Tutorial

Axions: Theory, cosmology and astrophysics

Day 2

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Exercise 1

Let us consider axions interacting with photons. The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} g_{a\gamma}\, a\, F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_a^2 a^2 \,. \tag{1}$$

- Derive Maxwell equations taking into account the latter interaction.
  [Hint: Derive the Eulero-Lagrange equations for the photon field A_ν. The second set of Maxwell equations is given by the Bianchi identity $\partial_\mu \tilde{F}^{\mu\nu} = 0$.]

- Express the Maxwell equations in terms of the electric and magnetic fields.
  *[Hint: In terms of $\boldsymbol{E}$ and $\boldsymbol{B}$, one can write $F^{i0} = E^i$, $F^{ij} = -\epsilon^{ijk} B_k$, $\tilde{F}^{i0} = B^i$, $\tilde{F}^{ij} = \epsilon^{ijk} E_k$.
  Express $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}_a$ and $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_a$ in terms of a background contribution ($\boldsymbol{E}_0$, $\boldsymbol{B}_0$) and an axion-induced one ($\boldsymbol{E}_a$, $\boldsymbol{B}_a$), then write the Maxwell equations at first order in $g_{a\gamma}$.]*

- Define the effective polarization $\mathbf{P}$ and magnetization $\mathbf{M}$ induced by axions.
  [Hint: Write $\nabla \cdot \boldsymbol{E}_a = -\nabla \cdot \boldsymbol{P}$ and $\nabla \times \boldsymbol{B}_a = \dot{\boldsymbol{E}}_a + \partial \boldsymbol{P}/\partial t + \nabla \times \boldsymbol{M}$.]

Exercise 2

1. Starting from the computed Maxwell equations, calculate the probability of the photon conversion into an axion (and viceversa) in an external transverse magnetic field $\mathbf{B} = \mathbf{B}_0$ through the following steps.

   - From Eq. (1) compute the equations of motion for the axion field a and show that $F_{\mu\nu} \tilde{F}^{\mu\nu} = -4\, \mathbf{E}_a \cdot \mathbf{B}_0$.
   - Simplify Maxwell equations taking into account that $\nabla a \perp \mathbf{B}_0$ and $\mathbf{E}_0 = 0$.
   - Write $\mathbf{B}_a = \nabla \times \mathbf{A}_a$ and $\mathbf{E}_a = -\dot{\mathbf{A}}_a$ and obtain the system of differential equations

$$\begin{cases} \Box \mathbf{A}_a - g_{a\gamma}\, \dot{a}\, \mathbf{B}_0 = 0 \\ (\Box + m_a^2)\, a + g_{a\gamma}\, \dot{\mathbf{A}} \cdot \mathbf{B}_0 = 0 \end{cases} . \tag{2}$$

- Only the component of $\mathbf{A}_a$ parallel to $\mathbf{B}_0$ mixes with a, so let us fix $A_a = A_\parallel$. Assuming the propagation is in the z axis, use the plane wave ansatz $A_\parallel, a \sim e^{-i(\omega t - k\,z)}$ and redefine a to absorb a relative phase between $A_\parallel$ and a. Linearize the obtained system of equations using $(\omega^2+\partial_z^2) \simeq 2\omega(\omega+i\partial_z)$ (what this the origin of this equation?). In this way you obtain

$$(\omega + i\partial_z - H)\begin{pmatrix} A_\parallel \\ a \end{pmatrix} = 0 \tag{3}$$

with

$$H = \begin{pmatrix} 0 & \Delta \\ \Delta & \Delta_a \end{pmatrix}, \qquad \Delta = \frac{g_{a\gamma}\,B_0}{2}, \quad \Delta_a = -\frac{m_a^2}{2\omega}\,. \tag{4}$$

- Find the eigenvalues $\Delta'_\parallel$, Δ'_a of the matrix H and diagonalize it by introducing the rotation matrix $R(\theta)$ which connects interaction eigenstates $(A_\parallel \;\; a)^T$ to propagation eigenstates $(A'_\parallel \;\; a')^T$:

$$\begin{pmatrix} A'_\parallel \\ a' \end{pmatrix} = R(\theta)\begin{pmatrix} A_\parallel \\ a \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} A_\parallel \\ a \end{pmatrix} \tag{5}$$

How will propagation eigenstates evolve? What about interaction eigenstates?

- Demonstrate the following relations

$$\frac{1}{2}\tan 2\theta = -\frac{\Delta}{\Delta_a}\,, \quad \sin 2\theta = \frac{2\Delta}{\sqrt{\Delta_a^2 + 4\,\Delta^2}}, \quad \cos 2\theta = -\frac{\Delta_a}{\sqrt{\Delta_a^2 + 4\,\Delta^2}} \tag{6}$$

- For a pure initial photon state, compute the probability $p(\gamma \to a)$ to produce an axion at a distance z. Analogously, compute the probability to produce a photon $p(a \to \gamma)$ for an initial pure axion state.

2. Find the expression for the oscillation length L_{osc} and show that when the propagation length L is shorter than L_{osc} (coherent conversions) the probability becomes

$$p(\gamma \to a) = p(a \to \gamma) = \frac{1}{4} g_{a\gamma}^2\, B^2\, L^2\,. \tag{7}$$

[Hint: Write $p(\gamma \to a) \propto \sin^2(\pi\, z/L_{\mathrm{osc}})$.]

3. Write the probability in the limit $\Delta_a^2 \gg \Delta^2$.

<u>Exercise 3</u>

Assume that the cross section for the Primakoff process $\gamma + p \to p + a$ is roughly

$$\sigma \sim \frac{\alpha\, g_{a\gamma}^2}{8\pi}\,. \tag{8}$$

- Estimate the energy loss of the Sun due to axion emission, assuming axions can freely escape once produced. Consider the Sun as made purely of hydrogen and assume an average temperature $T = 1$ keV. Express the result as a fraction of the solar photon luminosity $L_\odot = 4 \times 10^{33}$ erg s^{-1}. Note that the solar mass is $M_\odot = 2 \times 10^{33}$ g. Thus, the average nuclear energy generation rate is $\epsilon = L_\odot / M_\odot = 2$ erg g^{-1} s^{-1}.

[Hint: The energy-loss rate per unit volume is $Q = \Gamma\, \rho_\gamma$, with Γ the scattering rate of a photon in a proton gas of number density n_p and $\rho_\gamma = \pi^2/15\, T^4$ the thermal photon energy density in the solar interior. The energy-loss rate per unit mass is $\epsilon = Q/\rho$, with ρ the solar mass density.]

A more rigorous treatment, including screening effects in the Primakoff rate and integrating over a realistic solar model yields

$$L_a \sim g_{10}^2\, 1.85 \times 10^{-3}\, L_\odot\,, \tag{9}$$

similar to the simple estimate, where $g_{10} = g_{a\gamma}/(10^{-10}\ \mathrm{GeV}^{-1})$.

- Assuming the solar axion production cannot exceed its normal photon luminosity (why?), which limit on $g_{a\gamma}$ is implied?

- Verify that for the relevant range of axion-photon couplings it is indeed true that axions can escape freely once produced, approximating the Sun as a homogeneous body made purely of hydrogen, with mass $M_\odot$ and radius $R_\odot = 6.96 \times 10^{10}$ cm.

<u>Exercise 4</u>

Axions produced in the sun via the Primakoff effect can be detected in the laboratory using a long and strong dipole manget (helioscope experiments). An example is the CERN Axion Solar Telescope (CAST), which used a LHC prototype dipole magnet (with length $L = 9.26$ m and a magnetic field $B = 9$ T) with two parallel straight pipes with cross-sectional area $A = 2 \times 14.5$ cm^2. The magnet was mounted on a movable platform to follow the Sun. The probability of $a \to \gamma$ conversion is

$$p(a \to \gamma) = \left(g_{a\gamma}\, B\, \frac{\sin(q\, L/2)}{q} \right)^2 , \tag{10}$$

where $q = m_a^2/2\, E$, with m_a and E the axion mass and energy respectively.

- Knowing that solar axions have energy of a few keV, at which mass m_a^c coherence is lost?

The number flux of solar axions at Earth produced via Primakoff effect would be

$$F_a = g_{10}^2\, 3.75 \times 10^{11}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\,. \tag{11}$$

- For $m_a \ll m_a^c$, compute the expected number of axion-induced photon events per hour in CAST for a coupling $g_{a\gamma} = 0.66 \times 10^{-10}$ GeV^{-1}.

Exercise 5

In the Light-shining-through-a-wall (LSW) experiment ALPS II, 20 of the HERA magnets with a length of 8.8 m and a magnetic field of 5.3 T were used, 10 for the axion production region and 10 for the reconversion region each. The production region was equipped with a Fabry-Perot (FP) cavity with a power build-up factor $\beta_{\mathrm{PC}} \sim 5000$, while the reconversion region had a FP cavity with $\beta_{\mathrm{RC}} \sim 40000$. The probability $P(\gamma \to a \to \gamma)$ can be written

$$p(\gamma \to a \to a) = \frac{1}{16} \beta_{\mathrm{PC}}\, \beta_{\mathrm{RC}}\, (g_{a\gamma}\, B\, L)^4 \ . \tag{12}$$

Knowing that a 30 W laser at 1064 nm is used, how many photons per second would be expected for a coupling strength of $g_{a\gamma} = 0.2 \times 10^{-10}$ GeV^{-1}?