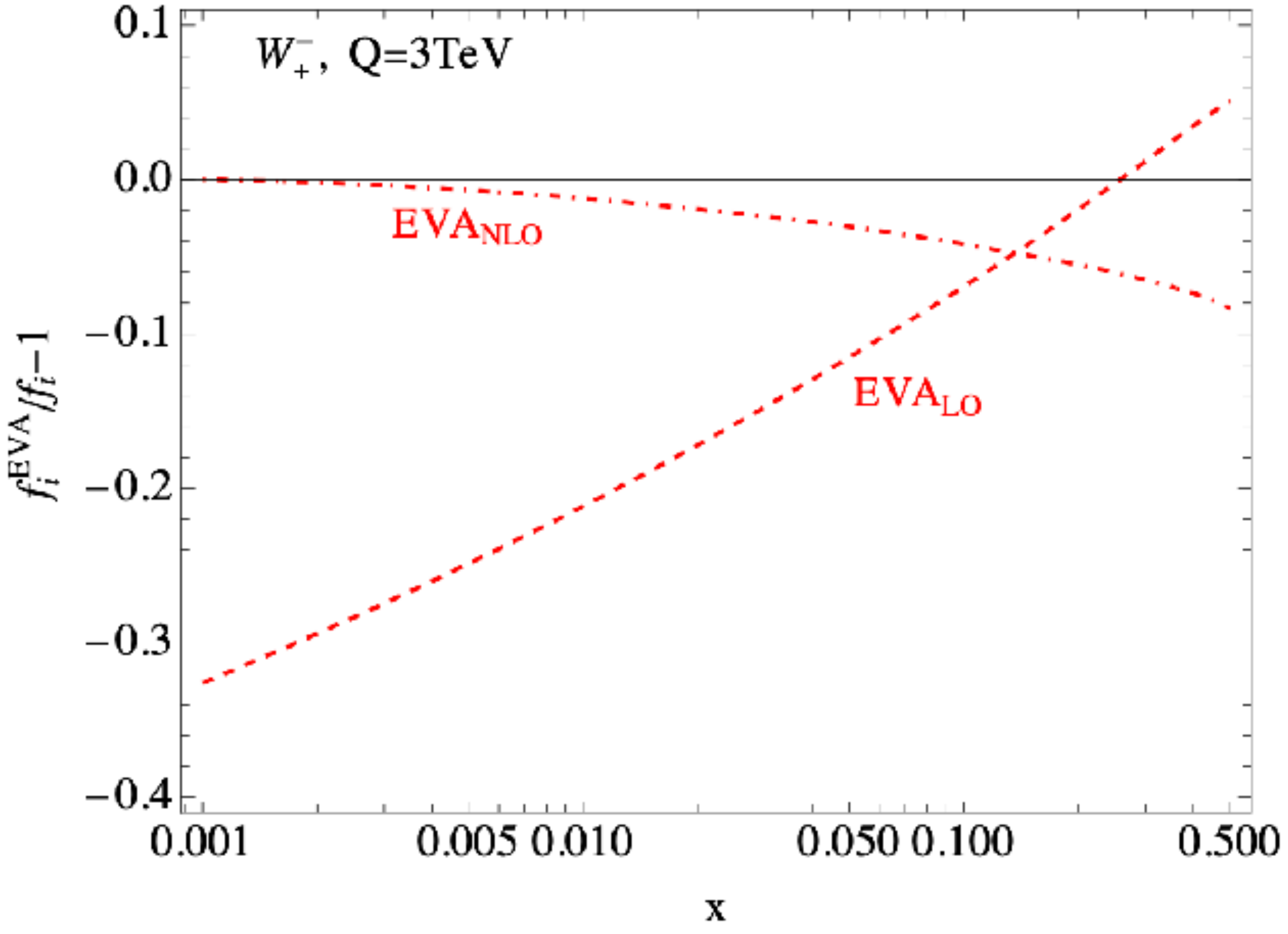


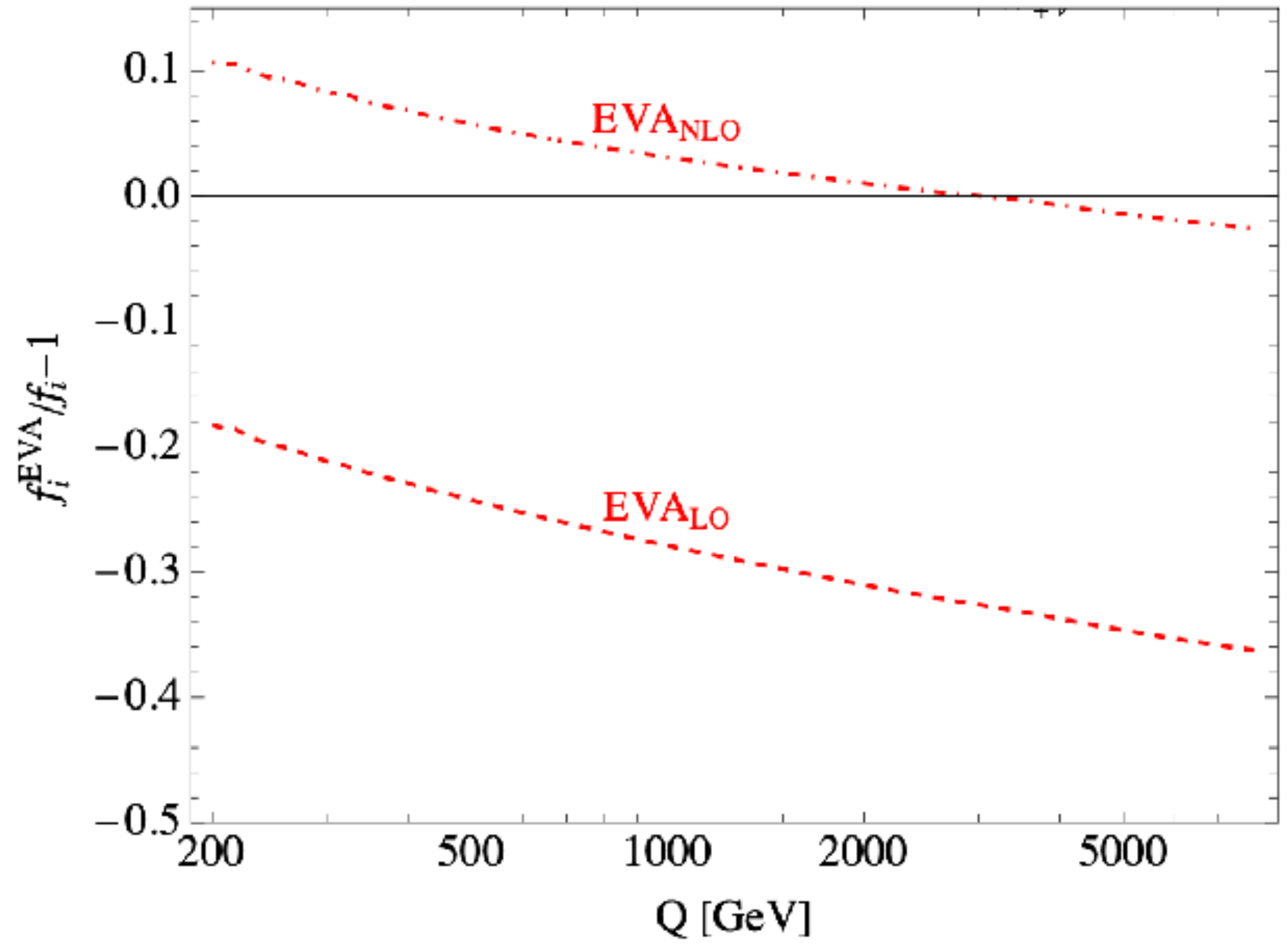


Some comments:

- The **very large $\gamma\gamma$ lumi** could dominate over Z and Z/γ contributions.
- **gluon and quark luminosities are small**: suppressed impact of QCD-induced backgrounds.

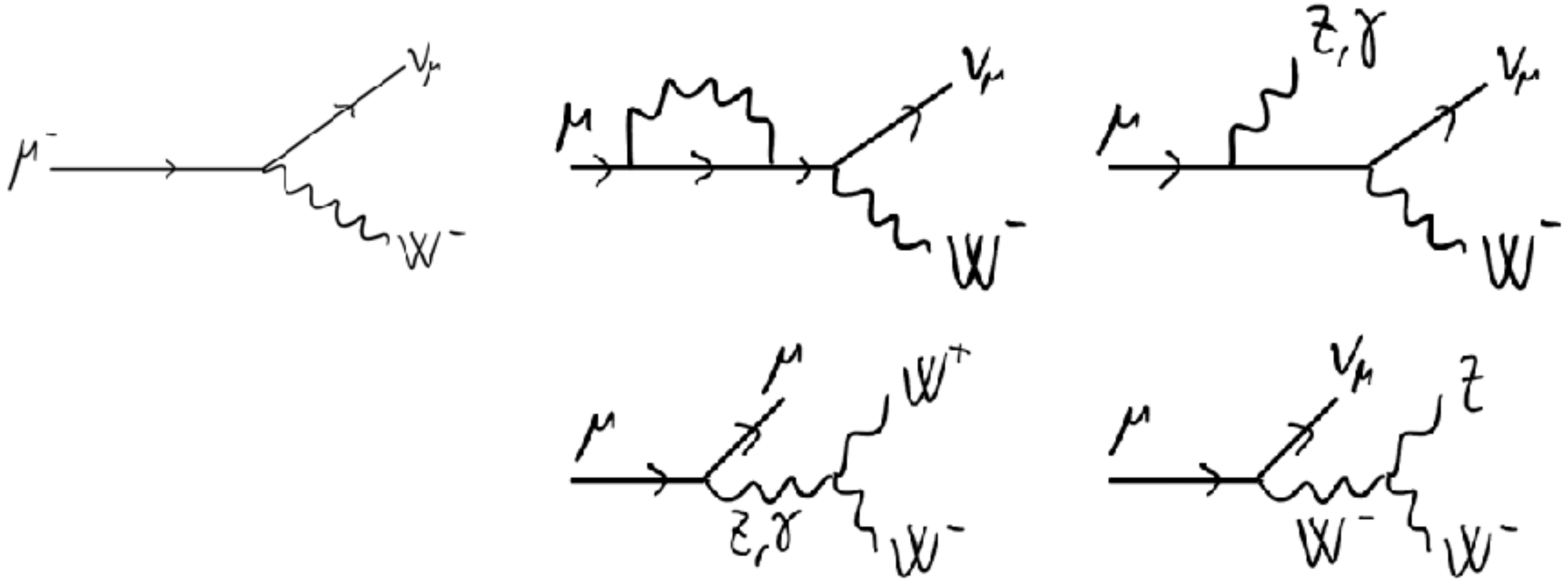
LePDF vs. EVA





The **deviation becomes larger at small x and at large scales** (Sudakov double logs are absent in EVA).

We improve EVA by computing iteratively the $\boldsymbol{W^-_+}$ **PDF at** $\mathbf{O(\alpha^2)}$. *



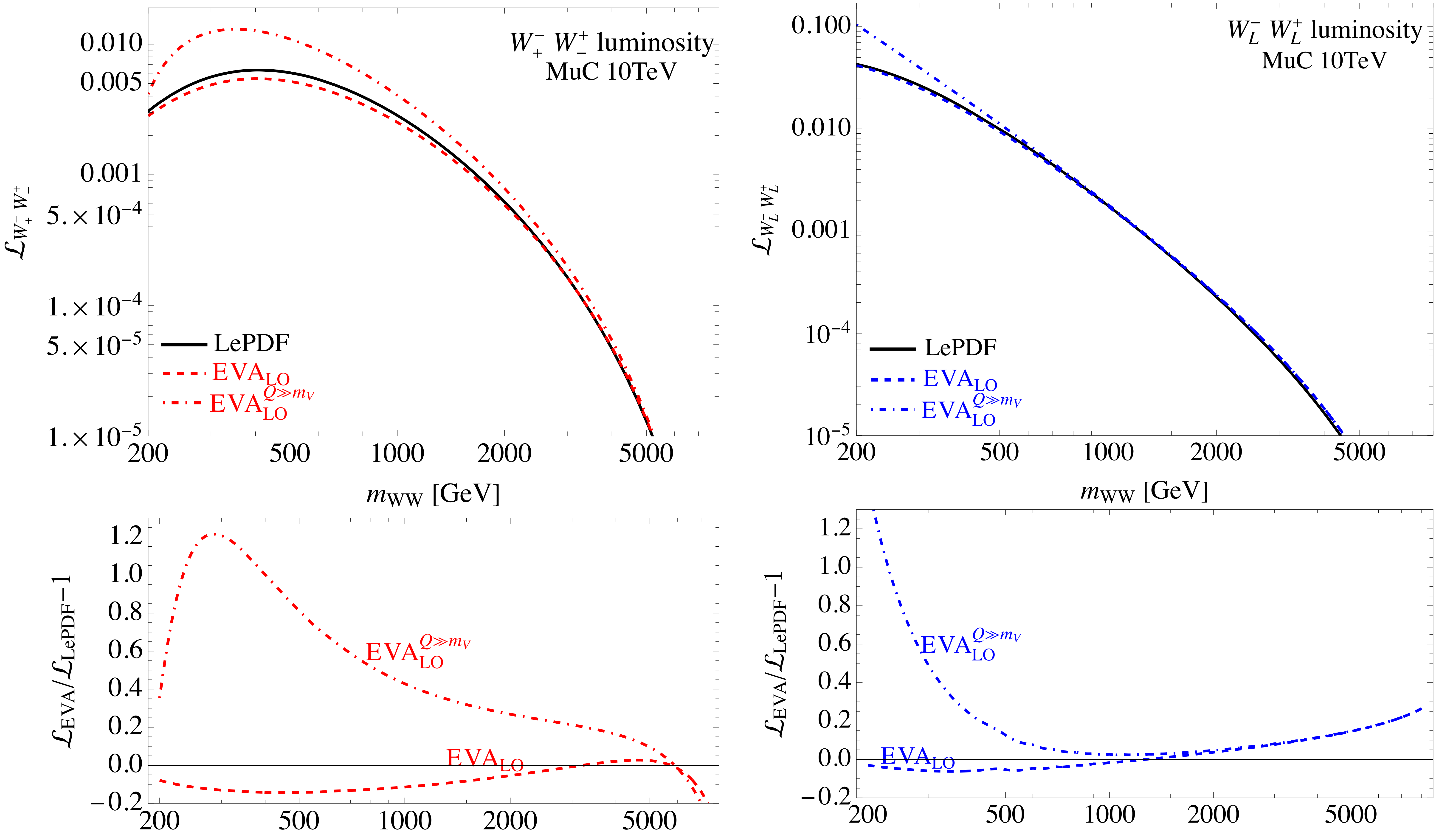
$$f^{(\alpha)}_{\mu_L}(x,t) \simeq \int_{t_{m_W}}^{t} dt' \left(\frac{1}{2}P^{v}_{\mu_L}(t')\delta(1-x) + \frac{\alpha_\gamma}{4\pi}P^{V}_{ff}(x) + \frac{\alpha_2}{4\pi c_W^2}(Q^Z_{\mu_L})^2 P^{V}_{ff}(x)\right) ,$$

$$f^{(\alpha^2)}_{W^-_+}(x,t) \simeq \int_{t_{m_W}}^{t} dt' \left(P^{v}_{W^-_+} f^{(\alpha)}_{W^-_+} + \frac{\alpha_2}{4\pi}P^{f}_{V_+ f_L} \otimes f^{(\alpha)}_{\mu_L} + \frac{\alpha_2}{2\pi}c_W^2 P_{V_+ V_s} \otimes (f^{(\alpha)}_{W^-_s} + f^{(\alpha)}_{Z_s}) + \right.$$
$$\left. + \frac{\alpha_\gamma}{2\pi}P_{V_+ V_s} \otimes (f^{(\alpha)}_{W^-_s} + f^{(\alpha)}_{\gamma_s}) + \frac{\sqrt{\alpha_\gamma \alpha_2}}{2\pi} c_W P_{V_+ V_s} \otimes f^{(\alpha)}_{Z/\gamma_s}\right) .$$

* for simplicity, in the NLO part we take the Q ≫ m_W and x ≪ 1 limit in the LO EVA expression.

Several **double logs appear at this order**, we find a **much improved agreement** with the LePDF resummation.

LePDF vs. EVA: WW Luminosity



At the level of **parton luminosity**:

- for **W_TW_T**: EVA$_{\text{LO}}$ is accurate to **~15%**
- for **W_LW_L**: EVA$_{\text{LO}}$ is accurate to **~5%**
- The **$Q \gg m_V$** approximation does not reproduce well the complete result, with **O(1) differences** up to large scales (particularly for transverse modes).

EVA$_{\text{LO}}$
$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) = \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\left(\log\frac{Q^2+(1-x)m_W^2}{m_\mu^2+(1-x)m_W^2} - \frac{Q^2}{Q^2+(1-x)m_W^2}\right)$$

EVA$_{\text{LO}}^{\text{mV}\to 0}$
$$f^{(\alpha)}_{W^-_\pm}(x,Q^2) \approx \frac{\alpha_2}{8\pi} P^f_{V_\pm f_L}(x)\log\frac{Q^2}{m_W^2}$$

Implemented in `MadGraph5_aMC@NLO`
Ruiz, Costantini, Maltoni, Mattelaer [2111.02442]

Top quark PDF

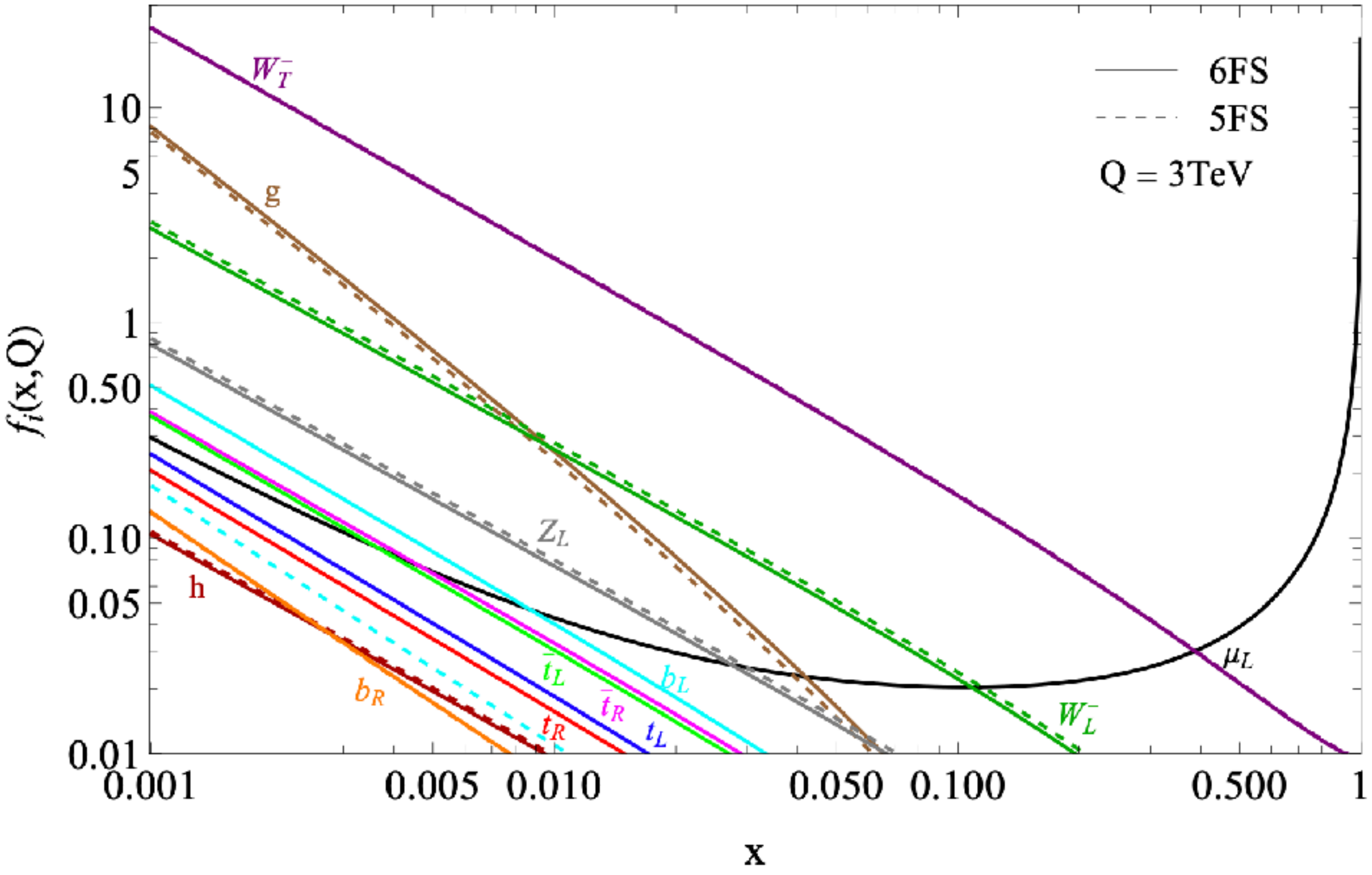
For hard scattering energies $E \gg m_t$, terms with $\log E/m_t$ due to collinear emission of top quarks can arise.

These can be resummed by including the **top quark PDF** within the DGLAP evolution, in a **6FS**.

Barnett, Haber, Soper '88; Olness, Tung '88

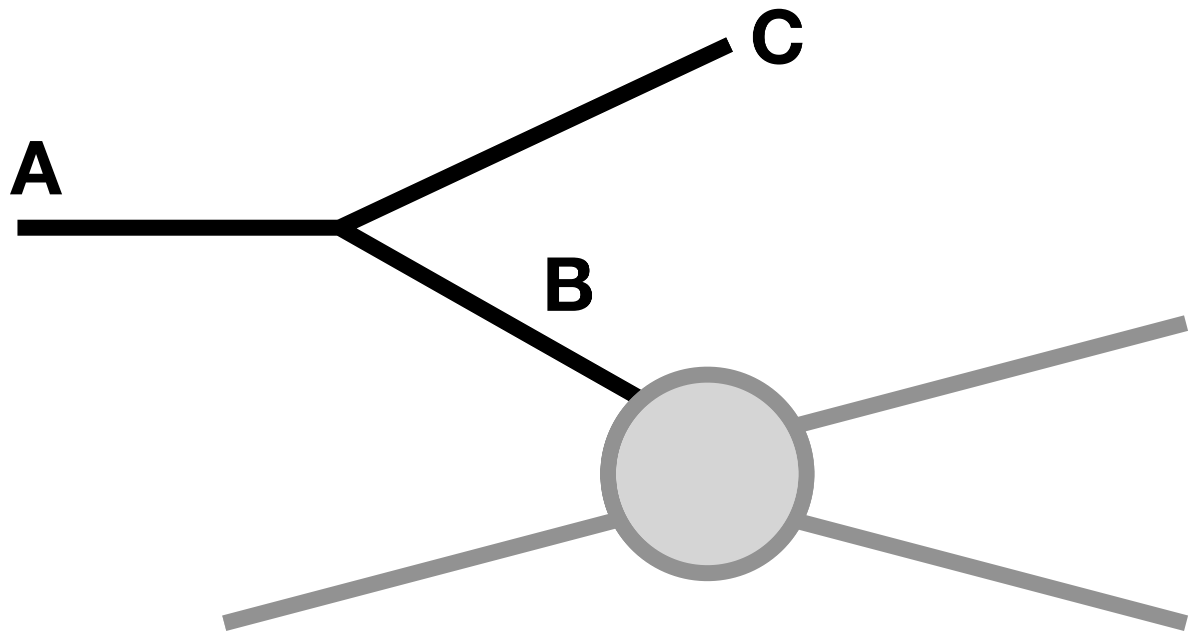
Whether or not this is useful depends on the process under consideration.

Dawson, Ismail, Low [1405.6211]
Han, Sayre, Westhoff [1411.2588]



We provide two version of the codes: **5FS** and **6FS**.
In the 6FS we keep **finite top quark mass** effects, like we do for other heavy SM states.

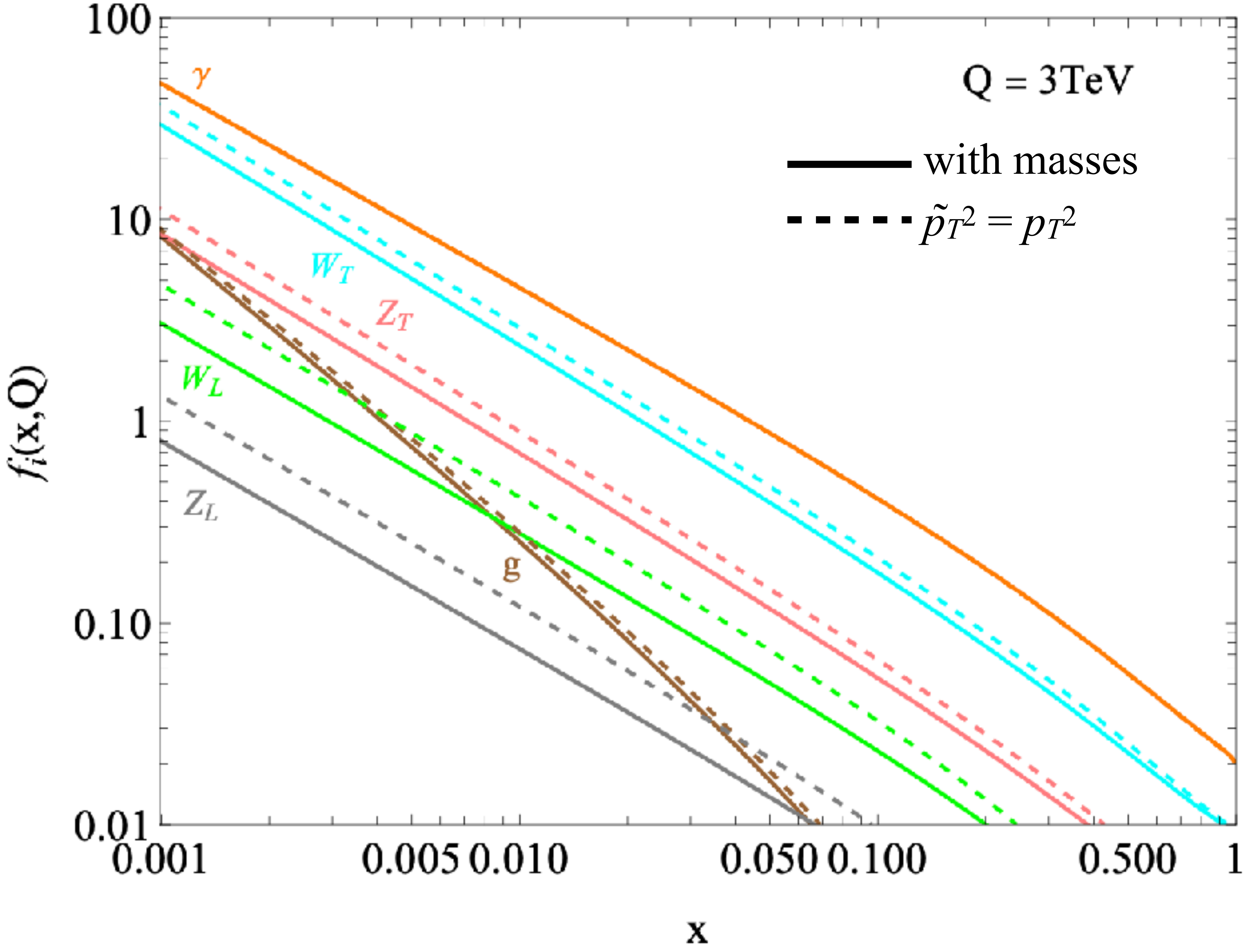
Mass effect



The mass modifies the propagator of the off-shell parton which then enters the hard scattering:

$$\widetilde{p}_T^2 \equiv \bar{z}(m_B^2 - q^2) = p_T^2 + z m_C^2 + \bar{z} m_B^2 - z\bar{z} m_A^2 + \mathcal{O}\left(\frac{m^2}{E^2}, \frac{p_T^2}{E^2}\right)$$

Chen, Han, Tweedie [1611.00788]



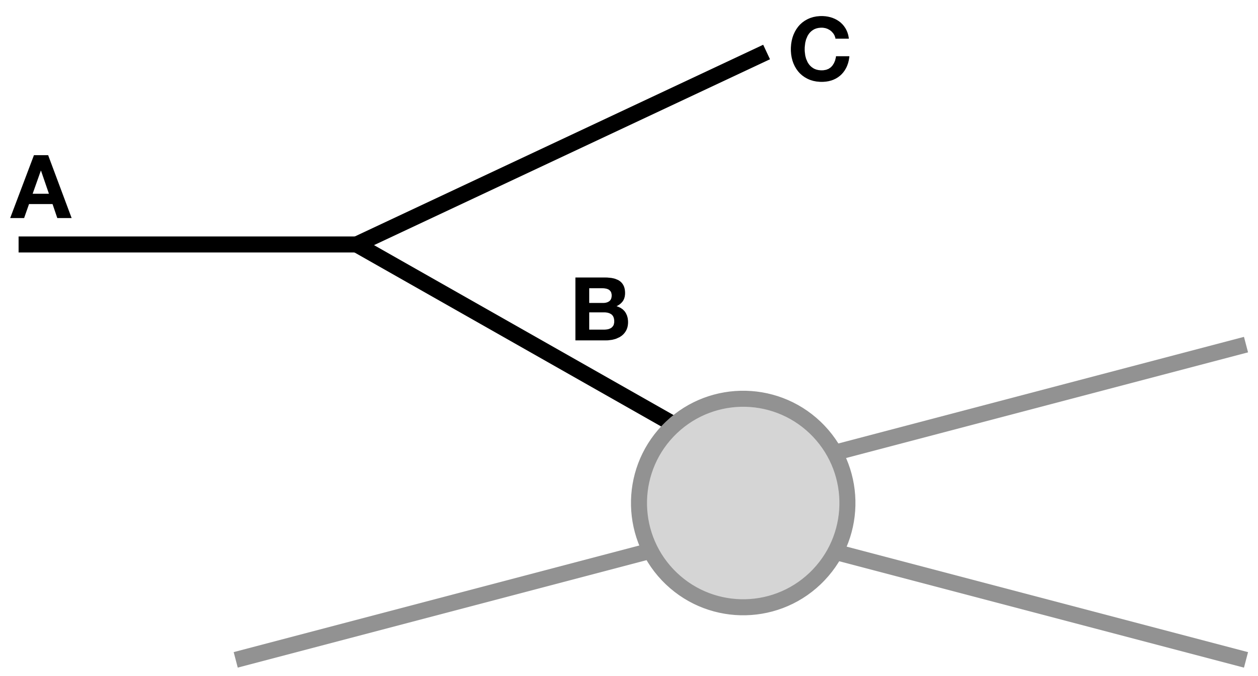
The **effect of finite EW masses is sizeable** even at TeV scales.

The kinematical effect of the mass of particle C is instead negligible in the collinear limit

$$E_C = (z\text{-}x)\, E > m_C \qquad z \geq x + \frac{m_C}{E}$$

For $E \gg p_T, m$, **we can neglect this effect**.

Ultracollinear splittings



In the unbroken phase, splitting matrix elements are proportional to $p_T{}^2$

$$|\mathcal{M}(A\to B+C)|^2 \equiv 8\pi\alpha_{ABC}\frac{p_T^2}{z\bar{z}}P_{BA}^C(z)$$

Ultra-collinear splitting function Chen, Han, Tweedie [1611.00788]

Upon EWSB, further splittings proportional to v^2 are generated.
They generalise the EWA splitting $f \to W_L f'$

$$|\mathcal{M}_{A\to B+C}|^2 \equiv \frac{v^2}{z\bar{z}}P_{BA,C}^{u.c.}(z)$$

For example: $$P^{u.c.}_{f_L^{(2)} f_L^{(1)},W_L}(z) = \left(y_{f_1}^2 z\bar{z} - y_{f_2}^2\bar{z} - \boxed{g_2^2 z}\right)^2\frac{1}{2\bar{z}_+}$$

coupling of massless fermions to W_L, with no chirality flip
(via coupling to remainder gauge field W_n in GEG)

The missing $p_T{}^2$ factor removes the log enhancement at high scales, making the **u.c. terms approach a constant value**.

The DGLAP equations are generalised as:

$$Q^2\frac{df_B(x,Q^2)}{dQ^2} = P_B^v\, f_B(x,Q^2) + \sum_{A,C}\frac{\alpha_{ABC}}{2\pi}\widetilde{P}_{BA}^C\otimes f_A + \frac{v^2}{16\pi^2Q^2}\sum_{A,C}\widetilde{U}_{BA}^C\otimes f_A$$

LePDF: Numerical Implementation

We solve the DGLAP numerically in x space. Due to the sharp behaviour of the muon PDF near x=1, the typical interpolation techniques used for PDFs of proton do not work.

We discretise x interval $[x_{min}=10^{-6},1]$ in N_x small intervals, denser for $x \approx 1$:

$$x_\alpha = 10^{-6((N_x-\alpha)/N_x)^{2.5}}$$

$$\alpha = 0, 1, \ldots, N_x$$

For the splitting functions divergent in $z \longrightarrow 1$ we us the "+" distribution

$$\int_x^1 dz \frac{f(z)}{(1-z)_+} = \int_x^1 dz \frac{f(z)-f(1)}{1-z} - f(1)\int_0^x \frac{dz}{1-z} = \int_x^1 dz \frac{f(z)-f(1)}{1-z} + f(1)\log(1-x)$$

The differential evolution is done in $t = \log Q^2/m_\mu^2$ with 4th order Runge-Kutta.

At x=1 we fix

$$f_{iN_x}(t) = \begin{cases} \frac{L(t)}{\delta x_{N_x}} & i = \mu \\ 0 & i \neq \mu \end{cases}$$

where L(t) is fixed imposing momentum conservation:

Han, Ma, Xie [2103.09844]

$$L(t) = 1 - \sum_{i=1}^{N_f} \sum_{\alpha=1}^{N_x-1} \delta x_\alpha x_\alpha f_{i\alpha}(t)$$

The uncertainties due to x and t discretisation are estimated to be of ~1% and ~0.1%, respectively, for $N_x=1000$.