EW evolution equations and isospin conservation

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with Dimitri Colferai, Denis Comelli and Giampaolo Co', on the ArXiv next week

Istituto Nazionale di Fisica Nucleare

OUTLOOK

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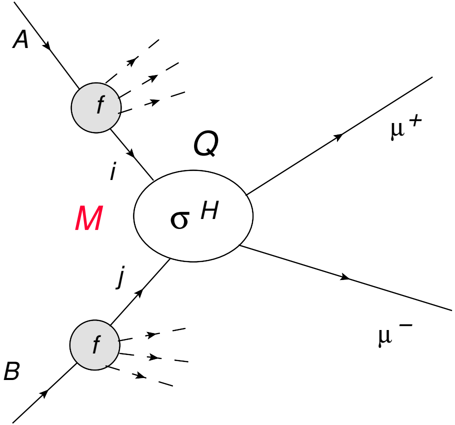
- In order to be compatible with isospin conservation, we propose to modify EWEEs with respect to what have been done until now in the literature
- These modifications have a sizeable impact on the Parton Distribution Functions (PDFs)

Factorization

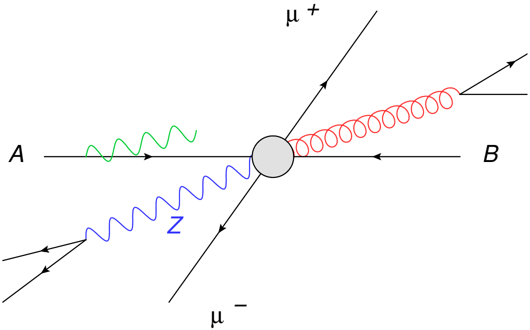


M=weak scale, $Q \gg M$ =c.m.

energy, f_{ij} = PDFs

$$\sigma(AB \to \mu^+\mu^- + X) = \sum_{i,j} \int dx_i dx_j \; f_{iA}(x_i, M) \; \sigma^H_{ij}(x_j x_i Q^2) \; f_{jB}(x_j, M)$$

What is X?



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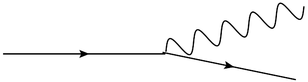

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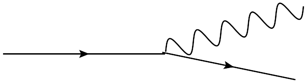

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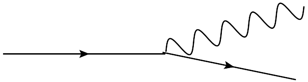

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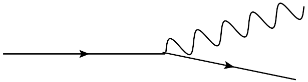

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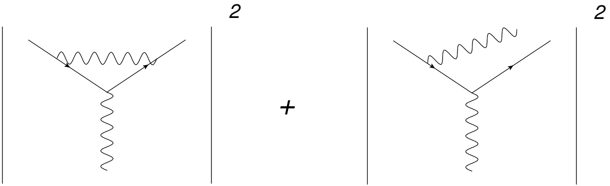
- P.C., D. Comelli, M. Ciafaloni, L. Lipatov, A.D. Martin, M. Melles, A.A. Penin, M. Beneke, S. Pozzorini, B. Webber, V.A. Smirnov, A.V. Manohar, A. Vergine, V. Fadin, S. Moch ...

Inclusive quantities



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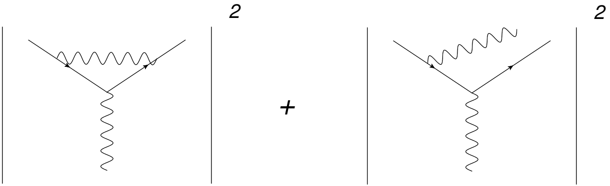

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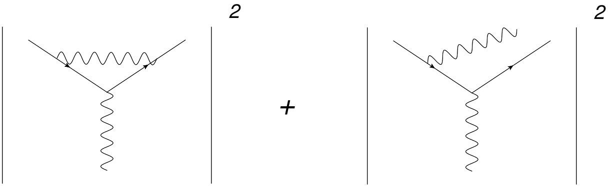

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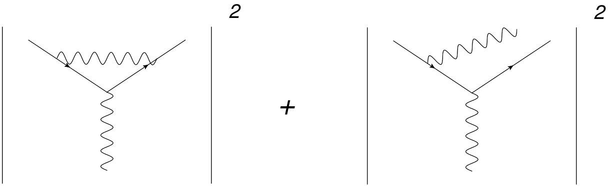

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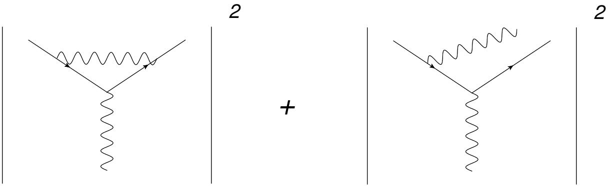

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- Large, energy-growing, double logs are ubiquitous

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How do we calculate PDFs f_{ij}?

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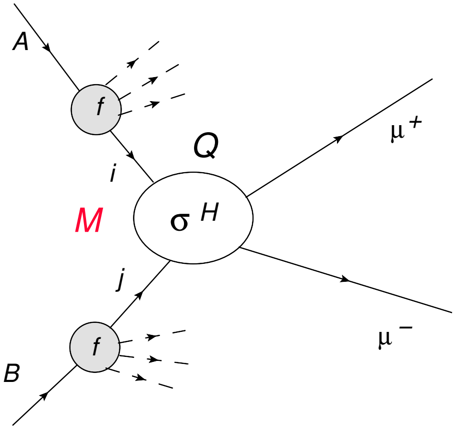

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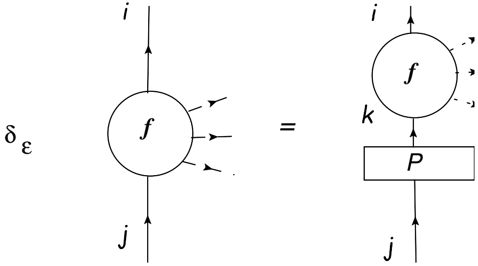
Electroweak Evolution Equations EWEE (DGLAP in QCD)



- $\varepsilon = \frac{\mu}{Q}$; $\quad -\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon} f_{ij}(x,\varepsilon) = \int_0^1 \frac{dz}{z} f_{ik}(z,\varepsilon) P_{kj}(\frac{x}{z},\varepsilon)$

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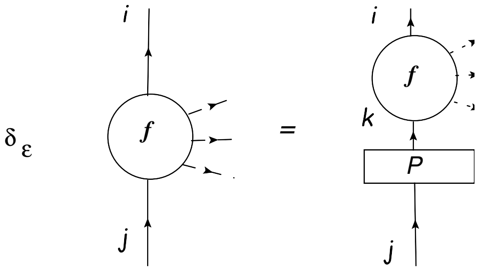

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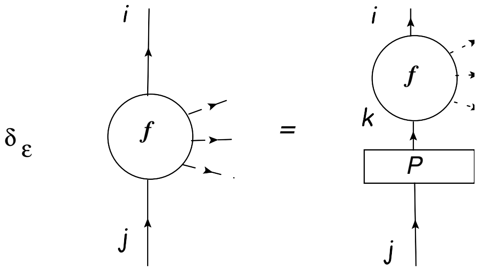

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- EW only, $i = \nu_L, e_L^-$ +antif. , W_T^+, W_T^-, W_T^0, $g' = 0$ (NOT the full SM)

EWEE in isospin basis - qualitative analysis

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- $I = 1$ "Genuinely EW"

$$f_1 = f_\nu - f_e$$

  EWEEs resum $\mathcal{O}(\alpha L^2)^n$ collinear/IR "EW Bloch-Nordsieck violation"

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$$\tilde{f}(N,\varepsilon) = \int_0^1 dz f(z,\varepsilon) z^{N-1} \quad -\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon}\tilde{f}_{ij}(N,\varepsilon) = \tilde{f}_{ik}(N,\varepsilon)\,\tilde{P}^G_{kj}(N,\varepsilon)$$

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- N=1:charge, fermion number N=2: momentum

SUM RULES - Fermion Number

$$1 = \int_0^1 dz[f_{e\nu}(z,\varepsilon) + f_{\nu\nu}(z,\varepsilon) - f_{\bar{\nu}\nu}(z,\varepsilon) - f_{\bar{e}\nu}(z,\varepsilon)] = \int_0^1 dz\, f_{L_0^- \, L_0^-}(z,\varepsilon)$$

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- STANDARD SUM RULE

$$\tilde{P}^R_{ff}(1,\varepsilon) + \tilde{P}^V_{ff}(1,\varepsilon) = \int_0^1 dz (P^R_{ff}(z,\varepsilon) + P^V_{ff}(z,\varepsilon)) = 2\varepsilon + \mathcal{O}(\varepsilon^2) \to 0$$

P. Ciafaloni, D. Comelli, JHEP 11 (2005), 022; C. W. Bauer, N. Ferland and B. R. Webber, JHEP 08 (2017), 036; F. Garosi, D. Marzocca and S. Trifinopolous, JHEP 09 (2023) 107...

SUM RULES - Isospin

- $t_\nu^3 = \frac{1}{2} = \sum_i t_i^3 \tilde{f}_{i\nu}(1,\varepsilon) = \frac{1}{2}(\tilde{f}_{L_1^- L_1^-}(1,\varepsilon) + \sqrt{2}\tilde{f}_{G_1^- L_1^-}(1,\varepsilon))$
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$$\int_0^1 dx \tilde{P}^R_{ff}(x,\varepsilon) = \int_0^1 dx \tilde{P}^R_{gf}(x,\varepsilon)$$

$$P^R_{gf}(z) = \frac{1+(1-z)^2}{z} \rightarrow P^R_{gf}(z,\varepsilon) = \frac{1+(1-z)^2}{z} \quad \theta(z-\varepsilon)$$

Splitting functions

$$P_{ff}^{V} = -\delta(1-z)\left(\log\frac{1}{\varepsilon^2} - \frac{3}{2}\right); \quad P_{ff}^{R} = \frac{1+z^2}{1-z}\theta(1-\varepsilon-z) \ ,$$

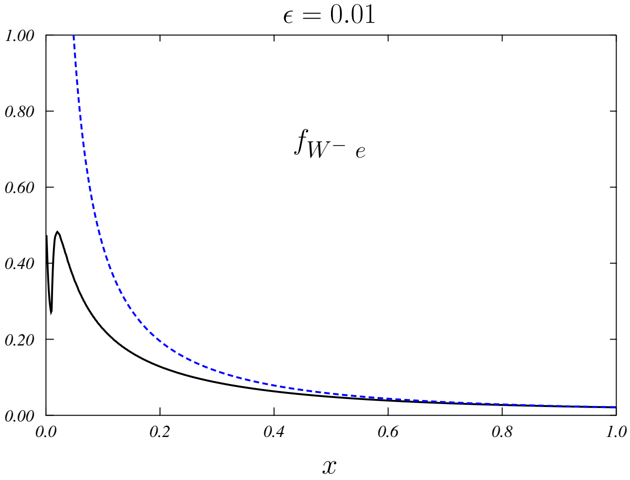
$$P_{gg}^{V} = -\delta(1-z)\left(\log\frac{1}{\varepsilon^2} - \frac{5}{3}\right);$$

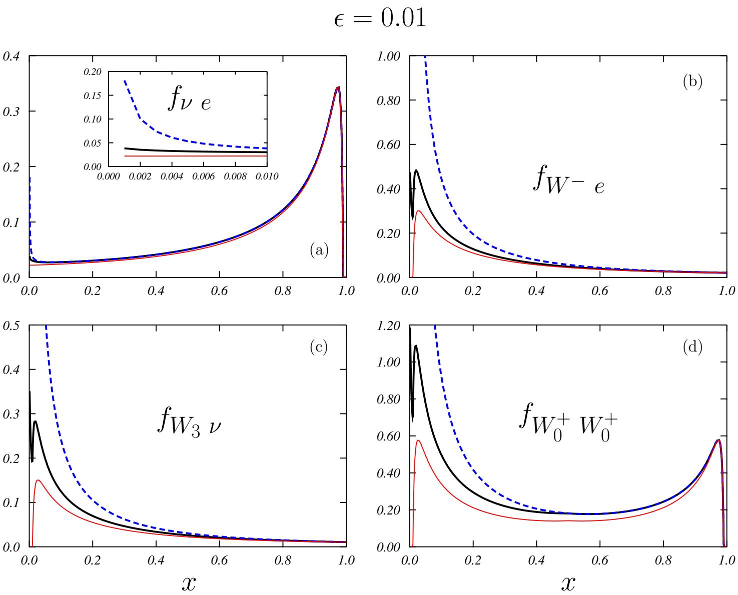
$$P_{gg}^{R} = 2\left(z\,(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right)\ \left[\theta(z-\varepsilon)\right]\ \theta(1-\varepsilon-z) \ ,$$

$$P_{gf}^{R} = \frac{1+(1-z)^2}{z}\ \left[\theta(z-\varepsilon)\right]; \quad P_{fg}^{R} = z^2 + (1-z)^2 \ ,$$

Impact on PDFs

$\epsilon = \frac{M}{Q} = 10^{-2}$,numerical $10^{-2} < x < 1$ ~ 20 % @ x=0.3





$\epsilon = 0.01$
$f_{\nu\ e}$
(a)
$f_{W^-\ e}$
(b)
$f_{W_3\ \nu}$
(c)
$f_{W_0^+\ W_0^+}$
(d)
x
x

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- The modifications described here will be particularly relevant if a 100 TeV Future Circular Collider and/or a TeV scale muon collider will see the light.