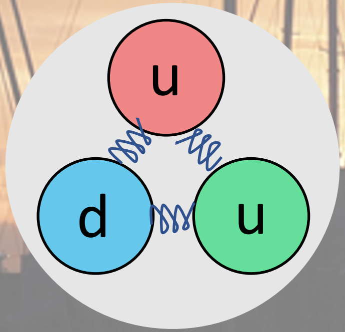
Transversity 2024

Trieste, 3-7 June 2024

# Gravitational form factors from lattice QCD




**Dimitra Anastasia Pefkou**

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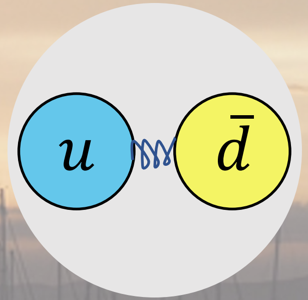


Berkeley UNIVERSITY OF CALIFORNIA

BERKELEY LAB

7th international workshop on transverse phenomena in hard processes

# In collaboration with:

1.[**DAP** Hackett Shanahan PRD (2022) arXiv:2107.10368]
2.[Hackett Oare **DAP** Shanahan PRD (2023) arXiv:2307.11707]
3.[Hackett **DAP** Shanahan PRL (2024) arXiv:2310.08484]

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# Energy-momentum tensor (EMT) and Noether's theorems

*The energy-momentum tensor is the conserved current under spacetime translational symmetry*

- Noether's theorems: Conserved current under $x^\mu \to x'^\mu$
  - 1st (global) : $x'^\mu = x^\mu + \epsilon^\mu$
  - 2nd (local) : $x'^\mu = x^\mu + \epsilon^\mu(x)$
- QCD : $\mathcal{L}_{QCD} = -\frac{1}{4}F^{a,\mu\nu}F^a_{\mu\nu} + \sum_f [\bar{\psi}_f i\,\gamma^\mu\, D_\mu \psi_f + m_f \bar{\psi}_f \psi_f]$
- $\mathcal{L}_{QCD} \xrightarrow[\text{1st theorem}]{\text{Noether's}}$ Canonical EMT (not symmetric)
- Canonical EMT $\xrightarrow[\text{improvement}]{\text{Belinfante}}$ Belinfante-improved EMT (symmetric)
- Belinfante improvement adds $c\,\partial_\alpha \Theta^{\alpha\mu\nu}$ term with $c = 1$, but could also have $c \neq 1$

→ Different operators (and form factors) for different choices (Hudson Schweitzer PRD 2017)

* Newer work: Freese PRD 2022: $\mathcal{L}_{QCD} \xrightarrow[\text{2nd theorem}]{\text{Noether's}}$ recover Belinfante-improved EMT ?

# Contents of this talk

- Introduction
- Bare gravitational form factors (GFFs) from lattice QCD
- Non-perturbative renormalization
- GFFs of the proton, pion, and other hadrons: selected results

[Hackett Oare **DAP** Shanahan PRD (2023) arXiv:2307.11707]
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# Gravitational form factors

*Gravitational form factors are the form factors of the energy-momentum tensor*

$$T^{\mu\nu} = \underbrace{-F_a^{\mu\alpha}F^{\nu}_{a,\alpha} + \frac{1}{4}g^{\mu\nu}F_a^{\alpha\beta}F_{a,\alpha\beta}}_{T_g^{\mu\nu}} + \underbrace{\sum_f i\bar{\psi}_f\gamma^\mu D^\nu \psi_f}_{T_q^{\mu\nu}}$$

\# increases with spin

- $\langle p', s'|T_i^{\mu\nu}|p, s\rangle \sim$ Kinematic coefficients (Lorentz structure) $\times$ Gravitational form factors $\mathcal{G}_i(t)$ (scalar functions of $t = -(p'-p)^2$)
- $\partial_\mu T_i^{\mu\nu} \neq 0$ → $\mathcal{G}_q(t), \mathcal{G}_g(t)$ renormalization scheme and scale dependent
- $\partial_\mu T^{\mu\nu} = 0$ → $\mathcal{G}(t) \equiv \mathcal{G}_{q+g}(t)$ scheme and scale independent
- Poincaré symmetry constraints, e.g., $\int d^3x\, T^{00}|p, s\rangle = m|p, s\rangle$, encoded in $\mathcal{G}(t)$
- Proton: $\langle p', s'|T_i^{\mu\nu}|p, s\rangle \sim A_i(t), J_i(t), D_i(t), \bar{c}_i(t)$

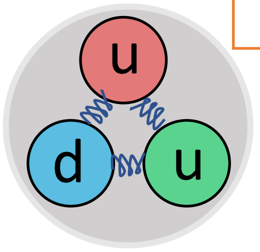
totals: $A(0) = 1$, $J(0) = \frac{1}{2}$, $\bar{c}(t) = 0$, $D(0) = ?$

| totals | | | |
|---|---|---|---|
| $A(0) = 1$ | $J(0) = \frac{1}{2}$ | $\bar{c}(t) = 0$ | $D(0) = ?$ |
| momentum | angular momentum | $T^{\mu\nu}$ conserved | ``The last global unknown'' Polyakov Schweitzer 2018 |

| | | | |
|---|---|---|---|
| $T^{00}$ | $T^{01}$ | $T^{02}$ | $T^{03}$ |
| $T^{10}$ | $T^{11}$ | $T^{12}$ | $T^{13}$ |
| $T^{20}$ | $T^{21}$ | $T^{22}$ | $T^{23}$ |
| $T^{30}$ | $T^{23}$ | $T^{32}$ | $T^{33}$ |

energy
momentum density
energy flux
pressure
shear stress
momentum flux

*Gravitational form factors encode the distribution of energy, angular momentum, and mechanical properties within hadrons*



# Gravitational FFs ∋ Generalized FFs

$$x = \frac{-q^2}{2p^\mu q_\mu}$$

$$q = p' - p$$

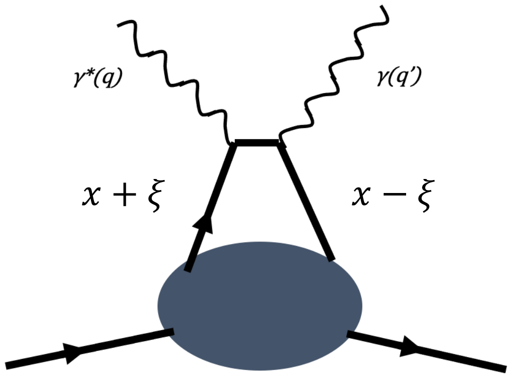
$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$

*The second Mellin moment of generalized parton distributions yields the gravitational form factors (generalized form factors)*

path-ordered gauge link

- Quark and gluon generalized parton distributions (GPDs)
  ~ matrix elements of $\bar{\psi}(-n/2)\gamma^\mu \mathcal{U}\,\psi(n/2), F^{\mu\alpha}(-n/2)\mathcal{U}F_\alpha^\nu(n/2)$

light-like vector

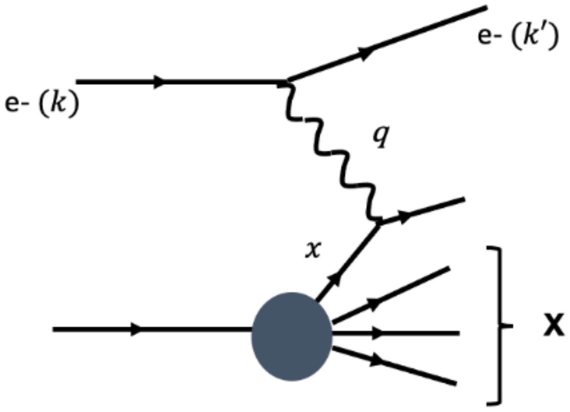



- Operator product expansion → tower of local operators
  lowest order: **traceless** $\hat{T}_q^{\mu\nu}, \hat{T}_g^{\mu\nu}$ (twist-2)

$B(t) = 2J(t) - A(t)$

- Proton: $\int_{-1}^{1} dx\; x\, H_i(x,\xi,t) = A_i(t) + \xi^2 D_i(t)$ , $\int_{-1}^{1} dx\; x\, E_i(x,\xi,t) = B_i(t) - \xi^2 D_i(t)$

- Forward limit $t = 0$: 2nd Mellin moment of parton distribution functions (PDFs)
  e.g. $\int_0^1 dx\; x\, f_i(x) = A_i(0)$



# Constraints on GFFs: examples

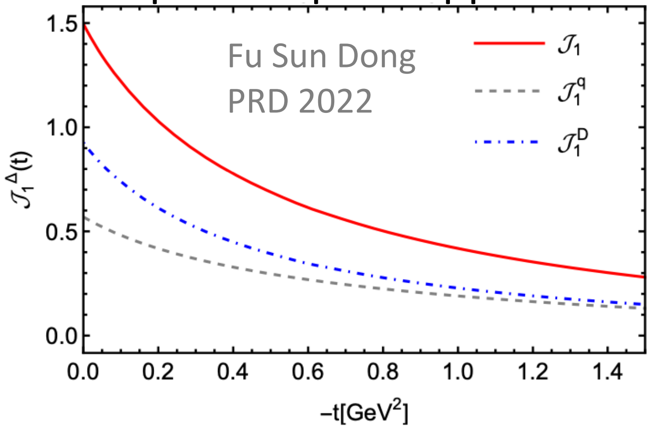
* see e.g. Burkert et al Rev.Mod.Phys. 2023 for review

*The GFFs have gained increasing interest in recent years, after their first phenomenological extractions*

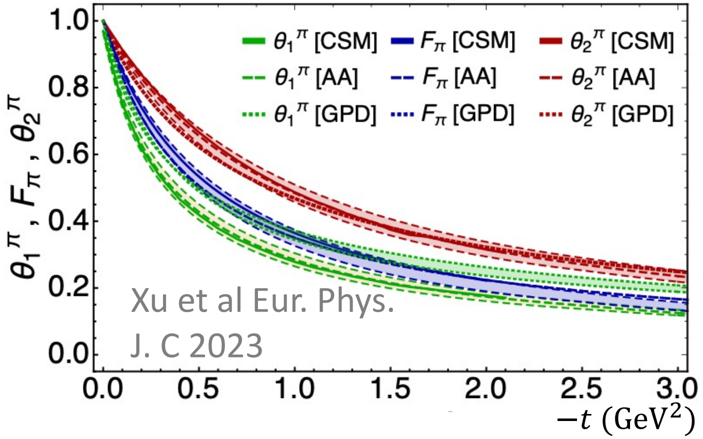
- Effective field theory and models

- chPT: $D(0) = -1$ for the Nambu Goldstone bosons in the chiral limit (generally unknown for hadrons)

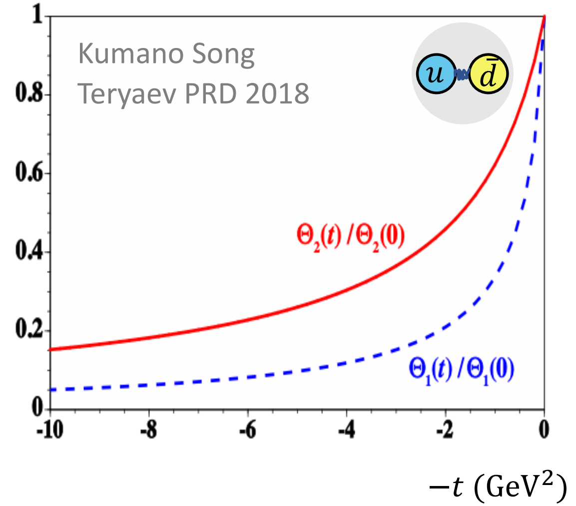
- Δ baryon in relativistic quark-diquark approach

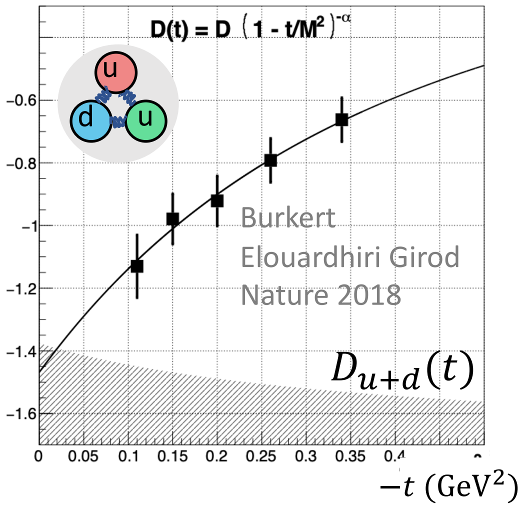


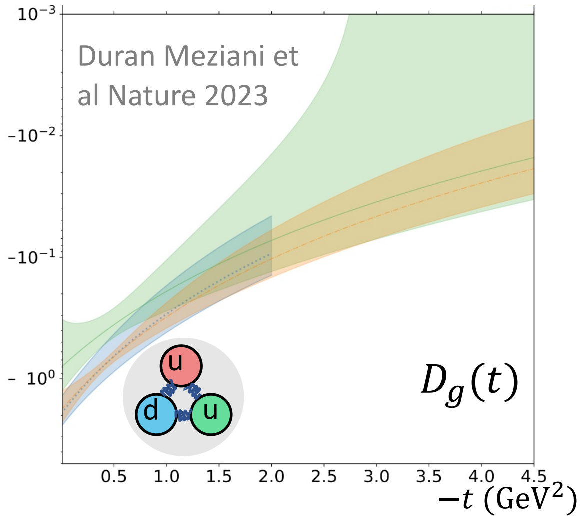

- Continuum Schwinger function methods

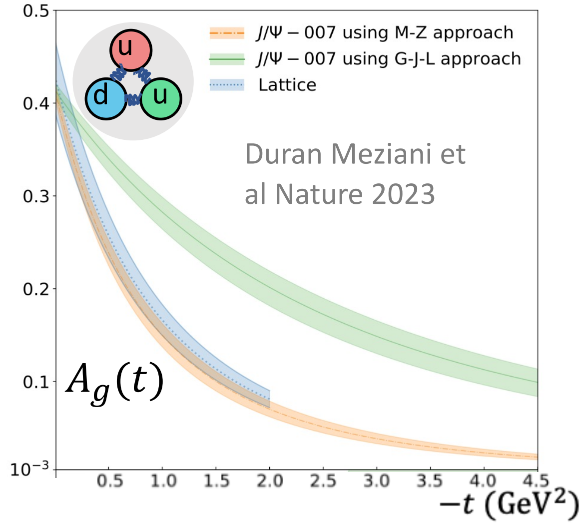



- Indirect experimental access (* Recent suggested direct access [Hatta PRD 2024] )



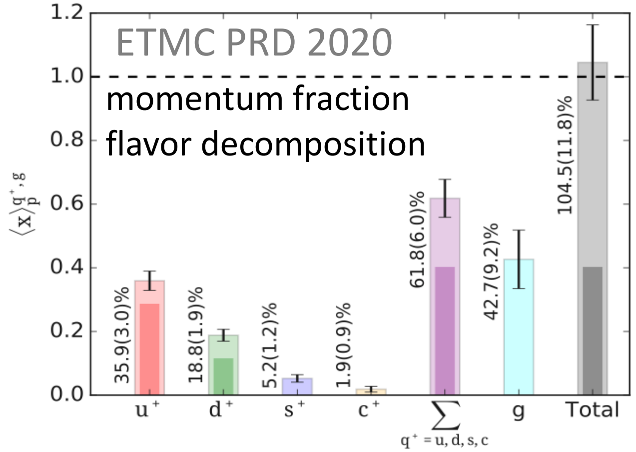



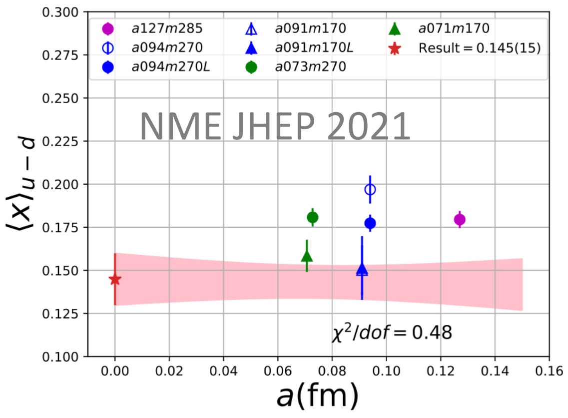



# Constraints on GFFs from lattice QCD: examples

*Older lattice QCD literature: connected quark momentum fraction and generalized form factors*
*More recently: disconnected contributions to momentum and spin fraction, gluon GFFs*

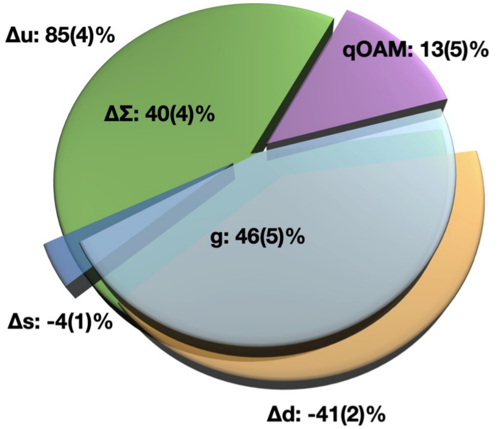
## Proton

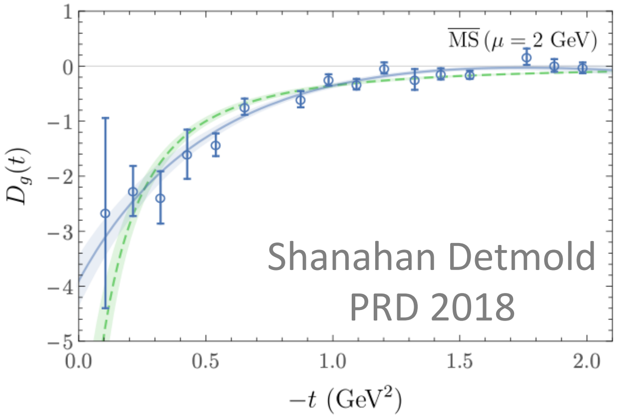


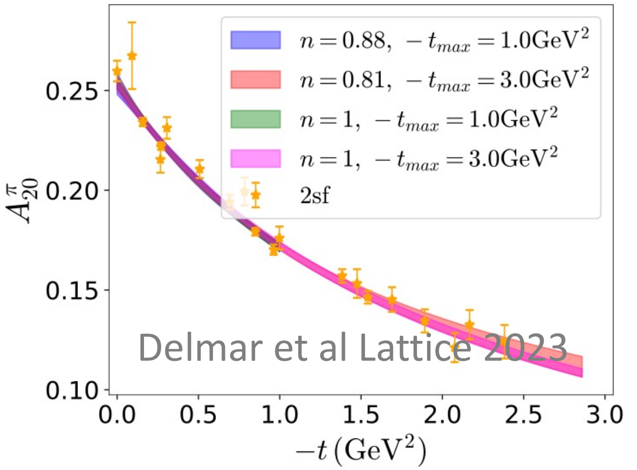

χQCD PRD 2022

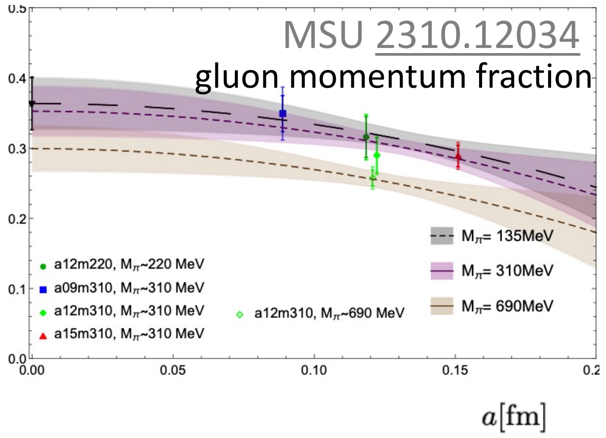
spin decomposition

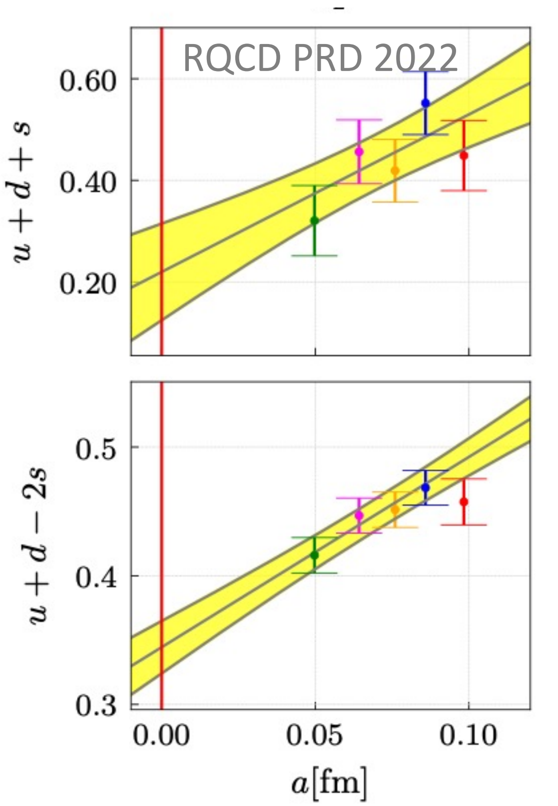



## Pion




quark momentum fraction

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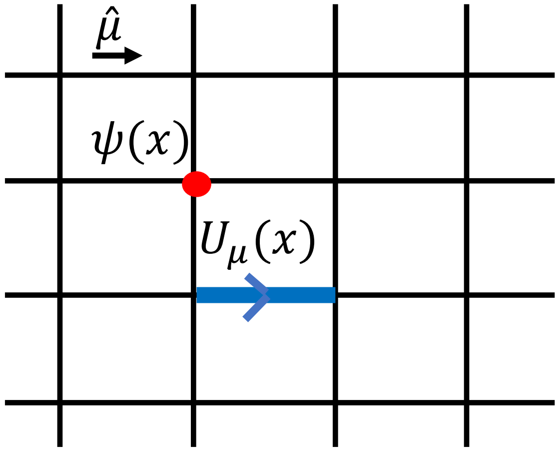
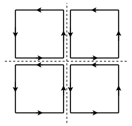
# GFFs from lattice QCD: EMT




$$T^{\mu\nu} = -F_a^{\mu\alpha} F_{a,\alpha}^{\nu} + \frac{1}{4} g^{\mu\nu} F_a^{\alpha\beta} F_{a,\alpha\beta} + \sum_f i\bar{\psi}_f \gamma^{\{\mu} D^{\nu\}} \psi_f$$
$$= \sum_{i\,\in\{q,g\}} T_i^{\mu\nu}$$

- $T_i^{\mu\nu}$ : write in terms of Euclidean lattice fields

  $F_{\mu\nu} \sim$ 

  $$(\vec{D}_\mu \psi)(x) = \frac{1}{2}\Big(U_\mu(x)\psi(x + a\hat{\mu}) - U_\mu^\dagger(x - a\hat{\mu})\psi(x - a\hat{\mu})\Big),$$
  $$(\bar{\psi}\,\overleftarrow{D}_\mu)(x) = \frac{1}{2}\Big(\bar{\psi}(x + a\hat{\mu})U_\mu^\dagger(x) - \psi(x - a\hat{\mu})U_\mu(x - a\hat{\mu})\Big)$$

- $T_i^{\mu\nu}$ : isotropic hypercubic lattice: Lorentz group → $H(4)$

  symmetric traceless components transform under $\tau_1^{(3)}$(diagonal), $\tau_3^{(6)}$(off-diagonal)

  Gockeler et al PRD 1996

- $T_i^{\mu\nu}$ : flavor singlet $q = u + d + s + \cdots$ mixes with $g$

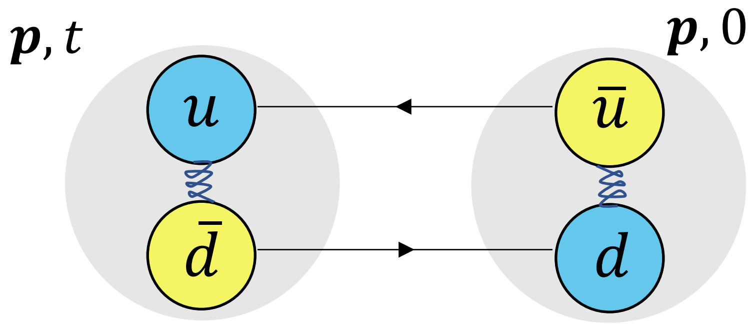
  non-singlet $u - d$, $u + d - 2s$ renormalize multiplicatively

# Lattice simulation

| | $m_\pi$ (MeV) | $a$ (fm) | $L^3\times T$ | $N_f$ |
|---|---|---|---|---|
| Ens. A | 450 | 0.12 | $32^3\times 96$ | $2+1$ |
| Ens. B | 170 | 0.09 | $48^3\times 96$ | $2+1$ |

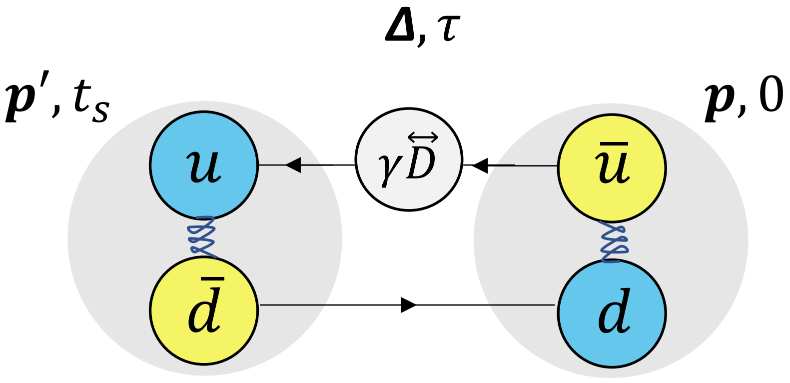
Clover-improved Wilson quarks, Lüscher-Weisz gauge action generated by JLab/LANL/MIT/WM groups

2-point functions ~ $e^{-E_{\boldsymbol{p}}t}, E_{\boldsymbol{p}} = \sqrt{m^2 + |\boldsymbol{p}|^2}$

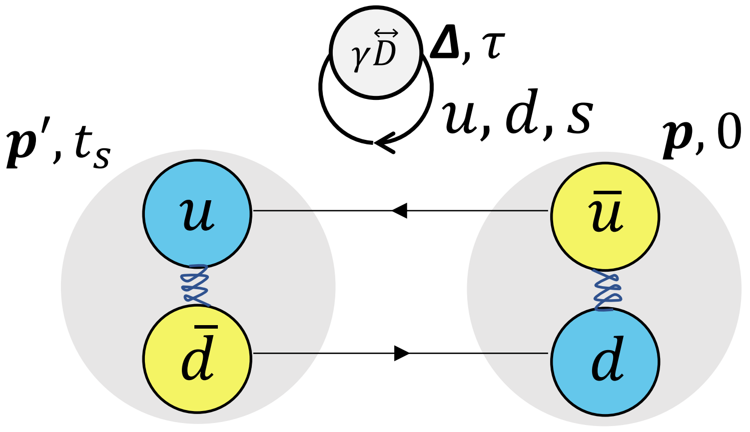



3-point functions ~ Matrix elements $\langle h(p',s')|T^{\mu\nu}_{q,g}|h(p,s)\rangle$

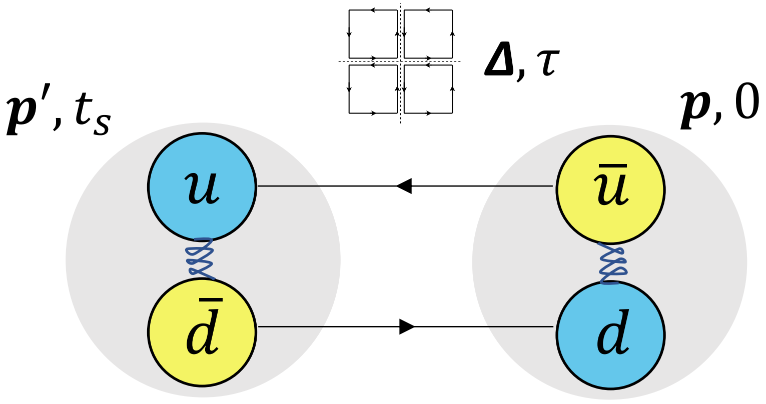
*Connected contribution*




*Disconnected contribution*




*Gluon contribution*

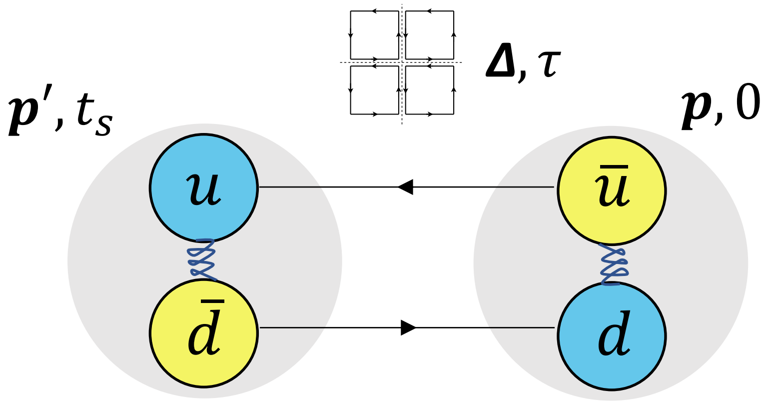


# Gluon GFFs of the pion, rho meson, proton, and delta baryon

**DAP**, Hackett, Shanahan PRD (2022)

| | $m_\pi$ (MeV) | $a$ (fm) | $L^3\times T$ | $N_f$ |
|---|---|---|---|---|
| Ens. A | 450 | 0.12 | $32^3\times 96$ | $2+1$ |

Clover-improved Wilson quarks, Lüscher-Weisz gauge action generated by JLab/LANL/MIT/WM groups




→ 2820 configurations

→ $\frac{t_{\text{flow}}}{a^2} = 1$

→ 235 sources

→ $|\boldsymbol{\Delta}|^2 \leq 18(\frac{2\pi}{L})^2$

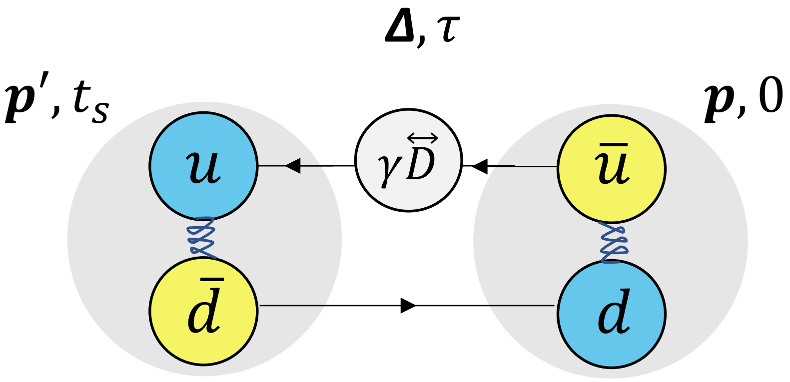
→ $|\boldsymbol{p}'|^2 \leq 10(\frac{2\pi}{L})^2$

# Quark and gluon GFFs

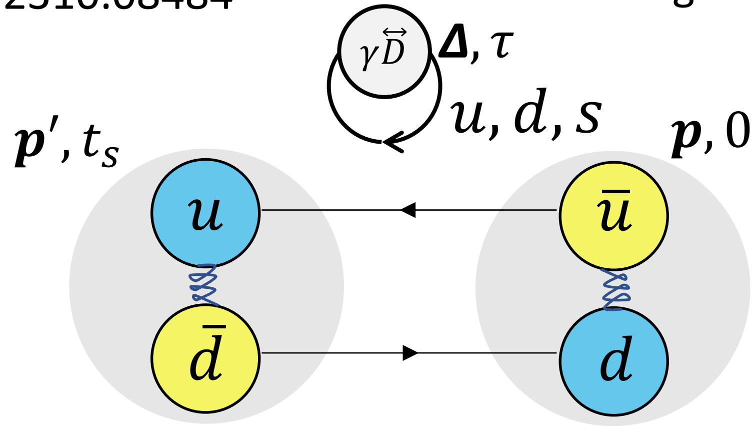
| | $m_\pi$ (MeV) | $a$ (fm) | $L^3 \times T$ | $N_f$ |
|---|---|---|---|---|
| Ens. B | 170 | 0.09 | $48^3 \times 96$ | $2+1$ |

Pion: Hackett, Oare, **DAP**, Shanahan PRD (2023)
Proton: Hacket, **DAP**, Shanahan 2310.08484

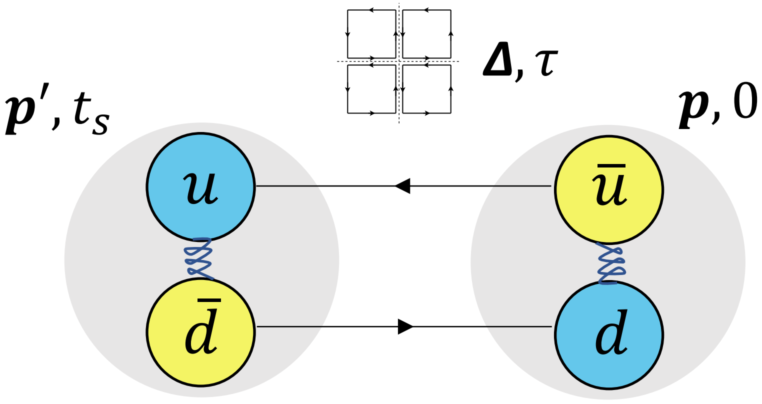
Clover-improved Wilson quarks, Lüscher-Weisz gauge action generated by JLab/LANL/MIT/WM groups




*Connected contribution*
- → 1381 configurations
- → sequential sources
- → $t_s \in \{6-18\}$
- → $|\boldsymbol{\Delta}|^2 \leq 25(\frac{2\pi}{L})^2$
- → $\boldsymbol{p}' \in \{(1,-1,0), (-2,-1,0), (-1,-1,-1)\} 2\pi/L$




*Disconnected contribution*
- → 1381 configurations
- → $Z_4$ noise, hierarchical probing, 512 Hadamard vectors
- → 1024 sources
- → $|\boldsymbol{\Delta}|^2 \leq 25(\frac{2\pi}{L})^2$
- → $|\boldsymbol{p}'|^2 \leq 10(\frac{2\pi}{L})^2$




*Gluon contribution*
- → 2511 configurations
- → $\frac{t_{\text{flow}}}{a^2} = 2$
- → 1024 sources
- → $|\boldsymbol{\Delta}|^2 \leq 25(\frac{2\pi}{L})^2$
- → $|\boldsymbol{p}'|^2 \leq 10(\frac{2\pi}{L})^2$

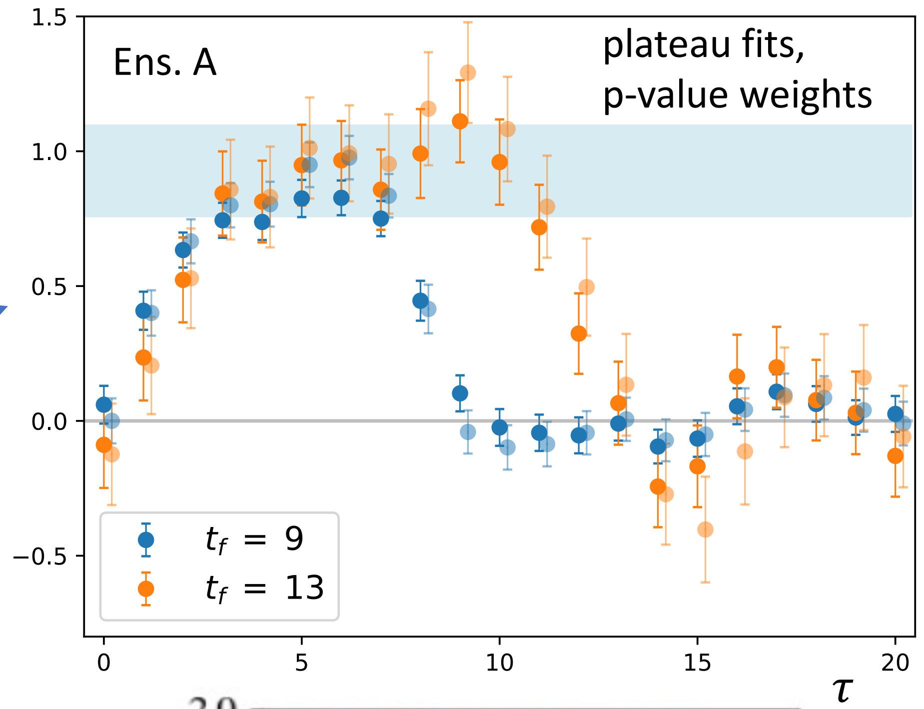
# Matrix elements → bare GFFs

- From 2- and 3-point functions, extract $\langle h(\boldsymbol{p},s)|T_i^{\mu\nu}|h(\boldsymbol{p}',s')\rangle$ for several kinematic combinations $\underline{\boldsymbol{p}', \boldsymbol{\Delta}, s, s', \mu, \nu}$

$$R_{\mu\nu}(\boldsymbol{p}', t_s, \boldsymbol{\Delta}, \tau) = \frac{C^{3pt}_{\mu\nu}(\boldsymbol{p}', t_s, \boldsymbol{\Delta}, \tau)}{C^{2pt}(\boldsymbol{p}', t_s)} \sqrt{\frac{C^{2pt}(\boldsymbol{p}, t_s-\tau)C^{2pt}(\boldsymbol{p}', t_s)C^{2pt}(\boldsymbol{p}', \tau)}{C^{2pt}(\boldsymbol{p}', t_s-\tau)C^{2pt}(\boldsymbol{p}, t_s)C^{2pt}(\boldsymbol{p}, \tau)}}$$

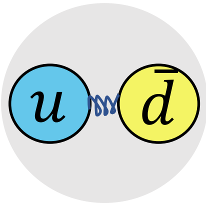
Model average over Euclidean time ranges

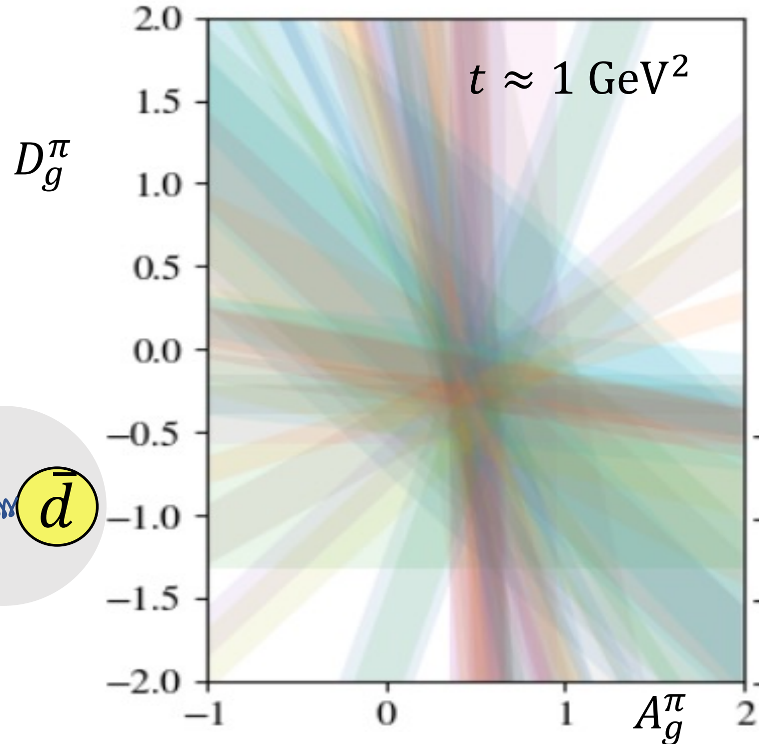
Jay Neil PRD 2021
Rinaldi et al PRL 2019
NPLQCD PRL 2015




- $\langle h(\boldsymbol{p},s)|T_i^{\mu\nu}|h(\boldsymbol{p}',s')\rangle \sim$ Coefficients × GFFs ($t = \Delta^2$)
  Partition into momentum bins with equal or similar values of $t$, solve over-constrained linear systems
  → bare GFFs at discrete values of $t$

  Connected contribution: sequential-source through the sink → limited $\boldsymbol{p}'$
  choose such that GFFs can be resolved

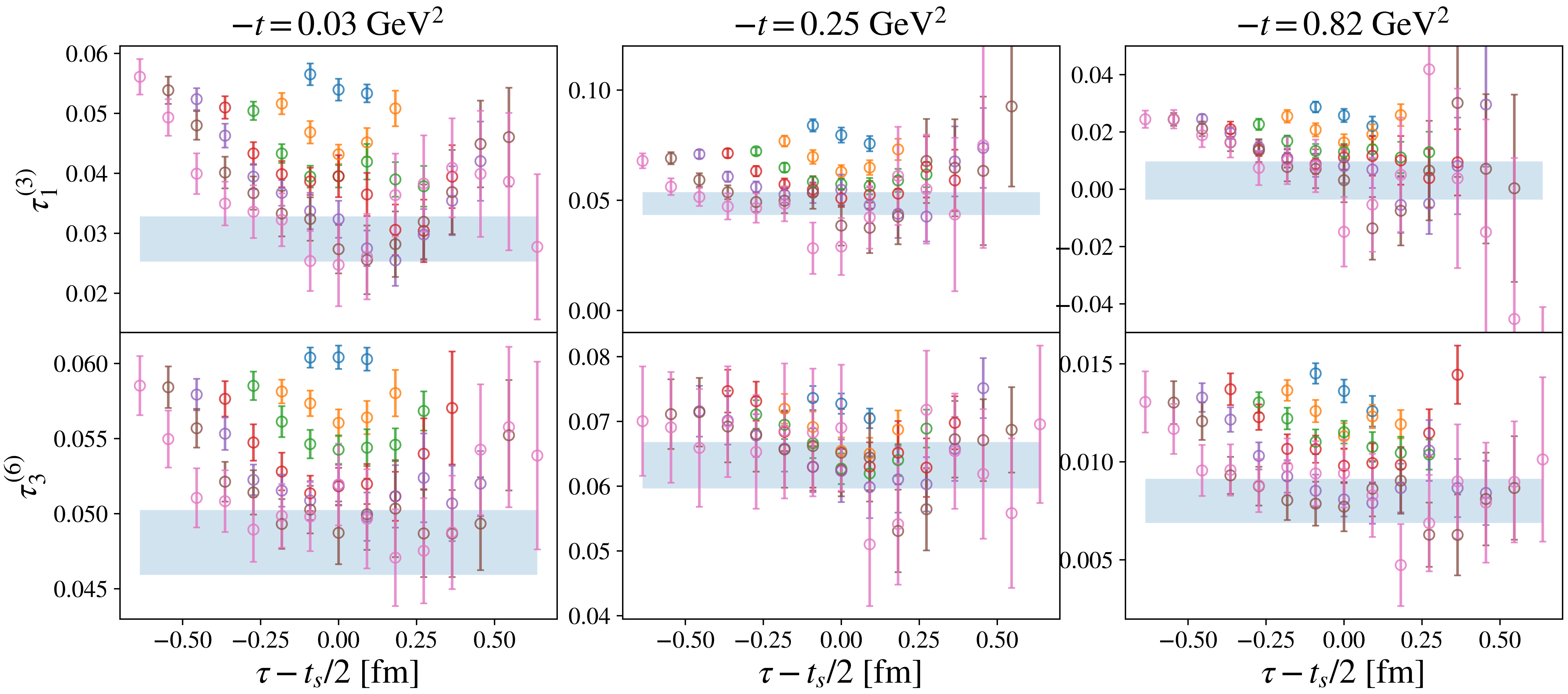


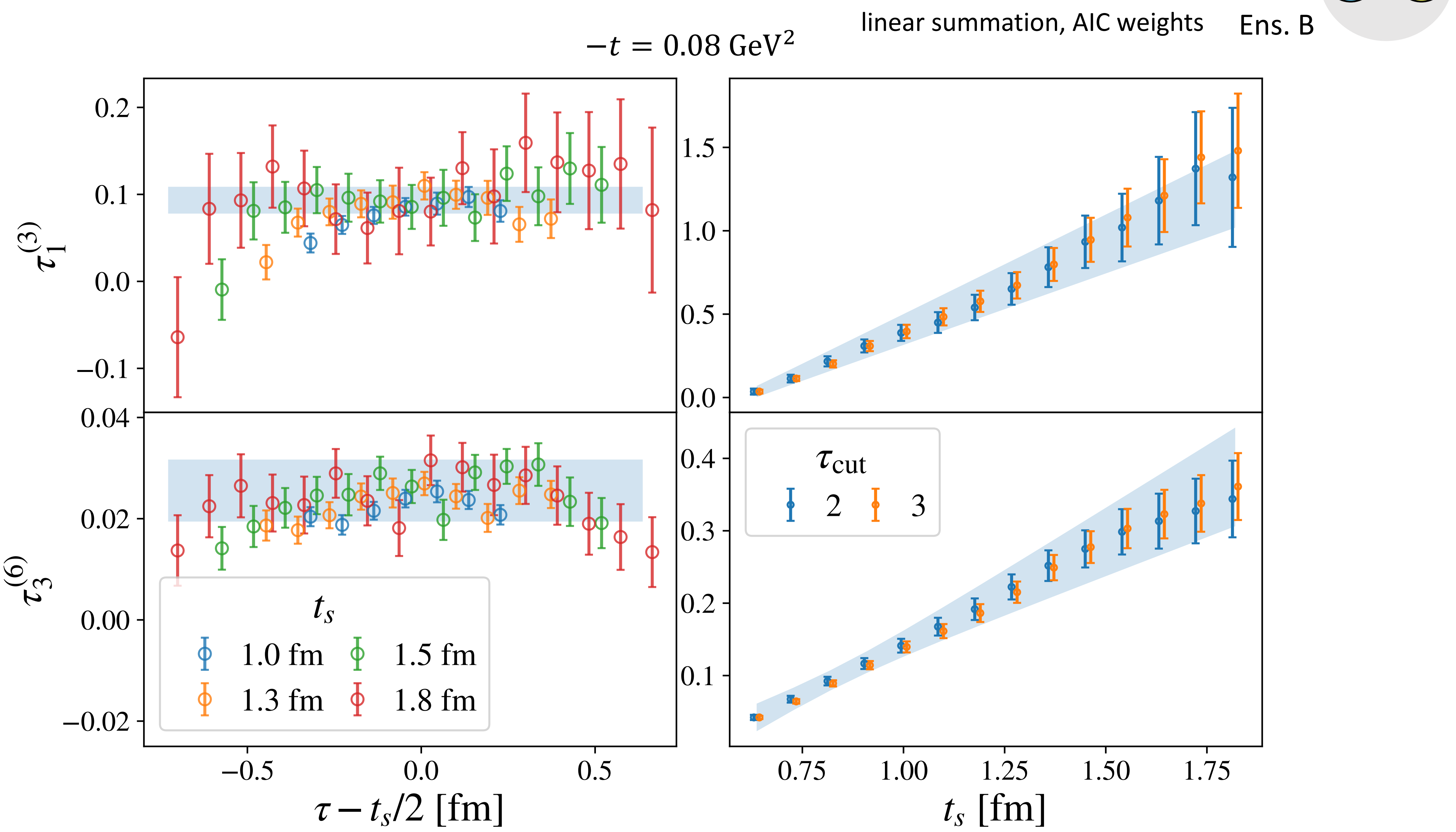
$\tau_1^{(3)}$: diagonal elements irrep
$\tau_3^{(6)}$: off-diagonal elements irrep

# Pion connected quark contribution

linear summation, summation + exponential, AIC weights  Ens. B

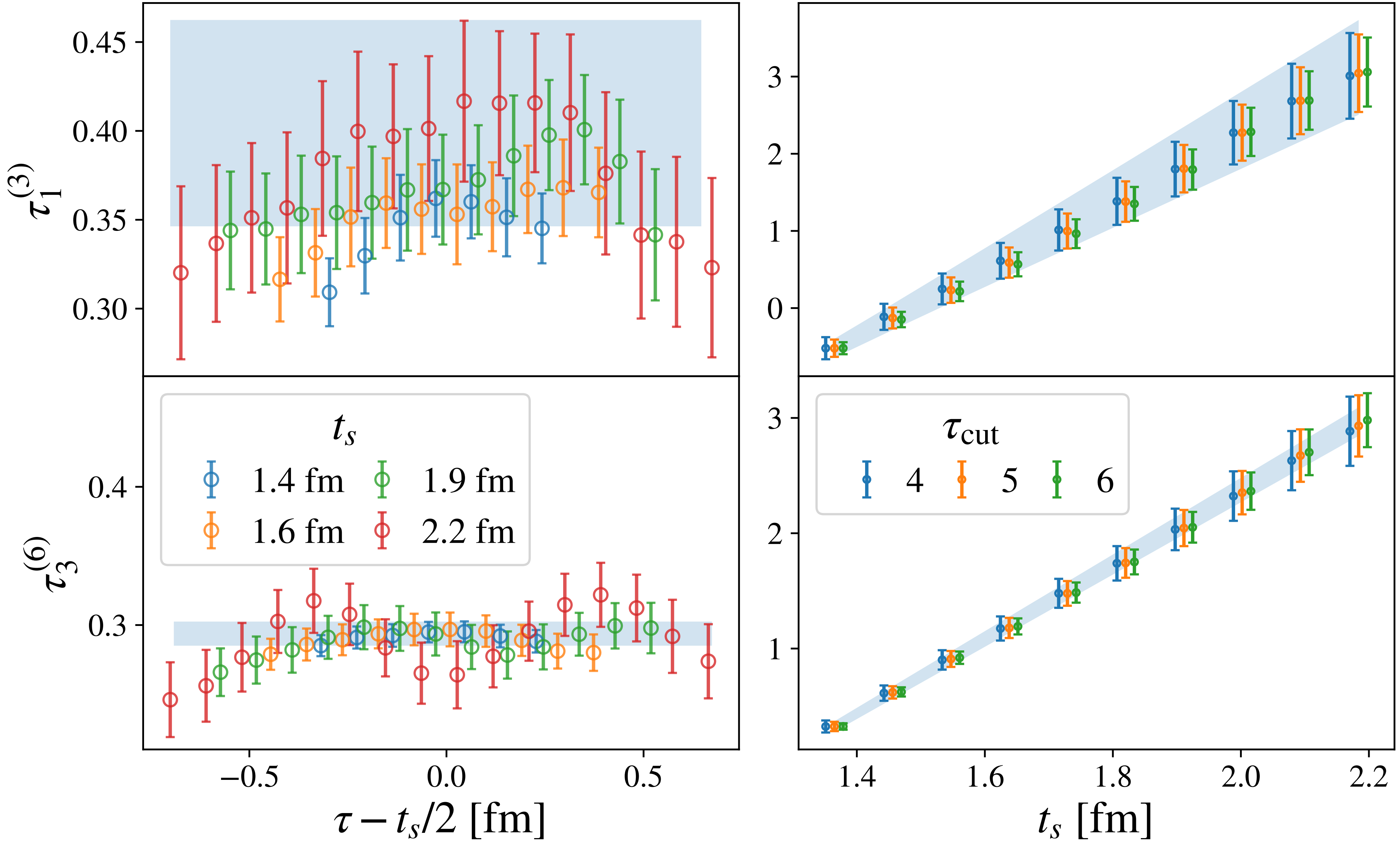


# Pion disconnected quark contribution

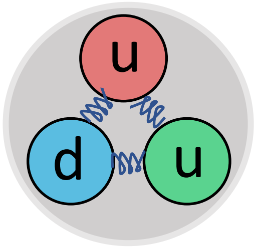


# Pion gluon contribution

$-t = 0.13\ \text{GeV}^2$ linear summation, AIC weights Ens. B



# Proton




Ens. B $\tau_1^{(3)}$

linear summation,
AIC weights

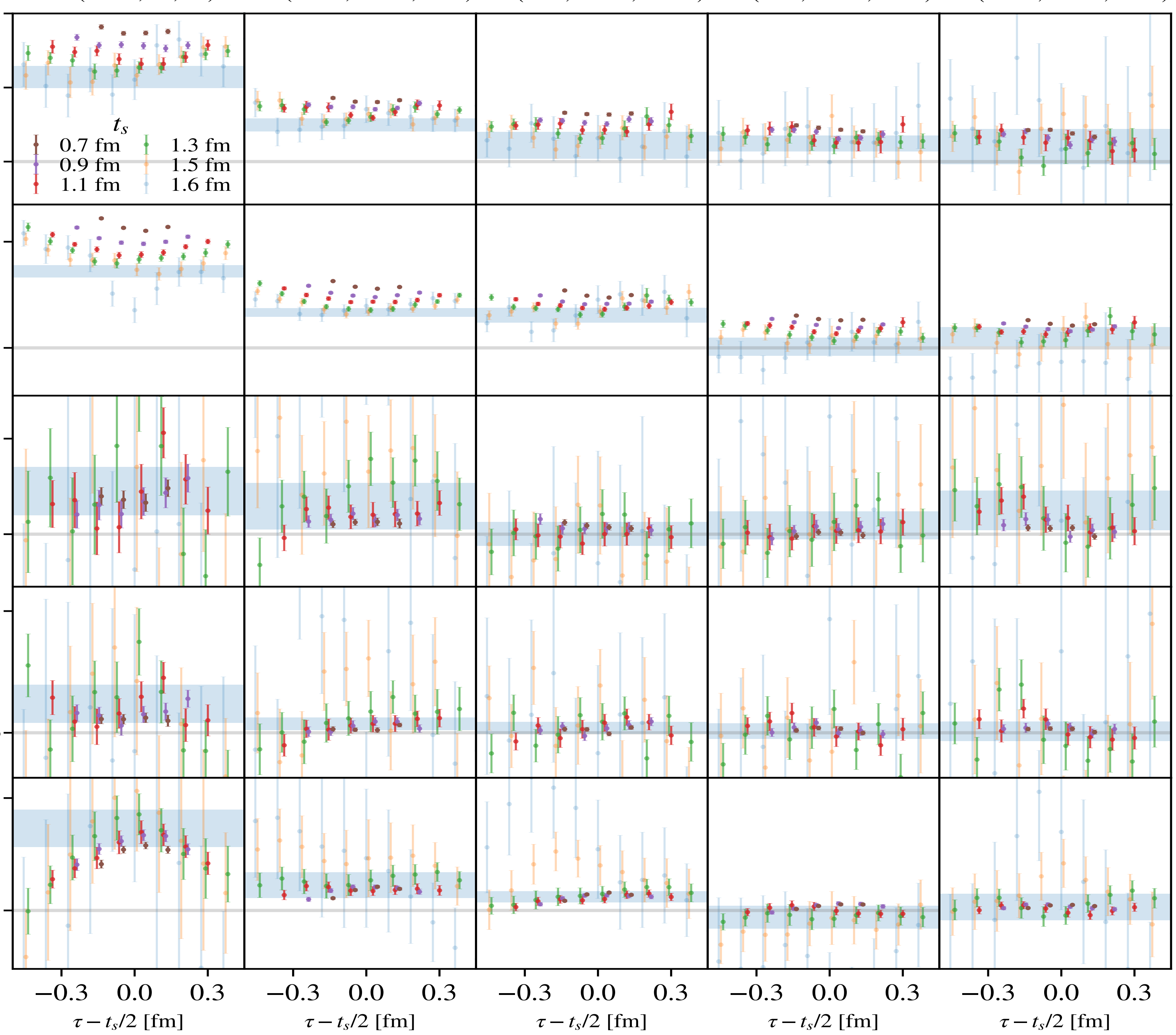
$u - d$

$u + d - 2s$ conn.

$u + d + s$ disco.

$u + d - 2s$ disco.

$g$



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# Renormalization

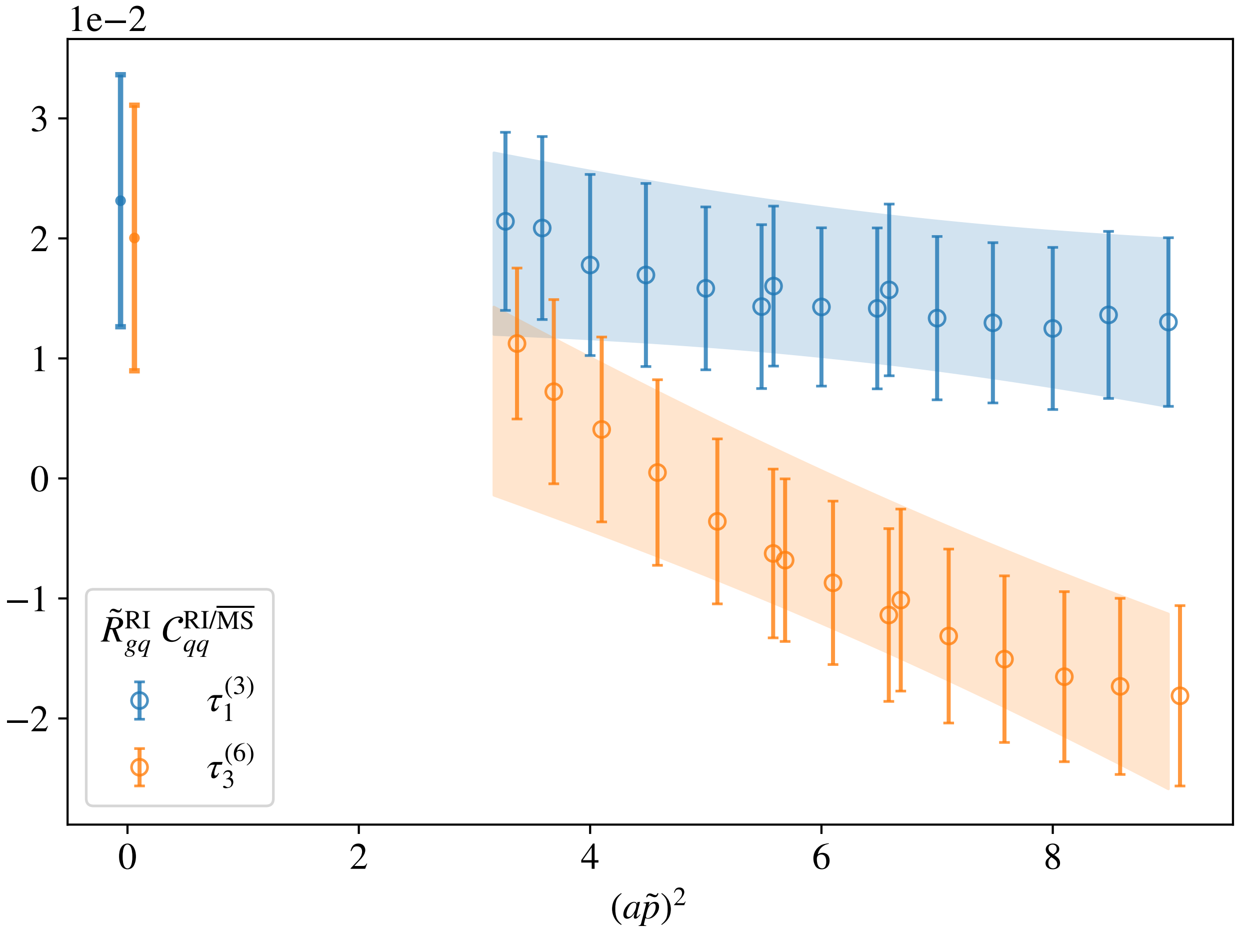
$\mathcal{R} \in \{\tau_1^{(3)}, \tau_3^{(6)}\}$

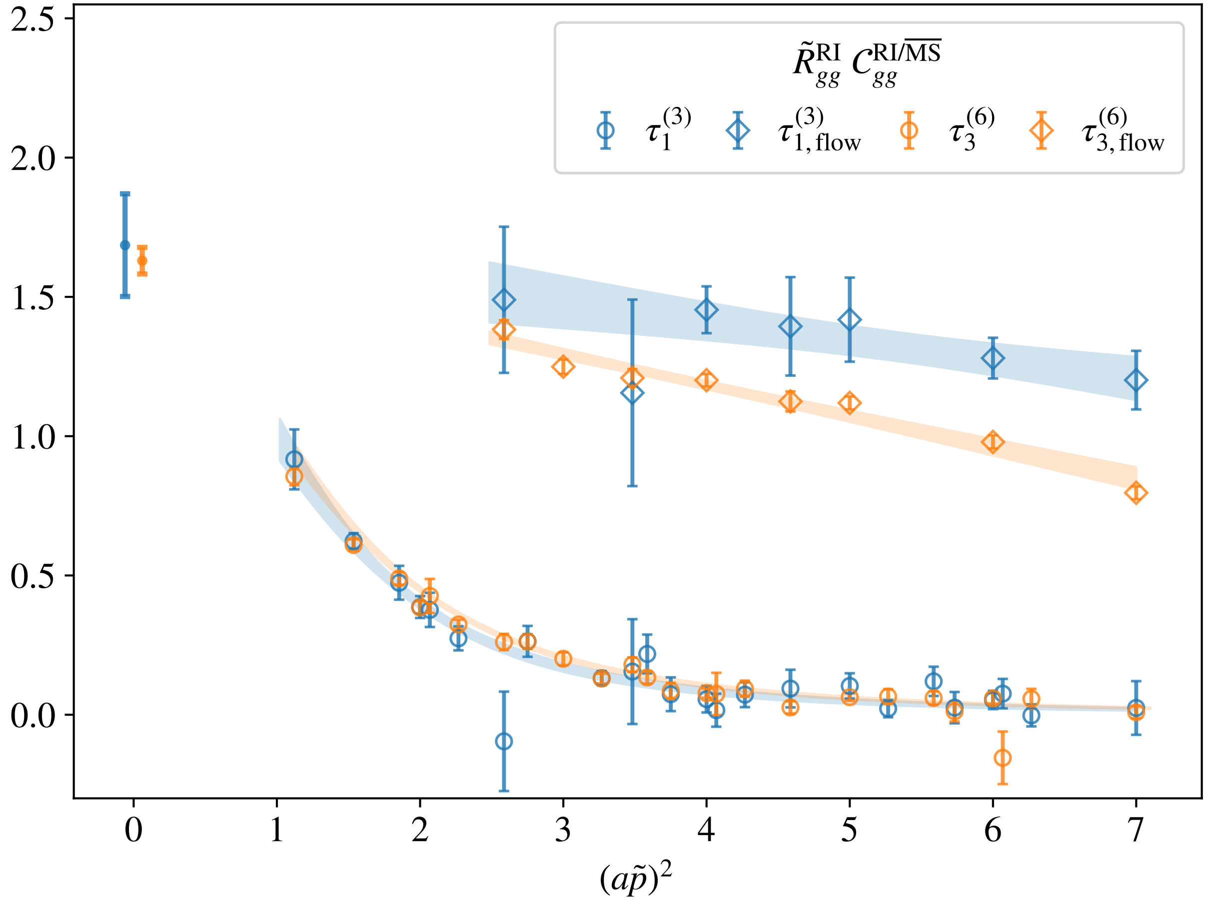
| $m_\pi$ (MeV) | $a$ (fm) | $L^3 \times T$ | $N_f$ |
|---|---|---|---|
| 450 | 0.12 | $12^3 \times 24$ | $2+1$ |

- $\begin{pmatrix} T_q^{\overline{\mathrm{MS}}} \\ T_g^{\overline{\mathrm{MS}}} \end{pmatrix} = \begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{qg\mathcal{R}}^{\overline{\mathrm{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{gg\mathcal{R}}^{\overline{\mathrm{MS}}} \end{pmatrix} \begin{pmatrix} T_{q\mathcal{R}}^{\mathrm{bare}} \\ T_{g\mathcal{R}}^{\mathrm{bare}} \end{pmatrix}$ : quark isosinglet and gluon mix under renormalization
- $T_v^{\overline{\mathrm{MS}}} = Z_{v\mathcal{R}}^{\overline{\mathrm{MS}}} T_{v\mathcal{R}}^{\mathrm{bare}}$ , $T_v = T_u + T_d - 2T_s$ : non-singlet does not mix in the chiral limit
- Compute non-perturbatively via the RI-MOM scheme, convert to $\overline{\mathrm{MS}}$ scheme at $\mu = 2$ GeV using two-loop matching coefficients (Panagopoulos et al PRD 2021)
- For regular volume ensembles, gluon and disconnected have intractable noise
  → Use smaller volume ensemble to get renormalization factors (different spacing)

$$\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{qg\mathcal{R}}^{\overline{\mathrm{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\mathrm{MS}}} & Z_{gg\mathcal{R}}^{\overline{\mathrm{MS}}} \end{pmatrix}^{-1} (\mu^2) = \begin{pmatrix} R_{qq\mathcal{R}}^{\mathrm{RI}} & R_{qg\mathcal{R}}^{\mathrm{RI}} \\ R_{gq\mathcal{R}}^{\mathrm{RI}} & R_{gg\mathcal{R}}^{\mathrm{RI}} \end{pmatrix} (\mu_R^2) \times \begin{pmatrix} \mathcal{C}_{qq}^{\mathrm{RI}/\overline{\mathrm{MS}}} & \mathcal{C}_{qg}^{\mathrm{RI}/\overline{\mathrm{MS}}} \\ \mathcal{C}_{gq}^{\mathrm{RI}/\overline{\mathrm{MS}}} & \mathcal{C}_{gg}^{\mathrm{RI}/\overline{\mathrm{MS}}} \end{pmatrix} (\mu^2, \mu_R^2)$$

# Extraction of renormalization coefficients

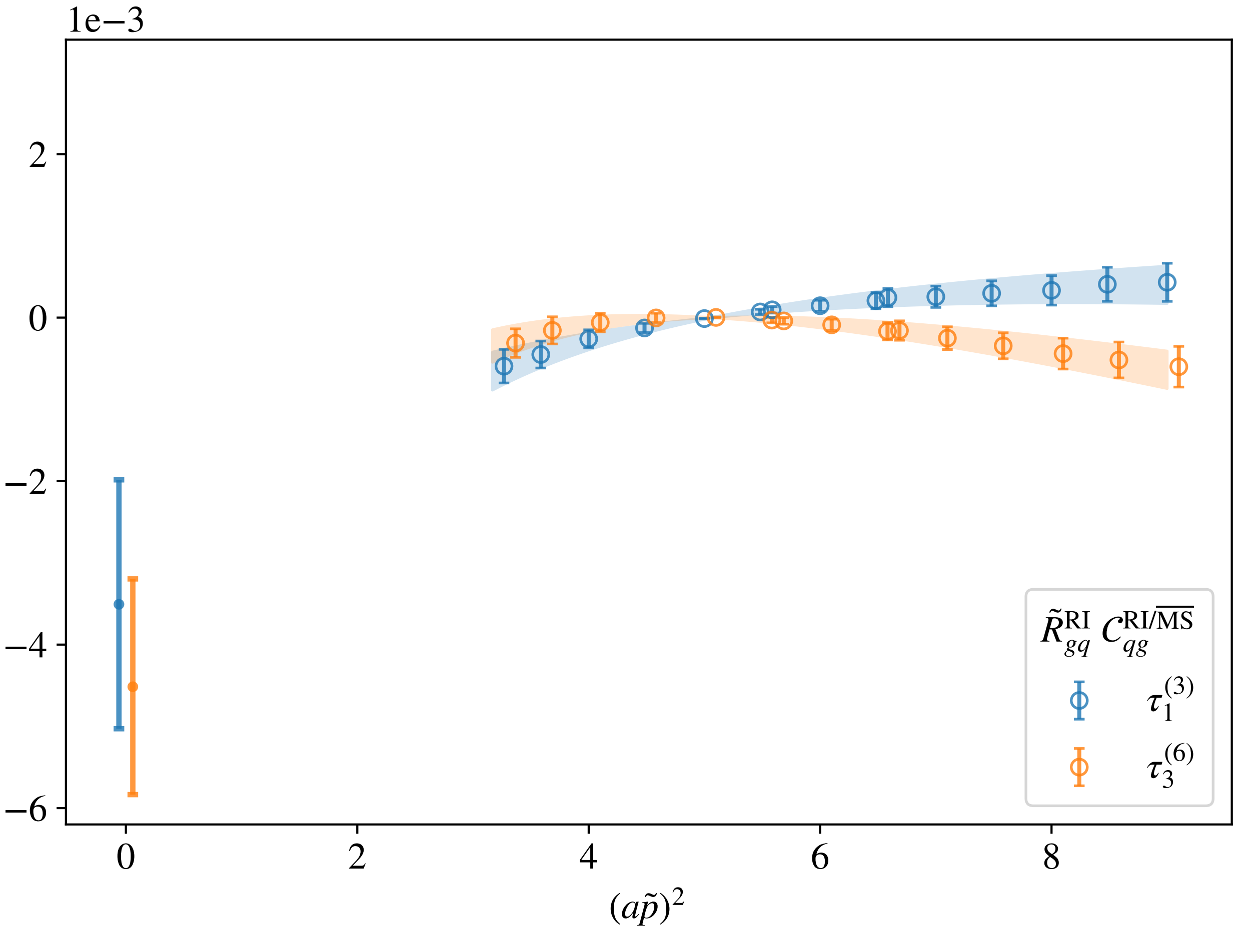


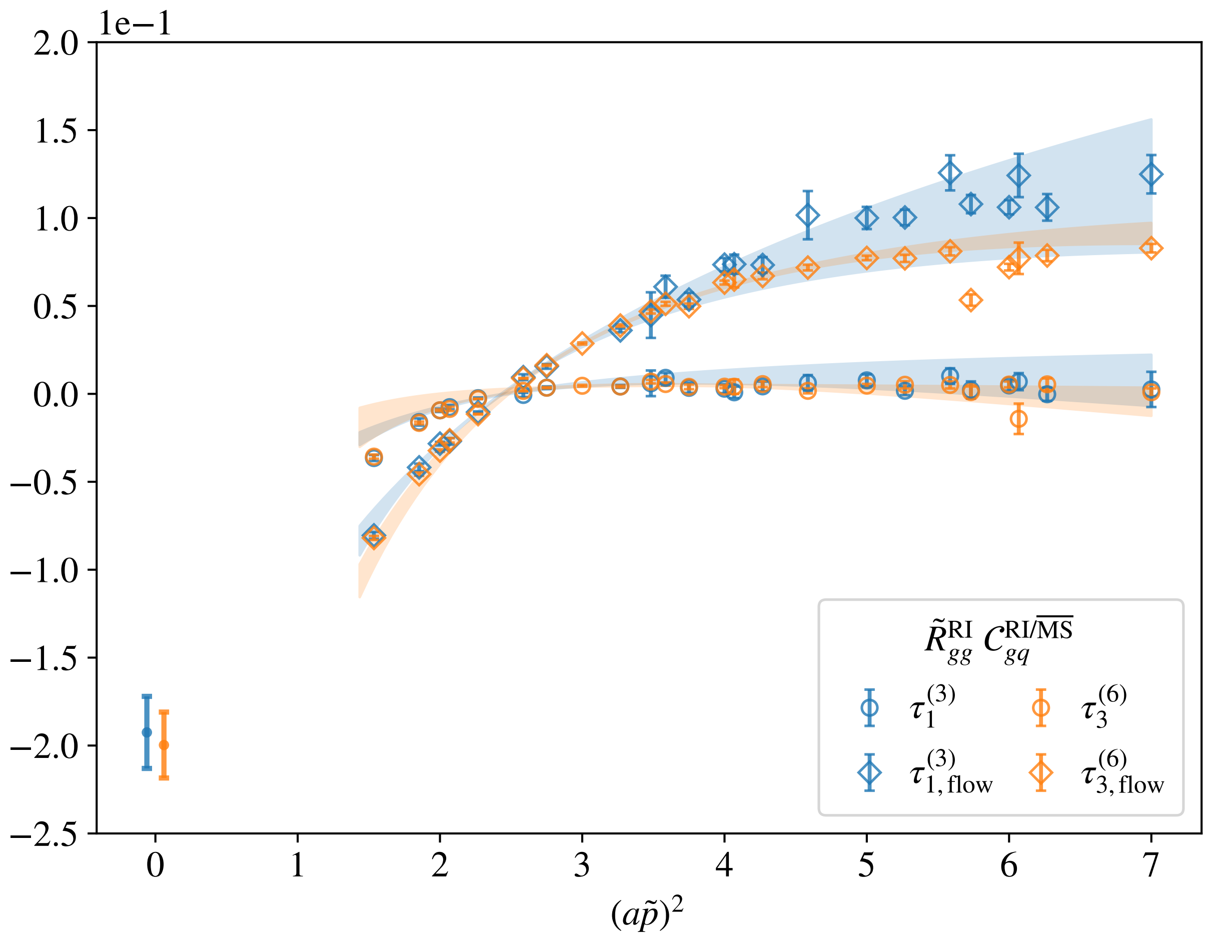




Fit $(a\tilde{p})$ dependence due to discretization artifacts, non-perturbative effects, etc.

(inverse) polynomial

# Extraction of renormalization coefficients




Fit $(a\tilde{p})$ dependence due to discretization artifacts, non-perturbative effects, etc.

logarithmic

# Finally: obtain renormalized GFFs

We have: 1) bare matrix elements $\langle h | T_i^{\mu\nu} | h \rangle, i \epsilon \{g, q, v\}$ grouped in t-bins for each irrep $\mathcal{R}$

2) mixing matrix renormalization $\begin{pmatrix} Z_{qq\mathcal{R}}^{\overline{\text{MS}}} & Z_{qg\mathcal{R}}^{\overline{\text{MS}}} \\ Z_{gq\mathcal{R}}^{\overline{\text{MS}}} & Z_{gg\mathcal{R}}^{\overline{\text{MS}}} \end{pmatrix}^{-1}$, non-singlet ${Z_{v\mathcal{R}}^{\overline{\text{MS}}}}^{-1}$ for each $\mathcal{R}$

→ recast into a simultaneous combined-irrep system of equations, solve by linear regression

Beware of d’Agostini bias!

D’Agostini Phys.Res.Sect.A 1994

Fit with 1) multipole : $F_n = \frac{\alpha}{(1+\frac{t}{\Lambda^2})^n}$,

2) z-expansion : $F = \sum_k \alpha_k [z(t)]^k$ (less restrictive)

# Contents of this talk

- Introduction
- Bare gravitational form factors (GFFs) from lattice QCD
- Non-perturbative renormalization
- GFFs of the proton, pion, and other hadrons: selected results

[Hackett Oare **DAP** Shanahan PRD (2023) arXiv:2307.11707]
[Hackett **DAP** Shanahan PRL (2024) arXiv:2310.08484]
[**DAP** Hackett Shanahan PRD (2022) arXiv:2107.10368]

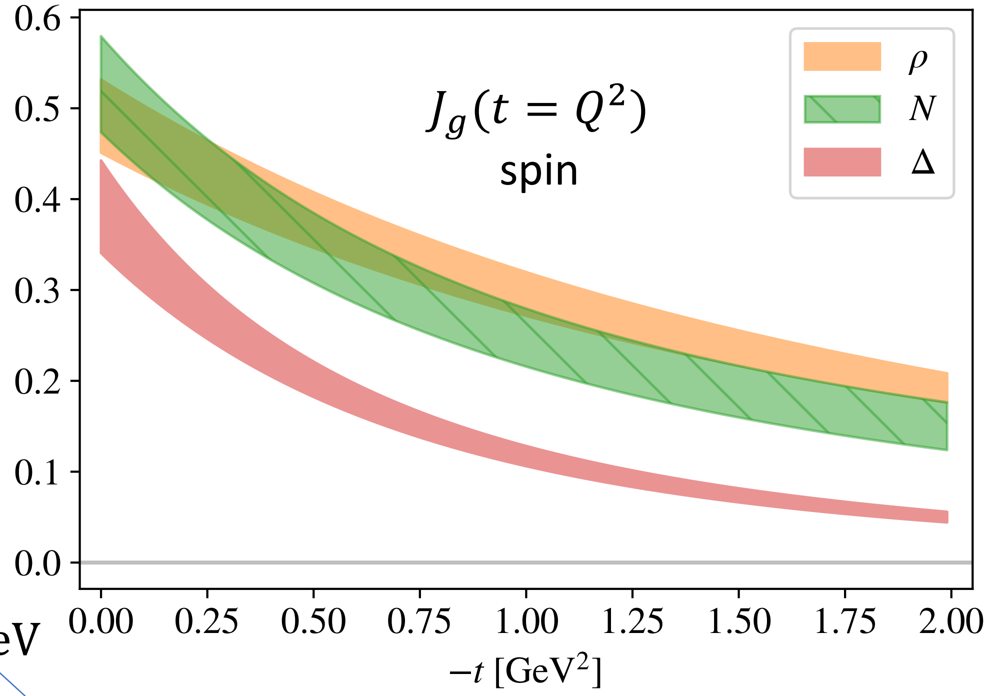
# Gluon gravitational structure hadrons of different spin

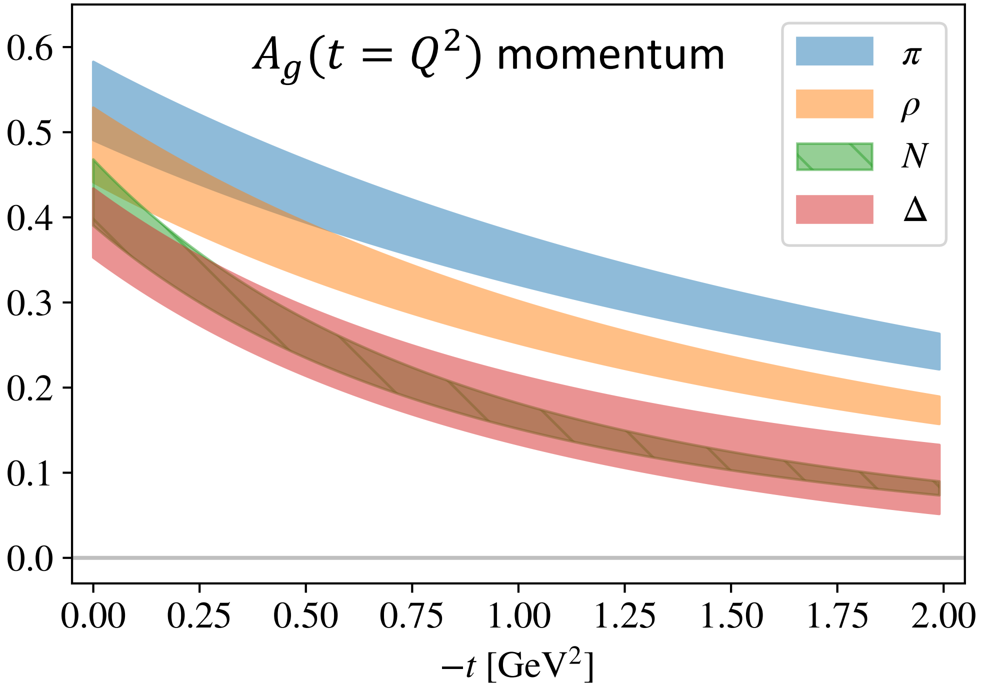
($m_\pi \approx 450$ MeV, mixing neglected)

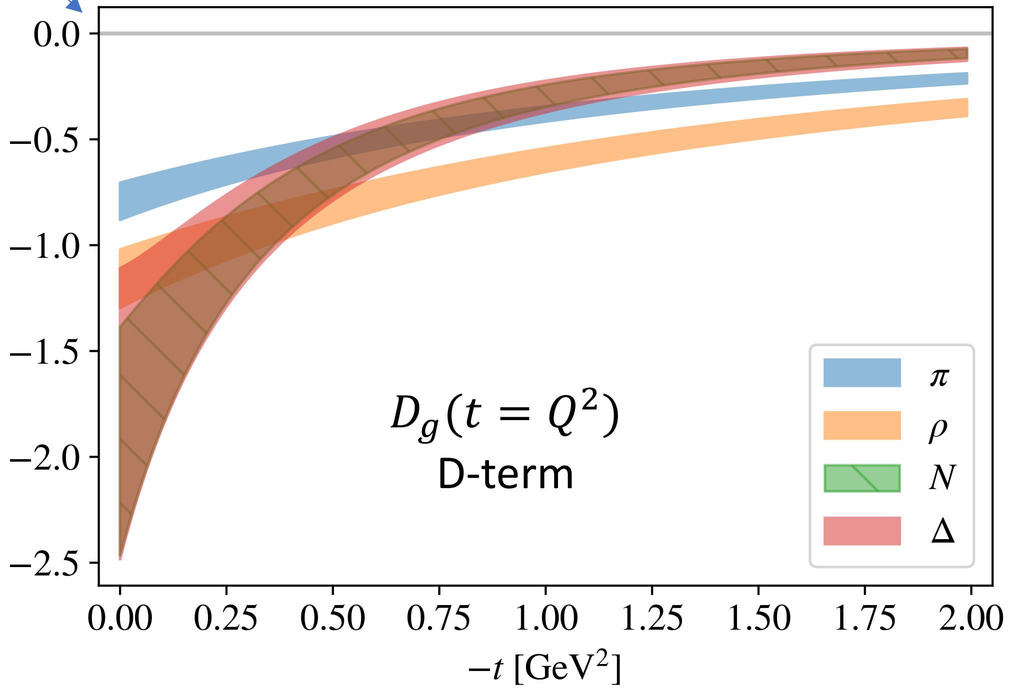
**DAP**, Hackett, Shanahan PRD (2022)

| Hadron | $\pi$ | $\rho$ | $N$ | $\Delta$ |
|---|---|---|---|---|
| Spin | 0 | 1 | 1/2 | 3/2 |
| GFF # | 2 | 7 | 3 | 8 |

$\overline{\text{MS}}, \mu = 2$ GeV



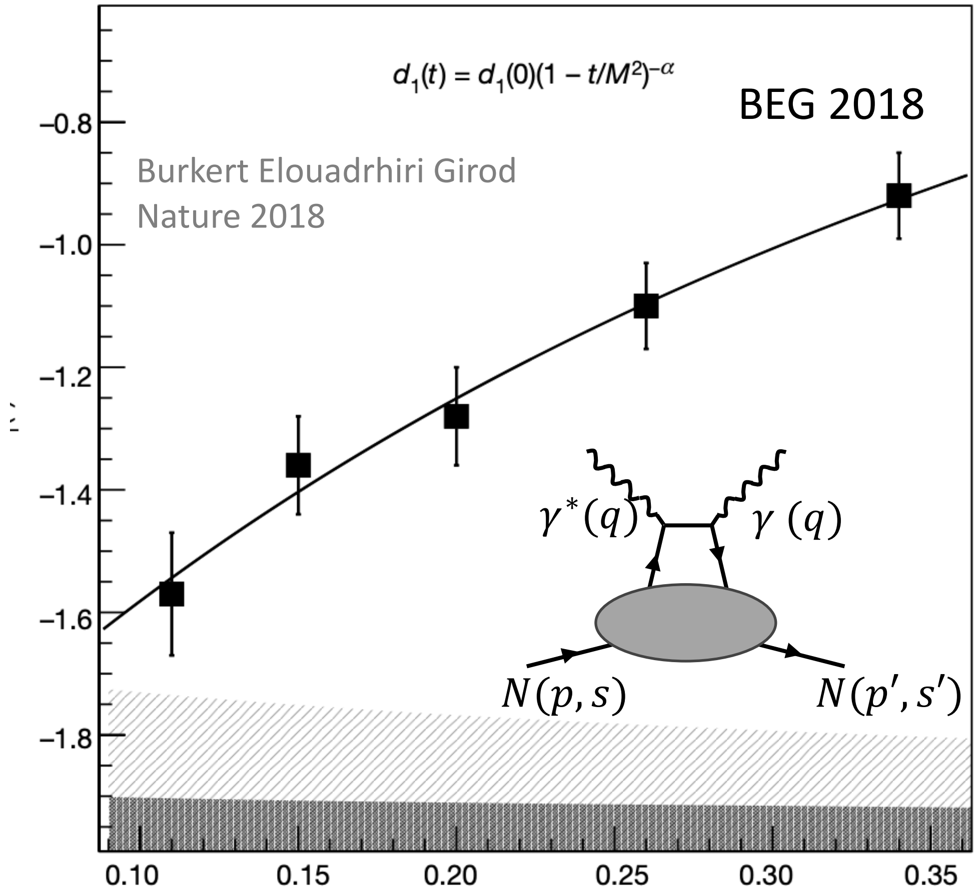



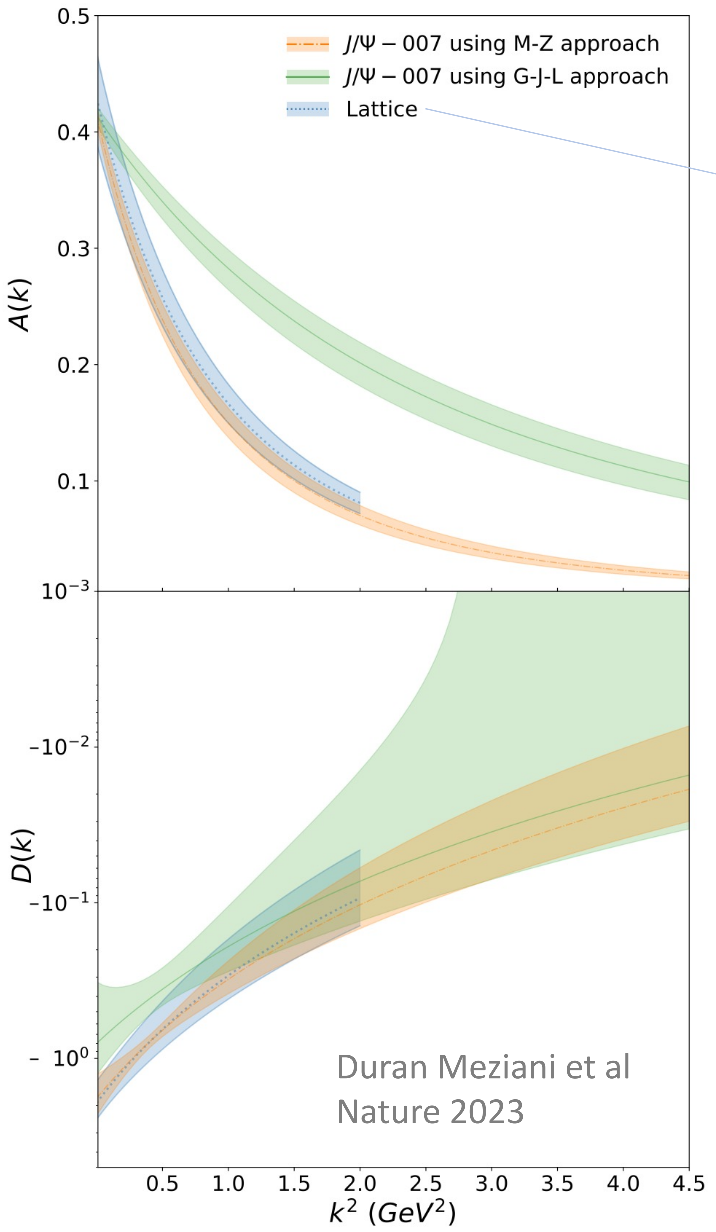
# Proton: first experimental results

model dependent

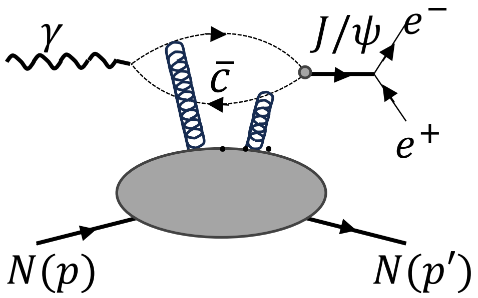
Quark $D_{u+d}^N$ from DVCS




Gluon $A_g^N$ and $D_g^N$ from $J/\psi$ photoproduction




Lattice: **DAP** Hackett Shanahan *PRD* (2022)
heavier pion mass + neglecting mixing with quark





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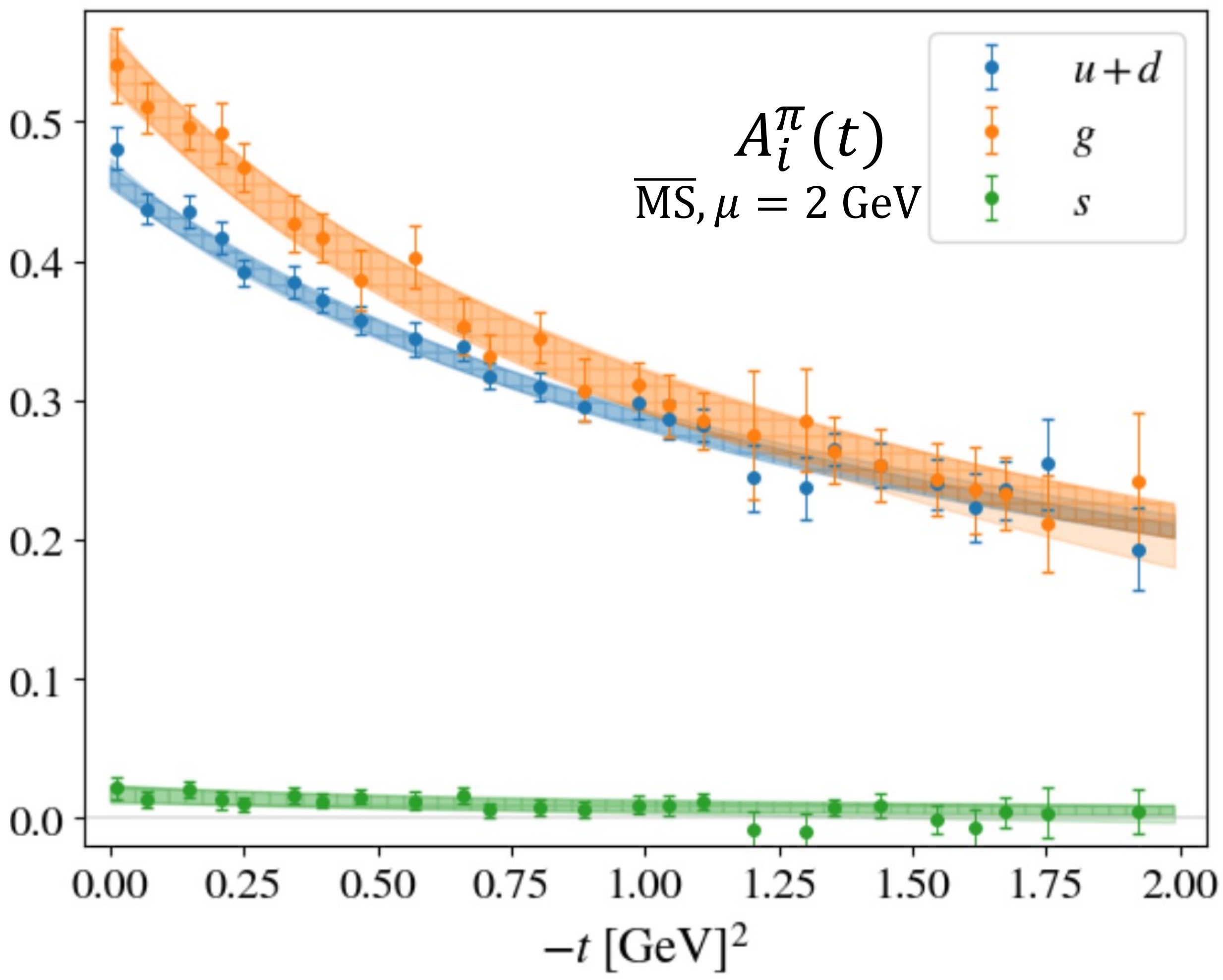
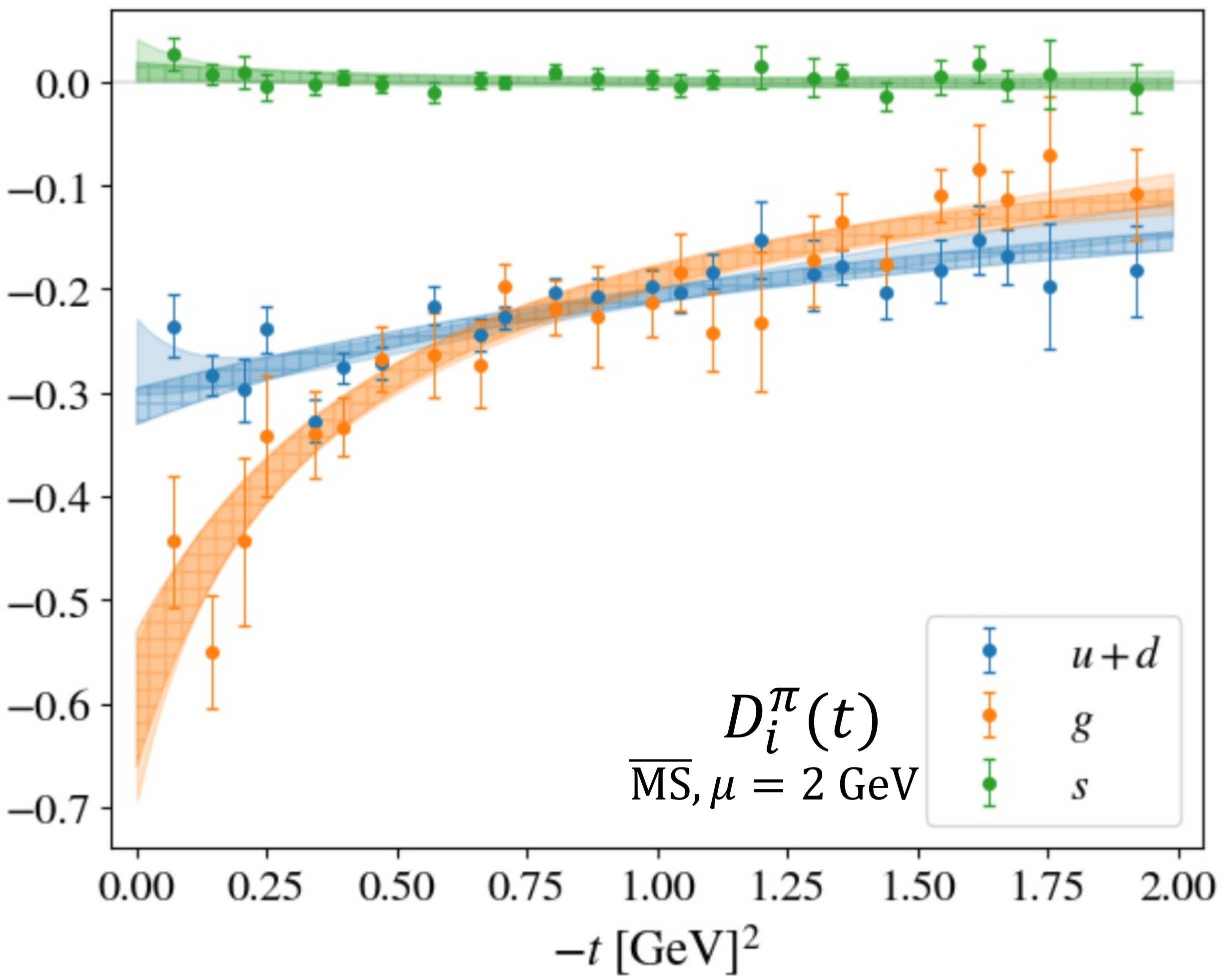
# Quark and gluon GFFs of the pion

($m_\pi \approx 170$ MeV, including mixing)
Hackett Oare **DAP** Shanahan PRD 2023



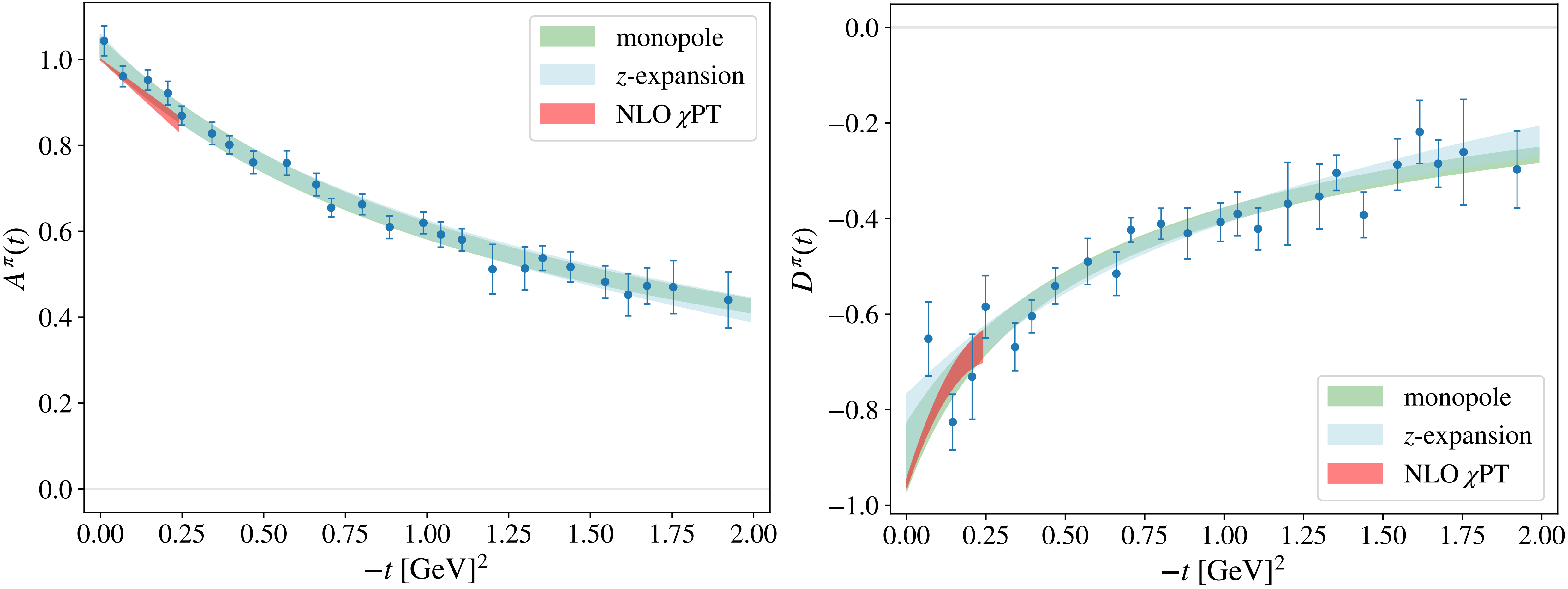




hatched bands : monopole, opaque bands : z-expansion with $k_{\text{max}} = 2$

# Pion : total GFFs

($m_\pi \approx 170$ MeV, including mixing)

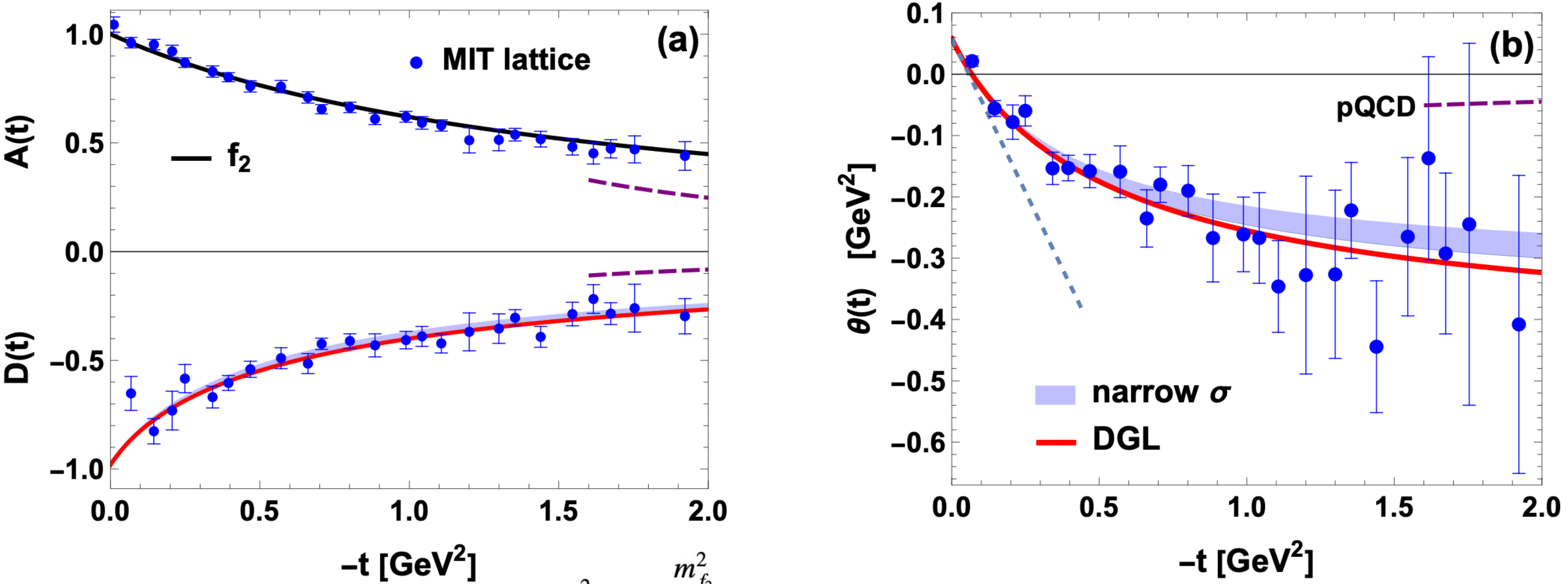
Hackett Oare **DAP** Shanahan PRD 2023




Red band spread due to different estimates for low energy constants [Donoghue Leutwyler Z.Phys.C 1991]

# Pion data in support of meson dominance principle

Broniowski Arriola arXiv:2405.07815



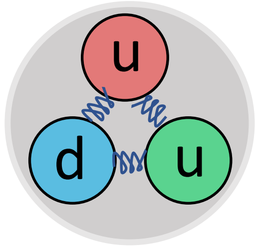

$$A(-Q^2) = \frac{m_{f_2}^2}{m_{f_2}^2 + Q^2},$$

$$\Theta(-Q^2) = 2m_\pi^2 - \frac{m_\sigma^2 Q^2}{m_\sigma^2 + Q^2},$$

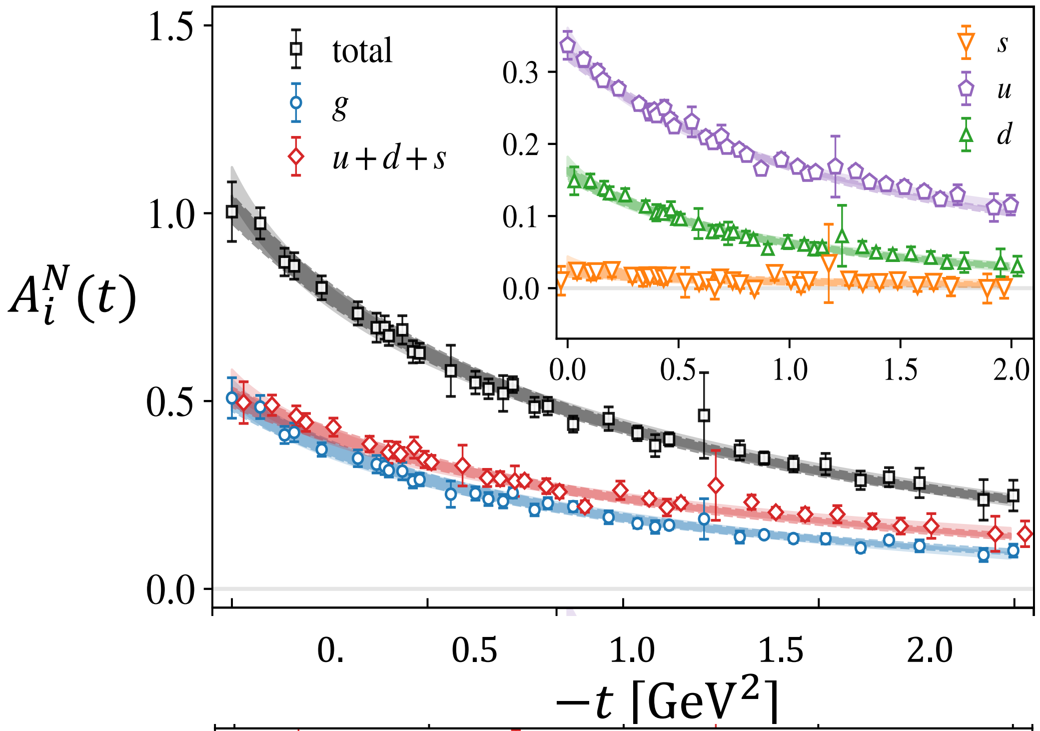
$$D = -\frac{2}{3t}\left[\Theta - \left(2m_\pi^2 - \tfrac{1}{2}\,t\right)A\right]$$

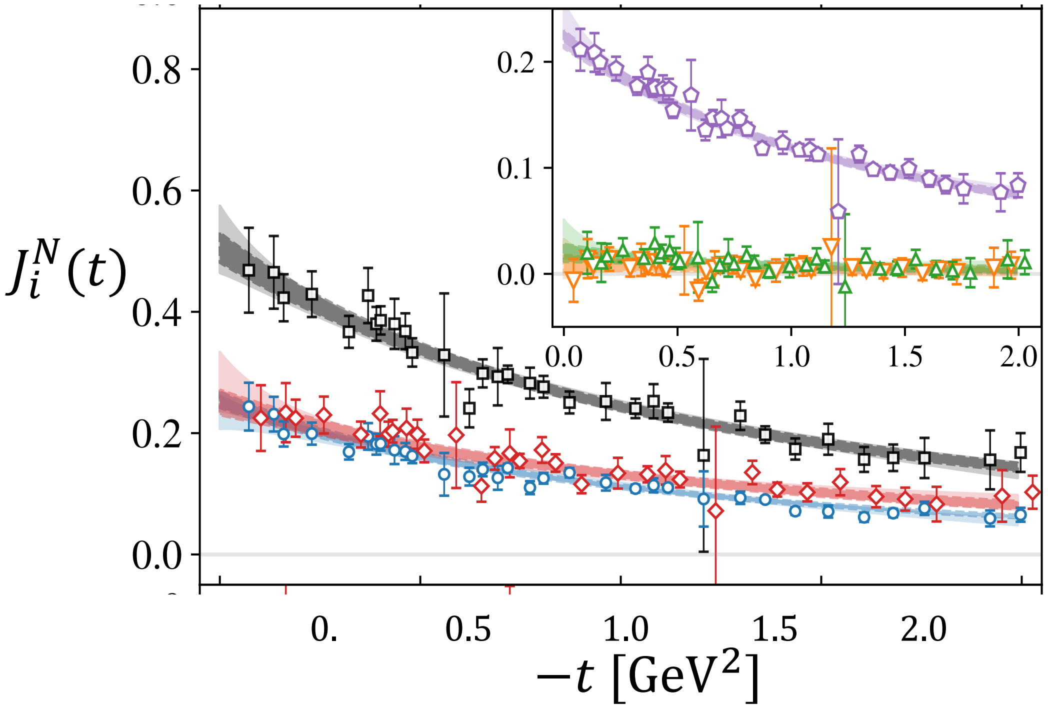
# Quark and gluon GFFs of the proton

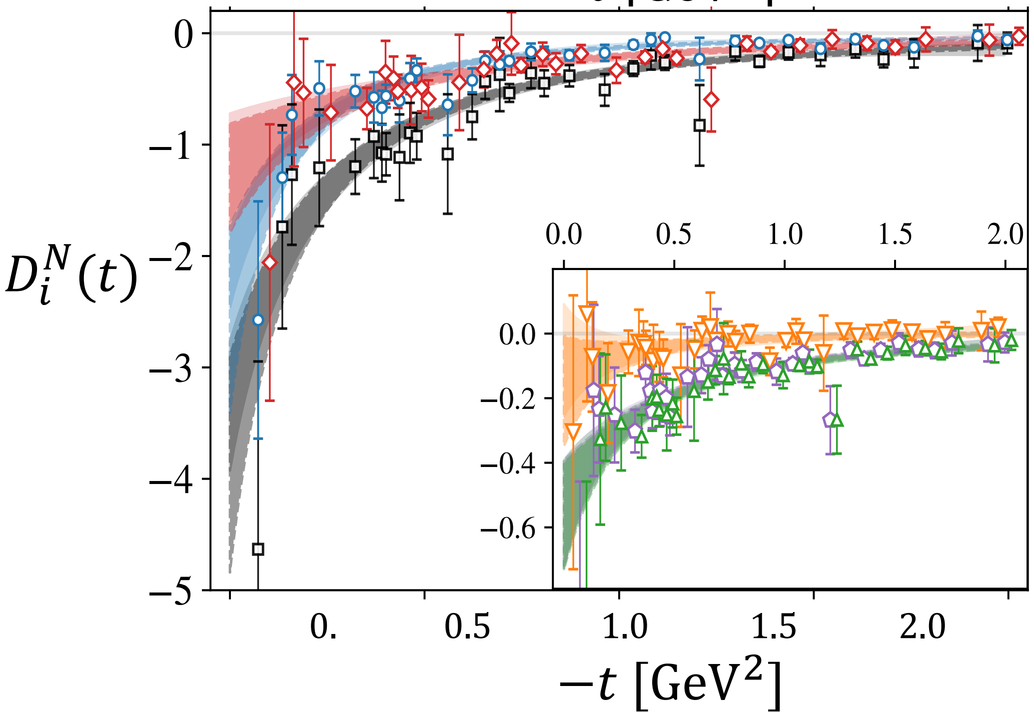
($m_\pi \approx 170$ MeV, including mixing)
Hackett **DAP** Shanahan PRL (2024)
arXiv:2310.08484



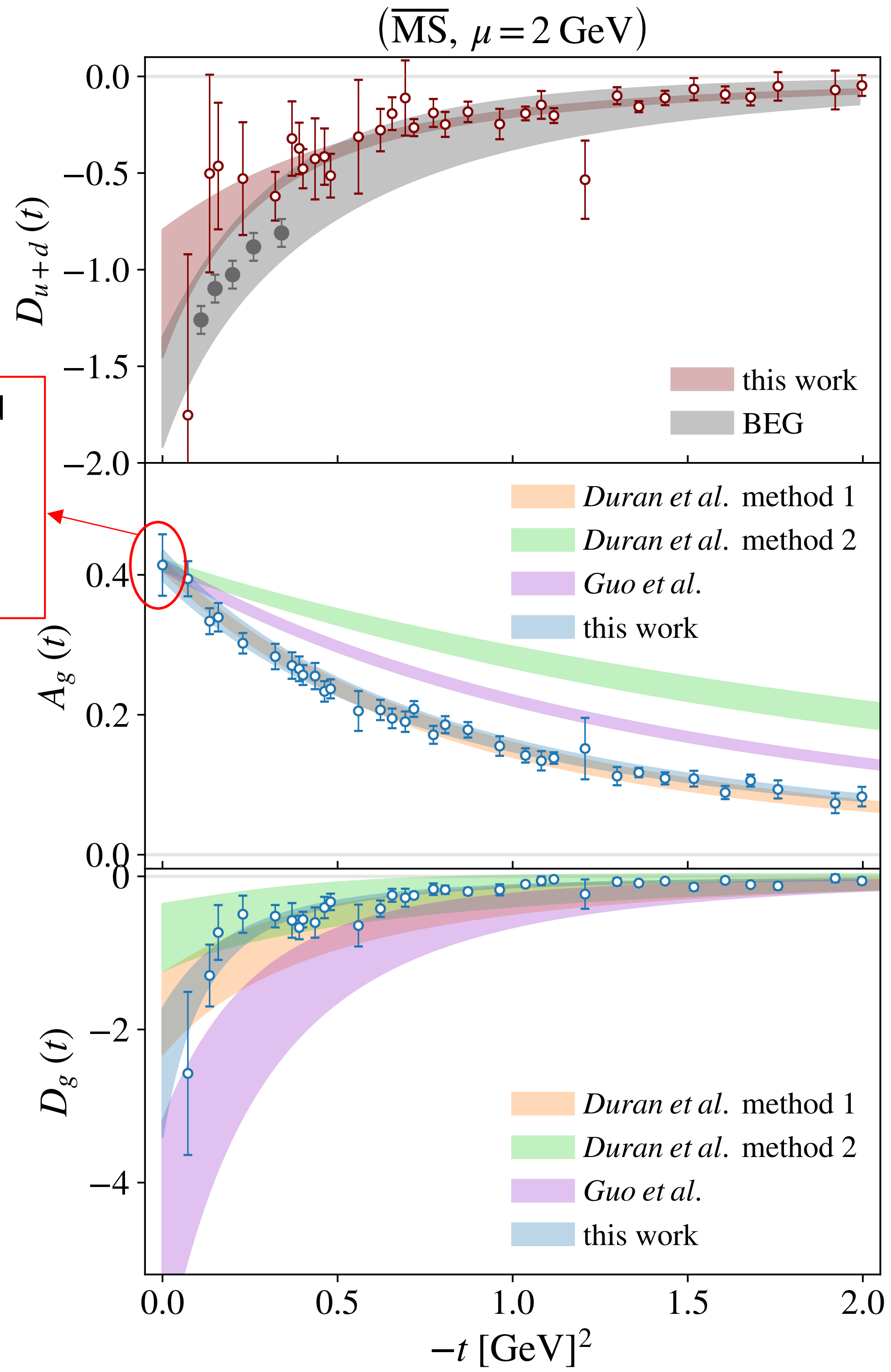

$\overline{\text{MS}}, \mu = 2$ GeV



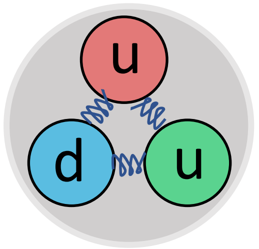



# Renormalized nucleon GFFs – comparison to experiments




all normalized to $\langle x \rangle_g$ from global fit
Hou et al PRD 2021



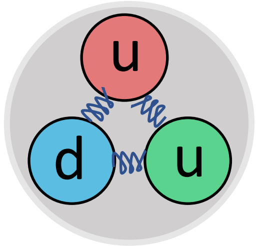

Burkert Elouardhiri Girod Nature 2018 (DVCS)

Duran et al Nature 2023 ($J/\psi$)
method 1: holographic QCD (Mamo Zahed PRD 2021+2022)
method 2: GPDs (Guo Ji Liu PRD 2021)

Guo et al PRD 2023 (+ GlueX data)
method 2 updated formula

FT = Fourier transform
3D Breit frame

# Nucleon size

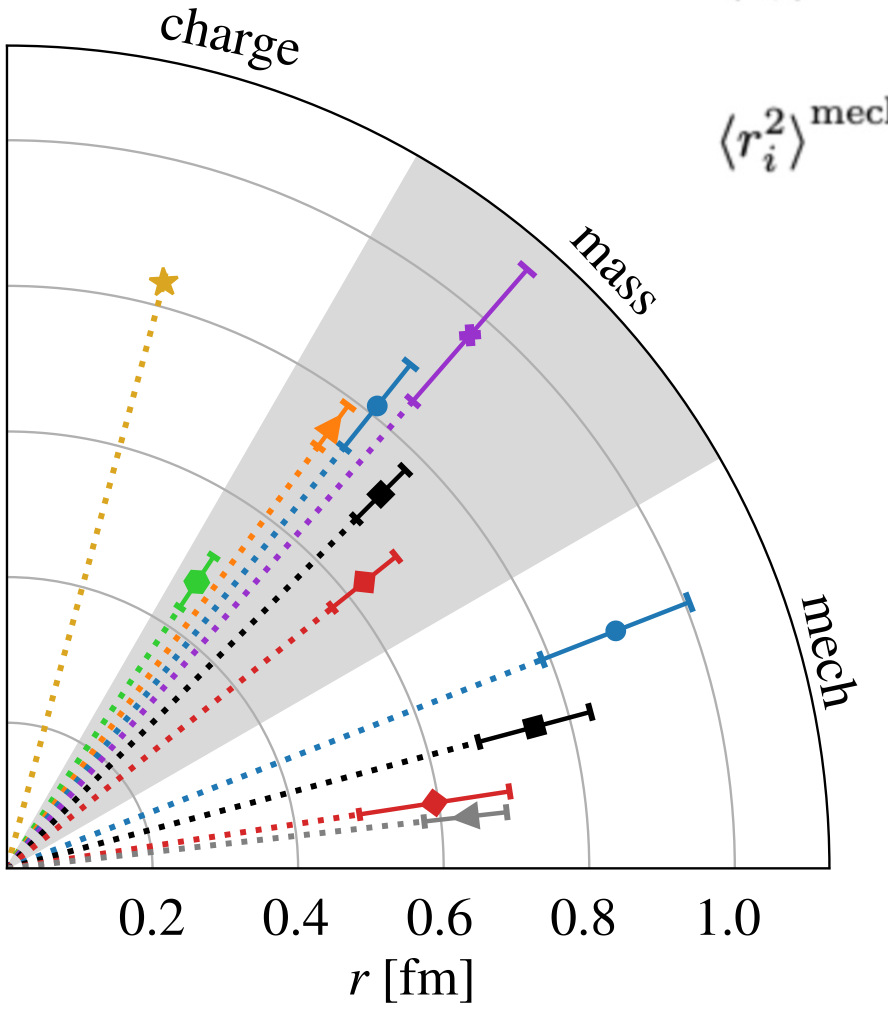



$$\langle r_i^2\rangle^{\text{mass}} = \frac{\int d^3\mathbf{r}\, r^2 \varepsilon_i(r)}{\int d^3\mathbf{r}\, \varepsilon_i(r)},$$

$$\varepsilon_i(r) = m\left[A_i(t) - \frac{t(D_i(t)+A_i(t)-2J_i(t))}{4m^2}\right]_{\text{FT}}$$

$$\langle r_i^2\rangle^{\text{mech}} = \frac{\int d^3\mathbf{r}\, r^2 F_i^{||}(r)}{\int d^3\mathbf{r}\, F_i^{||}(r)}$$

$$\begin{cases} p_i(r) = \dfrac{1}{6m}\dfrac{1}{r^2}\dfrac{d}{dr}r^2\dfrac{d}{dr}[D_i(t)]_{\text{FT}} \\ s_i(r) = -\dfrac{1}{4m}r\dfrac{d}{dr}\dfrac{1}{r}\dfrac{d}{dr}[D_i(t)]_{\text{FT}} \\ F_i^{||}(r) = p_i(r) + 2s_i(r)/3 \end{cases}$$



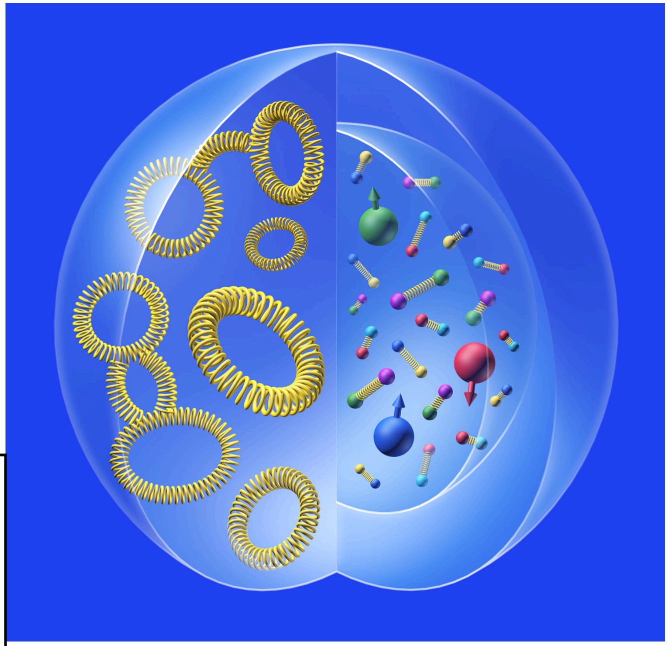
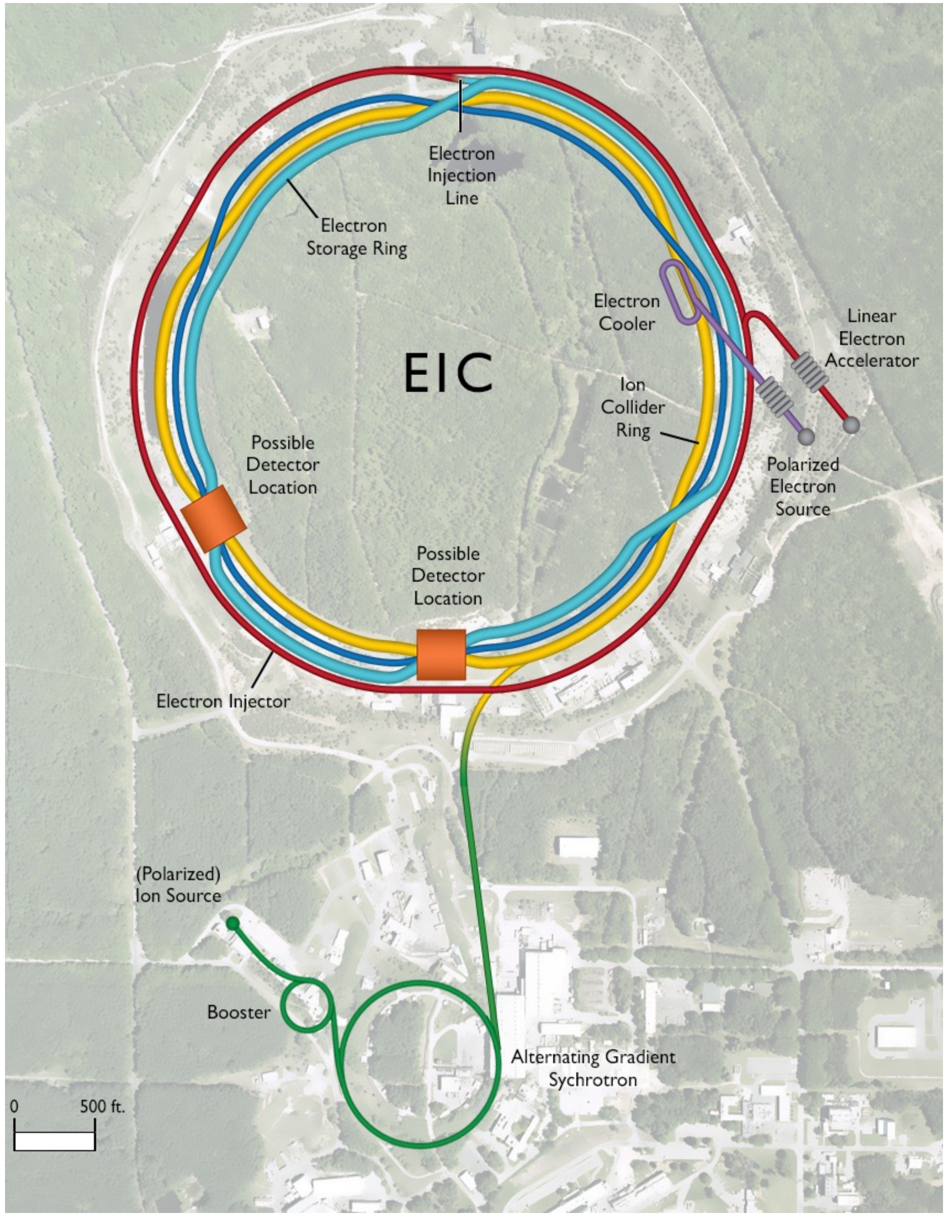



Illustration by Kent Leech for Lawrence Berkeley National Laboratory, Creative Services Office

# Summary and remarks

- Gravitational form factors: the form factors of the energy-momentum tensor.
- Encode how energy, angular momentum, and mechanical properties are distributed inside hadrons. Moments of GPDs (generalized form factors) and PDFs in the forward limit (e.g momentum fraction).
- Lattice QCD constraints to the GFFs of the pion, proton, … More results are coming from lattice and experiments!
- Beyond measuring: Much more to understand about the QCD EMT and GFFs



# Summary and remarks

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THANK YOU!

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