Basics of cosmology and thermodynamics of Early Universe

$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

$G_{\mu\nu} = 8\pi G_N T^{\mu\nu}$ Einstein eqns

$G_N = 1/m_{pl}^2$ $m_{pl} = 1.22 \times 10^{19}$ GeV

$ds^2 = g^{\mu\nu} dx_\mu dx_\nu = dt^2 - R^2(t)(dx^2 + dy^2 + dz^2)$

FLRW metric

Fluid $T^\mu{}_\nu = \mathrm{diag}(\rho, -p, -p, -p)$ $p = w\rho$ E.O.S

$$H^2 = \left(\frac{\dot R}{R}\right)^2 = \frac{8\pi}{3m_{pl}^2}\rho \quad \frac{8\pi}{3} G\rho - \frac{k}{R^2} \quad (k=0)$$

$$\dot H + H^2 = \frac{\ddot R}{R} = -\frac{4\pi}{3m_{pl}^2}(\rho + 3p)$$

Friedmann eq

$-\frac{4\pi G}{3}(\rho + 3p)$

$H = \dot R / R \propto t^{-1}$ Hubble rate

Conservation law $dE = -p\,dV$ { adiabatic eq

$$\dot\rho + 3H(p+\rho) = 0 \Rightarrow \frac{\partial}{\partial t}(\rho R^3) = -p \frac{\partial}{\partial t}(R^3)$$

Matter $w = 0$, $\rho_m \propto R^{-3}$

Radiation $w = 1/3$ • $\rho_{rad} \propto R^{-4}$ (additional R^{-1} redshift relativistic wavelength)

$\frac{\ddot R}{R} > 0$ accelerated $\rho + 3p < 0 \Rightarrow p < -\rho/3$

simple possibility $p_\Lambda = -\rho_\Lambda$ $(w = -1)$ $\rho_\Lambda =$ const vacuum energy

ΛCDM model : vacuum component 2/3 total energy

1/3 DM. $\rho_b \sim 0.2\, \rho_{DM}$ (baryons)

photons $\rho_{rad} \ll \rho_\Lambda, \rho_{DM}, \rho_B$

However EU was radiation dominated ($T \gtrsim$ eV)

~~Energy den.~~ Number density

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p})\, d^3 p$$

$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p})\, f(\vec{p})\, d^3 p$$

$$\rho = \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{1/2}}{\exp[(E-\mu)/T] \pm 1} E^2\, dE$$

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{(E^2 - m^2)^{1/2}}{\exp[(E-\mu)/T] \mp 1} E\, dE$$

Futnote

$T \gg m$

$$\rho = \begin{cases} (\pi^2/30)\, g\, T^4 & (\text{Bose}) \\ (7/8)(\pi^2/30)\, g\, T^4 & (\text{Ferm}) \end{cases}$$

$$n = \begin{cases} (\zeta(3)/\pi^2)\, g\, T^3 & (\text{Bose}) \\ (3/4)(\zeta(3)/\pi^2)\, g\, T^3 & (\text{Ferm}) \end{cases}$$

$$\rho = T^4 \sum_{i=\text{all species}} \left(\frac{T_i}{T}\right)^4 \frac{g_i}{2\pi^2} \int_{x_i}^{\infty} \frac{(u^2 - x_i^2)^{1/2} u^2\, du}{\exp(u - y_i) \pm 1}$$

$x_i = m_i / T \quad y_i = \mu_i / T$

~~$m \ll T$~~ ~~one~~ species relativistic
neglecting

(neglecting non-relativistic) $\quad \rho = \frac{\pi^2}{30} g_* T^4$

$$g_* = \sum_{i=\text{bos}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fer}} g_i \left(\frac{T_i}{T}\right)^4$$

T = photon temperature

$T \ll \text{MeV}$ only 3 species of ν and photons $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$

$g_*(\ll \text{MeV}) = 3.36$

$100\ \text{MeV} \gtrsim T \gtrsim 1\ \text{MeV}$, $e^\pm$ $\quad T_\nu = T_\gamma$, $g_* = 10.75$

$T \gtrsim 300\ \text{GeV}$, all SM, 8 gluons, $W^\pm$, Z, 3 quark legs

1 Higgs, $g_* = 106.75$.

$H = 1.66\, g_*^{1/2}\, T^2 / m_{pl}$ $\quad \Leftarrow$ Radiation dominated

$t \lesssim 4 \times 10^{10}$ s

$t = 0.301\, g_*^{-1/2} \frac{m_{pl}}{T^2} \sim \left(\frac{T}{\text{MeV}}\right)^{-2}$ sec

$\rho \simeq \rho_R \quad R \propto t^{1/2}$

$P_R = \rho_R / 3$

Entropy

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

$$g_{*s} = \sum_{i=bos} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=fer} g_i \left(\frac{T_i}{T}\right)^3$$

Entropy conservation

before $e^+e^- \xrightarrow{\gamma\gamma} \nu\bar{\nu}$ annihilation

$$s(a_1) = \frac{2\pi^2}{45} T_1^3 \left[\frac{2}{8} + \frac{7}{8}\left(\underset{e^+}{2} + \underset{e^-}{2} + \underset{\nu}{3+3}\right)\right]$$

$$= \frac{43}{90} \pi^2 T_1^3$$

After annihilation e^+e^- have gone away and ν γ have different T

$$s(a_2) = \frac{2\pi^2}{45}\left[2 T_\gamma^3 + \frac{7}{8} 6 T_\nu^3\right]$$

ν decoupled and so lost contact with plasma slightly before annihilation photons heated by e^+e^- annihilation

$$\Rightarrow \quad s(a_1) a_1^3 = s(a_2) a_2^3$$

$$\frac{43}{2} (a_1 T_1)^3 = 4\left[\left(\frac{T_\gamma}{T_\nu}\right)^3 + \frac{21}{8}\right] (T_\nu(a_2) a_2)^3$$

$$a_1 T_1 = a_2 T_2 \qquad (T_\nu \propto a^{-1})$$

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$$

$T: T_\gamma \lesssim 0.5$ MeV photons do not have enough energy to produce e^+e^- pairs and hence electron positron would annihilate and disappears. Neutrinos decoupled so can't receive any entropy

Boltzmann equation annihilation

$a + \bar{a} \rightarrow$ $\quad 1+2 \rightarrow 3+4$

$a+2 \rightarrow 3+4$

$$R^{-3} \frac{d(n_1 R^3)}{dt} = \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3}$$

$$\int \frac{d^3 p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |M|^2$$

$$\{ f_3 f_4 [1 \pm f_1][1 \pm f_2] - f_1 f_2 [1 \pm f_3][1 \pm f_4] \}$$

Ignore quantum statistics

$$f(E) \rightarrow e^{\mu/T} e^{-E/T}$$

and Pauli blocking / Bose enhancement factor can be neglected

$$f_3 f_4 [1 \pm f_1][1 \pm f_2] - f_1 f_2 [1 \pm f_3][1 \pm f_4] \rightarrow$$

$$\rightarrow e^{-(E_1+E_2)/T} \{ e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T} \}$$

Energy conservation $\quad E_1 + E_2 = E_3 + E_4$

$$n_i = g_i e^{\mu_i/T} \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T}$$

g_i = degeneracy e.g. 2 for two spin states of photon

$$n_i^{(0)} = g_i \int \frac{d^3 p}{(2\pi)^3} e^{-E_i/T} \rightarrow g_i \frac{T^3}{\pi^2} \quad m_i \ll T$$

equilibrium

$$g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T} \quad m_i \gg T$$

$\Gamma = \sum_i n_i \langle \sigma_i v \rangle$

with this definition $e^{\mu_i/T}$ can be rewritten as $n_i/n_i^{(0)}$ so that last line of Boltzmann eq can be write

$$e^{-(E_1+E_2)/T}\left\{\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}\right\}$$

With these approx. Boltzmann eq simplifies enormously. thermally averaged cross-sects

$$\langle \sigma v \rangle \equiv \frac{1}{n_1^{(0)} n_2^{(0)}} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} \int \frac{d^3p_3}{(2\pi)^3 2E_3}$$

$$\times \int \frac{d^3p_4}{(2\pi)^3 2E_4} e^{-(E_1+E_2)/T} \times (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)|\mathcal{M}|^2$$

then Boltzmann eq becomes

$$a^{-3}\frac{d(n_1 a^3)}{dt} = n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left\{\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}\right\}$$

chemical equilibrium $\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} = \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}$

Axions $a + 2 \to 3 + 4$ 2, 3, 4 very tightly coupled to plasma $n_2 = n_2^{(0)}$

$$n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left\{\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}\right\} =$$

$$= n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left\{1 - \frac{n_1}{n_1^{(0)}}\right\} = n_2^{(0)} \langle \sigma v \rangle = \left\{n_1^{(0)} - n_1\right\}$$

$$\Gamma = \sum_i n_i \langle \sigma_i v \rangle$$

$$\frac{d n_e^{th}}{dt} + 3 H n_e^{th} = \Gamma \left(n_a^{eq} - n_a^{th} \right)$$

$$\begin{cases} R^{-3} \dfrac{d(R^3 n_a^{th})}{dt} = \Gamma \left(n_a^{eq} - n_a^{th} \right) \\[2ex] R^{-3} \dfrac{d(R^3 n_e^{eq})}{dt} = 0 \end{cases} \Rightarrow$$

$$\frac{d\left[R^3 (n_e^{th} - n_e^{eq})\right]}{dt} = -\Gamma R^3 \left(n_a^{th} - n_e^{eq} \right)$$

scale out effect of expansion of universe
evolution in a comoving volume
($sR^3 = $ cost)

$$Y = n_a / s' \qquad Y_{EQ} = n_a^{EQ} / s'$$

$$x = m_N / T \qquad t = 0.301\, g_*^{-1/2} \frac{m_{pl}}{T^2} =$$

$$= 0.301\, g_*^{-1/2} \frac{m_{pl}}{m^2} x^2$$

$$\frac{dY}{dx} = -\left(\frac{\Gamma_{ABS}}{xH} \right) \left(Y - Y_{EQ} \right)$$

$$Y_{EQ} = \frac{n_e}{s'} , \qquad Y_{EQ} = \frac{n_e^{EQ}}{s} \simeq \frac{0.278\, g_{eff}}{g_{s'}^{*}}$$

$$n_e^{EQ} = \zeta(3) T^3 / \pi^2 \qquad \Gamma_{ABS} = n_1 \langle \sigma |v| \rangle_{ABS}$$

Solution

Basics of cosmology and thermodynamics of Early Universe

$$\frac{1 - Y(x)/Y_{EQ}}{1 - Y(0)/Y_{EQ}} = \exp\left[-\int_0^x \frac{\Gamma_{ABS}}{x' H}\, dx'\right]$$

$\frac{\alpha_S}{8\pi}$ α_S^2

thermal population, $Y(0) = 0$ $\frac{\alpha_S^2}{64\pi^2}$ α

$$\Rightarrow Y(x) = Y_{EQ}\left\{1 - \exp\left[-\int_0^x \frac{\Gamma_{ABS}}{x' H}\, dx'\right]\right\}$$

$\Rightarrow$ Equilibrium abundance if $\Gamma_{ABS} > H$ for at least an expansion time ($\Delta x / x = (\Delta T / T) \sim 1$)

Long before thermal axions become non-relativistic their interactions freeze-out ($\Gamma_{ABS}/H \leq 1$) $\rightarrow$ their relic abundance freeze in -

Relic abundance

$$Y_\infty \simeq \frac{0.278}{g_{*F}}\left\{1 - \exp\left[\int_0^\infty \frac{\Gamma_{ABS}}{x' H}\, dx'\right]\right\}$$

$$\frac{\cdots}{dt}$$

Counting the two eqs ...

$$\frac{d}{dt}\left[R^3\left(n_e^{th}-n_e^{eq}\right)\right] = -\Gamma R^3\left(n_e^{th}-n_e^{eq}\right)$$

this eq. implies that a thermal distribution of axions is approached exponentially fast whenever the condition

$$\Gamma > H \quad (**)$$

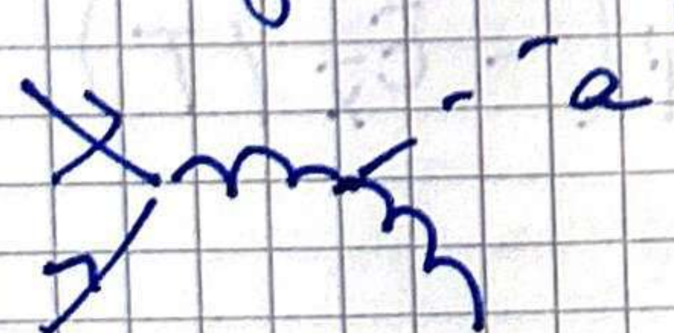
is satisfied

So we have a thermal population of axions today, provided (∗∗) prevails for a few expansion times at some point in the early universe and the thermal population of axions thus established did not get diluted away by the inflation or some other case of huge entropy release.

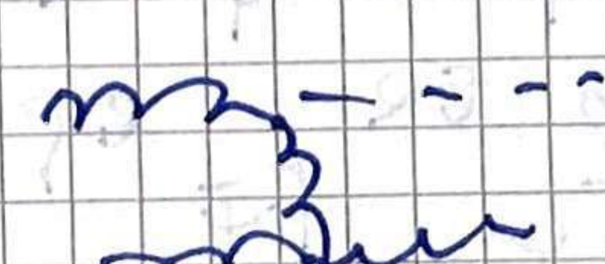
The least model-dependent process for thermalizing axions in the EU are

(a) $a + q(\bar{q}) \leftrightarrow g + q(\bar{q})$

(b) $a + g \leftrightarrow q + \bar{q}$



(c) $a + g \leftrightarrow g + g$



Combining the two eqs one obtains

$$\frac{d}{dt}\left[R^3\left(n_e^{th}-n_e^{eq}\right)\right] = -\Gamma R^3\left(n_e^{th}-n_e^{eq}\right)$$

this eq. implies that a thermal distribution of axions is approached exponentially fast whenever the condition

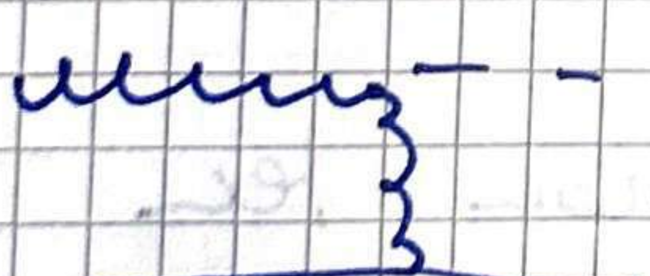
$$\Gamma > H \quad (\ast\ast)$$

is satisfied.

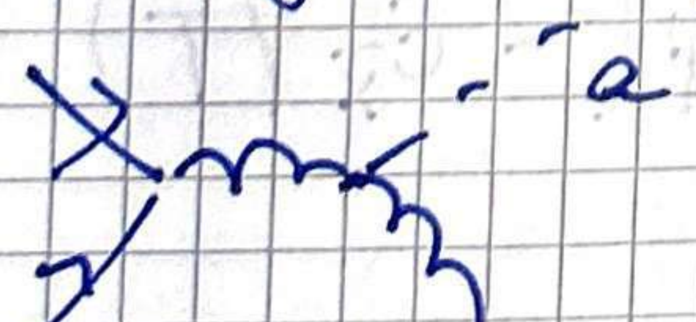
So we have a thermal population of axions today, provided $(\ast\ast)$ prevails for a few expansion times at some point in the early universe and the thermal population of axions thus established did not get diluted away by the inflation or some other case of huge entropy release.

The least model-dependent process for thermalizing axions in the EU are

(a) $a + q(\bar{q}) \leftrightarrow g + q(\bar{q})$



(b) $a + g \leftrightarrow q + \bar{q}$



(c) $a + g \leftrightarrow g + g$

These processes involve only the coupling of axions to gluons, present in any axion model, and the coupling of quarks to gluons.

Ref: Masso et al. hep-ph/0203221
Graf and Steffen 1008.4528
Salvio, Strumia, Xue 1310.6982

Rough estimation

$$\sigma \sim \frac{\alpha_s^3}{8\pi^2} \frac{1}{f_a^2} \qquad \left(\frac{\alpha_s}{8\pi}\frac{1}{f_a}\right)^2 \times 8 \text{ gluons}$$

$$\underbrace{\frac{g_s^2}{32\pi^2}}_{\text{axion gluon}} \quad \underbrace{\alpha_s}_{\text{gluon gluon}}$$

where $\alpha_s = \dfrac{g_s^2}{4\pi}$

A temperature $T > 1$ TeV, the densities of quarks, antiquarks and gluons are

$$n_q = n_{\bar q} = 27 \frac{\zeta(3)}{\pi^2} T^3 \qquad \left[\begin{array}{ccccc} \text{spin} & & & & \\ 6 & \times & 2 & \times & 3 \\ q & & \cancel{q\bar q} & & c \end{array}\right] = 36$$

$$n_g = 16 \frac{\zeta(3)}{\pi^2} T^3 \qquad \underset{g}{8} \times 2 = 16 \qquad 36 \cdot \frac{3}{4} = 27 \leftarrow \text{Fermi}$$

The Hubble rate is given by the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\Big[\underbrace{N_b(T) + \frac{7}{8}N_f(T)}_{g_*}\Big]\frac{\pi^2}{30}T^4$$

$N_b(T)$ and $N_f(T)$ = effective # of bosonic and fermionic spin degrees of freedom at T.

for $T > 1$ TeV

$$N = N_b + \frac{7}{8}N_f = 107.75$$

If we assume no d.o.f. other than those of SM + axions. Combining everything and setting $\alpha_s \simeq 0.03$ one finds

$$\sum_{i=1}^{3} \frac{n_i \langle \sigma_i v \rangle}{H} \sim 2 \left(\frac{10^{12}\,\mathrm{GeV}}{f_a} \right)^2 \frac{T}{10^{12}\,\mathrm{GeV}}$$

3 quark factor   3 process

$$T_D \sim 5 \times 10^{11}\,\mathrm{GeV} \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^2$$

Note that the calculation is not valid when $T \gtrsim v_a = N f_a$, as PQ symmetry is restored there.
We expect a thermal population only when $f_a \leq 2 N\, 10^{12}\,\mathrm{GeV}$
(We will see that $f_a \leq 10^{11}\,\mathrm{GeV}$ avoids overclose universe with cold axions)
We should keep in mind, however that this thermal axion population may be wiped out by a period of inflation with reheat temperature less than T_D. So it is interesting to search for processes that may reestablish a thermal axion population later on.

~~Let us consider~~
If axions were sufficiently strong interacting $f_a \leq 10^8\,\mathrm{GeV}$, they decouple after QCD phase transition ($T < 200\,\mathrm{MeV}$). The most general interaction involves hadrons rather than quarks and gluons relevant at earlier epochs
Among axions do not couple to charged leptons
The main thermalization processes in the post-QCD epochs involve hadrons.

$$a + \pi \leftrightarrow \pi + \pi$$

$$a + N \leftrightarrow N + \pi$$

Due to scarcity of nucleons relative to pions, the pionic processes are by far the most important

$$\mathcal{L} = \frac{C_{a\pi}}{f_\pi f_a}\left(\pi^0 \pi^+ \partial_\mu \pi^- + \pi^0 \pi^- \partial_\mu \pi^+ - 2\pi^+ \pi^- \partial_\mu \pi^0\right)\partial_\mu a$$

In hadronic axion model

$$C_{a\pi} = \frac{1-z}{3(1+z)} \qquad z = m_u/m_d \simeq 0.56$$

the cross section is of the order

$$\sigma_\pi \sim f_a^{-2}$$

Using $g_* \simeq 17.25$ we find

$$\frac{n_\pi \langle \sigma_\pi v \rangle}{H} \sim \left(\frac{3 \times 10^8\, \text{GeV}}{f_a}\right)^2$$

at $T \sim m_\pi$

The $\pi + \pi \leftrightarrow \pi + a$ process has the advantage of occurring very late so that any thermal population cannot be wiped out by inflation. However, it is ineffective unless the bound $f_a > 10^9$ GeV from SN 1987 A is saturated. For $f_a > 10^9$ GeV the axion lifetime exceeds by many orders of magnitudes the age of the Universe. Between their last decoupling at T_D

and today, the thermal axion population is merely diluted and redshifted by the expansion of the Universe. Their present number density is

$$n_a^{th}(t_0) = \frac{\zeta(3)}{\pi^2} T_D^3 \left(\frac{R_D}{R_0}\right)^3$$

where R_D/R_0 is the ratio of scale factors between the time t_D of decoupling and today.

Their average momentum is

$$\langle p \rangle = \langle E \rangle / c \quad \langle p_a^{th}(t_0) \rangle = \frac{\pi^4}{30\,\zeta(3)} T_D \frac{R_D}{R_0} = 2.701\, T_D \frac{R_D}{R_0}$$

$\langle E \rangle$

If $\langle p_a^{th}(t_0)\rangle \gg m_a$, the energy distribution is thermal with temperature

$$T_{a_0} = T_D \frac{R_D}{R_0}$$

If there is neither inflation nor any form of entropy release from t_D to present, T_{a0} is related to the present CMB temperature $T_{\gamma 0} = 2.735\,K$ by conservation of entropy. As electron-positron annihilation occurs after ν decoupling one finds

$$T_{a0} = \left(\frac{10.75}{N_D}\,\frac{4}{11}\right)^{1/3} T_{\gamma 0} = 0.905\,K \left(\frac{106.75}{N_D}\right)^{1/3}.$$

and the number density is

$$n_a = \frac{g_{*s}(T_0)}{g_{*s}(T_{a,d})}\,\frac{n_\gamma}{2}$$

Axion to photon T ratio from entropy $\quad \frac{T_a}{T} = \left[\frac{g_{*s}(T)}{g_{*s}(T_D)}\right]^{1/3}$

What is the axion number density today?

$$n_{a,0} = n_a(T_d)\frac{R_d^3}{R_0^3} = \frac{\zeta(3)}{\pi^2}\frac{T_d^3 R_d^3}{R_0^3} =$$

$$n_e(T_d) = \frac{\zeta(3)}{\pi^2} T_d^3$$

$$= \frac{\zeta(3)}{\pi^2}\frac{g_s(T_0)}{g_s(T_d)} T_0^3$$

entropy conservation

$$g_s(T_d)\, T_d^3 R_d^3 = g_s(T_0)\, T_0^3 R_0^3$$

in terms of photon number density today

$$n_{\gamma,0} = \frac{2\zeta(3)}{\pi^2} T_0^3 = 411\ \mathrm{cm}^{-3} \qquad T_0 = 2.73\ K$$

$$n_{a0} = \frac{g_s(T_0)}{g_s(T_d)}\frac{n_{\gamma 0}}{2}$$

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

$$\rho_{rad} = \rho_\gamma + \rho_\nu = g_\gamma \left(\frac{\pi^2}{30}\right) T_\gamma^4 + g_\nu \left(\frac{\pi^2}{30}\right)\left(\frac{7}{8}\right) T_\nu^4$$

$$\rho_{rad} = \left[1 + \left(\frac{7}{8}\right)\left(\frac{4}{11}\right)^{4/3}\left(\frac{g_\nu}{g_\gamma}\right)\right]\rho_\gamma$$

$$\rho_{rad} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right]\rho_\gamma$$

$N_{eff}^{SM} = 3$ [not-instantaneous decoupling $N_{eff}^{SM} = 3.046$
ν-decoupling not complete when e^+e^- annihilation happen

Hot axions and ΔN_{eff}

$$\rho_{rad} = \rho_\gamma + \rho_\nu + \rho_a$$

$$= \rho_\gamma\left[1 + \frac{\frac{7}{8}\, 2\, N_{eff}^{SM}\, T_\nu^4}{2\, T_\gamma^4} + \frac{T_a^4}{2\, T_\gamma^4}\right]$$

$$= \rho_\gamma\left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{eff}^{SM} + \frac{1}{2}\left(\frac{T_a}{T_\gamma}\right)^4\right]$$

$$\equiv \rho_\gamma\left[1 + \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{eff}\right]$$

$$\Rightarrow \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{eff}^{SM} + \frac{1}{2}\left(\frac{T_a}{T_\gamma}\right)^4 = \frac{7}{8}\left(\frac{T_\nu}{T_\gamma}\right)^4 N_{eff}$$

$$\Rightarrow N_{eff} = N_{eff}^{SM} + \frac{4}{7}\left(\frac{T_a}{T_\nu}\right)^4$$

$$\Delta N_{eff} = N_{eff} - N_{eff}^{SM} = \frac{4}{7}\left(\frac{T_a}{T_\nu}\right)^4$$

Planck 2018

$N_{eff} = 2.99 \pm 0.17$

$\Delta N_{eff} \leq 0.28$ (2σ)

T_e ~~at the~~ $> T_d$ $\qquad T_d = T_\gamma = T_\nu \neq T_e$

$T_2 \ll 1$ MeV

$$\begin{cases} T_\gamma = T_2 \\ T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \\ T_e \end{cases}$$

Apply entropy conservation ($g_s T^3 R^3 = \text{const}$)

$$\Big[g_s(T_d) + \overset{\text{axion}}{1}\Big](T_d R_d)^3 = \left[2\left(\frac{T_\gamma}{T_a}\right)^3 + \frac{7}{8}\times 6\left(\frac{T_\nu}{T_a}\right)^3 + 1\right](T_a R_2)^3$$

since axions do not get reheated by subsequent SM annihilations temperature scales as $T \propto R^{-1}$ and so

$T_d R_d = T_a R_a$

$$\Rightarrow g_s(T_d) = 2\left(\frac{T_\gamma}{T_e}\right)^3 + \frac{21}{4}\left(\frac{T_\nu}{T_e}\right)^3 = \qquad T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_\nu$$

$$= 2\cdot\frac{11}{4}\left(\frac{T_\nu}{T_a}\right)^3 + \frac{21}{4}\left(\frac{T_\nu}{T_a}\right)^3 = \frac{43}{4}\left(\frac{T_\nu}{T_e}\right)^3$$

$$\Rightarrow \frac{T_e}{T_\nu} = \left(\frac{43}{4}\,\frac{1}{g_s(T_d)}\right)^{1/3}$$

What ~~is axion~~

$$\Delta N_{eff} = \frac{4}{7}\left(\frac{T_a}{T_\nu}\right)^4 = \frac{4}{7}\left(\frac{43}{4\, g_s(T_d)}\right)^{4/3} \simeq 0.027\left[\frac{106.75}{g_s(T_d)}\right]^{4/3}$$

↑ typical reach of CMB-S4

<?xml version="1.0" encoding="UTF-8"?>
<!DOCTYPE xpif SYSTEM "xpif-v02025.dtd">

Axion field evolution

The thermal axions discussed in the previous sector are quantum fluctuations about the average bkg value of the axion field. The evolution of the average axion field, from the moment $U(1)_{PQ}$ get SB to the moment the axion acquire a mass, is the topic of this sector.

The $U(1)_{PQ}$ symmetry gets SB at a critical temperature $T_{PQ} \sim v_a$, where v_a is the v.ev of a complex field $\phi(x)$. The action density of the field is of the form

$$\mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi^\dagger \partial^\mu \phi - \frac{\lambda}{4}(\phi^\dagger \phi - v_a^2)^2 + \dots$$

At $T > T_{PQ}$ the free energy has its minim at $\phi = 0$.

At $T < T_{PQ}$ the minim is a circle. The radius which quickly approaches v_a as T decreases.

After reads

$$\langle \phi(x) \rangle = v_a \, e^{i a(x)/v_a}$$

where $a(x)$ is the axion field before mixing with π^0 and η mesons. The field $a(x)$ has random initial conditions. In particular, at two points outside each's other causal horizon, the values of $a(x)$ are completely uncorrelated.

It is well-known that the size of the causal horizon is hugely modified during cosmological inflation. Without inflation, the size of a causal horizon is of the order of t, the age of the universe. But, during inflationary epoch, the causal horizon grows exponentially fast and becomes enormous compared to t.

There are two cases to consider:

- Case 1 : Inflation occurs with reheat $T < T_{PQ}$, and the axion field is homogenized over enormous distances. The subsequent evolution of the zero-momentum mode is relatively simple (Pre-Inflationary PQ symm breaking)
- Case 2 : Inflation occurs with reheat temperature larger than T_{PQ}. In addition to zero mode the axion has non-zero modes and carries string and domain walls as topological defects. (post-inflationary PQ)

$$S = \int_\Omega d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

$$\partial_\mu \frac{\partial \sqrt{-g} \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{\partial \sqrt{-g} \mathcal{L}}{\partial \phi}$$

$\rightarrow$ e.o.m $\nabla_\mu \nabla^\mu \phi + V'(\phi) = 0$ E.L.

$$\Box \equiv \nabla_\mu \nabla^\mu = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi \right)$$

$$g_{\mu\nu} = (1, -a^2(t), -a^2(t), -a^2(t))$$

$$g^{00} = 1 \quad g^{ij} = -a^{-2} \delta_{ij} \qquad \sqrt{-g} = a^3$$

E.om $\Box = \frac{1}{a^3} \partial_t \left(a^{+3} \times 1 \times \partial_t \phi \right) = \partial_0^{\,2} + 3 \frac{\dot{a}}{a} \partial_0$

$\partial_0 \partial^0$

$$\nabla_i \nabla^i = \frac{1}{a^3} a^3 \partial_i \left(-a^{-2} \delta^{ii} \right) \partial_i$$

$$\delta S_\phi = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu}\partial_\mu \phi\, \partial_\nu(\delta\phi)\times 2 - V'(\phi)\,\delta\phi\right]$$

$$= \int d^4x \left[\partial_\nu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\mu \phi\right)(\delta\phi) - \sqrt{-g}\, V'(\phi)\,\delta\phi\right]$$

$$= \int d^4x \sqrt{-g}\left[\frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\mu \phi\right) - V'(\phi)\right]\delta\phi$$

$$\delta\phi = 0 \rightarrow \frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\mu\phi\right) - V'(\phi) = 0$$

$$g_{\mu\nu} = \mathrm{diag}\left(+1, -a^2(t), -a^2(t), -a^2(t)\right)$$

$$\sqrt{-g} = a^3(t)$$

$$\frac{1}{a^3}\partial_t\left(-a^3\dot\phi\right) + \frac{1}{a^3}\times a^3 \times a^{-2}\nabla^2\phi - V'(\phi) = 0$$

$$\ddot\phi + 3\frac{\dot a}{a}\dot\phi - \frac{1}{a^2}\nabla^2\phi + V'(\phi) = 0$$

$$S = \int_\Omega d^4x \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] \quad \Big|\ \partial_\mu \frac{\partial\sqrt{-g}\,\mathcal{L}}{\partial(\partial_\mu\phi)} = \frac{\partial\sqrt{-g}\,\mathcal{L}}{\partial\phi}$$

E.L.

$$\Rightarrow \text{E.o.M.} \quad \nabla_\mu\nabla^\mu\phi + V'(\phi) = 0$$

$$\Box = \nabla_\mu\nabla^\mu = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\phi\right)$$

$$g_{\mu\nu} = \left(1, -a^2(t), -a^2(t), -a^2(t)\right)$$

$$g^{00} = 1 \qquad g^{ij} = -a^{-2}\delta_{ij}, \qquad \sqrt{-g} = a^3$$

E.o.M. $\partial_0 g^{00}$

$$\Box = \frac{1}{a^3}\partial_t\left(a^{+3}\times 1\times\partial_t\phi\right) = \partial_0^2 + 3\frac{\dot a}{a}\partial_0$$

$\partial_0\partial^0$

$$\nabla_i\nabla^i = \frac{1}{a^3}a^3\partial_i\left(-a^{-2}\delta^{ij}\right)\partial_j$$

The early universe is assumed to be homogeneous and isotropic. Its curvature is negligible. The space-time metric can be therefore written in the FRW form

$$-ds^2 = dt^2 - R(t)^2 d\vec{x} \cdot d\vec{x}$$

The equation for the motion of $a(x)$ in this space time is

$$\partial_\mu \partial^\mu a(x) + V_a'[a(x)] =$$

$$= \left(\partial_t^2 + 3 \frac{\dot{R}}{R} \partial_t - \frac{1}{R^2} \nabla_x^2 \right) a(x) + V_a'[a(x)] = 0$$

where $V(a)$ is the effective potential for the axion field and $'$ indicates derivative w.r.t. a. V_a results from non-perturbative QCD effects associated with instantons. They break $U(1)_{PQ}$ symmetry to a discrete $Z(N)$ subgroup. $V(a)$ is therefore periodic with period $\Delta a = \frac{2\pi v_a}{N}$ $= 2\pi f_a$.

We may write the potential qualitatively as

$$V_a = f_a^2 m_a^2(t) \left[1 - \cos(a/f_a) \right]$$

where the axion mass $m_a(t) = m_a[T(t)]$ is a function of temperature and hence of time.

v.e.v implies that axion field has a range $0 \le a \le 2\pi v_a$. Hence there are N degenerate vacua. The discrete degeneracy implies the existence of domain walls, which will be discussed.

Sub. the potential in the e.o.m. becomes

$$\left(\partial_t^2 + 3\frac{\dot{R}}{R}\partial_t - \frac{1}{R^2}\nabla_x^2\right) a(x) + m_a^2(t) f_a \sin\left(\frac{a_0(x)}{f_a}\right) = 0$$

The non-perturbative QCD effects associated with instantons have amplitude proportional to

$$e^{-2\pi/\alpha_s(T)} \simeq \left(\frac{\Lambda_{QCD}}{T}\right)^{11 - 2N_f/3}$$

where N_f is the n. of quark flavors with mass less that T. Previous eq implies that the axion mass is strongly suppressed at T which are larger compared to QCD scale but turns on abruptly when the temperature approaches Λ_{QCD}.

Because the first term in e.o.m. are prop to t^{-2}, the axion mass is unimportant in the evolution of the axion field until $m_a(t)$ becomes of order t^{-1}. Let us define a critical time t_1 by

$$m_a(t_1)\, t_1 = 1$$

The axion mass effectively turns on at t_1. From a calculation of the effects of instantons at high T one obtains

$$m_a(T) \simeq 4 \times 10^{-9}\, eV \left(\frac{10^{12}\, GeV}{f_a}\right)\left(\frac{GeV}{T}\right)^4$$

when T is near 1 GeV.

the relation b.tw T and t follows from relation btw $H(T)$ and $H = 1/2t$. $H \simeq 1.66\, g_*^{1/2}\, T^2/m_{pl}$

~~the relation btw T and t~~ for the total number of effective d.o.f $\mathcal{N}$ of thermal spins is changing near 1 GeV from a value near 60, valid above the quark - hadron phase transition, to a value of order 30 below the transition. Using $\mathcal{N} = 60$, one has

$$(*) \quad m_a(t) \simeq 0.7 \times 10^{20}\, s^{-1} \left(\frac{t}{1s}\right)^2 \left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)$$

which implies

$$t_1 \simeq 2 \times 10^{-7}\, s \left(\frac{f_a}{10^{12}\,\mathrm{GeV}}\right)^{1/3}$$

the corresponding temperature is

$$T_1 \simeq 1\,\mathrm{GeV} \left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)^{1/6}$$

Eq $(*)$ implies $d \ln[m_a(t)]/dt < m_a(t)$ after t_1.

So, at least for a short while below 1 GeV, the axion mass changes adiabatically. The number of axions is an adiabatic invariant. Conservation of number of axions after t_1 allows after t_1 allows us to estimate the energy density of axions today from the estimate of their n. density at t_1. When the temperature drops below 1 GeV, the delicate interaction gas calculation that gave $(*)$ are no longer reliable. Complicated things happen such as confinement and chiral SB. However, because $m_a \gg H$,

Zero mode

In case 1, where inflation occurs after the PQ phase-transition, the axion field is homogeneous over enormous distances. E.O.M becomes

$$\left(\frac{d^2}{dt^2} + \frac{3}{2t}\frac{d}{dt}\right) a(t) + m_a^2(t) f_a \sin\left(\frac{a(t)}{f_a}\right) = 0$$

where we used $R(t) \propto t^{1/2}$. For $t \ll t_1$, we may neglect m_a. The solution is then

$$a(t) = a_0 + a_{1/2}\, t^{-1/2}$$

where a_0 and $a_{1/2}$ are constants. Previous eq implies that the expansion of the universe slows ~~down~~ the axion field down to a constant value (before t_1).

When t approaches t_1 the axion field start oscillating in response to turn-on of the axion mass. We will assume that the initial value of a is sufficiently small that $f_a \sin\left(\frac{a}{f_a}\right) \simeq a$. Let us define ψ by

$$a(t) = t^{-3/4}\psi$$

The eq for ψ is

$$\left[\frac{d^2}{dt^2} + \omega^2(t)\right]\psi(t) = 0$$

where $\omega^2(t) = m_a^2(t) + \frac{3}{16t^2}$

For $t > t_1$, we have $d\ln\omega/dt \ll \omega \simeq m_a$. That regime is characterized by the adiabatic invariant $\psi_0^2(t)\,\omega(t)$, where $\psi_0(t)$ is slowly the oscillation amplitude of $\psi(t)$.

We have therefore

$$\psi(t) \simeq \frac{C}{\sqrt{m_a(t)}} \cos\left[\int^t dt'\, \omega(t')\right]$$

where C is a constant. Hence

$$a(t) = A(t) \cos\left[\int^t dt'\, \omega(t')\right]$$

with $A(t) = \frac{C}{\sqrt{m_a(t)}}$ $t^{-3/4}$ $\propto R^{-3/2}$

Hence, during the adiabatic regime

$$A^2(t)\, m_a(t) \propto t^{-3/2} \propto R(t)^{-3}$$ { n_a density per comoving volume conserved }

The zero-momentum mode of the axion field has energy density $\rho_a = \frac{1}{2} m_a^2 A^2$ and describes a coherent state of axions at rest with number density $n_a = \frac{1}{2} m_a A^2$. Previous eq states, therefore, that the number of zero-momentum axions per comoving volume is conserved. The result holds as long as the changes in the axion mass are adiabatic.

We estimate the number density of axions in the zero-momentum mode at late times later by saying that the axion field has a random initial value $a(t_1) = f_a \alpha_1$ and evolves according the previous eq for $t \geq t_1$, where α_1 is called "initial misalignment angle". As the effective potential for a is periodic with period $2\pi f_a$, the relevant range of α_1 values is $-\pi$ to π. The number density of zero momentum axions at time t_1 is then

$$n_a^{\text{vac},0}(t_1) \sim \frac{1}{2} m_a(t_1)\, a^2(t_1) = \frac{f_a^2}{2t_1} \alpha_1^2$$

(using $m_a(t_1)\, t_1 = 1$)

Axion density

$f \lesssim 10^{11}$ GeV $\Rightarrow$ not overproduce DM by vacuum realignment

$m(t_1)t_1 = 2$ defined time $t_1 \simeq 2 \cdot 10^{-7}$ sec $\left(f/10^{12}\,\text{GeV}\right)^{1/3}$

before t_1 axion field $\phi \propto f$

After t_1, ϕ oscillates with decreasing amplitude, consistent with axion number conservation. The number density of axions produced by vacuum realignment is

$$n(t) \sim \frac{f^2}{t_1}\left(\frac{a(t_1)}{a(t)}\right)^3 = \frac{4\times10^{47}}{cm^3}\left(\frac{f}{10^{12}\,\text{GeV}}\right)^{5/3}\left(\frac{a(t_1)}{a(t)}\right)^3$$

Non-relativistic soon after their production

Their typical momenta at time t_1 are $\mathcal{O}(1/t_1)$ and vary as $1/a(t)$, so their velocity dispersion is

$$\delta v(t)\ \cancel{\delta(v)} \sim \frac{1}{m(t_1)}\frac{a(t_1)}{a(t)}$$

$$\delta p(t) \sim \frac{1}{t_1}\frac{R(t_1)}{R(t)}$$

momenta are order t_1^{-1}.

The average quantum state occupation number of CDM axions is therefore

$$N \sim \frac{(2\pi)^3 n(t)}{\frac{4\pi}{3}(m\,\delta v(t))^3} \sim 10^{61}\left(\frac{f}{10^{12}\,\text{GeV}}\right)^{8/3}$$

classical field

$$(\Delta x)^3(\Delta p)^3 \sim (2\pi)^3 \qquad n(\Delta x)^3 \sim \frac{(2\pi)^3}{(\Delta p)^3}$$

misalignment for generic scalar

$\phi \propto \phi_i \; R(t)^{-3/2} \cos(mt)$

$\rho_\phi = \frac{1}{2}\dot\phi^2 + \frac{1}{2}m^2\phi^2 \Rightarrow \rho_\phi \propto a^{-3}$ ~~like~~

we want to force it to be equal to abundance of DM today

$\Omega_\phi h^2 = \frac{\rho_\phi}{\rho_c} h^2 = \frac{\rho_\phi}{3H_0^2 M_{pl}^2} h^2 \to 0.12$ $\quad H_0 = h\,100\ \mathrm{Km\ s^{-1}\,Mpc^{-1}}$

the density of our scalar field is

$\rho_\phi^{today} = \frac{1}{2} m^2 \phi_i^2 \left(\frac{a_{today}}{a_{osc}}\right)^{-3}$ where $\left(\frac{a_{today}}{a_{osc}}\right)^{-3} = \left(\frac{T_{today}}{T_{osc}}\right)^{3}$

to get these temperatures, use Friedmann eq.

$3H^2 M_{pl}^2 = \frac{\pi^2}{30} g_* T^4 \to g_* = 3.4$

$\Omega_\phi h^2 = 0.12 \left(\frac{\phi_i}{4.7\times 10^{6}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-4}\,\mathrm{eV}}\right)^{1/2}$

$T m_e t_1 \sim 1$

$t \sim T^{-2}$

$m_e T^{-2} \sim 1$

$\left(\frac{1}{T^2}\right)^{\frac{1}{2}} \sim \frac{1}{(m_e)^{\frac{1}{2}}}$

$m_e^2 \frac{1}{m_e^{3/2}} \sim m_e^{1/2}$

Axion case

$\Phi(t) = \rho e^{i\vartheta}$

$V(\Phi) = V_{PQ}(\rho) + V_{QCD}(\vartheta) = \frac{\lambda}{8}(\rho^2 - f_a^2)^2 + \Lambda_{QCD}^4 (1-\cos\vartheta)$

$\phi \to f_a \vartheta \qquad m \to m_a \simeq \frac{\Lambda_{QCD}^2}{f_a}$

$\phi_i \to \vartheta_i f_a \simeq O(1) f_a \qquad m \to m_a \simeq 6\,\mathrm{meV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)$?

No! If you do this, you will not get what's quoted in literature

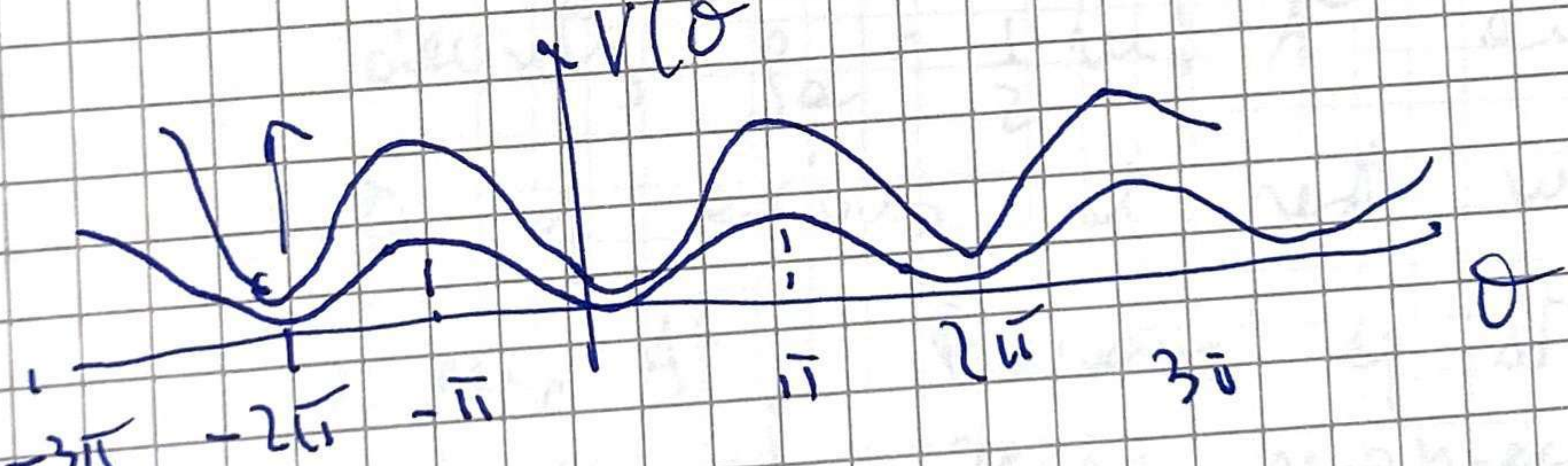
We have therefore

$$\Omega_a h^2 \simeq 0.12\, \theta_i^2 \left(\frac{f_a}{9\times10^{11}\,\mathrm{GeV}}\right)^{1.165} \rightarrow \text{Why?}$$

Mass is generated by instantons whose effects are T dependent. In literature this dependence is called "topological susceptibility" $\chi(T)$

$$V(\theta) \simeq \chi(T)(1-\cos\theta) = m_a^2(T) f_a^2 (1-\cos\theta)$$

Axion mass grows as T drops, nearly constant when $T < T_{QCD}$



$$\chi(T) \sim T^{-8} \Rightarrow m_a \propto T^{-4}$$

Isocurvature fluctuations in pre-inflationary scenario

In an axion-dominated universe, that undergoes inflation after, or during, PQ symmetry breaking isocurvature axion perturbation will exist in addition to the usual adiabatic (curvature) perturbation associated with inflation. These isocurvature fluctuations convey to fluctuations in the local axion-to-entropy ratio (n_a/s). In contrast, the adiabatic perturbations occur in all species and are characterized by $\delta(n_a/s) = 0$.

Fluctuations in the initial misalignment angle, α_I. Since the axion field is massless and weakly-coupled to inflaton, it has the same fluctuation spectrum

$$P_a(k) = \int \frac{d^3x}{(2\pi)^3} \langle \delta a(\vec{x},t)\, \delta a(\vec{x}',t) \rangle e^{-i\vec{k}(\vec{x}-\vec{x}')}$$

$$= \left(\frac{H_I}{2\pi}\right)^2 \frac{2\pi^2}{k^3}$$

where H_I is the expansion rate during inflation

The fluctuations in the axion field produce perturbations in the cold axion density

$$\frac{\delta n_a^{iso}}{n_a} = \frac{2\delta a}{a_i} = \frac{H_I}{\pi f_a \alpha_I} \qquad \alpha_I = \text{angle during QCD phase transition}$$

Fluctuations become matter perturbation when axion gets mass. Uncorrelated with curvature fluctuations from inflation. Planck bounds isocurvature power $< 4\%$ than primordial curvature perturbation

For a given f_a the scale of inflation must be below some maximum values or axions produces too much isocurvature

$$H_I \lesssim 2.8 \times 10^8 \, \mathrm{GeV} \times \theta_i \left(\frac{f_a}{10^{11} \, \mathrm{GeV}} \right)$$

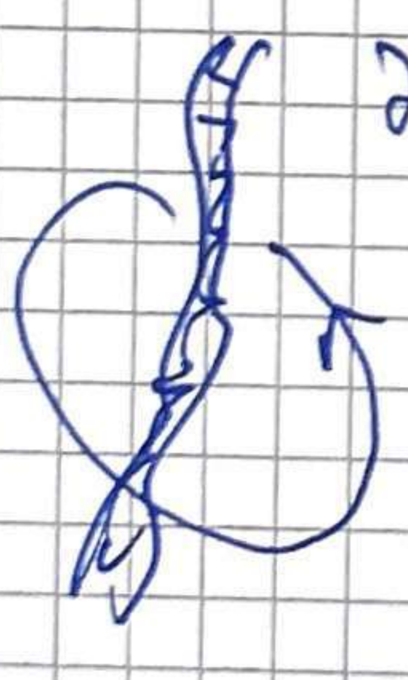
$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - \lambda(\phi^* \phi - \sigma^2/2)^2$$

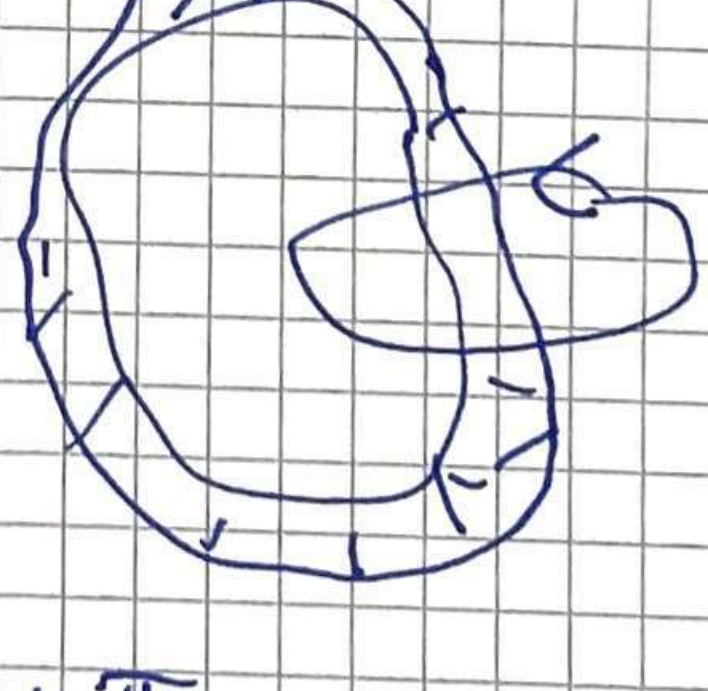
SSB $\langle |\phi|^2 \rangle = \sigma^2/2$

$\phi = (\varphi + i\varphi_2)/\sqrt{2}$ $\quad \langle |\phi \rangle = \langle \phi \rangle / \sqrt{2}$

$\langle \phi \rangle = \left(\frac{\sigma}{\sqrt{2}}\right) \exp(i\vartheta)$ $\quad \vartheta = \vartheta(\vec{x})$

ϕ must be single valued, total change in ϑ, $\Delta\vartheta$, around any closed path must be an integer multiple of 2π. Imagine a closed path with $\Delta\vartheta = 2\pi$. As the path shrunk to a point (assuming no singularity is encountered) $\Delta\vartheta$ cannot change continuously from $\Delta\vartheta = 2\pi$ to $\Delta\vartheta = 0$. There must therefore be one point contained within the path where the phase ϑ is undefined, i.e. $\langle \phi \rangle = 0$. The region of false vacuum within the path is part of a tube of false vacuum. Such tube must either be closed or infinite length, otherwise it would be possible to deform the path around the tube and contract it to a point without encountering the tube of false vacuum.





Change of phase : $\Delta\alpha = 2n\pi$

large distance $\phi \to \left(\frac{\sigma}{\sqrt{2}}\right) \exp(i N\vartheta)$

Axion string today decay

$\ddot\theta + 3H\dot\theta - \frac{1}{a^2}\nabla^2\theta + m_a^2\theta = 0$ ← Present

The final mechanism for axion production is far more intriguing: production through the decay of axion strings. In the post-inflationary scenario inflation had already happened before axion was born

→ Universe filled with many values of θ_i

→ Different value in every causal ~~patch~~ patch

→ Patches come into contact as horizon grows.

→ Non-trivial topology.

That is, mapping of α_i to 3D space cannot, in general, be smoothly deformed to a uniform value of α_i throughout space. The topological entities that arise are axionic strings. PQ SSB involves a spontaneous breakdown of $U(1)$ symmetry. One-D objects – strings – arise when $U(1)$ gauge symmetry is SB. The main difference btw global and gauge strings is that the energy per length of a long straight string diverges logarithmically. This is because there is no gauge field contribution to cancel the $\partial\theta\vec\phi$ piece of $D_\mu\vec\phi$ in the spatial gradient term; for a global symmetry $D_\mu = \partial_\mu$ (θ = angle around string axis). The energy per length of two anti parallel strings is finite and propto log of their separation. This is because at large distance $\vec\phi$ is constant and does not wind, so that $\partial_\mu\vec\phi \to 0$. The energy per length of axionic string is given by $\mu \sim f_{PQ}^2 \ln(f_{PQ}\, d)$ d distance btw strings.

Provided Universe did not inflate before or during PQ SSB, a network of axionic string will fill the Universe after PQ SB. We expect axionic string network will rapidly approach a scaling solution $\rho_s \sim \frac{\mu}{t^2}$, $\frac{\rho_s}{\rho_r} \sim G\mu$.

Axionic strings dissipate its energy primarily by radiation of axions. Decay of axionic string.

String populations

the population of strings can be divided into two subgroups: long strings and string loops.

Long strings are string-like defects of size comparable to the horizon scale. The string energy per unit length, called string tension μ, of an isolated string gets contribution from the string core where the divergence related to the singularity is cut off by the heavy degree of freedom. For a standard potential $f_a^2 \sim f_a^2$, at potential maximum, and string core is defined as a region of size $r_c \sim f_a^{-1}$ from the center. Integrating the volume energy density r_c provides a short-distance cut-off. There is, however, also a large-distance log divergence, cut off Hubble size $t \sim H^{-1}$

string tension $\mu(t) = \pi f_a^2 \log(\eta f_a t)$ characteristic and distance

$\eta \sim O(1)$.

string evolve in a configuration independent of early times. to minimize energy, bended strings tend to straighten emitting radiation, closed string tend to shrink further, long string crossing each other reconnect in configuration of reduced length.

Domain wall

As the Universe expands, once T lowers at ~ 1 GeV non perturbative QCD effects become important and axion acquires a potential which provides an explicit breaking of PQ symmetry. The potential has a periodicity $2\pi f_a$, with $f_a = v_a/N_{DW}$, and hence is characterized by N_{DW} equivalent minima, which are related to $Z_{N_{DW}}$ discrete symmetry. Axion models have an exact, SB, discrete $Z(N)$ symmetry. $Z(N)$ is subgroup of $U(1)_{PQ}$ that does not get broken by non-perturbative QCD effects. The SB $Z(N)$ symmetry implies N fold degeneracy of vacua. The N vacua are at equidistant points on the circle at the bottom of the "Mexican hat" potential for the PQ field ϕ. The axion relaxes to one of these minima thus breaking spontaneously $Z_{N_{DW}}$. In each causally disconnected Hubble patch a different minimum is randomly chosen, so that after few Hubble times, when several patches will have reentered the horizon, DW, form as field configurations that interpolate in space b/w neighbouring vacua. The DW tension (energy per unit area) for a QCD cosine potential is $\sigma = 8 m_a f_a^2$, and the contribution to the energy density from energy stored in such configurations largely exceeds the critical energy density

→ axion DW problem

When axion mass turn on each axion string becomes the edges of N domain well.

There are three solutions to axion domain-wall problem:

1) Have inflation with reheating temperature less than PQ scale. The axion field is then homogenized by inflation, and there are no strings or domain walls.

2) $N = 1$.

3) Postulate small explicit breaking of $Z(N)$ symmetry

2) $N_{DW} = 1$ axionic topological defects are unstable. Each string is attached to a single DW. This DW wall forms because, wrapping space around the string, at some point the value of the axion field must change abruptly by $2\pi v_a$. Since the system evolves to minimize energy stored on the DW, two strings attached to same walls are pulled together until they eventually annihilate, so that string and DW energy is released in low-momentum axions.

For $N_{DW} > 1$ instead each string is attached to more than one DW in different directions, reaching a stable equilibrium that prevents annihilation.

3) Postulate a small explicit breaking of $Z(N)$ symmetry

and hence of PQ symm. The symm-breaking must lift completely the degeneracy of vacuum and be large enough so that the unique true vacuum takes over before the walls dominate the energy density. On the other hand, it must be small enough so that the PQ mechanism still works. This solution does not offer very attractive and very little room.

Axion miniclusters

If there is not inflation after PQ transition, the initial misalignment angle α_1 changes by $O(1)$ from one QCD time horizon to the next. Hence, the fluid of cold axions produced by vacuum realignment is inhomogeneous with $\delta\rho_a/\rho_a = O(1)$ at time of QCD phase-transition, the streaming length of axions is too short for these inhomogeneities to get erased by free streaming before the time t_{eq} of equality between matter and radiation, when density perturbations start to grow in earnest by gravitational instability. At the t_{eq}, $\delta\rho_a/\rho_a = O(1)$ inhomog. in the axion fluid promptly form gravitationally bound objects, called axion miniclusters. Their properties are of concern to experimentalists attempting to detect axion

DM on Earth. Indeed these experiments would become even more challenging, if most of cold axions clusters into miniclusters.

The scale of minicluster masses is set by the total mass in axions within one Hubble volume of radius $2R_{osc} \sim H_{osc}^{-1} = \sqrt{\frac{3 M_{pl}^2}{8\pi \rho_{osc}}}$ at time when axion mass becomes relevant, since after the onset of oscillations the number of cold axions per comoving volume remains conserved. At T_{osc} the Universe energy density is radiation dominated so that $\rho_{osc} \simeq \rho_{rad} = \frac{\pi^2}{30} g_* T_{osc}^4$. The energy enclosed in Hubble volume then is

$$M_{rad}(T_{osc}) \simeq \frac{4\pi R_{osc}^3}{3} \rho_{rad} = \frac{3}{32\pi} \sqrt{\frac{5}{\pi g_*}} \frac{M_{pl}^3}{T_{osc}^2} \quad (a)$$

As the Universe expands, the energy in radiation gets redshifted, and at matter-radiation equality temperature T_{eq} it provides an estimate of the gravitationally bound minicluster mass $M_{MC} \simeq M_{rad}(T_{eq})$ that is

$$M_{MC} \simeq M_{rad}(T_{osc}) \frac{T_{eq}}{T_{osc}} = 1.3 \times 10^{46} \left(\frac{800\ \text{keV}}{T_{osc}}\right)^3 \times \frac{T_{eq}}{0.8\ \text{eV}}\ \text{GeV} \simeq 10^{-11} M_\odot$$

Matter-equality $T \sim 0.75\,\text{eV}$

$z = 3400$

where we have used $g_*(T_{osc}) = 61.75$ and $M_\odot = 2 \times 10^{30}\ \text{kg} = 1.3 \times 10^{57}\ \text{GeV}$.

From $R_{osc} \simeq 0.1\ \text{km}$ ~~and~~ (using Hubble flow at T_{eq}) and using entropy conservation, (as)

$$(R_{eq} T_{eq})^3 g_s(T_{eq}) = (R_{osc} T_{osc})^3 g_s(T_{osc})$$

gives

$$R_{MC} \simeq R_{eq} = R_{osc} \frac{T_{osc}}{T_{eq}} \left(\frac{g_s(T_{osc})}{g_s(T_{eq})} \right)^{1/3} \simeq 2.5 \times 10^8 \text{ km}$$

$$g_s(T_{eq}) = 3.91$$

Axion miniclusters would lead to important detection features. First, their typical density

$$\rho_{MC} \simeq \frac{3 M_{MC}}{4\pi R_{MC}^3} = 0.2 \times 10^6 \text{ GeV cm}^{-3}$$ is about a factor 10^6 larger than the local DM density, so that in an encounter with the Earth the rate of conversion of axions into photons in dedicated resonant cavities or helioscopes would be enhanced.

The expected signal would be time-dependent due to revolution and rotation of Earth (annual modulation).

Microlensing of halo formed of axion miniclusters.

Axion miniclusters can be disrupted by gravitational field of a nearby star encounter, or by mean galactic gravitational field, leading to a tidal streams of axions that would enhance the local CDM density by about one order of magnitude.

As we have seen axions an attractive self-interaction, and, although extremely tiny $\sim \left(\frac{m_a}{f_a}\right)^4$ it can still produce significant effects due to large phase space density, causing a relaxation of gravitationally bound axion clouds. If relaxation is efficient, axion star can form within time scale compatible with age of universe. Contrarily to axion miniclusters, the mass of axion stars is not fixed once setting the mass of QCD axions. It is fixed once a formation mechanism is specified.

What about ALPs? Arias et al 1201.5902

$$\mathcal{L} = -\frac{1}{4} g \phi F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$\frac{\phi}{f_\phi} \in (-\pi, \pi)$

$V(\phi) = m_\phi^2 f_\phi^2 (1 - \cos \phi/f_\phi)$

$$g = \frac{\alpha}{2\pi} \frac{1}{f_\phi} N$$

$$\phi_1 = \theta_1 \frac{\alpha N}{2\pi g}$$

$\phi/f_\phi < \pi \Rightarrow \phi < \pi f_\phi$

$m_1 > 3H(T_{eq})$ [m_1 value at which it starts oscillate]

If we want to mimic the behavior of standard CDM we should ensure that at matter-radiation equality, $T_{eq} \sim 1.3\,\mathrm{eV}$ the mass attains the current value m_0 and therefore DM density starts to scale as $1/a^3$. The field should have started to oscillate already at T_{eq}

Realization of time dependence of axion mass?

m_0 standard value

Standard ALP $m_1 = m_0$ constant $\theta_1 \sim \pi$

Summary of main relic production mechanisms

Product. mech	Pre-inflation model	Post-inflation model
Vacuum realignment	The ex. ax DM density produced depends on the value of the initial misalignment angle θ_i which is unique for the whole observable universe. One can fine tune θ_i to get the desired density for a very large value of m_a	θ_i takes randomly different values $[-\pi, \pi)$ in different patches of the universe, so the axion density can be reliably predicted corresponding to the average $\theta_i \sim \pi/\sqrt{3}$. If this were the only production channel, the totality of DM would be achieved for $m_a \sim 26\,\mu eV$
Decay of topological defects (TD)	Topological defects are wiped out by inflation so they do not contribute	Topological defects form and decay producing large amount of axion DM. Their contrib must be computed by complex simulations and is uncertain: current results range from a contribution of the same order of misalignment angle up to several times it
thermal production	Axions produced thermally (like the case of ν) are relativistic, and therefore they contribute to the hot DM density.	

Lee-Weinberg curve for axions

Detection of CDM axions

Haloscope

$m_a \sim 10^{-5}$ eV

$\rho_{HALO}^{local} \simeq 0.3$ GeV cm^{-3}

If halo is composed by 10^{-5} axions, this corresponds to a local axion density of $n_a(local) \simeq 3 \times 10^{13} cm^{-3}$

gravitationally produced velocities $v \sim 10^{-3} c$, local flux of axions $F_a = n_a(local)\, v_a \simeq 10^{21} cm^{-2} s^{-1}$.

$E_a = m_a \left(1 + \frac{1}{2} v_a^2\right) \simeq 10^{-5}$ eV $[1 + O(10^{-6})]$

because of axions coupling to $\vec{E} \cdot \vec{B}$ they can be "converted" by a strong B-field the interconvert to photons of energy E_a, corresponding to frequencies $\nu_a = 2\,GHz \left(\frac{m_a}{10^{-5} eV}\right)$

Sikivie → resonant cavity immersed in a very strong B-field. Enormous local density and long de Broglie length $\lambda_a = \frac{h}{m_a v_a} \simeq 10^4$ cm $\left(\frac{10^{-5} eV}{m}\right)$

locally axion can be described by a coherent, classical field

Provided cavity is smaller than λ, the coupling of cavity mode to galactic axion field is

$$g_{a\gamma\gamma}\, a\, \vec{B} \cdot \int_{Vol} \vec{E}_{ne}\, d^3x$$

$\vec{E}_{ne}$ is the electric field of the mode ne, and

$\vec{B}$ is a constant, applied magnetic field

$$P_{ne} \simeq 2\times10^{-22}\, W \frac{Vol}{10L} \left(\frac{B}{6T}\right)^2 C_{en} \frac{m_a}{10^{-5} eV} \min\left[\frac{Q_{ne}}{Q_a}, 1\right]$$

C_{ne} mode-dependent form factor

ADMX $B : 7.5T$, $V = 136L$

Dish antennas and dielectric haloscopes

Going to higher frequencies require different detection concepts.

Magnetized dish antennas

Dielectric haloscope

Dielectric interface (mirror, surface of dielectric slab) immersed in a magnetic field parallel to the surface should emit e.m. radiation ⊥ to its surface, due to the presence of DM field.

This tiny signal can be made detectable if emission of a large surface is made concentrated in a small point, like surface lens of spherical shape. Broad band (sensitivity to all axion mass) BRASS at Hamburg.

Large areas.

Dielectric haloscopes, several dielectric slabs are stacked together inside a B field placed in front of metallic mirror. Increase of emitting surface, constructive interference. MADMAX

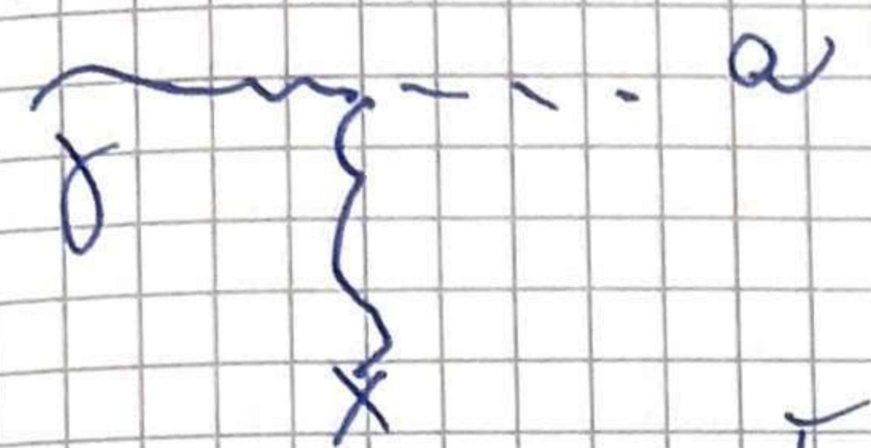
10 T field. Range 40 – 400 μeV. (should start by 2030)

DM radio

below μeV — resonance by LC circuit

ABRACADABRA

Primakoff Processes in Stars



Conversion in external electric or magnetic field

Stars: electric fields of nuclei and electrons

Primakoff process turns out to be important for non relativistic conditions where $T \ll m_e$ so that both electrons and nuclei can be treated as "heavy" relative to typical energies of the ambient photons

Ignoring recoils one finds the differential cross section

$$\frac{d\sigma_{\gamma\to a}}{d\Omega} = \frac{g_{a\gamma}^2 Z^2 \alpha}{8\pi} \frac{|\vec{k}_\gamma \times \vec{k}_a|^2}{q^4}$$

$q = \vec{k}_\gamma - \vec{k}_a$ momentum transfer

In plasma, long-range Coulomb is cut off by screening effects. Non-degenerate medium the screening Debye-Hückel formula

$$k_s^2 = \frac{4\pi\alpha}{T} n_B \left(Y_e + \sum_j Z_j^2 Y_j\right) \quad (*) \text{ paper ref}$$

$n_B = \rho / m_u$ $\quad Y_e, Y_j$ number fractions per baryon of electrons and nuclear species

transition rate

$$\Gamma_{\gamma\to a} = \frac{g_{a\gamma}^2 T k_s^2}{32\pi}\left[\left(1+\frac{k_s^2}{4\omega^2}\right)\ln\left(1+\frac{4\omega^2}{k_s^2}\right)-1\right]$$

(negelecting plasma mass)

$$Q = \int 2 \frac{d^3 k_\gamma}{(2\pi)^3} \frac{\Gamma_{\gamma\to a}\, \omega}{e^{\omega/T} - 1} = \frac{g_{a\gamma}^2\, \langle T^7 \rangle}{4\pi} F(k^2)$$

energy-loss per unit volume

HB stars $\rho = 10^4\, g/cm^3$ $T = 10^8\, K$ $k^2 \simeq 2.5$

$F = 1.85$

Sun $F = 0.98$

Solar emission spectrum Axion number flux at Earth

$$\Phi_a = \frac{R_\odot^3}{4\pi D_\odot^2} \int_0^1 dr\, 4\pi r^2 \int_{\omega_{pl}}^{\infty} dE \frac{4\pi k^2}{(2\pi)^3} \frac{dk}{dE}\, 2 f_B\, \Gamma_{\gamma\to a}$$

$f_B = (e^{E/T} - 1)^{-1}$ Bose-Einstein

$r = R/R_\odot$

$\omega_{pl}^2 = \frac{4\pi\alpha n_e}{m_e}$ $E^2 = k^2 + \omega_{pl}^2$ $\frac{dk}{dE} = \frac{E}{k}$

total

$\Phi_a = g_{10}^2\; 3.75 \times 10^{11}\; cm^{-2} s^{-1}$

$L_a = g_{10}^2\; 1.85 \times 10^{-3}\; L_\odot$

$\langle E \rangle = 4.20$ keV

$\langle E^2 \rangle = 22.7$ keV^2

$L_\odot = 3.84 \times 10^{33}$ erg s^{-1}

$$\frac{d\Phi_a}{dE} = 6.02 \times 10^{10}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{keV}^{-1}\ g_{10}^{2}\ E^{2.481}\ e^{-E/1.205}$$

$g_{10} = 1$ $\quad L_a \leq 1.3 \times 10^{-3} L_\odot$

~~Solar~~ SUN is ~ half its life, axion luminosity should not exceed the photon luminosity, otherwise nuclear fuel would have been spent before reaching an age $g_{a\gamma} \leq 2.4 \times 10^{-9}\ \mathrm{GeV}^{-1}$

CAST experiment

$B = 9\ \mathrm{T}$

$L = 9.26\ \mathrm{m}$

$A = 2 \times 14.5\ \mathrm{cm}^2$

peak $E \sim 4\ \mathrm{keV}$

$$P_{a\gamma} = \left(\frac{g_{a\gamma} B}{q}\right)^2 \sin^2\left(\frac{qL}{2}\right) \qquad q = \frac{m_a^2 \cancel{- m_\gamma^2}}{2E}$$

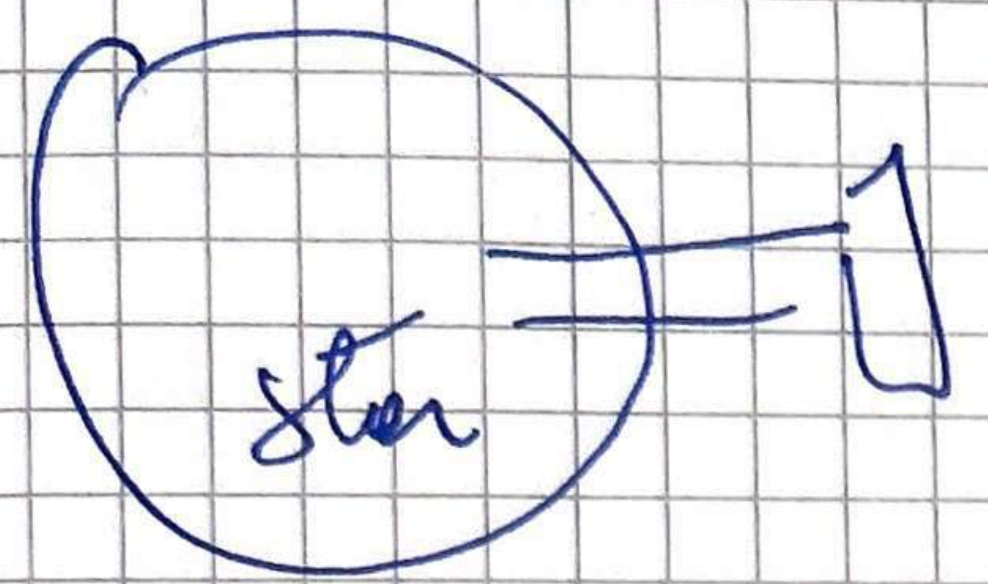
$$\rightarrow \left(\frac{g_a B}{2}\right)^2 \begin{cases} L^2 & qL \ll 1 \\ \frac{1}{2q^2} & qL \gg 1 \end{cases}$$

$$\frac{d\Phi_\gamma}{dE} = \frac{d\Phi_a}{dE} P_{a\gamma} = 0.088\ \mathrm{cm}^{-2}\ \mathrm{day}^{-1}\ \mathrm{keV}^{-1}\ g_{10}^{4}\ E^{2.481}\ e^{-E/1.205} \left(\frac{L}{9.26\,\mathrm{m}}\right)^2 \left(\frac{B}{9\,\mathrm{T}}\right)^2$$

coherent conversion $qL \lesssim 1 \rightarrow m_a \lesssim 0.02\ \mathrm{eV}$

low-Z gas a higher mass.

Energy-loss in stars



- Axions have very small mass
- Emission from stellar plasma not suppressed by threshold effect
- New energy-loss channel
- Back-reaction on stellar properties and evolution

Additional energy loss ("cooling"):

> loss of pressure
> contraction
> Heating
> Increased nuclear burning