Twist-3 contribution to deeply virtual electroduction of pions

Kornelija Passek-K.

Rudjer Bošković Institute, Croatia

Transversity 2024, June 7, 2024

Hrzz Croatian Science Foundation

STRONG 2020

Outline

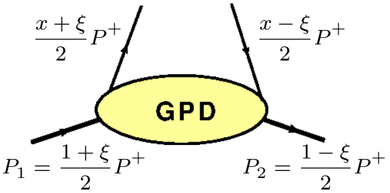
- Wide angle meson production at twist-3

[Kroll, P-K. '18, '21]

- Deeply virtual π^0 production at twist 3

[Duplančić, Kroll, P-K., Szymanowski '24]

Generalized Parton Distributions



$P = P_1 + P_2 \qquad \Delta = P_2 - P_1$

$\Delta^2 = t$ momentum transfer

$\xi = -\frac{\Delta^+}{P^+}$ longitudinal momentum transfer (skewness)

$$K^a(x,\xi,t;\mu) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\mathcal{O}^a(z)|P_1\rangle \Big|_{z^+=0,\, \mathbf{z}_\perp=\mathbf{0}}$$

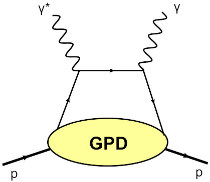
$a \in \{q, g\}$, $\mu\ldots$ factorization scale

- vector (H^a, E^a) and axial-vector GPDs ($\widetilde{H}^a$, $\widetilde{E}^a$)
  $\to$ chiral-even $\qquad \mathcal{O}^q = \bar{q}(z)\gamma^+(\gamma^+\gamma_5)q(-z)$
- transversity GPDs (H_T^a, E_T^a, $\widetilde{H}_T^a$, $\widetilde{E}_T^a$)
  $\to$ chiral-odd $\qquad \mathcal{O}^q = \bar{q}(z)i\sigma^{+i}q(-z)$

H^a, $\widetilde{H}^a$, $H_T^q \xrightarrow{\xi=0,t=0}$ PDFs

GPDs from deeply virtual exclusive processes

DVCS



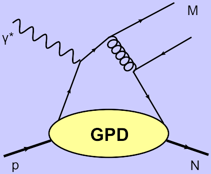
$$\gamma_T^* N \to \gamma N$$

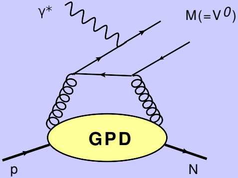
H^q, E^q, $\widetilde{H}^q$, $\widetilde{E}^q$

NLO:

H^G, E^G, $\widetilde{H}^G$, $\widetilde{E}^G$, K_T^G

DVMP





$$\gamma_L^* N \to M N'$$

$M = V_L$: H^{q_i}, E^{q_i}; H^G, E^G

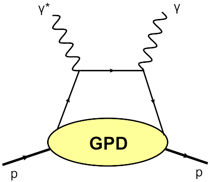
$M = PS$: $\widetilde{H}^{q_i}$, $\widetilde{E}^{q_i}$

$$\gamma_T^* N \to M N'$$

$M_{\textbf{twist-3}} \Rightarrow K_T^q$

GPDs from deeply virtual exclusive processes

DVCS



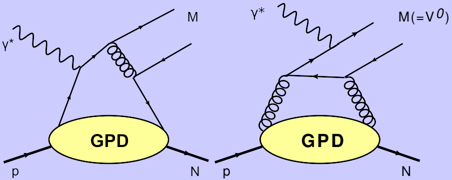
$$\gamma_T^* N \to \gamma N$$

H^q, E^q, $\widetilde{H}^q$, $\widetilde{E}^q$

NLO:

H^G, E^G, $\widetilde{H}^G$, $\widetilde{E}^G$, K_T^G

DVMP



$$\gamma_L^* N \to M N'$$

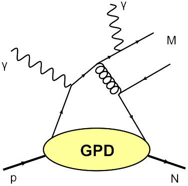
$M = V_L$: H^{q_i}, E^{q_i}; H^G, E^G

$M = PS$: $\widetilde{H}^{q_i}$, $\widetilde{E}^{q_i}$

$$\gamma_T^* N \to M N'$$

$M_{\textbf{twist-3}} \Rightarrow K_T^q$

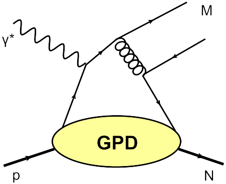
→ J. Qiu talk



$$\gamma N \to (\gamma M) N'$$

K^a, K_T^a

Meson production: handbag factorization



$\gamma^{(*)} N \to M N'$

DEEPLY VIRTUAL

$Q^2 \gg$, $-t \ll$

DVMP

[Collins, Frankfurt, Strikman '97]

- factorization
  $\mathcal{H}^a \otimes GPD$
- GPDs at small $(-t)$

WIDE ANGLE

$-t$, $-u$, $s \gg$, $Q^2 \ll$ or 0

WAMP

[Huang, Kroll '00]

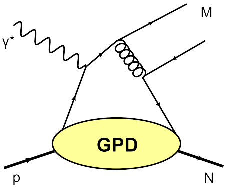
- arguments for factorization
  $\mathcal{H}^a(1/x \otimes GPD(\xi = 0))$
- GPDs at large $(-t)$

$\mathcal{H}^a$ ... parton subprocess helicity amplitudes

$\Rightarrow \mathcal{M}$ ... hadron helicity amplitudes

$\Rightarrow$ observables (cross sections, asymmetries)

Meson production: handbag factorization



$\gamma^{(*)} N \to M N'$

DEEPLY VIRTUAL

$Q^2 \gg, \ -t \ll$

DVMP

[Collins, Frankfurt, Strikman '97]

- factorization $\boxed{\mathcal{H}^a \otimes GPD}$
- GPDs at small $(-t)$
- tw2: γ_L^*, tw3: γ_T^*

WIDE ANGLE

$-t, \ -u, \ s \gg, \ Q^2 \ll$ or 0

WAMP

[Huang, Kroll '00]

- arguments for factorization $\boxed{\mathcal{H}^a (1/x \otimes GPD(\xi = 0))}$
- GPDs at large $(-t)$

large scale $\mathcal{Q}^2$ $(Q^2, \ -t, \ s, \ \ldots)$

- twist expansion: $\langle \mathcal{H} \rangle^{tw2} + \dfrac{\langle \mathcal{H} \rangle^{tw3}}{\mathcal{Q}} + \ldots$
- α_S expansion for each twist: $\alpha_S(\mathcal{Q}) \, \langle \mathcal{H} \rangle^{LO} + \alpha_S^2(\mathcal{Q}) \, \langle \mathcal{H} \rangle^{NLO}$

Meson production status

- DV (V_L) P:
  - data show dominance of γ^*_L contributions
    $\Rightarrow$ twist-2 predictions can describe the data
  - tw2 NLO corrections large
    $\Rightarrow$ global DIS+DVCS+DVV_LP fits at NLO [Čuić, Duplančić, Kumerički, P-K. '23]

Meson production status

- DV (V_L) P:
  - data show dominance of γ_L^* contributions

    $\Rightarrow$ twist-2 predictions can describe the data
  - tw2 NLO corrections large

    $\Rightarrow$ global DIS+DVCS+DVV_LP fits at NLO [Čuić, Duplančić, Kumerički, P-K. '23]
- DVπP:
  - data show suppression of γ_L^* contributions

    $\Rightarrow$ γ_T^* contributions with quark transversity GPDs and 2-body twist-3 ($\pi = q\bar{q}$) approximation (WW) [Goloskokov, Kroll '10, Goldstein, Hernandez, Liuti '13]

Meson production status

- DV (V$_L$) P:
  - data show dominance of γ_L^* contributions

    $\Rightarrow$ twist-2 predictions can describe the data
  - tw2 NLO corrections large

    $\Rightarrow$ global DIS+DVCS+DVV_LP fits at NLO [Čuić, Duplančić, Kumerički, P-K. '23]
- DVπP:
  - data show suppression of γ_L^* contributions

    $\Rightarrow$ γ_T^* contributions with quark transversity GPDs and 2-body twist-3 ($\pi = q\bar{q}$) approximation (WW) [Goloskokov, Kroll '10, Goldstein, Hernandez, Liuti '13]
- WAπP:
  - twist-2 results bellow the data for photoproduction

    $\&$ 2-body twist-3 contributions vanish

    $\Rightarrow$ 3-body tw3 contributions ($\pi = q\bar{q}g$) determined

    $\Rightarrow$ tw3 pion DA from photoproduction fits [Kroll, P-K. 18', '21]

Meson production status

- DV (V$_L$) P:
  - data show dominance of γ_L^* contributions

    $\Rightarrow$ twist-2 predictions can describe the data
  - tw2 NLO corrections large

    $\Rightarrow$ global DIS+DVCS+DVV_LP fits at NLO [Čuić, Duplančić, Kumerički, P-K. '23]
- DVπP:
  - data show suppression of γ_L^* contributions

    $\Rightarrow$ γ_T^* contributions with quark transversity GPDs and 2-body twist-3 ($\pi = q\bar{q}$) approximation (WW) [Goloskokov, Kroll '10, Goldstein, Hernandez, Liuti '13]
  - full (2- and 3-body) twist-3 contributions confronted with data

    [Duplančić, Kroll, P-K., Szymanowski '24]
- WAπP:
  - twist-2 results bellow the data for photoproduction

    & 2-body twist-3 contributions vanish

    $\Rightarrow$ 3-body tw3 contributions ($\pi = q\bar{q}g$) determined

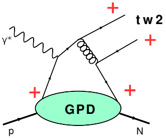
    $\Rightarrow$ tw3 pion DA from photoproduction fits [Kroll, P-K. 18', '21]

DVπP at twist-3

π production to twist-3

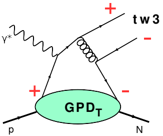
μ photon helicity, $\lambda \ldots$ quark helicities

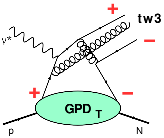
$\mathcal{H}^{\pi}_{0\lambda,\mu\lambda}$ ... non-flip subprocess amplitudes (twist-2)



$\widetilde{H},\ \widetilde{E}$

$\mathcal{H}^{\pi}_{0-\lambda,\mu\lambda}$ ... flip subprocess amplitudes (twist-3) $\boxed{\sim \mu_\pi/Q}$





$H_T,\ \bar{E}_T = 2\widetilde{H}_T + E_T,\ \ldots$

→ just pion DA tw-3 contributions $\Leftarrow \mu_\pi = m_\pi^2/(m_u + m_d) \cong 2$ GeV

(see S. Bhattacharya talk)

distribution amplitudes (DAs):

twist-2 $(q\bar{q})$: ϕ_π

2-body $(q\bar{q})$ twist-3 $\phi_{\pi p},\ \phi_{\pi\sigma}$ 3-body $(q\bar{q}g)$ twist-3 $\phi_{3\pi}$

→ connected by equations of motion (EOMs)

Subprocess amplitudes: twist-3

$$
\begin{aligned}
\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}}\, + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \qquad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \qquad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}
\end{aligned}
$$

- 2- and 3-body contributions necessary for gauge invariance

Subprocess amplitudes: twist-3

$$
\begin{aligned}
\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}} + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \qquad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \qquad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}
\end{aligned}
$$

- 2- and 3-body contributions necessary for gauge invariance
- WAMP:
    - photoproduction ($Q \to 0$): $\mathcal{H}^{\pi,\phi_{\pi p}} = 0$
    - no end-point singularities

Subprocess amplitudes: twist-3

$$\begin{aligned}\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\ &= \big(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\big) + \big(\mathcal{H}^{\pi,q\bar{q}g,C_F}} + \mathcal{H}^{\pi,q\bar{q}g,C_G}\big) \\ &= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \qquad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \qquad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}\end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance
- WAMP:
  - photoproduction ($Q \to 0$): $\mathcal{H}^{\pi,\phi_{\pi p}} = 0$
  - no end-point singularities
- DVMP ($t \to 0$):
  - end-point singularities in $\mathcal{H}^{\pi,\phi_{\pi p}}$:

$$\int_0^1 \frac{d\tau}{\tau}\phi_{\pi p}(\tau)\ \frac{1}{(x+\xi+i\epsilon)^2} \overset{x}{\otimes} H_T(\bar{E}_T)$$

$$\phi_{\pi p}(\tau) = 1 + \omega_{\pi p}\, C_2^{1/2}(2\tau - 1) + \dots$$

τ ... quark long. momentum fraction in π

Treatment of end-point singularities: MPA

$\Rightarrow$ Modified perturbative approach (MPA) [Goloskov, Kroll, '10]

- $k_\perp$ quark transverse momenta in pion

$$\frac{1}{((x+\xi)\tau - k_T^2/Q^2(2\xi) + i\epsilon)} \frac{1}{(x+\xi+i\epsilon)}$$

- $\phi_\pi \rightarrow$ light-cone wave function $\Psi_\pi \sim \phi_\pi \exp\left[-a_\pi^2 k_\perp^2\right]$
- $\int_0^1 d\tau \quad \rightarrow \int d^2\mathbf{k}_T \int_0^1 d\tau \quad \overset{\text{FT}}{\rightarrow} \int d^2\mathbf{b} \int_0^1 d\tau$
- Sudakov form factor $\exp\left[-S(\tau, \mathbf{b}, Q^2)\right]$

- consistently treated 2- and 3-body tw3 contributions, as well as tw2
- involved multidimensional integrations
- complicated calculation of NLO corrections

Treatment of end-point singularities: m_g^2

$\Rightarrow$ pure collinear picture with effective gluon mass m_g^2

[Schwinger '62, Cornwall '82, ..., Shuryak, Zahed '21]

$$\int_0^1 d\tau \phi_{\pi p}(\tau) \frac{1}{((x+\xi)\tau - m_g^2/Q^2(2\xi) + i\epsilon)} \frac{1}{(x-\xi+i\epsilon)} \overset{x}{\otimes} H_T(\bar{E}_T)$$

$$m_g^2(Q^2) = \frac{m_0^2}{1+(Q^2/M^2)^{1+p}}$$ [Aguilar, Binosi, Papavassiliou '14]

$m_g^2(0) = 0.01$ GeV^2

- proof of concept
- suitable for faster fitting
- easier determination of NLO corrections (already available for tw2)

Soft physics input

GPDs

- double distribution representation [Müller '94, Radyushkin '99], weight function $w_j^a \to$ generates ξ dependence double-distribution integral analytically evaluated [Goloskokov, Kroll '08]
- zero-skewness GPD: $$K_j^a(x,\xi=0,t) = k_j^a(x)\ \exp\left[(b_j^a - \alpha'^a_j \ln x)\,t\right]$$
  - H: $k_j^a(x)$ from PDFs (q, Δq, δq), E: $k_j^a(x) = N_j^a x^{-\alpha_j^a(0)}(1-x)^{\beta_j^a}$
- $\{\ N_j^a,\ b_j^a,\ \alpha'^a_j,\ \alpha_j^a(0),\ \beta_j^a\ \}$ parameters
- moments of H_T and $\bar{E}_T$ compared to lattice results

DAs

$$\begin{aligned}\phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F) \quad = \quad & 360\tau_a\tau_b\tau_g^2\Big[1+\omega_{1,0}(\mu_F)\,\frac{1}{2}(7\tau_g-3) \\ & +\omega_{2,0}(\mu_F)\,(2-4\tau_a\tau_b-8\tau_g+8\tau_g^2) \\ & +\omega_{1,1}(\mu_F)\,(3\tau_a\tau_b-2\tau_g+3\tau_g^2)\Big]\end{aligned}$$ [Braun, Filyanov '90]

$\to \phi_{\pi p}$ using EOMs [Kroll, P-K '18]

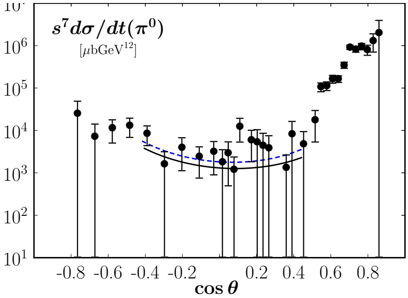
evolution taken into account

Results from photoproduction (π)

- complete tw-3 prediction for π_0 photoproduction fitted to CLAS data

  $\Rightarrow \phi_{3\pi}$ coefficients $\omega_{1,0}$, $\omega_{2,0}$, $\omega_{1,1}$ (set2)

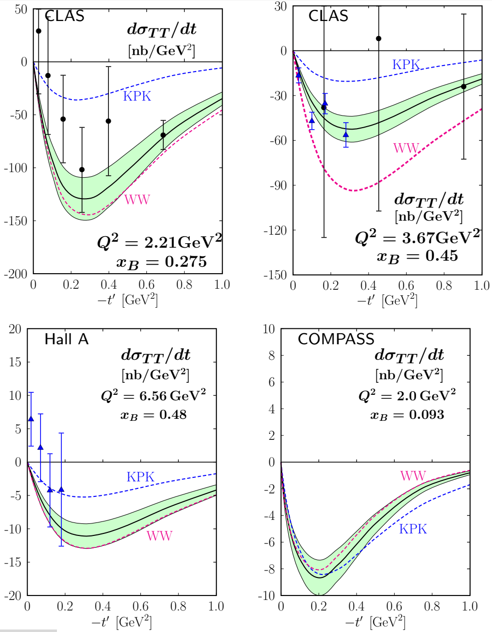
[Kroll, P-K '21]



solid curve: set1 (DVπ^0P)
dashed curve: set2

exp data:
full circles [CLAS '18]

Modified perturbative approach (MPA): $d\sigma_{TT}$



solid curves: set1

dashed curves: set2, WW

exp data:

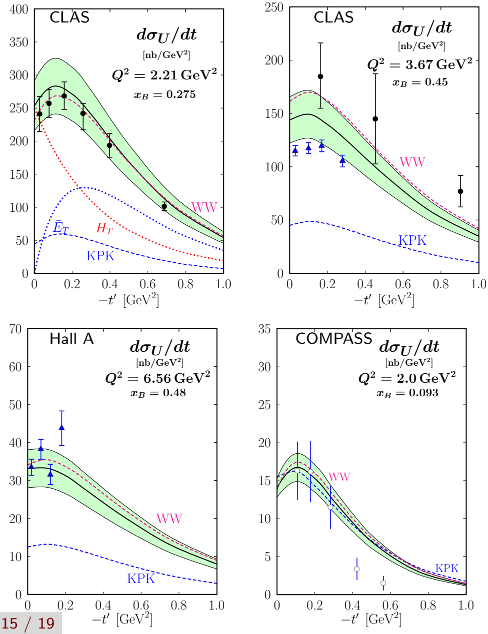
full circles [CLAS '14]

triangles [Hall A '20]

$$\frac{d\sigma_{TT}}{dt} : \bar{E}_T, \qquad \left|\frac{d\sigma_{TT}}{dt}\right| \leq \frac{d\sigma_T}{dt}$$

- $d\sigma_{TT}$ large
- good description with set1
- strong dependence on DA

Modified perturbative approach (MPA): $d\sigma_U$



solid curves: set1

dashed curves: set2, WW

exp data:

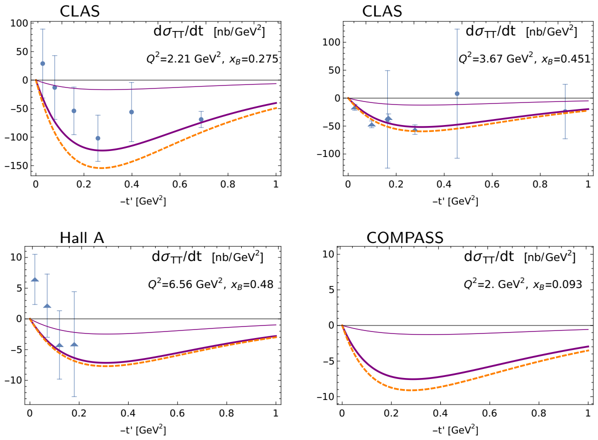
- full circles [CLAS '14]
- triangles [Hall A '20]
- open circles [COMPASS '19]

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}$$

$$\frac{d\sigma_T}{dt} : H_T, \bar{E}_T \qquad \frac{d\sigma_L}{dt} : \widetilde{H}, \widetilde{E}$$

- σ_L negligible except for COMPASS kin. (40%)

Collinear approach with m_g^2: $d\sigma_{TT}$



set1:purple solid

set2: thin solid

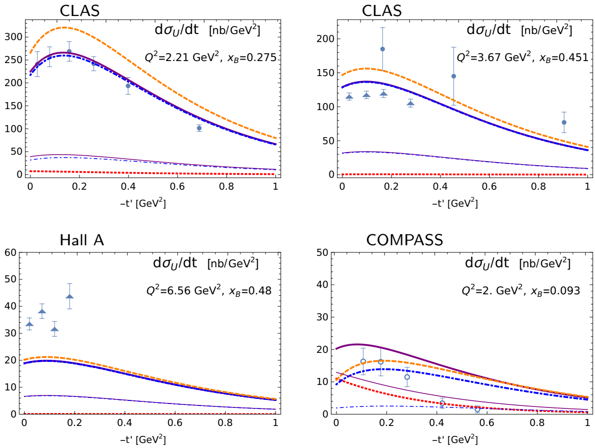
WW: orange dashed

exp data:

full circles [CLAS '14]

triangles [Hall A '20]

Collinear approach with m_g^2: $d\sigma_U$



set1:purple solid

set2: thin solid

WW: orange dashed

red curves: tw2

blue curves: tw3

exp data:

full circles [CLAS '14]

triangles [Hall A '20]

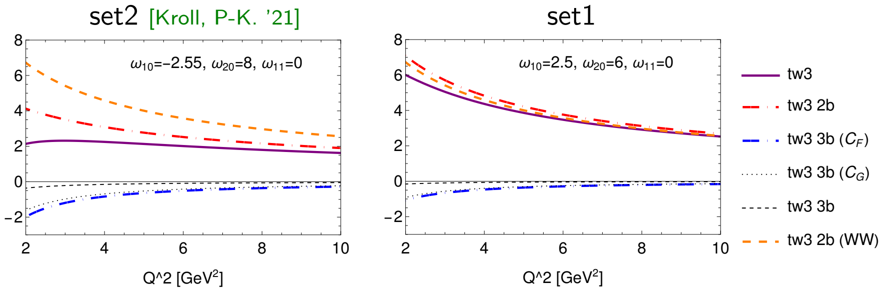
open circles [COMPASS '19]

- tw2 (σ_L) significant for COMPASS kinematics (small x_B)
- Q^2 dependence challenging

Collinear approach with m_g^2

illustration: approximate factorization of x and τ integration

$\Rightarrow \tau$ integrals:



- 3-body contributions smaller but influence the Q^2 behaviour

Concluding remarks

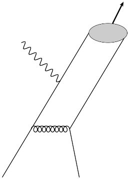
- 3-body twist-3 contributions:
  - important for gauge invariance
  - smaller than 2-body contributions
  - change 2-body twist-3 DA, and thus 2-body tw3 contributions, through EOM
- Improved twist-3 analysis shows twist-3 dominates in DVπ^0P at accessible energies, except for COMPASS kinematics (small xB).
- NLO corrections to twist-2 may be important for COMPASS kinematics.
- Wide-angle meson production also dominated by twist-3 and provides complementary information on pion DA and GPDs at large-t.
- Next steps: GPD fits (MPA and collinear), DA fits or both

Concluding remarks

- 3-body twist-3 contributions:
  - important for gauge invariance
  - smaller than 2-body contributions
  - change 2-body twist-3 DA, and thus 2-body tw3 contributions, through EOM
- Improved twist-3 analysis shows twist-3 dominates in DVπ^0P at accessible energies, except for COMPASS kinematics (small xB).
- NLO corrections to twist-2 may be important for COMPASS kinematics.
- Wide-angle meson production also dominated by twist-3 and provides complementary information on pion DA and GPDs at large-t.
- Next steps: GPD fits (MPA and collinear), DA fits or both

Thank you.

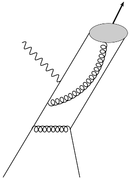
Subprocess amplitudes $\mathcal{H}$: projectors



$q\bar{q} \to \pi$ projector [Beneke, Feldmann '00]

$$(\tau q' + k_\perp) + (\bar{\tau} q' - k_\perp) = q'$$

$$\mathcal{P}_2^\pi \sim f_\pi \Big\{ \gamma_5\, \not{q}'\, \phi_\pi(\tau, \mu_F) + \mu_\pi(\mu_F) \Big[\gamma_5\, \phi_{\pi p}(\tau, \mu_F) - \frac{i}{6}\, \gamma_5\, \sigma_{\mu\nu}\, \frac{q'^\mu n^\nu}{q' \cdot n}\, \phi'_{\pi\sigma}(\tau, \mu_F) + \frac{i}{6}\, \gamma_5\, \sigma_{\mu\nu}\, q'^\mu \phi_{\pi\sigma}(\tau, \mu_F) \frac{\partial}{\partial k_{\perp\nu}} \Big] \Big\}_{k_\perp \to 0}$$



$q\bar{q}g \to \pi$ projector [Kroll, P-K '18]

$$\tau_a q' + \tau_b q' + \tau_g q' = q'$$

$$\mathcal{P}_3^\pi \sim f_{3\pi}(\mu_F)\, \frac{i}{g}\, \gamma_5\, \sigma_{\mu\nu} q'^\mu g_\perp^{\nu\rho}\, \frac{\phi_{3\pi}(\tau_a, \tau_b, \tau_g, \mu_F)}{\tau_g}, \quad f_{3\pi} \sim \mu_\pi$$

$\mu_\pi = m_\pi^2/(m_u + m_d) \cong 2$ GeV

DAs and EOMs

$$\tau\phi_{\pi p}(\tau) + \frac{\tau}{6}\,\phi'_{\pi\sigma}(\tau) - \frac{1}{3}\,\phi_{\pi\sigma}(\tau) \quad = \quad \phi_\pi^{EOM}(\bar{\tau})$$

$$\bar{\tau}\phi_{\pi p}(\tau) - \frac{\bar{\tau}}{6}\phi'_{\pi\sigma}(\tau) - \frac{1}{3}\phi_{\pi\sigma}(\tau) \quad = \quad \phi_\pi^{EOM}(\tau)$$

$$\phi_\pi^{EOM}(\tau) = 2\frac{f_{3\pi}}{f_\pi\mu_\pi}\int_0^{\bar{\tau}}\frac{d\tau_g}{\tau_g}\,\phi_{3\pi}(\tau,\bar{\tau}-\tau_g,\tau_g)$$

- EOMs and symmetry properties
  $\Rightarrow$ the subprocess amplitudes in terms of two twist-3 DAs
  and 2- and 3-body contributions combined
- combined EOMs $\rightarrow$ first order differential equation $\Rightarrow$ from known form of $\phi_{3\pi}$ [Braun, Filyanov '90] one determines $\phi_{\pi p}$ (and $\phi_{\pi\sigma}$)

Note: $q\bar{q}g$ projector and EOMs were derived using light-cone gauge for constituent gluon

Subprocess amplitudes: twist-3

General structure:

$$\begin{aligned}\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\ &= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}} + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right) \\ &= \mathcal{H}^{\pi,\phi_{\pi p}} + \mathcal{H}^{\pi,\phi_{3\pi},C_F} + \mathcal{H}^{\pi,\phi_{3\pi},C_G}\end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance
- WAMP
  - photoproduction ($Q \to 0$): $\mathcal{H}^{\pi,\phi_{\pi p}} = 0$
  - no end-point singularities for $\hat{t} \neq 0$!

Subprocess amplitudes: twist-3

General structure:

$$\begin{aligned}
\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}} + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \qquad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \qquad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}
\end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance
- DVMP ($\hat{t} \to 0$):
  - end-point singularities in $\mathcal{H}^{\pi,\phi_{\pi p}}$
  - $\mathcal{H}^{\pi,\phi_\pi^{EOM}} = 0$

Subprocess amplitudes: twist-3

$$\mathcal{H}^{\pi,tw3} = \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g}$$

$$= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}}\, + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right)$$

$$= \mathcal{H}^{\pi,\phi_{\pi p}} + \mathcal{H}^{\pi,\phi_{3\pi},C_F} + \mathcal{H}^{\pi,\phi_{3\pi},C_G}$$

- DVMP ($\hat{t} \to 0$): $\hat{s} = -\frac{\xi - x}{2\xi} Q^2, \hat{u} = -\frac{\xi + x}{2\xi} Q^2$

$$\mathcal{H}^{\pi,\phi_{\pi p}}_{0-\lambda,\mu\lambda} \sim (2\lambda + \mu)\ f_\pi \mu_\pi C_F \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2}\right) \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}} \phi_{\pi p}(\tau)}$$

$$\mathcal{H}^{\pi,\phi_{3\pi},C_F}_{0-\lambda,\mu\lambda} \sim -(2\lambda + \mu)\ f_{3\pi}\, C_F \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2}\right)$$

$$\times \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}^2} \int_0^{\bar{\tau}} \frac{d\tau_g}{\tau_g(\bar{\tau} - \tau_g)} \phi_{3\pi}(\tau, \bar{\tau} - \tau_g, \tau_g)}$$

$$\mathcal{H}^{P,\phi_{3\pi},C_G}_{0-\lambda,\mu\lambda} \sim -(2\lambda + \mu)\ f_{3\pi}\, C_G \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2} + \frac{e_a + e_b}{\hat{s}\hat{u}}\right)$$

$$\times \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}} \int_0^{\bar{\tau}} \frac{d\tau_g}{\tau_g(\bar{\tau} - \tau_g)} \phi_{3\pi}(\tau, \bar{\tau} - \tau_g, \tau_g)}$$

Pion distribution amplitudes

Twist-2 DA: $\phi_\pi(\tau,\mu_F) = 6\tau\bar{\tau}\left[1 + a_2(\mu_F)\, C_2^{3/2}(2\tau-1)\right]$

Twist-3 DAs:

$$\begin{aligned}\phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F) \quad &= \quad 360\tau_a\tau_b\tau_g^2\Big[1 + \omega_{1,0}(\mu_F)\,\frac{1}{2}(7\tau_g - 3) \\ &+\omega_{2,0}(\mu_F)\,(2 - 4\tau_a\tau_b - 8\tau_g + 8\tau_g^2) \\ &+\omega_{1,1}(\mu_F)\,(3\tau_a\tau_b - 2\tau_g + 3\tau_g^2)\Big]\end{aligned}$$

[Braun, Filyanov '90]

using EOMs [Kroll, P-K '18]:

$$\begin{aligned}\phi_{\pi p}(\tau,\mu_F) \quad &= \quad 1 + \frac{1}{7}\frac{f_{3\pi}(\mu_F)}{f_\pi\mu_\pi(\mu_F)}\Big(7\,\omega_{1,0}(\mu_F) - 2\,\omega_{2,0}(\mu_F) - \omega_{1,1}(\mu_F)\Big) \\ &\times\Big(10\,C_2^{1/2}(2\tau-1) - 3\,C_4^{1/2}(2\tau-1)\Big)\,, \quad \phi_{\pi\sigma}(\tau) = \ldots\end{aligned}$$

Parameters:

- $a_2(\mu_0) = 0.1364 \pm 0.0213$ at $\mu_0 = 2$ GeV [Braun et al '15] (lattice)
- $\omega_{10}(\mu_0) = -2.55\,, \omega_{10}(\mu_0) = 0.0$ and $f_{3\pi}(\mu_0) = 0.004\ \mathrm{GeV}^2$. [Ball '99]
- $\omega_{20}(\mu_0) = 8.0$ [Kroll, P-K '18] fit to π^0 photoproduction data [CLAS '17]

Evolution of the decay constants and DA parameters taken into account.

Form factors and GPDs at large t

$R_i \ldots 1/x$ moment of $\xi = 0$ GPD (K_i)

- $R_V(\leftarrow H)$, $R_T(\leftarrow E)$ from nucleon form factor analysis [Diehl, Kroll '13]
- $R_A(\leftarrow \tilde{H})$ form factor analysis and WACS KLL asymmetry [Kroll '17]
- $S_T(\leftarrow H_T)$, $\bar{S}_T(\leftarrow \bar{E}_T)$ low $-t$ from DVMP analysis [Goloskokov, Kroll '11]
- $S_S(\leftarrow \tilde{H}_T) \cong \bar{S}_T/2$ $(\bar{E}_T = 2\tilde{H}_T + E_T)$

GPD parameterization [Diehl, Feldmann, Jakob, Kroll '04, Diehl, Kroll '13]

$$K_j^a(x, \xi = 0, t) = k_j^a(x) \exp\left[t\, f_j^a(x)\right]$$

$$f_j^a(x) = \left(B_j^a - \alpha_i'^a \ln x\right)(1-x)^3 + A_j^a x (1-x)^2$$

- strong $x - t$ correlation
- power behaviour for large $(-t)$
- choice for transversity GPDs $A = 0.5$ GeV^{-2}

Parameterization of GPDs at small t

double distribution representation [Müller '94, Radyushkin '99]

$$K_j^a(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta \; \delta(\rho+\xi\eta-x)\, K_j^a(\rho,\xi=0,t)\, w_j^a(\rho,\eta)$$

- weight function w_j^a $\rightarrow$ generates ξ dependence
- zero-skewness GPD:

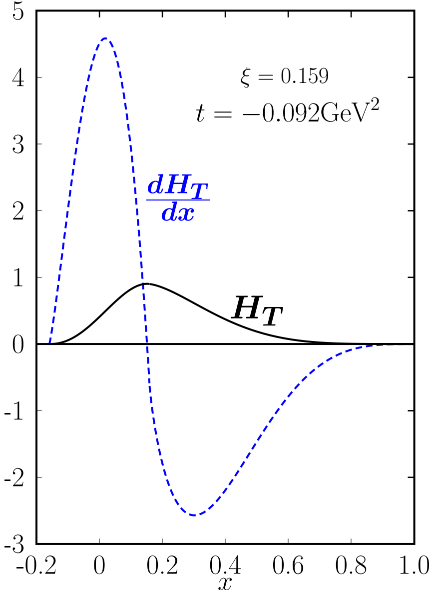
$$K_j^a(x,\xi=0,t) = k_j^a(x)\;\exp\left[(b_j^a - {\alpha'}_j^a \ln x)\, t\right]$$

  - H - GPDs: $k_j^a(x)$ from PDFs (q, Δq, δq)
  - E - GPDs: $k_j^a(x) = N_j^a x^{-\alpha_j^a(0)}(1-x)^{\beta_j^a}$
- double-distribution integral analytically evaluated [Goloskokov, Kroll '08]

Parameters:

- $\{\; N_j^a,\, b_j^a,\, {\alpha'}_j^a,\, \alpha_j^a(0),\, \beta_j^a \;\}$ [Goloskokov, Kroll '11, '14]
  [Duplančić, Kroll, P-K., Szymanowski '24]
- moments of H_T and $\bar{E}_T$ compared to lattice results

Parameterization of GPDs at small t

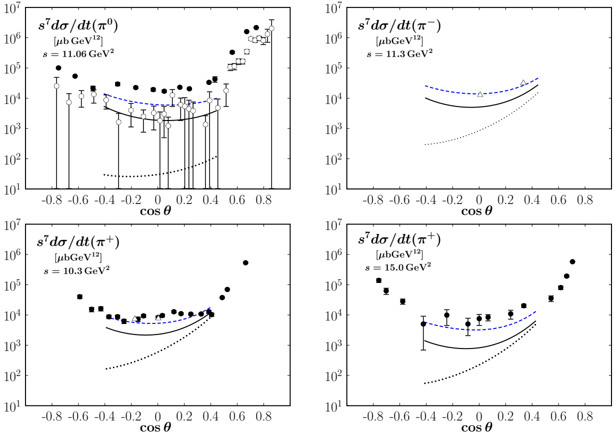


$\mu_0 = 2$ GeV

Photoproduction (π)

- complete tw-3 prediction for π_0 photoproduction fitted to CLAS data and obtained predictions for $\pi^{\pm}$

[Kroll, P-K '21]



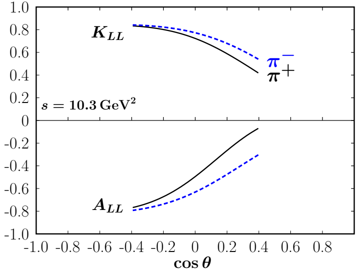
solid curves:
complete twist-3
dotted curves: twist-2

exp data:
full circles [SLAC '76]
open circles [CLAS '17]
triangles [JLab, Hall A '05]

- twist-2 prediction well below the data

Spin effects - photoproduction

[Kroll, P-K '21]: $\pi^{\pm}$

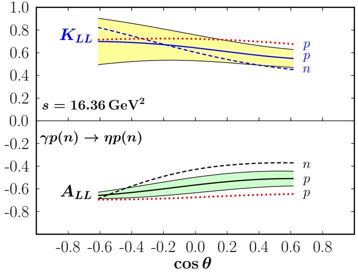


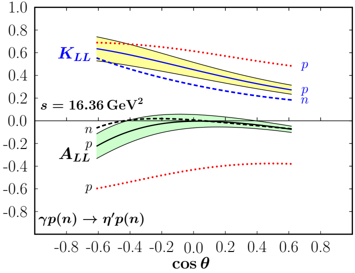
$A_{LL}(K_{LL})\ldots$ correlation of the helicities of the photon and incoming (outgoing) nucleon

$$A_{LL}^{P,tw2} = K_{LL}^{P,tw2}$$
$$A_{LL}^{P,tw3} = -K_{LL}^{P,tw3}$$

→ characteristic signature for dominance of twist-3 (like $\sigma_T \gg \sigma_L$ in DVMP)

[Kroll, P-K '22]: η, η'





→ in contrast to π and η, for η' dominance of twist-2 and sensitivity to gluons

Cross-sections

$$\frac{d^4\sigma}{dW^2dQ^2dtd\varphi} = \frac{\alpha_{em}(W^2-m_N^2)}{16\pi^2E_L^2m_N^2Q^2(1-\varepsilon)}\left(\frac{d\sigma_T}{dt}+\varepsilon\frac{d\sigma_L}{dt} + \varepsilon\cos(2\varphi)\,\frac{d\sigma_{TT}}{dt}+\sqrt{2\varepsilon(1+\varepsilon)}\,\cos\varphi\,\frac{d\sigma_{LT}}{dt}\right)$$

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt}+\epsilon\frac{d\sigma_L}{dt}$$

$$\frac{d\sigma_L}{dt}:\widetilde{H},\widetilde{E} \qquad \frac{d\sigma_T}{dt}:H_T,\bar{E}_T \qquad \frac{d\sigma_{TT}}{dt}:\bar{E}_T \qquad \frac{d\sigma_{LT}}{dt}:\widetilde{E},H_T$$

$$\left|\frac{d\sigma_{TT}}{dt}\right| \le \frac{d\sigma_T}{dt}$$