Recent topics in the theory of rare B decays

La Thuile 2024 - Rencontres de Physique de la Vallée d'Aoste – 06/03/2024

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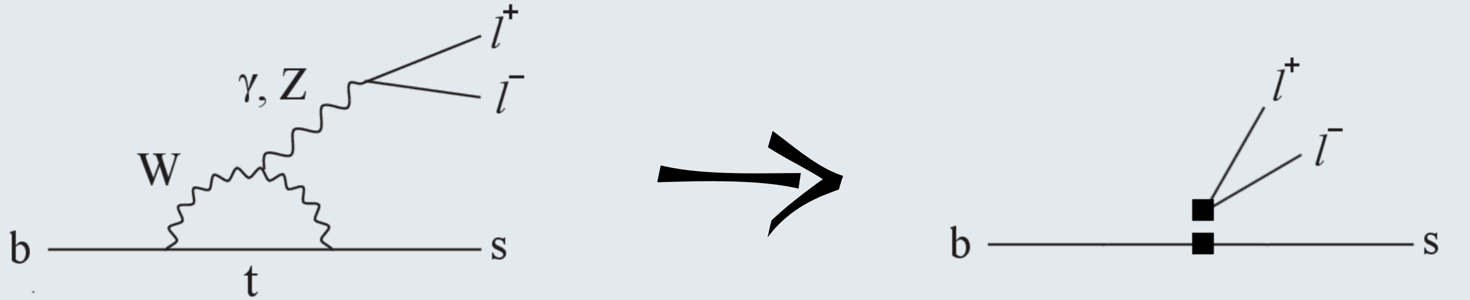
cnrs

Weak Effective Theory

- FCNC processes take place at a scale $m_b < m_W, m_t$



- Allows for a generic calculation of the observables (in and beyond SM) through

$$\boxed{\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)}$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2}\left(\bar{s}_L\gamma_\mu b_L\right)\left(\bar{\ell}\gamma^\mu(\gamma_5)\ell\right)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\left(\bar{s}_L\sigma_{\mu\nu}b_R\right)F^{\mu\nu}$$

- Avoids the appearance of large logarithm in the calculations of observables

Theory uncertainties in rare b decays

- I focus on **SM predictions**
  - Plenty of global fits of the WC: see e.g. the recent review [Capdevilla '23]
  - See the BSM talks by Admir G. and Jason A.

Theory uncertainties in rare b decays

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  - See the BSM talks by Admir G. and Jason A.
- Main sources of **SM uncertainties** are
  - QCD → decay constants and form-factors
  - CKM (not discussed in this talk, see talk by Marzia B.)
  - (further uncertainties come from: SM WC, lifetimes, radiative corrections…)

→ Hadronic effects are a **blocker** for the extraction of SM and BSM parameters

Theory uncertainties in rare b decays

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  - (further uncertainties come from: SM WC, lifetimes, radiative corrections…)
- I will focus on **b → s transitions**, but the method applies equally well to
  - b → d [see e.g. Biswas, Nandi, Patra, Ray '22; Marshall, McCann et al' 23]
  - b → {u, c} → see talk by Marzia B.
  - But also charm physics (many papers in preparation)

Form factors in b → sℓℓ

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

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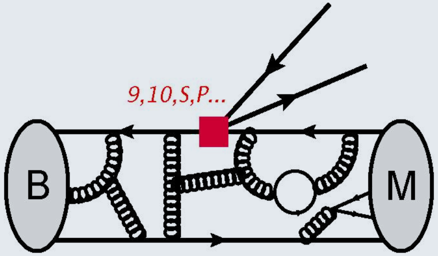
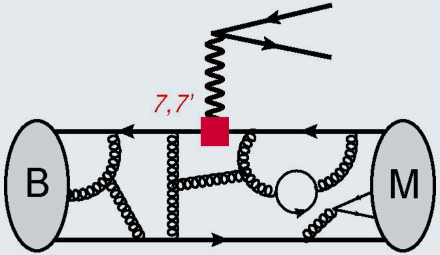




$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda\ell\ell) = \mathcal{N}_\lambda\left\{(C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_bM_B}{q^2}\left[C_7\mathcal{F}_\lambda^T(q^2) \qquad\qquad\right]\right\}$$

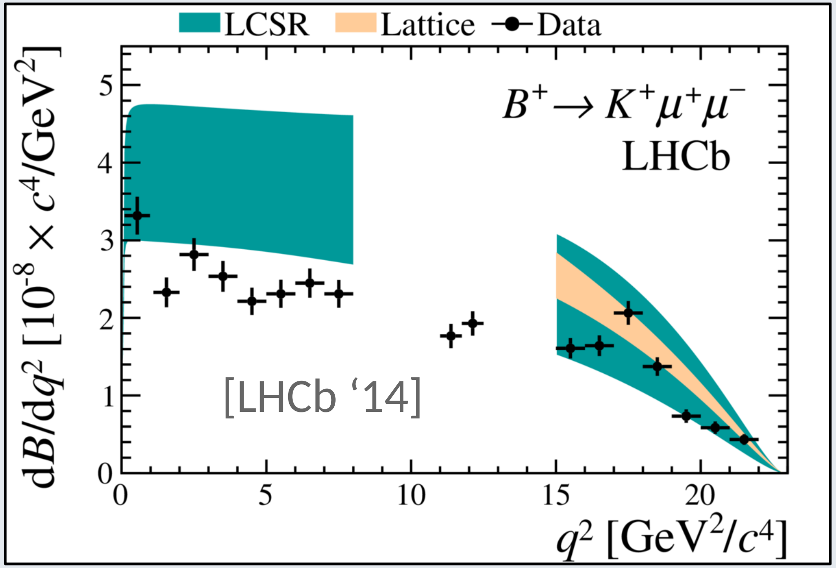
- B → K$^{(*)}$ μμ
- B$_s$ → φ μμ, ...

Local form-factors,

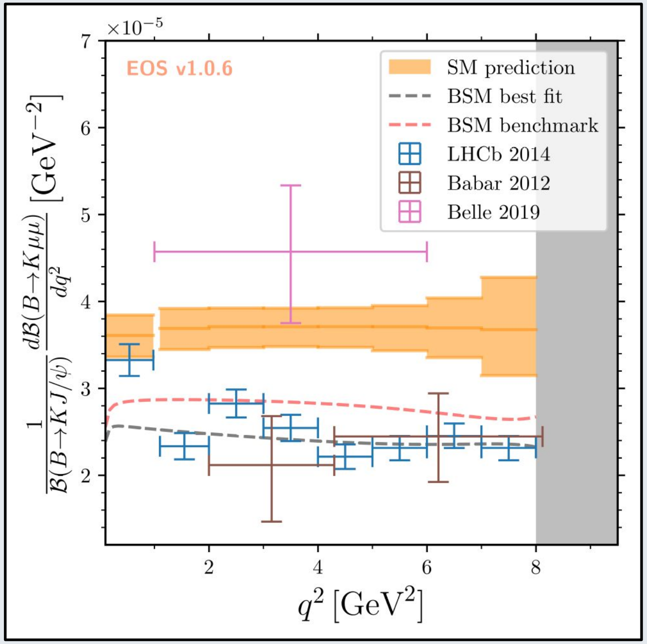
involves e.g. $\mathcal{F}_\lambda(k,q) = \mathcal{P}_\lambda^\mu\,\langle\bar{M}(k)|\bar{s}\gamma_\mu b_L|\bar{B}(q+k)\rangle$

Status of the Local Form Factors

- Parametrizations based on the analyticity properties provide excellent fits to both:
  - **Lattice QCD** [recent review: Meinel '24]
  - **Light-cone Sum Rules** estimates [recent review: Khodjamirian, Melic, Wang '23]



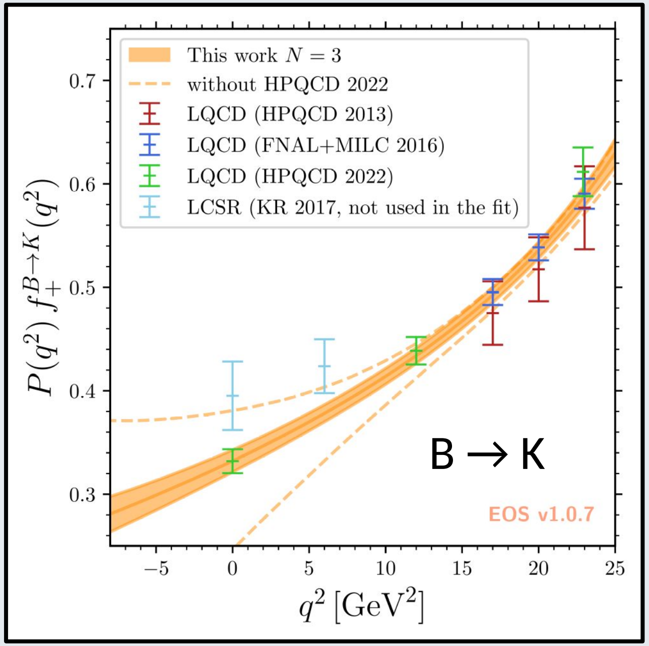
9 years later

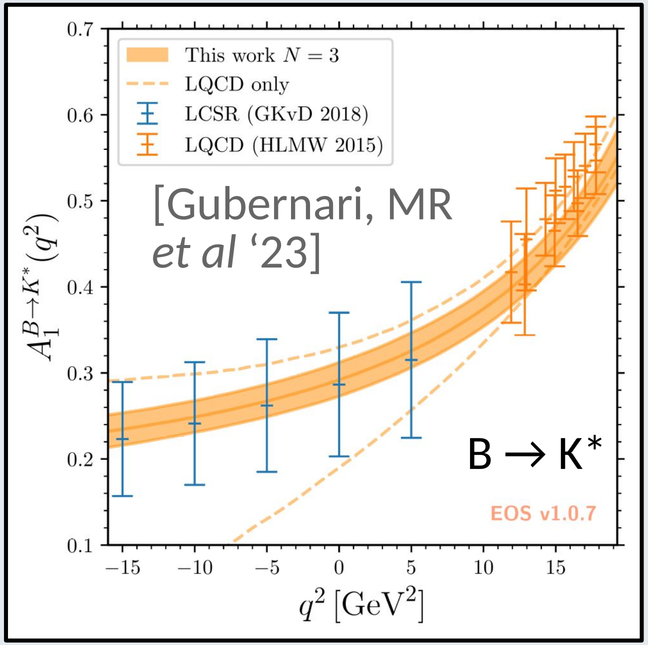


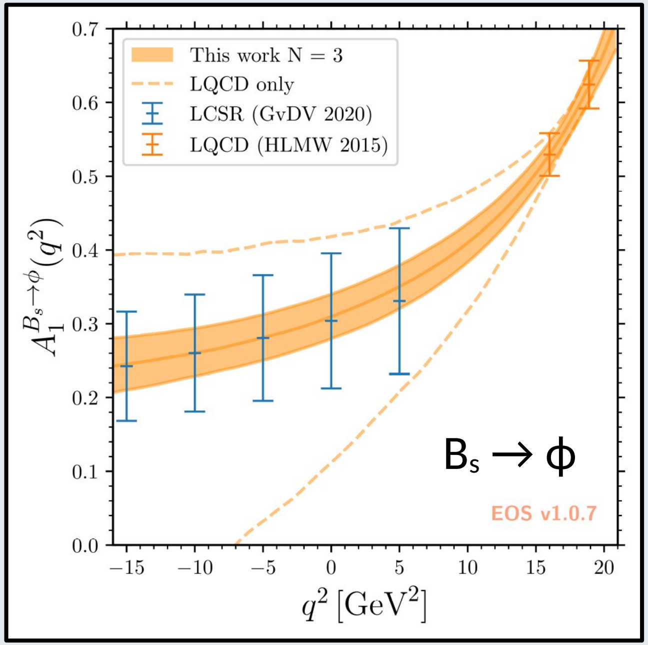
+ [CMS '23] in perfect agreement with [LHCb '14]

Analyticity and unitarity

- State-of-the-art form-factors are obtained by a combined fit of all available channels ensuring **analyticity** and **unitarity (dispersive bounds)**:
  - ✅ Systematically improvable
  - ✅ Controlled interpolation/extrapolation uncertainties







List of inputs and references in the backup!

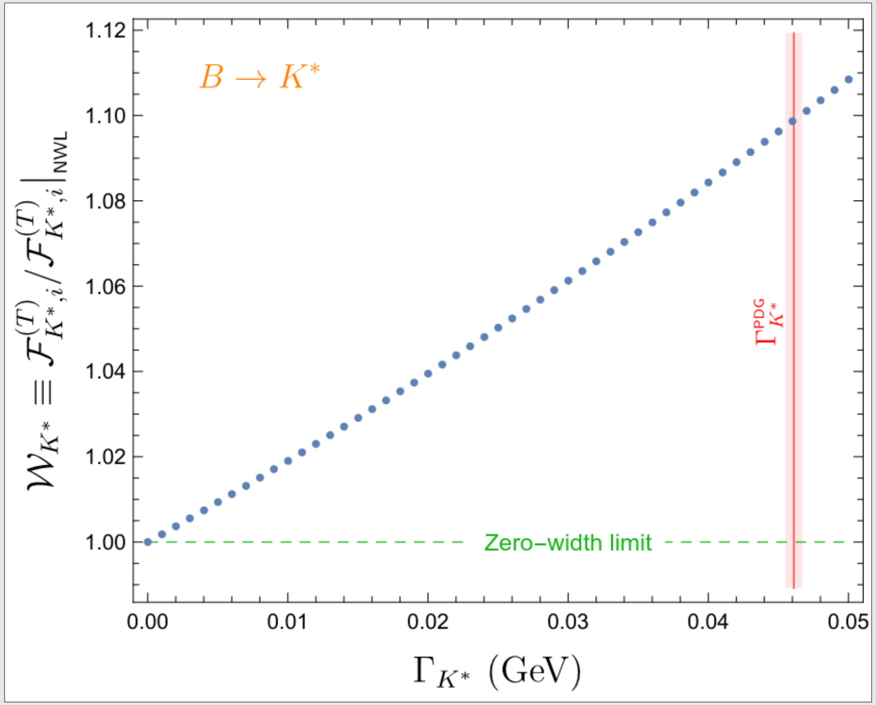
Caveat: finite width effects in B → K*

- $\Gamma_{K^*} / M_{K^*} \sim 5\%$ is not very small

- **Finite width effects** have to be accounted for in the LQCD and LCSR calculations
  - Universal 20% correction to the observables [Descotes-Genon, Khodjamirian, Virto '19]
  - Computable in LQCD [Leskovec '24]

- B → Kπμμ decays also have a large **S-wave component** [LHCb '16]
  - LCSR inputs for the S-wave are now available [Descotes-Genon, Khodjamirian, Virto, Vos '23]

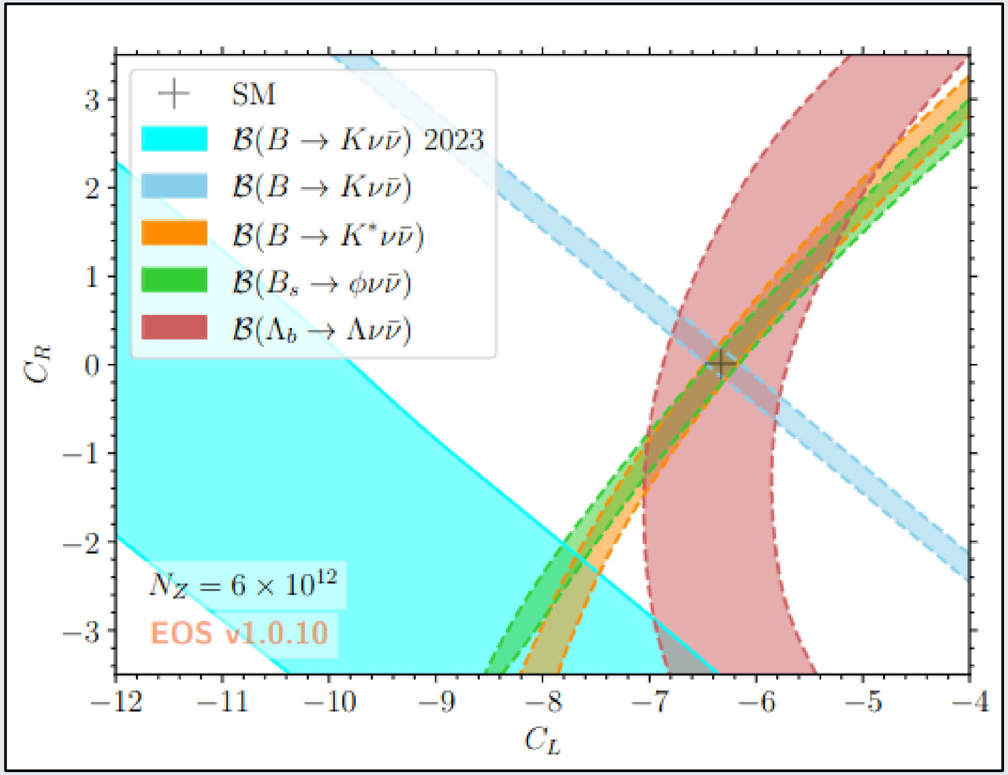
- Need for a generic parametrization for B → Kπ form factors [Gustafson, Herren et al '23 (B → Dπ)]



b → sνν decays

- Recent interest triggered by the **3.5σ evidence** for $B^+ \to K^+\nu\bar{\nu}$ [Belle II '23]

- b → sνν decays only have local contributions, which makes them particularly **clean probe of the SM** [Altmannshofer et al. '09; Buras et al. '14]
  - Combined study of b → sνν and b → sℓℓ [Becirevic et al. '23]
  - Golden channels for FCC-ee [Amhis, Kenzie, MR, Wiederhold, '23]



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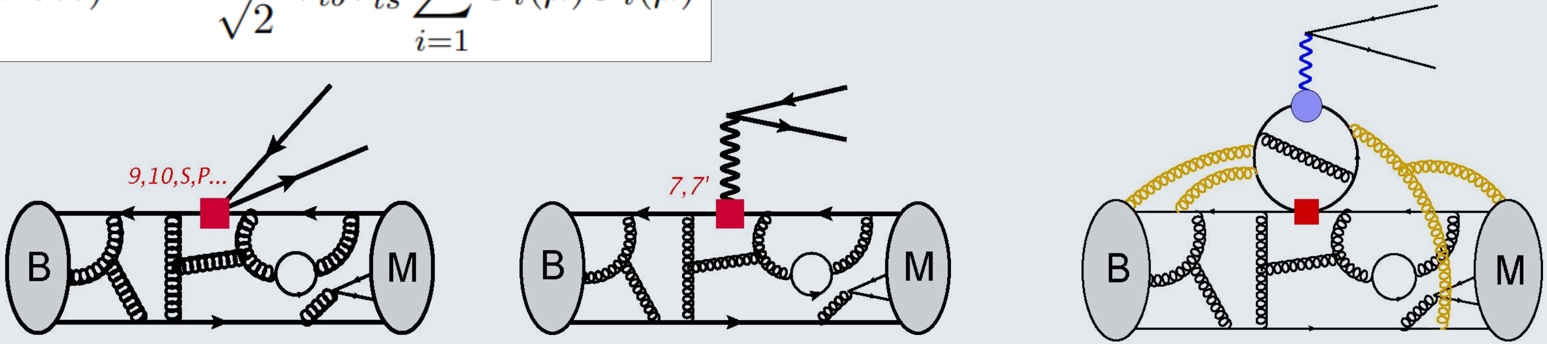

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Non-local form-factors:

$$\mathcal{H}_\lambda(k,q) = i \int d^4x\, e^{iq\cdot x} \mathcal{P}_\lambda^\mu \langle \bar{M}(k) | T\{ Q_c [\bar{c}\gamma_\mu c](x), \mathcal{C}_i \mathcal{O}_i \} | \bar{B}(q+k) \rangle$$

Long story short

1) The contribution is **dominated by the charm loops** due to O_{1c} and O_{2c}

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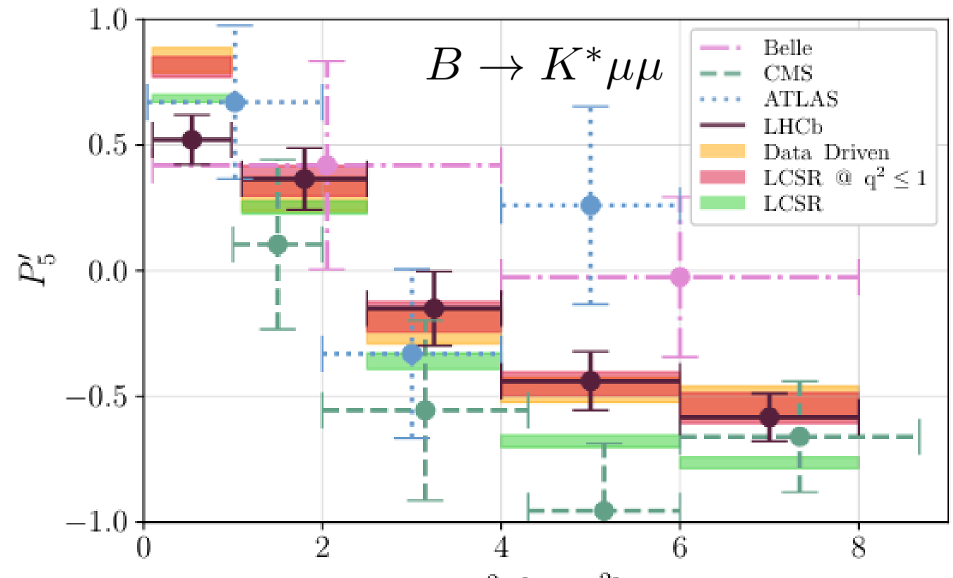

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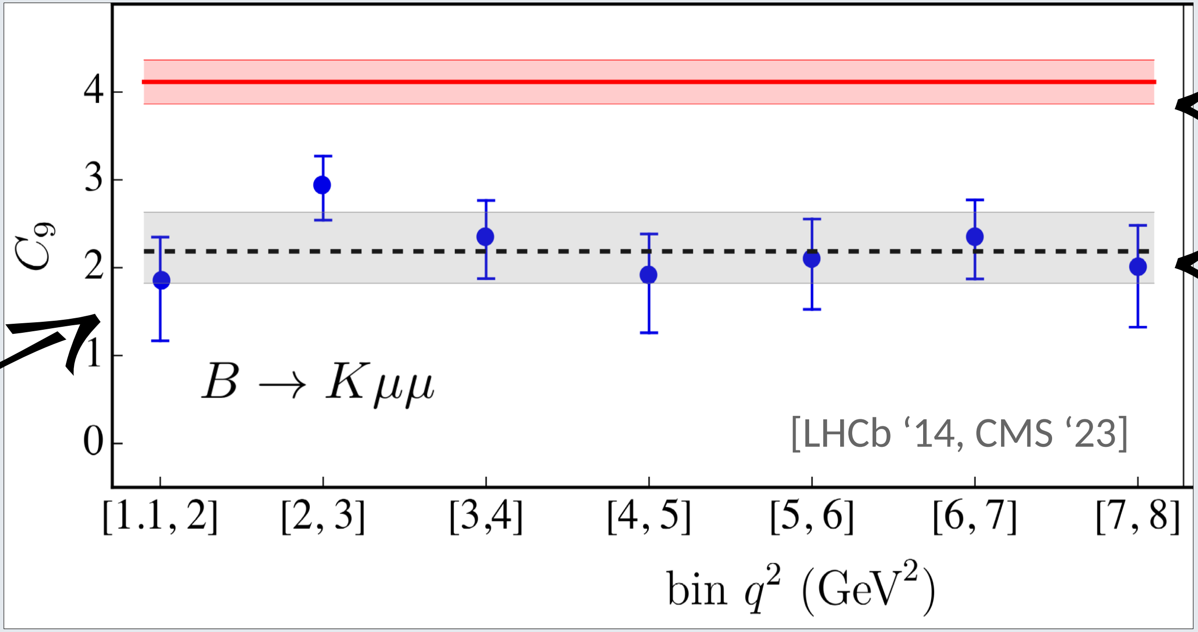
→ Data favors a constant shift in C_9 [Bordone, Isidori, Maechler, Tinari '24]



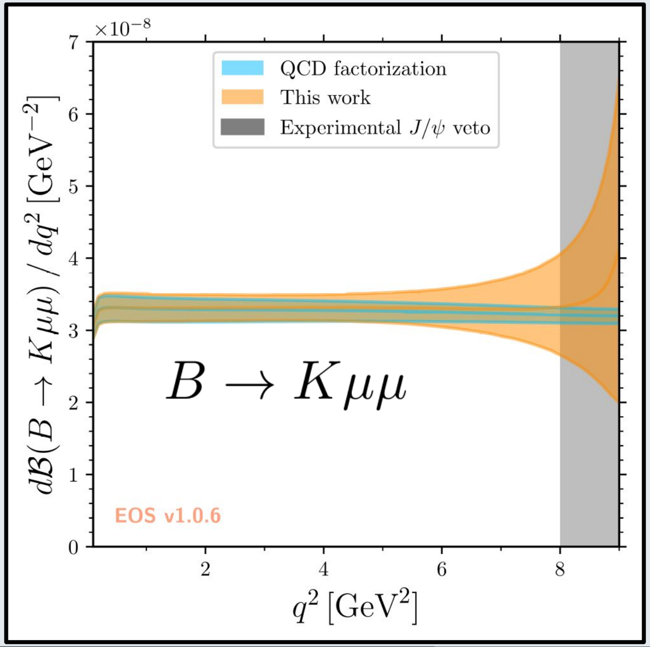
SM prediction

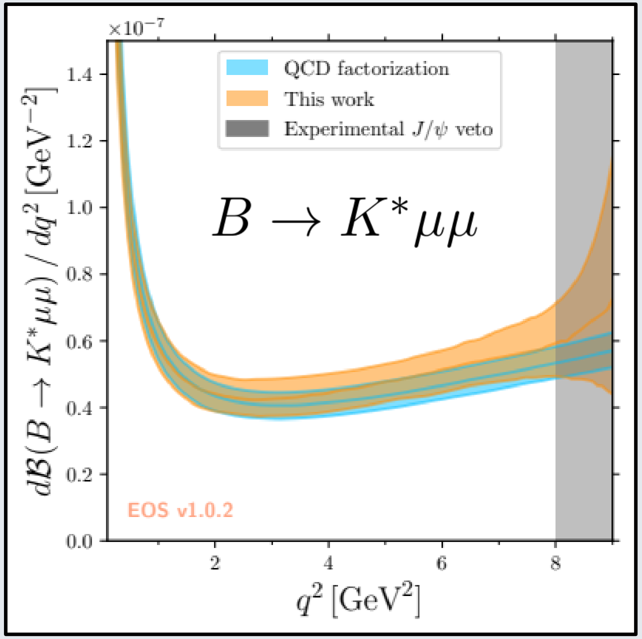
Constant-C_9 fit

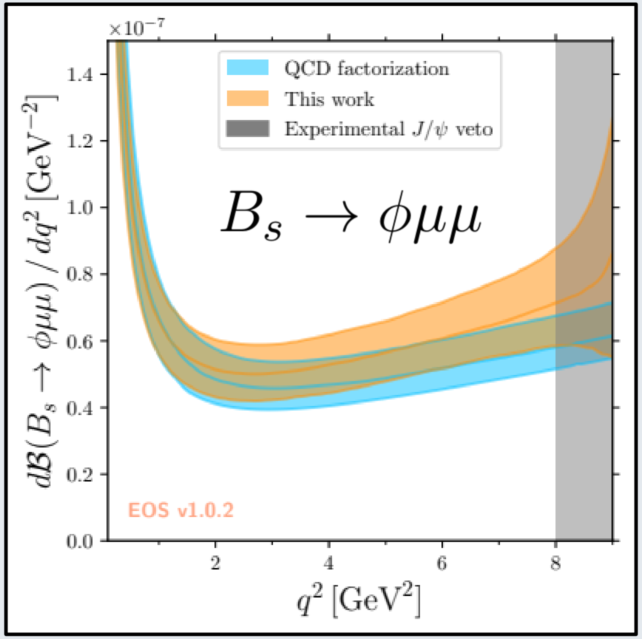
Data from LHCb and CMS

Long story short

1) The contribution is **dominated by the charm loops** due to O_{1c} and O_{2c}
2) The contribution **mimics new physics** by shifting C_9
3) *Assuming that the analytic structure is well understood*, dispersive bounds and explicit calculation at negative q^2 allows to **control the charm-loop below the DD threshold** [Gubernari, MR, van Dyk, Virto '22]







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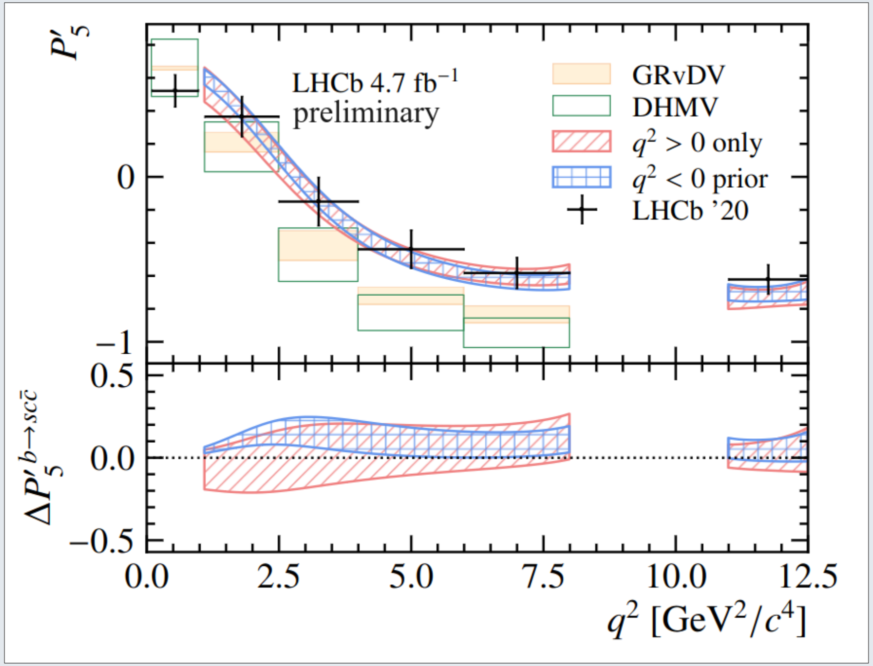
2) The contribution **mimics new physics** by shifting C_9
3) *Assuming that the analytic structure is well understood*, dispersive bounds and explicit calculation at negative q^2 allows to **control the charm-loop below the DD threshold** [Gubernari, MR, van Dyk, Virto '22]
4) The corresponding parameters can be fitted directly from data [LHCb '23]

Contribution of H_μ to the optimized angular observable P_5':

- With data at $q^2 < 0$
- Without data at $q^2 < 0$

The GRvDV parametrization describes the data well!

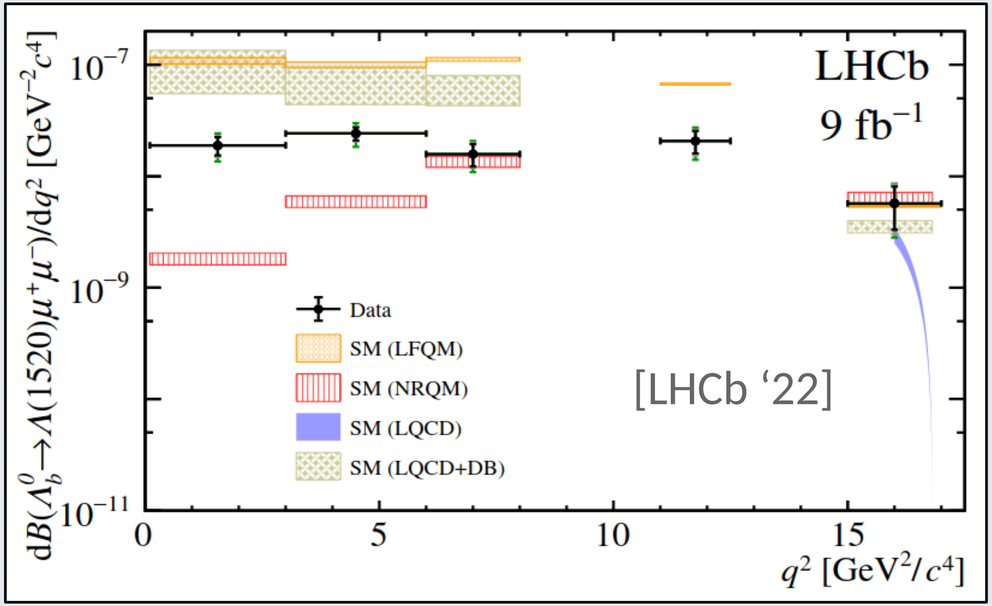


$\Lambda_b \to \Lambda^{(*)}\ell\ell$ decays

- **Baryonic decays** follow the same pattern but with a richer helicity structure:
  - ✅ They offer more **complementary** probe of the SM
  - ❌ They require **more hadronic inputs**

- Local form factors:
  - Lattice inputs [Detmold, Meinel '16, Meinel, Rendon '21]
  - Dispersive analysis [Amhis, Bordone, MR '22; Black, Meinel, Rahimi, van Dyk '22]

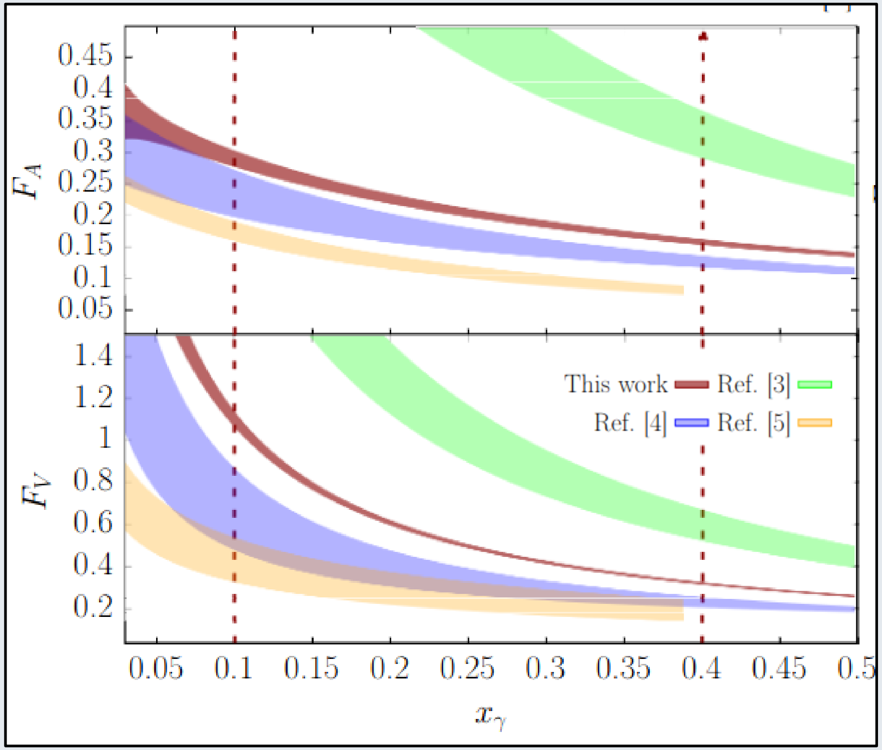
- Non-factorizable contributions [Feldmann, Gubernari '23]



Further probes of the SM

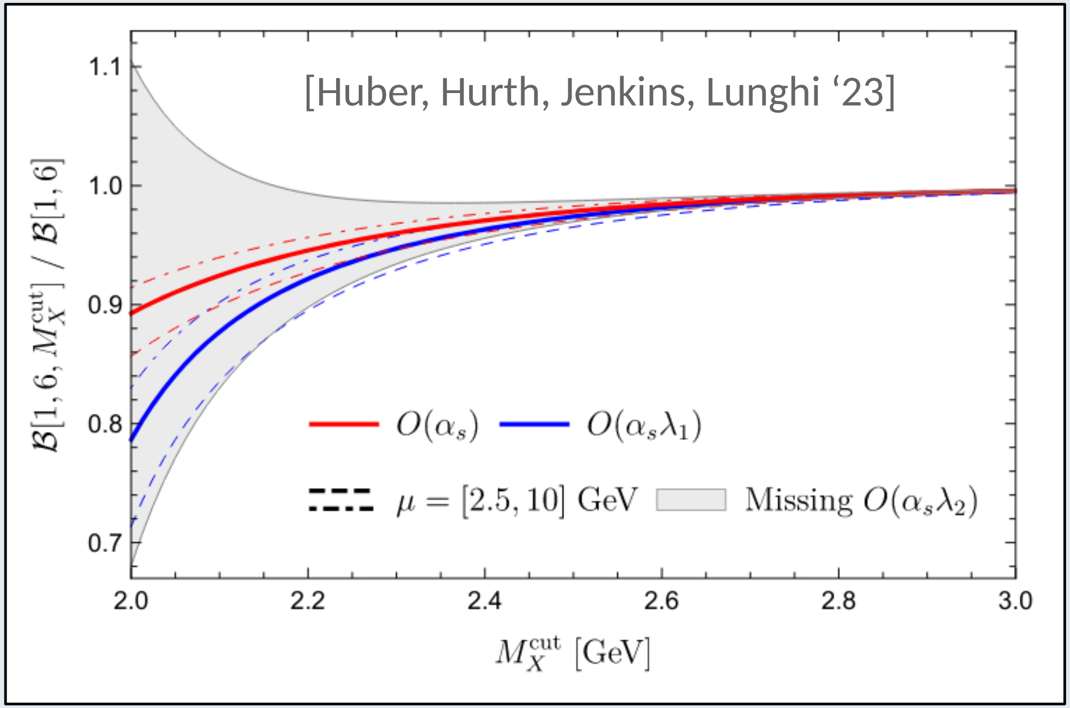
- **$B_{(s)} \to \mu\mu\gamma$**
  - Long standing interest [Melikhov *et al* '04, '17; Guadagnoli *et al* '16 '21 '23]
  - Workshop on radiative leptonic decays in Marseille [https://indico.in2p3.fr/event/31709/]
  - $B_s \to \gamma$ form-factors from Lattice QCD [Frezzoti *et al* '24]
- **$B_{(s)} \to \gamma\gamma$**
  - Offers a different probe of charm loops contribution [Belov *et al* '23]

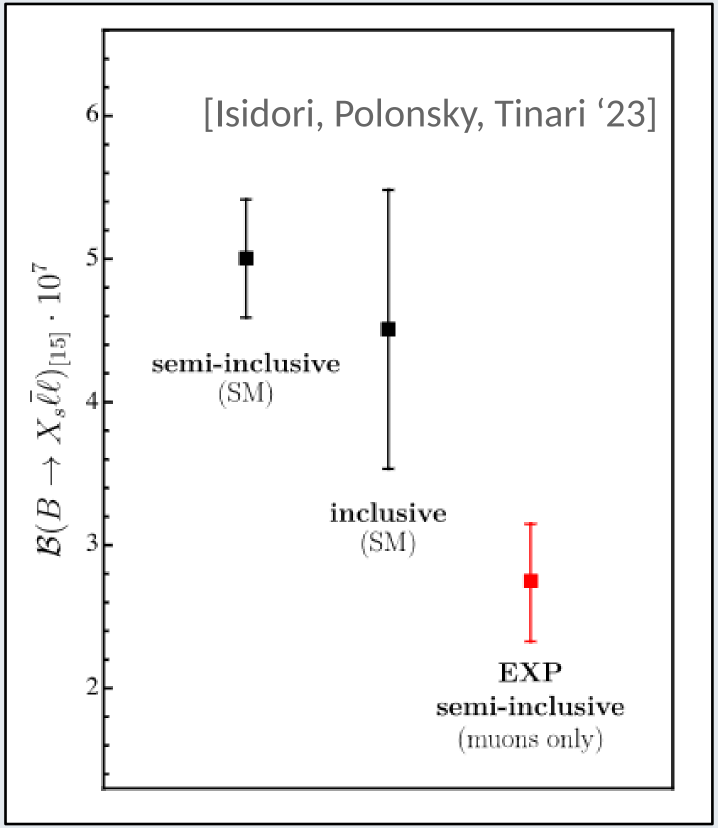
→ See the **dedicated experimental talks** by Irene B. and Shubhangi K. M.



Further probes of the SM

- Inclusive $B \to X_s \ell\ell$ [Isidori, Polonsky, Tinari '23, Huber, Hurth, Jenkins, Lunghi '23]





Conclusion & Outlook

- Hadronic form-factors limits the full interpretation of rare B decays observables
- Recent lattice results and improvement of the parametrization allowed us to reduce the theory uncertainties and to **confirm the current tensions**
- Non-local contributions are still subject to intense discussions and a consensus will have to emerge to fully benefit from the **upcoming results from Belle II and LHC Run III:**
  - Many upcoming $b \to d$ decays
  - Additional results for $B \to K^{(*)}\nu\bar{\nu}$
  - Many updates for $b \to s$ modes
  - …

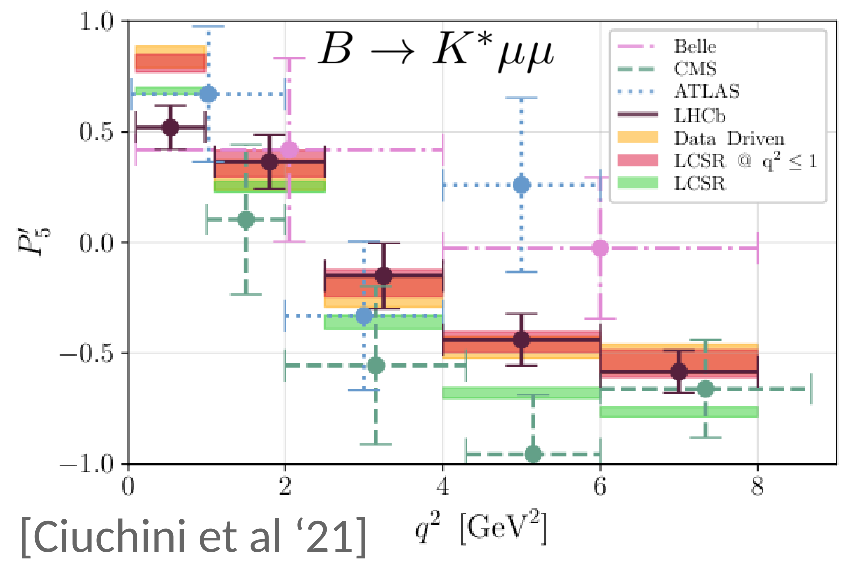
Back-up

q² parametrization

- **Simple q² expansion** [Jäger, Camalich '12; Ciuchini et al. '15]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\text{QCDF}}(q^2) + h_\lambda(0) + \frac{q^2}{m_B^2} h'_\lambda(0) + \dots$$

Computed in [Beneke, Feldman, Seidel '01]



[Ciuchini et al '21]

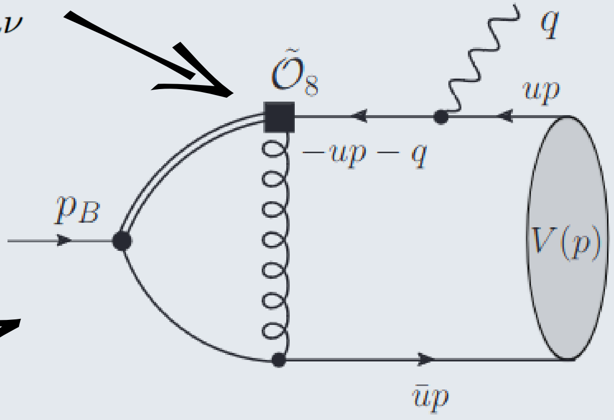
- The h_λ terms can be fitted or varied
- Fitting the h_λ terms on data gives a satisfactory fit but lacks predictive power
- This parametrization **cannot account** for the analyticity properties of $\mathcal{H}_\lambda$

Anatomy of H_μ in the SM

$C_1(\mu_b)$	$C_2(\mu_b)$	$C_3(\mu_b)$	$C_4(\mu_b)$	$C_5(\mu_b)$	$C_6(\mu_b)$	$C_7(\mu_b)$	$C_8(\mu_b)$	$C_9(\mu_b)$	$C_{10}(\mu_b)$
-0.2906	1.010	-0.0062	-0.0873	0.0004	0.0011	-0.3373	-0.1829	4.2734	-4.1661

- The contribution of O_8 is **negligible** [Khodjamirian, Mannel, Wang, ‘12; Dimou, Lyon, Zwicky ‘12]

$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$



One of the non-factorizable contributions

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- The contribution of O_8 is **negligible** [Khodjamirian, Mannel, Wang, '12]
- The contributions of $O_{3,\,4,\,5,\,6}$ are suppressed by **small Wilson coefficients**

$$\mathcal{O}_3 = (\bar{s}_L \gamma_\mu b_L) \sum_p (\bar{p}\gamma^\mu p)\,, \qquad \mathcal{O}_4 = (\bar{s}_L \gamma_\mu T^a b_L) \sum_p (\bar{p}\gamma^\mu T^a p)\,,$$

$$\mathcal{O}_5 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho b_L) \sum_p (\bar{p}\gamma^\mu \gamma^\nu \gamma^\rho p)\,, \qquad \mathcal{O}_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \sum_p (\bar{p}\gamma^\mu \gamma^\nu \gamma^\rho T^a p)\,,$$

Anatomy of H_μ in the SM

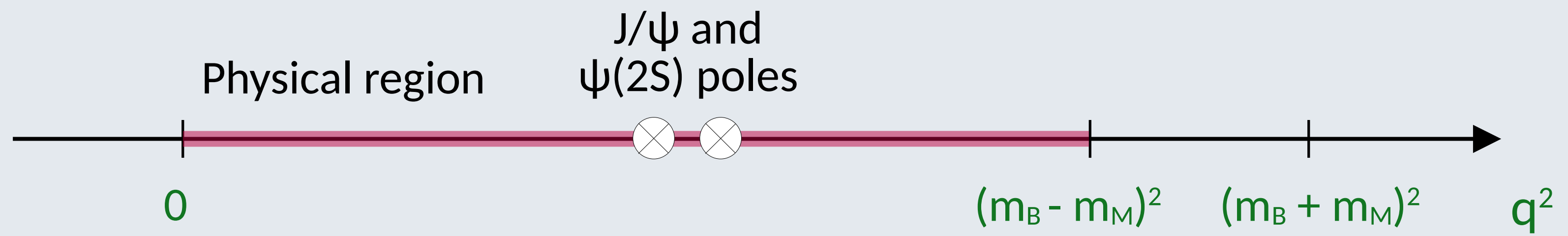
$$\mathcal{O}_1^q = (\bar{s}_L \gamma_\mu T^a q_L)(\bar{q}_L \gamma^\mu T^a b_L)\,, \qquad \mathcal{O}_2^q = (\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu b_L)$$

- Light-quark loops are CKM suppressed → **small contributions** even at the resonances [Khodjamirian, Mannel, Wang, '12]

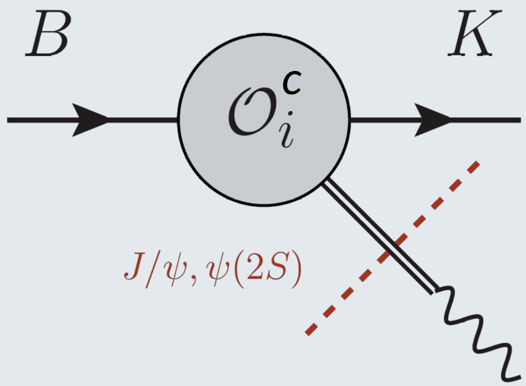
Vector meson	ρ	ω	ϕ	J/ψ	$\psi(2S)$
f_V	221^{+1}_{-1}	195^{+3}_{-4}	228^{+2}_{-2}	416^{+5}_{-6}	297^{+3}_{-2}
$\lvert A_{\bar{B}^0 V \bar{K}^0}\rvert$	$1.3^{+0.1}_{-0.1}$	$1.4^{+0.1}_{-0.1}$	$1.8^{+0.1}_{-0.1}$	$33.9^{+0.7}_{-0.7}$	$44.4^{+2.2}_{-2.2}$
$\lvert A_{B^- V K^-}\rvert$	$1.2^{+0.1}_{-0.1}$	$1.5^{+0.1}_{-0.1}$	$1.8^{+0.1}_{-0.1}$	$35.6^{+0.6}_{-0.6}$	$42.0^{+1.2}_{-1.2}$

→ The main contribution comes from $\mathbf{O_1^c}$ and $\mathbf{O_2^c}$: **"charm loop"**

Analyticity properties of H_μ



- Poles due to the narrow charmonium resonances



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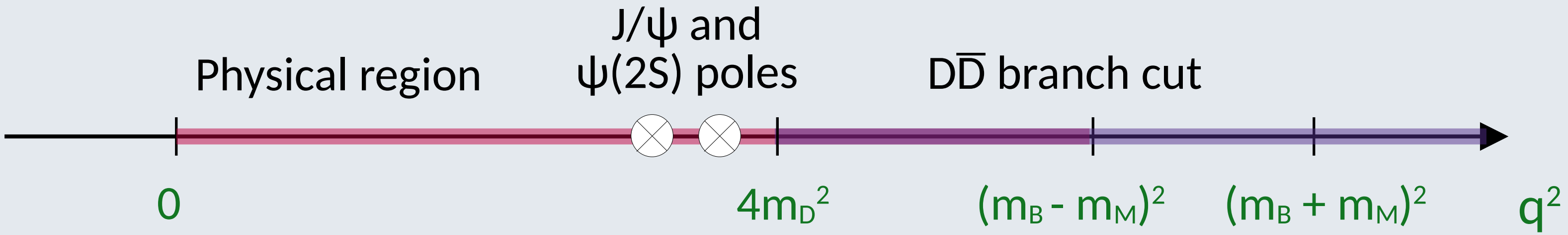
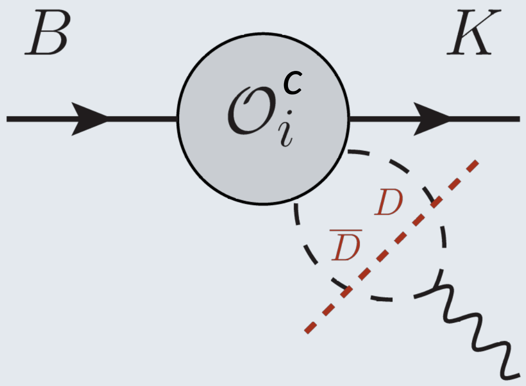


- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$



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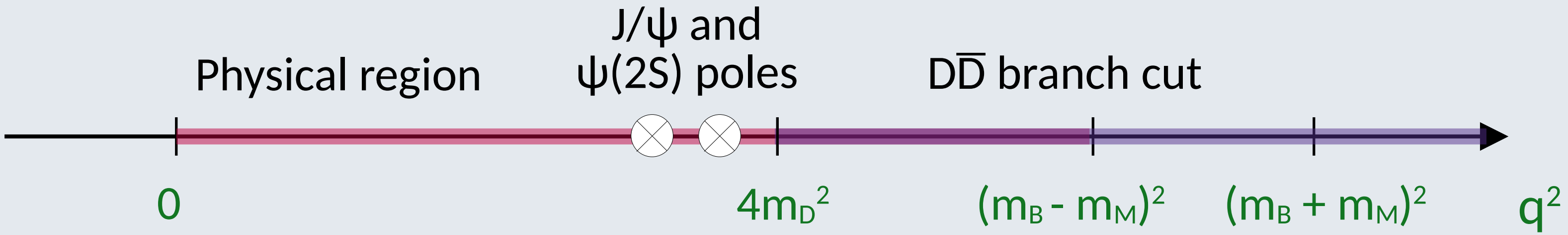
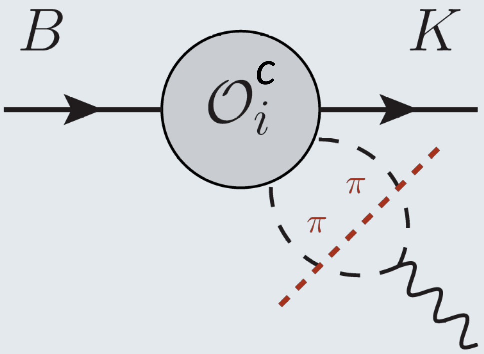
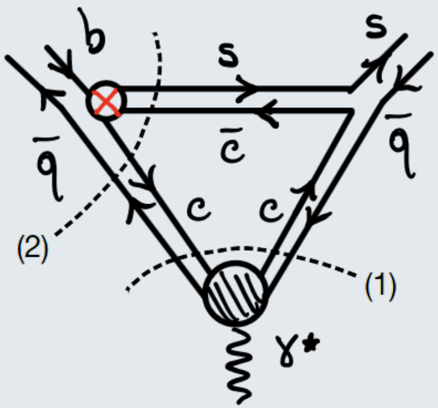
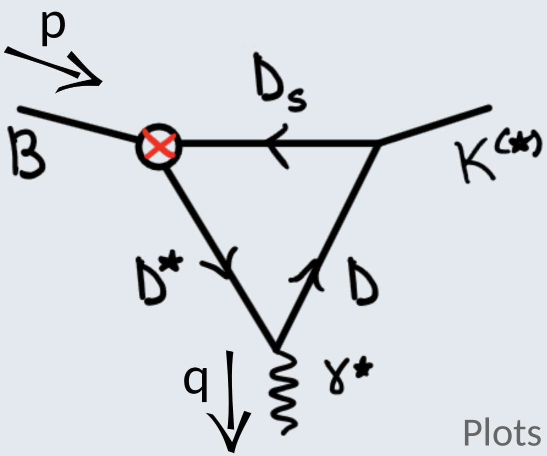


- Poles due to the narrow charmonium resonances
- Branch-cut starting at $4m_D^2$
- Branch-cut starting at $4m_\pi^2$ → negligible (OZI suppressed)



More involved analytic structure?





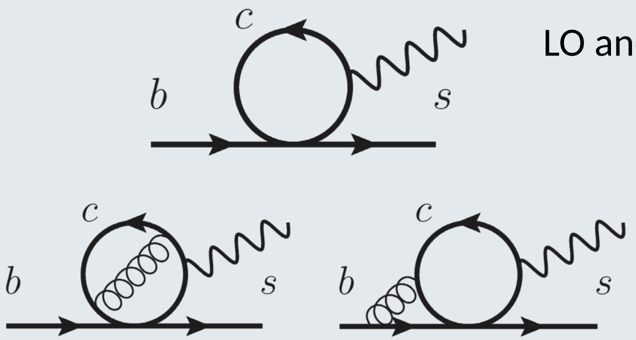
Plots from [Ciuchini et al. '22]

- $M_B > M_{D^*} + M_{D_s}$ $\rightarrow$ The function $H_\lambda(p^2,q^2)$ has a branch cut in p^2 and the physical decay takes place on this branch cut: **H_λ is complex-valued!**
- Triangle diagrams are known to create *anomalous* branch cuts in q^2 [e.g. Lucha, Melikhov, Simula '06] $\rightarrow$ Does this also apply here? We have no Lagrangian nor power counting!
- The presence and the impact of such a branch cut in our approach is under investigation

Theory inputs

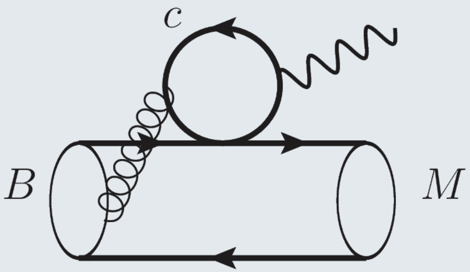
$\mathcal{H}_\lambda$ can be calculated in **two kinematics regions**:

- **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol '04; Beylich, Buchalla, Feldmann '11]
- **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang '10]



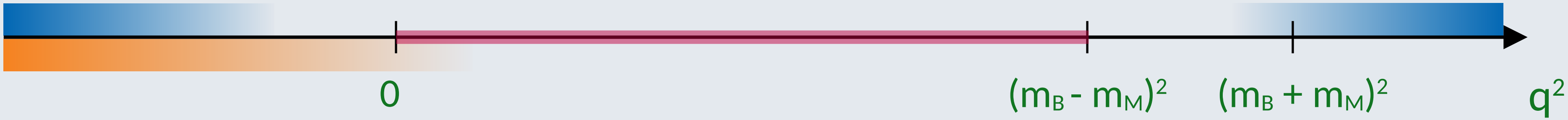
LO and α_s corrections

[Asatrian, Bieri, Greub, Walker '04; de Boer '17; **Asatrian, Greub, Virto '19**]



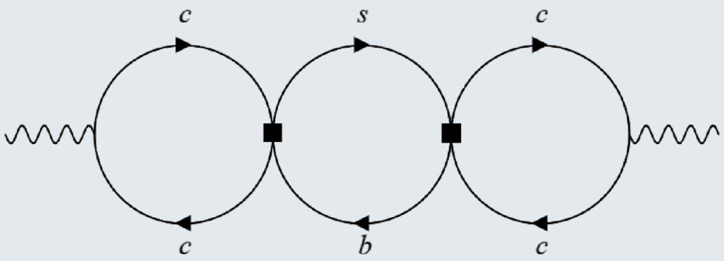
Non-perturbative soft gluon corrections

[Khodjamirian, Mannel, Pivovarov, Wang '10; **Gubernari, van Dyk, Virto '20**]



Dispersive bound

- **Main idea:** Compute the charm-loop induced, inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to $\mathcal{H}_\lambda$ [Gubernari, van Dyk, Virto '20]



+ other diagrams…

- The optical theorem gives a **shared bound** for **all the b → s processes**:

$$1 > 2 \int_{(m_B+m_K)^2}^{\infty} \left|\hat{\mathcal{H}}_0^{B\to K}(t)\right|^2 dt + \sum_\lambda \left[2 \int_{(m_B+m_{K^*})^2}^{\infty} \left|\hat{\mathcal{H}}_\lambda^{B\to K^*}(t)\right|^2 dt + \int_{(m_{B_s}+m_\phi)^2}^{\infty} \left|\hat{\mathcal{H}}_\lambda^{B_s\to \phi}(t)\right|^2 dt \right]$$

$$+ \Lambda_b \to \Lambda^{(*)} \ldots$$

$\text{known functions} \times \mathcal{H}_0^{B\to K}(t)$

GRvDV parametrization

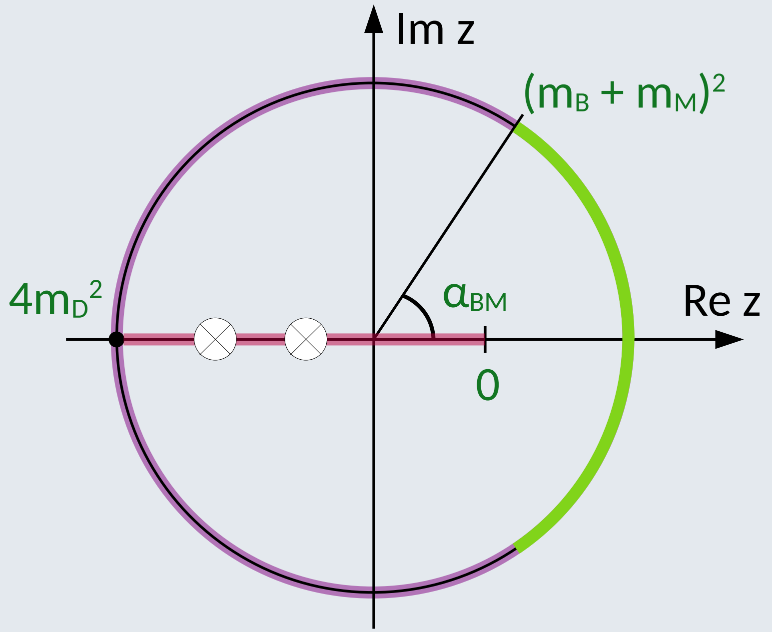
- The bound can be "**diagonalized**" with **orthonormal polynomials** of the arc of the unit circle [Gubernari, van Dyk, Virto '20]

$$z(s) = \frac{\sqrt{4m_D^2 - s} - \sqrt{4m_D^2 - s_0}}{\sqrt{4m_D^2 - s} + \sqrt{4m_D^2 - s_0}}$$

$$\mathcal{H}_\lambda(z) = \frac{1}{\phi(z)\mathcal{P}(z)} \sum_{k=0}^{N} a_{\lambda,k}\, p_k(z)$$

- The coefficients respect the **simple bound**:

$$\sum_{n=0}^{\infty} \left\{ 2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0} \left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2 \right] \right\} < 1$$



Numerical analysis

- The parametrization is fitted to

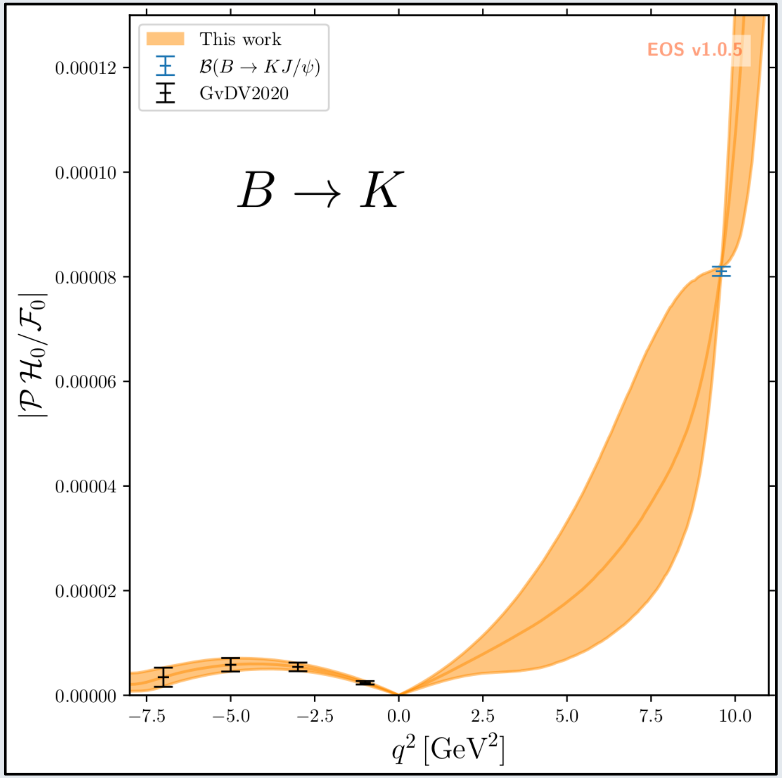
  B → K, B → K*, B_s → φ

  using:
  - 4 theory point at negative q^2 from the light cone OPE
  - Experimental results at the J/ψ
  - Use an **under-constrained fit** and allow for **saturation of the dispersive bound**

→ The uncertainties are **truncation order-independent**, i.e., increasing the expansion order does not change their size

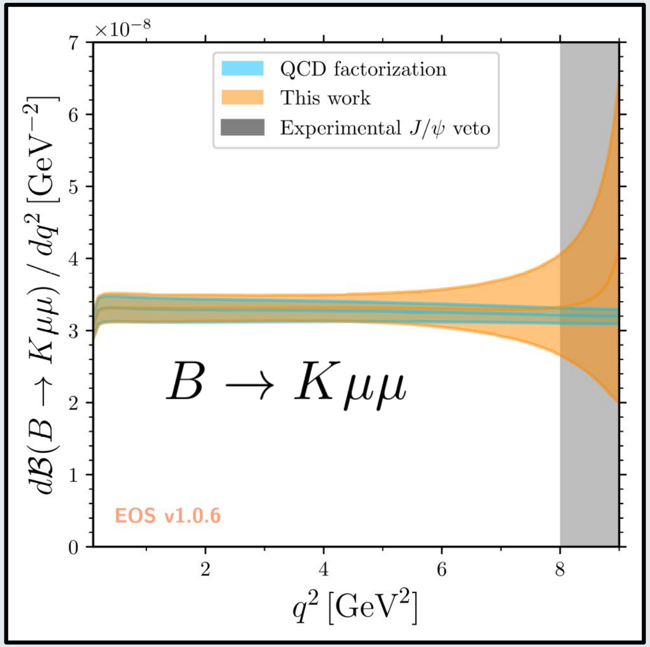
→ All p-values are larger than 11%

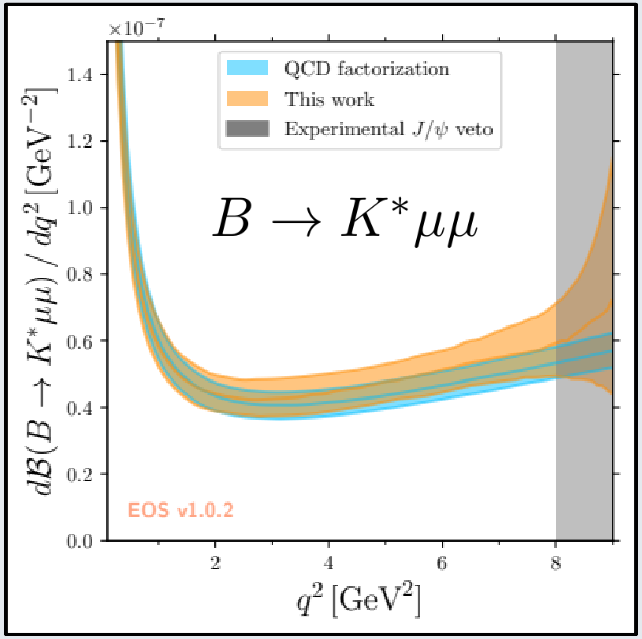
[Gubernari, MR, van Dyk, Virto '22]

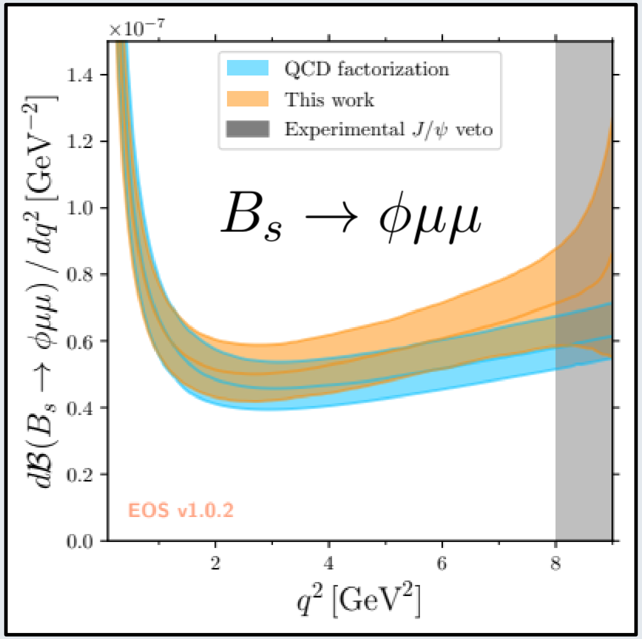


SM predictions

- **Good overall agreement** with previous theoretical approaches
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- **Larger** but **controlled** uncertainties especially near the J/ψ
  - The approach is **systematically improvable** (new channels, ψ(2S) data...)



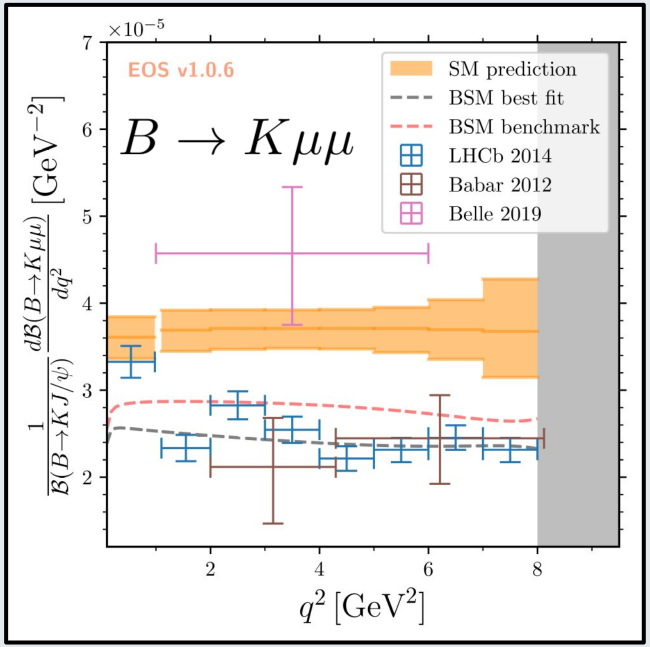


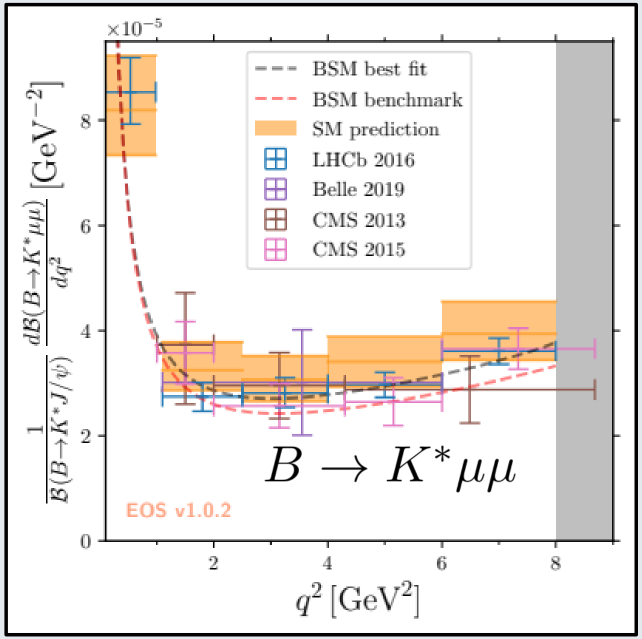


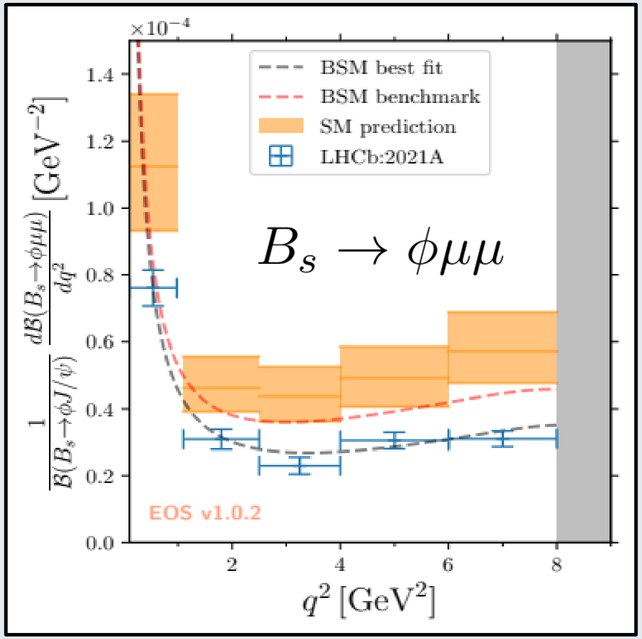
Confrontation with data

- This approach of the non-local form factors **does not solve the “B anomalies”**.
- In this approach, the greatest source of theoretical uncertainty now comes from **local form factors**.

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]







Additional plots can be found in the paper: 2206.03797

Local form factors fit

- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD '13 and '22; FNAL/MILC '17]
    - ([Khodjamiriam, Rusov '17]) $\to$ large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate '15]
    - [Gubernari, Kokulu, van Dyk '18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate '15]
    - [Gubernari, van Dyk, Virto '20] (B-meson LCSRs)
- Adding $\Lambda_b \to \Lambda^{(*)}$ form factors is possible and desirable

Details on the fit procedure

- The fit is performed in two steps...
  - Preliminary fits:
    - **Local** form factors:
      - BSZ parametrization (**8 + 19 + 19 parameters**)
      - Constrained on LCSR and LQCD calcultations
    - **Non-local** form factors:
      - order 5 GRvDV parametrization (**12 + 36 + 36 parameters**)
      - 4 points at negative q^2 + $B \to M\, J/\psi$ data

    $\to$ **130 nuisance parameters**
  - ‘Proof of concept’ fit to the WET’s **Wilson coefficients**
- ... using **EOS**: eos.github.io

BSM analysis

- A combined BSM analysis would be **very CPU expensive** (130 correlated, non-Gaussian, nuisance parameters!)
- Fit **separately** C_9 and C_{10} for the three channels:
  - $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
  - $B \to K^*\mu^+\mu^-$
  - $B_s \to \varphi\mu^+\mu^-$