Transverse momentum dependent distributions of the pion and the nucleus

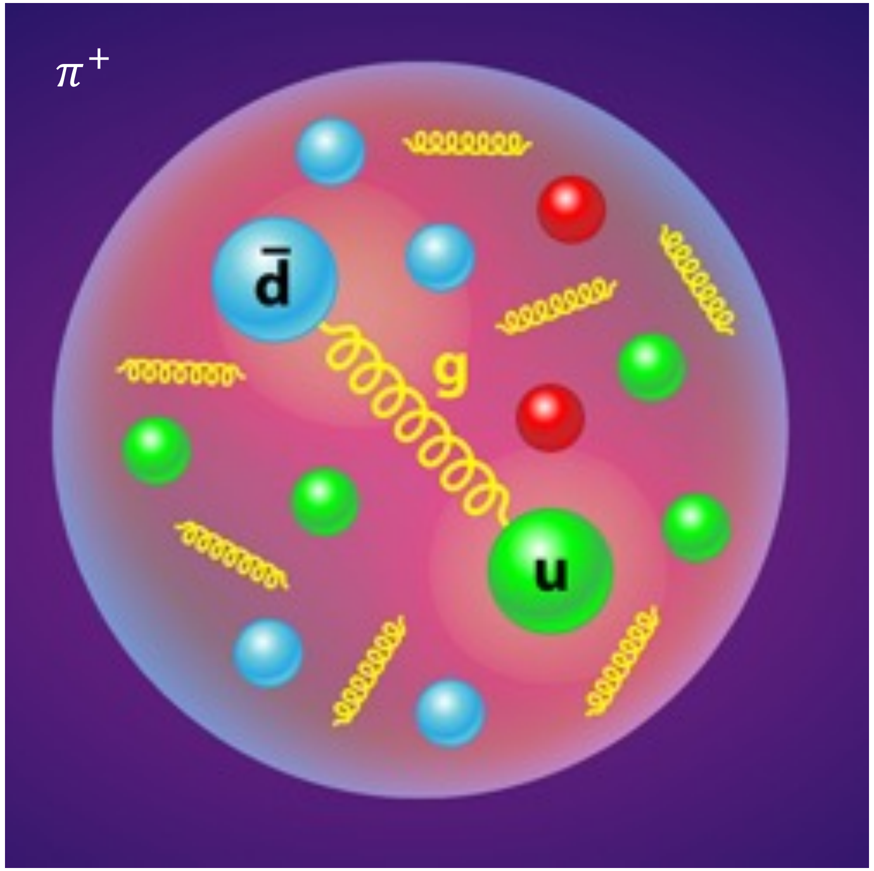
Patrick Barry

In collaboration with: Leonard Gamberg, Wally Melnitchouk, Eric Moffat, Daniel Pitonyak, Alexei Prokudin, and Nobuo Sato

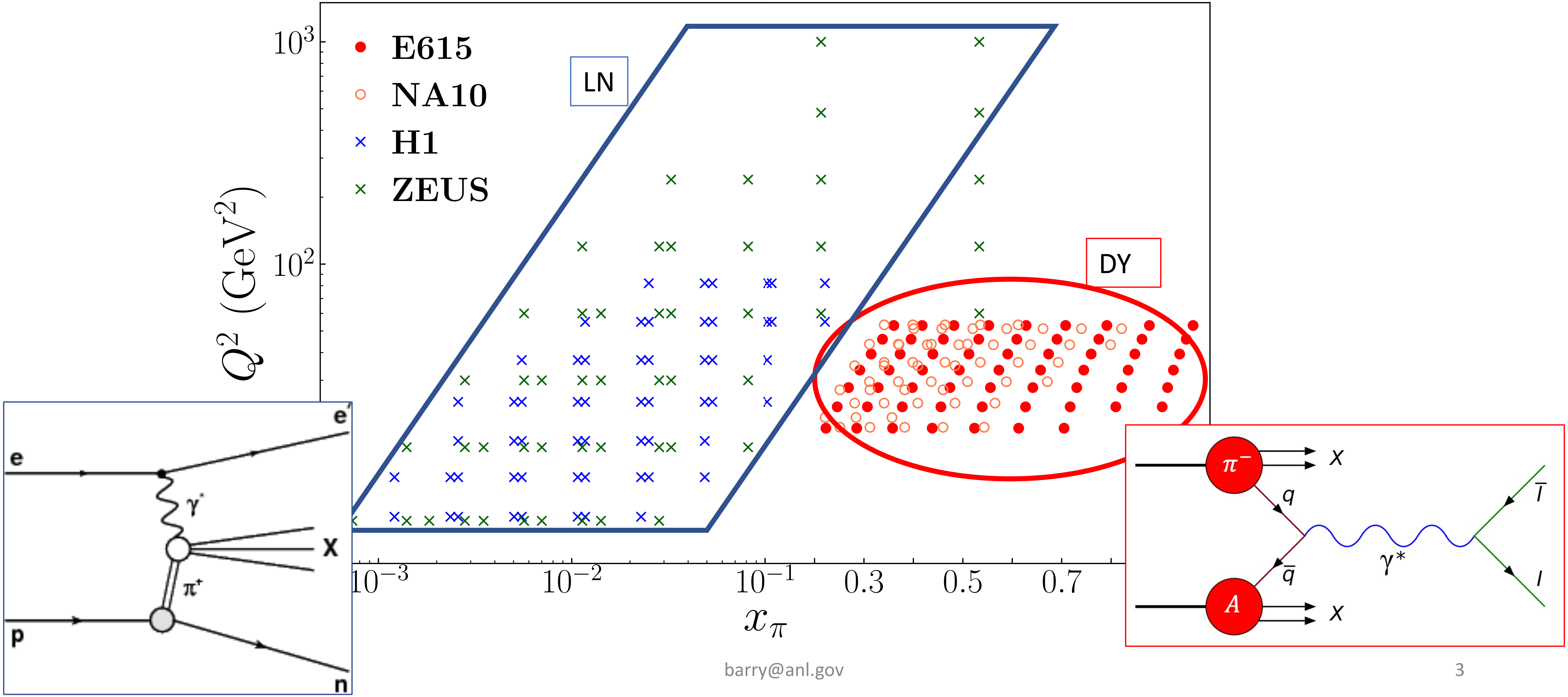
Based on: Phys. Rev. D **108**, L091504 (2023).

Non-proton structures - **Pions**

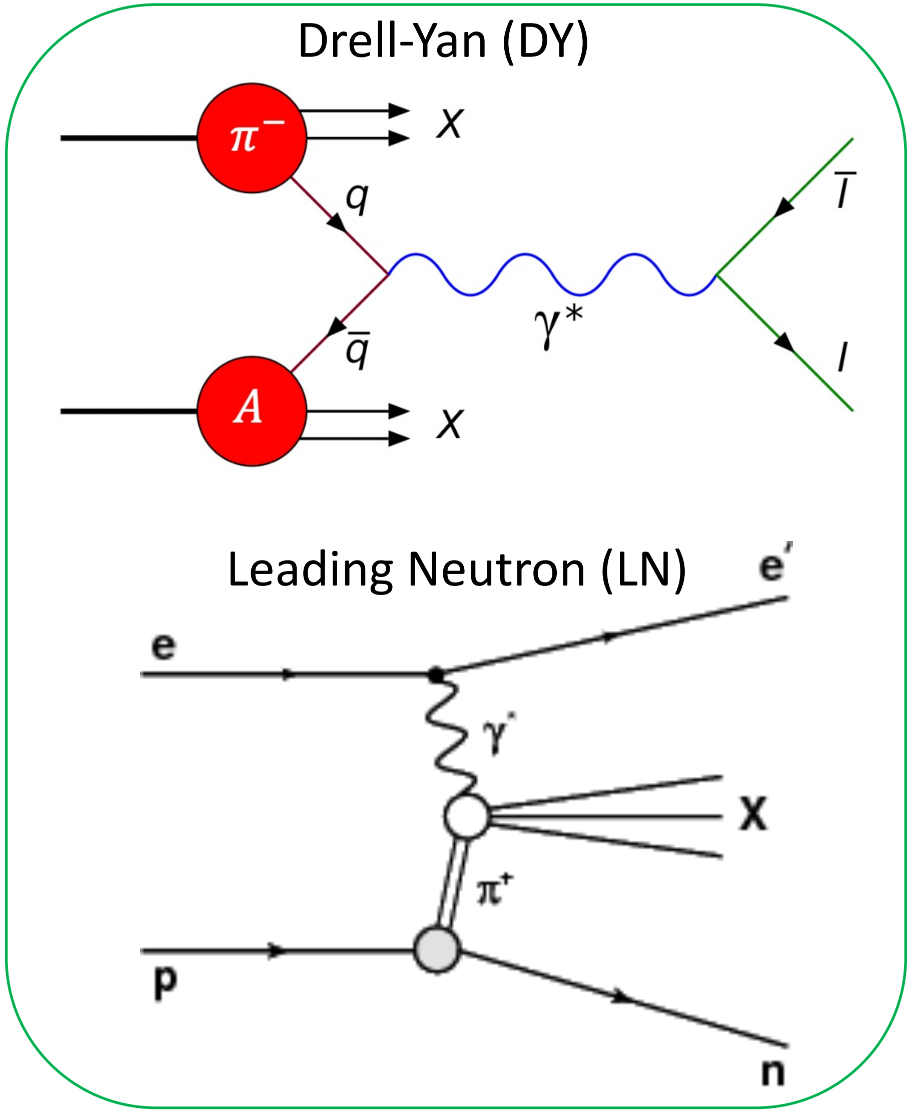
- Pion is the Goldstone boson associated with SU(2) chiral symmetry breaking
- Simultaneously a $q\bar{q}$ bound state
- Studying pion structures provides another angle to probe QCD and effective confinement scales
- More available data is desperately needed



Available datasets for pion structures



Pion PDFs in JAM

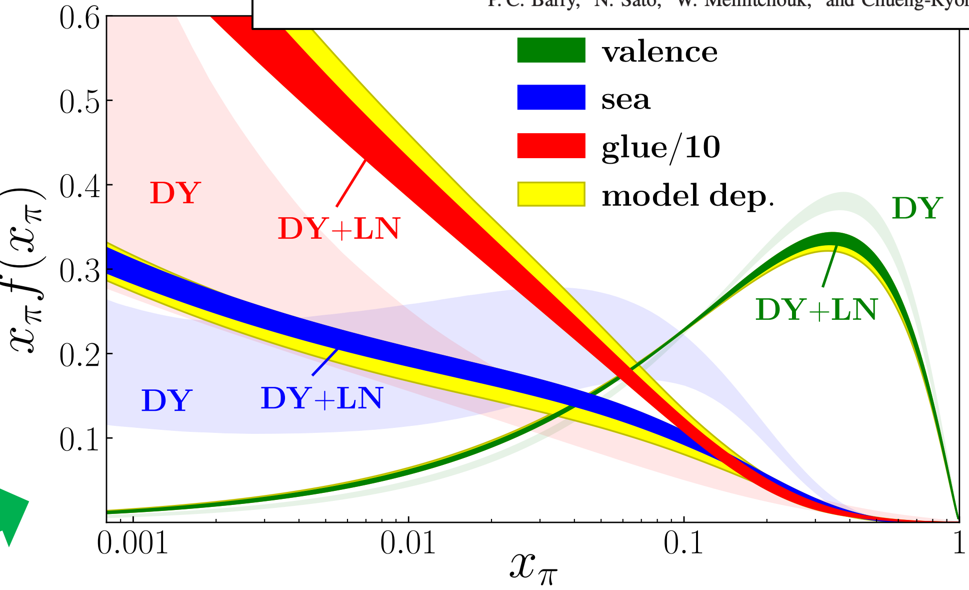


PHYSICAL REVIEW LETTERS 121, 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,[1] N. Sato,[2] W. Melnitchouk,[3] and Chueng-Ryong Ji[1]

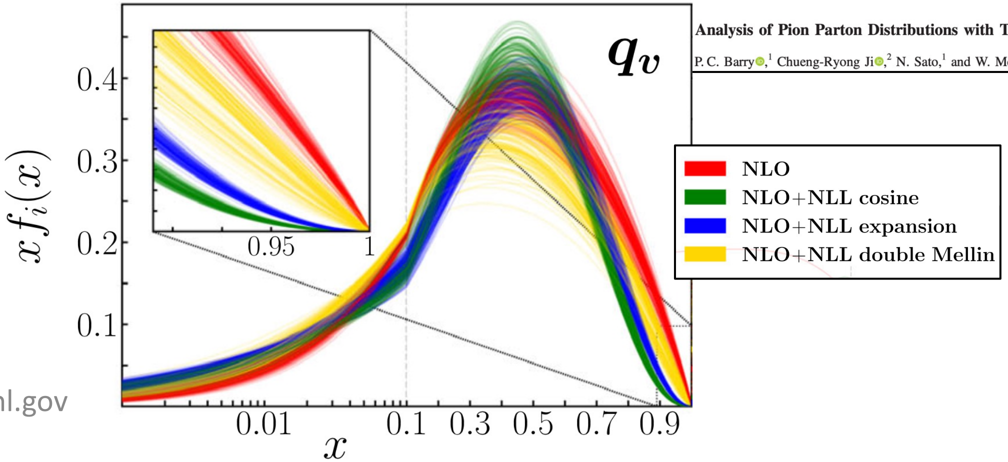


Threshold resummation in DY

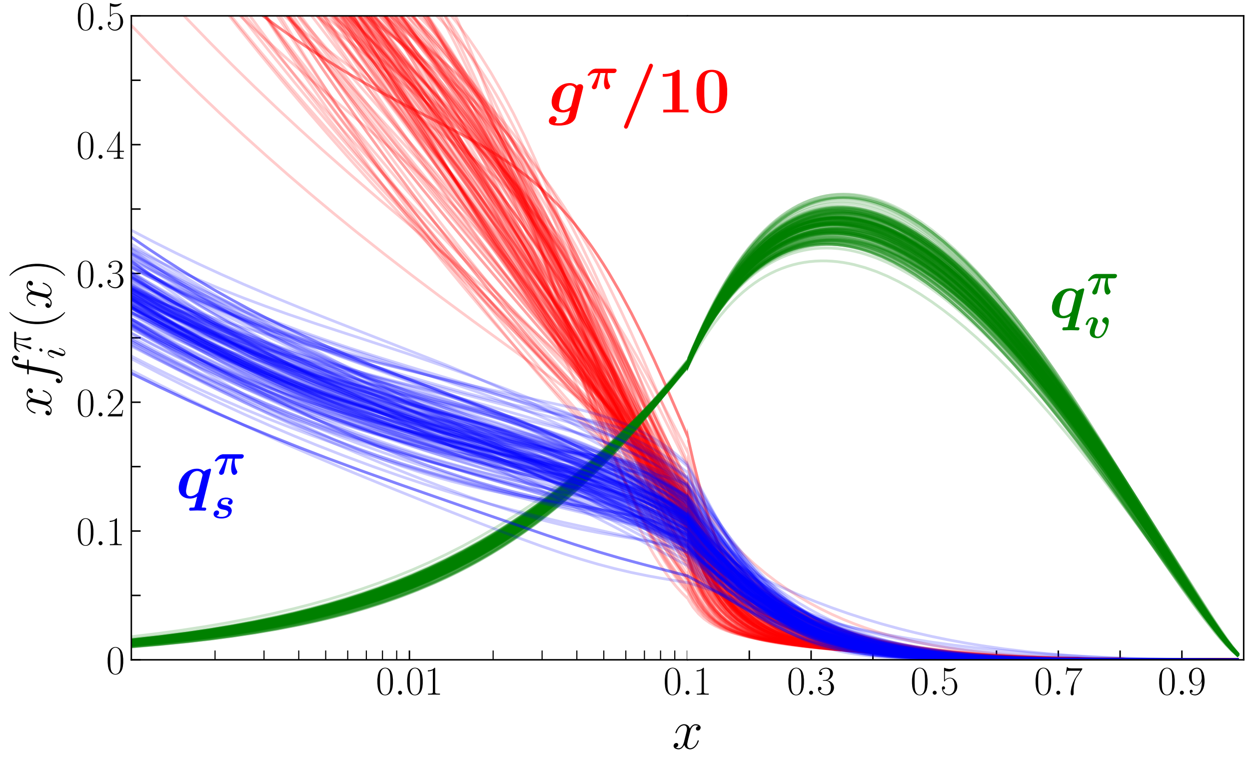
PHYSICAL REVIEW LETTERS 127, 232001 (2021)

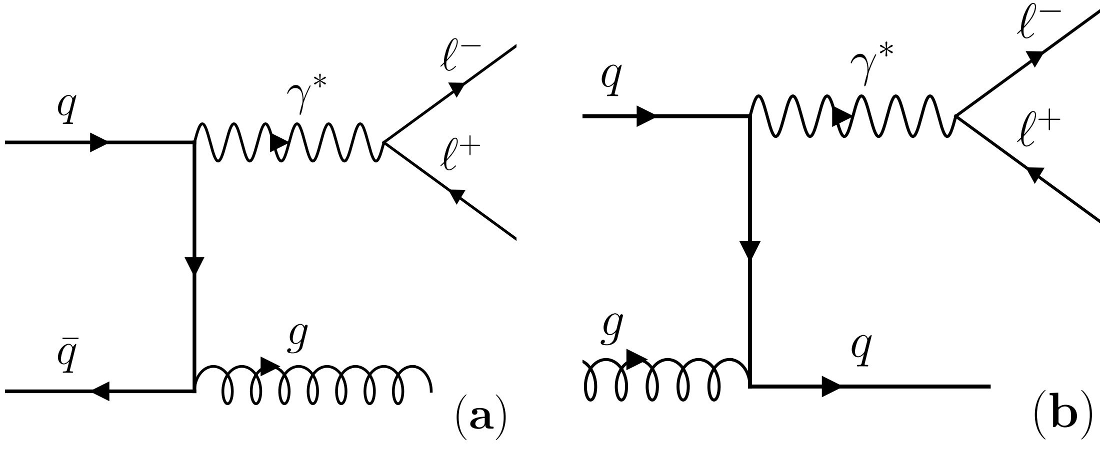
Analysis of Pion Parton Distributions with Threshold Resummation

P. C. Barry,[1] Chueng-Ryong Ji,[2] N. Sato,[1] and W. Melnitchouk[1]



Large-p_T DY data





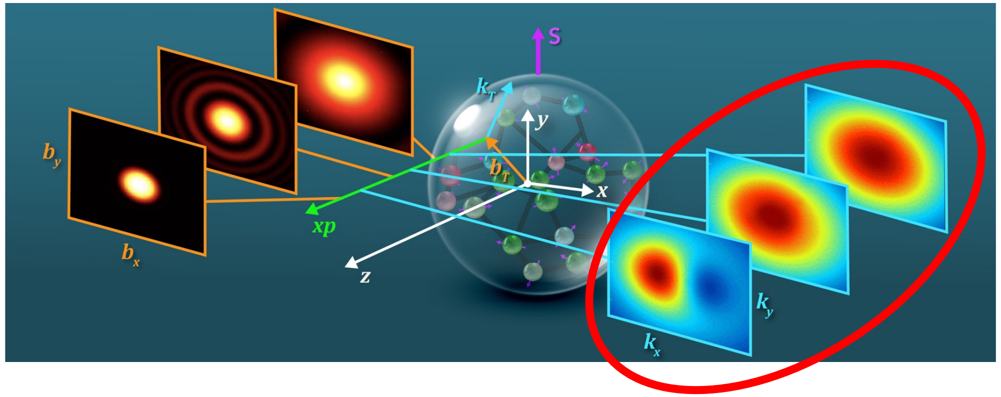
- Does not dramatically affect the PDF
- Successfully describe data with a scale $\mu = p_\mathrm{T}/2$

PHYSICAL REVIEW D **103,** 114014 (2021)

Towards the three-dimensional parton structure of the pion: Integrating transverse momentum data into global QCD analysis

N. Y. Cao,[1] P. C. Barry,[2,3] N. Sato,[3] and W. Melnitchouk[3]

Unpolarized TMD PDF



$$\tilde{f}_{q/\mathcal{N}}(x,b_T) = \int \frac{\mathrm{d}b^-}{4\pi} e^{-ixP^+b^-} \mathrm{Tr}[\langle \mathcal{N} \,|\, \bar{\psi}_q(b)\gamma^+ \mathcal{W}(b,0)\psi_q(0) \,|\, \mathcal{N}\rangle]$$

$$b \equiv (b^-, 0^+, \boldsymbol{b_T})$$

- $\boldsymbol{b_T}$ is the Fourier conjugate to the intrinsic transverse momentum of quarks in the hadron, $\boldsymbol{k_T}$
- Coordinate space correlations of quark fields in hadrons can tell us about their transverse momentum dependence
- Modification needed for UV and rapidity divergences; acquire regulators: $\tilde{f}_{q/\mathcal{N}}(x,b_T) \rightarrow \tilde{f}_{q/\mathcal{N}}(x,b_T;\mu,\zeta)$

Factorization for low-q_T Drell-Yan

- Cross section has hard part and two functions that describe structure of beam and target
- So called "W"-term, optimized at low-q_T

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\tau \mathrm{d}Y \mathrm{d}q_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2, \mu) \int \mathrm{d}^2 b_T \, e^{i b_T \cdot q_T}$$

$$\times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \, \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2) + \mathcal{O}\left(\frac{q_T}{Q}\right)$$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}.$$

Low-b_T: perturbative
high-b_T: non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*)$$
$$\times \exp\left\{-g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q)\right\}$$

Relates the TMD at small-b_T to the **collinear** PDF
$\Rightarrow$ TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative TMD structure of the hadron $\mathcal{N}(A)$
g_K: universal non-perturbative Collins-Soper kernel – same in all hadrons

- In this analysis, we use the MAP collaboration's parametrizations JHEP 10 (2022) 127

Controls the perturbative evolution of the TMD

A few details

- Nuclear TMD model linear combination of bound protons and neutrons

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A} \tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A - Z}{A} \tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Include an additional A-dependent nuclear parameter

$$g_{q/\mathcal{N}/A} = g_{q/\mathcal{N}} \left(1 - a_{\mathcal{N}} \left(A^{1/3} - 1\right)\right)$$

Alrashed, et al., Phys. Rev. Lett **129**, 242001 (2022).

- Fit to fixed target pA and πA q_T-dependent DY data and collinear π data
- We **simultaneously** fit: π *and* p TMDs, π collinear PDFs, CS kernel, and nuclear TMD parameter

Note about E615 πA Drell-Yan data

- Provides both $\frac{d\sigma}{dx_F d\sqrt{\tau}}$ (p_T-integrated) **and** $\frac{d\sigma}{dx_F dp_T}$ (p_T-dependent)
  - Large constraints on π collinear PDFs from p_T-integrated
  - Large constraints on π TMD PDFs from p_T-dependent
- Projections of same events ⇒ correlated measurements
- They have the **same luminosity** uncertainty, so they have the **same** overall **normalization** uncertainty
- To account for this, we *equate* the fitted normalizations of the two otherwise independent measurements
  - No other guidance from experiment how the uncertainties are correlated

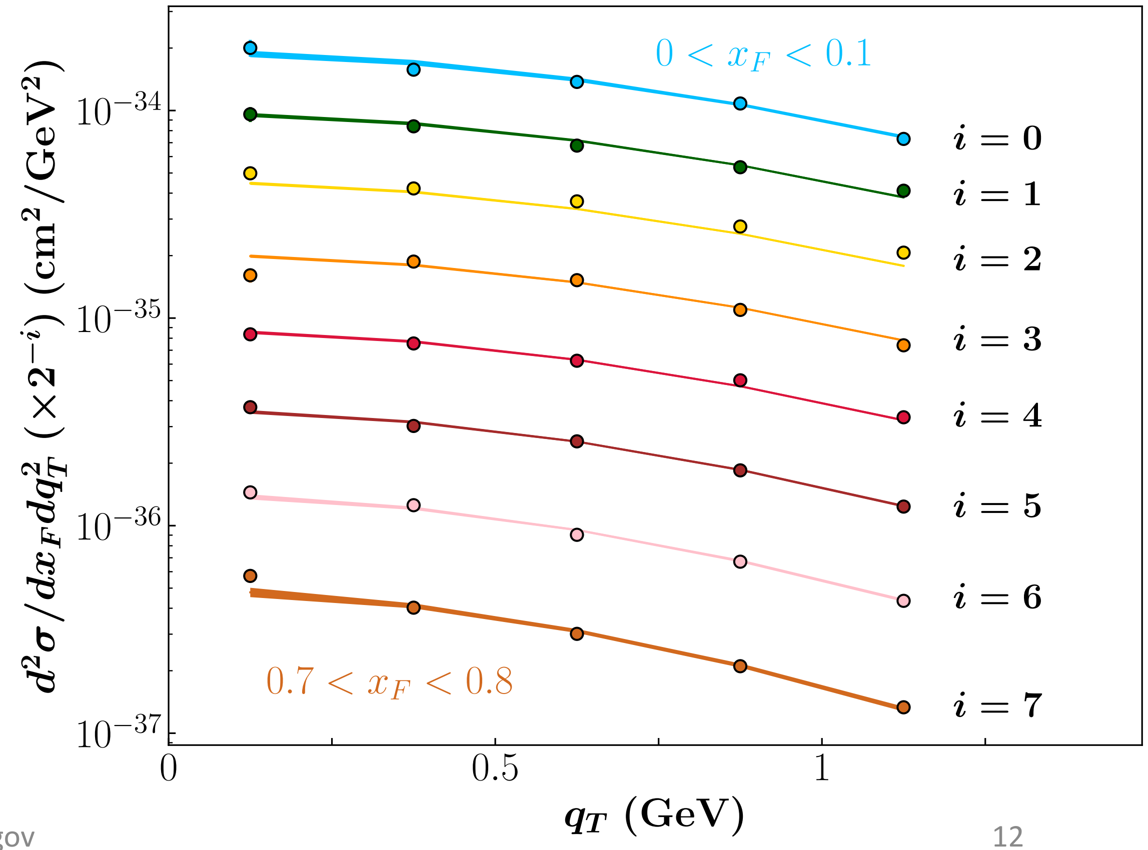
Note on collinear DY theory

- When equating the normalizations, we found
  - ***Tension*** when using **NLO+NLL** threshold resummed theory on the collinear observables
  - ***Agreement*** when using **NLO** theory on the collinear observables
- We note that in the OPE part of the TMD formalism, we use **NLO** accuracy
  - We do not use any *threshold enhancements* on the p_T-dependent observables

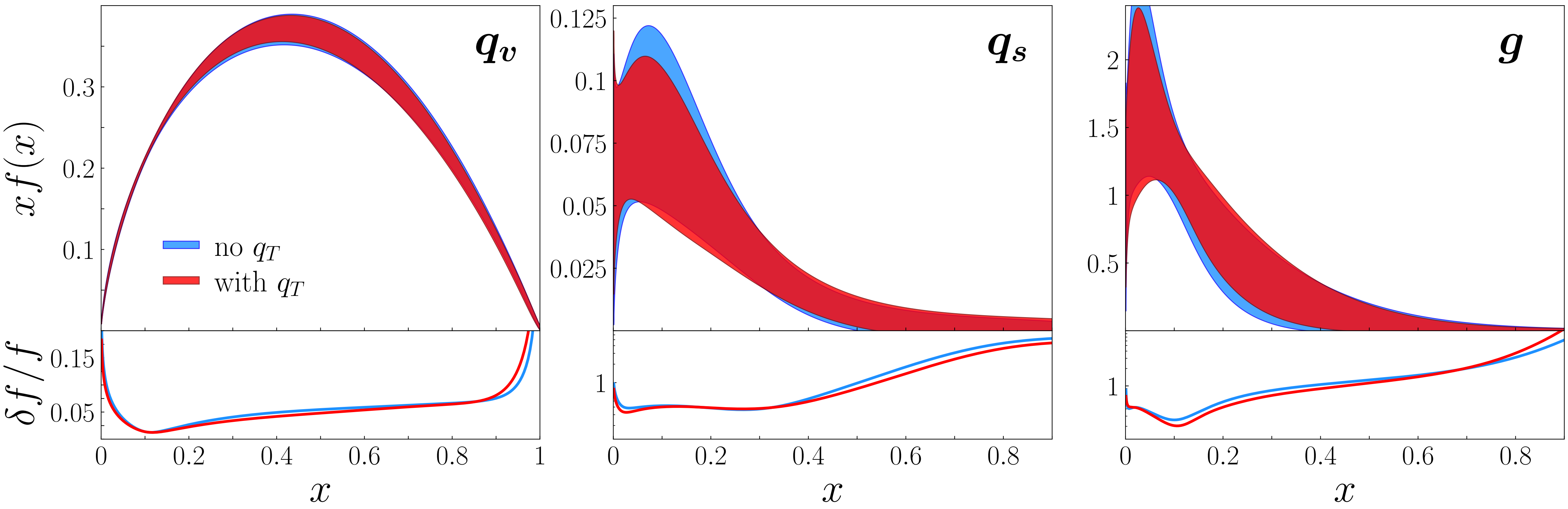
Data and theory agreement

- Fit both pA and πA DY data and achieve good agreement to both

Process	Experiment	$\sqrt{s}$ (GeV)	χ^2/N	Z-score
TMD				
q_T-dep. pA DY	E288 [90]	19.4	1.07	0.34
$pA \to \mu^+\mu^- X$	E288 [90]	23.8	0.99	0.05
	E288 [90]	24.7	0.82	0.99
	E605 [91]	38.8	1.22	1.03
	E772 [92]	38.8	2.54	5.64
(Fe/Be)	E866 [93]	38.8	1.10	0.36
(W/Be)	E866 [93]	38.8	0.96	0.15
q_T-dep. πA DY	E615 [94]	21.8	1.45	1.85
$\pi W \to \mu^+\mu^- X$	E537 [95]	15.3	0.97	0.03
collinear				
q_T-integr. DY	E615 [94]	21.8	0.90	0.48
$\pi W \to \mu^+\mu^- X$	NA10 [96]	19.1	0.59	1.98
	NA10 [96]	23.2	0.92	0.16
leading neutron	H1 [97]	318.7	0.36	4.59
$ep \to enX$	ZEUS [98]	300.3	1.48	2.15
Total			1.12	1.86



Extracted pion PDFs

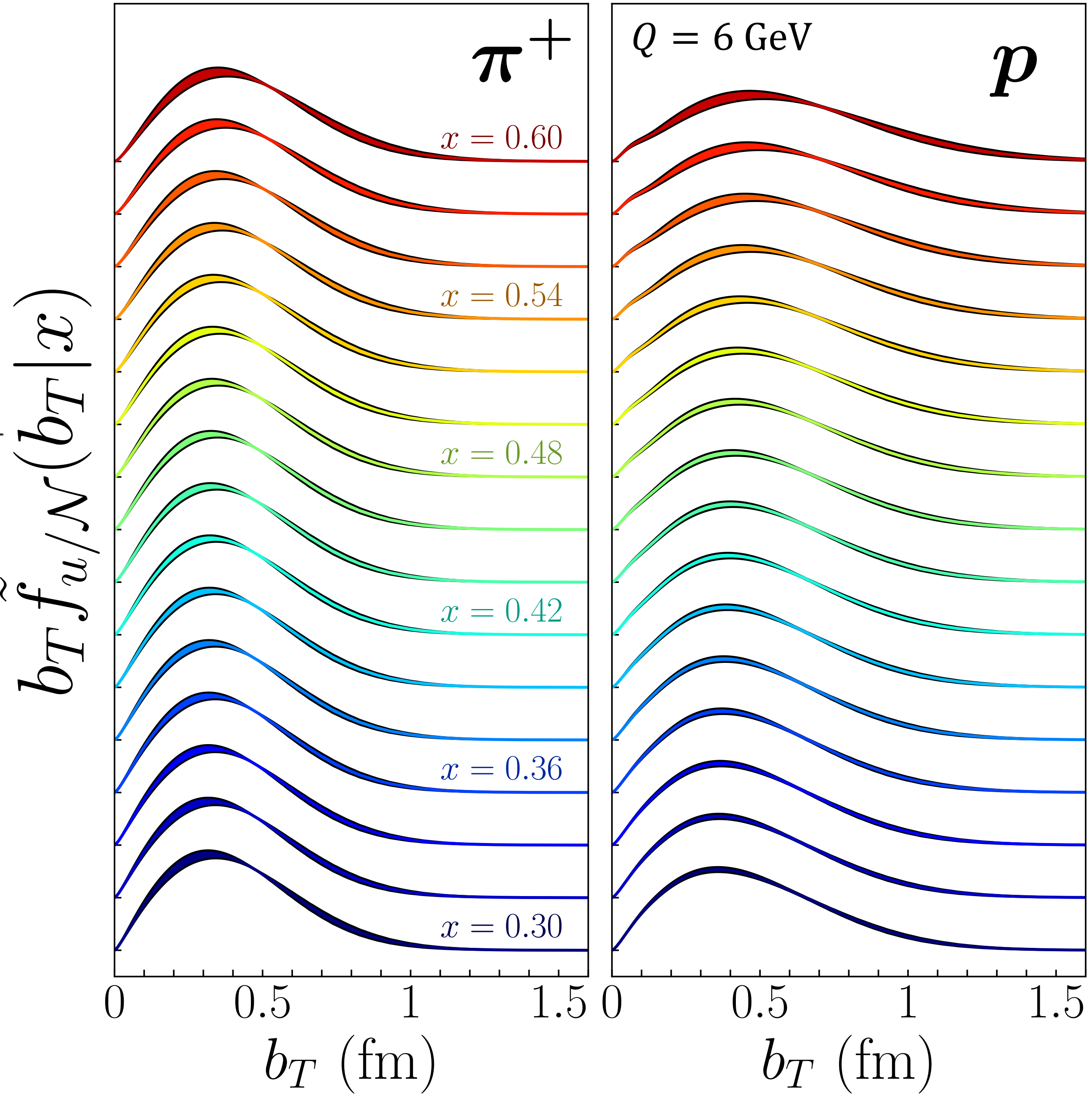


- The small-q_T data do not constrain much the PDFs

Resulting TMD PDFs of proton and pion

$$\tilde{f}_{q/\mathcal{N}}(b_T|x;Q,Q^2) \equiv \frac{\tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}{\int \mathrm{d}^2\boldsymbol{b}_T \tilde{f}_{q/\mathcal{N}}(x,b_T;Q,Q^2)}$$

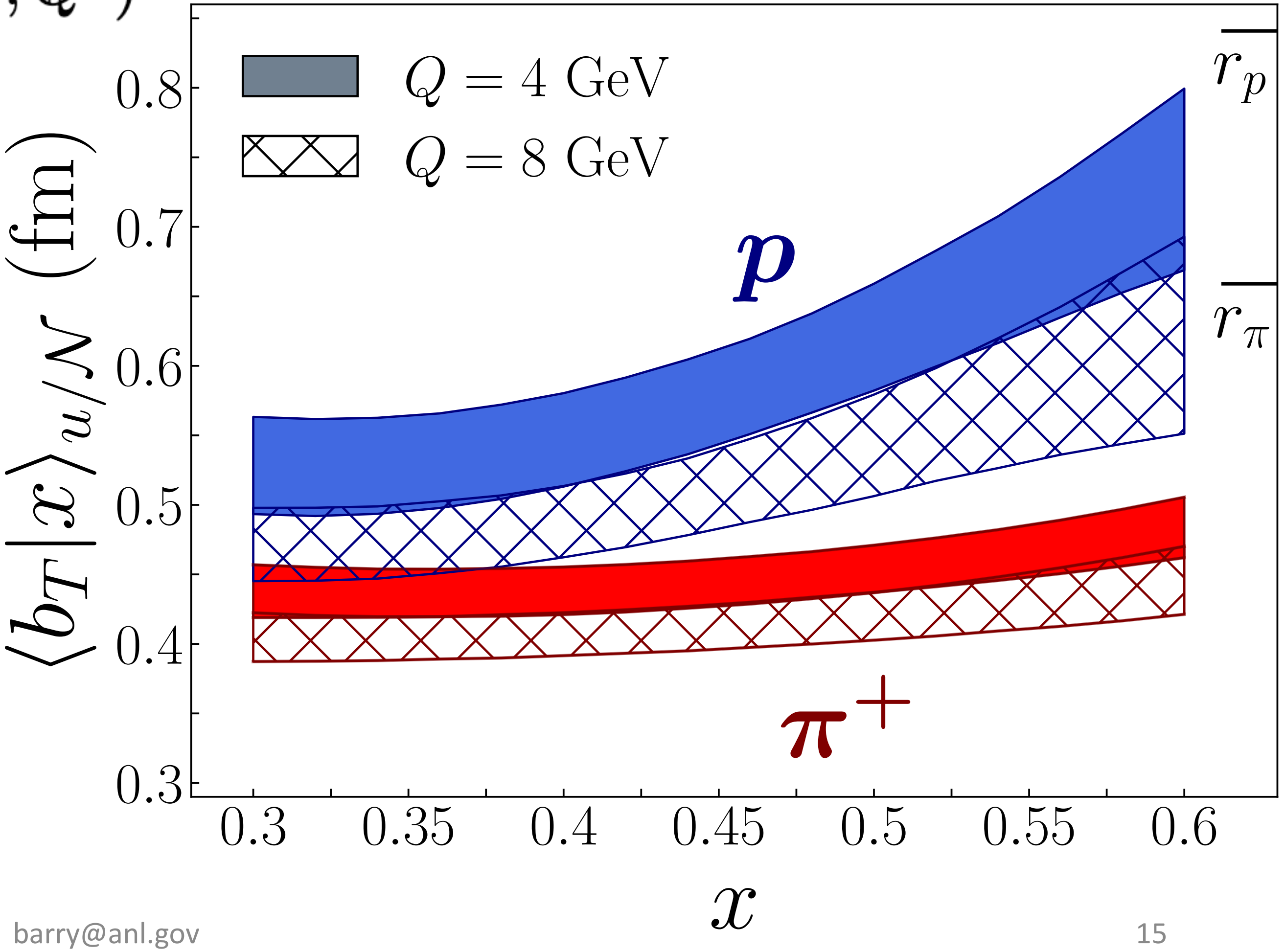
- Broadening appearing as x increases
- Up quark in pion is narrower than up quark in proton



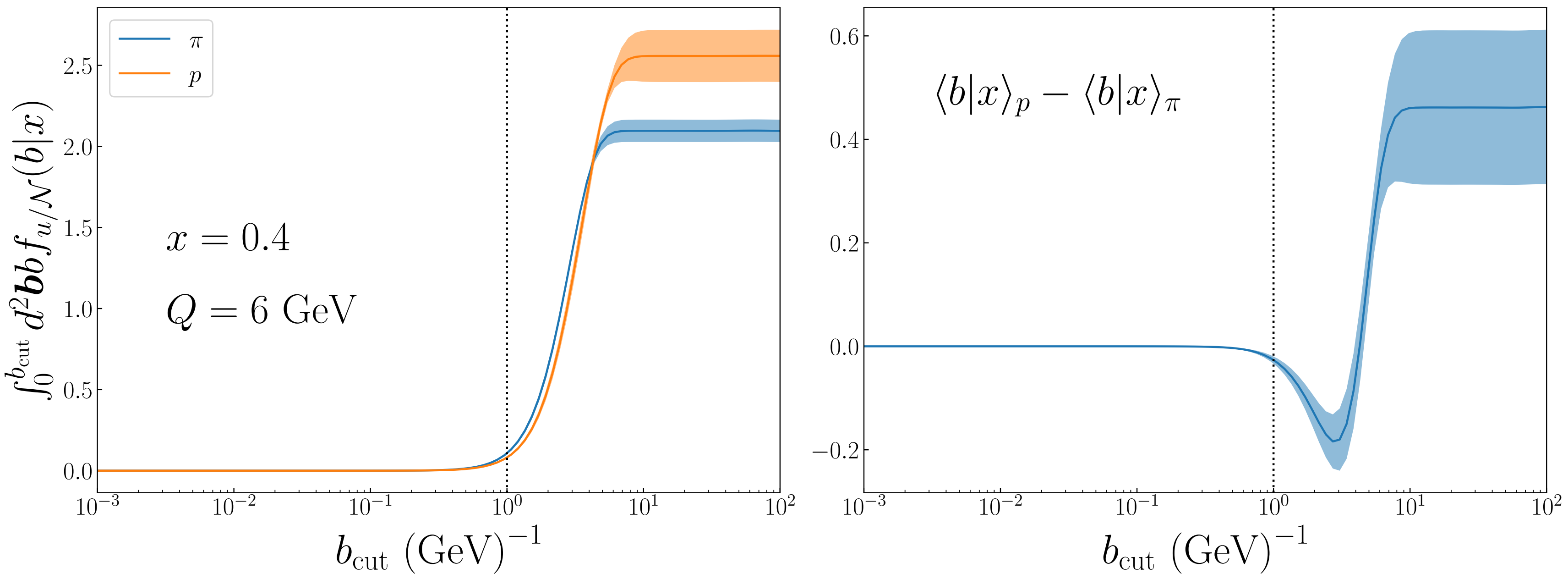
Resulting average b_T

$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int \mathrm{d}^2 \boldsymbol{b}_T \, b_T \, \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$

- Average transverse spatial correlation of the up quark in proton is $\sim$ 1.2 times bigger than that of pion
- Pion's $\langle b_T | x \rangle$ is $4 - 5.2\sigma$ smaller than proton in this range
- Decreases as x decreases



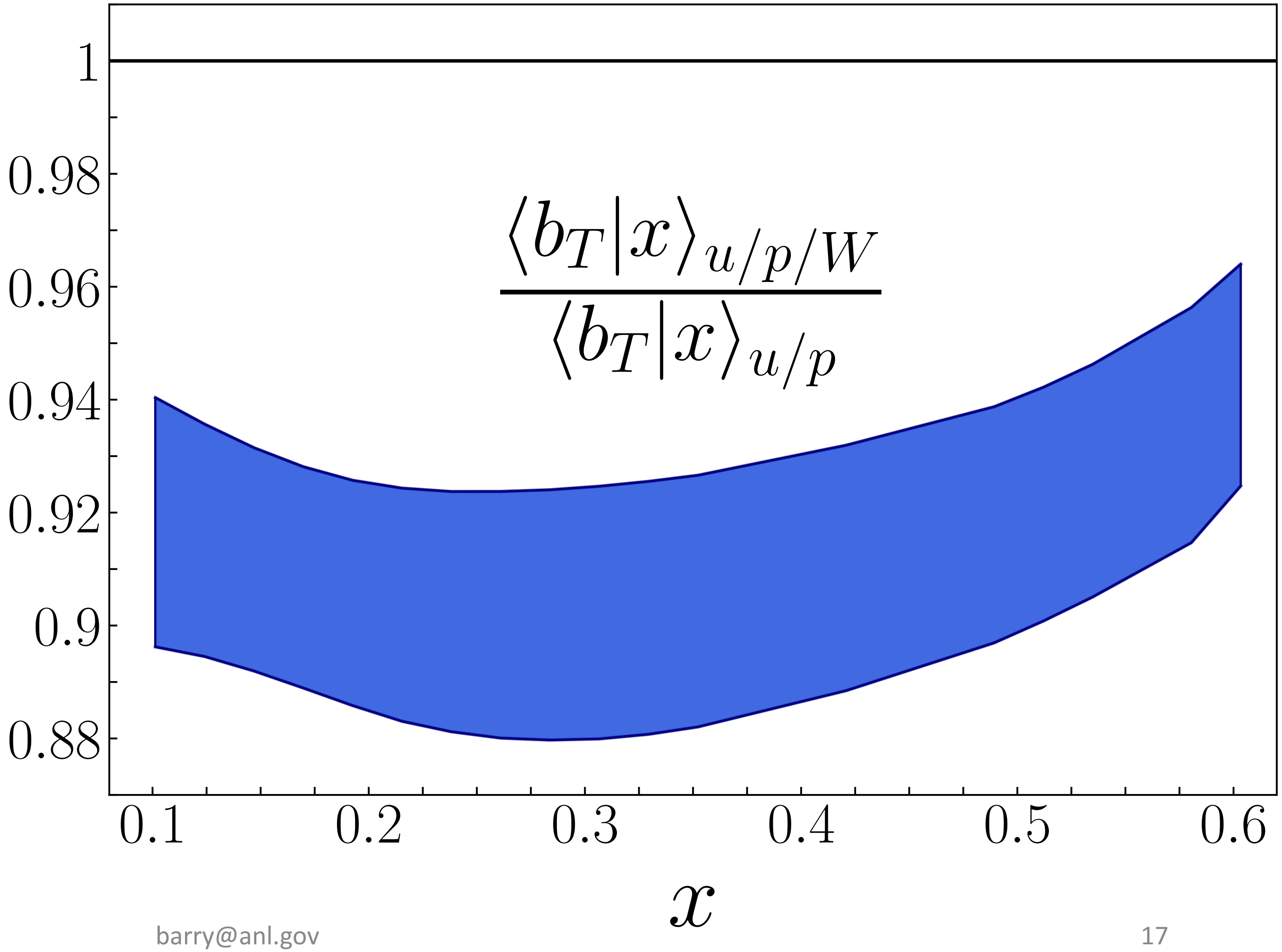
Emphasis on nonperturbative effects



- The $\langle b_T|x\rangle$ grows appreciably in the large-b_T region
- Saturation well beyond a perturbative scale
- Differences between proton and pion are in the nonperturbative region

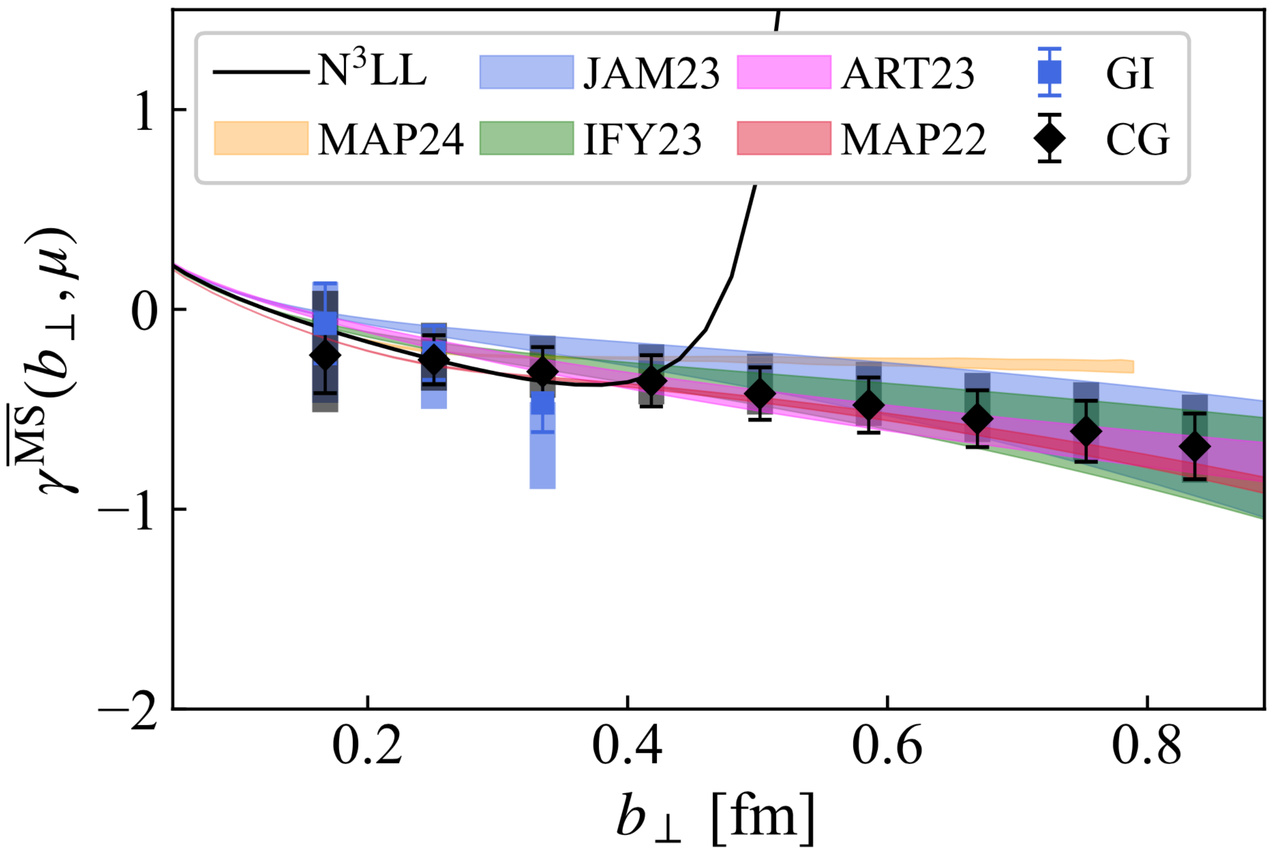
Transverse EMC effect

- Compare the average b_T given x for the up quark in the bound proton to that of the free proton
- Less than 1 by $\sim 5 - 12\%$ over the x range



CS kernel

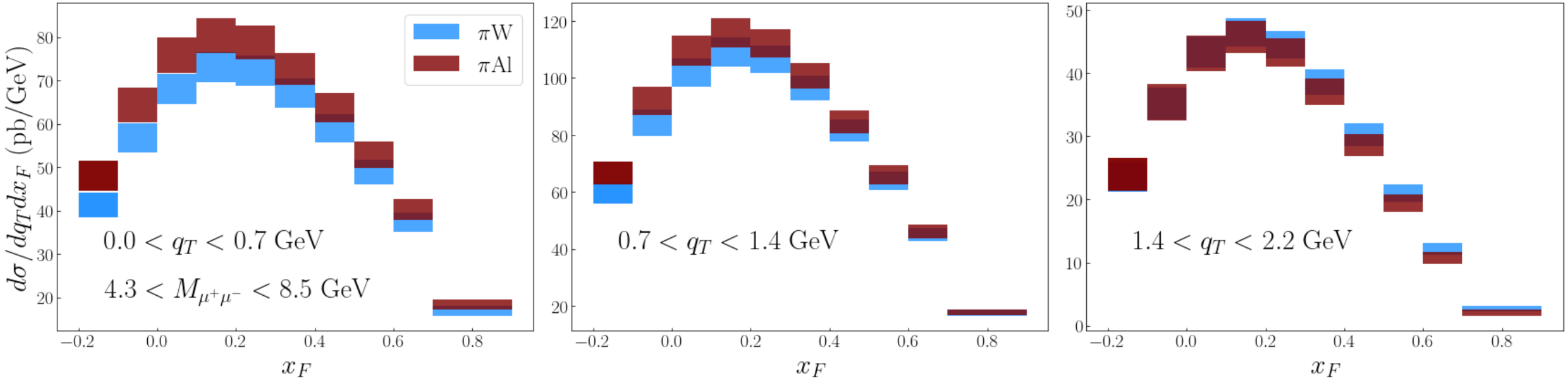
- Agreement with other phenomenological analyses, but with larger errors
- Good agreement with recent lattice data Phys. Lett. B **852**, 138617 (2024)



Courtesy of Xiang Gao

Predictions for COMPASS: tungsten and aluminum targets

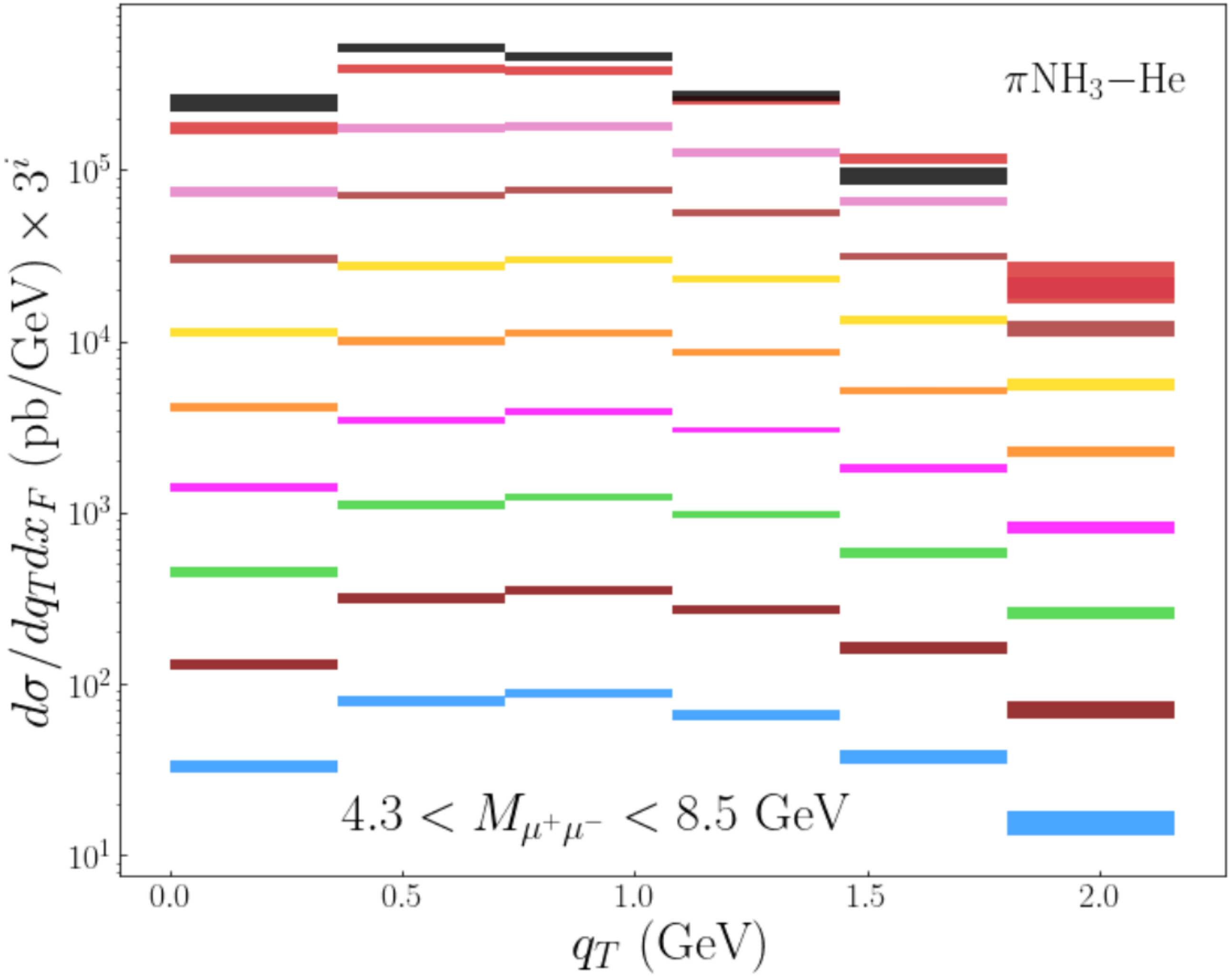
- Integrated over the M range, and bin averaged in x_F (horizontal axis) and q_T (panels)



- The πAl spectrum appears wider in q_T-space, consistent with the transverse EMC effect

Predictions for COMPASS: NH_3-He target

- Each color represents a different x_F bin (smallest x_F at the bottom)
- Much more finely binned in q_T than heavier nuclei!
- It should be noted that this is still a projection onto (q_T, x_F) and the triply differential measurement will be very useful



Takeaways and Outlook

- Pions and protons have significantly different **nonperturbative TMD structure** as evidenced from the low-energy data
- High energy data from the TeVatron and LHC provide further constraints on the proton TMDs and potentially collinear PDFs
- In order to fully trust the entire q_T spectrum, we should work towards including the full $W + Y$ theory

Backup

MAP parametrization

- The MAP collaboration (JHEP 10 (2022) 127) used the following form for the non-perturbative function

$$f_{1\,NP}(x, \boldsymbol{b}_T^2; \zeta, Q_0) = \frac{g_1(x)\, e^{-g_1(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda^2\, g_{1B}^2(x) \left[1 - g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}\right] e^{-g_{1B}(x)\frac{\boldsymbol{b}_T^2}{4}} + \lambda_2^2\, g_{1C}(x)\, e^{-g_{1C}(x)\frac{\boldsymbol{b}_T^2}{4}}}{g_1(x) + \lambda^2\, g_{1B}^2(x) + \lambda_2^2\, g_{1C}(x)} \left[\frac{\zeta}{Q_0^2}\right]^{g_K(\boldsymbol{b}_T^2)/2}, \quad (38)$$

$$g_{\{1,1B,1C\}}(x) = N_{\{1,1B,1C\}} \frac{x^{\sigma_{\{1,2,3\}}} (1-x)^{\alpha^2_{\{1,2,3\}}}}{\hat{x}^{\sigma_{\{1,2,3\}}} (1-\hat{x})^{\alpha^2_{\{1,2,3\}}}},$$

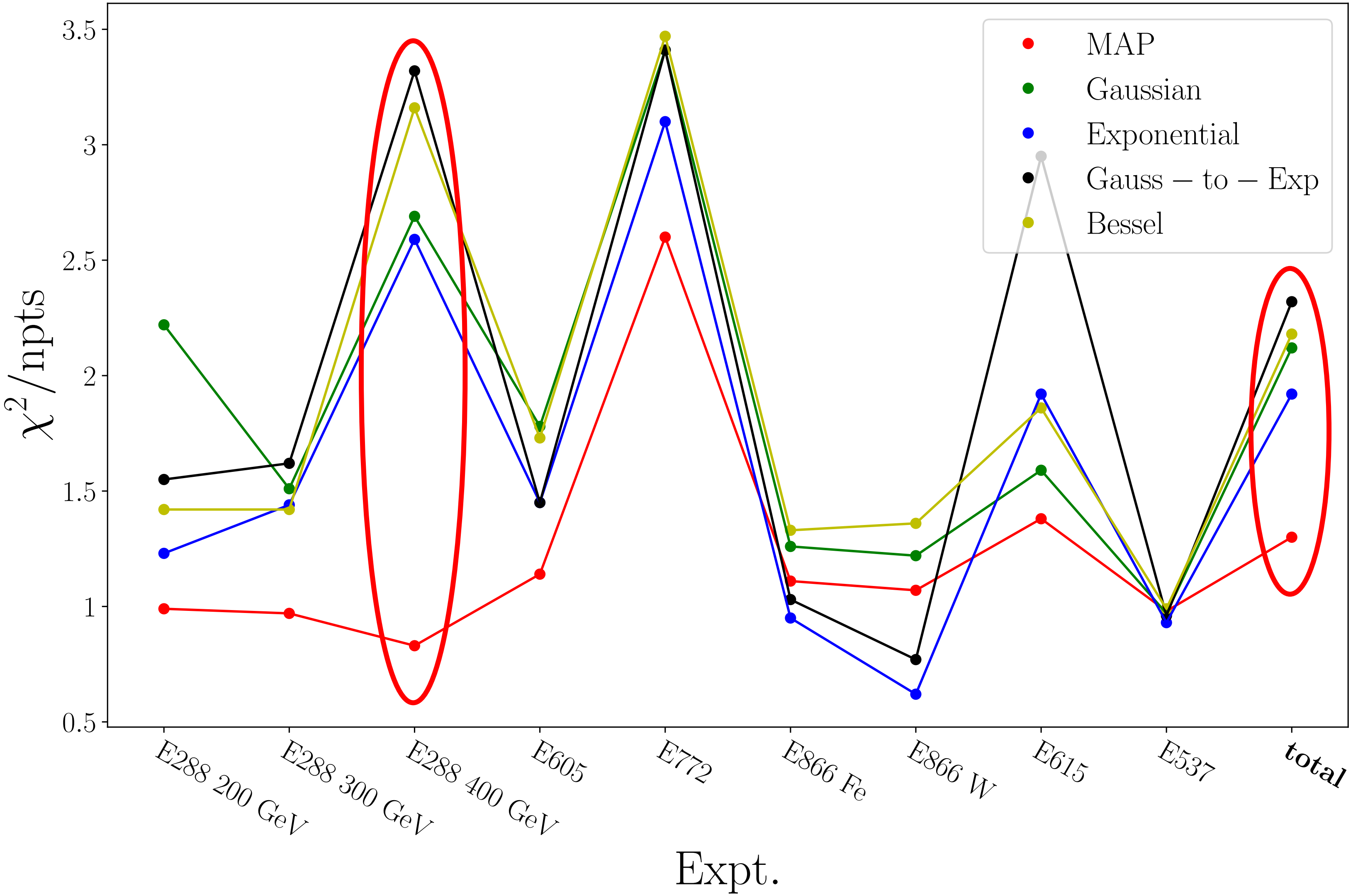
$$g_K(\boldsymbol{b}_T^2) = -g_2^2 \frac{\boldsymbol{b}_T^2}{2}$$

CS kernel

- 11 free parameters for each hadron (flavor dependence not necessary) (12 if we include the nuclear TMD parameter)

Resulting χ^2 for each parametrization

- Tried multiple parametrizations for non-perturbative TMD structures
- MAP parametrization is able to describe better all the datasets



Bayesian Inference

- Minimize the χ^2 for each replica

Normalization parameter

$$\chi^2(\boldsymbol{a}, \mathrm{data}) = \sum_e \left(\sum_i \left[\frac{d_i^e - \sum_k r_k^e \, \beta_{k,i}^e - t_i^e(\boldsymbol{a})/n_e}{\alpha_i^e} \right]^2 + \left(\frac{1 - n_e}{\delta n_e} \right)^2 + \sum_k \left(r_k^e \right)^2 \right)$$

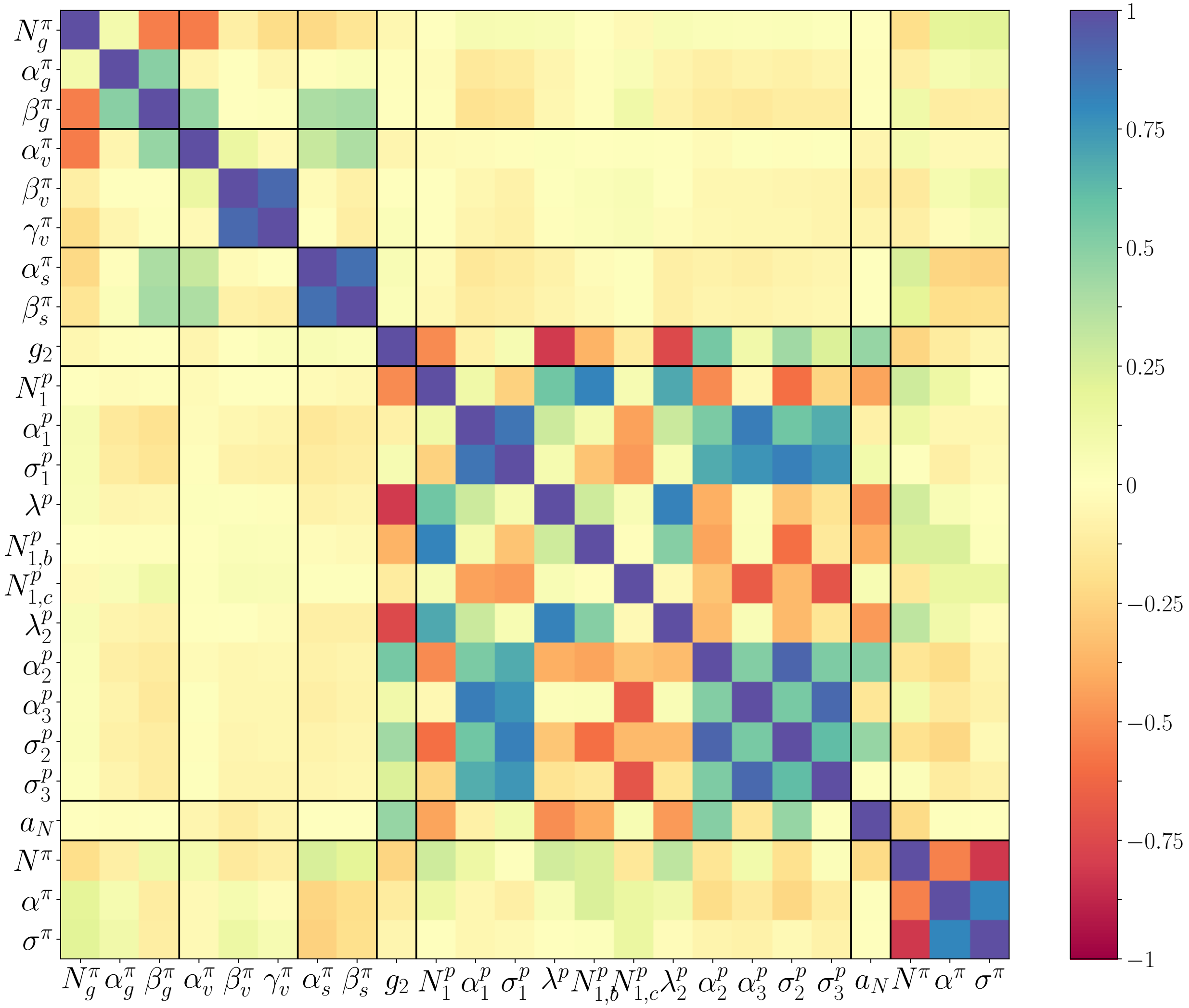
- Perform N total χ^2 minimizations and compute statistical quantities

Expectation value $$\mathrm{E}[\mathcal{O}] = \frac{1}{N} \sum_k \mathcal{O}(\boldsymbol{a}_k),$$

Variance $$\mathrm{V}[\mathcal{O}] = \frac{1}{N} \sum_k \left[\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}] \right]^2,$$

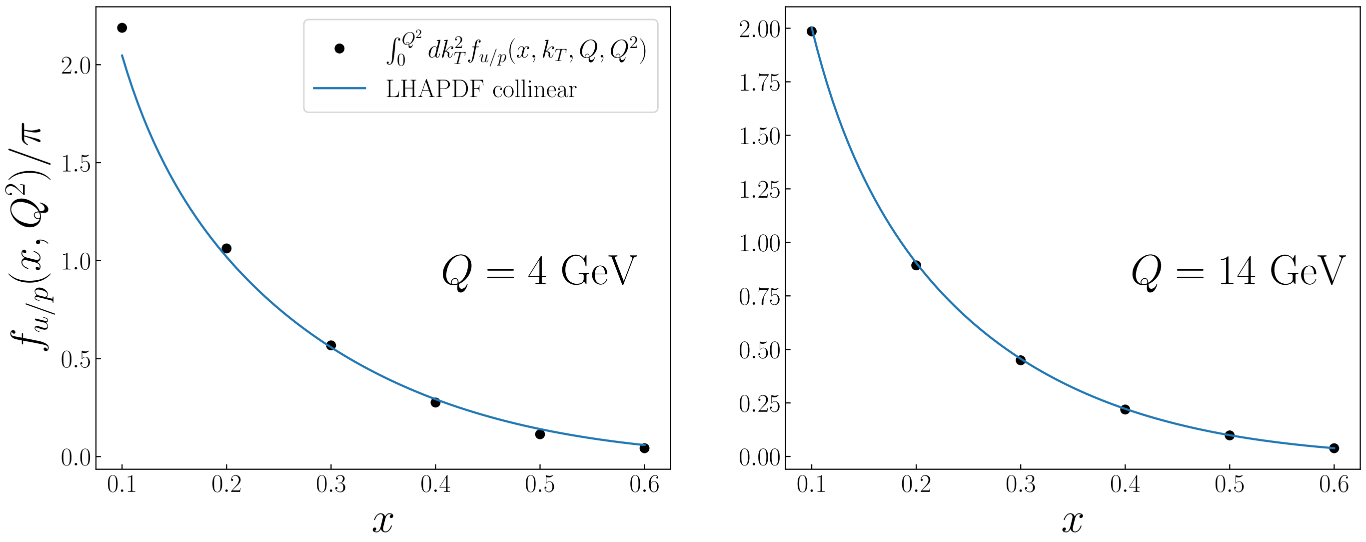
Correlations

- Level at which the distributions are correlated with each other
- Different distributions are largely correlated only within themselves



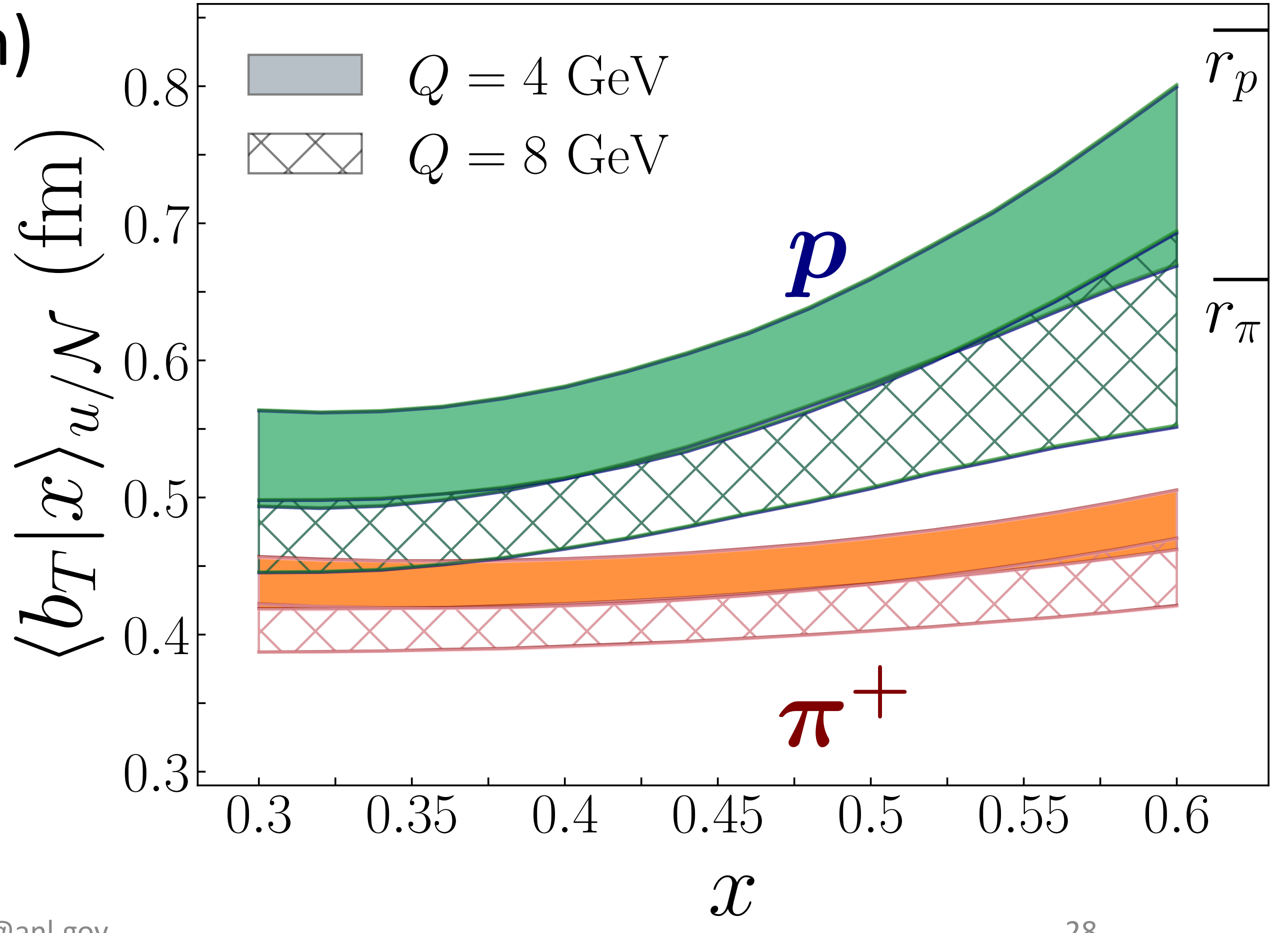
Collinear relation

- The TMD formalism requires that the integral over k_T^2 of the TMD gives the collinear PDF up to higher order corrections
- We demonstrate this for example in the proton case
- At larger Q, the power corrections are less important



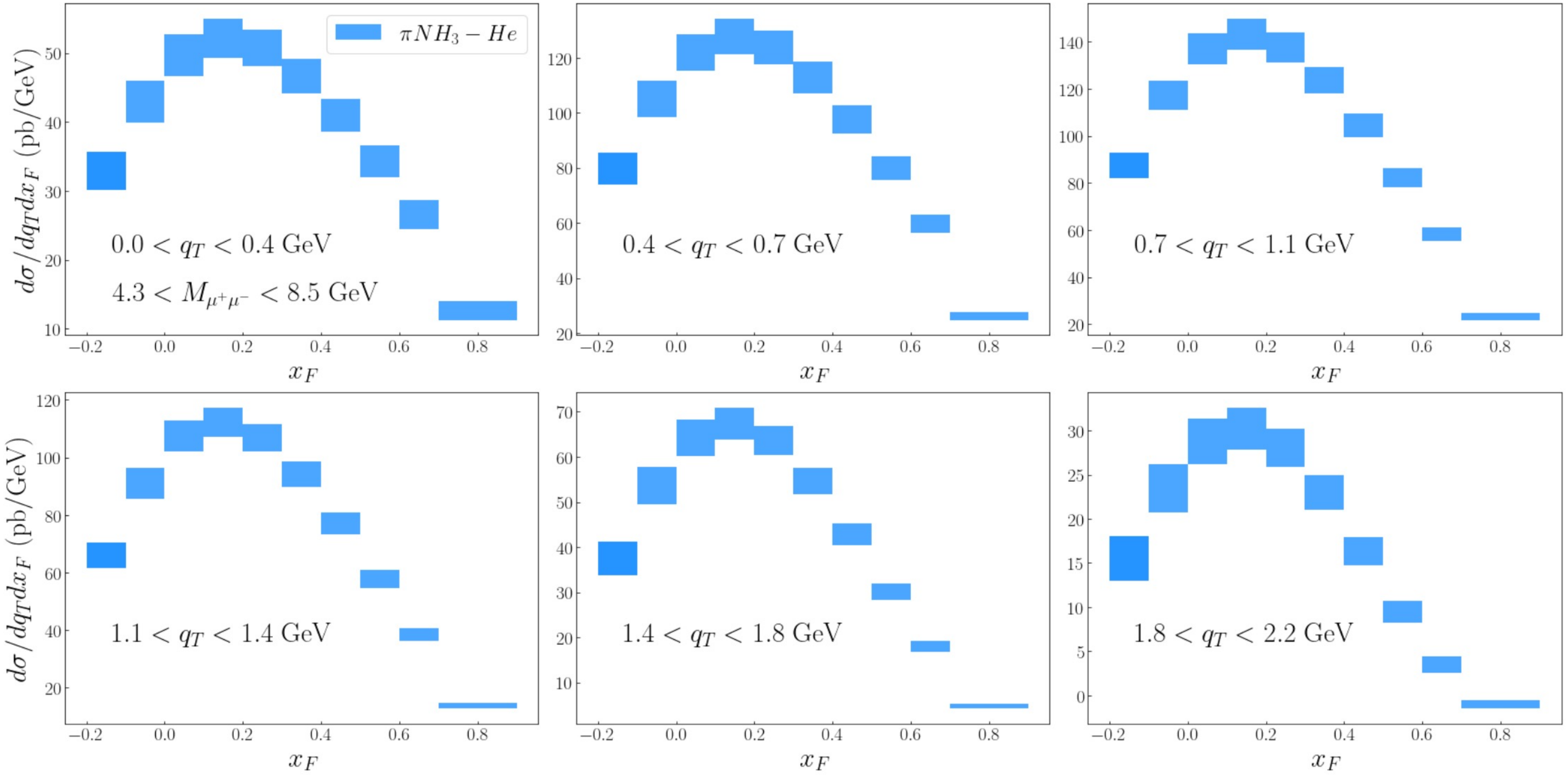
Emphasis on nonperturbative effects

- We vary the collinear PDFs
  p: CT14nlo (blue) → MMHT14 (green)
  π: JAM (red) → xFitter (orange)
- No change in the quantity!



Predictions for COMPASS: NH_3-He target

- Used a weighted average of N, H, and He parton distributions in the Drell-Yan formalism



Light nuclei from spin average polarised target:
mixture of $\mathbf{NH_3}$ & **LHe**:

molar fraction of nucleons:

H	He	N
15.7%	11.1%	73.2%

V. Andrieux from SPIN 2023