Recent advances in axion thermal production

La Thuile 2024

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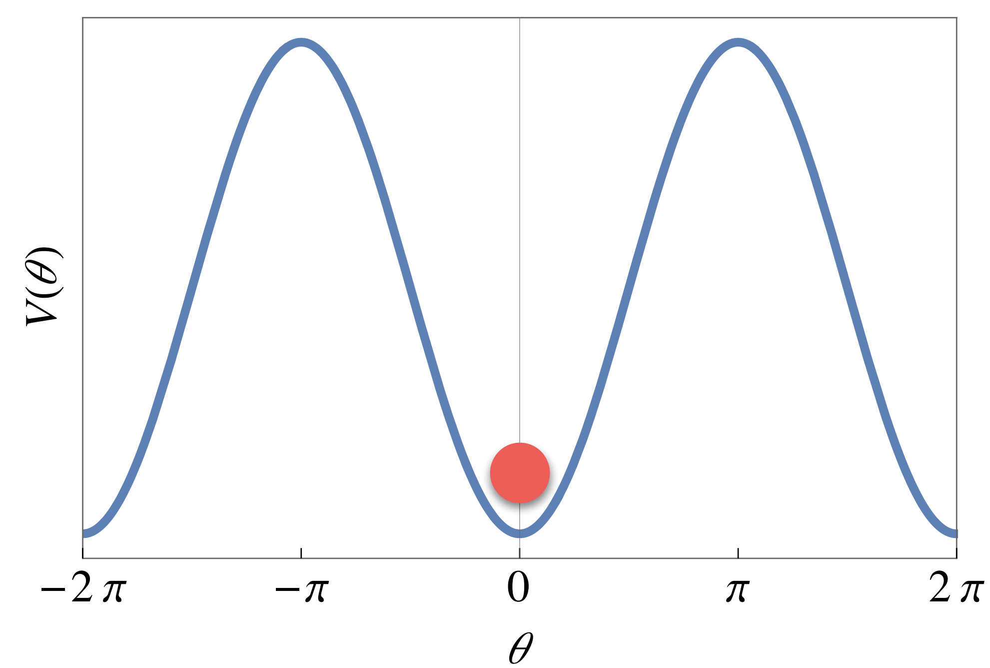
The QCD axion

- Introduced to address the strong CP problem [Peccei, Quinn '77, Weinberg '78, Wilczek '78]

$$\delta\mathcal{L}_{\rm QCD} = \theta \frac{g_s^2}{32\pi^2} G\tilde{G} \qquad |\theta| \lesssim 10^{-10}$$

- promote θ to a dynamical field (axion): $\theta \to \frac{a}{f_a}$

- it acquires a potential and relaxes dynamically to zero



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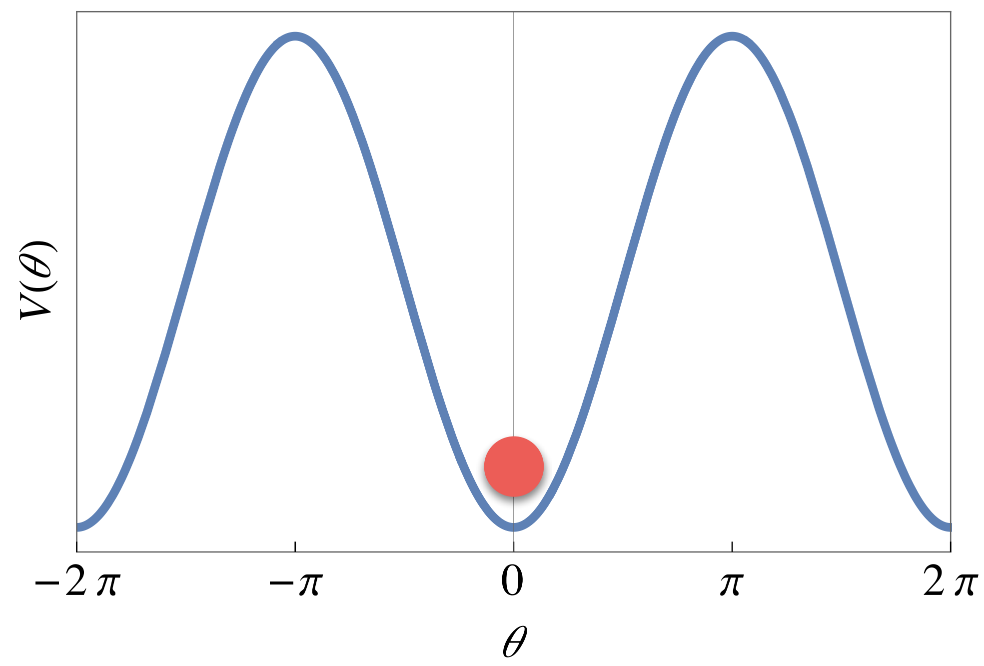
- Unavoidably contributes to the energy density of the universe

$\Omega_{\rm DM}$ (non-thermal production)

i) misalignment mechanism (axion oscillations) [Preskill, Wise, Wilczek '83, Abbott, Sikivie '83, Dine, Fischler '83]



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i) misalignment mechanism (axion oscillations) [Preskill, Wise, Wilczek '83, Abbott, Sikivie '83, Dine, Fischler '83]

ii) topological defects (axion strings, …) [Davies '86, Harari Sikivie '87, …]

→ absent if PQ symmetry is broken before inflation (Pre-inflationary)

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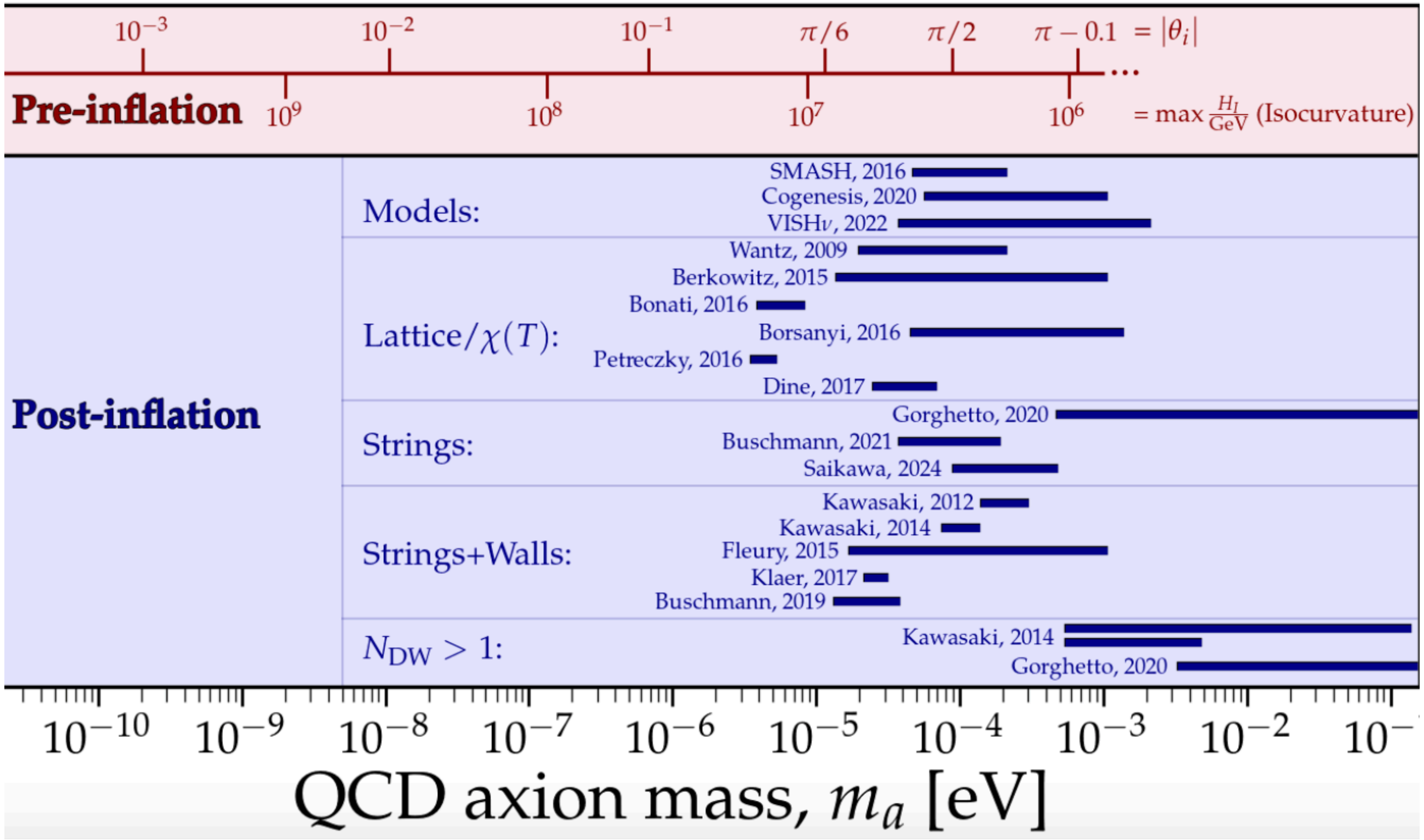
Ω_{DM} (non-thermal production)

[From https://cajohare.github.io/AxionLimits]



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$\Omega_{\rm DM}$ (non-thermal production)

$\Omega_{\rm rad}$ (thermal production) [Turner '87, …]

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu + \rho_a = \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}^{\rm SM} + \frac{1}{2}\left(\frac{T_a}{T_\gamma}\right)^4\right]\rho_\gamma \equiv \left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{\rm eff}\right]\rho_\gamma$$

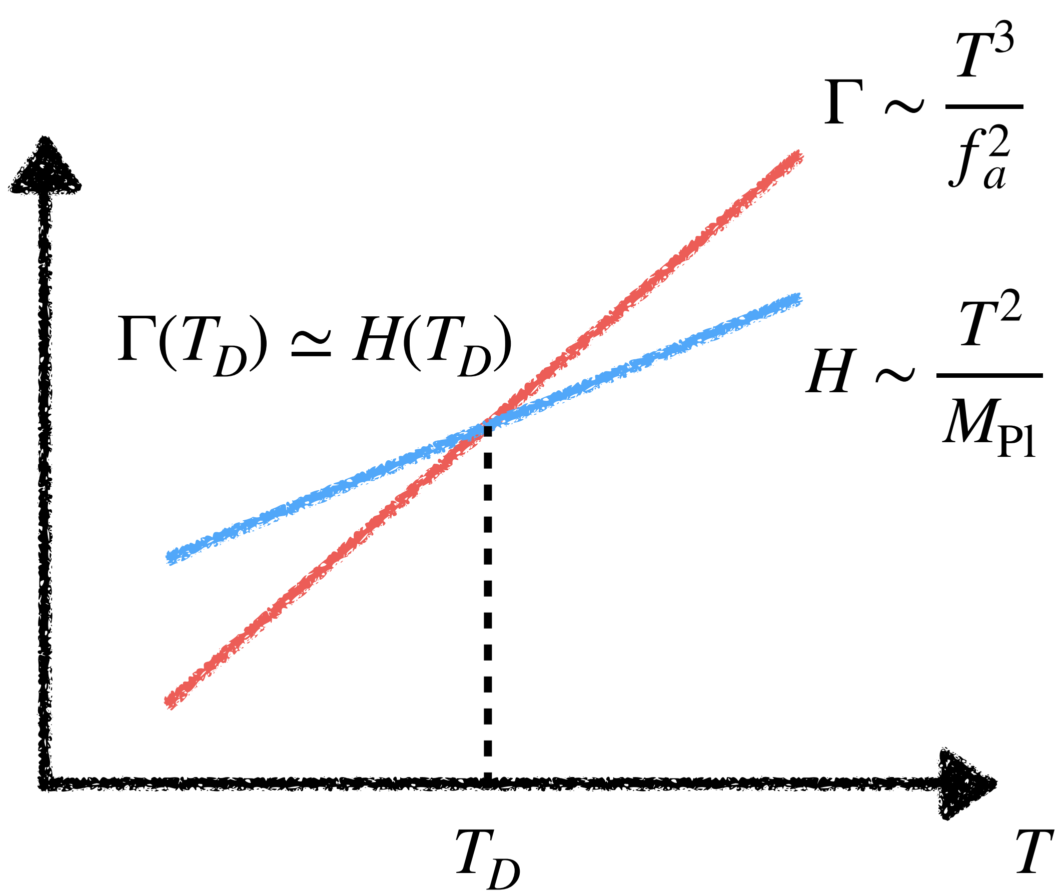
$$\longrightarrow \quad \Delta N_{\rm eff} \equiv N_{\rm eff} - N_{\rm eff}^{\rm SM} = \frac{4}{7}\left(\frac{T_a}{T_\nu}\right)^4 \simeq 0.027\left(\frac{106.75}{g_S(T_D)}\right)^{4/3}$$

Axion thermal production

- Axions can be thermally produced in the early universe

$$\mathscr{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \dots$$

$$T_D \sim \frac{f_a^2}{M_p} \sim \Lambda_{\rm QCD}\left(\frac{f_a}{10^8\ \text{GeV}}\right)^2$$



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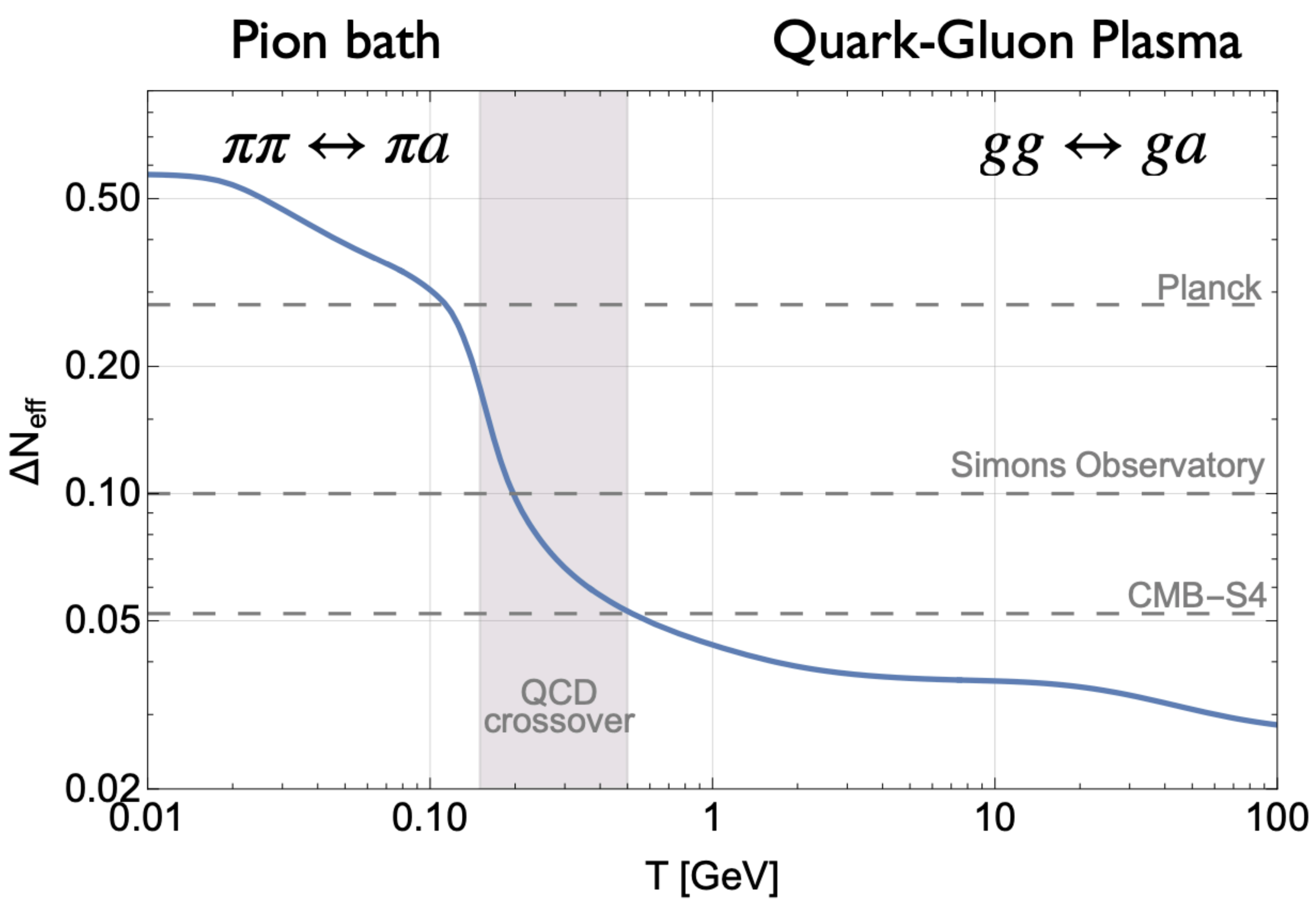

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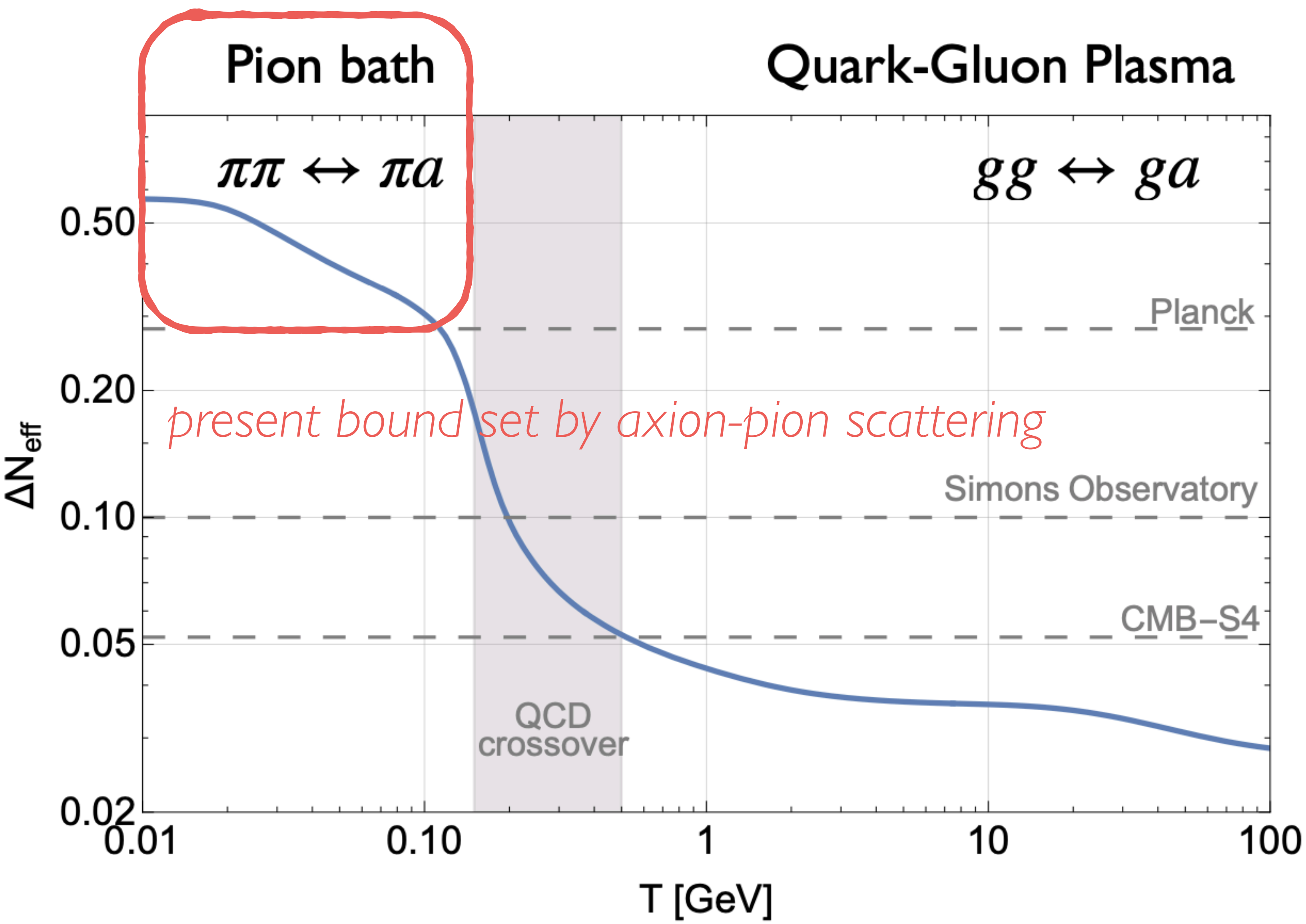


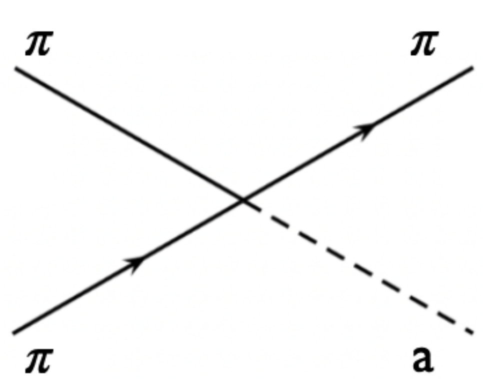
Axion-pion scattering

- Common lore circa 2021

$$\mathscr{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \dots \quad \longrightarrow \quad \frac{C_{a\pi}}{f_a f_\pi}\partial a\, \partial\pi\, \pi\, \pi \qquad C_{a\pi} = \frac{1}{3}\frac{m_d - m_u}{m_u + m_d} + \dots$$

$$\sum |\mathcal{M}|^2_{\rm LO} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4}\left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$

[Chang, Choi - hep-ph/9306216
Hannestad, Mirizzi, Raffelt - hep-ph/0504059]



$$\Gamma_a = \frac{1}{n_a^{\rm eq}}\int \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1}\frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2}\frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3}\frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4}$$
$$\times \sum |\mathcal{M}|^2 (2\pi)^4 \delta^4 (p_1 + p_2 - p_3 - p_4)$$
$$\times f_1 f_2 (1 + f_3)(1 + f_4)$$
$$= \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.212\, T^5 \left[h_{\rm LO}(m_\pi/T)\right]$$

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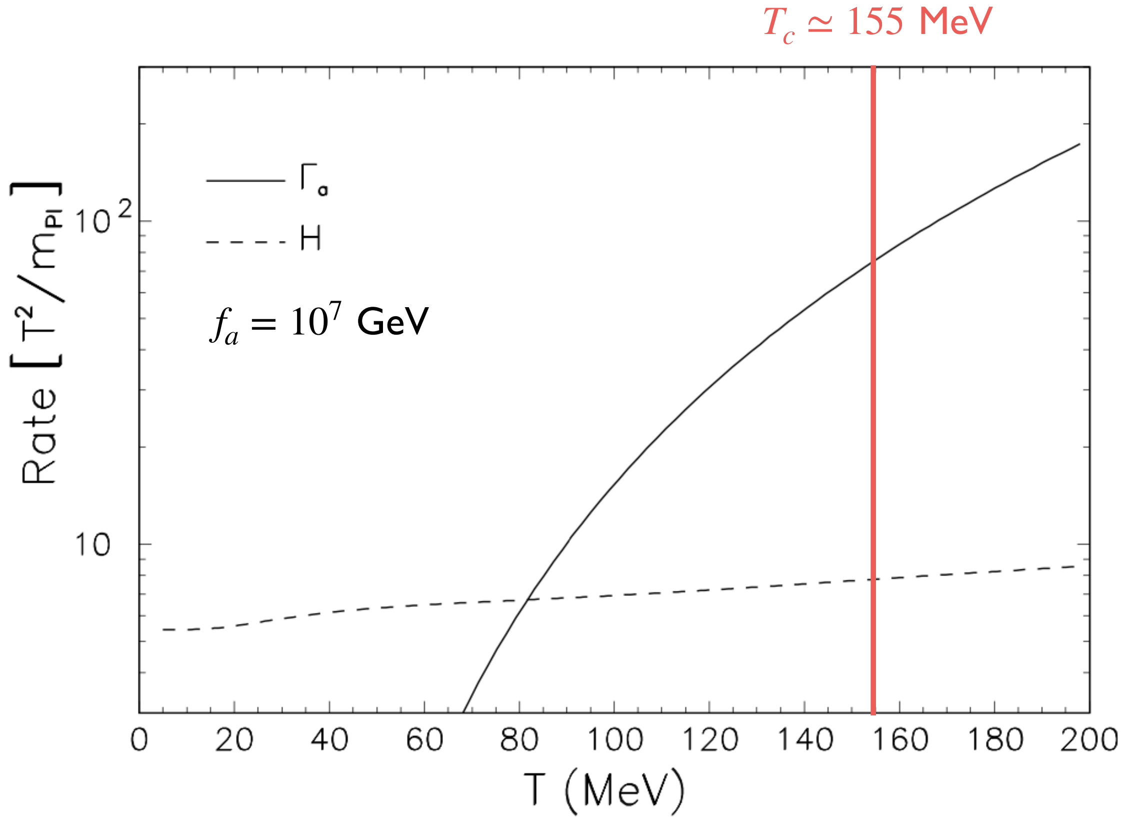
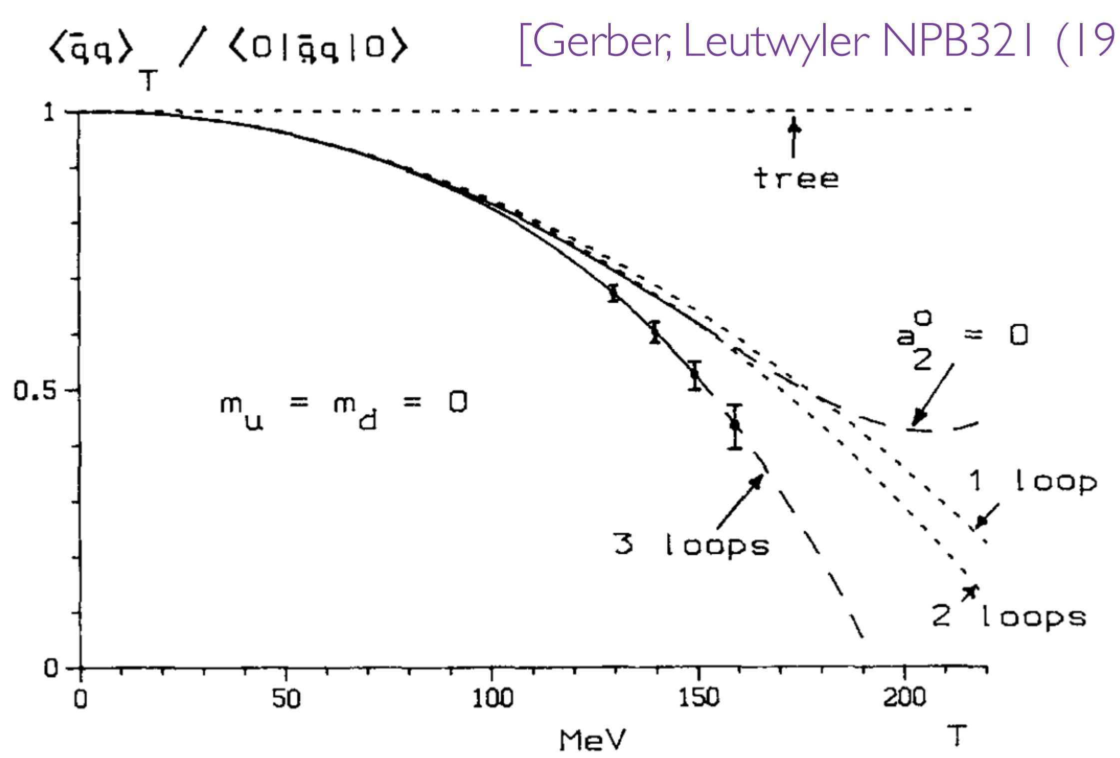


f_a [GeV]	T_D [MeV]	$g_*(T_D)$	n_a [cm^{-3}]
3×10^3	13.37	10.84	74.14
1×10^4	15.30	10.93	73.50
3×10^4	17.63	11.10	72.39
1×10^5	21.21	11.46	70.11
3×10^5	26.06	12.06	66.63
1×10^6	34.75	13.15	61.08
3×10^6	49.12	14.54	55.24
1×10^7	81.61	16.43	48.88
3×10^7	145.31	21.10	38.08

ChiPT breakdown @ finite T

- Qualitative argument #1: melting of chiral condensate

$$\langle \bar{q}q \rangle = \langle 0|\bar{q}q|0\rangle \left(1 - \frac{T^2}{8F^2} - \frac{T^4}{384F^4} - \frac{T^6}{288F^6}\ln\frac{\Lambda_q}{T} + O(T^8)\right)$$



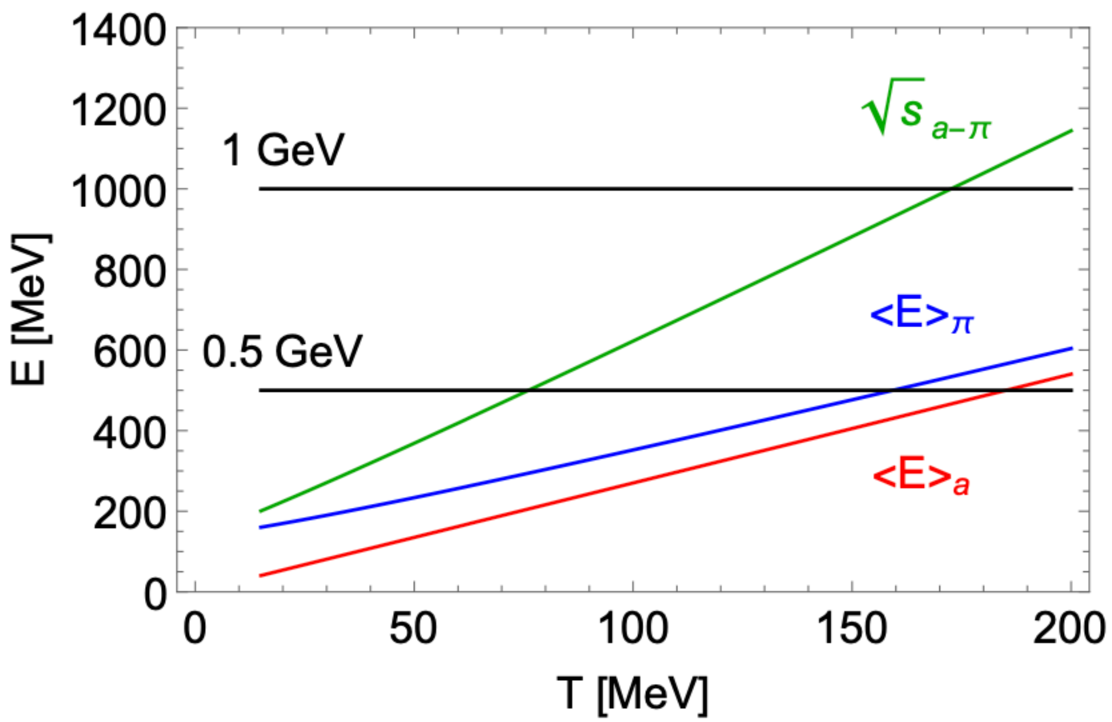
[Gerber, Leutwyler NPB321 (1989)]

→ ChiPT breaks down much earlier than T of chiral symmetry restoration

ChiPT breakdown @ finite T

- Qualitative argument #2: scattering energy in the thermal bath

$$\langle E_a \rangle \sim \frac{\rho_a(T)}{n_a(T)} = \frac{\frac{\pi^2}{30}T^4}{\frac{\zeta(3)}{\pi^2}T^3} \simeq 2.7\ T \qquad \langle E_\pi \rangle \sim \frac{\rho_\pi(T)}{n_\pi(T)} = \dots$$



$$\sqrt{s_{a-\pi}} \sim \langle E_\pi \rangle + \langle E_a \rangle \simeq 500 \text{ MeV } @\ T \simeq 70 \text{ MeV}$$

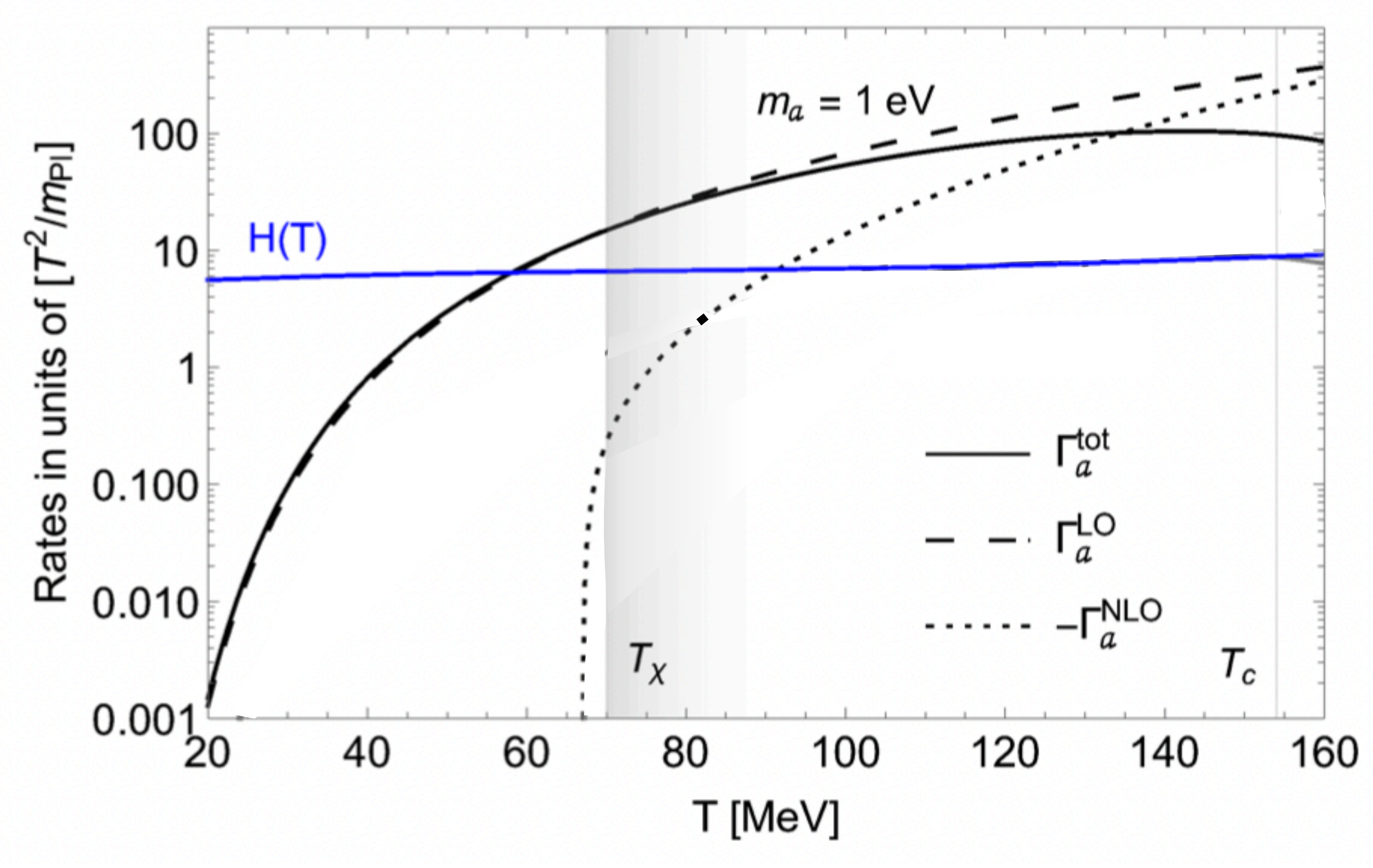
Axion-pion scattering at NLO

- NLO thermalization rate

[LDL, Piazza, Martinelli - 2101.10330]
[LDL, Camalich, Martinelli, Oller, Piazza, Martinelli - 2211.05073]

$$\sum |\mathcal{M}|^2 = |\mathcal{M}_{\rm LO}|^2 + 2{\rm Re}[\mathcal{M}_{\rm LO}\mathcal{M}^*_{\rm NLO}]$$

$$\Gamma_a(T) = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 0.163\, T^5 \left[h_{\rm LO}(m_\pi/T) - 0.251 \frac{T^2}{f_\pi^2}\, h_{\rm NLO}(m_\pi/T)\right]$$



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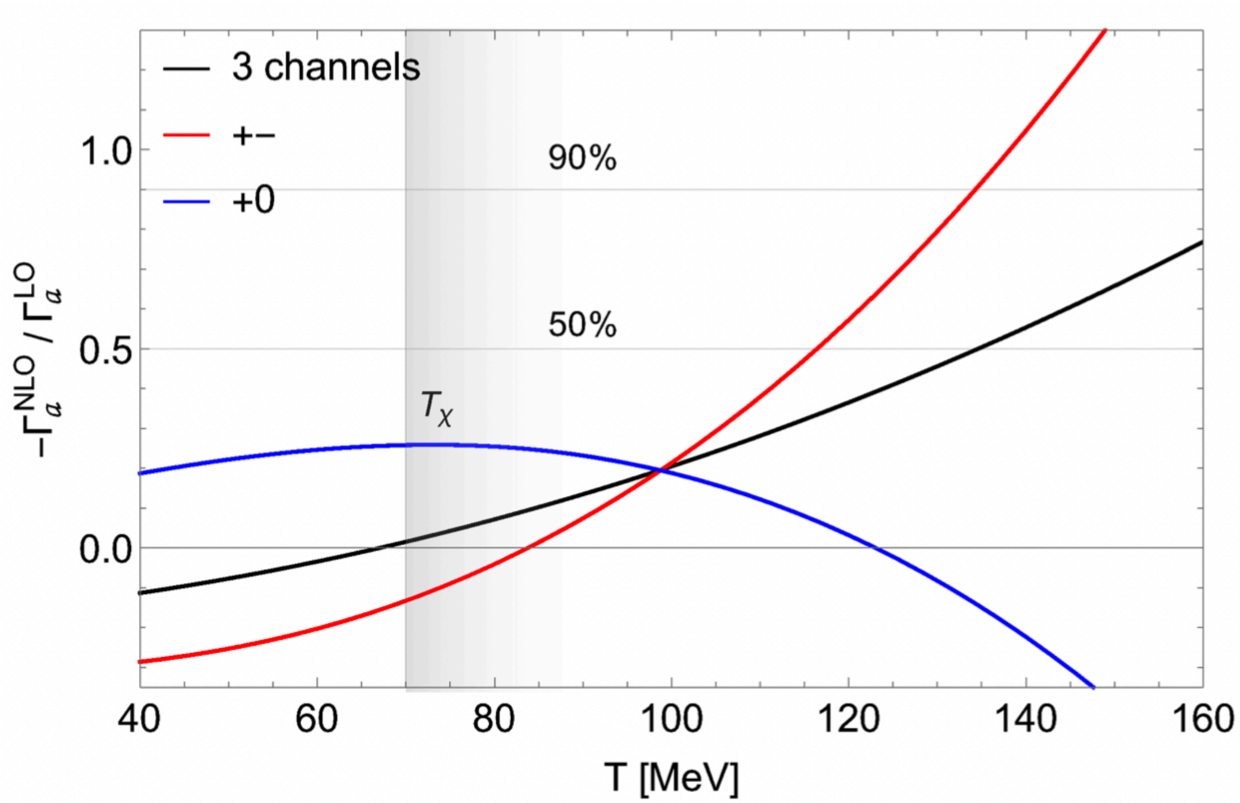
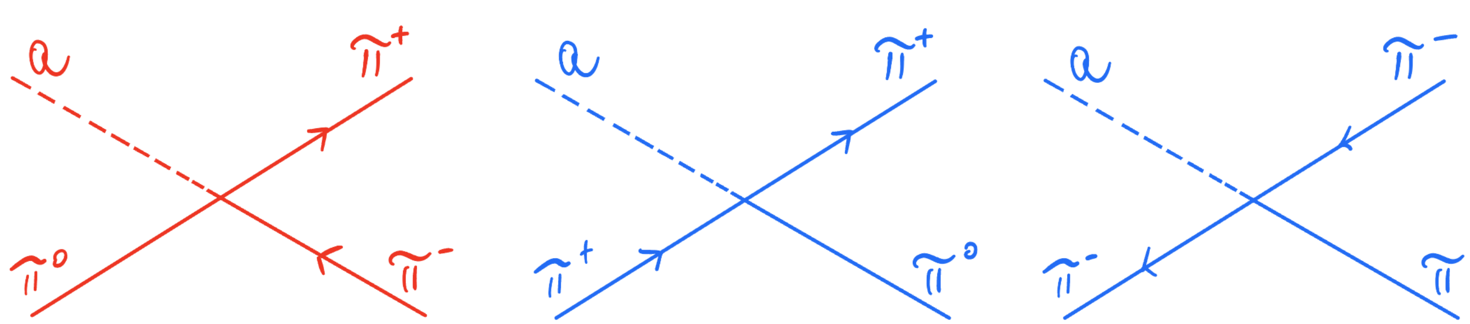


NLO correction to total Γ reaches 50% x LO at $T \simeq 135$ MeV (due to accidental cancellations)

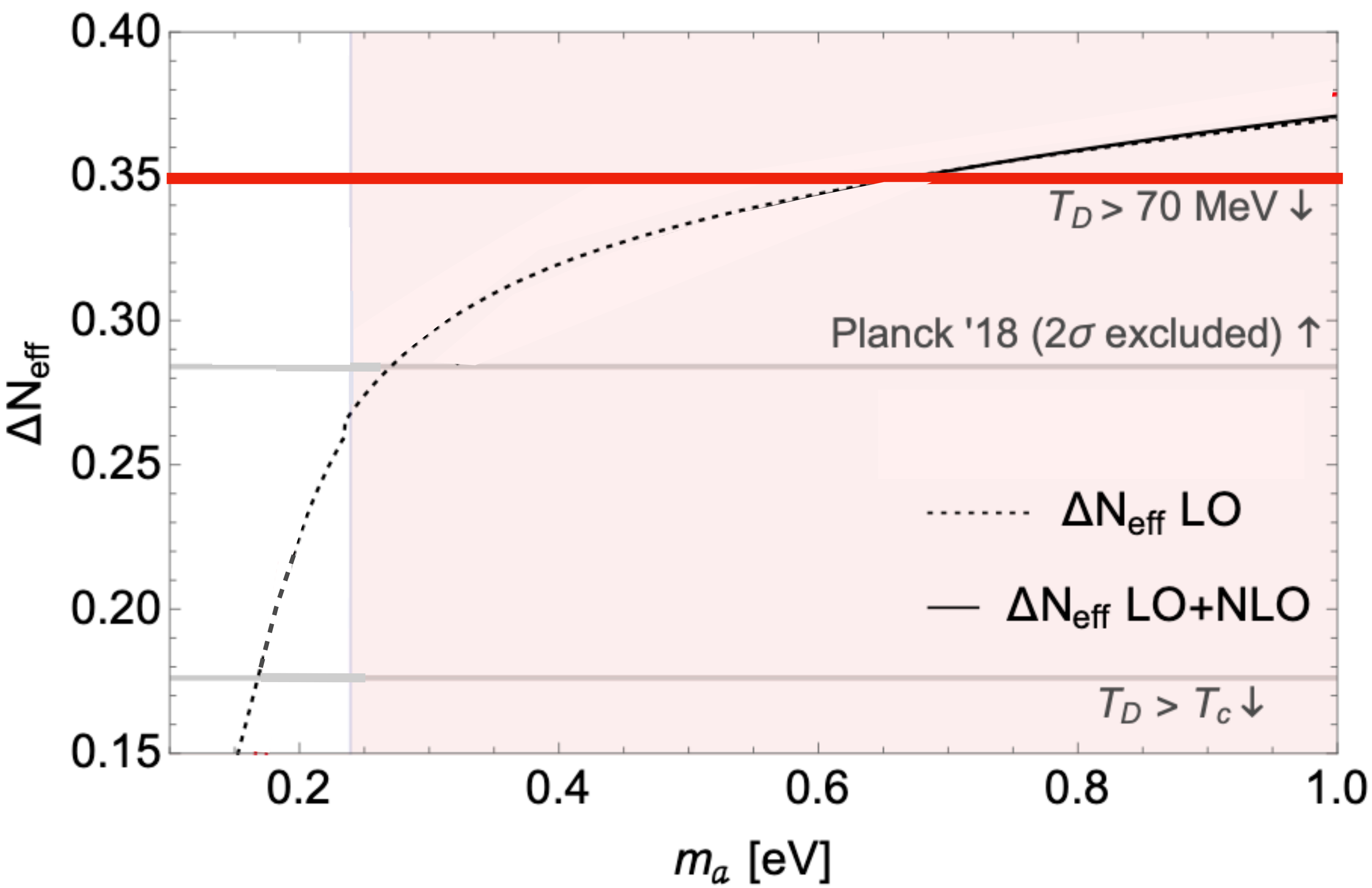
More realistic estimate of ChiPT validity by looking at first exclusive channel with large NLO correction

In $\pi^+\pi^0$ large correction at $T_\chi \simeq 70$ MeV



$\Delta N_{\rm eff}$ with NLO corrections

[LDL, Piazza, Martinelli - 2101.10330]
[LDL, Camalich, Martinelli, Oller, Piazza, Martinelli - 2211.05073]



present Planck bound is beyond the region of validity of ChiPT !

Beyond ChiPT

- Inverse Amplitude Method (IAM) [Truong - PRL61 (1988), …]

i) project scattering amplitude on partial waves, with angular momentum $\boldsymbol{J}$ *and iso-spin* $\boldsymbol{I}$

$$\mathscr{M} \to A_{IJ} \qquad\qquad A_{IJ} = A_{IJ}^{(2)} + A_{IJ}^{(4)} + \dots \qquad \text{(ChiPT expansion)}$$

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ii) construct IAM amplitude, based on a dispersion relation [See Salas-Bernárdez et al - 2010.13709 for theoretical uncertainties of IAM]

$$A_{IJ}^{\mathrm{IAM}} = \frac{A_{IJ}^{(2)}}{1 - A_{IJ}^{(4)}/A_{IJ}^{(2)}} \quad \longleftarrow \quad \frac{1}{A_{IJ}} = \frac{1}{A_{IJ}^{(2)} + A_{IJ}^{(4)} + \dots}$$

LECs from fit to $\pi\pi$ scattering data (O(10%) error)

[Dobado, Pelaez - hep-ph/9604416]

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$$A_{IJ}^{\mathrm{IAM}} = \frac{A_{IJ}^{(2)}}{1 - A_{IJ}^{(4)}/A_{IJ}^{(2)}}$$

→

- *exact elastic unitarity*
- *reproduces σ and ρ resonances*
- *matches to ChiPT at low energy*

↓

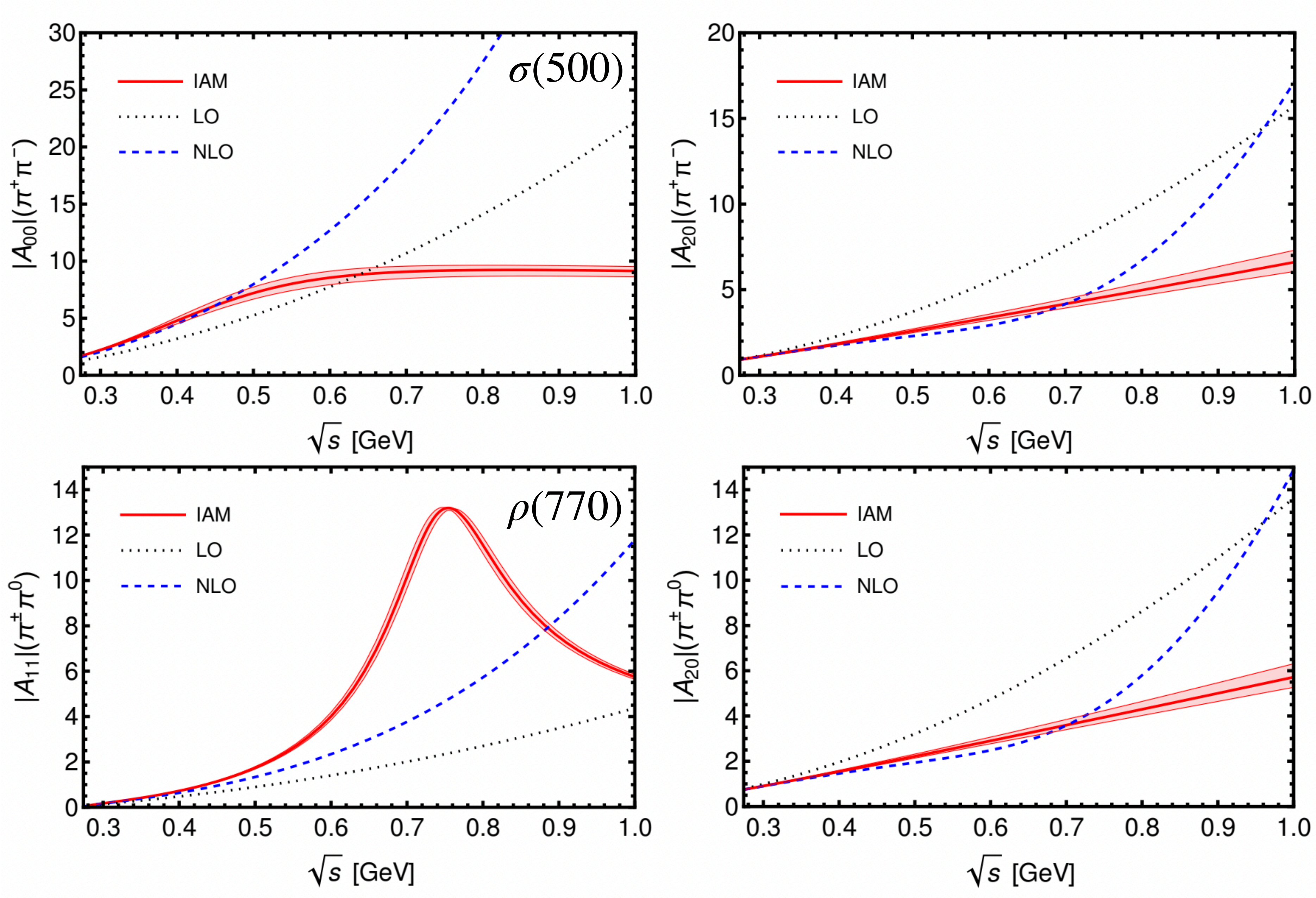
LECs from fit to $\pi\pi$ scattering data (O(10%) error)

[Dobado, Pelaez - hep-ph/9604416]

Partial wave amplitudes

- Growth in energy of ChiPT amplitudes tamed by unitarization

[LDL, Camalich, Martinelli, Oller, Piazza, Martinelli - 2211.05073]

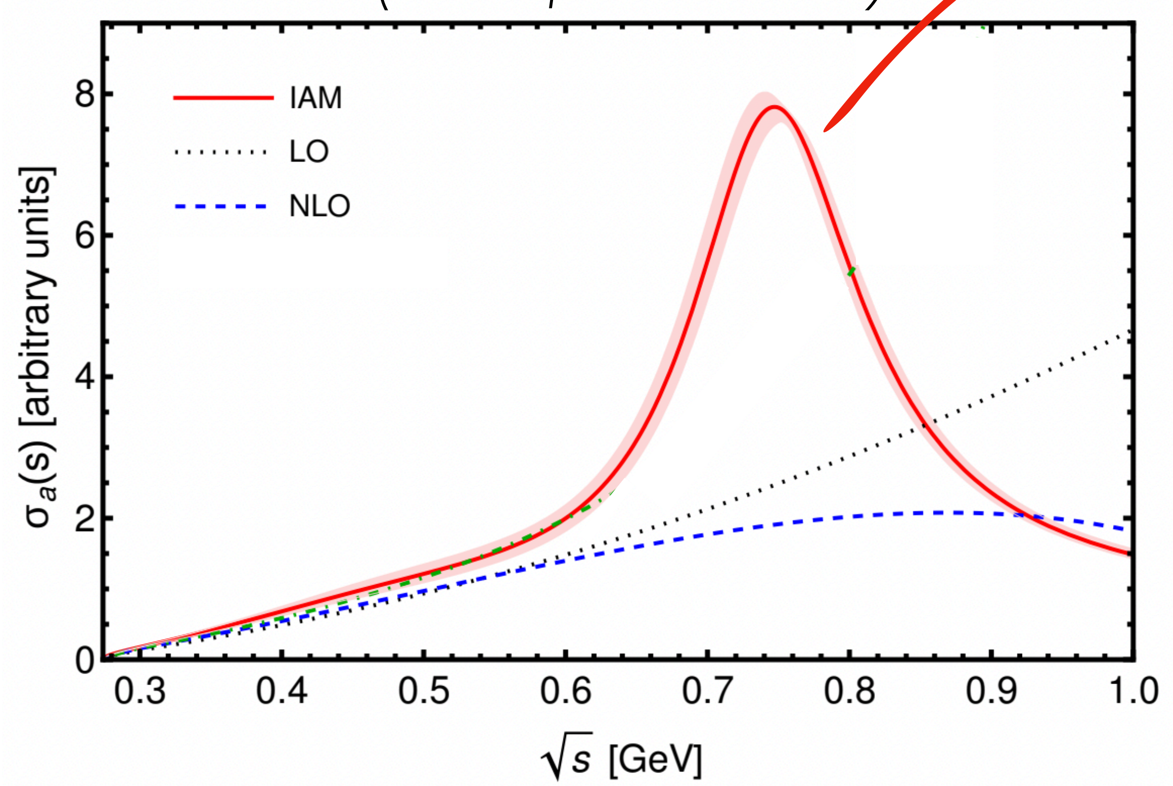


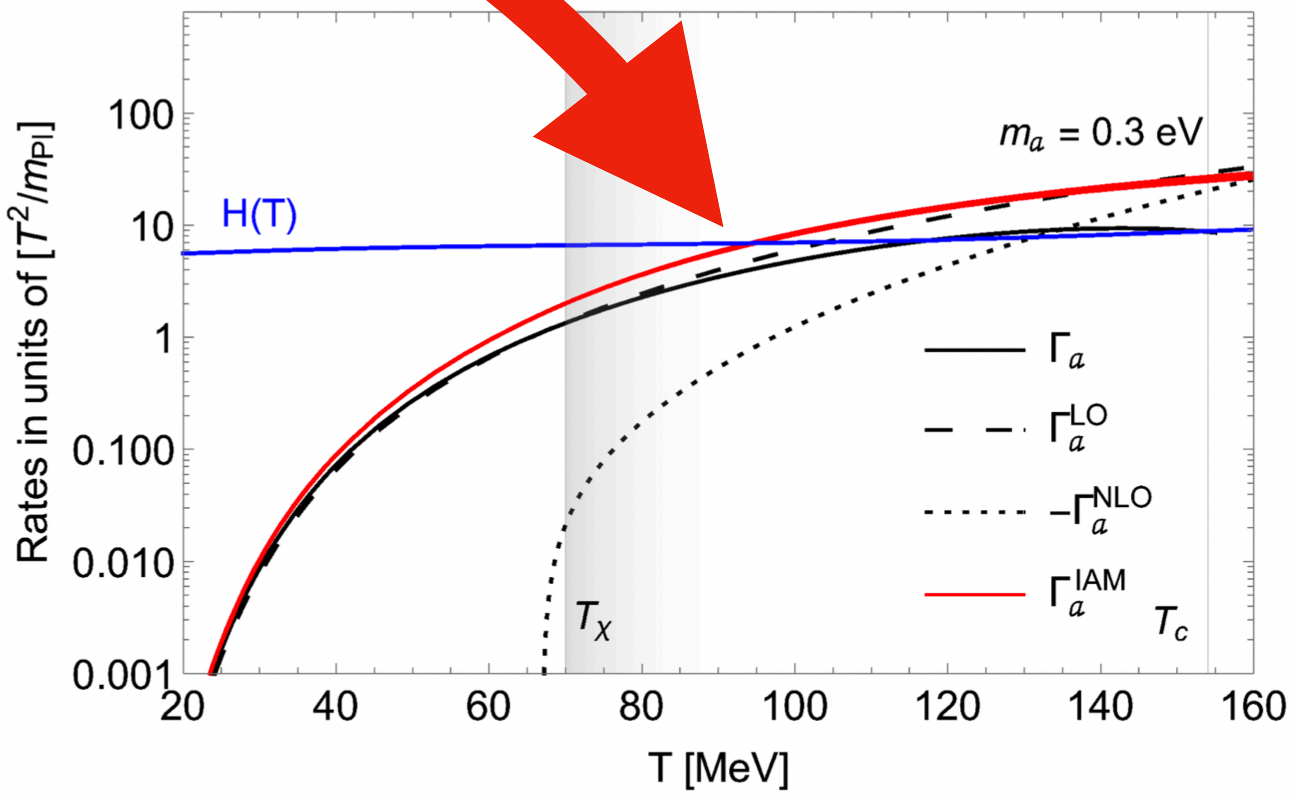
IAM thermalization rate

- Feature of ρ resonance (770 MeV) manifests around $T \sim 100$ MeV

[LDL, Camalich, Martinelli, Oller, Piazza, Martinelli - 2211.05073]

$\pi\pi \to \pi a$ *(sum of all channels)*



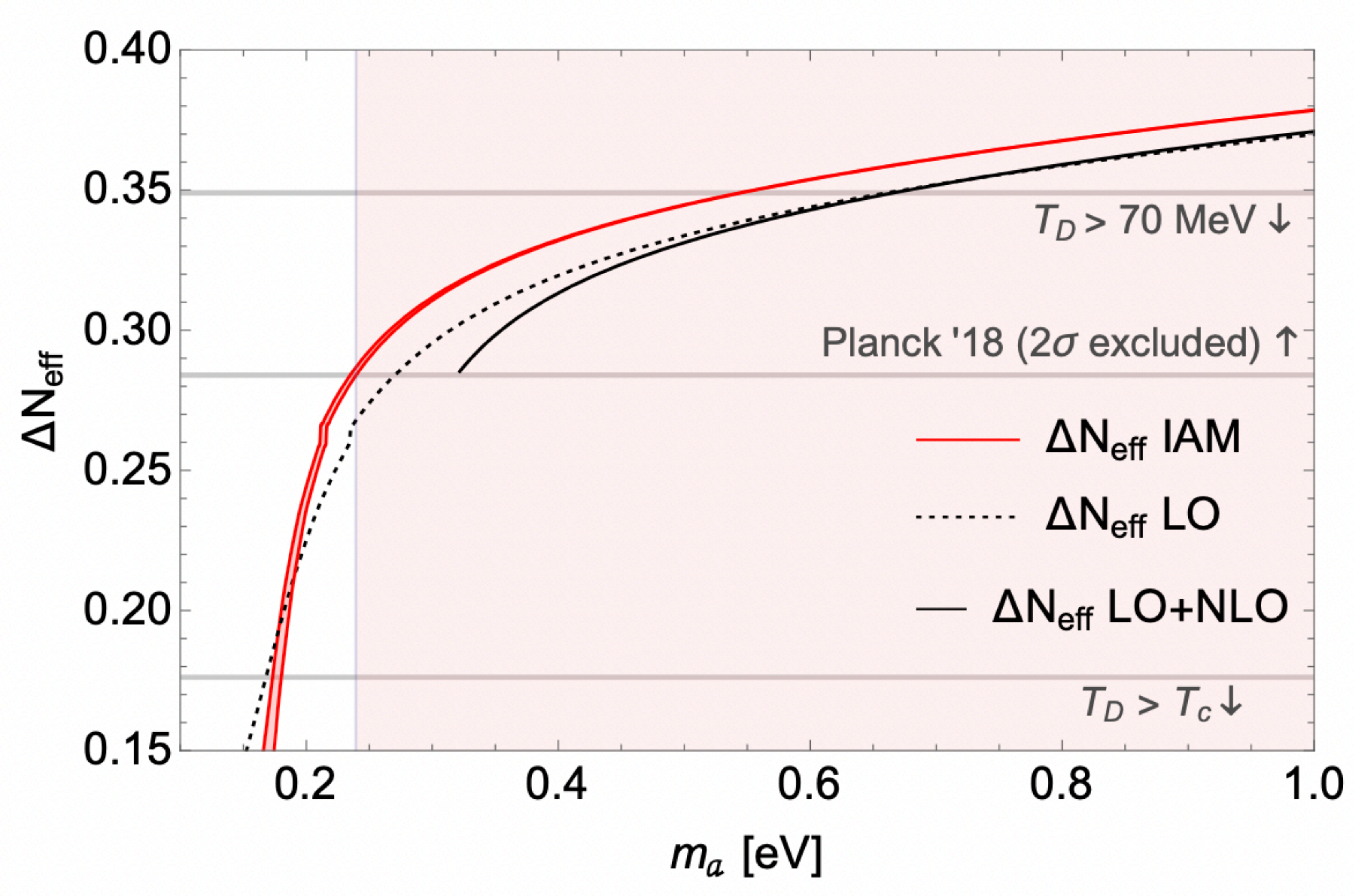


[Similar results in Notari, Rompineve, Villadoro - 2211.03799
based on a different unitarization approach]

$\Delta N_{\rm eff}$: IAM vs LO

- IAM bound: $m_a \lesssim 0.24$ eV

[LDL, Camalich, Martinelli, Oller, Piazza, Martinelli 2211.05073]



Towards a robust bound

- Axion thermalization rate at $T \lesssim T_C \simeq 155$ MeV

i) thermal corrections in axion-pion scattering

[Recently addressed in Wang, Guo, Zhou - 2312.15240. They amount to an O(10%) shift on the bound on the axion mass]

ii) 3-flavour analysis to extend IAM above $\sqrt{s}$ ~ 800 MeV (see backup slides)

iii) check contribution extra channels (kaons, etc)

[$K\pi \to Ka$ implemented at LO in ChiPT in Notari, Rompineve, Villadoro - 2211.03799. Comparable to $\pi\pi \to \pi a$ at T ~ Tc]

iv) axion couplings model-dependency

Towards a robust bound

- Axion thermalization rate at $T \lesssim T_C \simeq 155$ MeV
- Boltzmann equation

$$\frac{dY}{d\log x} = (Y^{\rm eq} - Y)\frac{\Gamma}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*S}}{d\log x}\right)$$

[Salvio, Strumia, Xue -1310.6982, …]

$$\Downarrow$$

Momentum dependence is important:

1. x-section depends on momenta, which decouple at different times
2. # of d.o.f. decreases rapidly around Tc, higher momenta less diluted
3. production might be never in thermal equilibrium

$$\frac{\partial \mathcal{F}_a}{\partial t} - H\,|\mathbf{k}|\frac{\partial \mathcal{F}_a}{\partial |\mathbf{k}|} = \Gamma_a\,(\mathcal{F}_a^{\rm eq} - \mathcal{F}_a)$$

[Notari, Rompineve, Villadoro - 2211.03799
Bianchini, Grilli di Cortona, Valli - 2310.08169]

$$\Gamma_a \equiv {\rm Im}(\Pi_a^{\rm R})/(E f_a^2)$$

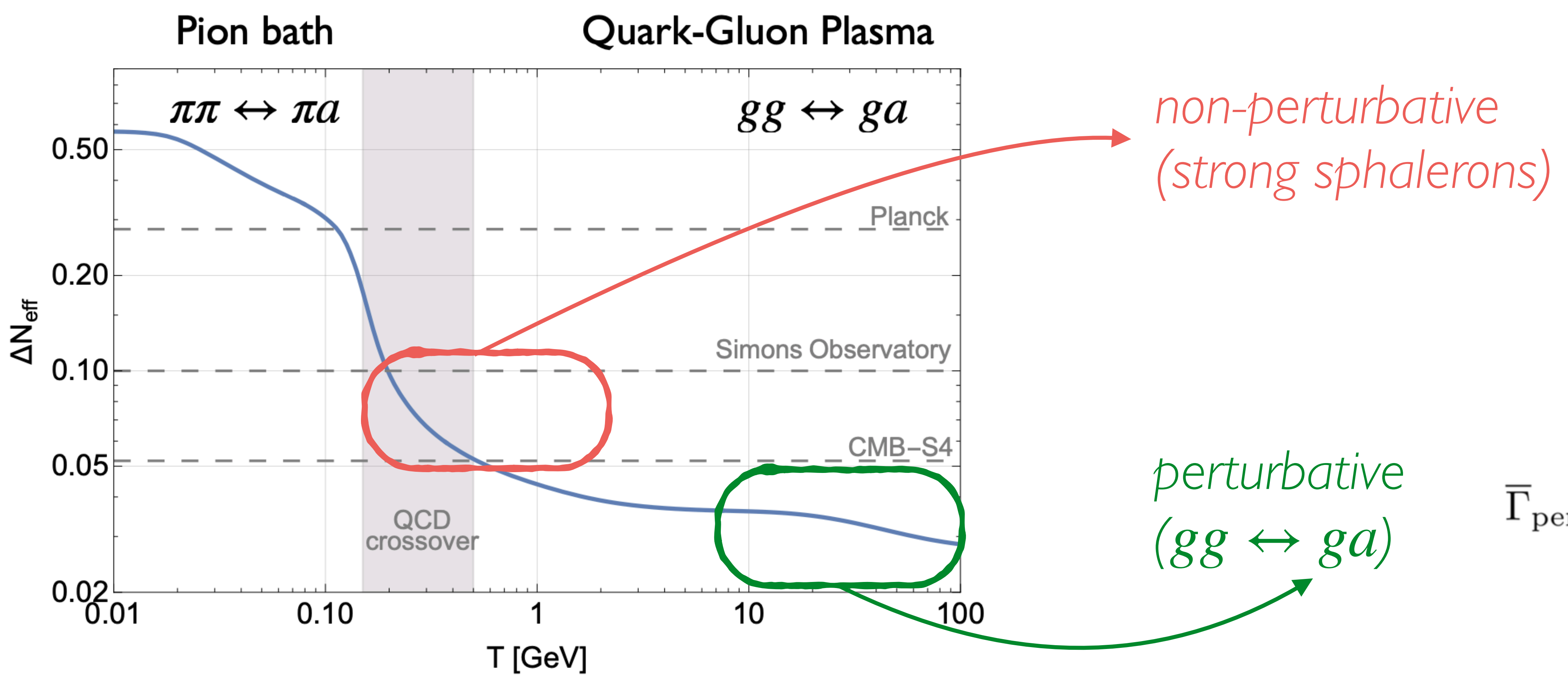
$$\Pi_a^{\rm R} = i\int d^4x\, e^{i\,xk}\,\langle\,[\,\mathcal{Q}(x),\,\mathcal{Q}(0)]\Theta(t)\,\rangle$$

$$\mathcal{Q} \equiv \alpha_s/(8\pi)\,G\tilde{G}$$

Towards a robust bound

- Axion thermalization rate at $T \lesssim T_C \simeq 155$ MeV
- Boltzmann equation
- Axion thermalization rate at $T \gtrsim T_C \simeq 155$ MeV

→ *needed to assess sensitivity of future CMB surveys*



$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

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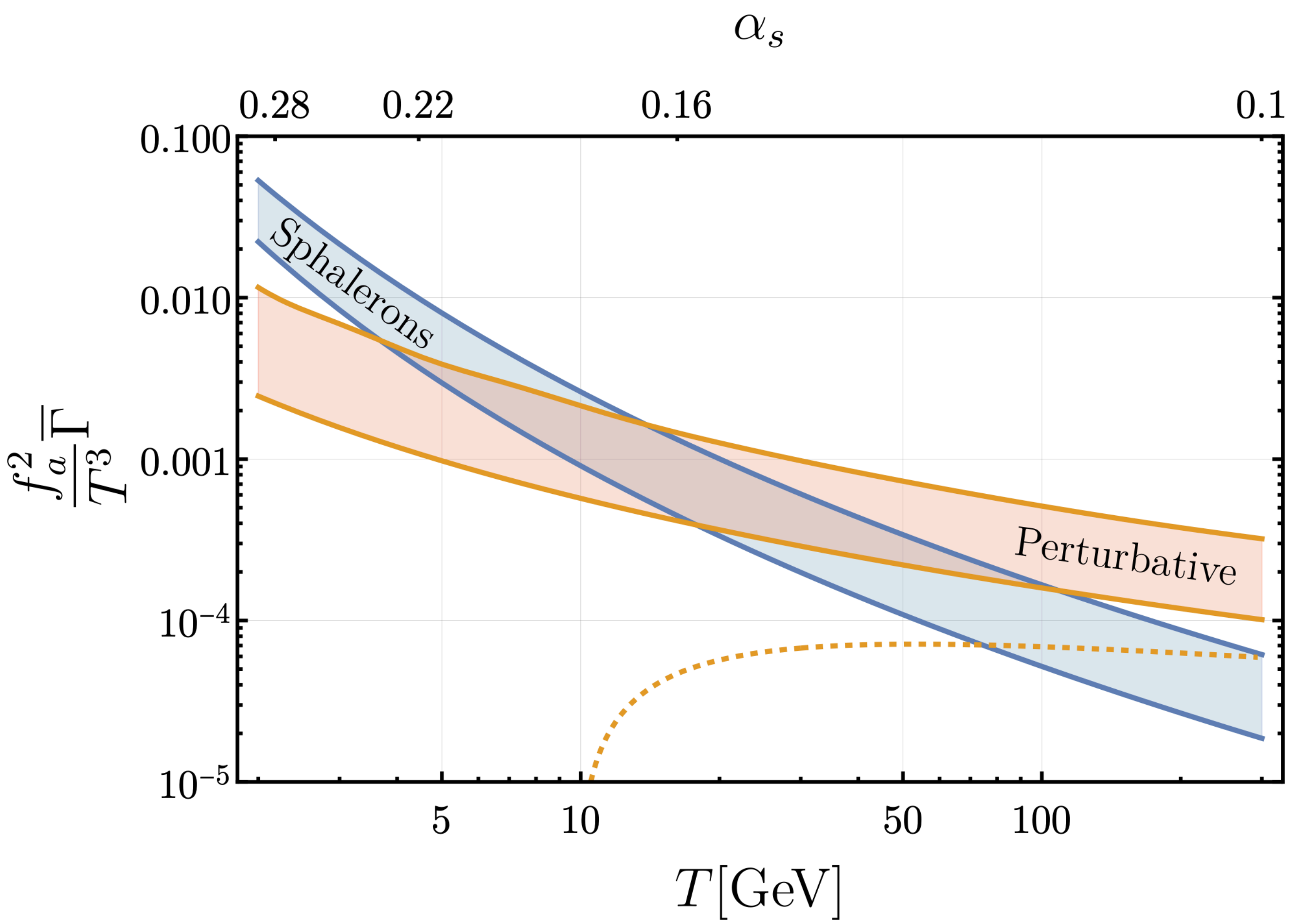
[D'Eramo, Hajkarim, Yun - 2211.03799 → interpolation strategy between perturbative and ChiPT regions]
[Notari, Rompineve, Villadoro - 2211.03799 → strong sphalerons dominate until T ~ 10 GeV]



non-perturbative (strong sphalerons)

perturbative ($gg \leftrightarrow ga$)

$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

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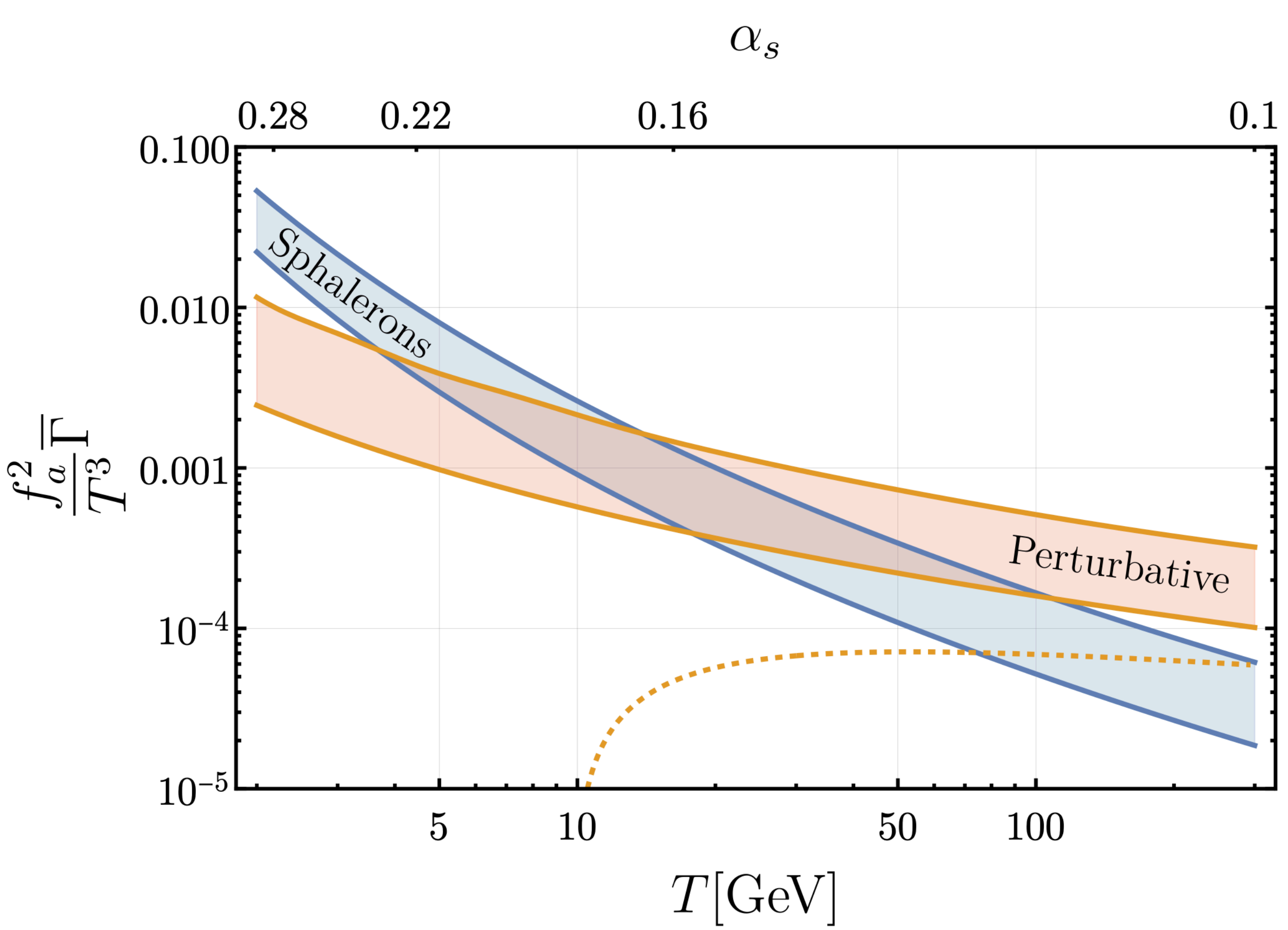


non-perturbative (strong sphalerons)

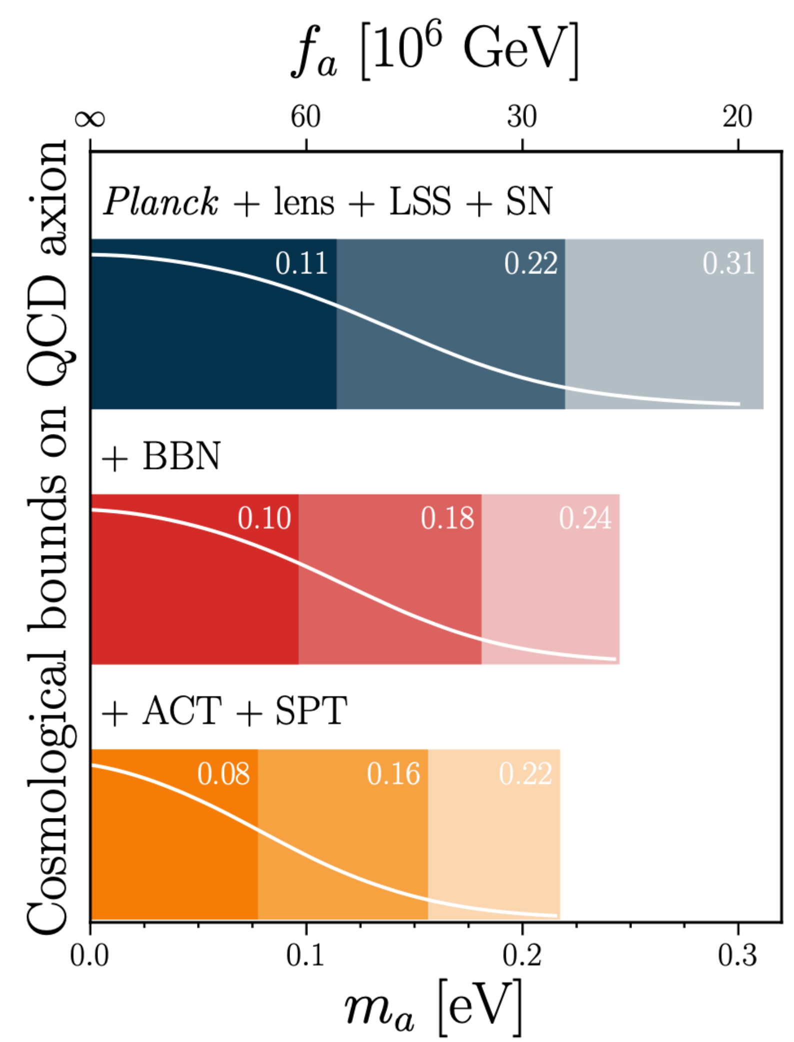
[Bonanno, D'Angelo, D'Elia, Naviglio, Maio - 2308.01287 ➝ first Nf = 2+1 lattice QCD result at k = 0 for 200 MeV ≲ T ≲ 600 MeV]

$$\Gamma_{\mathrm{Sphal}} = \lim_{\substack{V_s\to\infty \\ t_M\to\infty}} \frac{1}{V_s t_M} \left\langle \left[\int_0^{t_M} dt'_M \int_{V_s} d^3x\, q(t'_M, \vec{x}) \right]^2 \right\rangle$$

Towards a robust bound

- Axion thermalization rate at $T \lesssim T_C \simeq 155$ MeV
- Boltzmann equation
- Axion thermalization rate at $T \gtrsim T_C \simeq 155$ MeV
- Cosmological observables

[…
Caloni, Gerbino, Lattanzi, Visinelli - 2205.01637
D'Eramo, Di Valentino, Giarè, Hajkarim, Melchiorri, Mena, Renzi, Yue - 2205.07849
Di Valentino, Gariazzo, Giarè, Melchiorri, Mena, Renzi - 2212.11926
Bianchini, Grilli di Cortona, Valli - 2310.08169]



[Bianchini, Grilli di Cortona, Valli - 2310.08169]

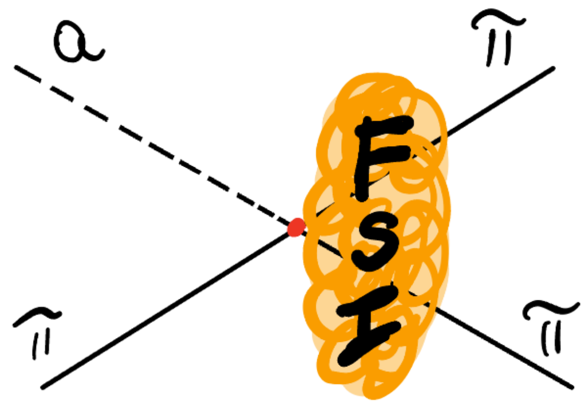
Conclusions

- Resurgence of interest in axion thermal production
- Robust bound in the ballpark of $m_a \lesssim 0.2$ eV
- Future CMB exp.'s will provide an axion discovery channel beyond astrophysical limits

Backup slides

Elementary Watson !

- $\pi\pi$ final state interactions (FSI) are resonant (ρ in $I = J = 1$ and σ in $I = J = 0$)



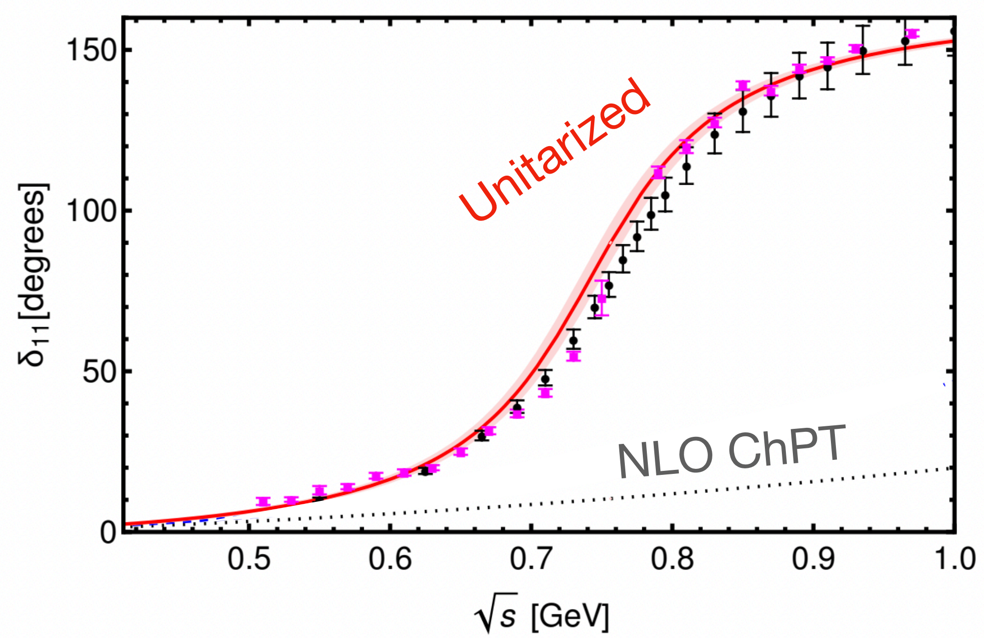
$$\text{Im}\ T_{IJ} = \frac{\sigma(s)}{32\pi}|T_{IJ}|^2 \longleftrightarrow T_{IJ} = \frac{32\pi}{\sigma(s)} e^{i\delta_{IJ}} \sin\delta_{IJ}$$

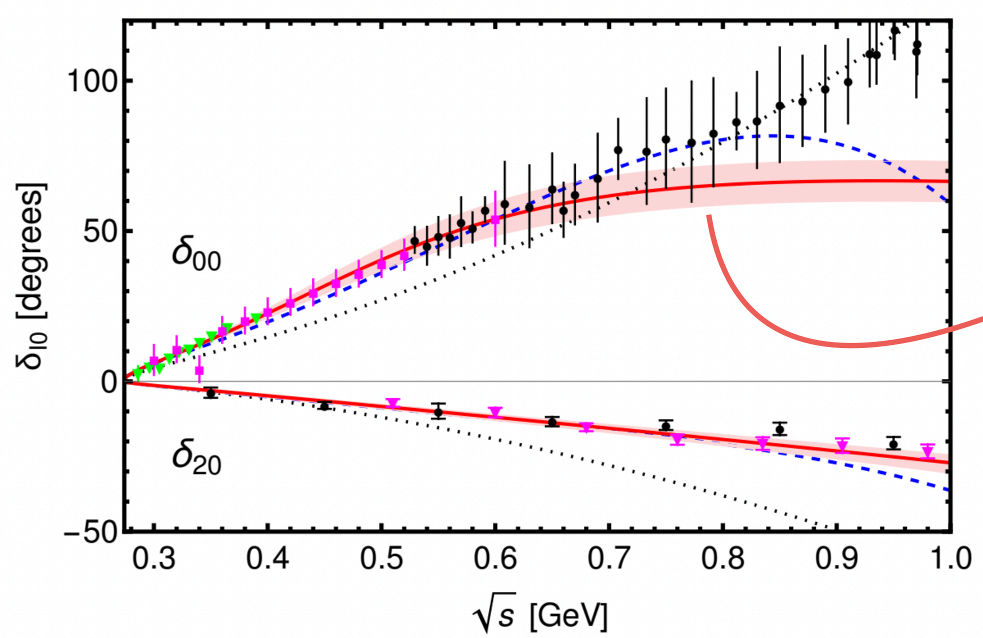
[Watson - Phys. Rev. 88 (1952)]

Unitarity $\longrightarrow$ Watson's theorem: $(\delta_{a\pi})_{IJ} = (\delta_{\pi\pi})_{IJ}$

- Phase shifts from IAM reproduce $\pi\pi$ data up to $\sqrt{s} \simeq 800$ MeV

[LDL, Camalich, Martinelli, Oller, Piazza, Martinelli - 2211.05073]





K-Kbar threshold - $f_0(980)$ requires 3-flavour analysis