# Transversity 2024

Trieste, 3-7 June 2024

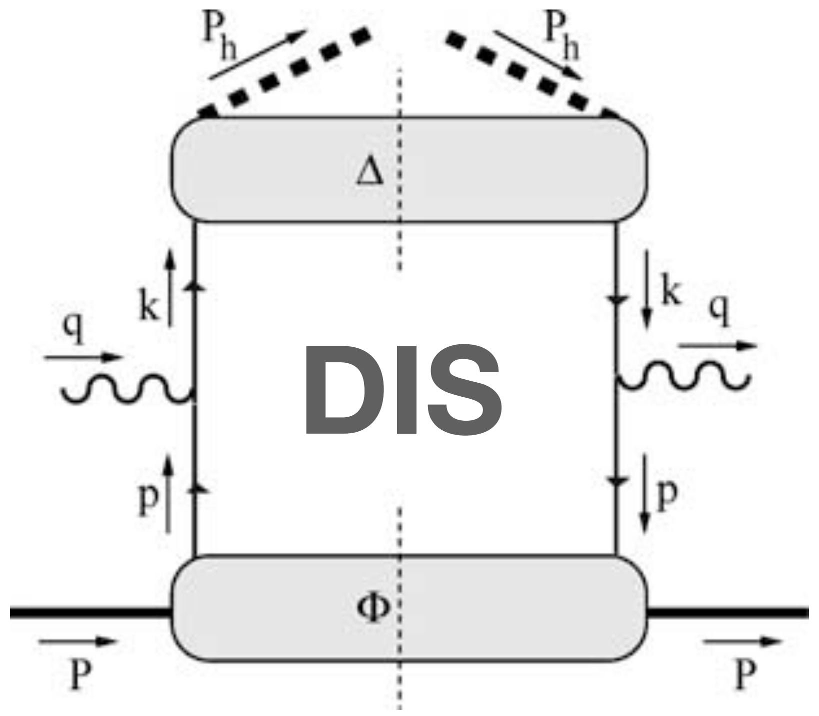
# Di–hadron fragmentation in reduced dimensionality

## 7th international workshop on transverse phenomena in hard processes

Gunar Schnell

STRONG-2020

# Transversity - a global approach



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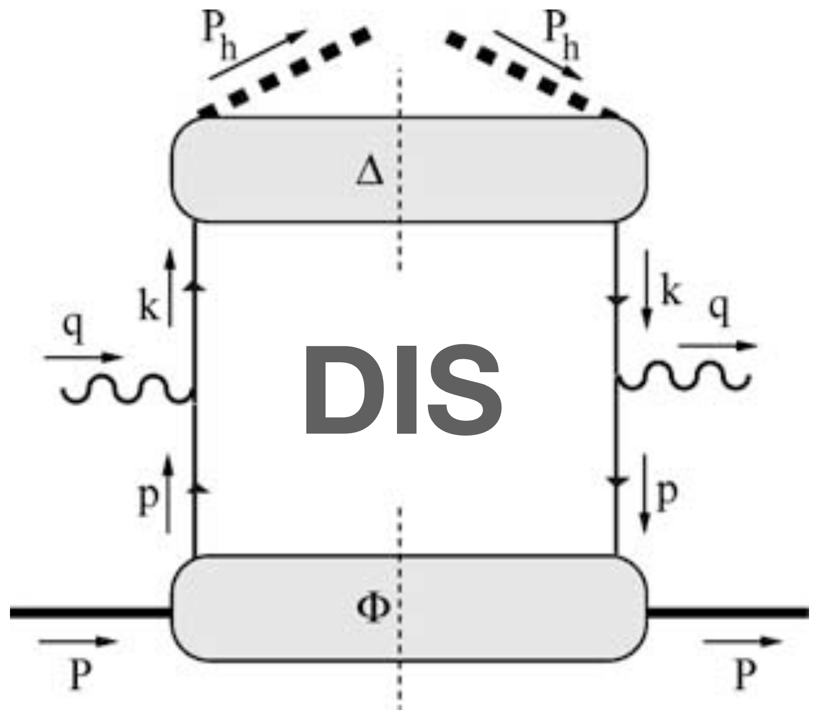

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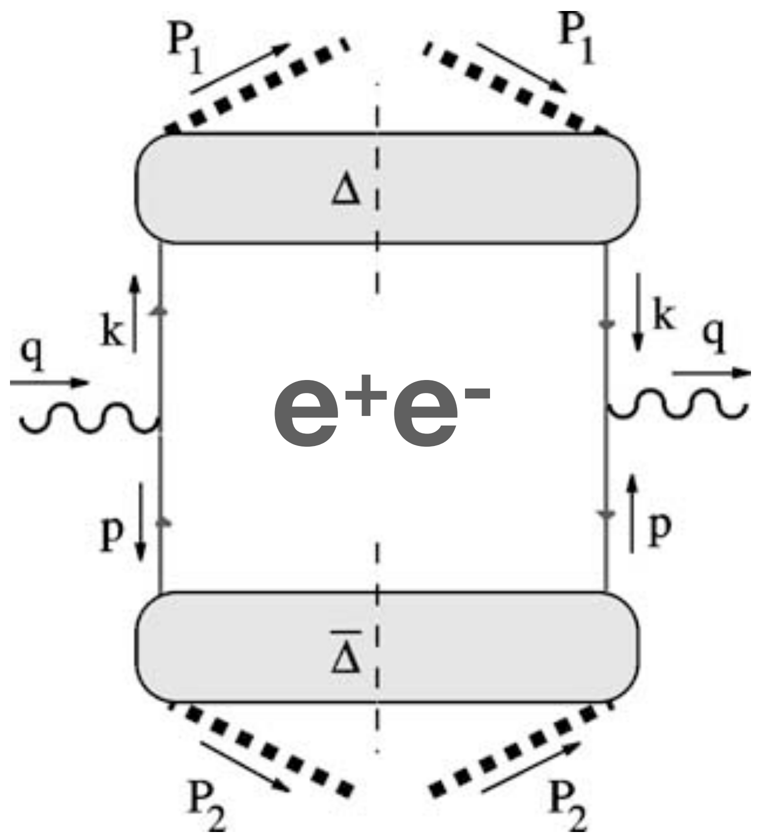

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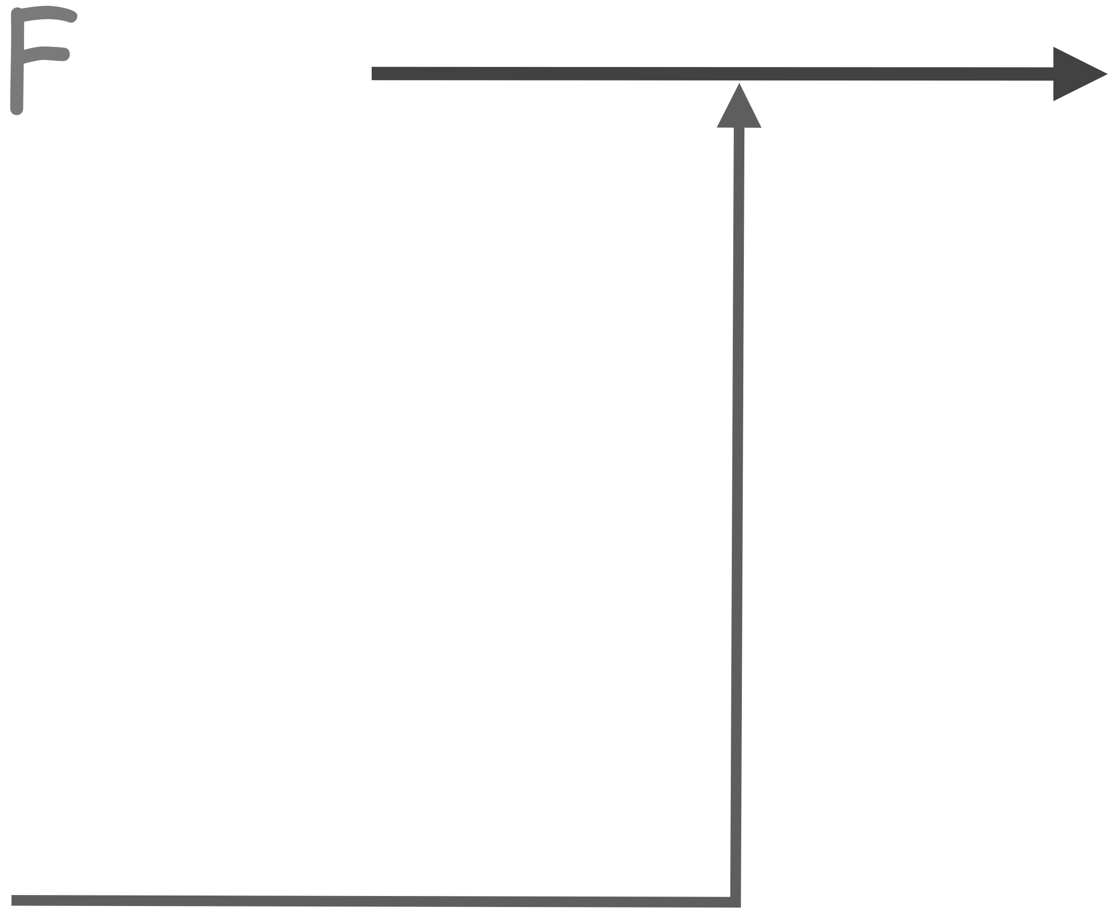

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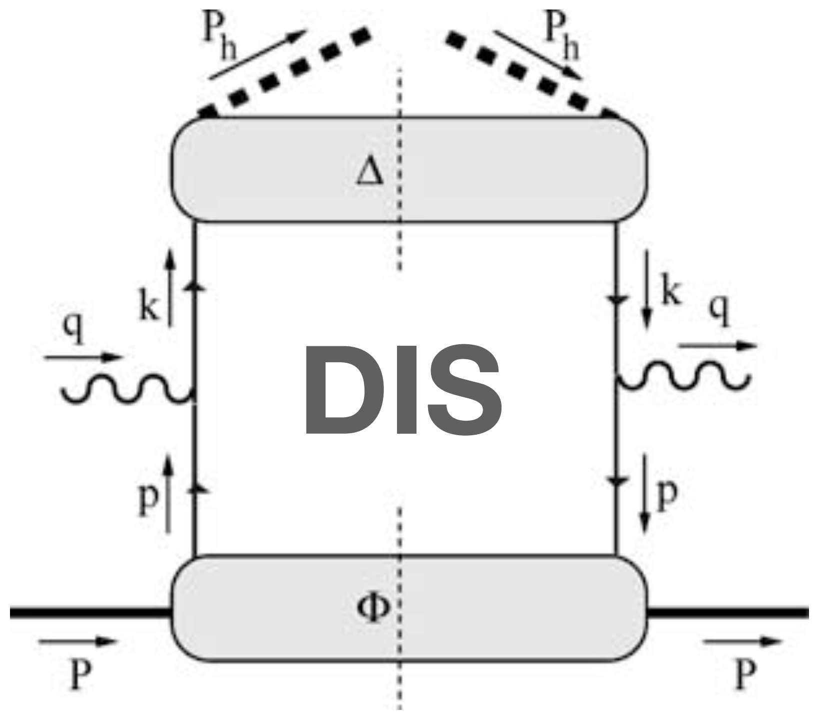

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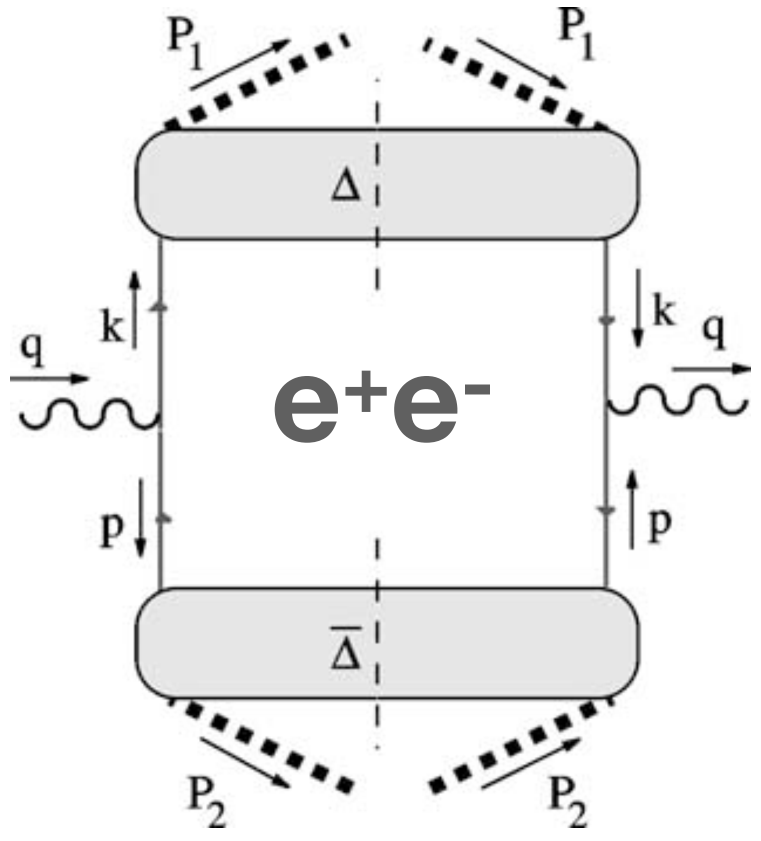

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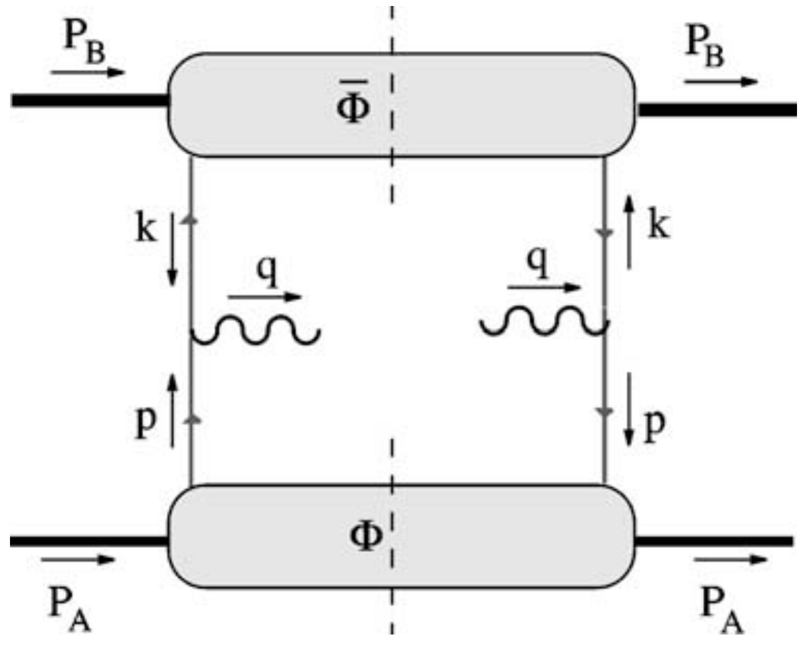

**Drell-Yan**

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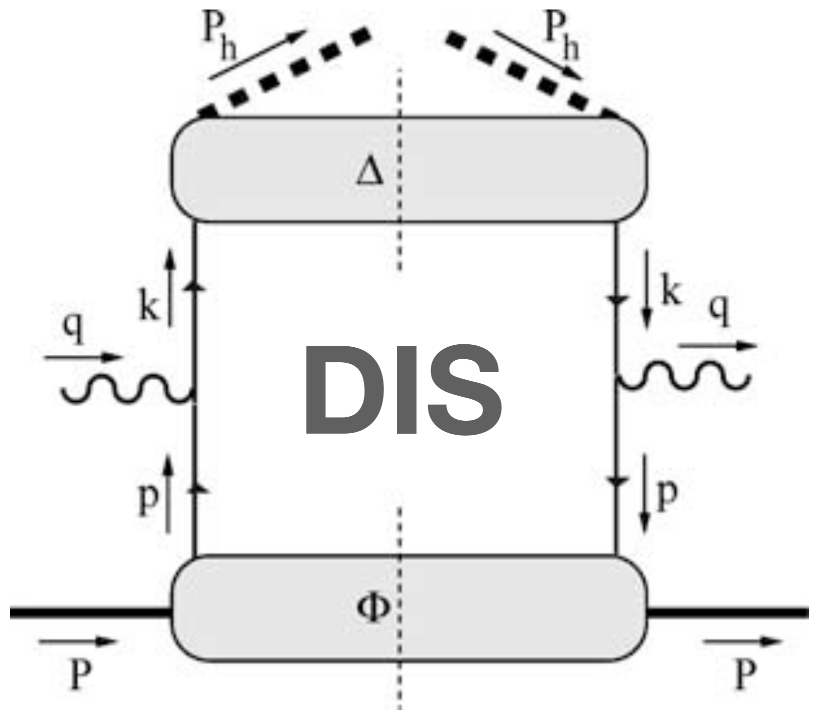
# Transversity - a global approach



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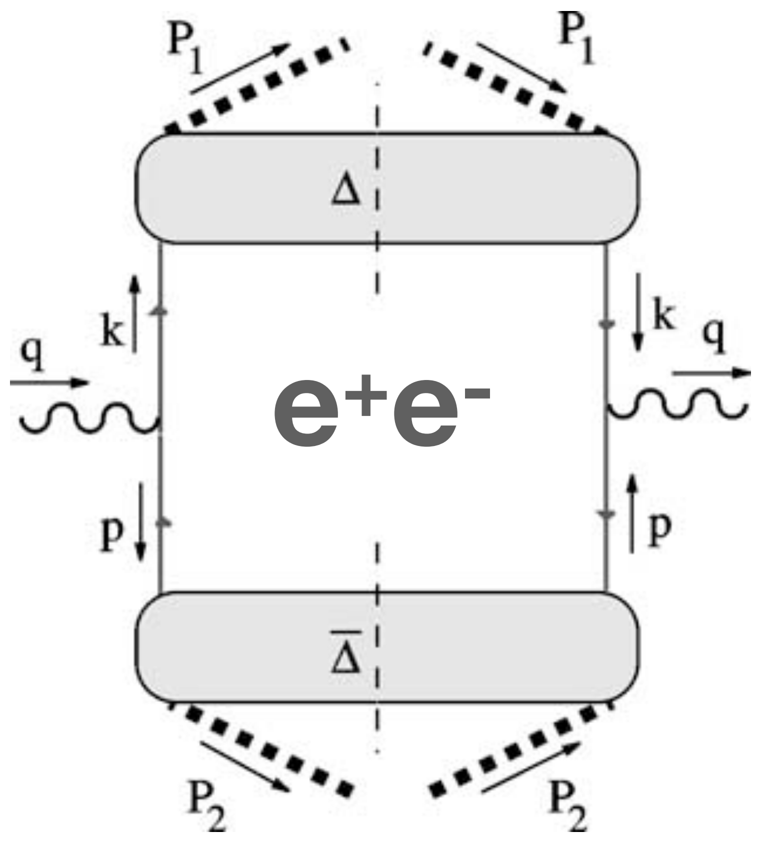

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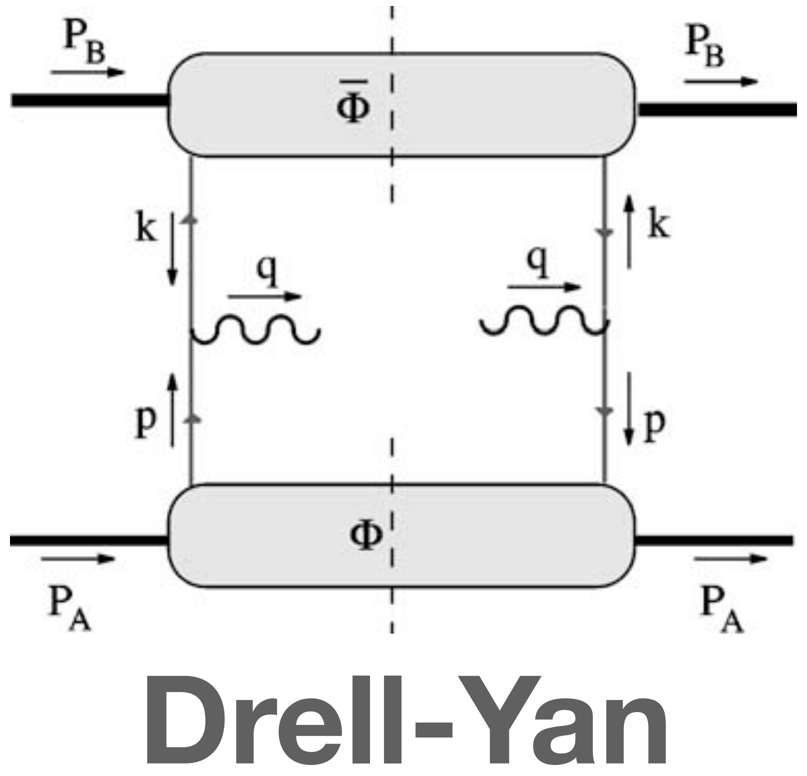

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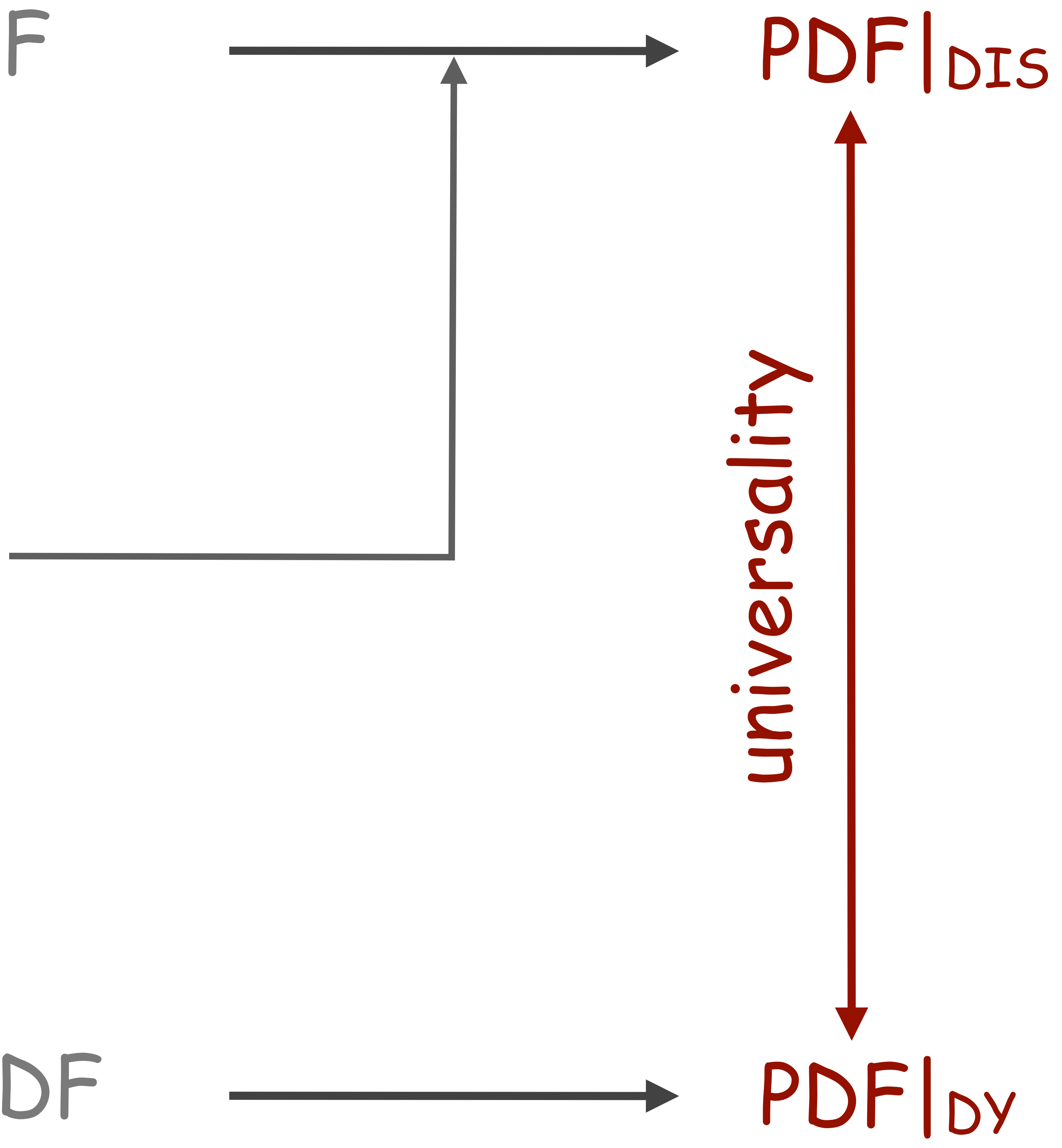

$\text{PDF}|_{\text{DIS}}$

universality

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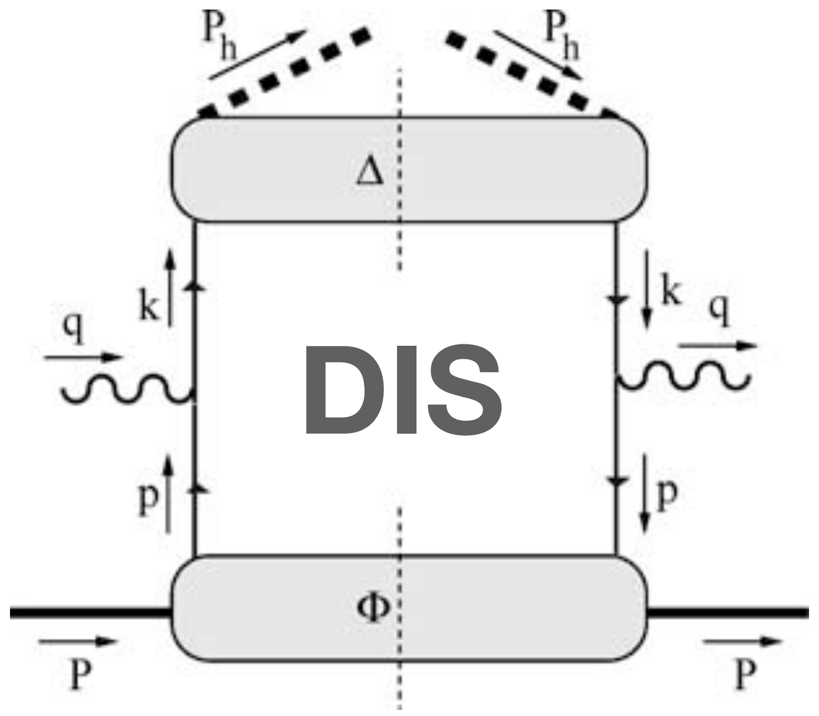

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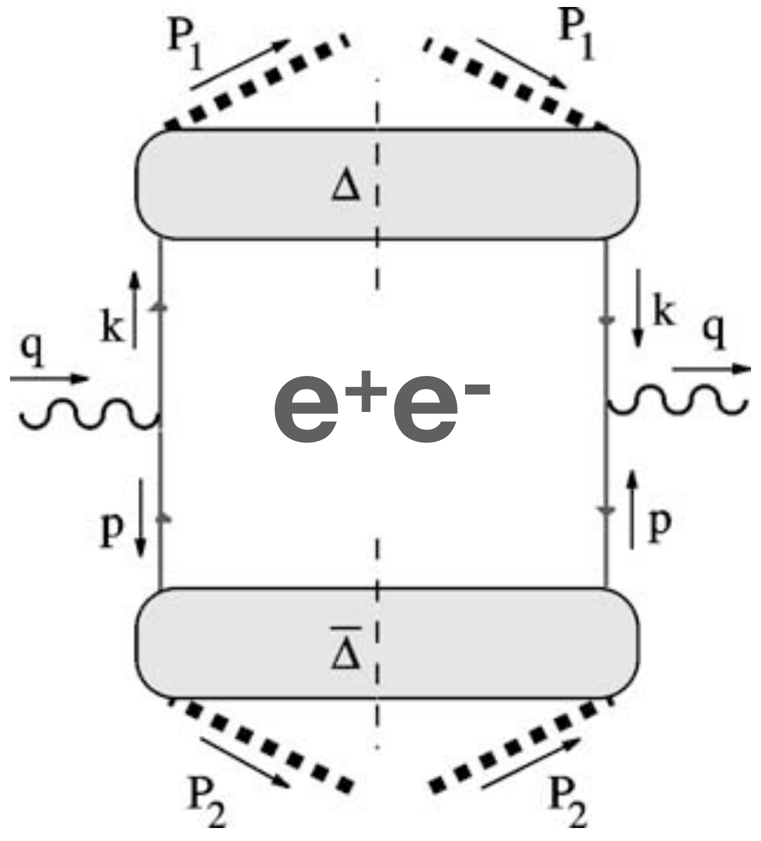
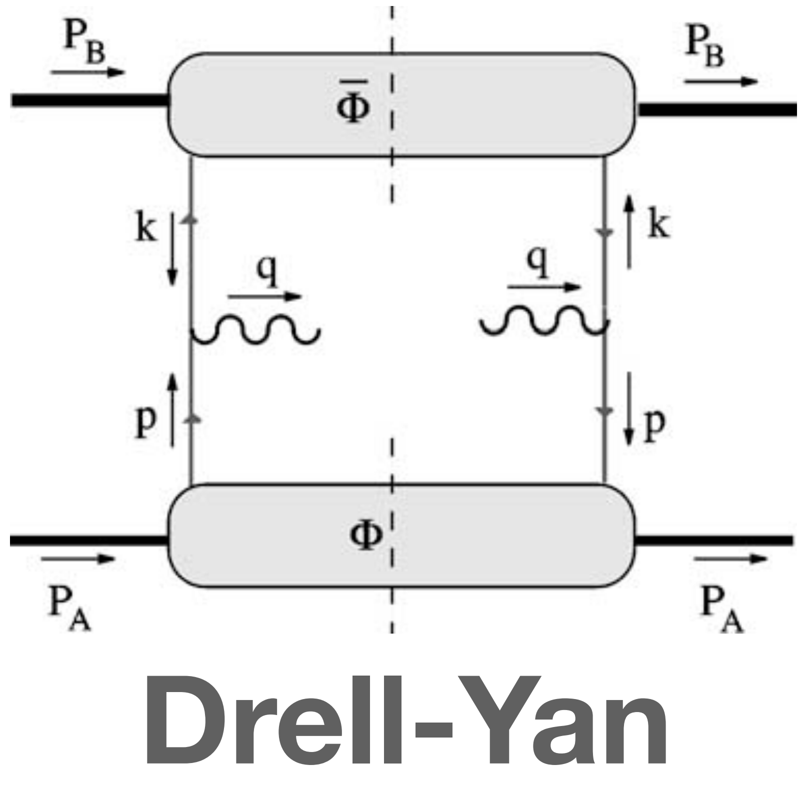


: FF

FFs also needed/ interesting for many other studies!



: PDF ⊗ PDF → PDF|DY

universality

# single-hadron TMD*) fragmentation functions

*) TMD … transverse-momentum dependent

| | | quark pol. | | |
|---|---|---|---|---|
| | | U | L | T |
| hadron pol. | U | $D_1$ | | $H_1^\perp$ |
| | L | | $G_1$ | $H_{1L}^\perp$ |
| | T | $D_{1T}^\perp$ | $G_{1T}^\perp$ | $H_1$ $H_{1T}^\perp$ |

☚ relevant for unpolarized final state

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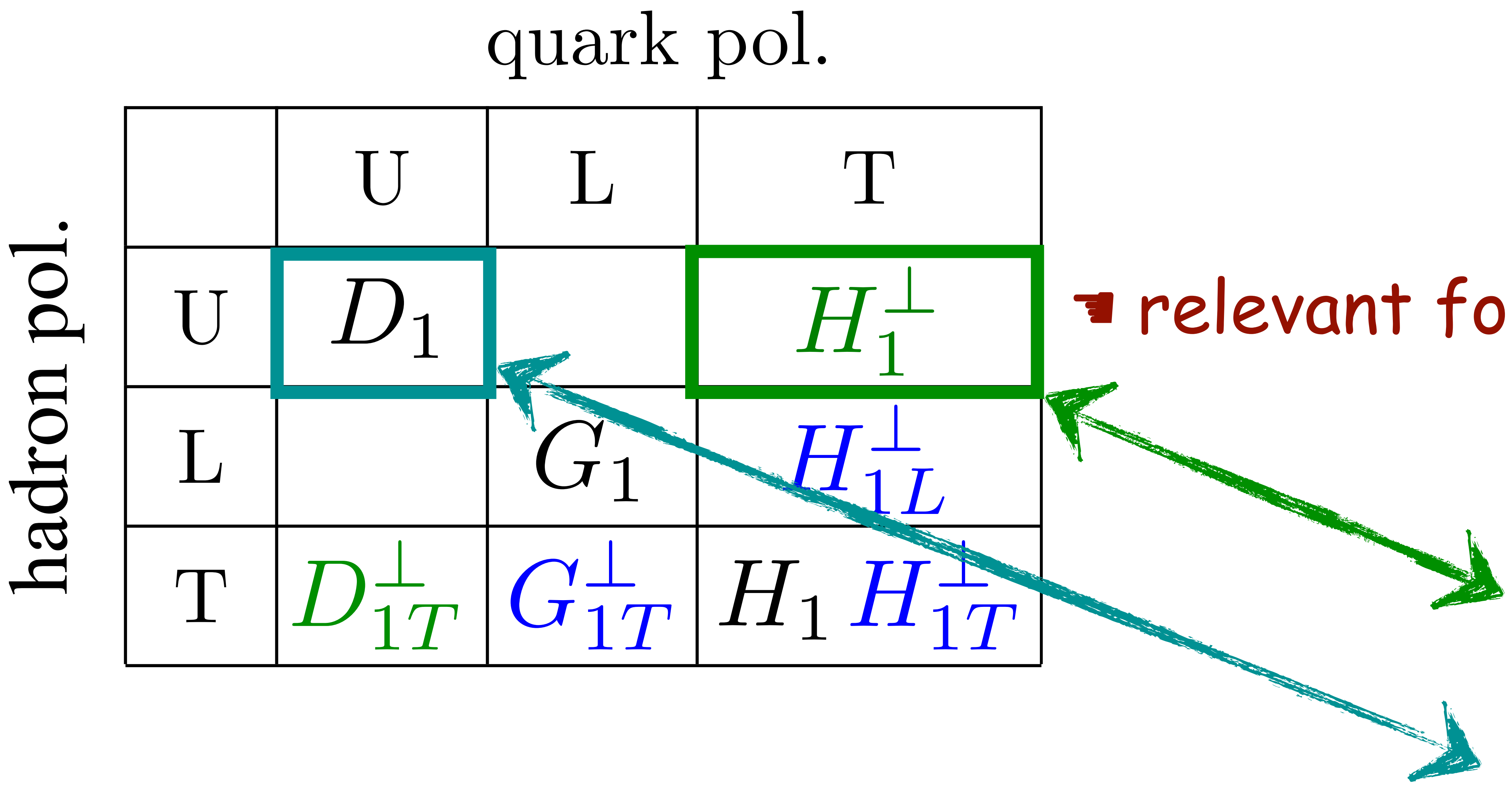

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☛ relevant for unpolarized final state

Collins FF: $H_1^{\perp,q\to h}$

ordinary FF: $D_1^{q\to h}$

FF … fragmentation function

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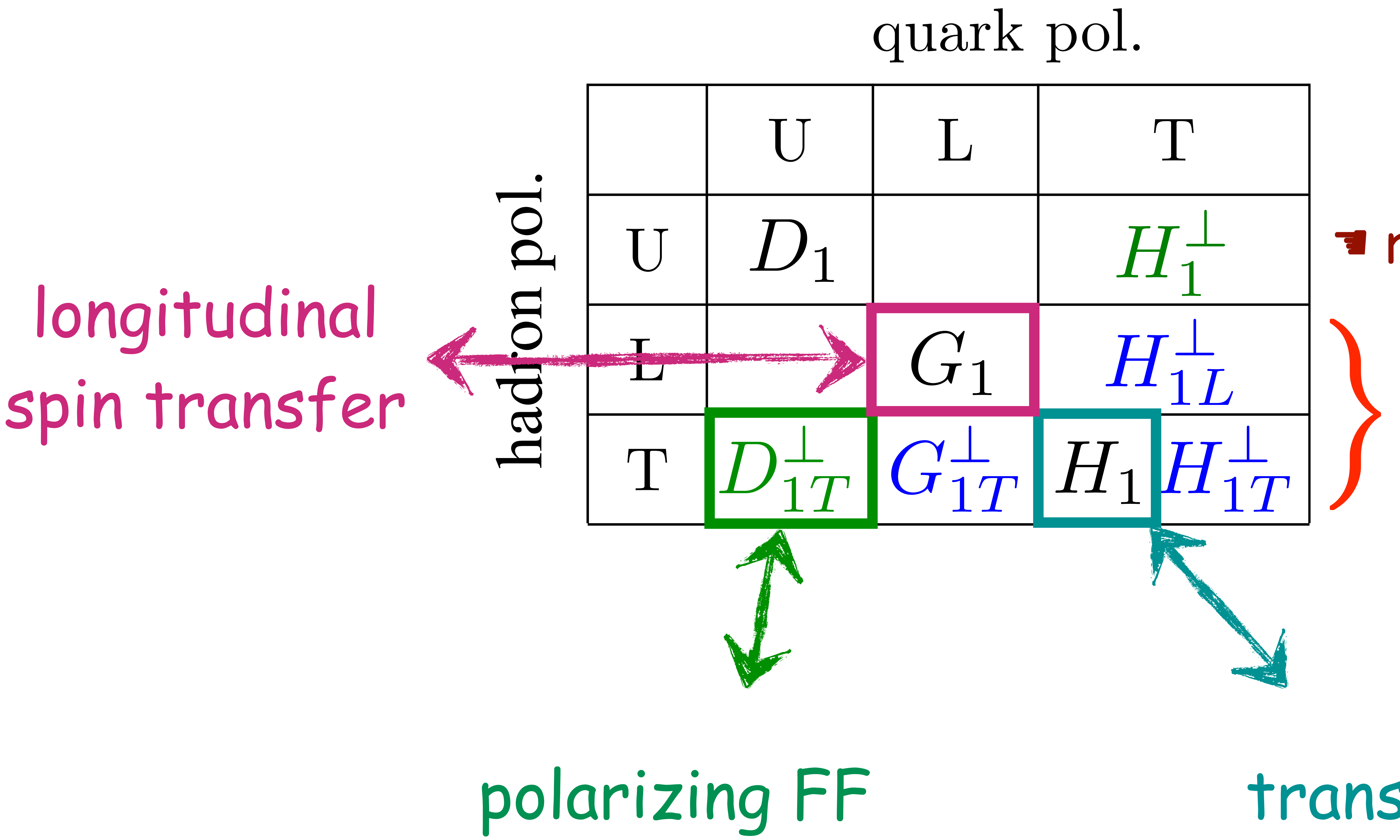

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☚ relevant for unpolarized final state

polarized final-state hadrons

longitudinal spin transfer

polarizing FF

transversity FF

FF … fragmentation function

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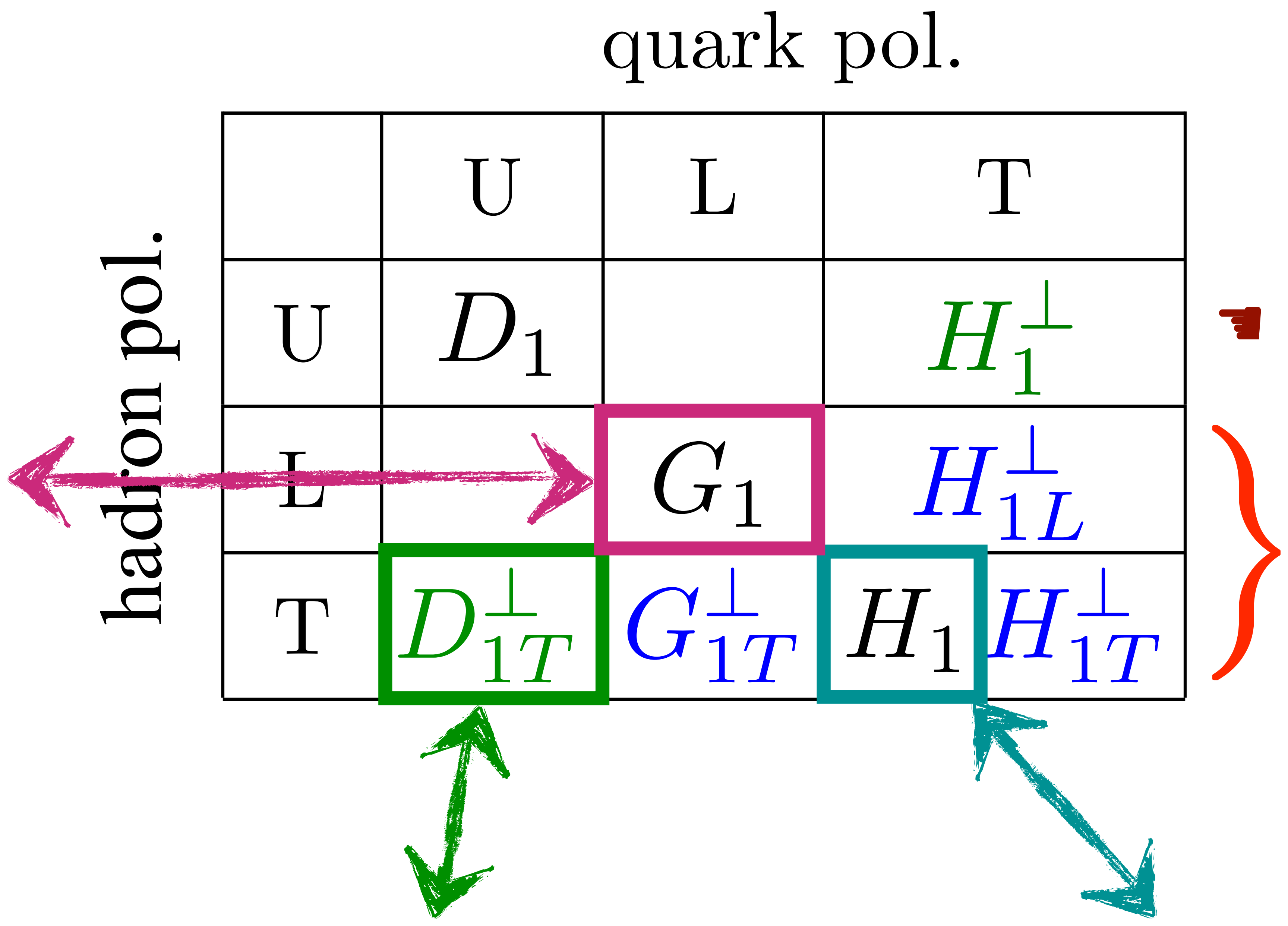

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☛ relevant for unpolarized final state

polarized final-state hadrons

longitudinal spin transfer

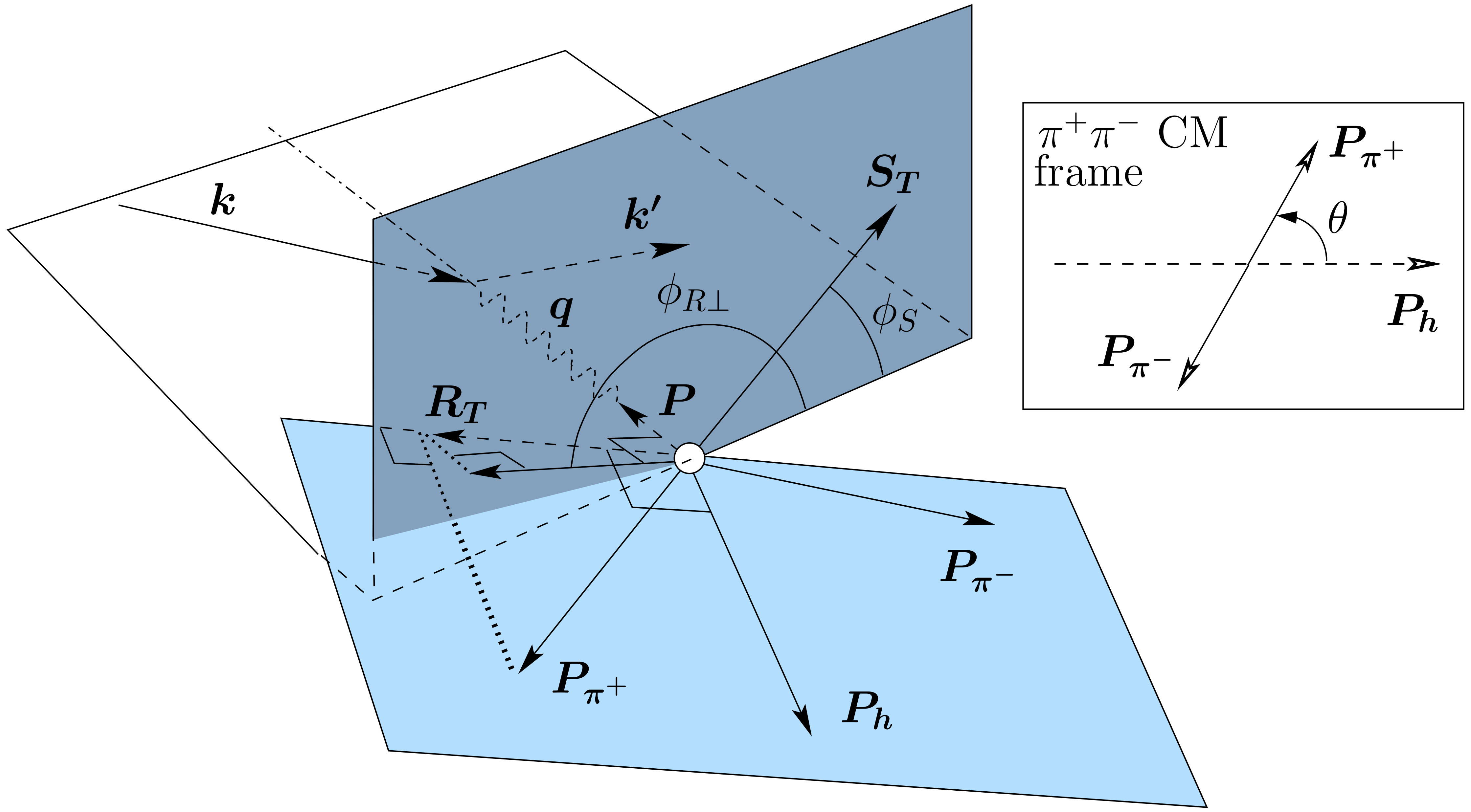
polarizing FF

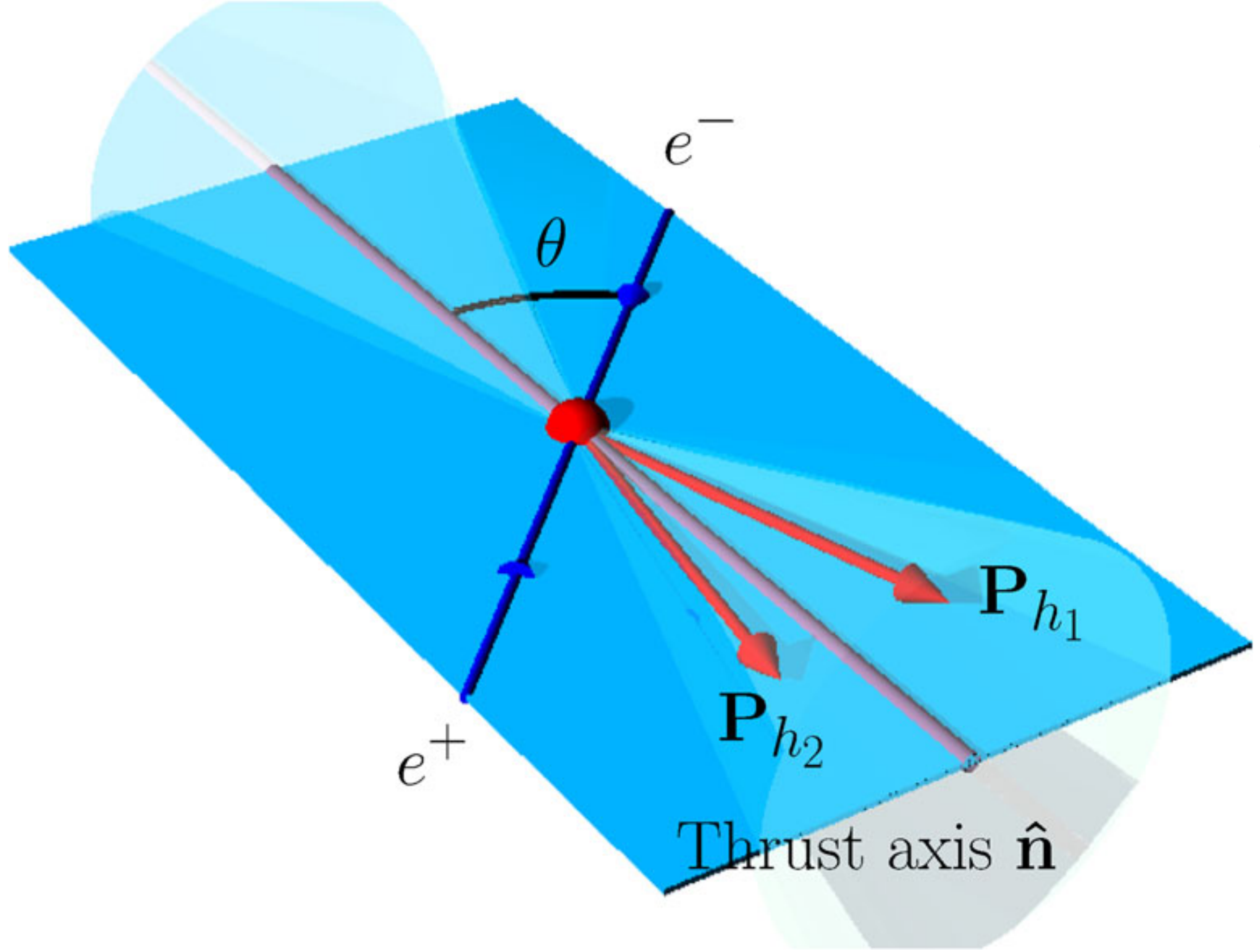
transversity FF

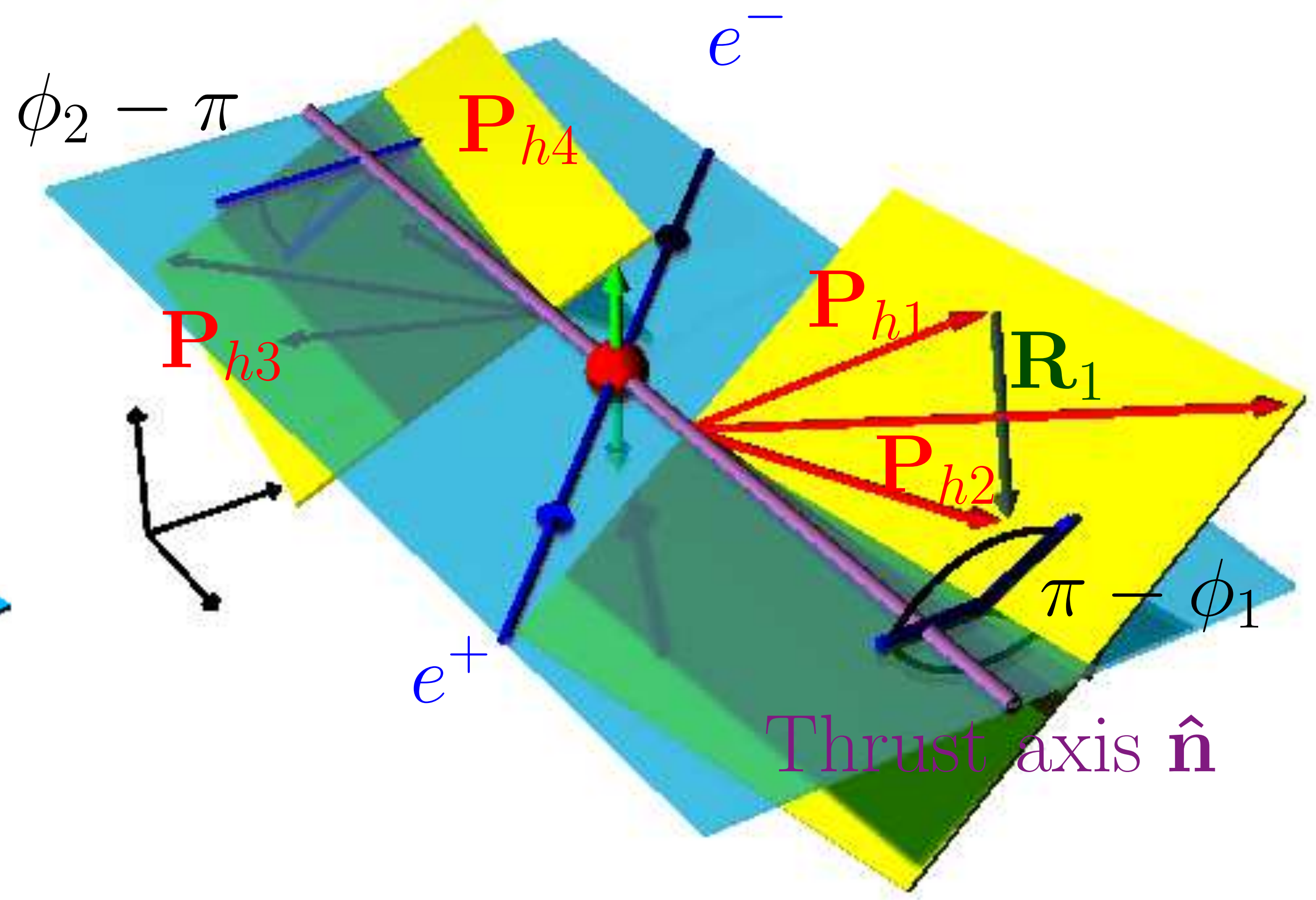
➟ FFs act as quark flavor-tagger and polarimeter

FF … fragmentation function

# hadron-pair production







- instead of looking at final-state hadron polarization:
  - use angular distribution of two hadrons to tag quark polarisation
  - dihadron fragmentation a la Collins, Heppelmann & Ladinsky [Nucl. Phys. B 420 (1994) 565]; Boer, Jacobs & Radici [Phys. Rev. D 67 (2003) 094003]

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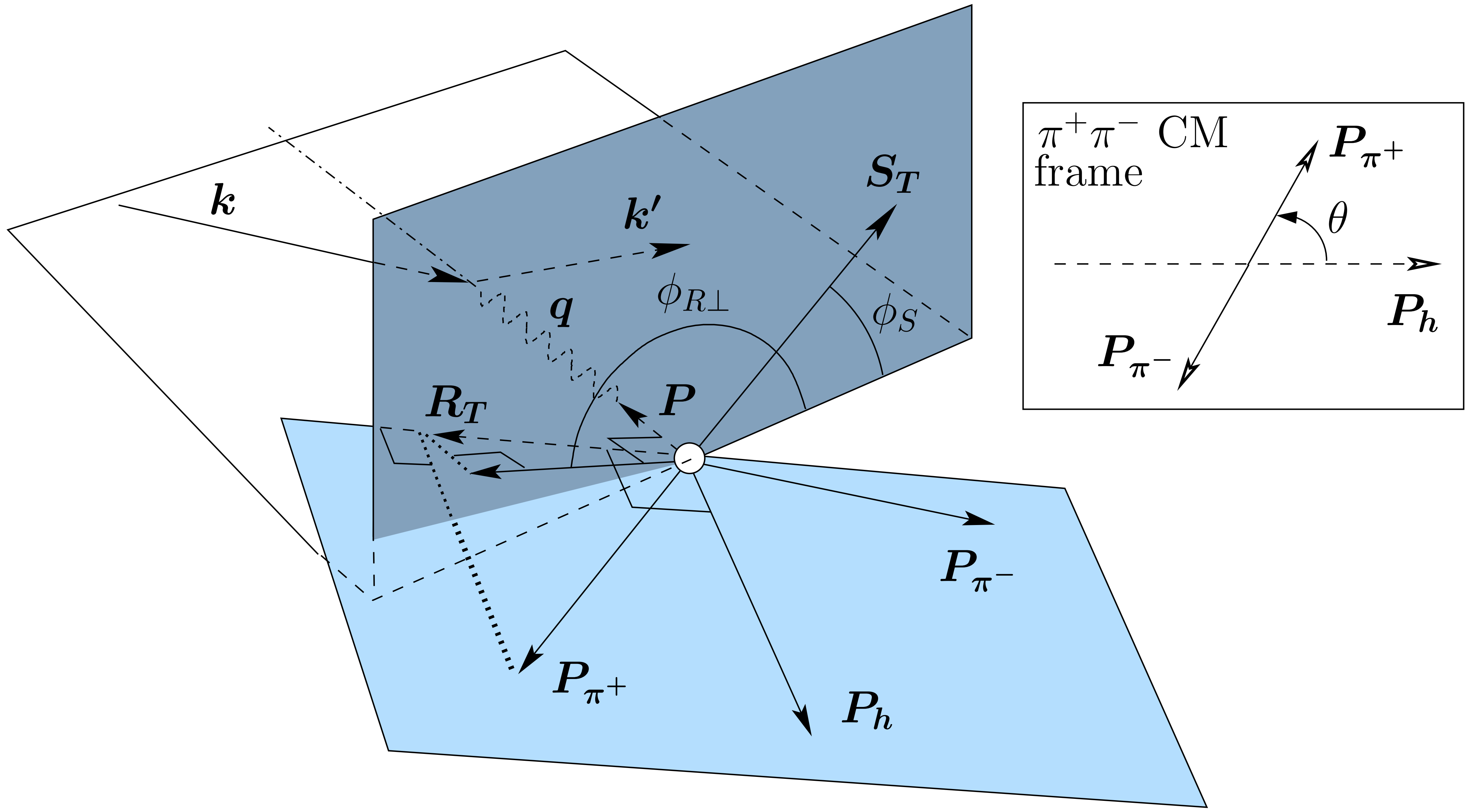

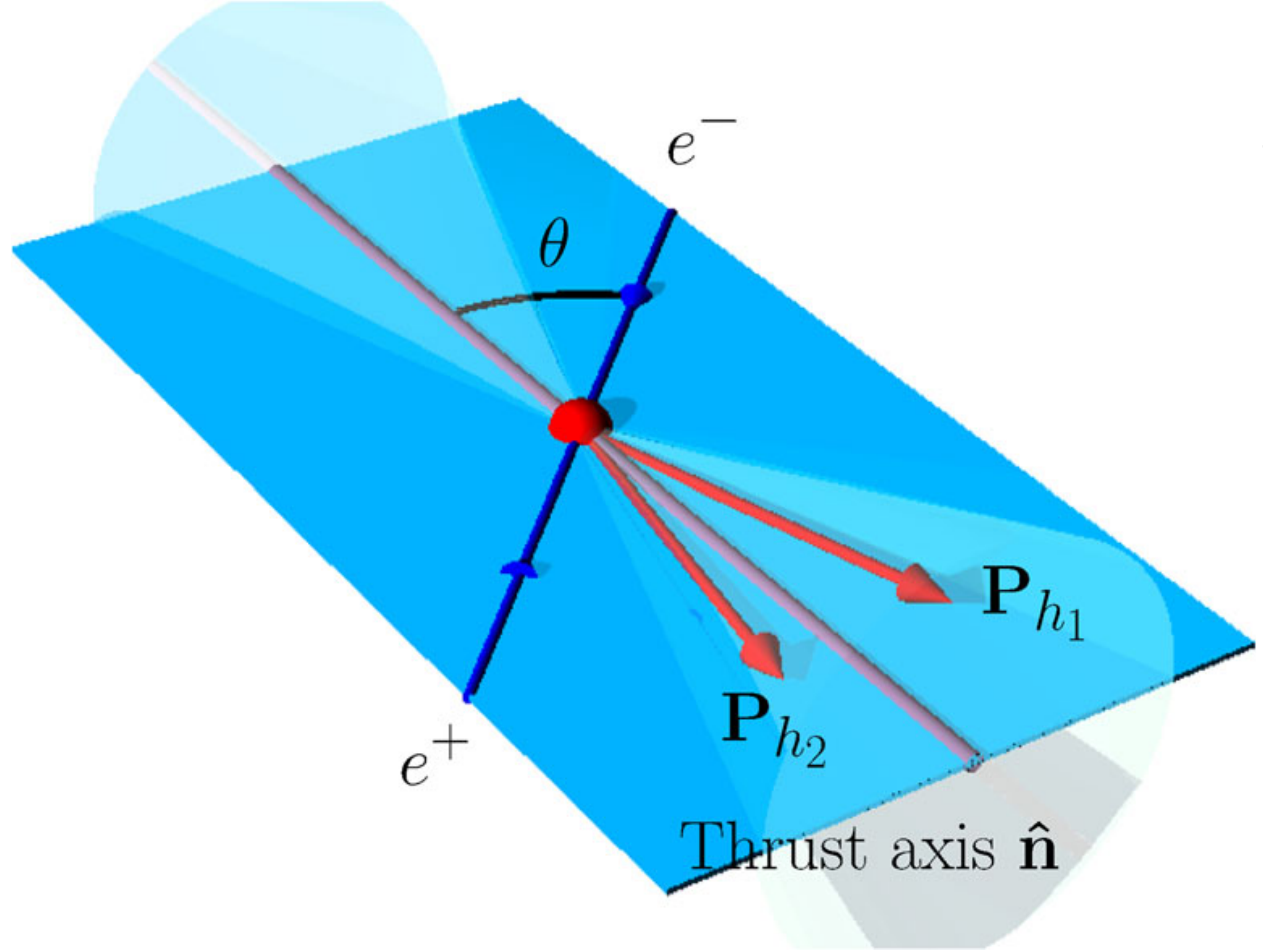

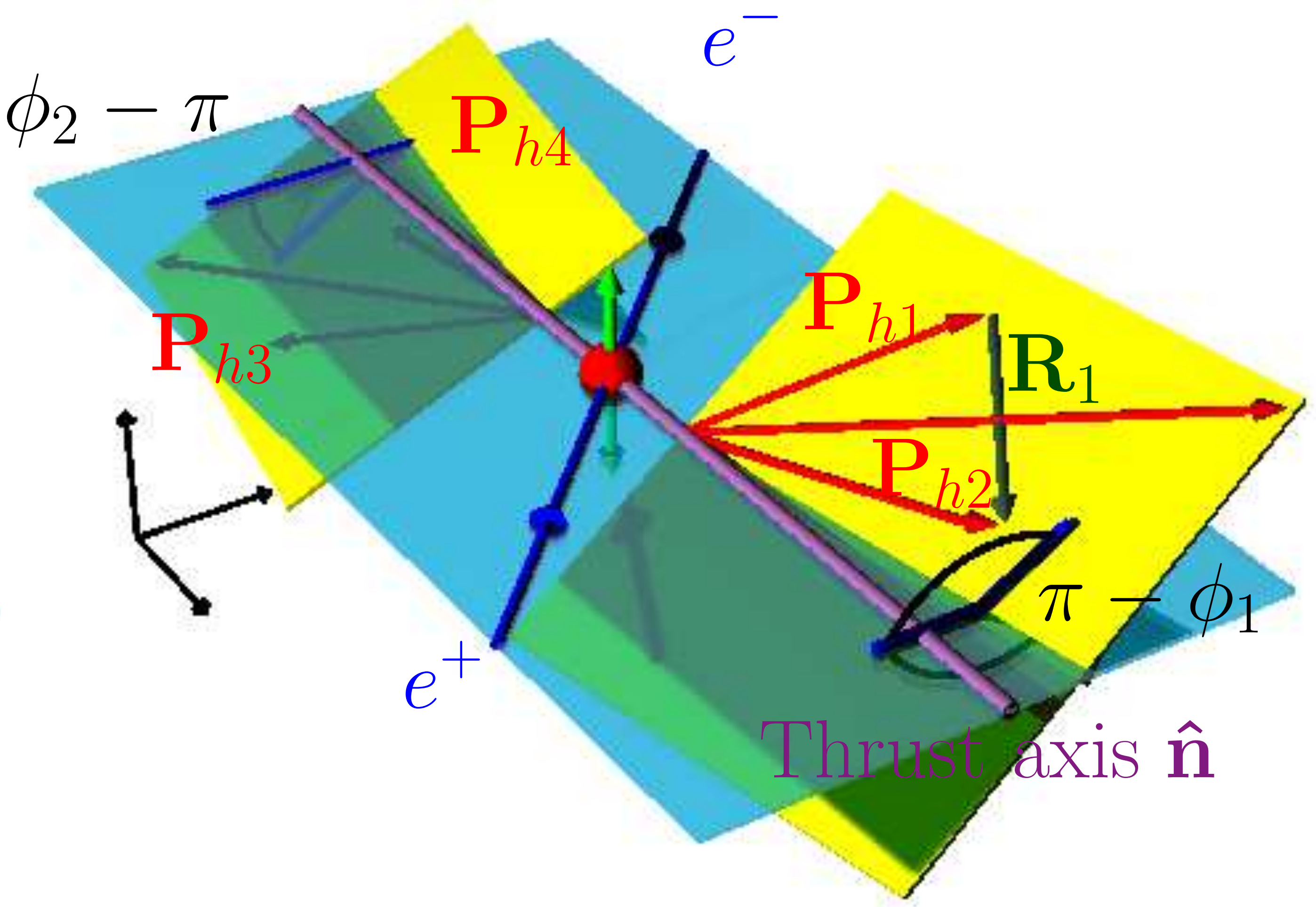

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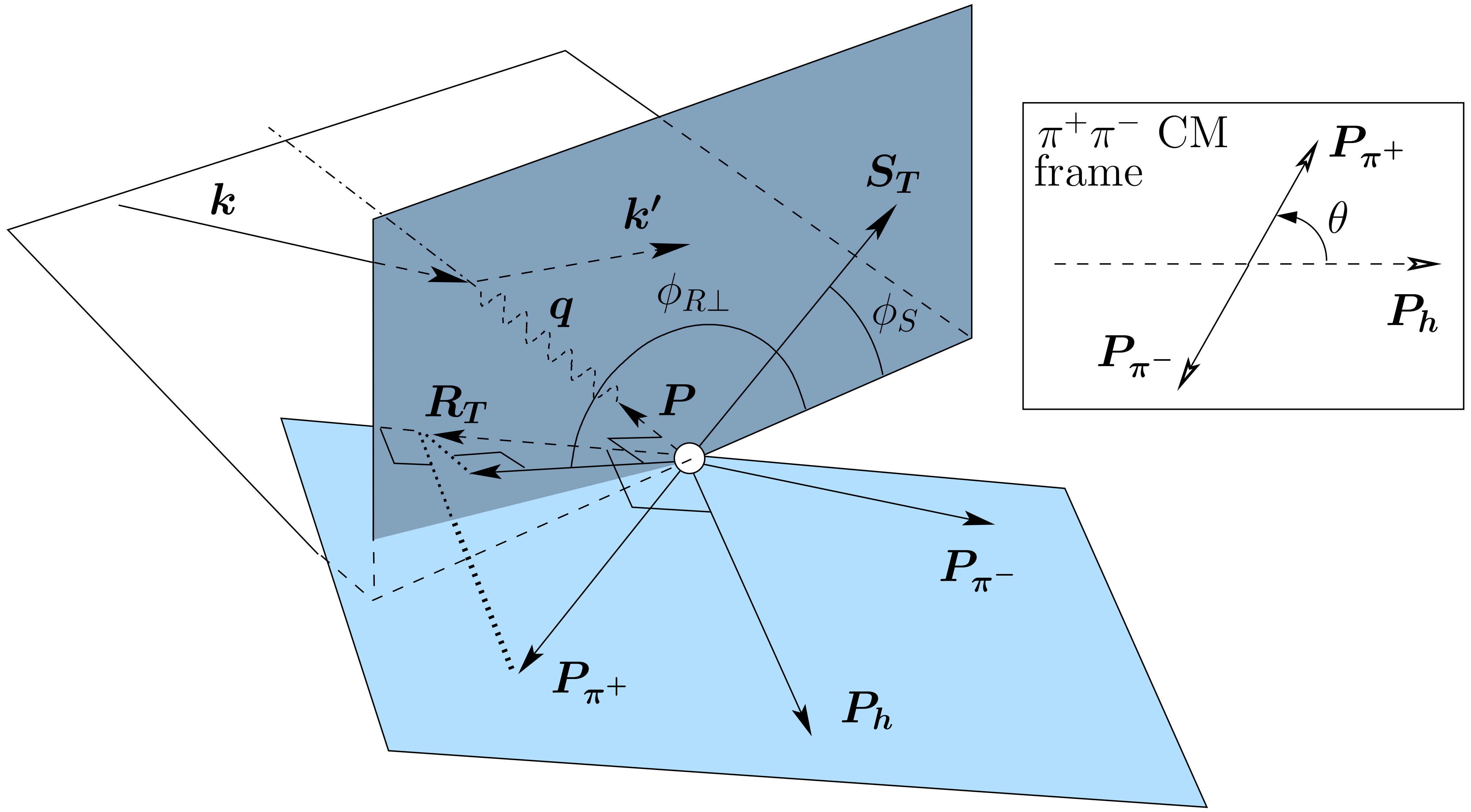
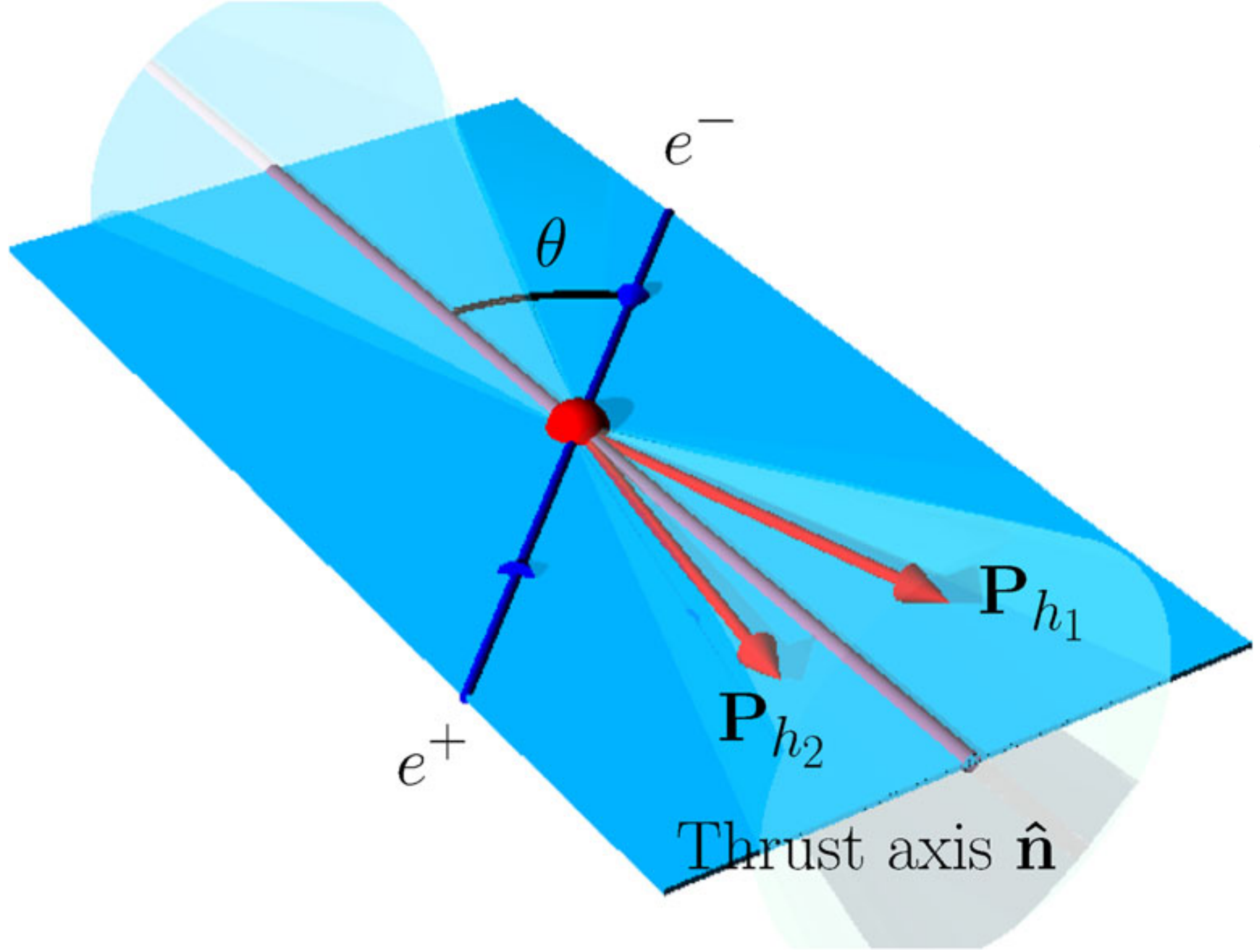
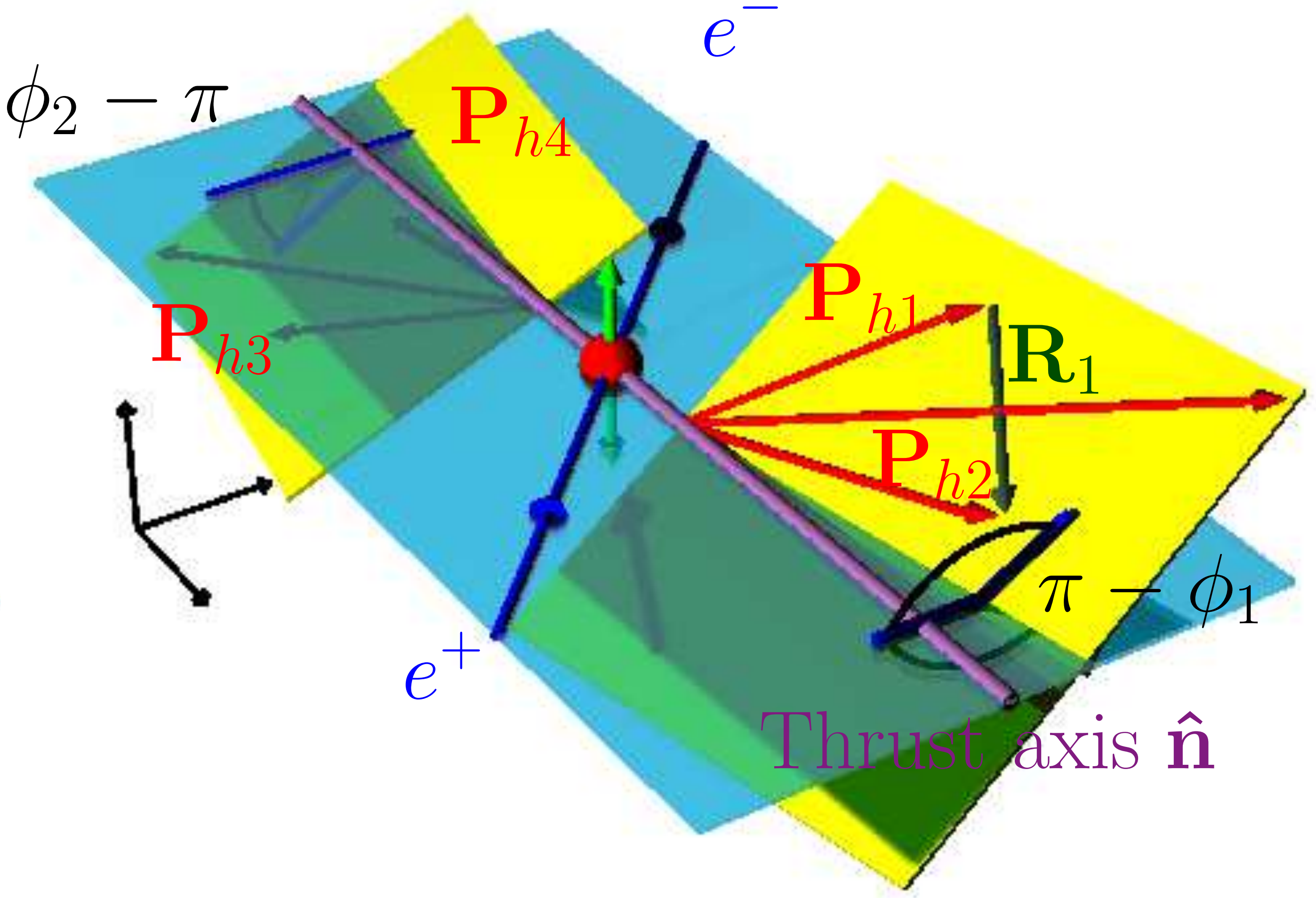
- dihadron FFs: alternative path to extract (even collinear!) transversity
  - exploit orientation of hadron’s **relative** momentum, correlate with target polarization

# hadron-pair production



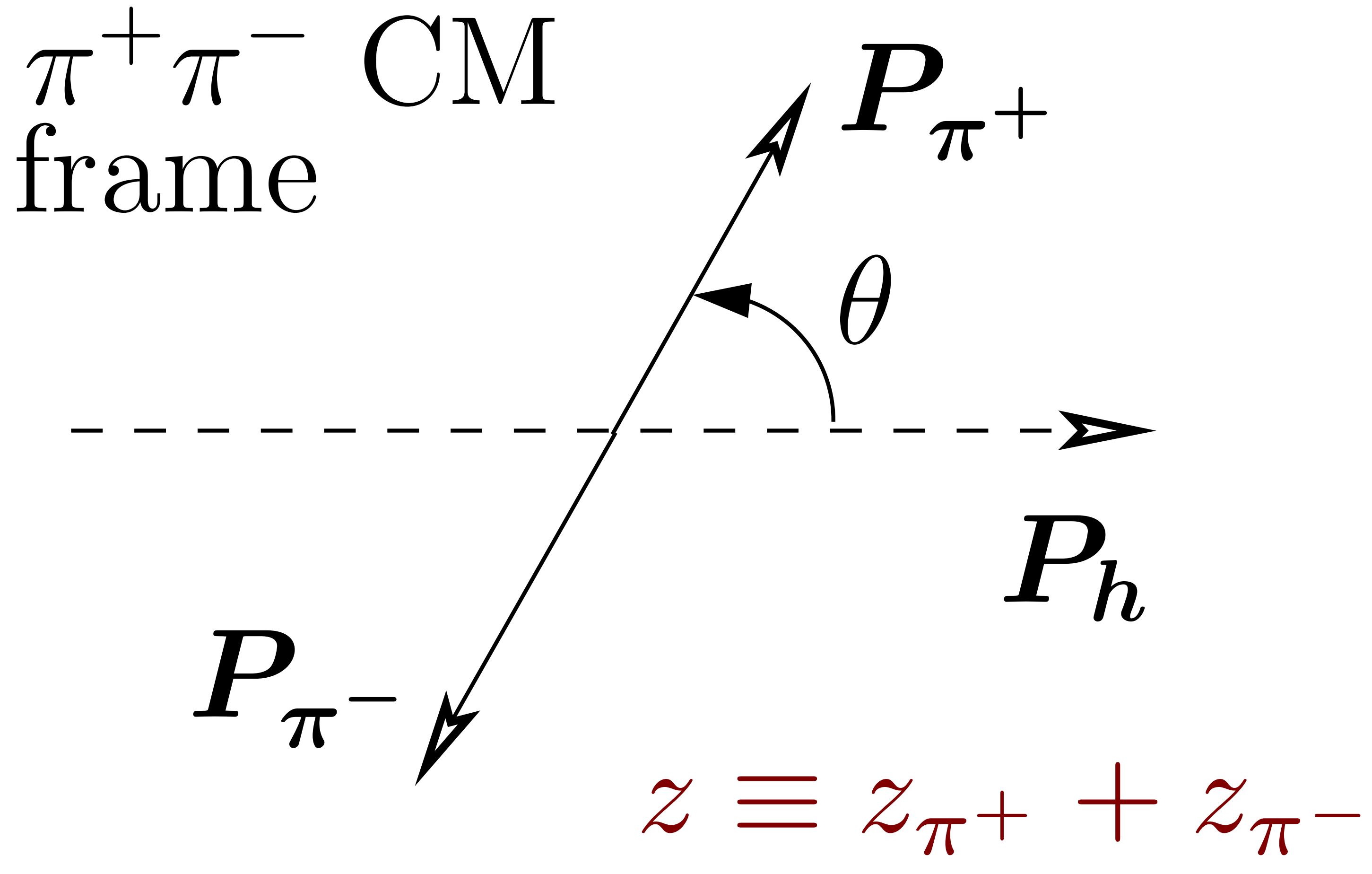




- **complication**: semi-inclusive DIS cross section with transverse-target polarization
  now **differential in 9(!) variables**
  (even more for back-to-back hadron pairs in e⁺e⁻ annihilation)
  - first step: consider only **collinear** case -> 7 variables

$$\frac{\mathrm{d}^7\sigma_{UT}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_S\,\mathrm{d}\phi_{R\perp}\,\mathrm{d}\cos\theta\,\mathrm{d}M_{\pi\pi}} = -|\boldsymbol{S_T}|\sum_q \frac{\alpha^2 e_q^2}{2\pi s x y^2}(1-y)\frac{1}{2}\sqrt{1-4\frac{M_\pi^2}{M_{\pi\pi}^2}}\,\sin(\phi_{R\perp}+\phi_S)\sin\theta\;h_1^q(x)H_{1,q}^{\sphericalangle}(z,M_{\pi\pi},\cos\theta)$$

# partial-wave expansion



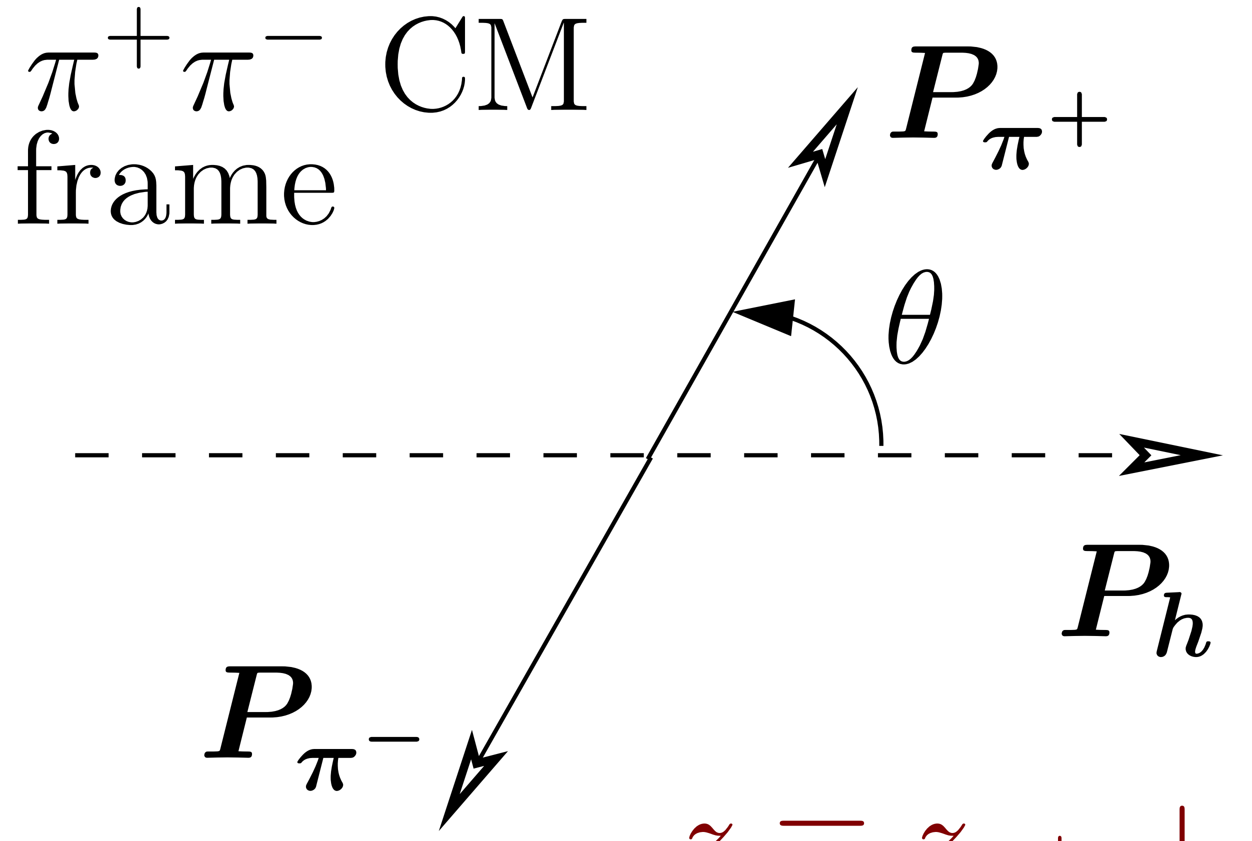
$z \equiv z_{\pi^+} + z_{\pi^-}$

- Legendre expansion in cos $\theta$:

$$\frac{\mathrm{d}^7\sigma_{UU}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_S\,\mathrm{d}\phi_{R\perp}\,\mathrm{d}\!\cos\theta\,\mathrm{d}M_{\pi\pi}} = \sum_q \frac{\alpha^2 e_q^2}{2\pi s x y^2}(1-y+\frac{y^2}{2})f_1^q(x)D_{1,q}(z,M_{\pi\pi},\cos\theta)$$

$$D_{1,q}(z,M_{\pi\pi},\cos\theta) \simeq D_{1,q}(z,M_{\pi\pi}) + D_{1,q}^{sp}(z,M_{\pi\pi})\cos\theta + D_{1,q}^{pp}(z,M_{\pi\pi})\frac{1}{4}(3\cos^2\theta-1)$$

# partial-wave expansion



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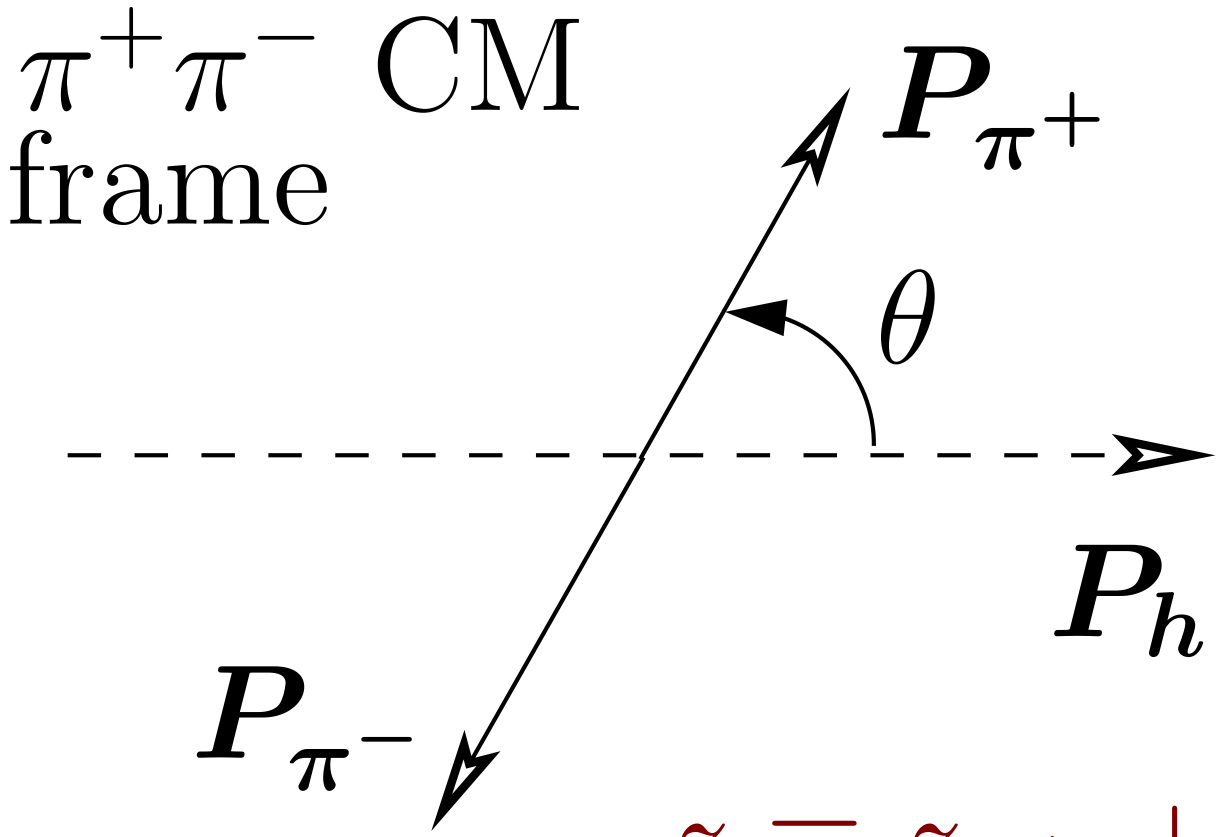
$$\frac{\mathrm{d}^7\sigma_{UT}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_S\,\mathrm{d}\phi_{R\perp}\,\mathrm{dcos}\,\theta\,\mathrm{d}M_{\pi\pi}} = -|\boldsymbol{S_T}| \sum_q \frac{\alpha^2 e_q^2}{2\pi s x y^2}(1-y)\frac{1}{2}\sqrt{1-4\frac{M_\pi^2}{M_{\pi\pi}^2}}\, \sin(\phi_{R\perp}+\phi_S) \sin\theta\ h_1^q(x) H_{1,q}^{\sphericalangle}(z, M_{\pi\pi}, \cos\theta)$$

$$H_{1,q}^{\sphericalangle}(z, M_{\pi\pi}, \cos\theta) \simeq H_{1,q}^{\sphericalangle,sp}(z, M_{\pi\pi}) + H_{1,q}^{\sphericalangle,pp}(z, M_{\pi\pi}) \cos\theta$$

# partial-wave expansion



$z \equiv z_{\pi^+} + z_{\pi^-}$

- Legendre expansion in cos $\theta$:

$$\frac{\mathrm{d}^7\sigma_{UU}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_S\,\mathrm{d}\phi_{R\perp}\,\mathrm{dcos}\,\theta\,\mathrm{d}M_{\pi\pi}} = \sum_q \frac{\alpha^2 e_q^2}{2\pi s x y^2}(1-y+\frac{y^2}{2})f_1^q(x)D_{1,q}(z, M_{\pi\pi}, \cos\theta)$$

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- next step: integration over cos $\theta$ -> 6 remaining variables and less FFs to worry about

# partial-wave expansion

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$$\pi\pi)\frac{1}{4}(3\cos^2\theta - 1)$$

$$\frac{\mathrm{d}^7\sigma_{UT}}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_S\,\mathrm{d}\phi_{R\perp}\,\mathrm{dcos}\,\theta\,\mathrm{d}M_{\pi\pi}} = -|S$$

$$y^2(1-y)\frac{1}{2}\sqrt{1-4\frac{M_\pi^2}{M_{\pi\pi}^2}}\sin(\phi_{R\perp}+\phi_S)\sin\theta\; h_1^q(x) H_{1,q}^{\sphericalangle}(z, M_{\pi\pi}, \cos\theta)$$

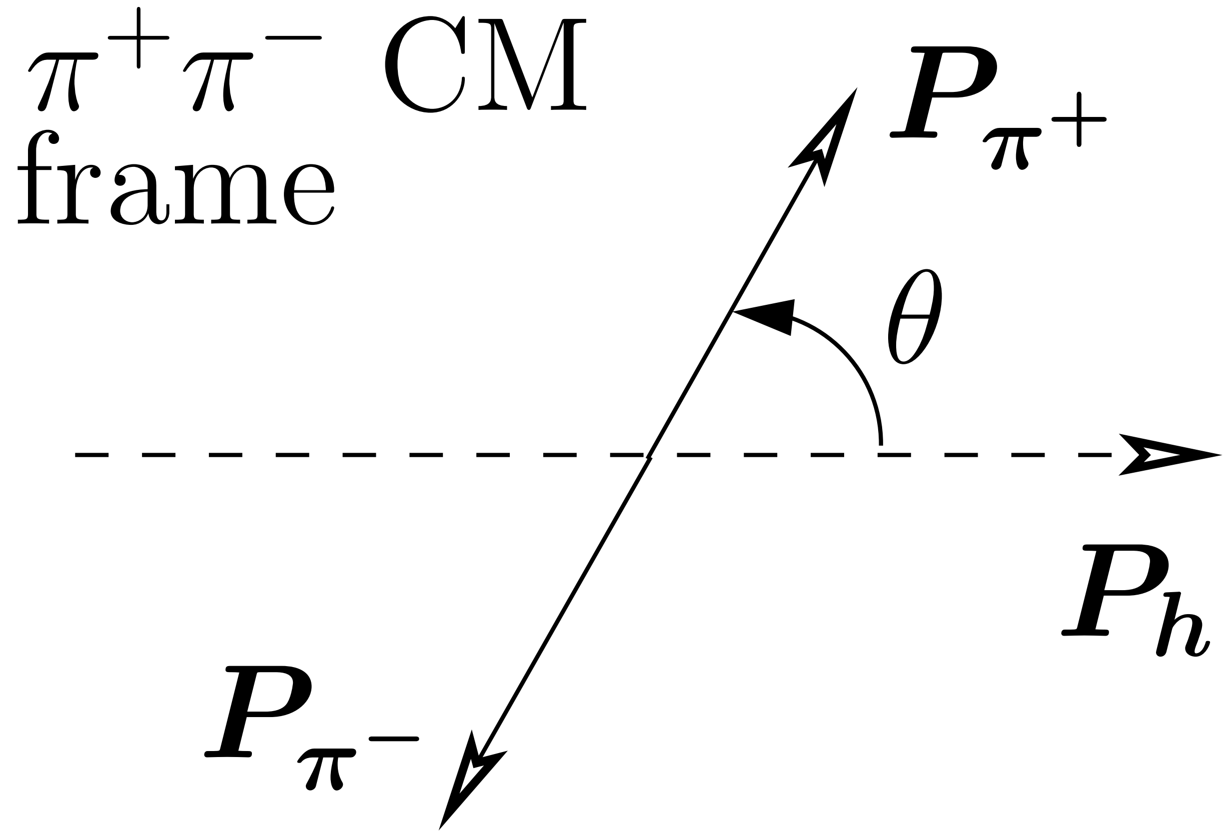
$$H^{\sphericalangle}$$

$$) \simeq H_{1,q}^{\sphericalangle,sp}(z, M_{\pi\pi}) + H_{1,q}^{\sphericalangle,pp}(z, M_{\pi\pi})\cos\theta$$

- p: integration over cos $\theta$ -> 6 remaining variables and less FFs to worry about

CAUTION: experimental constraints limit acceptance in polar angle!

# simple case study: $e^+e^-$ annihilation



basic assumptions:

- for simplicity: dihadron pair with equal-mass hadrons, e.g., pions
- $e^+e^-$ annihilation, thus energy fraction z translates directly to energy/momentum of particles/system as primary energy is “fixed” (-> simplifies Lorentz boost)
- without loss of generality, focus on B factory and use primary quark energy $E_0$ = 5.79GeV
- **minimum energy** of each pion in lab frame: 0.1 $E_0$ (i.e., $z_{min}$ = 0.1)

# application of Lorentz boost

- can easily apply Lorentz boost using the invariant mass of the dihadron M and its energy $zE_0$ to arrive at condition on $\theta$, e.g., polar angle of pions in center-of-mass frame:

$$\cos\theta \leq \frac{z-2z_{\text{min}}}{\sqrt{[(zE_0)^2-M^2)(M^2-4m_\pi^2)]}}E_0 M$$

- as both pions have to fulfil the constraint on the minimum energy:

$$\cos(\pi-\theta) = -\cos\theta \leq \frac{z-2z_{\text{min}}}{\sqrt{[(zE_0)^2-M^2)(M^2-4m_\pi^2)]}}E_0 M$$

thus:

$$|\cos\theta| \leq \frac{z-2z_{\text{min}}}{\sqrt{[(zE_0)^2-M^2)(M^2-4m_\pi^2)]}}E_0 M$$

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thus:

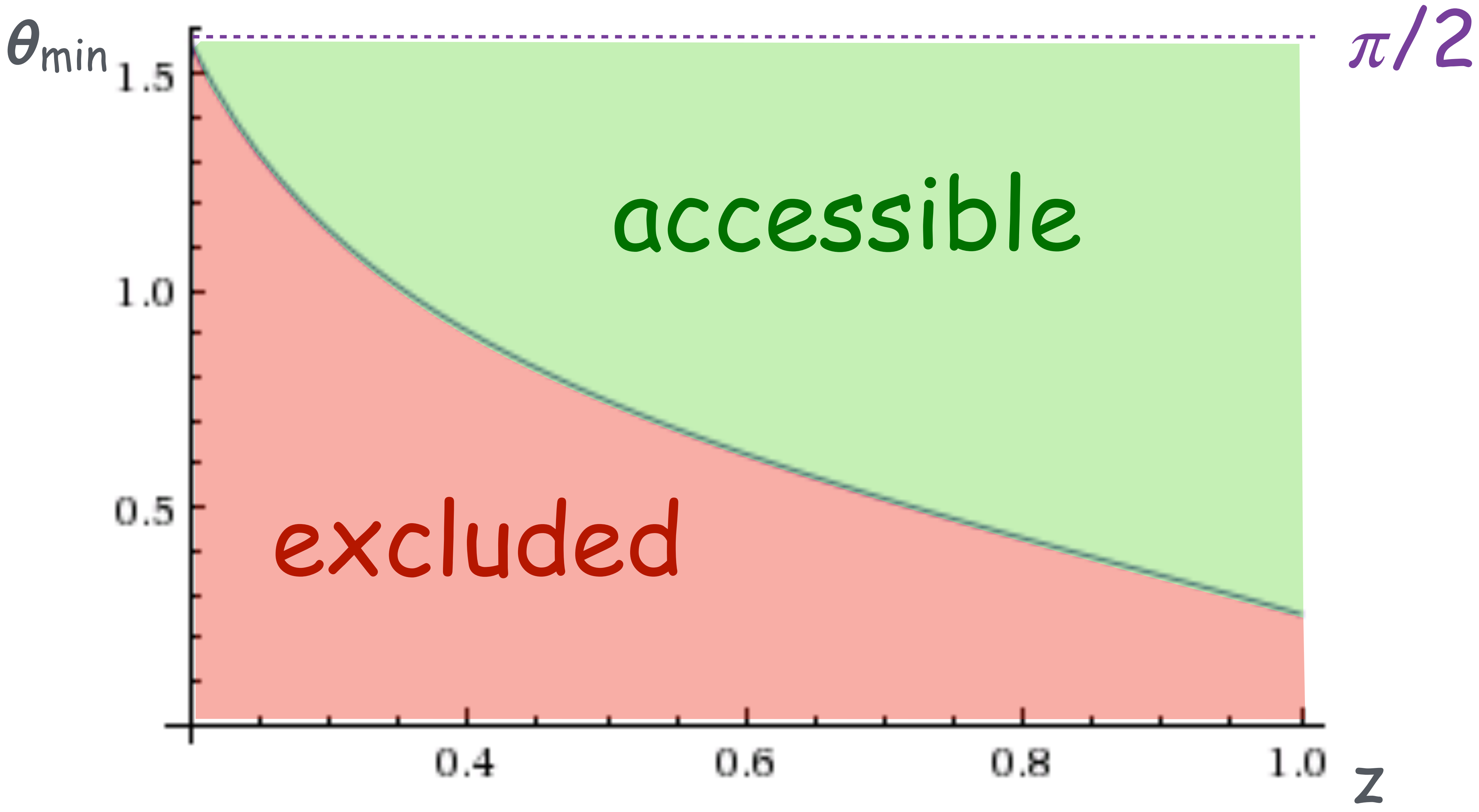
$$|\cos\theta| \leq \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M^2)(M^2 - 4m_\pi^2)]}} E_0 M$$

- translates to a symmetric range around $\pi/2$

(can be easily understood because at $\pi/2$ the pions will have both the same energy in the lab and easily pass the $z_{\min}$ requirement, while in the case of one pion going backward in the CMS, that pion will have less energy in the lab frame … and maybe too little)

# impact of $z_{min}$=0.1 on accepted polar range

- (again without loss of generality) let's assume M=0.5 GeV :



- all theta below curve (and **symmetrically** above its mirror curve relative to dashed line at $\pi/2$) are **excluded**
- clearly limited, especially at low z

# partial-wave expansion of dihadron FF

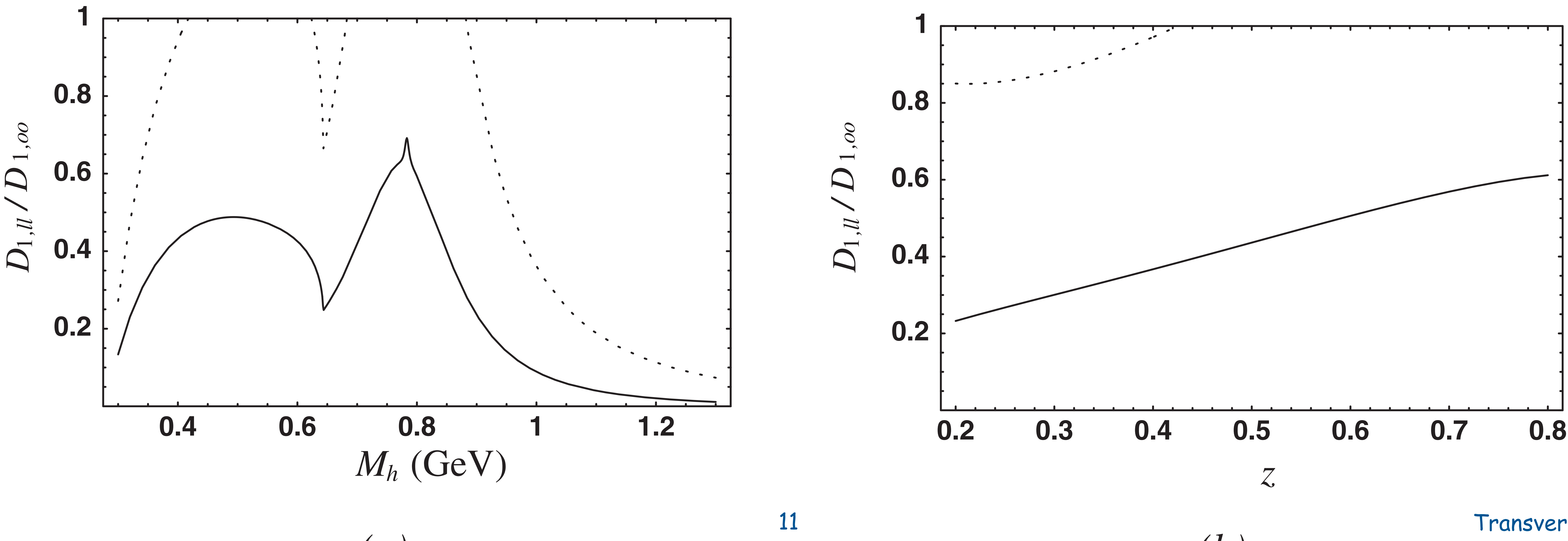
- partial-wave expansion worked out in Phys. Rev. D67 (2003) 094002
- for the particular case here, use Phys. Rev. D74 (2006) 114007, in particular Eq. (12) [and later on Figure 5]:

$$D_1^q(z, \cos\theta, M_h^2) \approx D_{1,oo}^q(z, M_h^2) + D_{1,ol}^q(z, M_h^2)\cos\theta + D_{1,ll}^q(z, M_h^2)\tfrac{1}{4}(3\cos^2\theta - 1), \qquad (12)$$

- it is the first contribution ($D_{1,oo}$) that is used in “collinear extraction” of transversity
    - it is also the only one surviving the integration over $\theta$
- $D_{1,ol}$ contribution vanishes upon integration over $\theta$ as long as the theta range is symmetric around $\pi/2$ [as it is the case here]
- the $D_{1,ll}$ term, however, will in general contribute in case of only partial integration over $\theta$
➥ the question is how much?

# $D_{1,ll}$ contribution to dihadron fragmentation

- $D_{1,ll}$ is unknown and can't be calculated using first principles
- it can not be extracted from cross sections integrated over $\theta$
- upon (partial) integration there is no way to disentangle the two contributions
- in PRD74 (2006) 114007, a model for dihadron fragmentation was tuned to PYTHIA and used to estimate the various partial-wave contributions
- its Figure 5 gives an indication about the relative size of $D_{1,ll}$ vs. $D_{1,oo}$:



# effect of partial integration

- as both contributions — $D_{1,ll}$ and $D_{1,oo}$ — will be affected by the partial integration, look at relative size of the $D_{1,ll}$ to $D_{1,oo}$ modulations when subjected to integration:

$$\frac{D_{1,ll}}{D_{1,oo}} \; \frac{\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta \, \frac{1}{4}(3\cos^2\theta - 1)}{\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta} = -\frac{1}{4}(1-\cos^2\theta_0) \, \frac{D_{1,ll}}{D_{1,oo}}$$

- without limit in the polar-angular range ($\theta_0$ =0) -> no contribution from $D_{1,ll}$ [sanity check!]

- the relative size of the partial integrals reaches a maximum of 25% for z=0.2
  [i.e., pions at 90 degrees in center-of-mass system]

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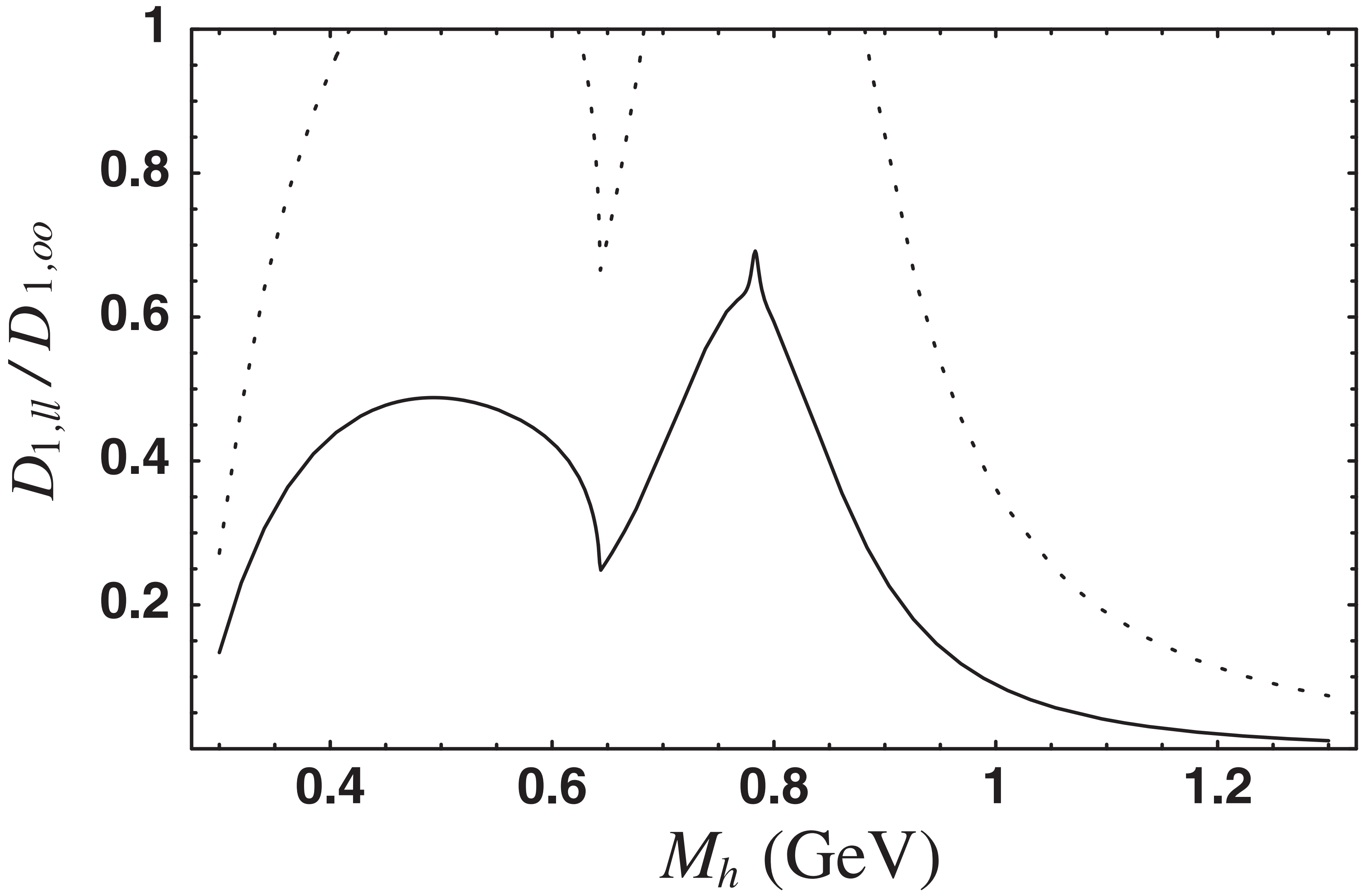
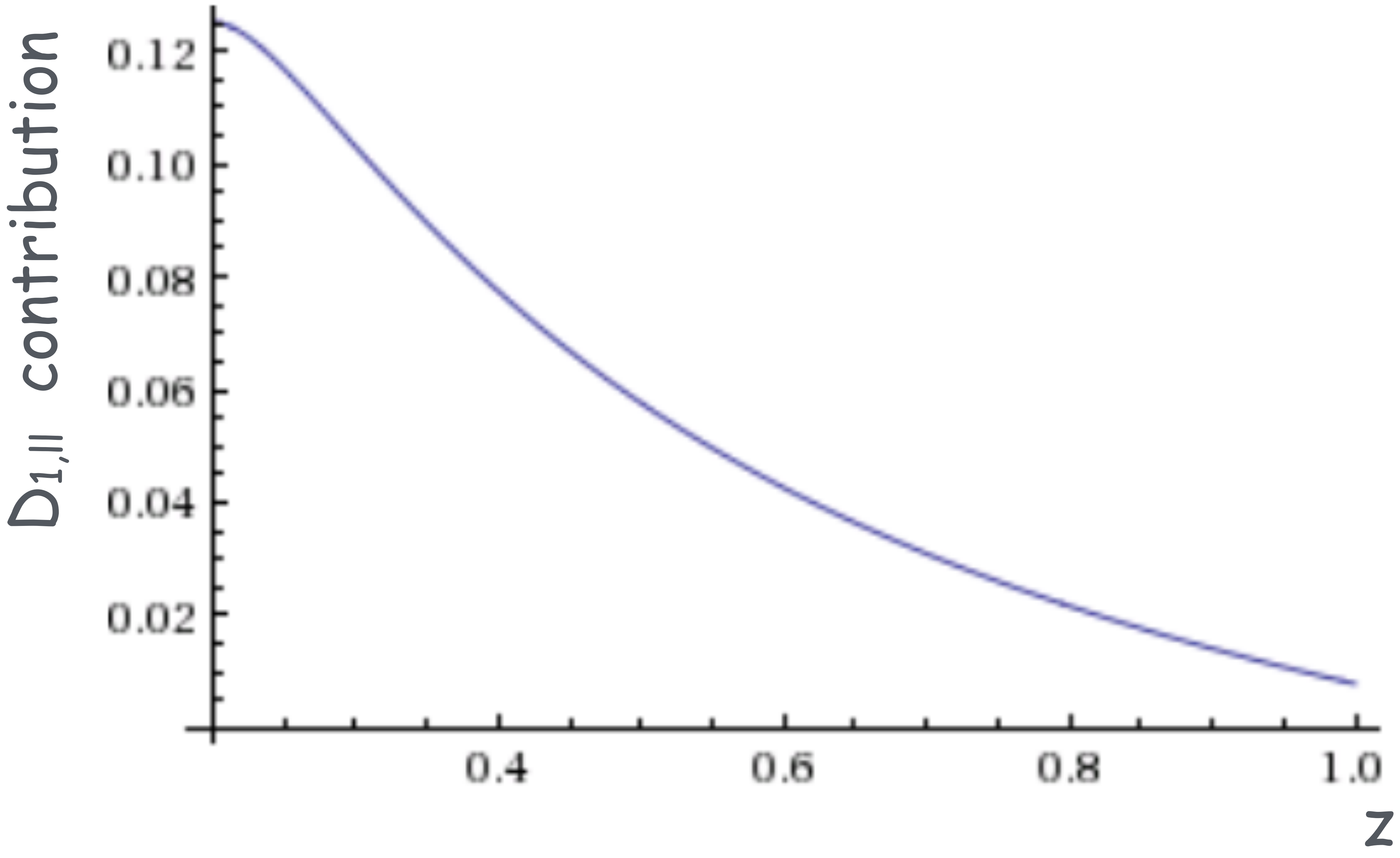
- in order to estimate the $D_{1,ll}$ contribution, one "just" needs the relative size of $D_{1,ll}$ vs. $D_{1,oo}$, e.g., Figure 5 of PRD74 (2006) 114007
  - let's take for that size 0.5 (rough value for M=0.5 GeV)



# effect of partial integration

- … $D_{1,ll}$ / $D_{1,oo}$ ~0.5 results in an up to O(10%) effect on the measured cross section:



- depending on the sign of $D_{1,ll}$, the partial integration thus leads to a systematic **underestimation** (positive $D_{1,ll}$) or **overestimation** (negative $D_{1,ll}$) **of the "integrated" dihadron cross section**

➡ **leads to overestimate/underestimate of extracted transversity**

# switching gears: hyperon polarisation in DIS

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hermes is still alive 😃

# Lambda production in DIS

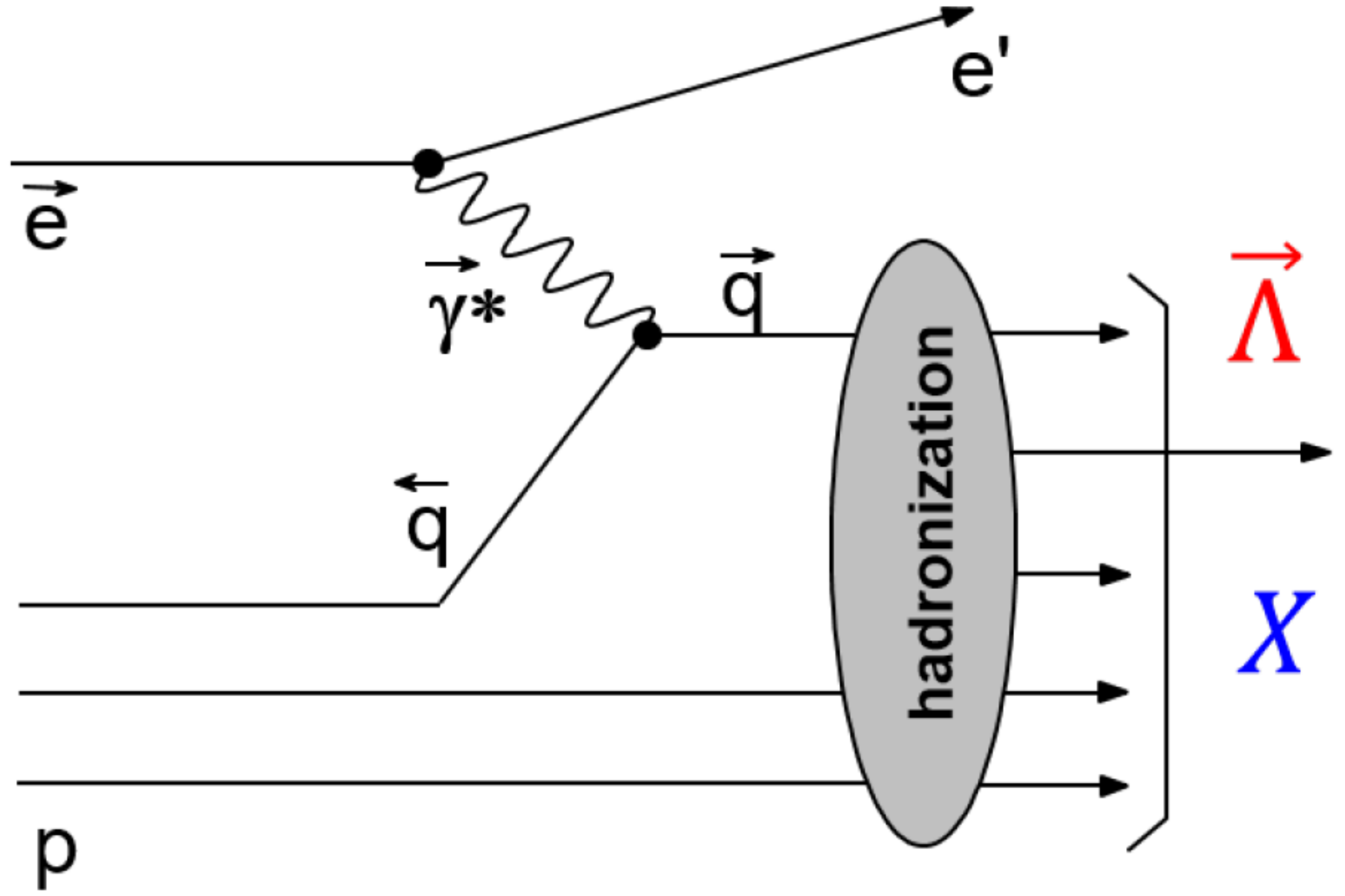
$$\vec{\gamma}^* + p \to \vec{\Lambda} + X$$

$$\Lsh p\pi^-$$

$$\frac{dN}{d\Omega_p} = \frac{dN_0}{d\Omega_p}\left(1 + \alpha\, P_{L'}^{\Lambda} \cos\theta_{pL'}\right)$$

unpolarised (uniform) distribution

angle between proton momentum and $\Lambda$ spin in $\Lambda$ rest frame



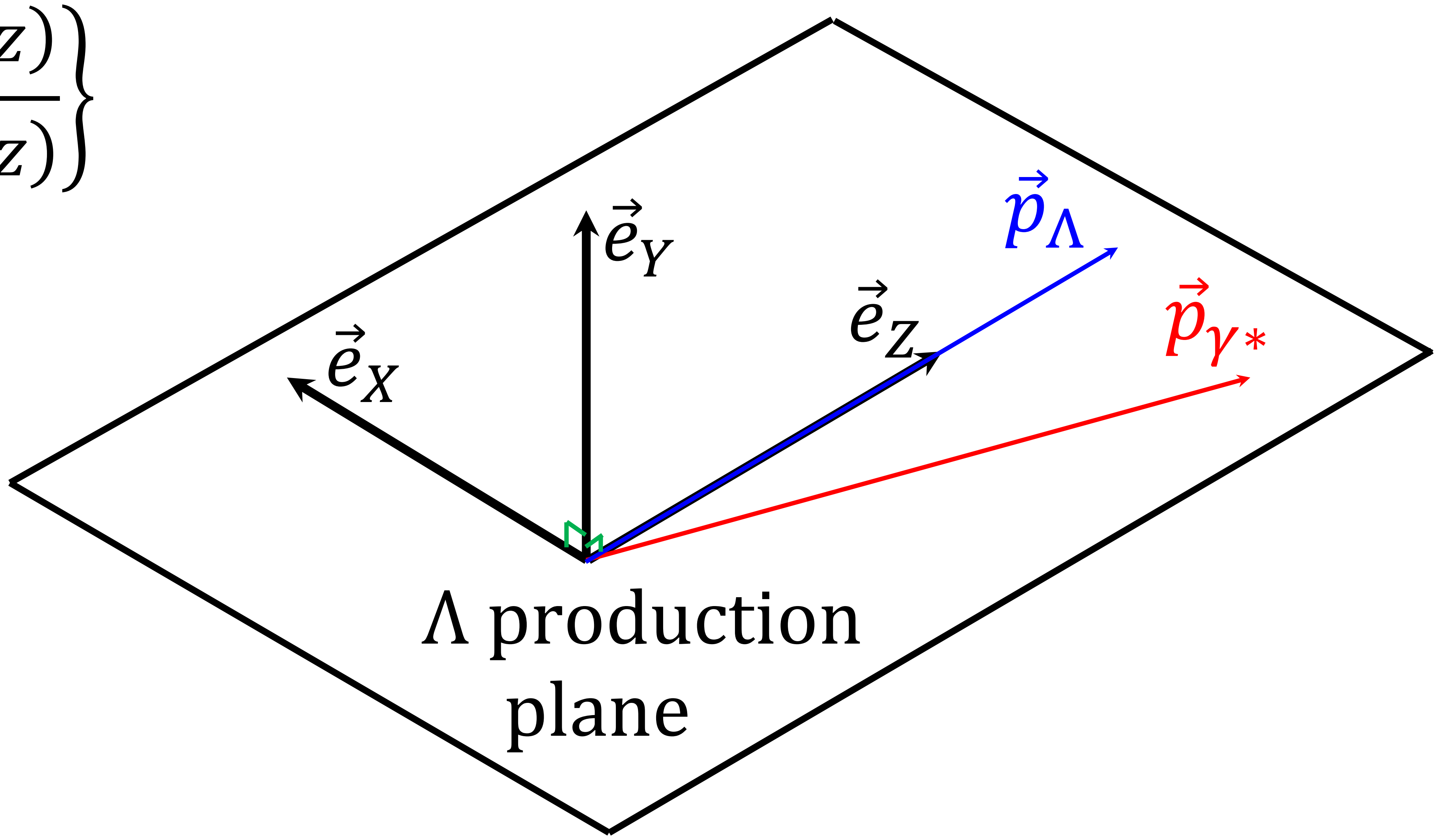
- "self-analyzing" particle due to its parity-violating decay
- polarization can be extracted just by measuring angular distribution of decay protons
  - in praxis distorted by instrumental effects (cf. first part of talk)

gunar.schnell @ desy.de

# Lambda production in DIS

Mulders &Tangerman, Nucl. Phys. B 461 (1996), 197

$$P_X^\Lambda = -P_B D_X(y) \left\{ \frac{M}{Q} \frac{\sum_q e_q^2 x_B f_1^q(x_B) H_1^q(z)}{\sum_q e_q^2 x_B f_1^q(x_B) D_1^q(z)} + \frac{M^\Lambda}{Q} \frac{\sum_q e_q^2 x_B f_1^q(x_B) \tilde{G}_T^q(z)}{\sum_q e_q^2 x_B f_1^q(x_B) D_1^q(z)} \right\}$$



$$P_Y^\Lambda = D_Y(y) \frac{M}{Q} \frac{\sum_q e_q^2 x_B f_1^q(x_B) D_{1T}^{\perp(1)q}(z)}{\sum_q e_q^2 x_B f_1^q(x_B) D_1^q(z)}$$

$$P_Z^\Lambda = P_B D_Z(y) \frac{\sum_q e_q^2 x_B f_1^q(x_B) G_1^q(z)}{\sum_q e_q^2 x_B f_1^q(x_B) D_1^q(z)}$$

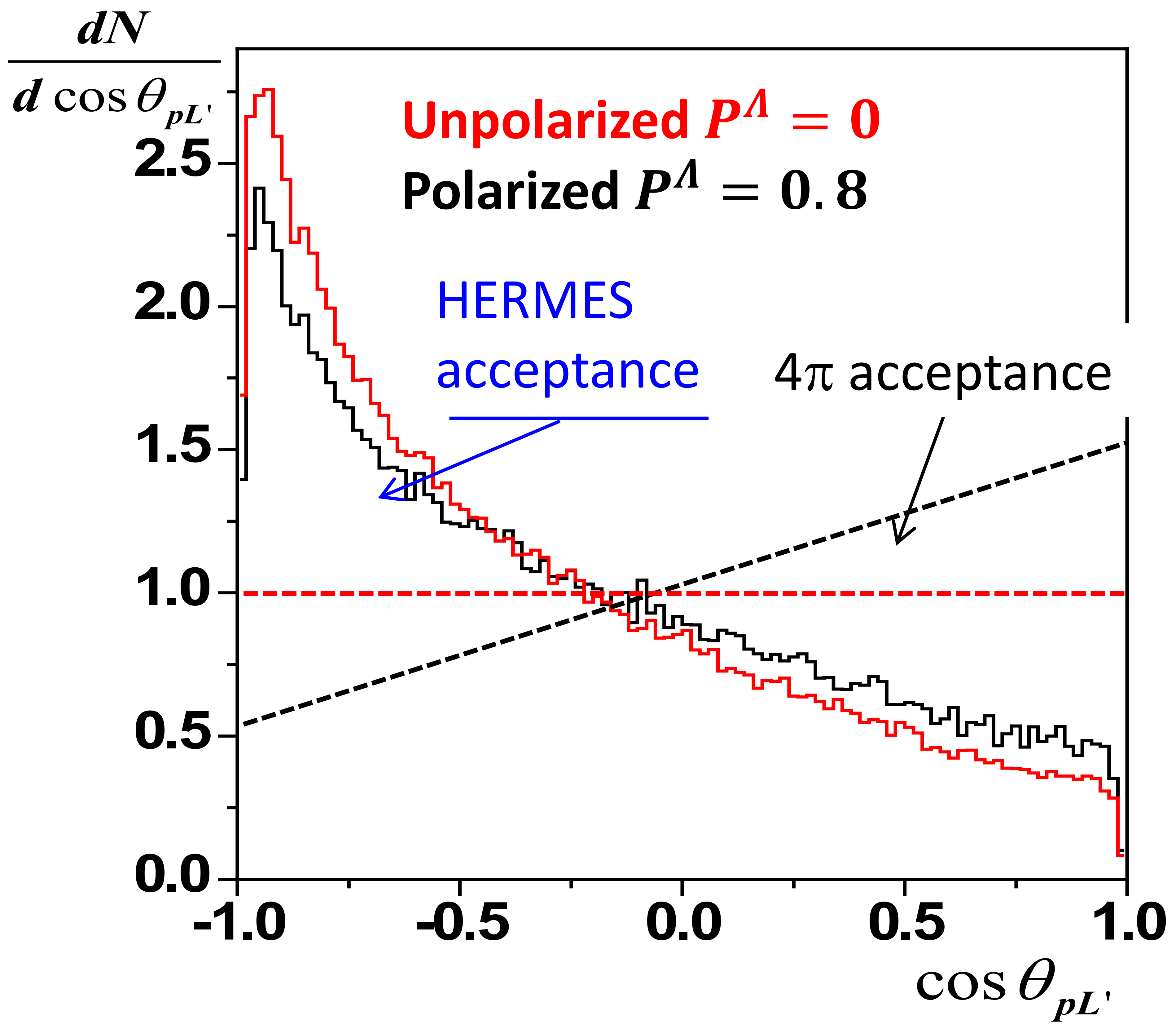
- access to several novel spin-dependent FFs
- Y-component of polarization (“self-polarization”) not correlated with beam polarization
  - ➥ drops out in beam-spin asymmetries
- concentrate on longitudinal and transverse spin transfer

# polarization measurement

$$\frac{dN}{d\Omega_p} = \frac{dN_0}{d\Omega_p}\left(1 + \alpha\, P_{L'}^{\Lambda} \cos\theta_{pL'}\right)$$

unpolarised (uniform) distribution

tilts the uniform distribution



- acceptance distorts distribution
  - heavy use of Monte Carlo to correct for acceptance
  - major source of systematic uncertainty

# polarization measurement

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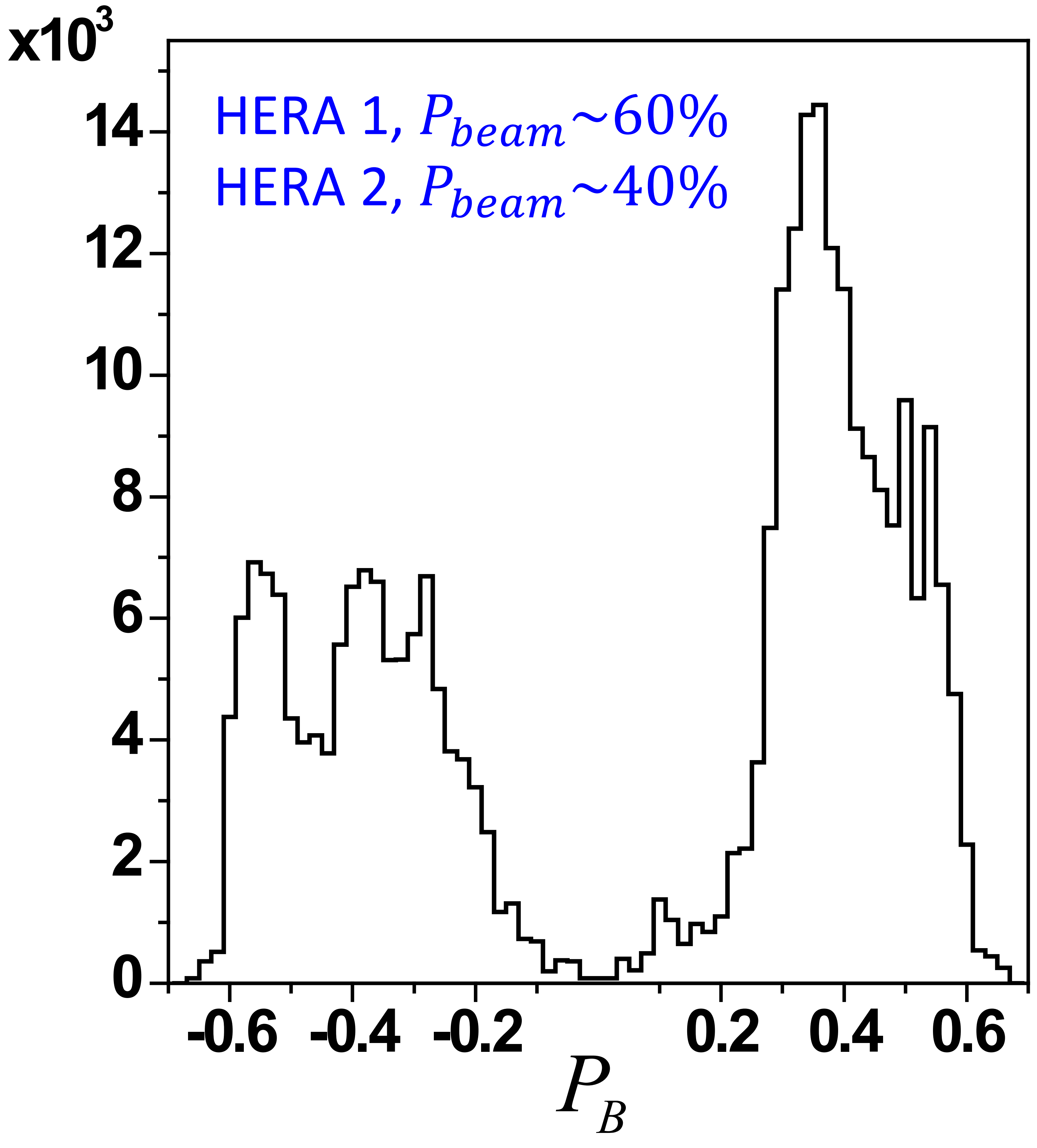
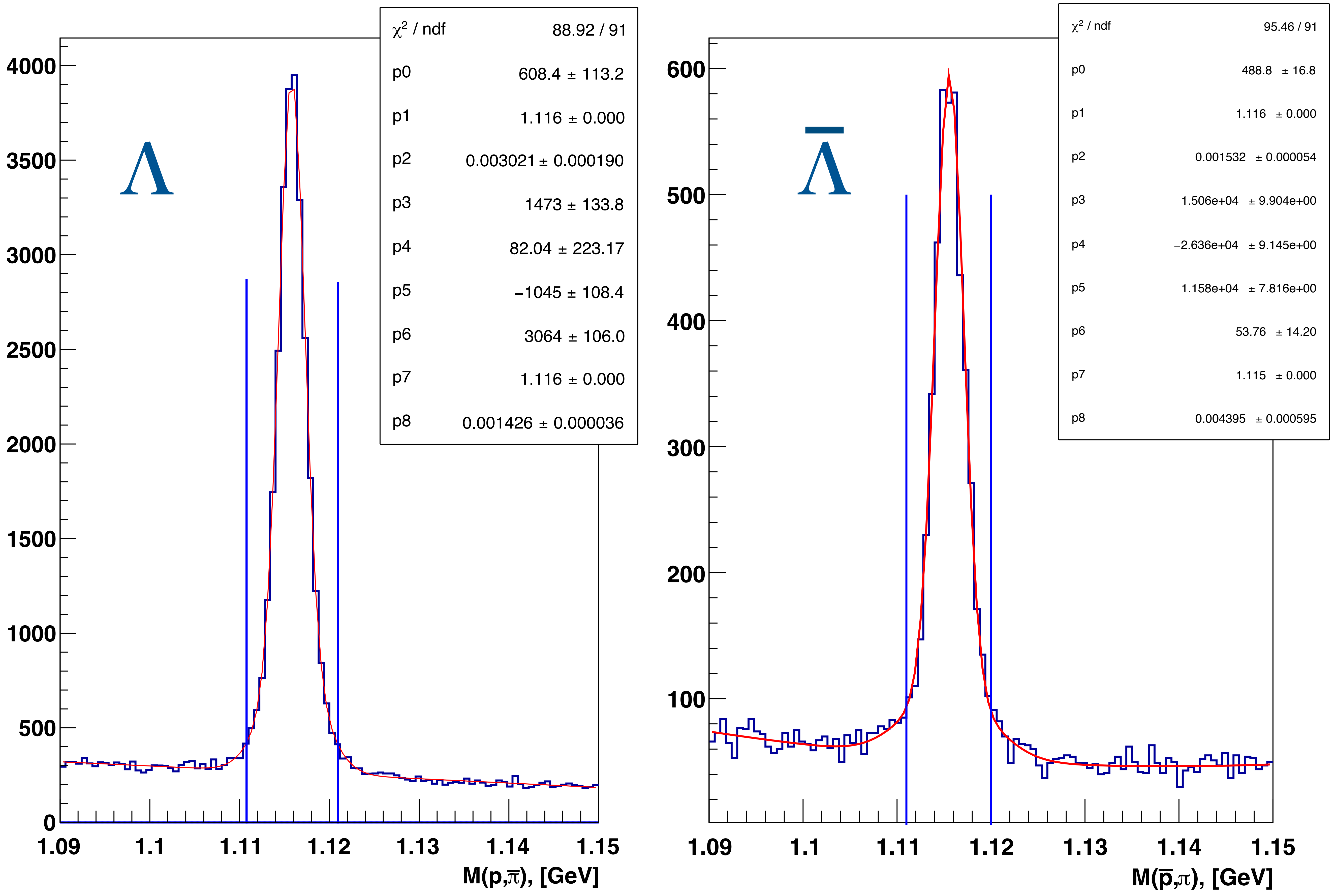


- NOMAD: basically $4\pi$ acceptance
- COMPASS: MC simulation of acceptance
- HERMES: cancel acceptance effect using two beam helicity states

# (anti-)Lambda yields at HERMES

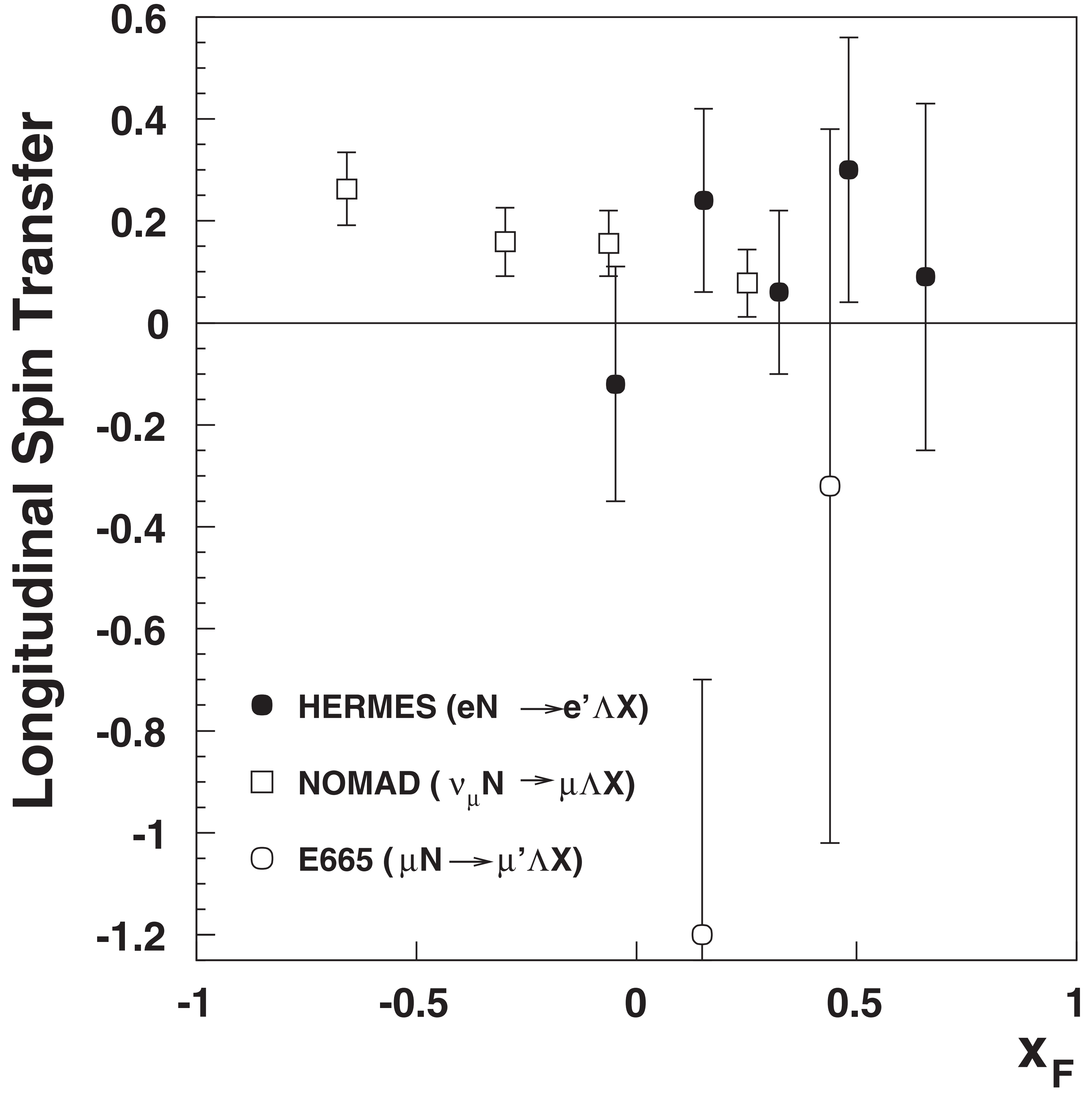
- HERA-I data of 1999-2000
- HERA-II data of 2003-2007
- DIS cuts
  - $W^2 > 10$ GeV²
  - $0.2 < y < 0.85$
  - Q2 > 0.8 GeV²
- **total number of (anti)$\Lambda$ is about 50k (6k)**

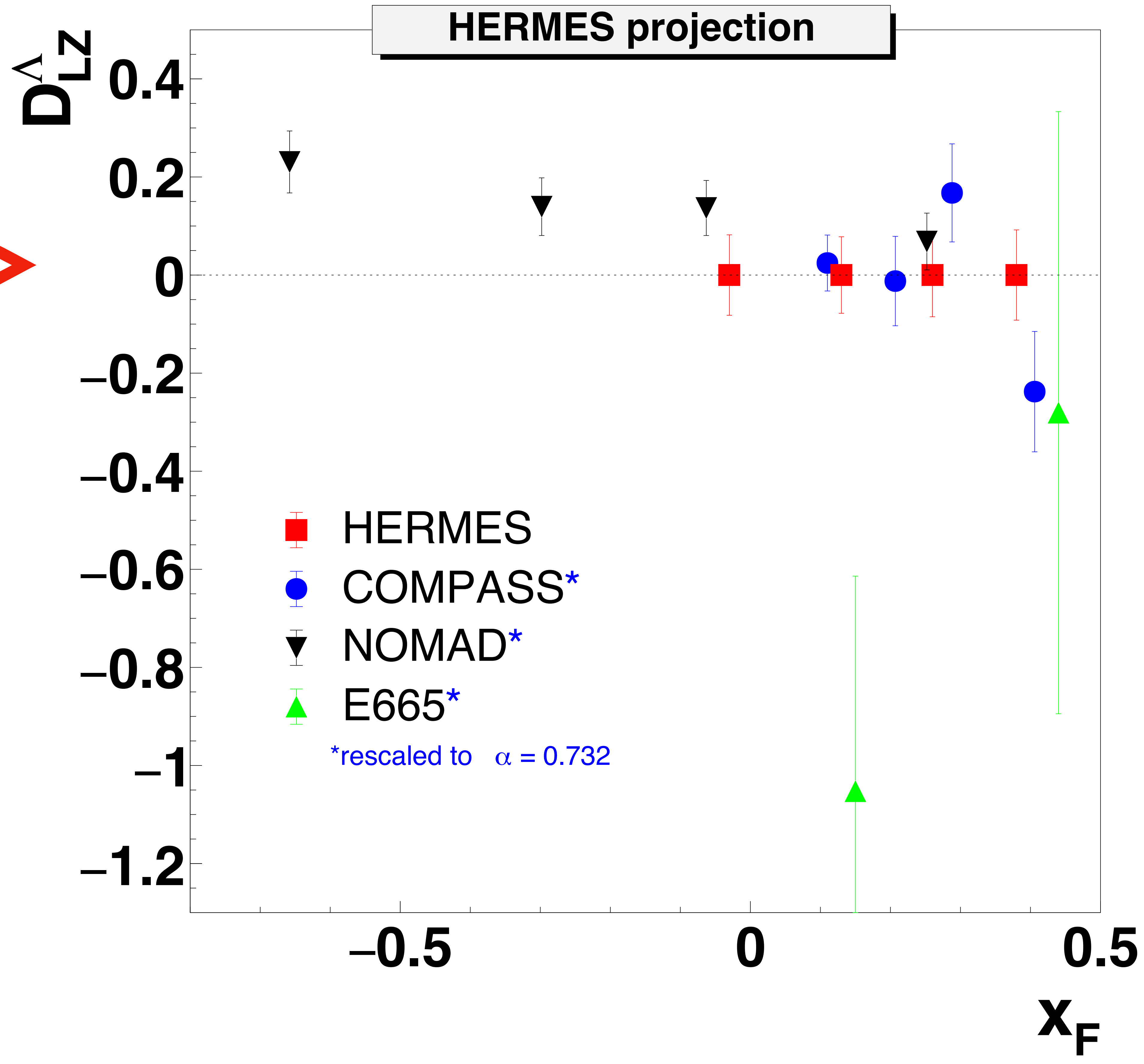


[one data-taking period]

# Projected precision

PHYSICAL REVIEW D **74,** 072004 (2006)

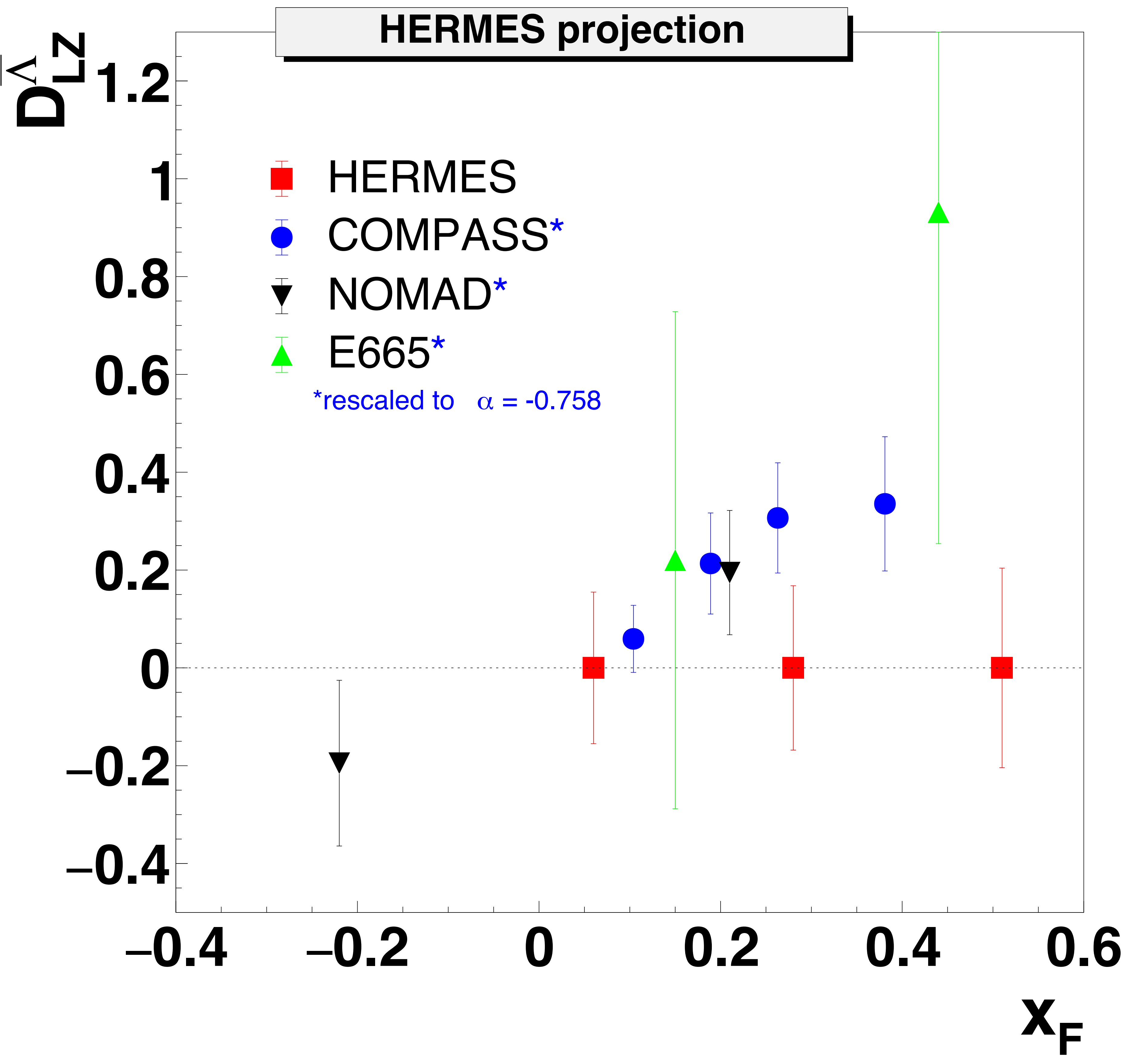




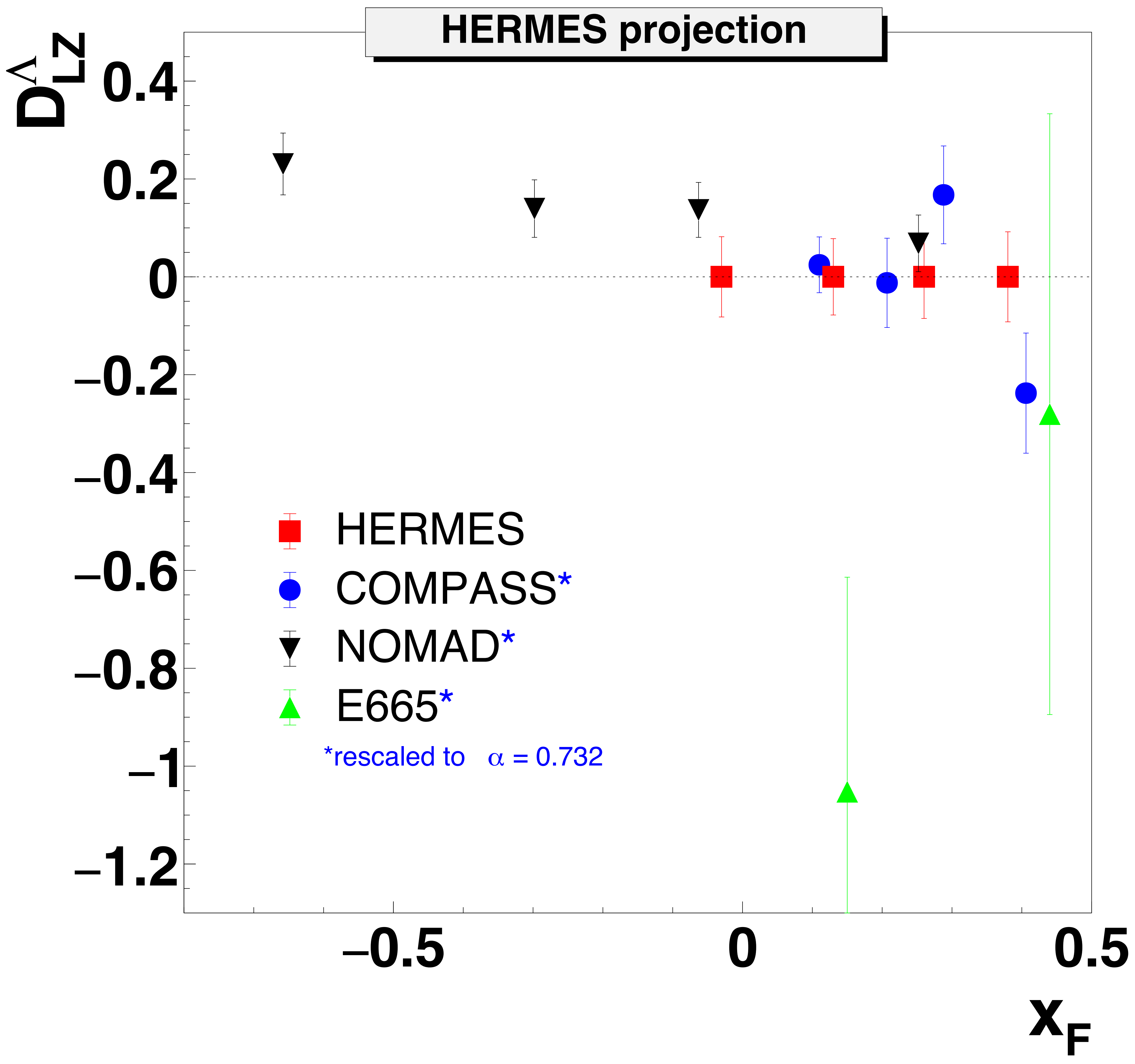
- PDG updated values of asymmetry parameter
- rescaled older results that used previous ±0.642

# Projected precision

$\bar{\Lambda}$



$\Lambda$



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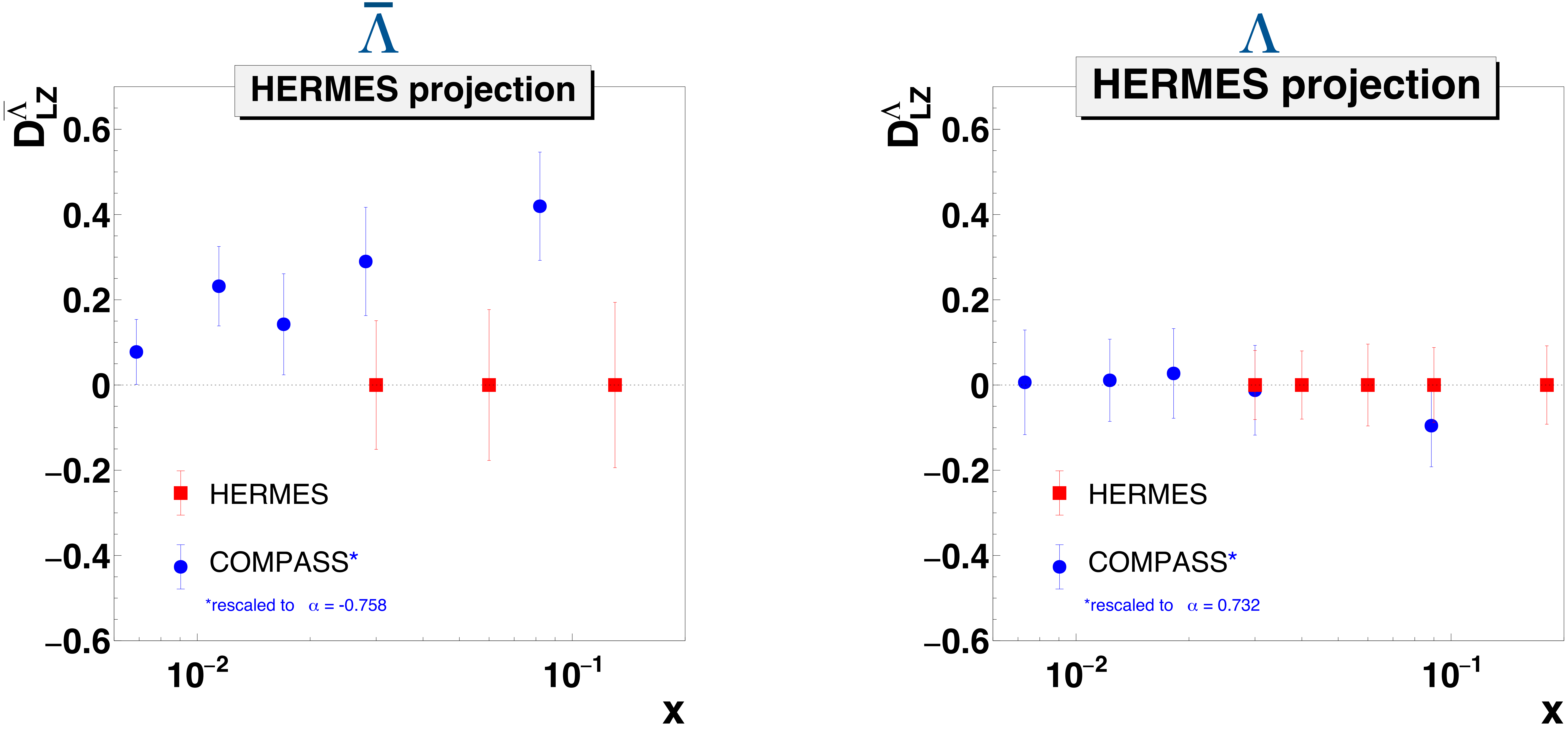

- results foreseen to be released for upcoming STRONG-2020 workshop “Present and future perspectives in Hadron Physics” -> to be presented by D. Veretennikov

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  - however, reduction of fully differential cross section comes with a price tag: not all terms that vanish in theory vanish in practice due to experimental requirements
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# conclusions

- two-hadron final states are a powerful albeit in parts more challenging tool to access spin-dependent distributions
- dihadron FFs very useful for collinear transversity extraction
  - however, reduction of fully differential cross section comes with a price tag: not all terms that vanish in theory vanish in practice due to experimental requirements
  - might lead to over-/underestimates of true size of extracted transversity
  - important to keep in mind when aiming for precision measurements
- hyperons and their two-hadron final states yet another way of accessing transversity and friends, as well as spin-dependent FFs
  - HERMES analysis of 1999-2007 data on longitudinal and longitudinal-to-transverse spin transfer to Lambda and anti-Lambda to be released

Thank you!