Tutorial

Axions: Theory, cosmology and astrophysics

Day 1

Tutor: Giuseppe Lucente

Email: giuseppe.lucente@ba.infn.it

Exercise 1

Using chiral perturbation theory in the two quark approximation, show that the leading order effective axion potential is

$$V(a) = -m_\pi^2 \, f_\pi^2 \sqrt{1 - \frac{4\, m_u\, m_d}{(m_u + m_d)^2} \, \sin^2\left(\frac{a}{2\, f_a}\right)} \, . \tag{1}$$

Exercise 2

The QCD axion coupling to photons

$$\mathcal{L} = -\frac{1}{4}\, g_{a\gamma}\, a\, F_{\mu\nu}\, \tilde{F}^{\mu\nu} \tag{2}$$

has two contributions

$$g_{a\gamma} = \frac{\alpha}{2\,\pi\, f_a} \left(\frac{E}{N} - 1.92(4)\right) \, . \tag{3}$$

The first one depends on the fermionic content of the specific model (and their contribution to the EM and QCD anomaly coefficients, E and N, respectively) while the second contribution is model-independent and originates from the axion mixing with neutral mesons. Using chiral perturbation theory in the two quark approximation, compute the model independent contribution as a function of the light quark masses m_u and m_d.

Exercise 3

Due to the interaction in Eq. (2) the axion can decay in two photons $a \to \gamma\gamma$. Show that in the axion rest frame the decay rate is

$$\Gamma_{a\to\gamma\gamma} = \frac{1}{64\,\pi}\, g_{a\gamma}^2\, m_a^3 \, . \tag{4}$$

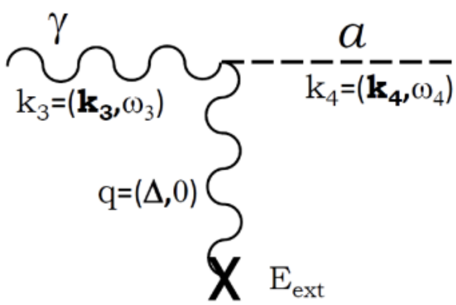

Figure 1: Primakoff process

Exercise 4

The interaction in Eq. (2) may lead to the production of an axion via the Primakoff effect, which is the conversion of a photon into an axion of the same energy in presence of an external electric field $\gamma + Z\,e \to Ze + a$. Show that the differential cross section for this process is

$$\frac{d\sigma}{d\Omega} = \frac{g_{a\gamma}^2\, Z^2\, \alpha}{8\pi} \frac{|\mathbf{p}\times\mathbf{k}|^2}{|\mathbf{p}-\mathbf{k}|^4}\,, \tag{5}$$

where $\alpha = e^2/4\pi$ and $\mathbf{p}$ and $\mathbf{k}$ are the axion and photon momenta, respectively. In a plasma, the long-range Coulomb potential is cut off by screening effects, implying that the differential cross section is modified with the substitution

$$\frac{1}{|\mathbf{p}-\mathbf{k}|^4} \to \frac{1}{|\mathbf{p}-\mathbf{k}|^4} \frac{|\mathbf{p}-\mathbf{k}|^2}{\kappa_s^2 + |\mathbf{p}-\mathbf{k}|^2}\,, \tag{6}$$

The screening scale κ_s in a non-degenerate plasma is given by the Debye-Huckel formula

$$\kappa_s^2 = \frac{4\pi\alpha}{T} \sum_i n_i Z_i^2\,, \tag{7}$$

where n_i is the number density of the i-th nuclear species with charge $Z_i\,e$. Taking into account this change, show that in stars the Primakoff production rate is given by

$$\Gamma_{\gamma\to a} = \frac{g_{a\gamma}^2\, T\, \kappa_s^2}{32\pi} \left[\left(1 + \frac{\kappa_s^2}{4\omega^2}\right) \log\left(1 + \frac{4\omega^2}{\kappa_s^2}\right) - 1\right]. \tag{8}$$

[Hint: To compute $d\sigma/d\Omega$ apply Feynman rules from Fig. 1, or follow the formalism in arXiv:1707.00701 [hep-ph]. The Lagrangian in Eq. (2) can be written as $\mathcal{L} = g_{a\gamma}\mathbf{E}_{\rm ext}\cdot\mathbf{B}_{\rm rad}$]