New tests of quantum mechanics at the LHC

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Basic question:
why?

Why?

☑ Quantum mechanics is a fundamental pillar of modern physics!
We have to test QM at all times!

In quantum information jargon, entanglement is usually studied for:

- qubits: systems with 2 possible states.

  ➡ Example: spin of top quarks, tau leptons

- qutrits: idem, with 3 states.

  ➡ Example: spin of W, Z bosons

- …

All the tests, formalism, etc. developed there can be applied to spins of particles produced at LHC and other colliders

Why?

LHC offers a variety of processes to test QM at the energy frontier.

- Top pair production

Afik, Nova 2003.02280, 2203.05582, 2209.03969
Fabbrichesi et al. 2102.11883
Maltoni et al. 2110.10112
JAAS , Casas 2205.00542
Dong et al. 2305.07075
Han, Low, Wu 2310.17696

- Higgs decays $H \to WW$

Barr 2106.01377
JAAS 2208.14033
Fabbri, Howarth, Maurin 2307.13783

- Higgs decays $H \to ZZ$

JAAS, Bernal, Casas, Moreno 2209.13441

- Electroweak production

Ashby-Pickering, Barr, Wierzchucka 2209.13990
Fabbrichesi, Floreanini, Gabrielli, Marzola 2302.00683

- VBF

Morales 2306.17247

- Other

JAAS 2307.06991, 2401.10988, 2402.14725
JAAS, Casas 2401.06854

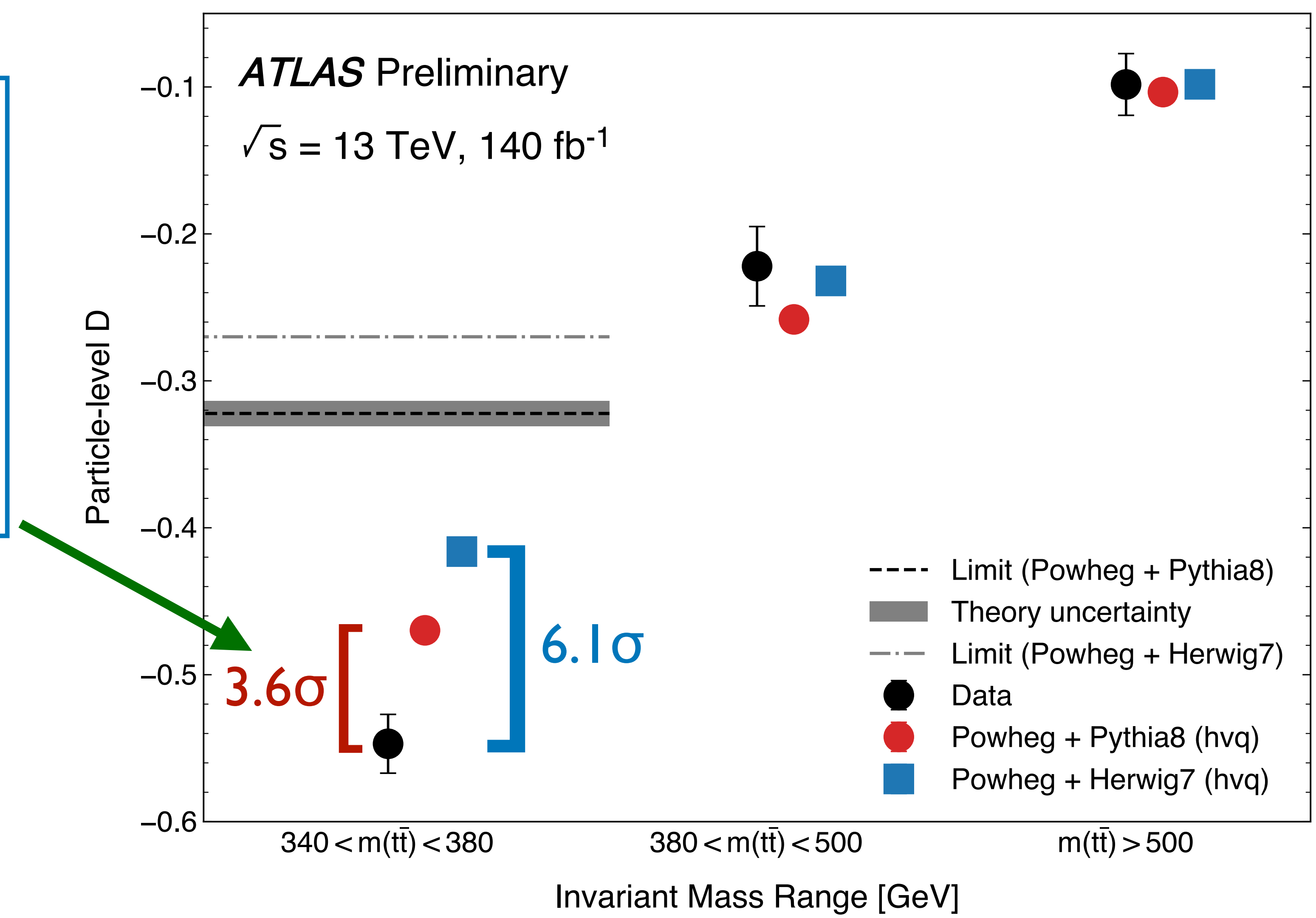
Why?

Entanglement measurements are quite demanding, and provide a stress test of our current understanding of

- theoretical modeling
- experimental systematic uncertainties

Example: ATLAS entanglement measurement in top pair production

Even if ATLAS does not make such claim, everybody can see that predictions and measurement are quite off and digitise the plot



Why?

☑ Novel entanglement tests that were not possible before.

What is genuinely new in particle physics with respect to experiments with electrons and photons? Particle decay.

JAAS 2307.06991
JAAS, Casas 2401.06854
JAAS 2401.10988

- Post-decay entanglement:

A and B entangled
$A \to A_1\ A_2$

➡

A_1, A_2 and B entangled
A_1 and B entangled

JAAS 2308.07412

- Entanglement and post-selection:

time ↓

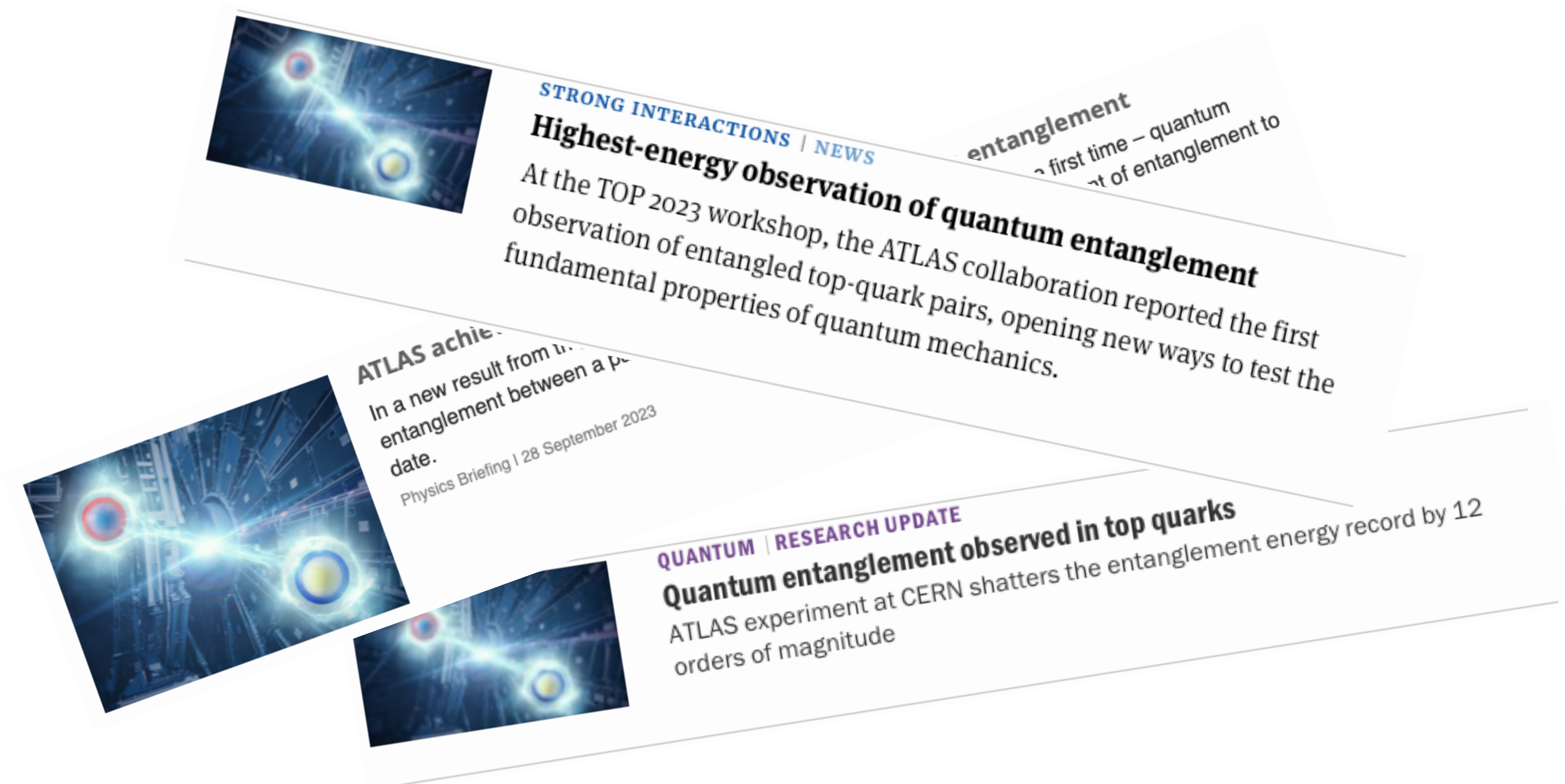
A and B entangled
$A \to A_1\ A_2$
Measurement on B

➡

$\asymp$ spin selection on A,
which already has decayed

Why?

☑ It is a new topic that gets headlines and publicity



STRONG INTERACTIONS | NEWS

Highest-energy observation of quantum entanglement

At the TOP 2023 workshop, the ATLAS collaboration reported the first observation of entangled top-quark pairs, opening new ways to test the fundamental properties of quantum mechanics.

ATLAS achie...

In a new result from th... entanglement between a p... date.

Physics Briefing | 28 September 2023

QUANTUM | RESEARCH UPDATE

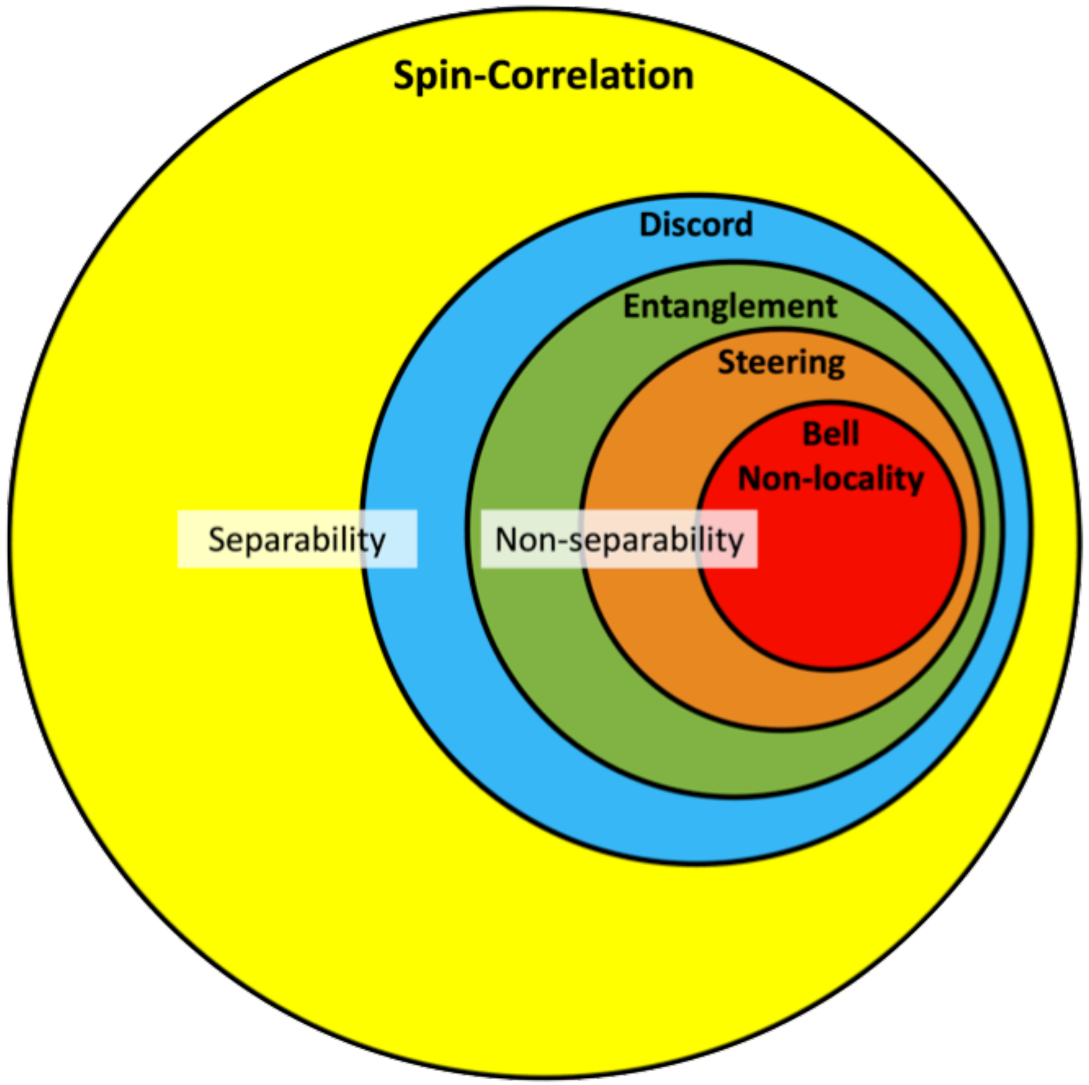
Quantum entanglement observed in top quarks

ATLAS experiment at CERN shatters the entanglement energy record by 12 orders of magnitude

So, what is to be looked for?

What?

There are many levels of quantum correlations



Captured from Yoav Afik talks

- Spin correlation: statistical correlation between spins, classical
- Discord: quantum correlations yet in separable states
- Entanglement: subsystems are not separable
- Steering: measurement in one subsystem influences the other
- Bell non-locality: correlation cannot be described by local hidden variables

What?

Example: top pair production

- q_L q_L-bar → *t t*-bar at threshold gives a spin configuration $|\leftarrow\rangle \otimes |\leftarrow\rangle$ that is obviously separable [in the q direction]

- q_R q_R-bar → *t t*-bar at threshold gives a spin configuration $|\rightarrow\rangle \otimes |\rightarrow\rangle$ that is separable too [in the q direction]

⇒ q q-bar → *t t*-bar gives 50% of each [density operator], separable.

We do have a *classical* spin correlation

- *g g* → *t t*-bar at threshold gives $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle\right)$

This one is entangled [actually, it is maximally entangled, violates Bell inequalities, etc.]

The mathematical formulation for e.g. entanglement in mixed states are complicated, so I skip it. If curious, see backup.

OK, but how?

How?

If we want to study quantum information stuff with the spin of elementary particles, we have to measure it. All of it! → density operator

As we all know, top quarks, *W/Z* bosons, … even τ leptons decay before one can pass them through a Stern-Gerlach experiment to measure spin.

But: the spin leaves its imprint in angular distributions.

$$\rho = \frac{1}{2}\begin{pmatrix} 1+P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1-P_3 \end{pmatrix}$$

density operator for spin 1/2

take daughter particle as spin analyser

constant

momentum direction of daughter particle

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi}(1+\alpha\vec{P}\cdot\hat{n})$$

$$\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

How?

Top pair: two spin-1/2 particles, simplest example of quantum correlation

$$\rho = \frac{1}{4}\left(1 \otimes 1 + \sum_i B_i^+ \sigma_i \otimes 1 + \sum_i B_i^- 1 \otimes \sigma_i + \sum_{ij} C_{ik} \sigma_i \otimes \sigma_j\right)$$

normalisation

$$\hat{n}_a = (\sin\theta_a \cos\varphi_a, \sin\theta_a \sin\varphi_a, \cos\theta_a)$$
$$\hat{n}_b = (\sin\theta_b \cos\varphi_b, \sin\theta_b \sin\varphi_b, \cos\theta_b)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}\left[1 + \alpha_a \vec{B}^+ \cdot \hat{n}_a + \alpha_b \vec{B}^- \cdot \hat{n}_b + \alpha_a \alpha_b \hat{n}_a^T C \hat{n}_b\right]$$

3 coefficients corresponding to top polarisation

3 coefficients corresponding to anti-top polarisation

9 spin correlations

Measured by ATLAS and CMS since some time

How?

For two qubits [e.g. spin-1/2 fermions] sufficient entanglement conditions are

$$|C_{11} + C_{22}| > 1 + C_{33} \quad \text{or} \quad |C_{11} - C_{22}| > 1 - C_{33}$$

Afik, Nova 2003.02280
Maltoni et al. 2110.10112
JAAS , Casas 2205.00542

And Bell-like inequalities are violated if

$$|C_{ii} + C_{jj}| > \sqrt{2} \quad \text{or} \quad |C_{ii} - C_{jj}| > \sqrt{2}$$

Maltoni et al. 2110.10112
JAAS , Casas 2205.00542

For *H* → *VV* [spin 1, extra symmetry] sufficient entanglement conditions are

$$C_{212-1} \neq 0 \quad \text{or} \quad C_{222-2} \neq 0$$

JAAS, Bernal, Casas, Moreno 2209.13441

And [optimised] sufficient condition for violation of Bell-like inequalities

$$I_3 = \frac{1}{36}\left[(18 + 16\sqrt{3}) - \sqrt{2}(9 - 8\sqrt{3})A^1_{20} - 8(3 + 2\sqrt{3})C_{212-1} + 6C_{222-2}\right] > 2$$

JAAS, Bernal, Casas, Moreno 2209.13441

For different dimensions, fall back into Peres-Horodecki criterion [backup]

How?

To take away:

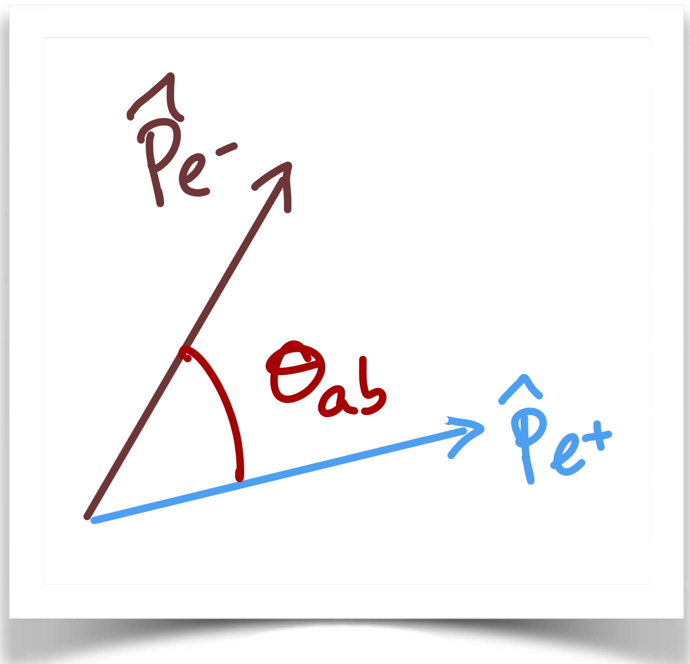
- ☑ You need to measure elements of spin density operator of composite system
- ☑ The spin can be accessed through distributions of decay products
- ☑ But for that you need to reconstruct rest frame, leaving only top, *W* and *Z* as candidates at LHC
- ☑ For τ leptons it is possible too, at e^+e^- colliders
- ☑ Orbital angular momentum cannot directly be accessed but this is another story…

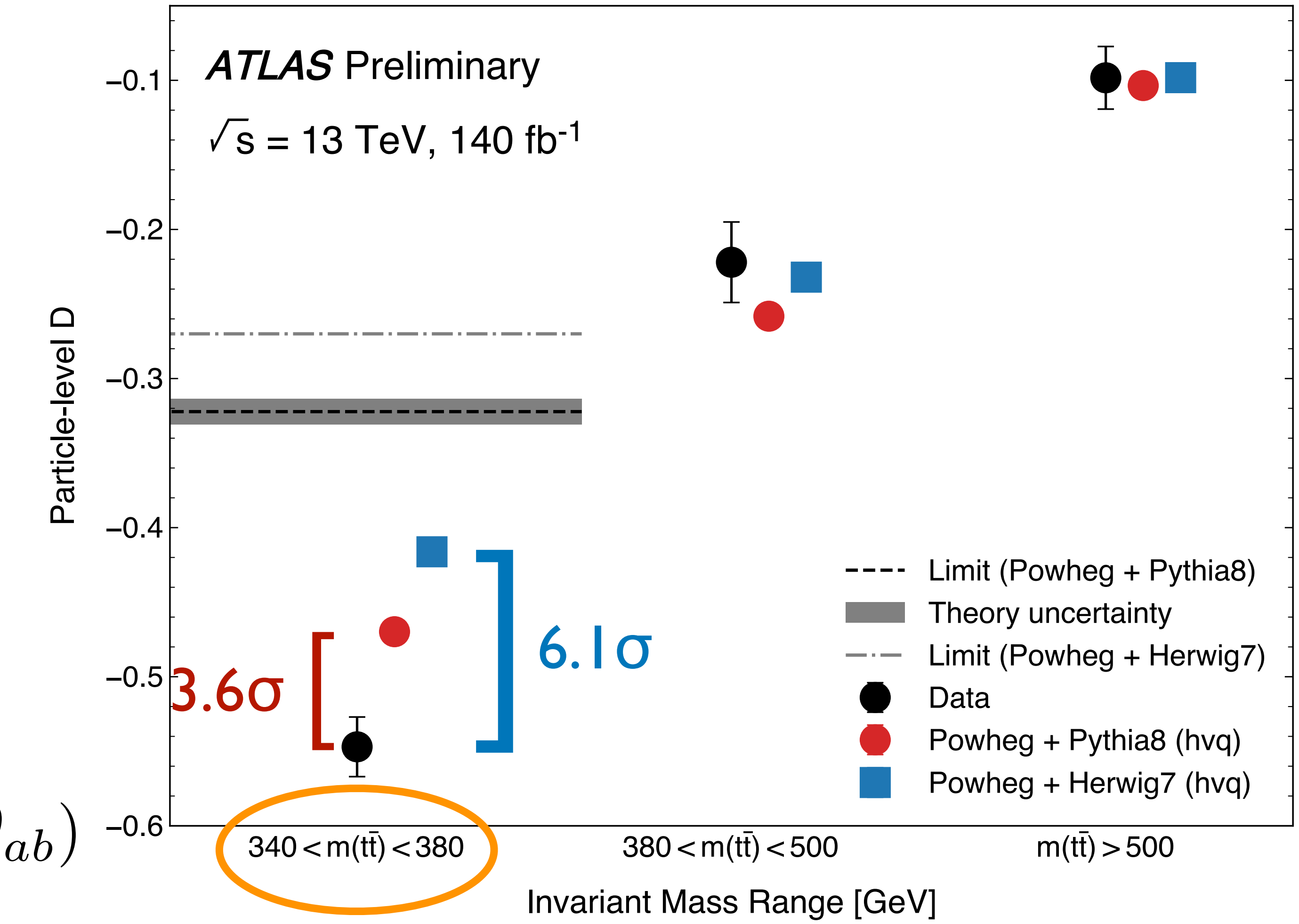
JAAS 2402.14725

Current status

Current status

ATLAS has performed [and CMS is pursuing] a measurement at threshold using the D observable, related to the angle between the two leptons





$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{ab}} = \frac{1}{2}\left(1+\alpha_a\alpha_b D\cos\theta_{ab}\right)$$

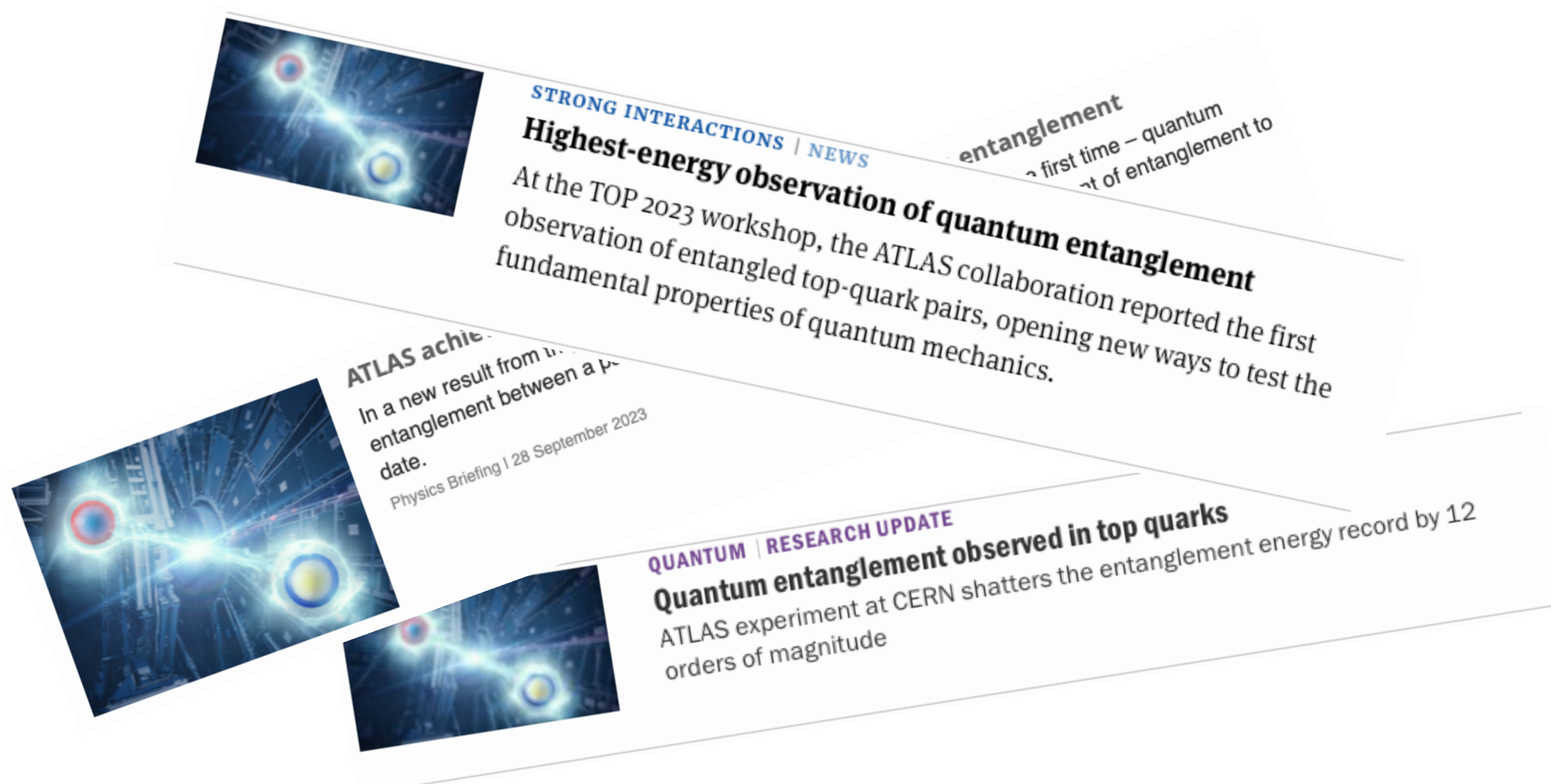
$$D = \frac{1}{3}(C_{11}+C_{22}+C_{33})$$

Entanglement test near threshold: $-3D-1>0$

Bottom line: we know there are spin correlations since a decade, but entanglement is a stronger condition

Current status

Testing basic properties of quantum mechanics with different particles, and higher energies, is very nice.

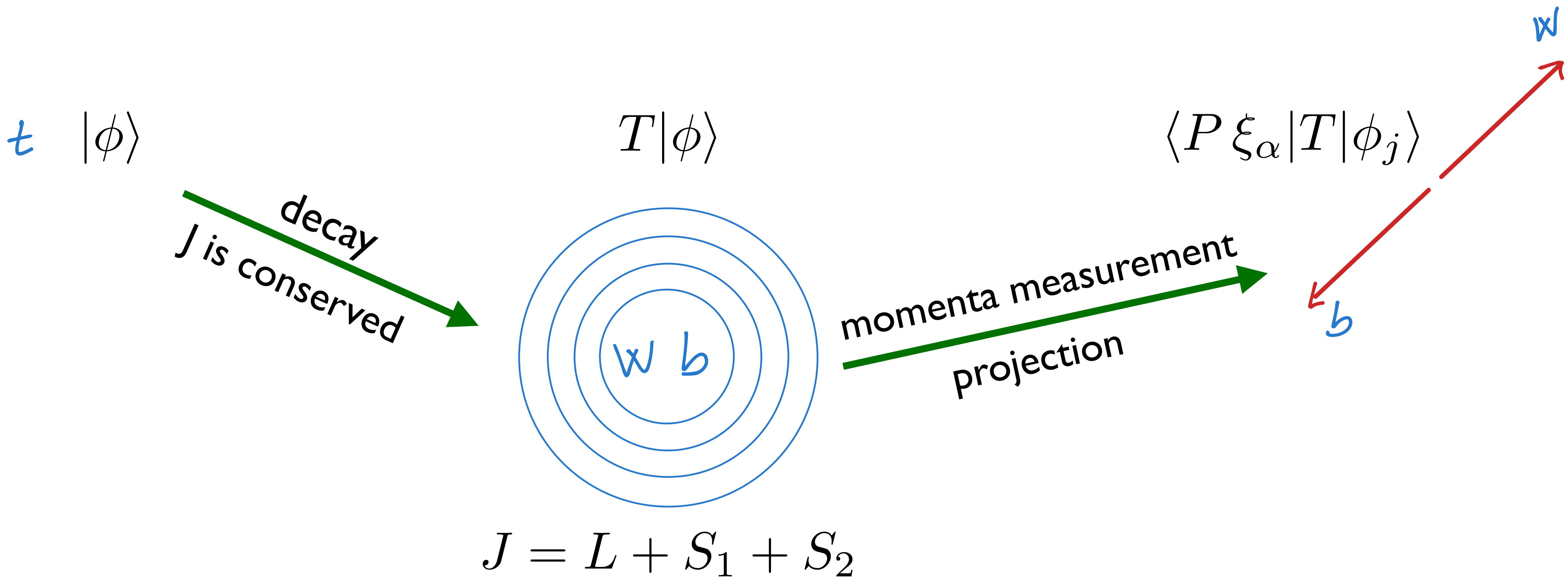


But as I have stressed, there are some tests that can be performed at colliders that cannot be [and have not been] done anywhere else: entanglement and decay.

Novel tests: decay and entanglement

Deconstructing particle decay

Example: top quark decay $t \rightarrow Wb$

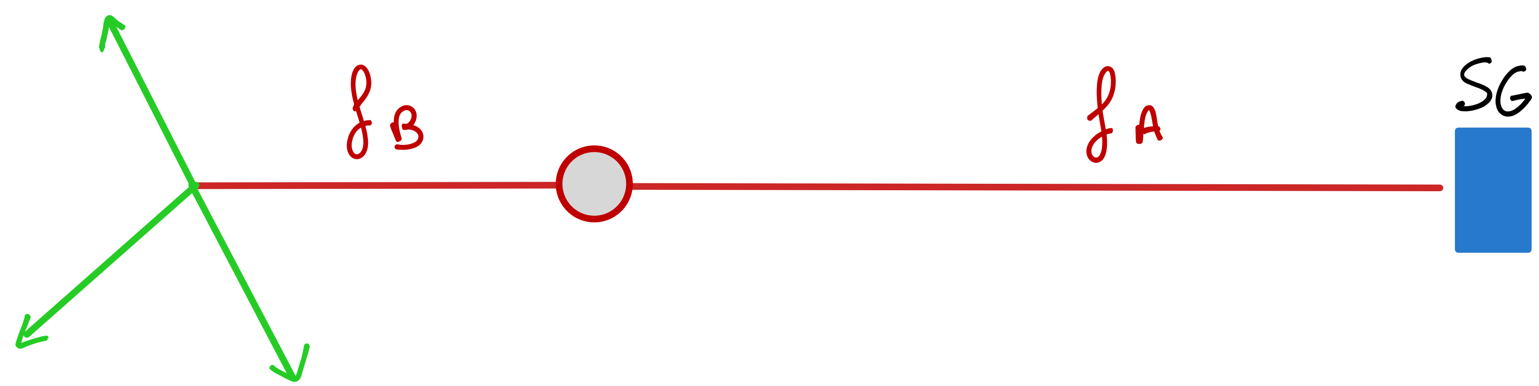


The measurement of momenta influences the spin state but in general it does not collapse it as a Stern-Gerlach experiment would do.

A post-selection experiment

Fermion pairs f_A f_B produced in an entangled state, say $\frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right]$

f_B decays and after that, we perform a Stern-Gerlach experiment on f_A



We select the subset of f_B for which the result of the SG experiment on f_A gives $|\uparrow\rangle$

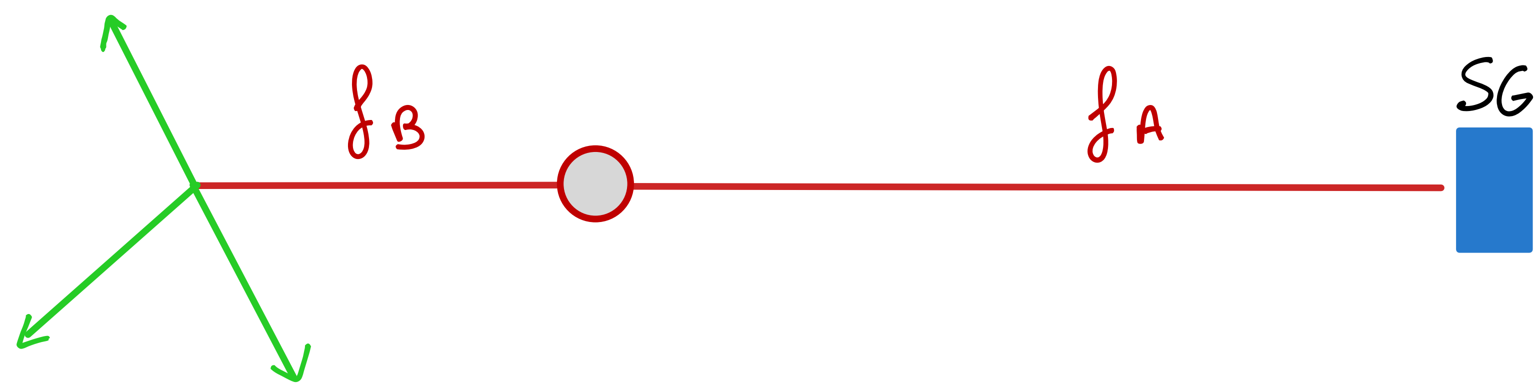
Then, the decay distribution of those f_B that had decayed before the outcome of the SG experiment corresponds to having spin $|\downarrow\rangle$

Magic? Spooky EPR action to the past? Not really. It is due to the projection.

A post-selection experiment

The initial state is $\frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right]$ and if we do a SG on f_A before f_B decays, we get up or down with equal probability.

The decay of f_B projects f_A into a state $a_+|\uparrow\rangle + a_-|\downarrow\rangle$ with a_+, a_- depending on the decay configuration. The probability to have SG up or down is not the same.



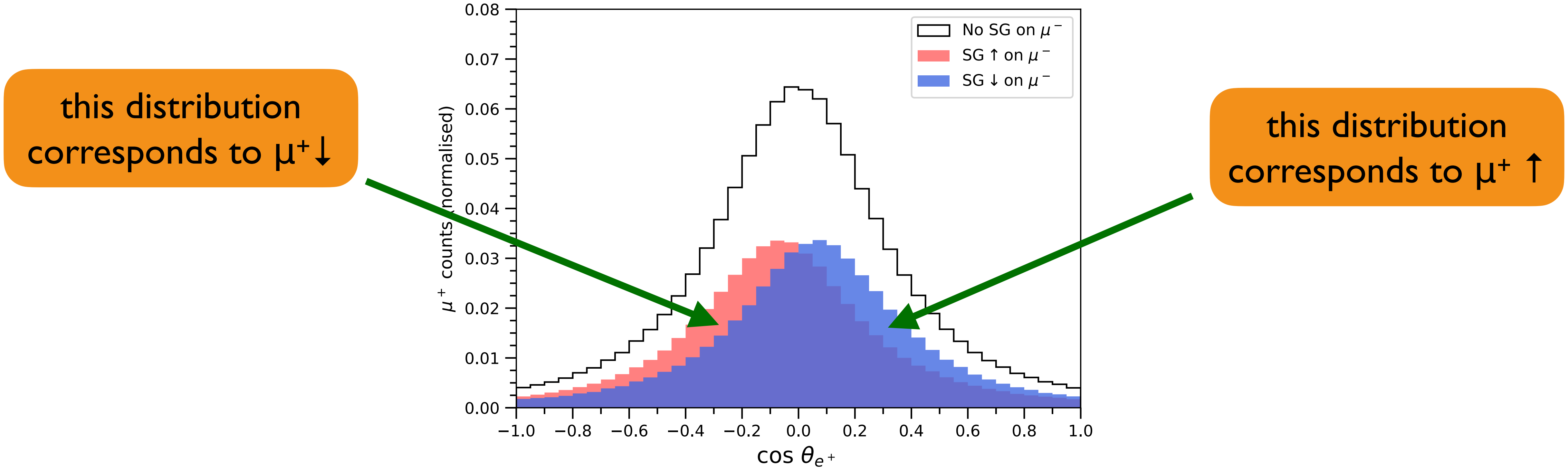
Because of this projection, if we post-select events where SG gives $|\uparrow\rangle$, we recover f_B decay distributions just as if f_B had spin $|\downarrow\rangle$ when it decayed.

This is a genuine entanglement effect. We can set our SG in any direction and even violate Bell inequalities.

A post-selection experiment

This experiment can be performed with low-energy $\mu^+\mu^-$ pairs produced in Drell-Yan or from the decay of a η meson

The muon polarisation can be measured from the daughter electron



Related: neutral kaon post-tag [Bernabéu, di Domenico 1912.04798] but the correlation presented [# decays vs time] does not seem a genuine quantum correlation <u>in my opinion</u> [the discussion is complicated]

Post-decay entanglement: formalism

Consider a system of two particles *A*, *B*, with spin state described by

$$\rho = \sum_{ijkl} p_{ij}^{kl} |\phi_i\, \chi_k\rangle\langle\phi_j\, \chi_l| \qquad |\phi_i\rangle \in \mathcal{H}_A\,, \quad |\chi_k\rangle \in \mathcal{H}_B$$

$\mathcal{H}$ are the spin spaces

Let *A* decay $A \to A_1\, A_2\, \dots$ with amplitudes

$$M_{\alpha j} = \langle P\,\xi_\alpha|T|\phi_j\rangle \qquad |\xi_\alpha\rangle \in \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \dots$$

Then, the spin state of $A_1\, A_2\, \dots$ and *B* is described by

these come from the projector

$$\rho' = \frac{1}{\sum_{\alpha k} (M\rho^{kk} M^\dagger)_{\alpha\alpha}} \sum_{\alpha\beta kl} (M\rho^{kl} M^\dagger)_{\alpha\beta} |\xi_\alpha\, \chi_k\rangle\langle\xi_\beta\, \chi_l|$$

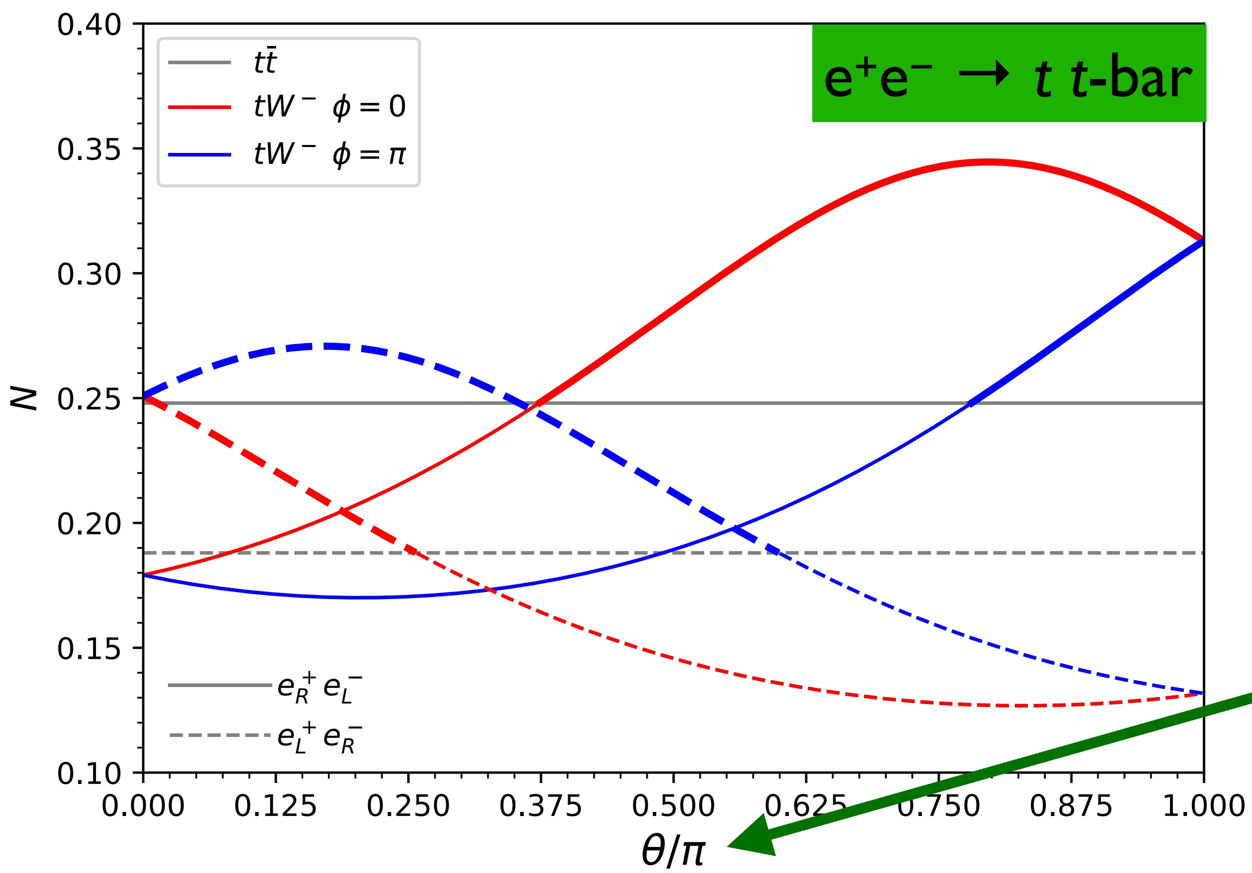
Entanglement between *A* and *B* is inherited by the decay products of *A*

Entanglement autodistillation

Entanglement decreases by measurements [collapse], interaction with environment [decoherence] …

Methods are known [distillation] to manipulate a sub-system and, if lucky, increase entanglement

Most remarkably, the decay can increase entanglement spontaneously.



Unique effect that requires large luminosity to be observed

polar angle between W momentum in top rest frame and top direction in c.m. frame

Post-decay entanglement at LHC

Post-decay entanglement can be measured in top pair production at LHC

When t t-bar are entangled and t-bar decays into W^- b-bar,
t is entangled with the W^- b-bar pair

Potential problem:

When we have several entangled particles and trace over [unobserved] degrees of freedom, entanglement may be lost.

→ b-bar has RH helicity up to small mass effects, trace maintains entanglement

Sufficient condition for entanglement: [...finally I had to name it 😖]

The operator ρ^{T2} where the transpose is taken in the W spin sub-space, has a negative eigenvalue.

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

Post-decay entanglement at LHC

Threshold region $m_{tt} \leq 390$ GeV, $\beta \leq 0.9$, beamline basis z = (0,0,1)

θ 👉 angle between W^- momentum in t-bar rest frame and z axis *or any fixed axis*

entanglement measure

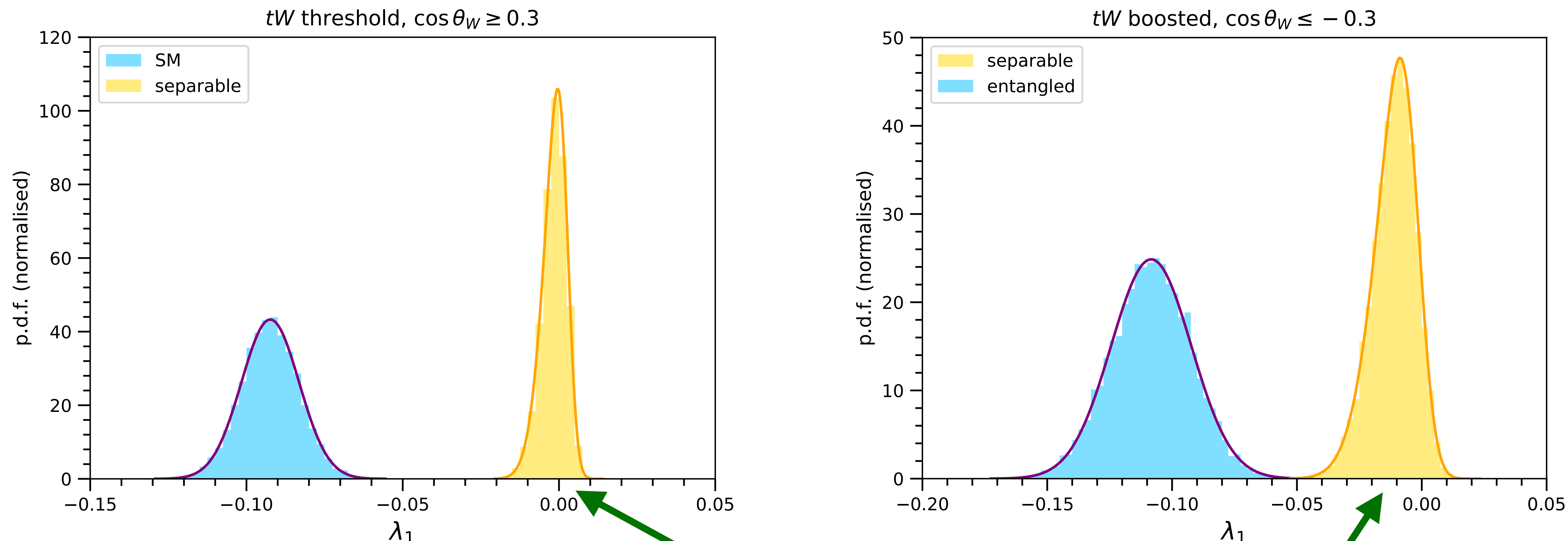
	N(ρ)
θ = 0	0.13
cos θ > 0.9	0.12
cos θ > 0.5	0.10
cos θ > 0	0.07
all θ	0

The amount of entanglement is the same in any direction but the quantum state is not, so integration washes out entanglement

Post-decay entanglement at LHC

Entanglement indicator:
lowest eigenvalue λ_1 of the ρ^{T2} matrix for *tW*

$\boxed{\lambda_1 < 0 \Leftrightarrow \text{Entanglement}}$



Run 2	Significance [stat + 10% sys + bias]
Threshold	7.0 σ
Boosted	5.0 σ

Bias: even if $\lambda_1 > 0$, in a small sample we may find it negative

Novel tests: decay and entanglement

To take away

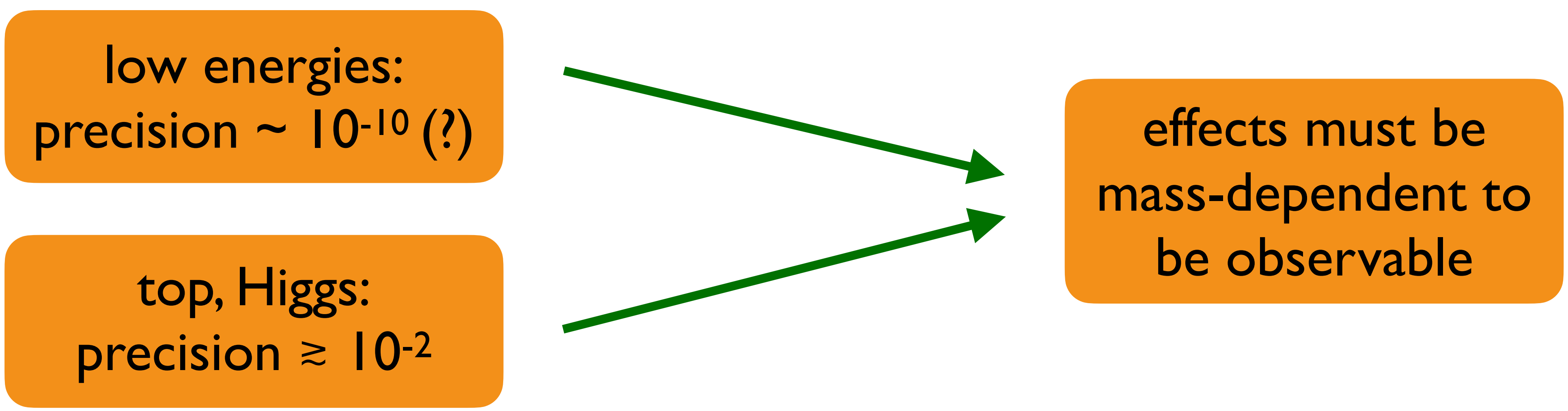
- Particle decay and subsequent momenta projection is a very special kind of "measurement"
- Unique QM effects:
  - post-selection
  - autodistillation
- Post-decay entanglement never tested, test is possible at LHC with current data

End

Why?

Q: Should we see any breaking of QM at the LHC?

A: it is not clear that we should see any effect at LHC even if QM has to be corrected (e.g. with non-linear terms)



… and it remains to be shown that effects should precisely be seen in entanglement measurements!

Why?

☑ Looking for new physics

Yes, but only if we use dedicated observables.

Example: ATLAS and CMS measured spin-correlation coefficients C_{kk}, C_{rr}, C_{nn} in t t-bar production.

If we consider entanglement observables [explanations later]

$$
\begin{aligned}
C_{kk} + C_{rr} + C_{nn} &\equiv 3D \\
C_{kk} + C_{rr} - C_{nn} &\equiv 3D_3
\end{aligned}
$$

and measure them indirectly from C_{kk}, C_{rr}, C_{nn}, it is unlikely to have any sensitivity gain.

The way to improve sensitivity is to consider observables that directly measure D and D_3 from distributions.

[an observable for D is known since long]

Why?

Entanglement observables involve spin correlations, which are sensitive to new physics.

→ we can parameterise deviations from SM in terms of dim-6 operators, which provide a definite framework for comparisons

Spin correlations are measured with angular distributions, with a relation that may be modified by new physics

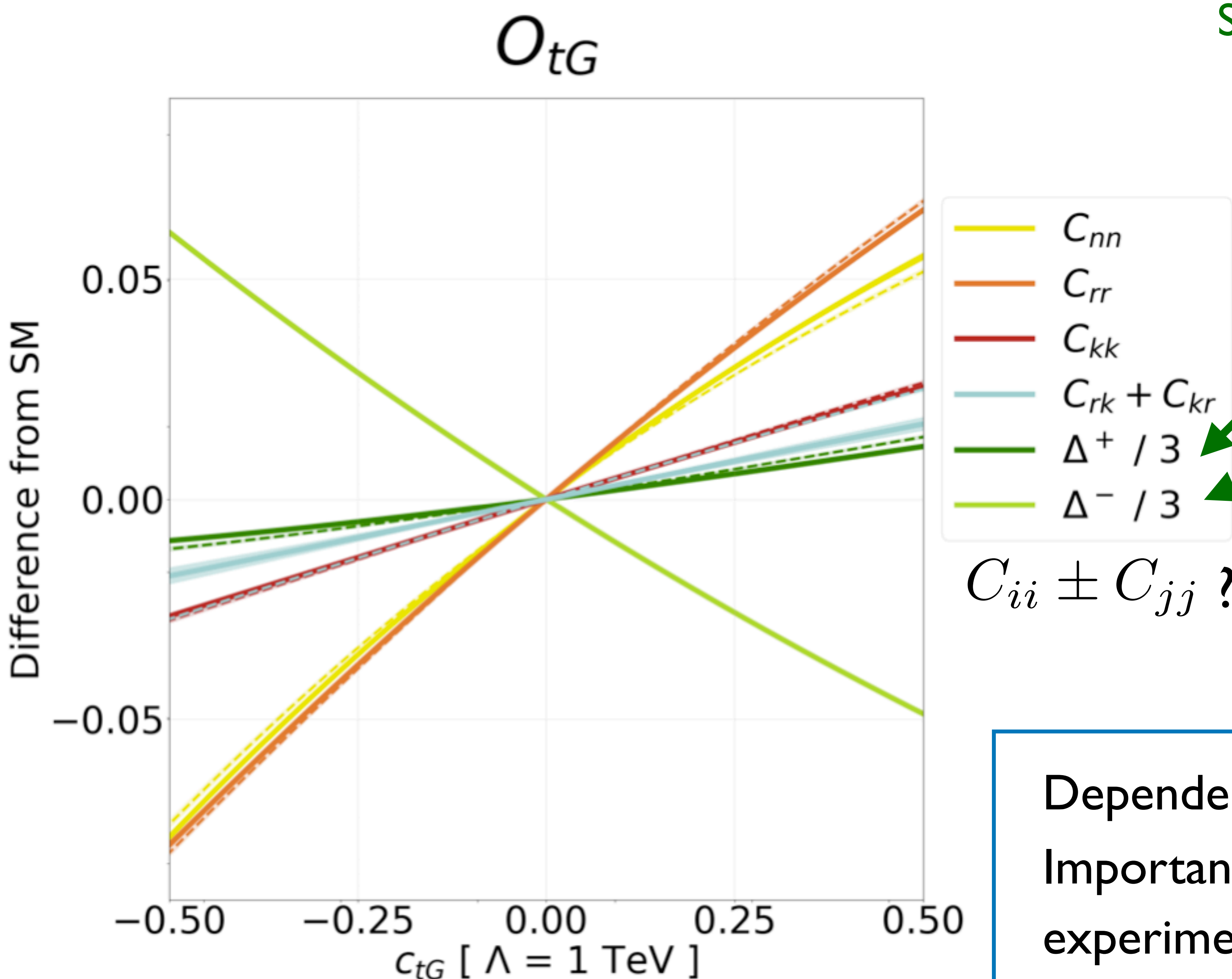
→ we can also introduce dim-6 operators for the decay of top, *W*, *Z*, but typically there are better ways to constrain them

EFT is <u>not</u> a model. When evaluating sensitivity, one should beware flat directions, which may be natural in actual models

Why?

t t-bar example: top chromomagnetic dipole operator

Severi, Vryonidou, 2210.09330



$C_{ii} \pm C_{jj}$???

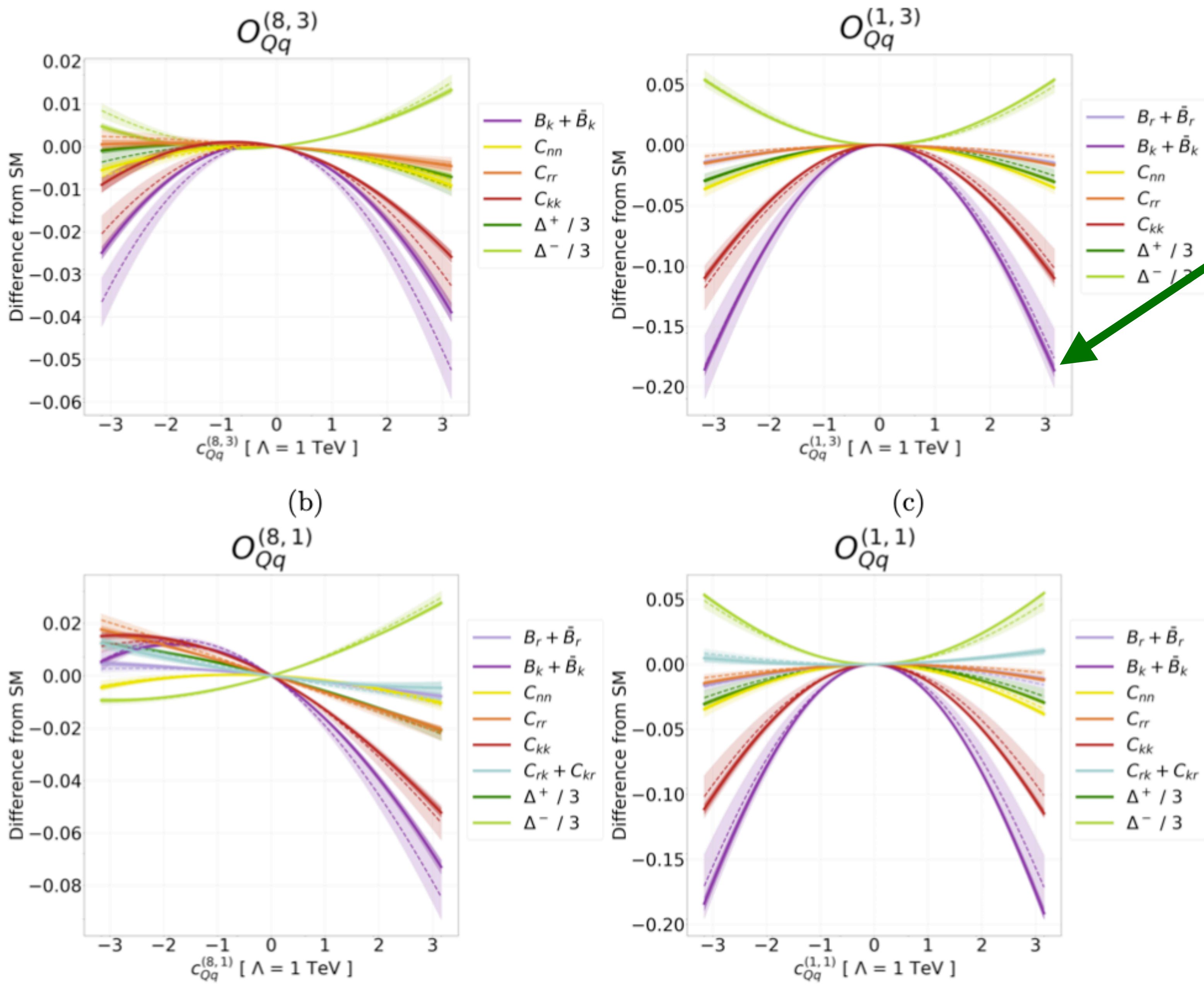
this is a new observable

this is essentially the old good D

Dependence on c is the first step. Important missing piece: expected experimental error bars for these quantities

Why?

t t-bar example: some four-fermion operators



polarisation in helicity direction

Polarisation seems to outperform the rest of observables [note that experimental uncertainties are likely smaller] but this statement is basis-dependent (!)

Why?

$H \to ZZ$ example: test anomalous HZZ interaction Fabbrichesi et al. 2304.02403

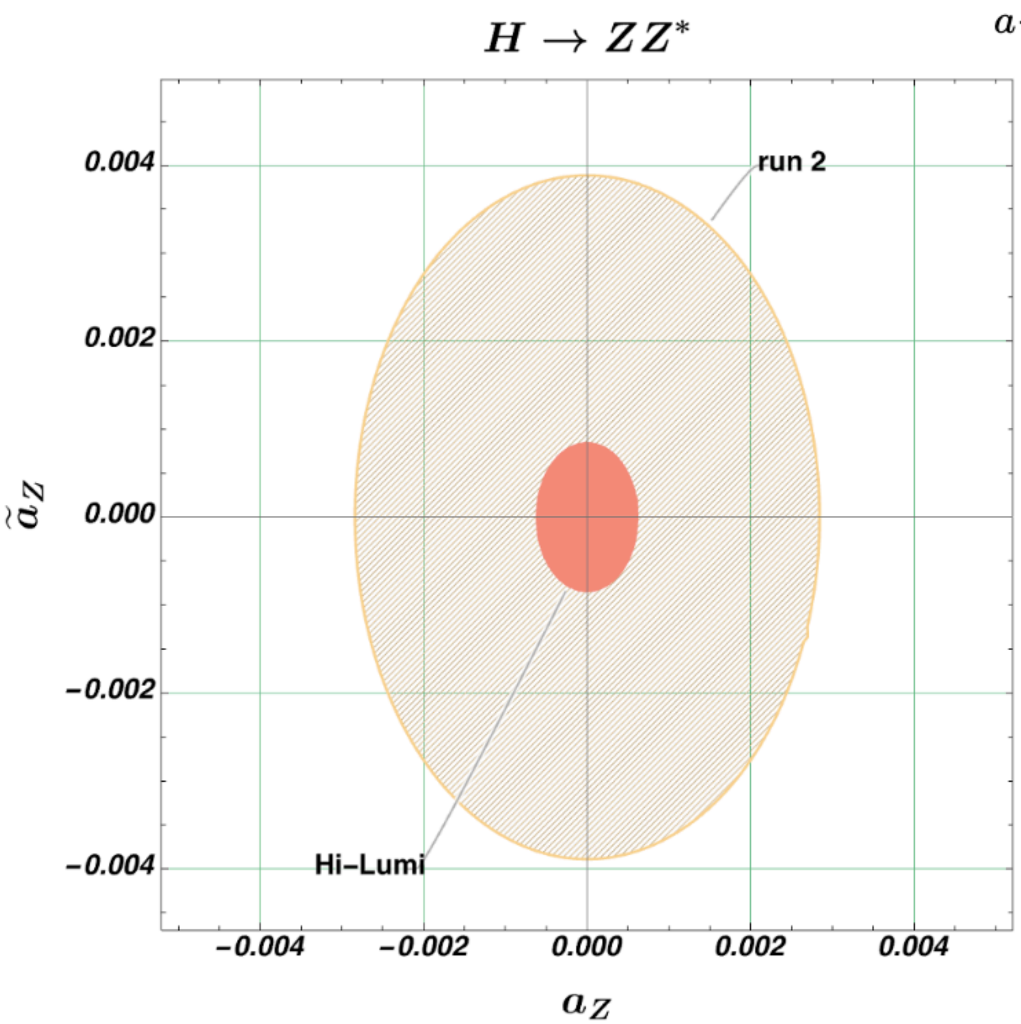
entanglement witness

$$\mathscr{C}_2 = 2\max\Big[0,\ -\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,,$$
$$-\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2\Big]$$

nothing to do with entanglement

$$\mathscr{C}_{\rm odd} = \frac{1}{2}\sum_{\substack{a,b\\ a<b}}\big|h_{ab} - h_{ba}\big|$$

parameters of ZZ density matrix



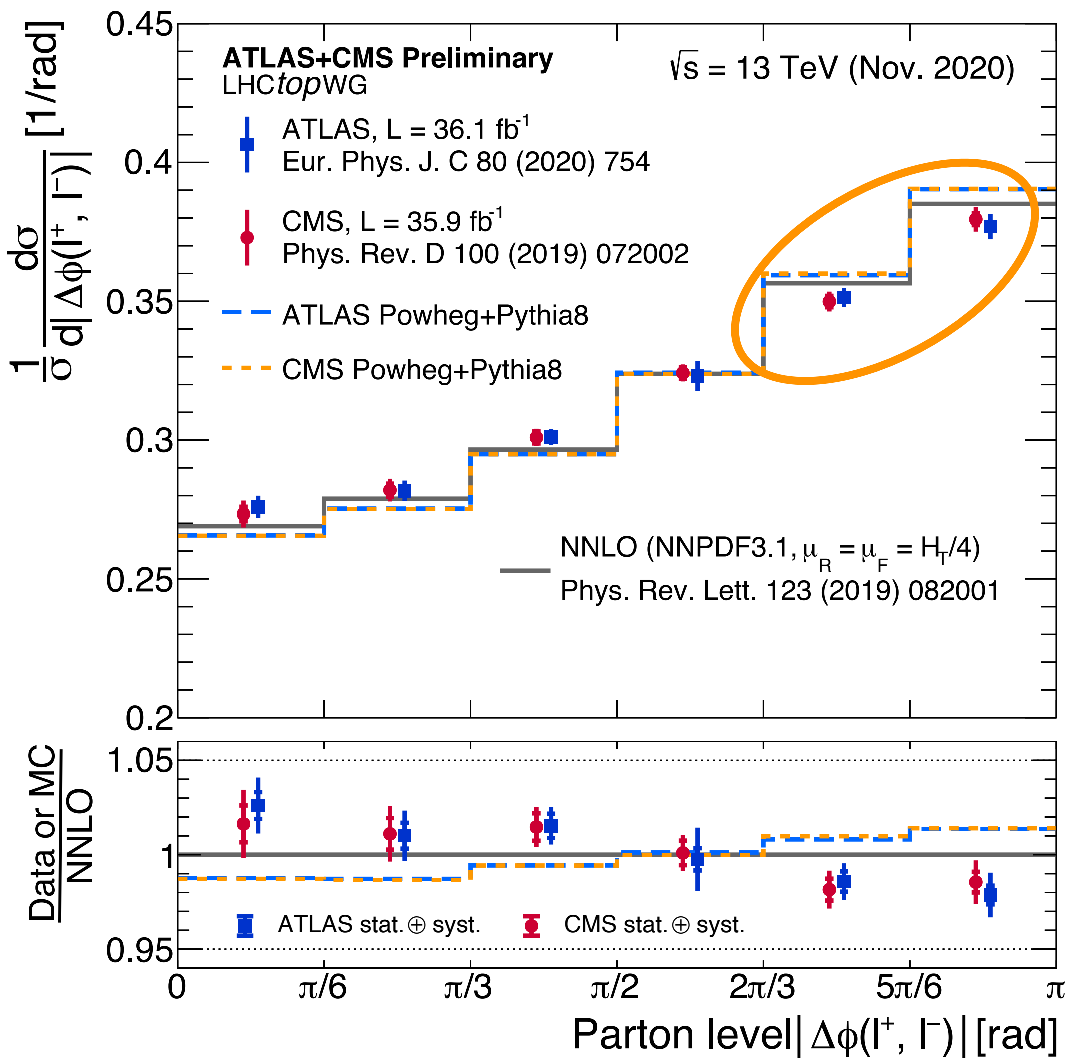
- Why not using ZZ density matrix elements instead ofx $\mathscr{C}_2$?
- Why use $\mathscr{C}_{\rm odd}$ and not dedicated triple-product observables?
- Same applies to EW diboson production Aoude et al. 2307.09675

Why?

Remember the ΔΦ anomaly in top pair production

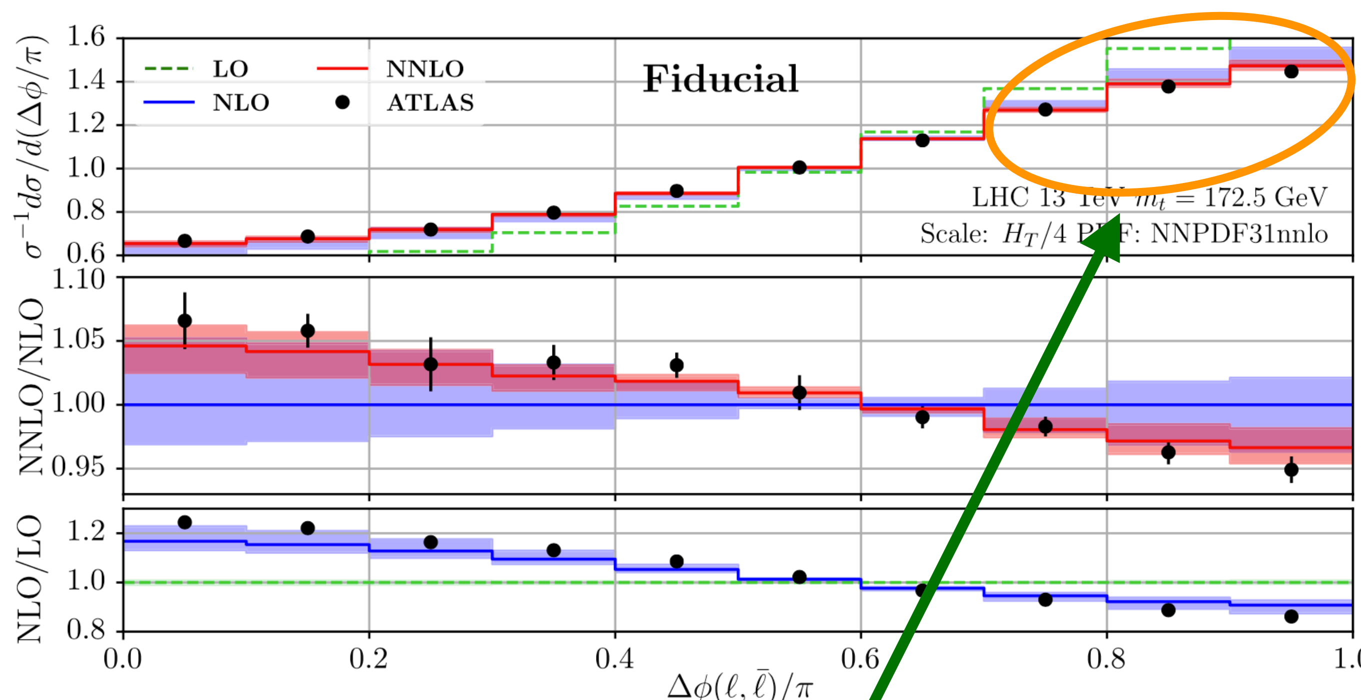
lab-frame azimuthal angle between leptons

Parton level full phase space



Parton level fiducial

Behring et al. 1901.05407



better agreement

New physics explanations break Δη and σ, see here and here

What?

Any operator cannot be a density operator. A valid density operator has several characteristics:

- Unit trace
- Hermitian
- Positive semidefinite: eigenvalues ≥ 0

A density operator describing a composite system is separable if it can be written as

$$\rho_{\rm sep} = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

Note: in general, one has something like

$$\rho = \sum_{ijkl} p_{ij}^{kl} |\psi_i\rangle\langle\psi_j| \otimes |\psi_k\rangle\langle\psi_l|$$

What?

Necessary criterion for separability:

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

taking the transpose in subspace of B [for example] the resulting density operator is valid.

Example: composite system A ⊗ B with dim $\mathcal{H}_A$ = n, dim $\mathcal{H}_B$ = m

P_{ij} are m x m matrices, $(P_{ij})^{kl} = p_{ij}^{kl}$

$$\rho = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & & \\ \vdots & & \ddots & \\ P_{n1} & & & P_{nn} \end{pmatrix} \Rightarrow \rho^{T_2} = \begin{pmatrix} P_{11}^T & P_{12}^T & \cdots & P_{1n}^T \\ P_{21}^T & P_{22}^T & & \\ \vdots & & \ddots & \\ P_{n1}^T & & & P_{nn}^T \end{pmatrix}$$

(n∗m) × (n∗m) matrix

Not easily tractable!

What?

To take away:

- It is quite complicated to prove [analytically] that a composite system is in a separable state.
- Numerically, it can be done but there may be a bias [see later]
- However, we are interested in showing that the system is entangled.
- To prove that, in some systems there are simple sufficient conditions that do the work

ρ^{T2} non-positive $\Rightarrow$ ρ^{T2} not valid $\Rightarrow$ system entangled

Showing this for a single vector is enough $\rightarrow$ simple conditions

What?

A useful formulation of Bell-like inequalities for spin-1/2 systems is provided by the so-called CHSH inequalities for two systems A (Alice) and B (Bob).

Clauser, Horne, Shimony, Holt, '69

Alice measures two spin observables A, A'. Bob measures two spin observables B, B'. [Both normalised to unity]. Then, clasically:

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \leq 2$$

these are spin correlation observables!

One can show violation of CHSH inequalities if one finds spin observables A, A' for Alice and B, B' for Bob such that the inequality is violated.

in a given quantum state!

What?

The CHSH inequalities involve spin correlations. Therefore, for a particle of spin 1/2, they involve the C_{ij} spin-correlation coefficients [already measured for top pair production]

It can be shown that the maximum of the l.h.s.

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle|$$

is given by

$$2\sqrt{\lambda_1 + \lambda_2}$$

where λ_1 and λ_2 are the two largest eigenvalues of the positive definite matrix C^TC

Horodecki, Horodecki, Horodecki, ' 95

What?

Simpler but equally effective: Take judicious choice of [non-commuting] spin observables

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(2S_i + 2S_j)$$
$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(-2S_i + 2S_j)$$

$$|C_{ii} + C_{jj}|$$

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle|$$

$$|C_{ii} - C_{jj}|$$

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(-2S_i - 2S_j)$$
$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(2S_i - 2S_j)$$

CHSH violation is probed by testing if $|C_{ii} \pm C_{jj}| > \sqrt{2}$
These estimators are optimal when off-diagonal C_{ij} vanish

What?

For spin-1 systems there is an inequality that is stronger than CHSH. For any observables A_1, A_2 [on system A], B_1, B_2 [on system B] CGLMP PRL '02

$$
\begin{aligned}
I_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\
& - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \leq 2
\end{aligned}
$$

if the systems are classical.

There is a well-known choice of A_1, A_2, B_1, B_2 that is believed to maximise I_3 for the spin-singlet state

$$|\psi\rangle = \frac{1}{\sqrt{3}} (| + -\rangle - |00\rangle + | - +\rangle)$$

However, it is not optimal for the mixed spin state of the *VV* pair resulting from *H* decay

$$\rho = \int d\beta \ \mathcal{P}(\beta) |\psi_\beta\rangle\langle\psi_\beta| \qquad |\psi_\beta\rangle = \frac{1}{\sqrt{1+\beta^2}} (| + -\rangle - \beta|00\rangle + | - +\rangle)$$

How?

With spin-1 particles V=W,Z it is the same but more complicated

$$\rho = \frac{1}{9}\left(1_{9\times9} + A^1_{LM} T^L_M \otimes 1_{3\times3} + A^2_{LM} 1_{3\times3} \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right)$$

8 polarisations for V_1

8 polarisations for V_2

64 spin correlations

where T^L_M [L = 1,2] are irreducible tensors

$$T^1_1 = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_0 = \sqrt{\frac{3}{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$T^2_2 = \sqrt{3}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_2 = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^2_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^1_{-1} = -(T^1_1)^\dagger$$
$$T^2_{-2} = -(T^2_2)^\dagger$$
$$T^2_{-1} = -(T^2_1)^\dagger$$

Alternative: Gell-Mann matrices

How?

… which translates into

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y^{M_1}_{L_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y^{M_2}_{L_2}(\Omega_2) + B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\Big]$$

$$\eta_\ell = \begin{cases} \dfrac{g_L^2 - g_R^2}{g_L^2 + g_R^2} & Z \\ 1 & W^- \\ -1 & W^+ \end{cases}$$

$$\Omega_1 = (\theta_1, \varphi_1)$$
$$\Omega_2 = (\theta_2, \varphi_2)$$

$$B_1 = -\sqrt{2\pi}\eta_\ell\,, \quad B_2 = \sqrt{\frac{2\pi}{5}}$$

Simpler than it looks because spherical harmonics are orthogonal functions

constants you can calculate

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1)^* Y^{M_2}_{L_2}(\Omega_2)^* = \frac{1}{(4\pi)^2} B_{L_1} B_{L_2} C_{L_1 M_1 L_2 M_2}$$

data follow this distribution

calculate the average of this quantity on your data

the quantity you want

Not yet measured neither in Higgs decays nor EW diboson production

Novel tests: qutrits

$H \to VV$ is a decay $0 \to 1 + 1$. Angular momentum conservation implies that many A and C coefficients are zero. The non-zero ones are

$$A^1_{10} = -A^2_{10}\,, \quad A^1_{20} = A^2_{20}$$
$$C_{1010}\,, \quad C_{2020}\,, \quad C_{1020}\,, \quad C_{2010}$$
$$C_{111-1} = C^*_{1-111}\,, \quad C_{222-2} = C^*_{2-222}\,, \quad C_{212-1} = C^*_{2-121}\,,$$
$$C_{112-1} = C^*_{1-121}\,, \quad C_{211-1} = C^*_{2-111}$$

$$\rho = \frac{1}{9}\left(1_{9\times 9} + A^1_{LM} T^L_M \otimes 1_{3\times 3} + A^2_{LM} 1_{3\times 3} \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right)$$

and the 9×9 ρ matrix is sparse [relations among coefficients used below]

$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 - C_{2020} & 0 & C_{212-1} & 0 & C_{222-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C^*_{212-1} & 0 & -1 + 2C_{2020} & 0 & C_{212-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C^*_{222-2} & 0 & C^*_{212-1} & 0 & 2 - C_{2020} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Separability $\xrightarrow{\text{Peres-Horodecki}}$ $C_{212-1} = 0\,, \quad C_{222-2} = 0$

$\xleftarrow{H \to VV \text{ special case}}$

Novel tests: qutrits

Prospects for $H \to ZZ \to 4\ell$

JAAS, Bernal, Casas,Moreno, 2209.13441

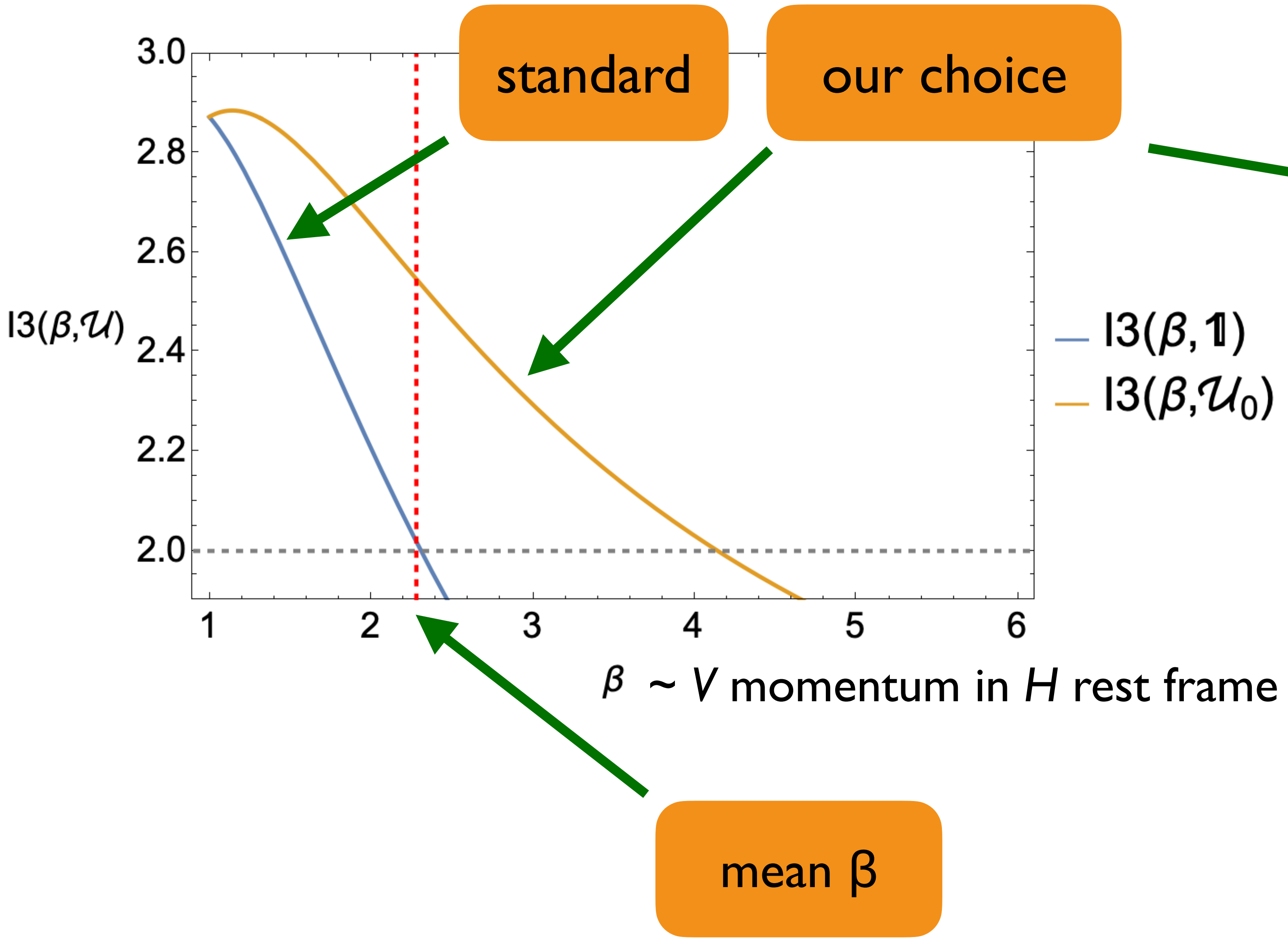
- Parton level, no detector simulation, approximate eff [0.25] injected
- Background not included [1/4 size of signal]
- Only statistical uncertainties, estimated with pseudo-experiments

	$C_{212\text{-}1}$	$C_{222\text{-}2}$	Significance
Run 2 + 3 : 300 fb^{-1}	-0.98 ± 0.31	0.60 ± 0.37	3σ
HL-LHC : 3 ab^{-1}	-0.95 ± 0.10	0.60 ± 0.12	many σ

Novel tests: qutrits

We saw earlier that for a spin singlet there is a \`standard´ Bell operator that is believed to be optimal. But this is not the case for $H \to VV$

[V not at rest in H rest frame]



$$I_3 = \frac{1}{36}\Big[\left(18+16\sqrt{3}\right) - \sqrt{2}\left(9-8\sqrt{3}\right) A^1_{2,0} - 8\left(3+2\sqrt{3}\right) C_{2,1,2,-1} + 6\, C_{2,2,2,-2}\Big]$$

	I_3	Significance
Run 2 + 3 : 300 fb^{-1}	2.66 ± 0.46	1.4σ
HL-LHC : 3 ab^{-1}	2.63 ± 0.15	4.2σ

Novel tests: qutrits

The *ZZ* final state is clean and easy to reconstruct…

… but the *WW* final state is clearly superior in terms of both statistics and spin analysing power

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y^{M_1}_{L_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y^{M_2}_{L_2}(\Omega_2) + B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\Big]$$

$$B_1 = -\sqrt{2\pi}\,\eta_\ell \longrightarrow \eta_\ell = \pm 1\ (W)\ ;\ 0.13\ (Z)$$

- Coefficients A_{lM}, $C_{lM2M'}$ have a suppression 1/10 for *Z* $\longrightarrow$ Δ_{stat} 3x penalty
- Coefficients $C_{lMlM'}$ have a suppression 1/100 for *ZZ* $\longrightarrow$ Δ_{stat} 10x penalty

Efforts needed towards realistic reconstruction methods for *WW*!

Novel tests: qutrits

Full reconstruction of $H \to WW \to \ell\nu qq$ possible by using *c*-tagging to distinguish jets

Fabbri, Howarth, Maurin, 2307.13783

Penalties of full reconstruction:

- 1/2 BR because $W \to ud$ is not usable
- 1/2 BR because $W \to cs$ is assumed on shell, $W \to \ell\nu$ off shell
- 0.4 efficiency for charm tagging

to reduce bkg

Still 20% more statistics than $WW \to 2\ell\, 2\nu$

- Detector simulation and unfolding
- Background included
- Only statistical uncertainties

standard operator [could be better]

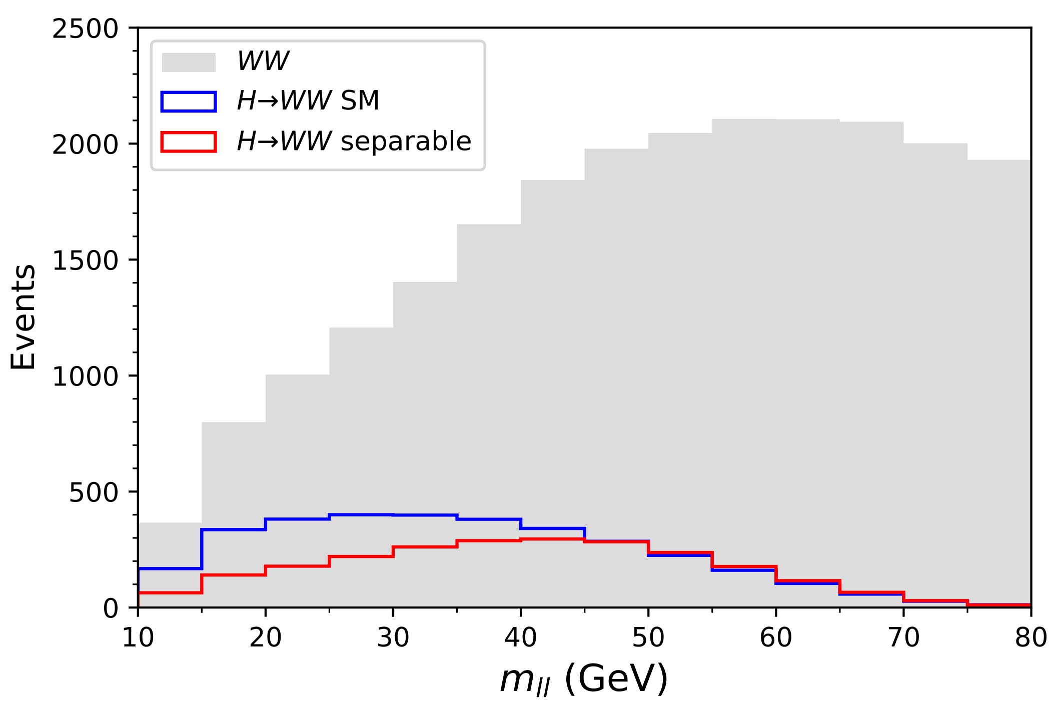
	Entanglement	Bell inequalities
Run 2 : 139 fb^{-1}	?	1.8σ
Run 2 + 3 : 300 fb^{-1}	??	2.7σ
HL-LHC : 3 ab^{-1}	???	many σ

Novel tests: qutrits

For $H \to WW \to 2\ell\, 2\nu$, entanglement conditions can be recast into a binary test using lab-frame dilepton kinematical distributions. JAAS, 2209.14033

SM vs separability hypothesis $\equiv$ $C_{212-1} = 0$, $C_{222-2} = 0$

$$\hat{h}_0 \hat{h}_-^* = h_{16} + i\,(h_{17} - h_{26}) + h_{27}$$
$$\hat{h}_+ \hat{h}_-^* = h_{44} + i\,(h_{45} - h_{54}) + h_{55}$$



	Run 2 Significance
stat only	7.1σ
stat + modeling syst	6.1σ

now this is the end