

Department of Physics

Trieste, Italy

Theory and Phenomenology of GPDs - Overview

(Can't be comprehensive in 30 minutes)

- Inclusive vs. Exclusive Explore Hadron's Partonic Structure without Breaking it!
- GPDs, 3D Tomographic Images & Beyond
- **Exclusive Processes for Extracting GPDs**
- **QCD** Factorization, Angular Modulations, ...
- **Summary and Outlook**



See talks by Nicole D'Hose, Charlotte Van Hulse, Valerio Bertone, Cedric Mezrag, Silvia Niccolai, ... See also talk by Herve Dutrieux @ QCD Evolution







Jianwei Qiu Jefferson Lab, Theory Center

"See" Internal Structure of Hadron without seeing quarks/gluons directly?

3D hadron structure:

1

NO quarks and gluons can be seen in isolation!



□ Need new observables with two distinctive scales:

- $Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$
- Hard scale: Q₁ to localize the probe to see the particle nature of quarks/gluons
- "Soft" scale: Q₂ to be more sensitive to the emergent regime of hadron structure ~ 1/fm



Inclusive vs. Exclusive – Partonic structure without breaking the hadron!



Jefferson Lab

Inclusive vs. Exclusive – Partonic structure without breaking the hadron!



Inclusive vs. Exclusive – Partonic structure without breaking the hadron!



Generalized Parton Distributions (GPDs)

Definition:

5

$$\begin{split} F^{q}(x,\xi,t) &= \int \frac{\mathrm{d}z^{-}}{4\pi} e^{-ixP^{+}z^{-}} \langle p' | \bar{q}(z^{-}/2) \gamma^{+}q(-z^{-}/2) | p \rangle \\ &= \frac{1}{2P^{+}} \left[H^{q}(x,\xi,t) \, \bar{u}\left(p'\right) \gamma^{+}u(p) - E^{q}(x,\xi,t) \, \bar{u}\left(p'\right) \frac{i\sigma^{+\alpha}\Delta_{\alpha}}{2m}u(p) \right] \\ \widetilde{F}^{q}(x,\xi,t) &= \int \frac{\mathrm{d}z^{-}}{4\pi} e^{-ixP^{+}z^{-}} \langle p' | \bar{q}(z^{-}/2) \gamma^{+}\gamma_{5}q(-z^{-}/2) | p \rangle \\ &= \frac{1}{2P^{+}} \left[\widetilde{H}^{q}(x,\xi,t) \, \bar{u}\left(p'\right) \gamma^{+}\gamma_{5}u(p) - \widetilde{E}^{q}(x,\xi,t) \, \bar{u}\left(p'\right) \frac{\gamma_{5}\Delta^{+}}{2m}u(p) \right]. \end{split}$$

Combine <u>*PDF*</u> and <u>*Distribution Amplitude* (DA):</u>

Forward limit $\xi = t = 0$: $H^q(x, 0, 0) = q(x)$, $\tilde{H}^q(x, 0, 0) = \Delta q(x)$



D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, Fortsch. Phys. 42 (1994) 101



$$P^{+} = \frac{p^{+} + p'^{+}}{2}$$
$$\Delta = p - p' \qquad t = \Delta^{2}$$

Jefferson Lab

Similar definition for gluon GPDs



Proton radii from quark and gluon spatial density distribution, $r_q(x)$ & $r_g(x)$



□ Impact parameter dependent parton density distribution:

$$q(x,b_{\perp},Q) = \int d^2 \Delta_{\perp} e^{-i\Delta_{\perp} \cdot b_{\perp}} H_q(x,\xi=0,t=-\Delta_{\perp}^2,Q)$$

Quark density in $dx d^2 \boldsymbol{b}_T$





 $p \qquad p'$ Measurement of p' fixes (t, ξ) x = momentum flowbetween the pair

- Should r_q(x) > r_g(x), or vice versa?
- Could $r_g(x)$ saturates as $x \to 0$
- How do they compare with known radius (EM charge radius, mass radius, ...), & why?
- How the image correlate to hadron spin, ... ?



QCD energy-momentum tensor:

Ji, PRL78, 1997 V. D. Burkert, et al. RMP 95 (2023) 041002

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \text{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \, \psi_q - g^{\mu\nu} \bar{\psi}_q \left(i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \text{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,\,\mu\nu} + \frac{1}{4} g^{\mu\nu} \left(F^a_{\rho\eta} \right)^2$$

Gravitational" form factors:

$$\langle p' | T_i^{\mu\nu} | p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^{\mu} P^{\nu}}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^{\mu} \Delta^{\nu} - g^{\mu\nu} \Delta^2}{4m} + m \,\bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

Connection to GPD moments:

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p'|T_i^{++}|p\rangle \quad \propto \quad \bar{u}(p') \begin{bmatrix} \left(A_i + \xi^2 D_i\right) \gamma^+ + \left(B_i - \xi^2 D_i\right) \frac{i\sigma^{+\Delta}}{2m} \end{bmatrix} u(p)$$
$$\int_{-1}^{1} dx \, x \, H_i(x,\xi,t) \quad \int_{-1}^{1} dx \, x \, E_i(x,\xi,t)$$

□ Angular momentum sum rule:

$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \left[H_i(x,\xi,t) + E_i(x,\xi,t) \right]$$

i = q, g

8

3D tomography Relation to GFFs Angular Momentum $C_i(t) \leftrightarrow D_i(t)/4$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018) Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

x-dependence of GPDs!

Jefferson Lab

Need to know the x-dependence of GPDs to construct the proper moments!

Known Physical Processes for Extracting GPDs

 \Box Hit the proton hard without breaking it \Rightarrow Diffractive scattering to keep proton intact



HERA discovery:

~ 10-15% of HERA events with the Proton stayed intact

□ Known exclusive processes for extracting GPDs:



Why is the GPD's *x*-dependence so *difficult* to measure?





Why is the GPD's *x*-dependence so *difficult* to measure?



How to Find Physical Processes to be Sensitive to GPDs?





How to Find Physical Processes to be Sensitive to GPDs?



Classification of SDHEPs

□ Electro-production (JLab, EIC, ...)







□ Photo-production (JLab, EIC, ...)



□ Meso-production (AMBER, J-PARC, ...)





SDHEP: Two-stage Paradigm and Power Expansion



≥ 3 parton connection: further Power suppressed



SDHEP: Two-stage Paradigm and Power Expansion



Where does the *x*-sensitivity come from?



 \Box *x*-sensitivity \Leftrightarrow 2 \rightarrow 2 hard scattering:

Kinematics:

1.
$$\hat{s} = 2 \xi s / (1 + \xi)$$
 \leftarrow ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2) \sin\theta$ \leftrightarrow x
3. ϕ (A^*B) spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_{A} e^{i(\lambda_A - \lambda_B)\phi} \cdot \int_{-1}^{1} dx \, F_A(x) \, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$
[suppressing *t* and ξ dependence]



Where does the *x*-sensitivity come from?



 \Box *x*-sensitivity \Leftrightarrow 2 \rightarrow 2 hard scattering:

Kinematics:

1.
$$\hat{s} = 2 \xi s / (1 + \xi)$$
 \leftarrow ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2) \sin\theta$ \leftarrow x
3. ϕ \leftarrow (A^*B) spin states

 $\mathcal{M}(Q,\phi) \simeq \sum_{A} e^{i(\lambda_{A}-\lambda_{B})\phi} \cdot \int_{-1}^{1} dx \ F_{A}(x) \ C_{A}(x;Q) \qquad (Q = \theta \text{ or } q_{T})$ [suppressing t and ξ dependence] $\text{Moment-type sensitivity:} \ C(x;Q) = G(x) \cdot T(Q) \implies F_{G} = \int_{-1}^{1} dx \ G(x) \ F(x,\xi,t) \qquad \text{Independent of } Q$ Scaling for F_{G} Inversion problem: <u>shadow GPD</u> $S_{G} = \int_{-1}^{1} dx \ G(x) \ S(x,\xi) = 0$ [Bertone et al. PRD `21]

• Enhanced sensitivity: $C(x; Q) \neq G(x) \cdot T(Q)$ \longrightarrow $d\sigma/dQ \sim |C(x; Q) \otimes_x F(x, \xi, t)|^2$ Jefferson Lab

Moment-type Sensitivity: $h(p) + B(p_2) \rightarrow h'(p') + C(q_1) + D(q_2)$



The x-integration is NOT sensitive to externally measured hard scale, q_T or Q^2 ! Need a very large range of Q^2 , but, cross section is strongly suppressed!

PRD56 (1997) 5524; PRD58 (1998) 094018; PRD59 (1999) 074009



Moment-type Sensitivity: $h(p) + B(p_2) \rightarrow h'(p') + C(q_1) + D(q_2)$

DVCS:

DVMP:

20

$h(p) = \operatorname{Proton}(p), \ h'(p') = \operatorname{Proton}(p'), \ B(p_2) = \operatorname{electron}(p_2), \ C(q_1) = \operatorname{electron}(q_1), \ D(q_2) = \operatorname{photon}(q_2)$ Factorization: $\xi = \frac{(p - p')^+}{(p + p')^+}$ $t = (p - p')^2$ $P_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$ $p_1 \longrightarrow q_2$ $p_2 \longrightarrow q_2$

The x-integration is NOT sensitive to externally measured hard scale, q_T or Q^2 ! Need a very large range of Q^2 , but, cross section is strongly suppressed!

PRD56 (1997) 5524; PRD58 (1998) 094018; PRD59 (1999) 074009



What Kind of Process Could be Sensitive to the x-Dependence?

Create an entanglement between the internal *x* and an externally measured variable?

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}\boldsymbol{x} \frac{F(\boldsymbol{x},\xi,t)}{x - x_p(\xi,\boldsymbol{q}) + i\varepsilon}$$

Change external *q* to sample different part of **x**.

Double DVCS (two scales):

$$x_p(\xi, q) = \xi\left(\frac{1-q^2/Q^2}{1+q^2/Q^2}\right) \to \xi \text{ same as DVCS if } q \to 0$$



Production of two back-to-back high pT particles (say, two photons):

 $\pi^{-}(p_{\pi}) + P(p) \rightarrow \gamma(q_{1}) + \gamma(q_{2}) + N(p')$ Hard scale: $q_{T} \gg \Lambda_{\text{QCD}}$ Soft scale: $t \sim \Lambda_{\text{OCD}}^{2}$

Qiu & Yu JHEP 08 (2022) 103

 $x \leftrightarrow q_T$

$$\mathcal{M}(t,\xi,q_T) = \int_{-1}^{1} \mathrm{d}x \, F(x,\xi,t;\mu) \cdot C(x,\xi;q_T/\mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/q_T) \longrightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}t \, \mathrm{d}\xi \, \mathrm{d}q_T} \sim |\mathcal{M}(t,\xi,q_T)|^2$$

$$q_T \text{ distribution is "conjugate" to x distribution}$$

Qiu & Yu, PRD 109 (2024) 074023



In addition to

$$F_0(\xi, t) = \int_{-1}^{1} \frac{dx F(x, \xi, t)}{x - \xi + i\epsilon}$$

When two photons are radiated from the same charged line

 $i\mathcal{M}$ also contains

$$I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx F(x,\xi,t)}{x - \rho(z;\theta) + i\epsilon \operatorname{sgn}\left[\cos^2(\theta/2) - z\right]}$$

$$\rho(z;\theta) = \xi \cdot \left[\frac{1-z+\tan^2(\theta/2)z}{1-z-\tan^2(\theta/2)z}\right] \in (-\infty,-\xi] \cup [\xi,\infty)$$







Qiu & Yu, PRD 109 (2024) 074023

Diphoton process:
$$N\pi \to N'\gamma\gamma$$
: (1) $p\pi^- \to n\gamma\gamma$; (2) $n\pi^+ \to p\gamma\gamma$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = 2\pi \left(\alpha_e \alpha_s \frac{C_F}{N_c}\right)^2 \frac{1}{\xi^2 s^3} \cdot \left[(1-\xi^2) \sum_{\alpha=\pm} \left(|\mathcal{M}_{\alpha}^{[\widetilde{H}]}|^2 + |\widetilde{\mathcal{M}}_{\alpha}^{[H]}|^2 \right) - \left(\xi^2 + \frac{t}{4m^2}\right) \sum_{\alpha=\pm} |\widetilde{\mathcal{M}}_{\alpha}^{[E]}|^2 - \frac{\xi^2 t}{4m^2} \sum_{\alpha=\pm} |\mathcal{M}_{\alpha}^{[\widetilde{E}]}|^2 - 2\xi^2 \sum_{\alpha=\pm} \operatorname{Re}\left(\widetilde{\mathcal{M}}_{\alpha}^{[H]} \widetilde{\mathcal{M}}_{\alpha}^{[E]*} + \mathcal{M}_{\alpha}^{[\widetilde{H}]} \mathcal{M}_{\alpha}^{[\widetilde{E}]*} \right) \right]$$

Nucleon transition GPDs

23



Qiu & Yu, PRD 109 (2024) 074023





Qiu & Yu, PRD 109 (2024) 074023



Exclusive Drell-Yan dilepton production

 $N + \pi \to N' + \gamma^* \left[\to \ell^+ + \ell^- \right]$



- Lower rate
- Blind to shadow GPDs



Enhanced x-Sensitivity: (2) γ - π Pair Photoproduction



Enhanced x-Sensitivity: (2) γ - π Pair Photoproduction (at JLab Hall D)





D Polarization asymmetries:

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi} \frac{d\sigma}{d|t|d\xi\,d\cos\theta} \cdot \left[1 + \lambda_N \lambda_\gamma \,A_{LL} + \zeta \,A_{UT}\cos2\left(\phi - \phi_\gamma\right) + \lambda_N \zeta \,A_{LT}\sin2\left(\phi - \phi_\gamma\right)\right]$$
$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi \left(\alpha_e \alpha_s\right)^2 \left(\frac{C_F}{N_c}\right)^2 \frac{1 - \xi^2}{\xi^2 s^3} \Sigma_{UU}$$

$$\begin{split} \Sigma_{UU} &= |\mathcal{M}_{+}^{[\widetilde{H}]}|^{2} + |\mathcal{M}_{-}^{[\widetilde{H}]}|^{2} + |\widetilde{\mathcal{M}}_{+}^{[H]}|^{2} + |\widetilde{\mathcal{M}}_{-}^{[H]}|^{2}, \\ A_{LL} &= 2 \, \Sigma_{UU}^{-1} \, \mathrm{Re} \left[\mathcal{M}_{+}^{[\widetilde{H}]} \, \widetilde{\mathcal{M}}_{+}^{[H]*} + \mathcal{M}_{-}^{[\widetilde{H}]} \, \widetilde{\mathcal{M}}_{-}^{[H]*} \right], \\ A_{UT} &= 2 \, \Sigma_{UU}^{-1} \, \mathrm{Re} \left[\widetilde{\mathcal{M}}_{+}^{[H]} \, \widetilde{\mathcal{M}}_{-}^{[H]*} - \mathcal{M}_{+}^{[\widetilde{H}]} \, \mathcal{M}_{-}^{[\widetilde{H}]*} \right], \\ A_{LT} &= 2 \, \Sigma_{UU}^{-1} \, \mathrm{Im} \left[\mathcal{M}_{+}^{[\widetilde{H}]} \, \widetilde{\mathcal{M}}_{-}^{[H]*} + \mathcal{M}_{-}^{[\widetilde{H}]} \, \widetilde{\mathcal{M}}_{+}^{[H]*} \right]. \end{split}$$

Neglecting: (1) E and \widetilde{E} ; (2) gluon channel



Qiu & Yu, PRL 131 (2023), 161902

Enhanced x-sensitivity: (2) γ - π pair photoproduction (at JLab Hall D)



Enhanced x-sensitivity: (2) γ - π pair photoproduction (at JLab Hall D)



29

Enhanced x-sensitivity: (2) γ - π pair photoproduction (at upgraded energy)



Angular Modulations – Separation of Different GPDs & Global Analyses

GPDs depend on the choice of Frame (light-cone n-vector to define the "+" component):



Angular modulation between "diffractive" and "scattering" planes to select the spin-state of A* - different GPDs

Experimental Breit frame is not ideal:





BH is not a "t"-channel process:



Propagators of $k_1 \& k_2$ have ϕ -dependence!



Angular modulation between "leptonic" and "hadronic" planes do not necessarily select the definite spin-state of A* - different GPDs!

DVCS Compton Form Factors (CFFs)

Like structure functions, CFFs include all power contributions:

Qiu, Sato & Yu in preparation



In terms of EM gauge invariance, this virtual photon Compton scattering amplitude can be decomposed into 18 scalar Compton Form Factors.

Extraction of GPDs from QCD global analyses:

- Universality of GPDs unlike PDFs, the choice of the light-cone vector to define "+" component is not unique! When considering the power corrections, it is critical to carefully define the GPDs!
- 2) Angular modulation is critically important for separating different GPDs, but, consistent choice of the "angle" to define the modulation is crucial!



Summary and Outlook

GPDs are fundamental, carrying rich information on:

- Tomographic images of confined quarks and gluons
- Underline dynamics of hadronic properties

□ It is challenging to extract the *x*-dependence of GPDs

The relative momentum fraction x needs to be entangled with externally measured hard scales!

QCD Global analyses to extract GPDs:

- Need more processes, more observables (see Silvia's talk)
- Need input from lattice QCD (see Herve's talk at QCD Evolution)
- Controlled GPD evolution (see Valerio's talk)
- Need to solve the end point issues of exclusive processes (momentum of active parton goes to zero)

A long but challenging & exciting way to go!

Many on-going efforts:

PARTON Collab, QuantOM Collab, QGT Collab, FemtoNET, ...





...