Andrea Simonelli

In collaboration with M. Boglione

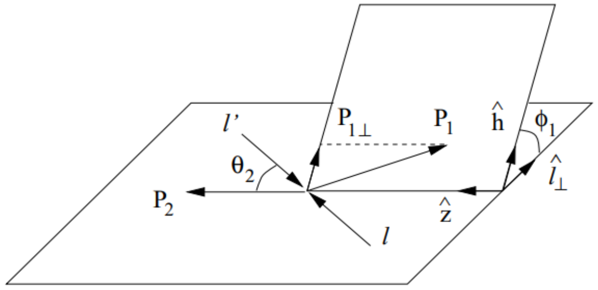
Fragmentation Functions in e^+e^-

OLD DOMINION UNIVERSITY

Jefferson Lab

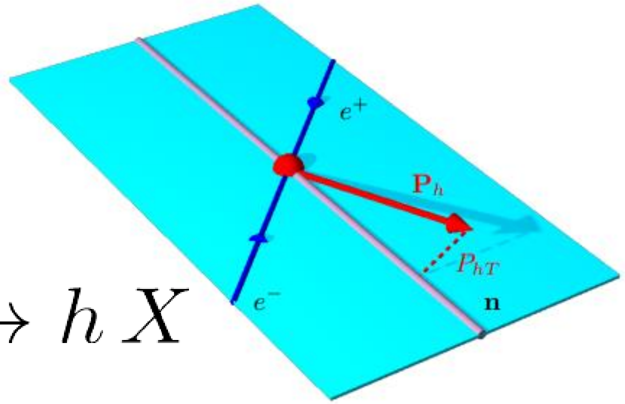
TMD Fragmentation Functions

D. Boer, R. Jakob, P. Mulders, Phys.Lett.B 424 (1998) 143-151



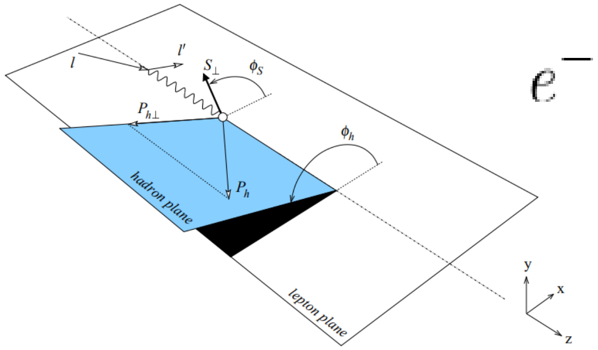
$$e^+e^- \to h_1 h_2 X$$

VS



$$e^+e^- \to h\, X$$

BELLE collab., Phys.Rev.D 99 (2019) 11, 112006



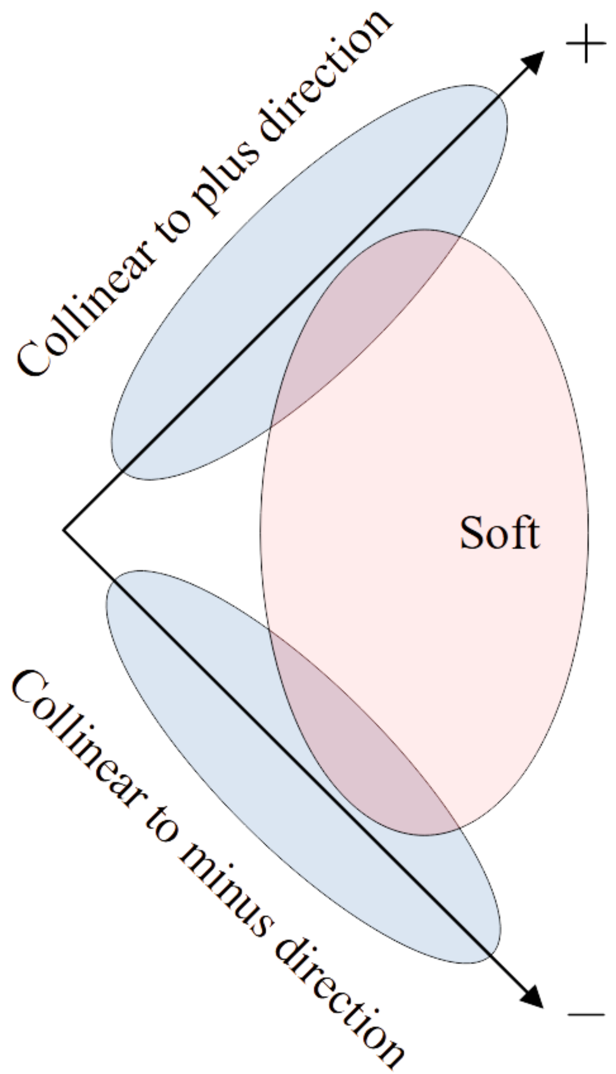
$$e^- P \to h\, X$$

A.Bacchetta, U.D'Alesio, M.Diehl, *Phys.Rev.D* 70 (2004) 117504

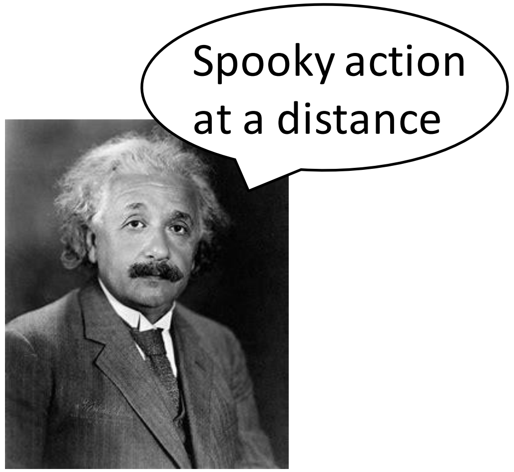
What happens to universality?

Crucial role of soft radiation

Soft Entanglement



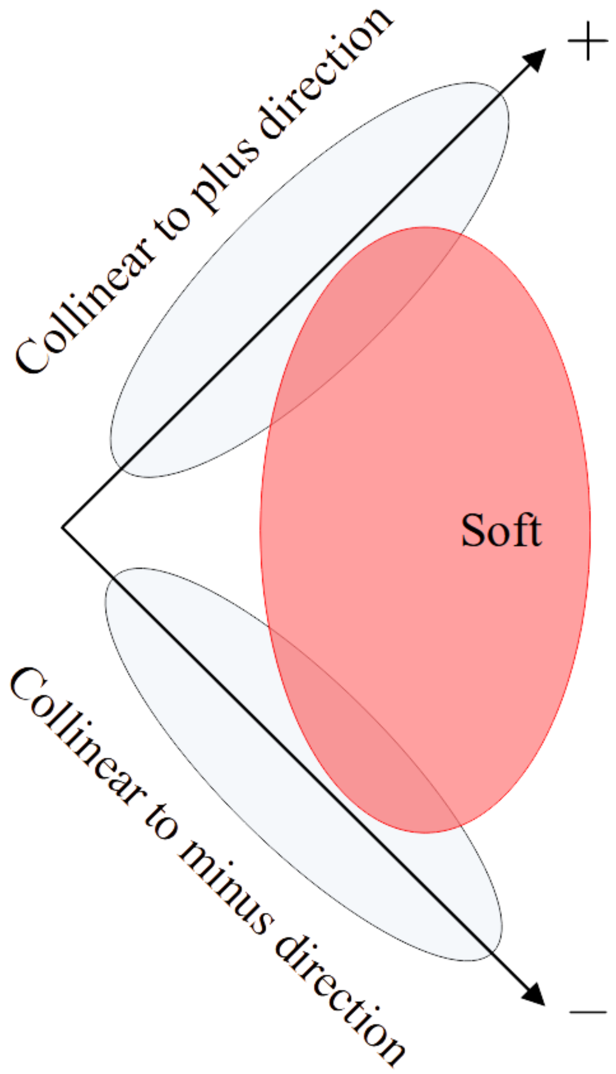
*Two opposite light-cone directions **entangled** by soft radiation*



Many examples:

- Two back-to-back hadrons produced in e^+e^- annihilation
- Semi-Inclusive DIS at low $q_T = P_T/z$
- Drell-Yan with lepton pair almost back-to-back
- DIS at threshold
- Thrust distribution in the 2-jet limit
- Single hadron production from e^+e^- annihilation, reconstructing the thrust in the 2-jet limit

The world as seen by a soft particle:



Large and positive rapidity, ultimately $+\infty$

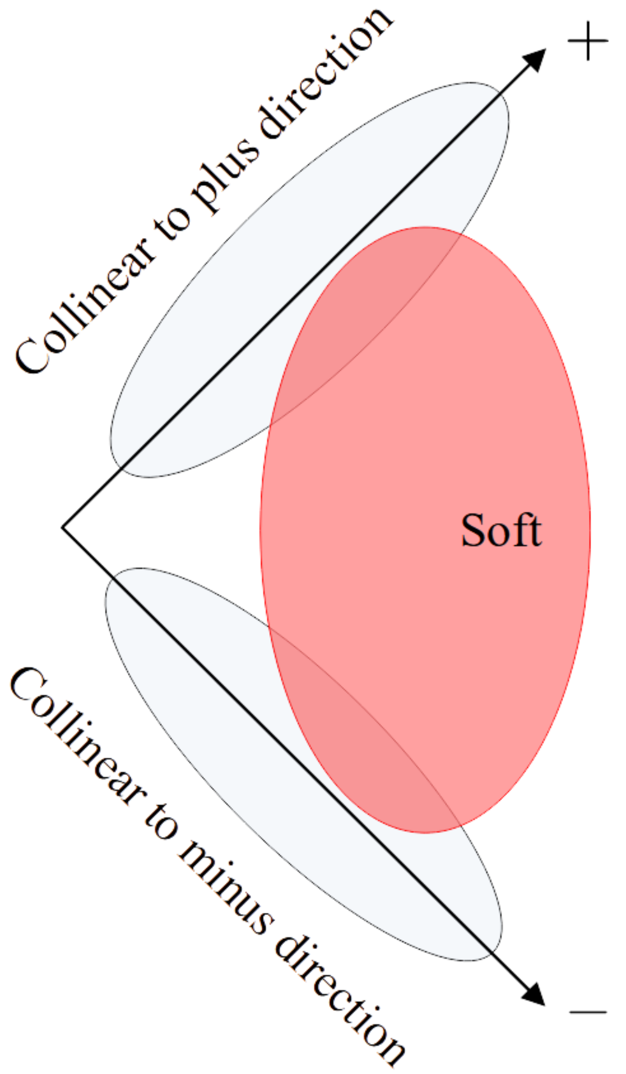
$$\left(1, -e^{-2y_1}, \vec{0}_T\right)$$

Tilts off the light-cone, ultimately $\begin{cases} y_1 \to +\infty \\ y_2 \to -\infty \end{cases}$

$$\left(-e^{2y_2}, 1, \vec{0}_T\right)$$

Large and negative rapidity, ultimately $-\infty$

The world as seen by a soft particle:

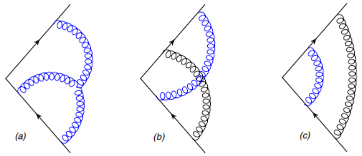


Large and positive rapidity, ultimately $+\infty$

$$\left(1, -e^{-2y_1}, \vec{0}_T\right)$$

Non-Abelian Exponentiation Theorem

See works by E. Gardi, E. Leanen, L. Magnea, C. White etc...



ON light-cone effects
(leading divergence on the light-cone)

$$S = \exp\left\{ \underline{K\text{-term}} + \underline{P\text{-term}} + \begin{matrix}\text{suppressed}\\ \text{terms}\end{matrix} \right\}$$

OFF light-cone effects
(sub-leading divergence on the light-cone)

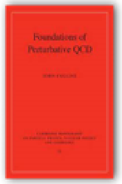
$$\left(-e^{2y_2}, 1, \vec{0}_T\right)$$

Large and negative rapidity, ultimately $-\infty$

$$S = \exp\left\{ K\text{-term} + P\text{-term} + \begin{matrix}\text{suppressed}\\ \text{terms}\end{matrix} \right\}$$

Many examples:

$$S(b_T) = \exp\{(y_1 - y_2)K(a_S, L_b) + P(a_S, L_b)\}$$



- Two back-to-back hadrons produced in e^+e^- annihilation
- Semi-Inclusive DIS at low $q_T = P_T/z$
- Drell-Yan with lepton pair almost back-to-back

$$S(N) = \exp\{\int_{y_2}^{y_1} dy\, K(a_S, L_N + y) + \\ + \frac{1}{2}\left[P(a_S, L_N + y_1 + i\pi/2) + P(a_S, L_N + y_2 + i\pi/2)\right]\}$$

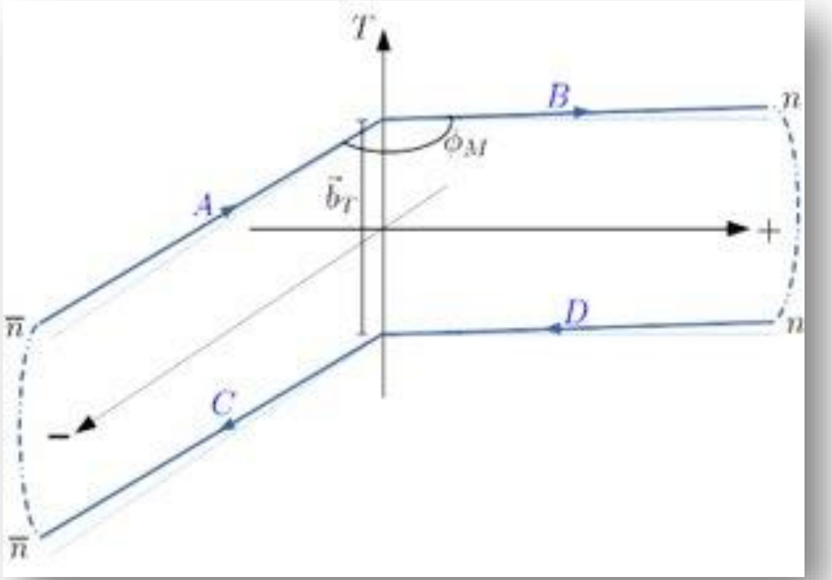
- DIS at threshold

$$S(u) \propto \exp\{\int_{0}^{y_1} dy\, K(a_S, L_u + y) + \int_{y_2}^{0} dy\, K(a_S, L_u - y) + \\ + \frac{1}{2}\left[P(a_S, L_u + y_1 + i\pi/2) + P(a_S, L_u - y_2 + i\pi/2)\right]\}$$

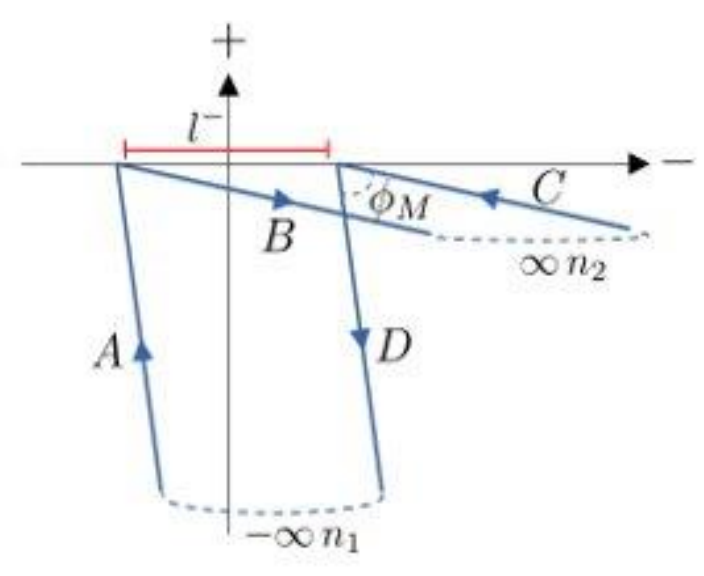
- Thrust distribution in the 2-jet limit
- Single hadron production from e^+e^- annihilation,
  reconstructing the thrust in the 2-jet limit

Why all these similarities? **geometry**

In TMD cases:



In DIS at threshold:



All configurations depend on a large cusp angle

$$\phi_M = y_1 - y_2 \to \infty$$

$$S = Z_{\rm UV} \langle 0 | \mathcal{P} \exp \left\{ -ig_0 \oint_\Gamma dx^\mu A_\mu^{(0)}(x) \right\} |0\rangle$$

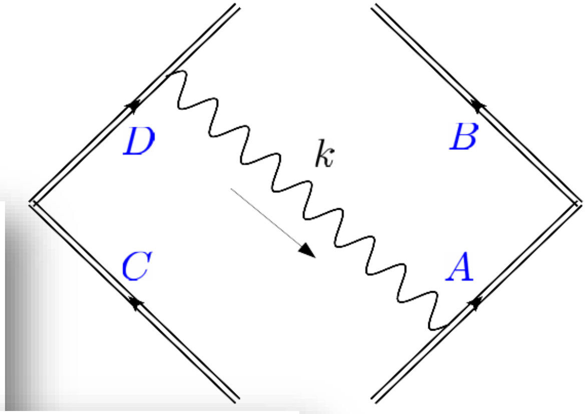
In the light-cone limit:

$\phi_M \to \infty$

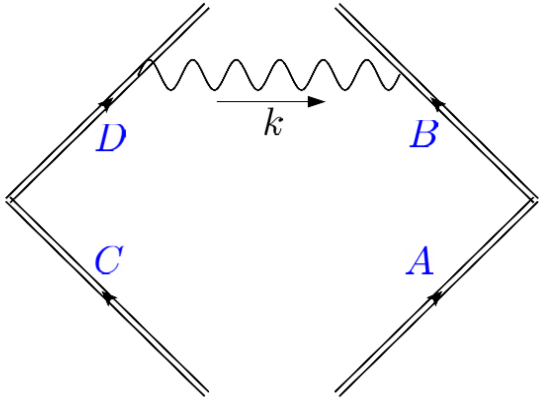
$$S(b_T;\phi_M) = e^{\phi_M K(b_T) + P(b_T)}$$

Studied a lot in TMD factorization...

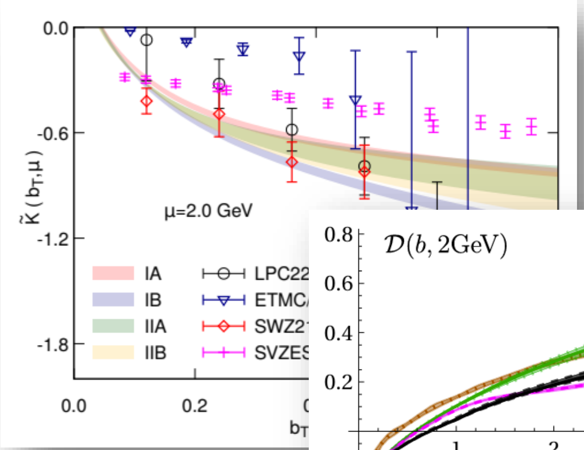
Collins-Soper kernel

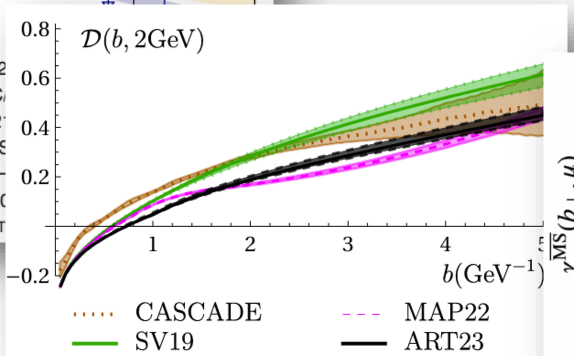


"Constant" P-term



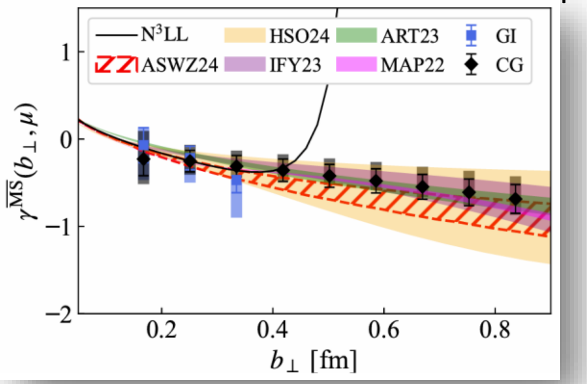
2206.08076 [hep-ph]





2305.07473 [hep-ph]

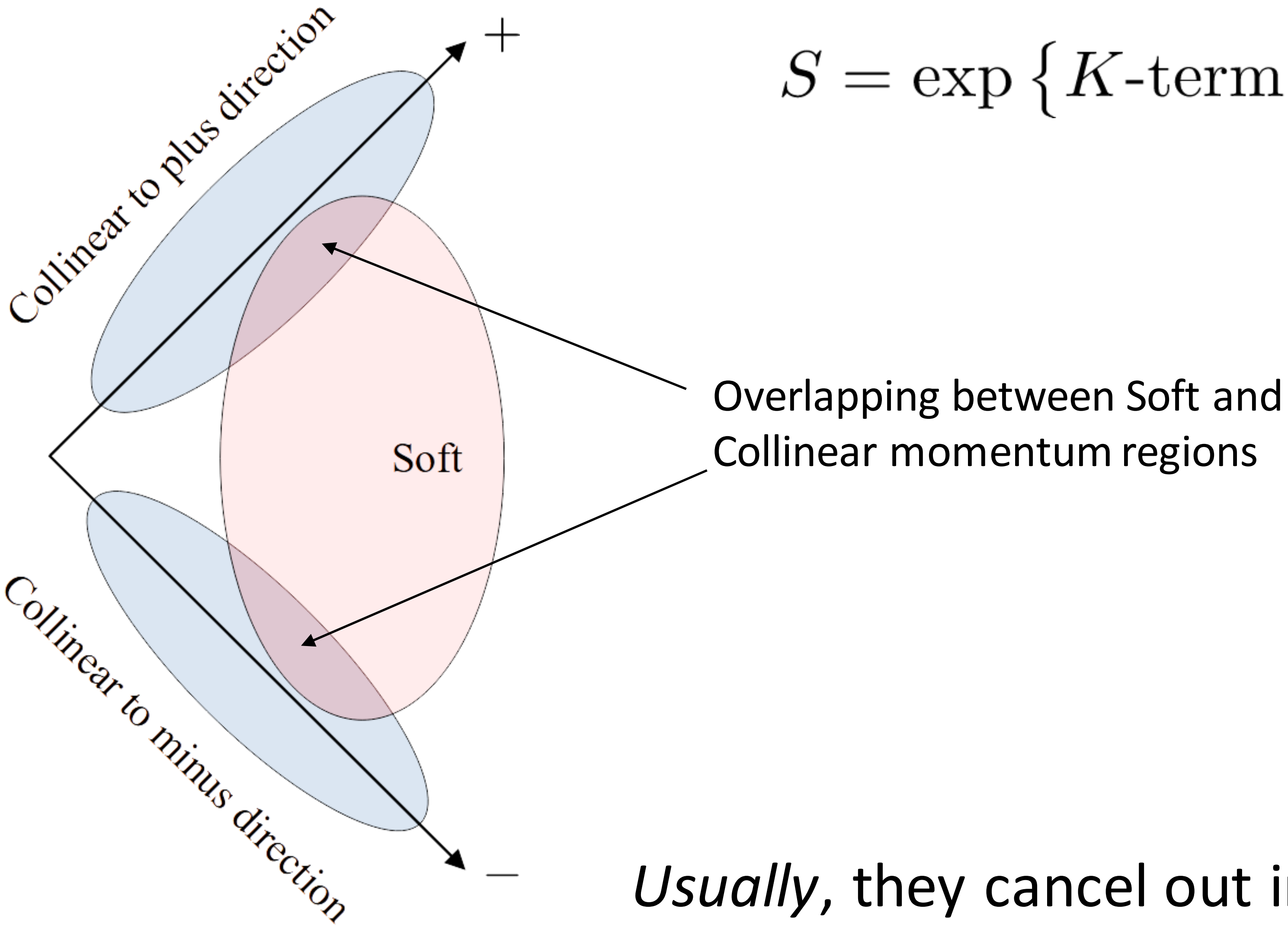
2403.00664 [hep-lat]



Where is it and **how** can we access it?

Where are the P-terms and how can we access them?

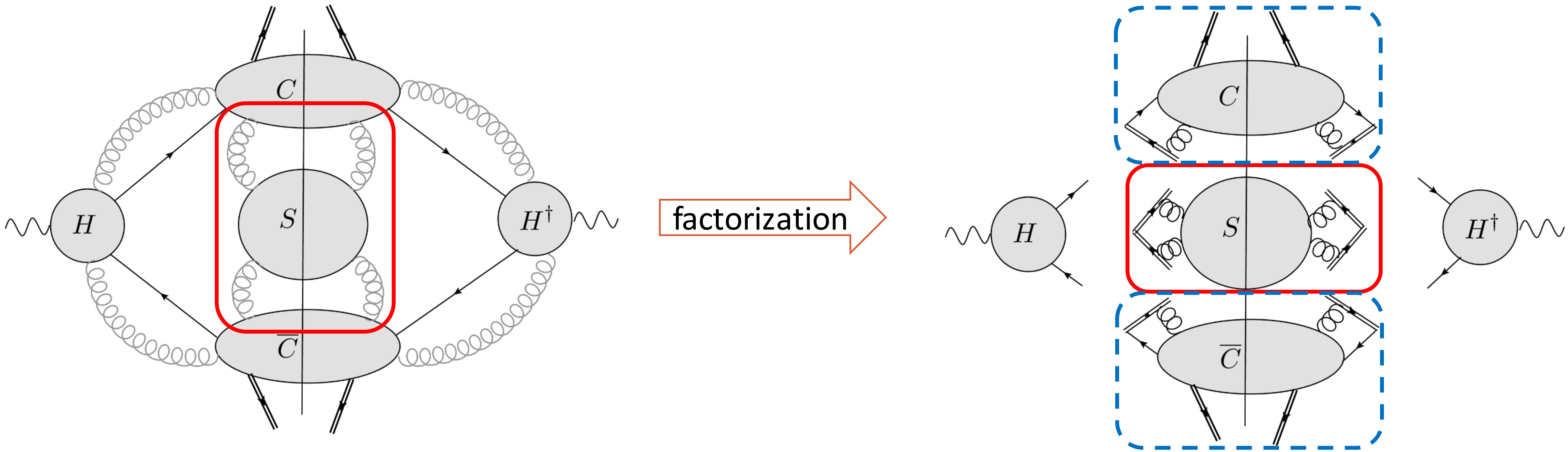
$$S = \exp\left\{K\text{-term} + \boxed{P\text{-term}} + \begin{matrix}\text{suppressed}\\ \text{terms}\end{matrix}\right\}$$



Usually, they cancel out in the **subtraction mechanism**

Consider the TMD cross section for e^+e^- annihilation in two back-to-back hadrons:

$$d\sigma = |H|^2 \int \frac{d\vec{b}_T}{(2\pi)^2} e^{-i\,\vec{q}_T\cdot\vec{b}_T} D_A^\star(z_A, b_T, y_A - y_1)\, S(b_T, y_1 - y_2)\, D_B^\star(z_B, b_T, y_2 - y_B)$$



Soft factor entangling the two collinear groups

$$d\sigma = |H|^2 \int \frac{d\vec{b}_T}{(2\pi)^2} e^{-i\,\vec{q}_T\cdot\vec{b}_T} \frac{D_A^{\text{uns.}}(z_A, b_T, y_A - (-\infty))}{S(b_T, y_1 - (-\infty))} S(b_T, y_1 - y_2) \frac{D_B^{\text{uns.}}(z_B, b_T, \infty - y_B)}{S(b_T, \infty - y_2)}$$

$$D_A^{\star}(z_A, b_T, y_A - y_1)$$

$$D_B^{\star}(z_A, b_T, y_2 - y_B)$$

Where:

$$D^{\text{uns}}(z, b_T, y_{\text{had}} - (-\infty)) = \frac{\text{Tr}_c}{N_c}\frac{\text{Tr}_D}{4}\sum_X \frac{1}{z}\int \frac{dx^-}{2\pi} e^{ik^+x^-} \qquad x = \left(0, x^-, \vec{b}_T/2\right)$$

$$\langle 0|\gamma^+ W_-\,(x/2 \to \infty)\,|P; X\rangle\langle P; X|W_-^\dagger\,(-x/2 \to \infty)\,|0\rangle$$

$$d\sigma = |H|^2 \int \frac{d\vec{b}_T}{(2\pi)^2} e^{-i\,\vec{q}_T\cdot\vec{b}_T}$$

$$\frac{D_A^{\text{uns.}}(z_A, b_T, y_A - (-\infty))}{S(b_T, y_1 - (-\infty))} \; S(b_T, y_1 - y_2) \; \frac{D_B^{\text{uns.}}(z_B, b_T, \infty - y_B)}{S(b_T, \infty - y_2)}$$

Same functional form!

$$D(z, b_T, y_{\text{had}} - y_1) = D^{\text{uns}}(z, b_T, y_{\text{had}} - (-\infty)) \sqrt{\frac{S(b_T, \infty - y_1)}{S(b_T, \infty - (-\infty))\, S(b_T, y_1 - (-\infty))}}$$

Optimal definition for standard TMD cases

The P-terms **disappear** in the standard TMD factorization...

$$\frac{\mathcal{D}_A^{\text{uns.}}(z_A, b_T; y_A - (-\infty))}{\mathcal{S}(b_T; y_1 - (-\infty))} \times \mathcal{S}(b_T; y_1 - y_2) \times \frac{\mathcal{D}_B^{\text{uns.}}(z_B, b_T; \infty - y_B)}{\mathcal{S}(b_T; \infty - y_2)}$$

$$= \frac{\mathcal{D}_A^{\text{uns.}}(z_A, b_T; y_A - (-\infty))}{e^{(y_1-(-\infty))K+\cancel{\frac{1}{2}P}}} \times e^{(y_1-y_2)K+\cancel{P}} \times \frac{\mathcal{D}_B^{\text{uns.}}(z_B, b_T; \infty - y_B)}{e^{(\infty-y_2)K+\cancel{\frac{1}{2}P}}}$$

...as well as in the standard TMD definition:

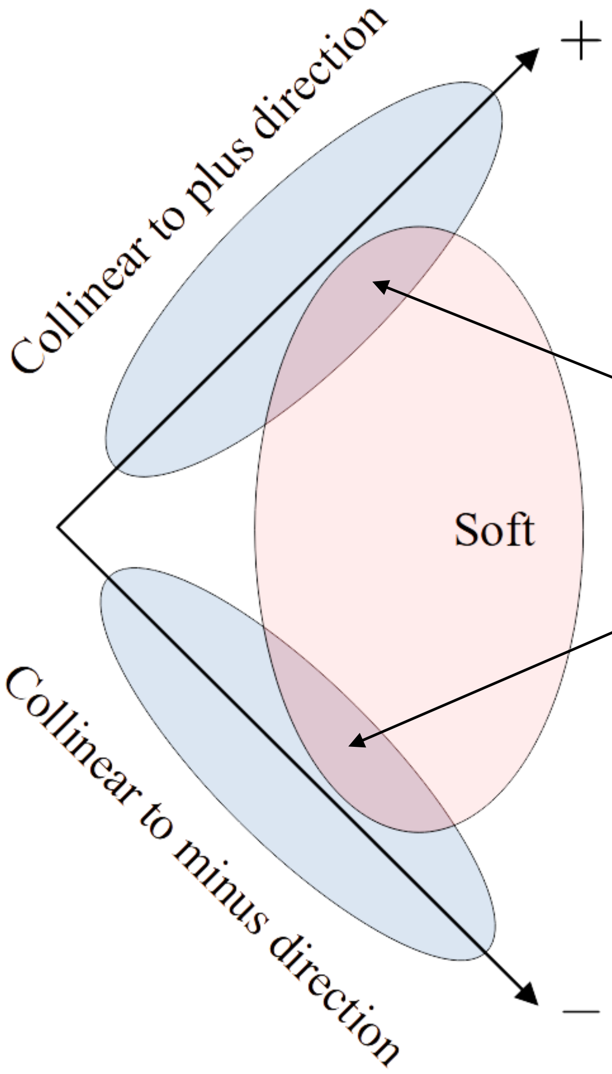
$$D(y_{\text{had}} - y_1) = D^{\text{uns.}}(y_{\text{had}} - (-\infty))\sqrt{\frac{\mathcal{S}(\infty - y_1)}{\mathcal{S}(\infty - (-\infty))\,\mathcal{S}(y_1 - (-\infty))}}$$

$$= D^{\text{uns.}}(y_{\text{had}} - (-\infty))\sqrt{\frac{e^{(\infty-y_1)K+\cancel{\frac{1}{2}P}}}{e^{(\infty-(-\infty))K}\; e^{(y_1-(-\infty))K+\cancel{\frac{1}{2}P}}}}$$

We can forget about the existence of the P-term **in the standard TMD cases**

Where are the P-terms and how can we access them?

$$S = \exp\left\{ K\text{-term} + \boxed{P\text{-term}} + \begin{matrix}\text{suppressed}\\ \text{terms}\end{matrix} \right\}$$



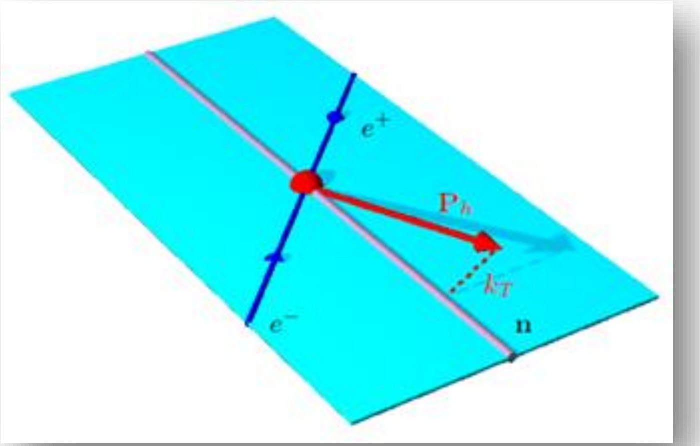
Overlapping between Soft and Collinear momentum regions

*When Soft and Soft-Collinear terms have **all** the same functional form (describe the same physics)*

Usually, they cancel out in the subtraction mechanism

A non-standard case

Single-Inclusive Annihilation (SIA) with thrust $e^+e^- \to h\,X$



The transverse momentum of the detected hadron is measured w.r.t. the thrust axis

$$z_h = \frac{E}{Q/2}, \quad T = \frac{\sum_i |\vec{P}_{(\mathrm{c.m.}),\,i} \cdot \widehat{n}|}{\sum_i |\vec{P}_{(\mathrm{c.m.}),\,i}|}, \quad P_T \text{ w.r.t } \vec{n}$$

Data available since 2019

Transverse momentum dependent production cross sections of charged pions, kaons and protons produced in inclusive e^+e^- annihilation at $\sqrt{s} =$ 10.58 GeV #11

Belle Collaboration • R. Seidl (RIKEN BNL) et al. (Feb 5, 2019)

Published in: *Phys.Rev.D* 99 (2019) 11, 112006 • e-Print: 1902.01552 [hep-ex]

pdf DOI cite claim reference search 34 citations

Complete theoretical treatment and first phenomenology is now available

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h\,X$ processes #1

M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)

Published in: *JHEP* 09 (2023) 006 • e-Print: 2306.02937 [hep-ph]

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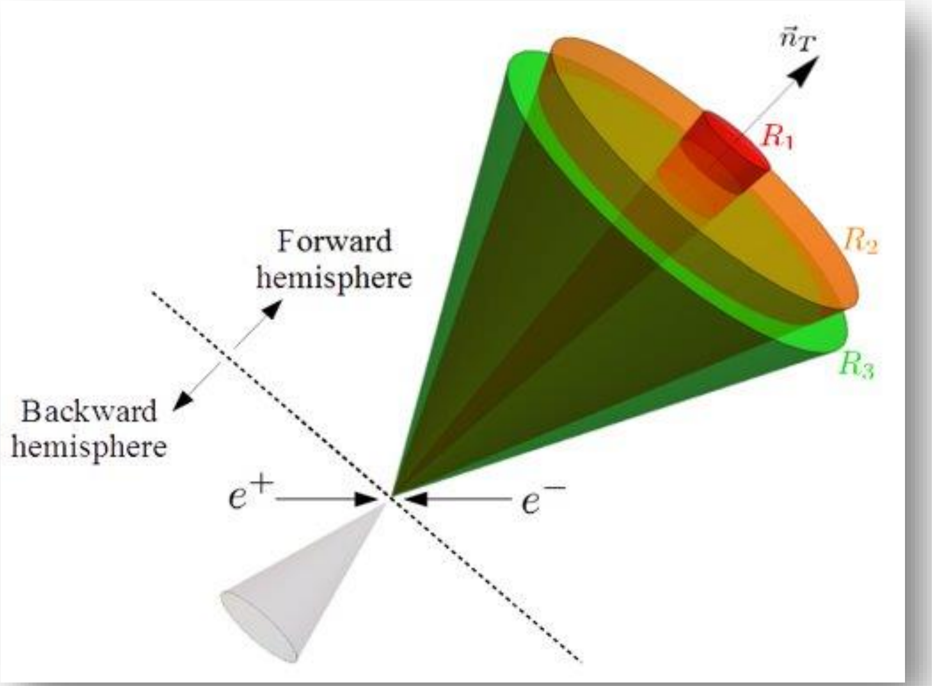


The hadron is detected very close to the **axis** of the jet.

- ❑ Extremely small P_T
- ❑ Soft radiation affects significantly the transverse deflection of the hadron from the thrust axis

The hadron is detected in the **central region** of the jet.

- ❑ Most common scenario
- ❑ Majority of experimental data expected to fall into this case

The hadron is detected near the **boundary** of the jet.

- ❑ Moderately small P_T
- ❑ The hadron transverse momentum affects the topology of the final state directly

Different kinematics leads to different factorized cross sections

Factorization works in the same way for all the three regions, but it produces different results depending on the underlying kinematics

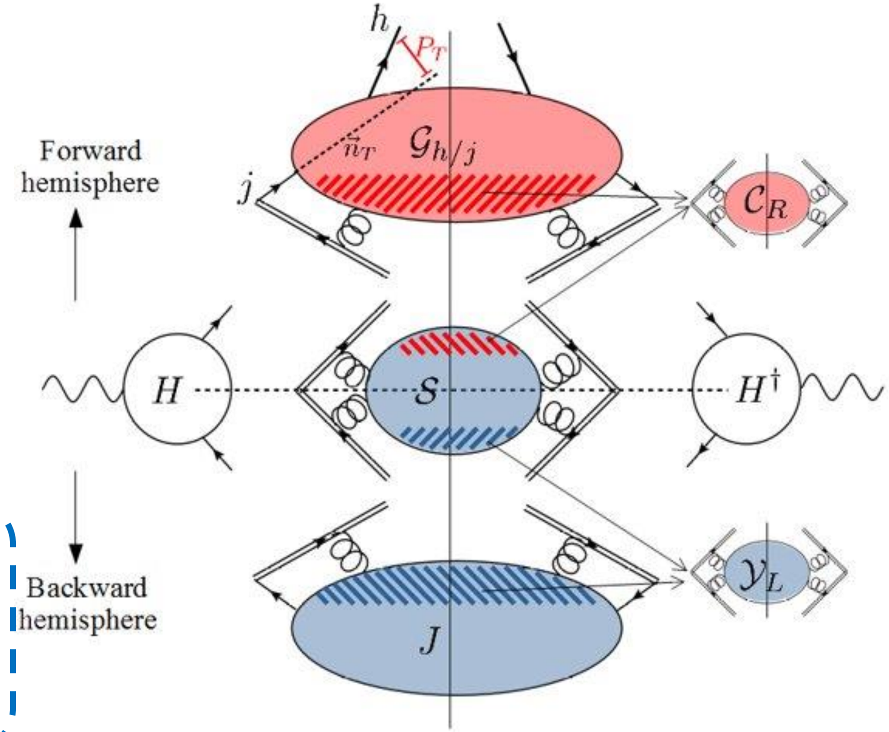
	soft	soft-collinear	collinear
R_1	TMD-relevant	TMD-relevant	TMD-relevant
R_2	TMD-irrelevant	TMD-relevant	TMD-relevant
R_3	TMD-irrelevant	TMD-irrelevant	TMD-relevant

$$d\sigma_{R_2} = |H|^2 \int \frac{du}{2\,i\,\pi} e^{u\tau} \int \frac{d\vec{b}_T}{(2\pi)^2} e^{iz\vec{P}_T\cdot\vec{b}_T}$$

$$J(u, \infty - y_j) \frac{\widehat{S}(u, y_1 - y_2)}{\widehat{S}(u, \infty - y_2)} D^{\star}(z, b_T, y_{\mathrm{had}-y_1})$$

$$\frac{D^{\mathrm{uns}}(z, b_T, y_{\mathrm{had}} - (-\infty))}{S(b_T, y_1 - (-\infty))}$$

Not same functional form!
No magic this time...



Red blobs are TMD-relevant
Blue blobs are TMD-irrelevant

We are forced to use the definition*.
There are disadvantages but also...

- More universal (no soft contamination)
- Inclusion of P-term effects (new physics!)

Factorization theorem in the central region

$$d\sigma_{R_2} \sim H\ J(u)\, \frac{\mathcal{S}(u,\overline{y}_1,y_2)}{\mathcal{Y}_L(u,y_2)}\, \widetilde{D}_{h/j}(z,b_T,\overline{y}_1)$$

$$= H\, J\, \frac{\mathcal{S}}{\mathcal{Y}_L}\Big|_{\text{ref. scale}} \exp\left\{\int_{\mu_J}^{Q}\frac{d\mu'}{\mu'}\gamma_J + \frac{1}{2}\int_{\mu_S}^{Q}\frac{d\mu'}{\mu'}\gamma_S\right\} \times \widetilde{D}_{h/j}(z,b_T)\Big|_{y_1=0}$$

$$\times \exp\left\{\frac{1}{2}\int_{\mu_S}^{\mu_S e^{\overline{y}_1}}\frac{d\mu'}{\mu'}\left[\widehat{g} - \gamma_K \log\left(\frac{\mu'}{\mu_S}\right)\right] - \overline{y}_1\ \widetilde{K}\Big|_{\mu_S}\right\}$$

Genuinely thrust. Exponent is *half* of standard thrust distributionin e+e- annihilation

Genuinely TMD. Reference scales as* in standard TMD factorization

Correlation part. It encodes the correlations between the measured variables

The function g_K does not only appear into the TMD FF!

$$\frac{d\sigma_{R_2}}{dz\, dT\, d^2\vec{P}_T} = -\frac{\sigma_B\, N_C}{1-T}\sum_j e_j^2\left(1 + a_S\, H^{[1]}\right)$$

$$\times \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{b}_T\cdot\vec{P}_T/z}\, e^{L_{b^\star}\, n_1+n_2}\, \widetilde{D}^{\text{NLL}}_{h/j}(z,b_T)\Big|_{\substack{\mu=Q\\ y_1=0}} (1 + a_S\, C_1)\, \frac{e^{Lf_1+f_2+\frac{1}{L}f_3}}{\Gamma\left(1-g_1\right)}\left(g_1 + \frac{1}{L}g_2\right)$$

Phenomenology $e^+e^- \to \pi X$

BELLE collaboration
Phys.Rev.D 99 (2019) 11, 112006

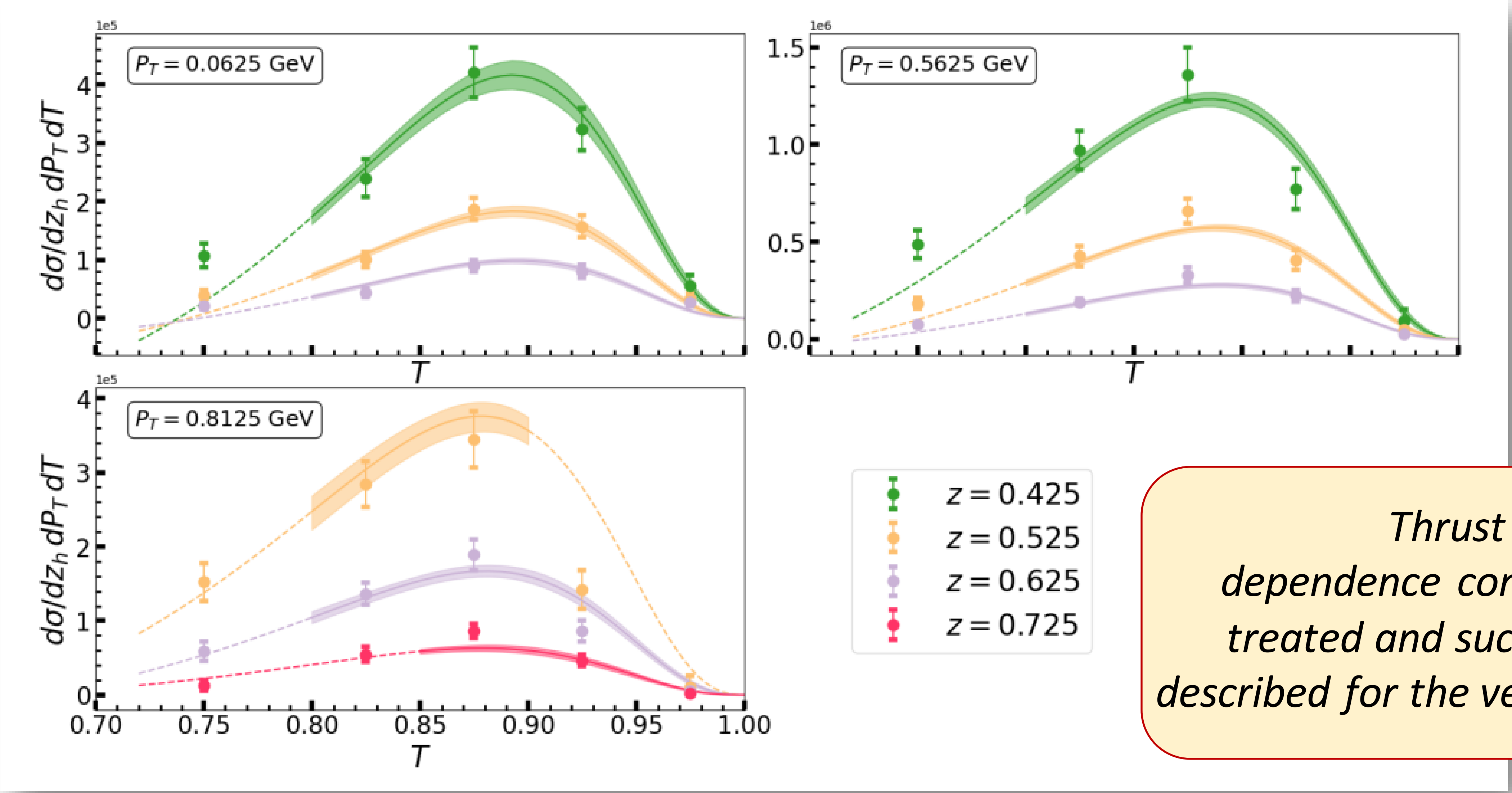
T	z												P_T/z max	N
	0.20 – 0.25	0.25 – 0.30	0.30 – 0.35	0.35 – 0.40	0.40 – 0.45	0.45 – 0.50	0.50 – 0.55	0.55 – 0.60	0.60 – 0.65	0.65 – 0.70	0.70 – 0.75	0.75 – 0.80		
0.80 – 0.85													$0.16\,Q$	57
0.85 – 0.90													$0.15\,Q$	60
0.90 – 0.95													$0.14\,Q$	61
0.95 – 1.00													$0.13\,Q$	52

Avoiding Region 1

Avoiding Region 3

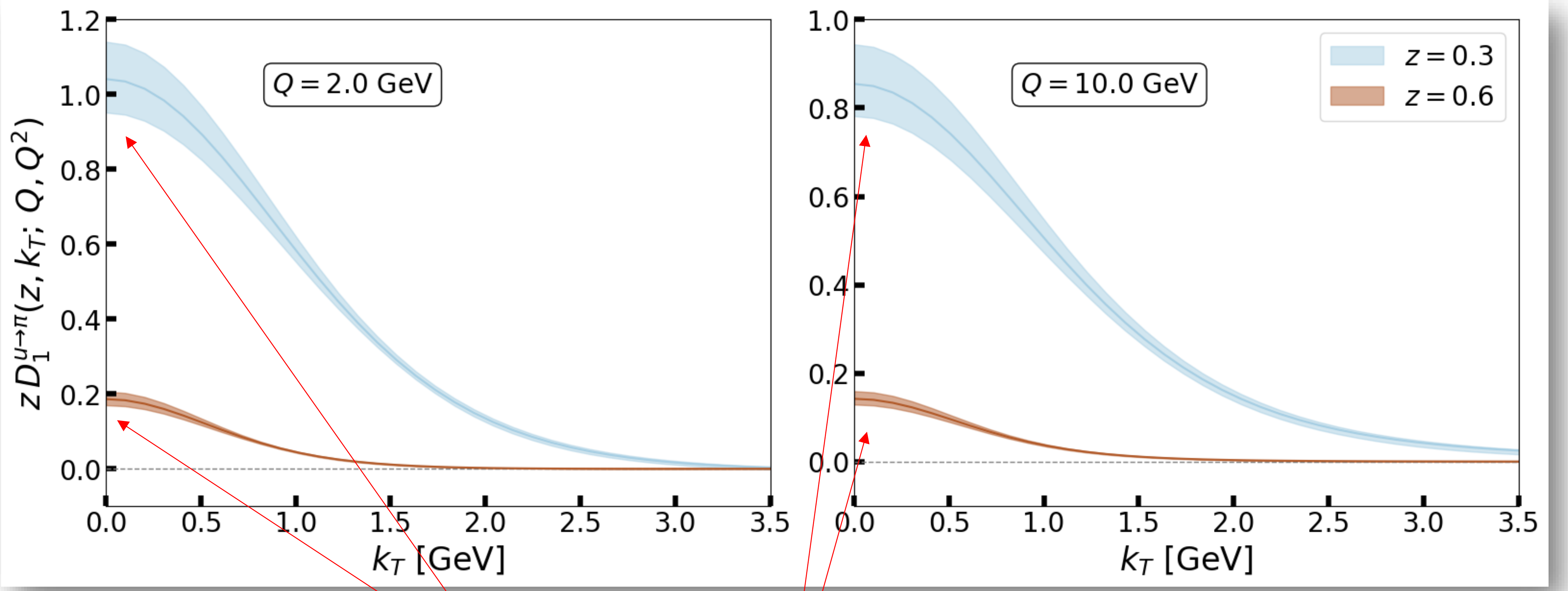
230 Data in total

Description of BELLE data



Thrust dependence consistently treated and successfully described for the very first time

Unpolarized TMD Fragmentation Function



Expected different behavior at low kT compared to TMD FFs extracted from standard processes (e.g. SIDIS)

Accessing the P-term

From comparing **operators**:

$$D(z, b_T, y_{\rm had} - y_1) = D^\star(z, b_T, y_{\rm had} - y_1) e^{-\frac{1}{2}P(b_T)}$$

Explicit P-term

TMD extraction from standard process

TMD extraction from beyond standard process (SIA R_2)

Oss: $\dfrac{dP(b_T,\mu)}{d\log\mu} = -\gamma_P(a_S(\mu))$

→ D^* has different evolution equations

$$\frac{\partial \log D^\star(z, b_T; \mu, y_1)}{\partial y_1} = -K(b_T, \mu)$$

$$\frac{\partial \log D^\star(z, b_T; \mu, y_1)}{\partial \log\mu} = \gamma_d(a_S(\mu)) + \frac{1}{2}\gamma_P(a_S(\mu)) - \gamma_K(a_S(\mu)) \log\frac{Qe^{-y_1}}{\mu}$$

$$R = \frac{\text{TMD extraction from beyond standard process}}{\text{TMD extraction from standard process}} = e^{-\frac{1}{2}P(b_T,\mu)}$$

This relation is *exact,* provided that:

1. Both extractions are performed @ same perturbative accuracy

2. Both extractions use the same model for the non-perturbative behavior of the CS-kernel

... in practice, given the extractions available today, none of the assumptions above are satisfied at the same time.

Also, the comparison is cleaner if both extractions use the same prescription for separating out the non-perturbative effects in b_T-space (CSS b*, HSO...)

Simplifications at the numerator:

Neglecting orders $\mathcal{O}\left(\frac{1}{y_1}\right)$ as in

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Published in: JHEP 09 (2023) 006 • e-Print: 2306.02937 [hep-ph]
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$$d\sigma_{R_2} \propto e^{I_R(u,y_1,\mu)-\frac{1}{2}P(b_T,\mu)} \approx e^{\frac{1}{2}g_P(b_T)}$$

P-terms for thrust and TMD sector

CSS b*-prescription

$$P(b_T,\mu) = P(a_S(\mu_b^\star)) - \int_{\mu_b^\star}^{\mu} \frac{d\mu'}{\mu'}\gamma_P(a_S(\mu')) - g_P(b_T)$$

The function g_P for the P-term is the counterpart of g_K for the CS-kernel.

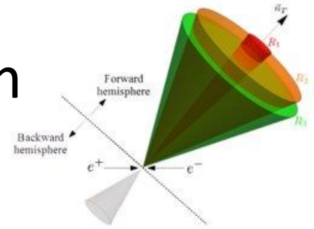
Then, effectively:

$$D^\star_{i/h}(z,b_T,\mu,y_1) = \frac{1}{z^2} C_{i/j} \otimes d_{j/h}(z,\mu_b)$$

$$e^{\frac{1}{2}K(a_S(\mu_b^\star))\log\frac{Qe^{-y_1}}{\mu_b^\star} + \int_{\mu_b^\star}^{\mu}\frac{d\mu'}{\mu'}\gamma_D\left(a_S(\mu'),\log\frac{Qe^{-y_1}}{\mu'}\right)}$$

$$M_D(z,b_T)e^{\frac{1}{2}g_P(b_T)} e^{-\frac{1}{2}g_K(b_T)\log\frac{Qe^{-y_1}}{M_{\rm had}}}$$

Effective combination extracted from



Ideally:

BS23 (NP-model)

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$$R = \frac{\text{BS23 (NP-model)}}{\text{SV19 (NP-model) OR MAP22 (NP-model)}} = e^{\frac{1}{2} g_P(b_T)}$$

SV19 (NP-model)

Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum #
Ignazio Scimemi (Madrid U.), Alexey Vladimirov (Regensburg U.) (Dec 13, 2019)
Published in: *JHEP* 06 (2020) 137 • e-Print: 1912.06532 [hep-ph]
pdf DOI cite claim reference search 138 citation

OR

MAP22 (NP-model)

Unpolarized transverse momentum distributions from a global fit of Drell-Yan and semi-inclusive deep-inelastic scattering data #4
MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (Jun 15, 2022)
Published in: *JHEP* 10 (2022) 127 • e-Print: 2206.07598 [hep-ph]
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The plan is:

- ❑ Check z-independence of R (insensitive to collinear physics)
- ❑ Infer information on g_P

Beware! This would be an extraction of an extraction!

Ratio w.r.t SV19

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h\,X$ processes

M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)

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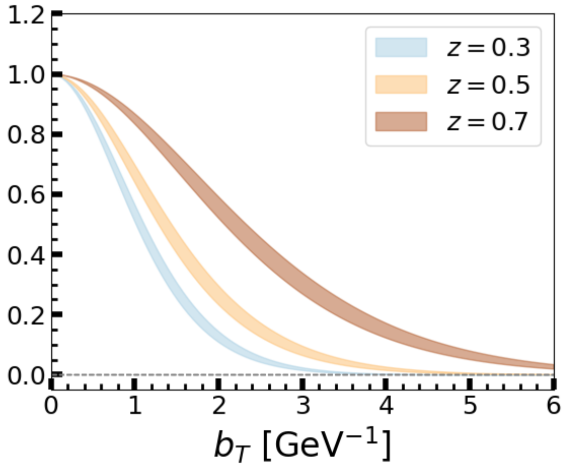
2 free parameters

$$M_D(z, b_T) = \frac{2}{\Gamma(p(z)-1)} \left(\frac{b_T\, m(z)}{2} \right)^{p(z)-1} K_{p(z)-1}\left(b_T\, m(z)\right)$$



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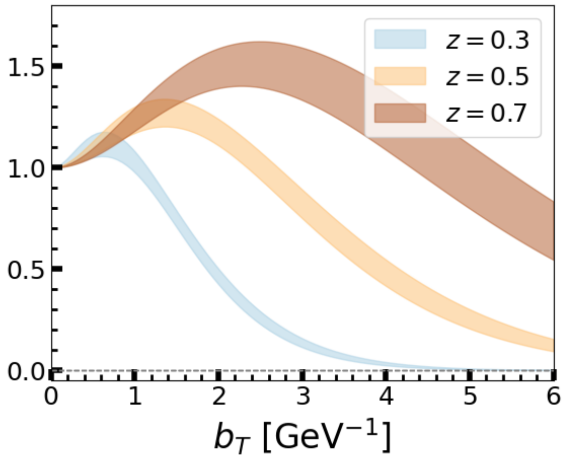
4 free parameters

$$D_{NP}(x, b) = \exp\left(-\frac{\eta_1 z + \eta_2 (1-z)}{\sqrt{1+\eta_3 (\boldsymbol{b}/z)^2}} \frac{\boldsymbol{b}^2}{z^2} \right) \left(1 + \eta_4 \frac{\boldsymbol{b}^2}{z^2} \right)$$

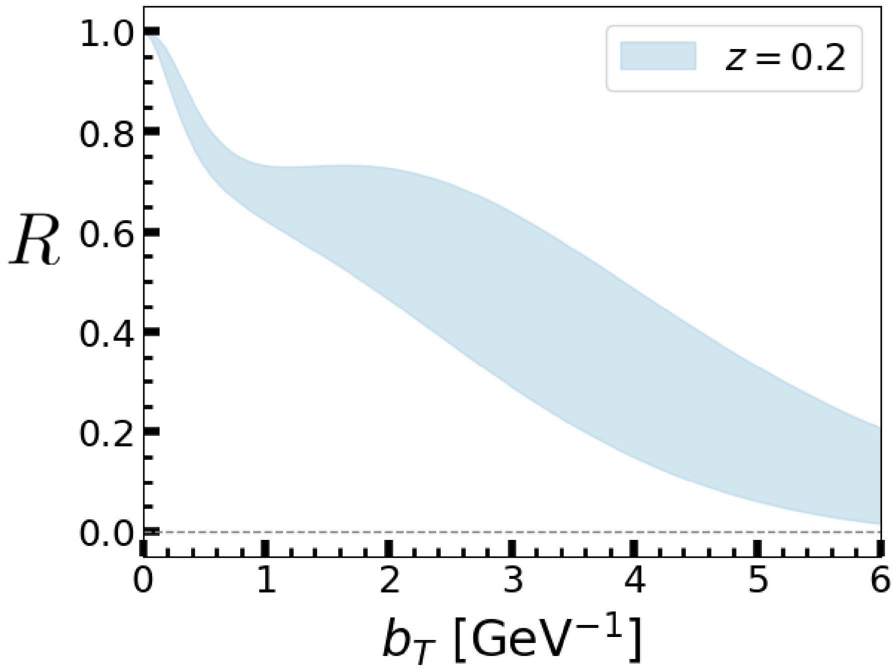


Test the ansatz:

- Extractions @ same perturbative accuracy ✗
- Extractions use the same g_K ✗

BS23	SV19	
NLL	N2LL	→ z-independence affected
~ constant	~ linear	→ b_T-dependence affected

Still:



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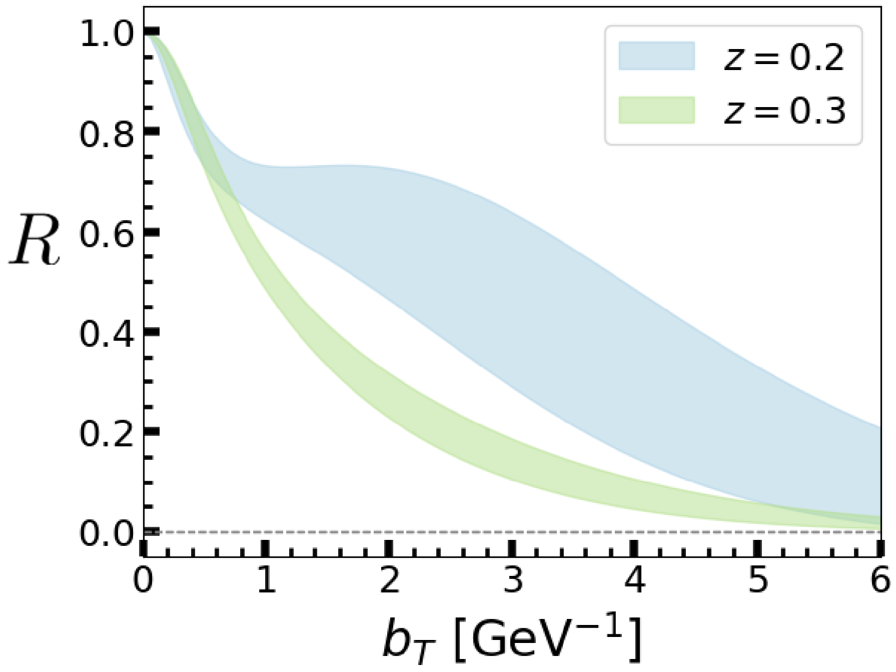

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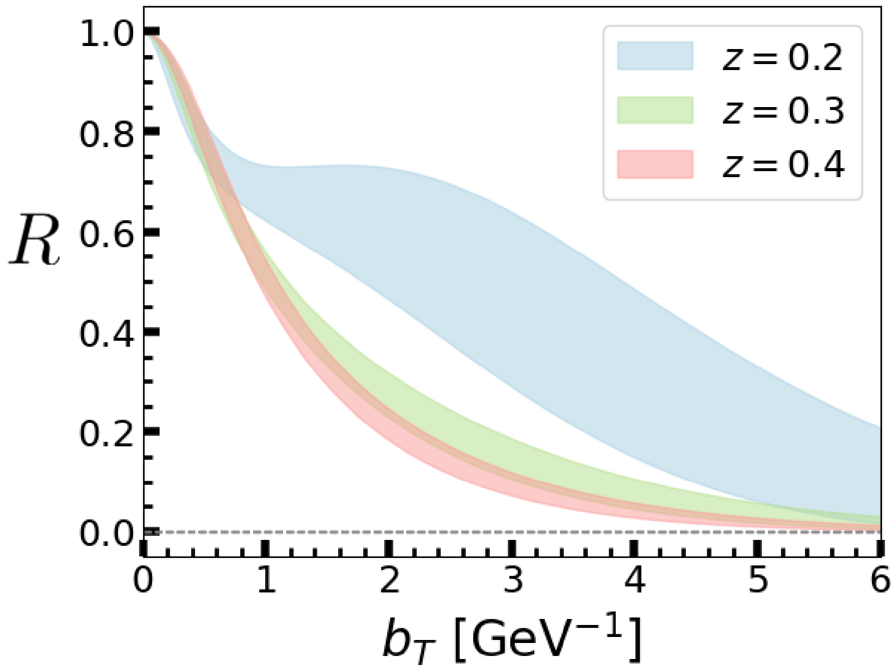

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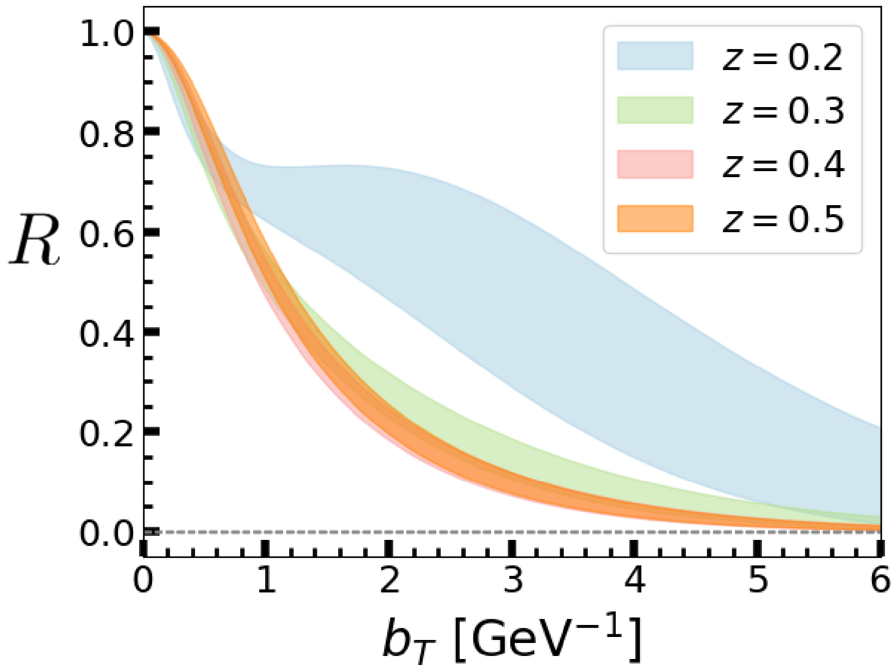

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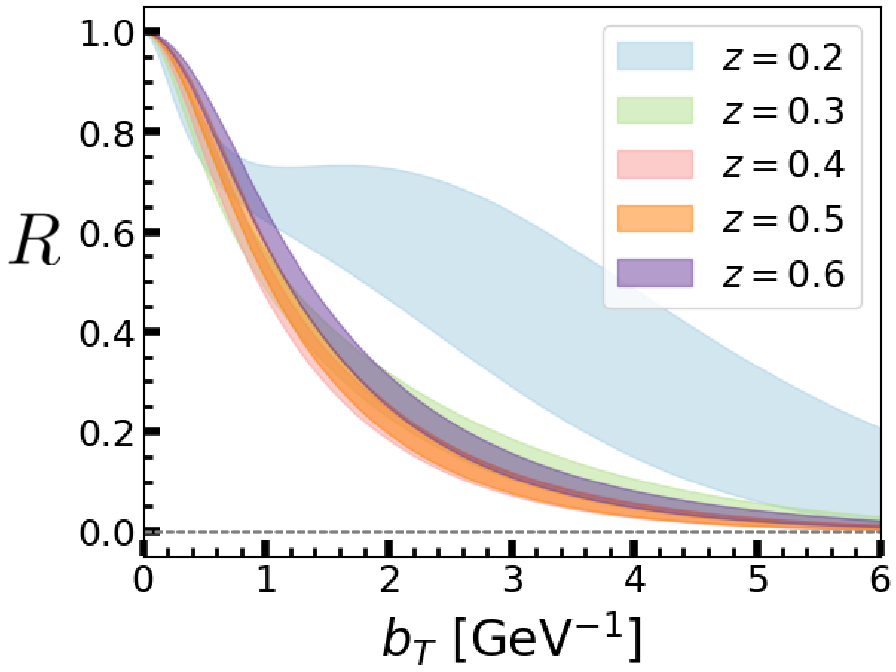

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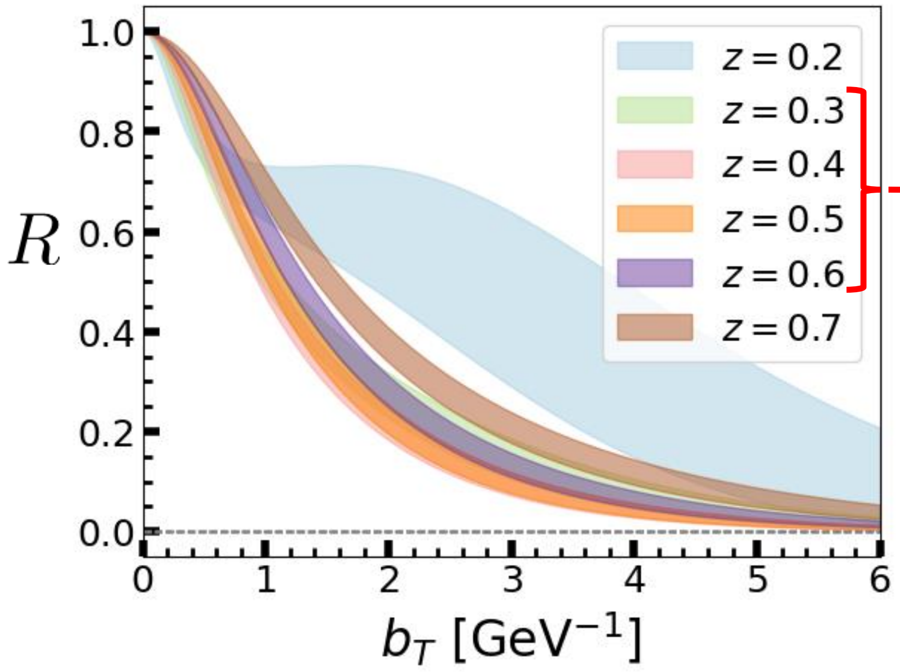

→ Quite not dependent on z

Ratio w.r.t MAP22

Full treatment of the thrust distribution in single inclusive $e^+e^- \to h\,X$ processes #1

M. Boglione (Turin U. and INFN, Turin), A. Simonelli (Old Dominion U. and Jefferson Lab) (Jun 5, 2023)

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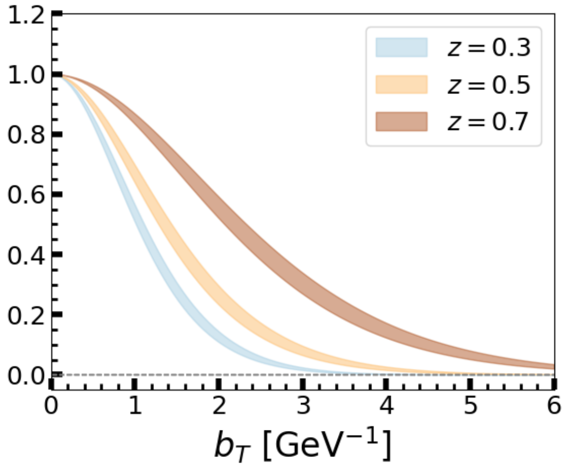

Unpolarized transverse momentum distributions from a global fit of Drell-Yan and semi-inclusive deep-inelastic scattering data #4

MAP (Multi-dimensional Analyses of Partonic distributions) Collaboration • Alessandro Bacchetta (Pavia U. and INFN, Pavia) et al. (Jun 15, 2022)

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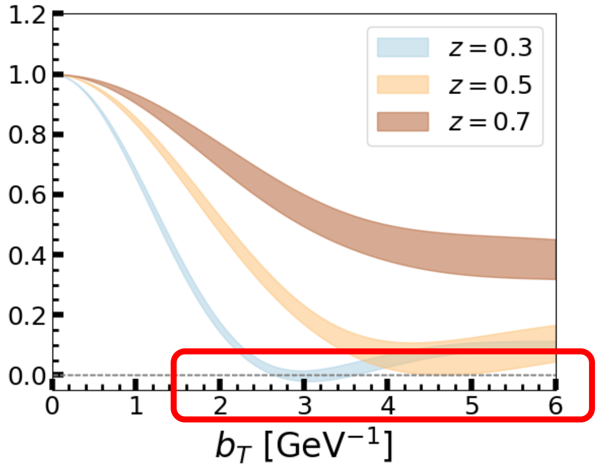
9 free parameters

$$D_{1\,NP}(z,b_T^2;\zeta,Q_0) = \frac{g_3(z)\,e^{-g_3(z)\frac{b_T^2}{4z^2}} + \frac{\lambda_F}{z^2}\, g_{3B}^2(z)\left[1 - g_{3B}(z)\frac{b_T^2}{4z^2}\right] e^{-g_{3B}(z)\frac{b_T^2}{4z^2}}}{g_3(z) + \frac{\lambda_F}{z^2}\, g_{3B}^2(z)}$$

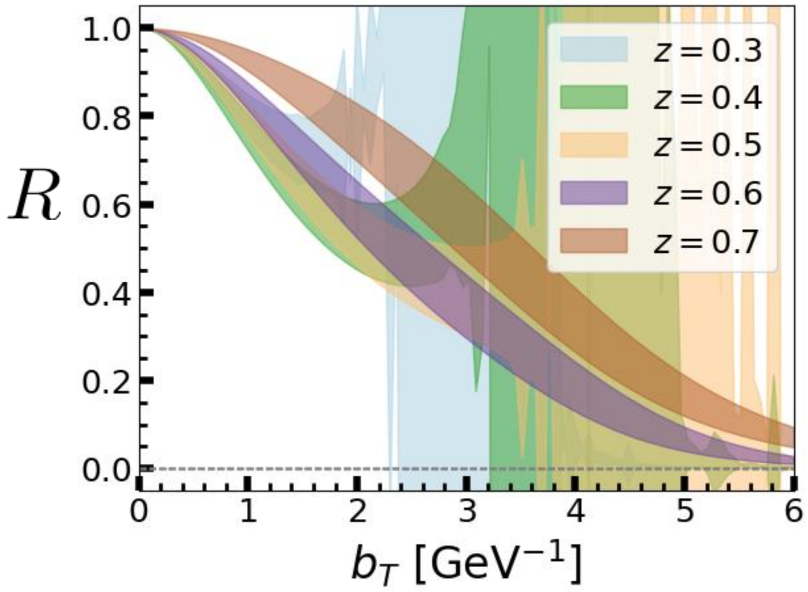


Test the ansatz:

- Extractions @ same perturbative accuracy ✓
- Extractions use the same g_K ✗

	BS23	MAP22	
Extractions @ same perturbative accuracy	NLL	NLL	→ z-independence not affected
Extractions use the same g_K	~ constant	~ quadratic	→ b_T-dependence affected

However:



z-independence ?

Not satisfied, or, at least, not constrained.

BS23 (BELLE data $e^+e^- \to \pi^\pm X$ with thrust)

T	z												P_T/z max	N
	0.20 – 0.25	0.25 – 0.30	0.30 – 0.35	0.35 – 0.40	0.40 – 0.45	0.45 – 0.50	0.50 – 0.55	0.55 – 0.60	0.60 – 0.65	0.65 – 0.70	0.70 – 0.75	0.75 – 0.80		
0.80 – 0.85													$0.16\,Q$	57
0.85 – 0.90													$0.15\,Q$	60
0.90 – 0.95													$0.14\,Q$	61
0.95 – 1.00													$0.13\,Q$	52

SV19

Experiment	Reaction	ref.	Kinematics	N_{pt} after cuts
HERMES	$p \to \pi^+$	[66]	0.023<x<0.6 (6 bins)	24
	$p \to \pi^-$		0.2<z<0.8 (6 bins)	24
	$p \to K^+$		$1.0<Q<\sqrt{20}$GeV	24
	$p \to K^-$			24
	$D \to \pi^+$			24
	$D \to \pi^-$		$W^2 > 10\text{GeV}^2$	24
	$D \to K^+$		0.1<y<0.85	24
	$D \to K^-$			24
COMPASS	$d \to h^+$	[67]	0.003<x<0.4 (8 bins)	195
	$d \to h^-$		0.2<z<0.8 (4 bins)	195
			1.0<Q≃ 9GeV (5 bins)	
Total				582

The two extractions overlapping core is

$$0.3 \lesssim z \lesssim 0.7$$

z-independence ✔

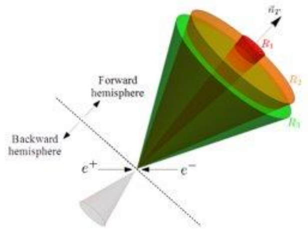
Keep in mind that:

- Very different functional forms at numerator and denominator
- Extractions from completely different data sets (BELLE vs SIDIS/DY)

Conclusions

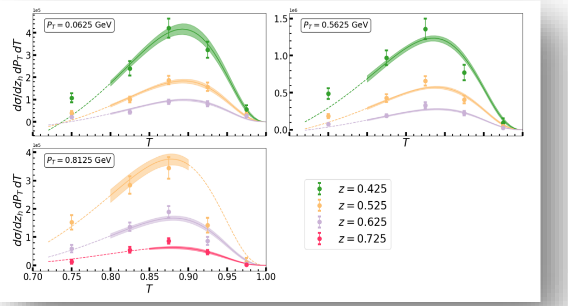
- Extensions of TMD factorization require to deal with extension of **universality**.
- The role of soft physics is crucial:

$$S(b_T;\phi_M) = e^{\phi_M K(b_T)+P(b_T)}$$



- Need to go beyond standard processes to have sensitivity on P-terms
- The relevant quantity is the ratio:

$$R = \frac{\text{TMD extraction from beyond standard process}}{\text{TMD extraction from standard process}} = e^{-\frac{1}{2}P(b_T,\mu)}$$



- Lattice applications

Thank You!

Back-up

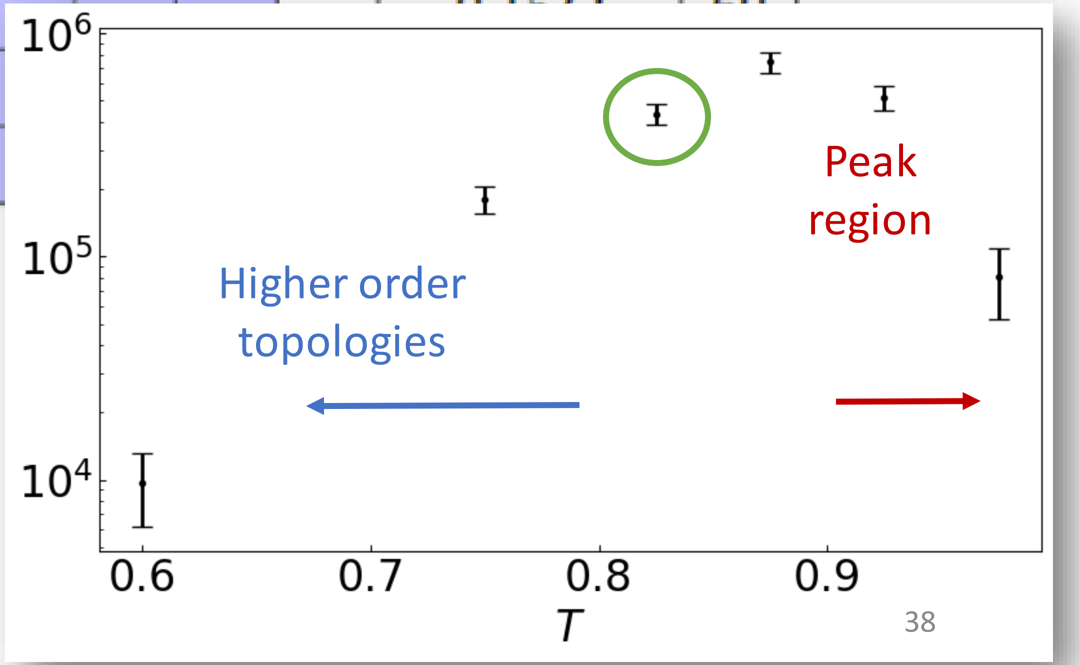
Phenomenology $e^+e^- \to \pi X$

T	z												P_T/z max	N
	$0.20-0.25$	$0.25-0.30$	$0.30-0.35$	$0.35-0.40$	$0.40-0.45$	$0.45-0.50$	$0.50-0.55$	$0.55-0.60$	$0.60-0.65$	$0.65-0.70$	$0.70-0.75$	$0.75-0.80$		
$0.80-0.85$													$0.16\,Q$	57
$0.85-0.90$													$0.15\,Q$	[illegible]
$0.90-0.95$													[illegible]	[illegible]
$0.95-1.00$													[illegible]	[illegible]

Subsample of 57 data points

"Pure" TMD extraction

A preliminary fit in the subsample fixes the functional form of the non-perturbative content of the TMD FF



Non-perturbative content of the TMD FF

1. function, describing the long-distance behavior of the Collins-Soper kernel.

- Even function of b_T
- Parabolic behavior at small $g_K \sim g_2 b_T^2 + \ldots$ for $b_T \to 0.$
- Constant behavior at large $g_K \to g_0$ for $b_T \to \infty.$

$$g_K^A(b_T) = g_0 \tanh\left(\beta^2 \frac{b_T^2}{b_{MAX}^2}\right),$$

$$g_K^B(b_T) = g_0 \tanh\left(\beta^2 b_T^\star b_T\right).$$

2. model for the (unpolarized) TMD FF, describing its characteristic long-distance behavior.

- Gaussian behavior at small $M_D \sim e^{-cb_T^2} \times \ldots$ for $b_T \to 0.$
- Exponential decay at large $M_D \sim e^{-db_T} \times \ldots$ for $b_T \to \infty.$

$$M_D\left(z_h, P_T, M, p\right) = \frac{\Gamma(p)}{\pi\Gamma(p-1)} M^{2\,(p-1)} \left(M^2 + \frac{P_T^2}{z_h^2}\right)^{-p}$$

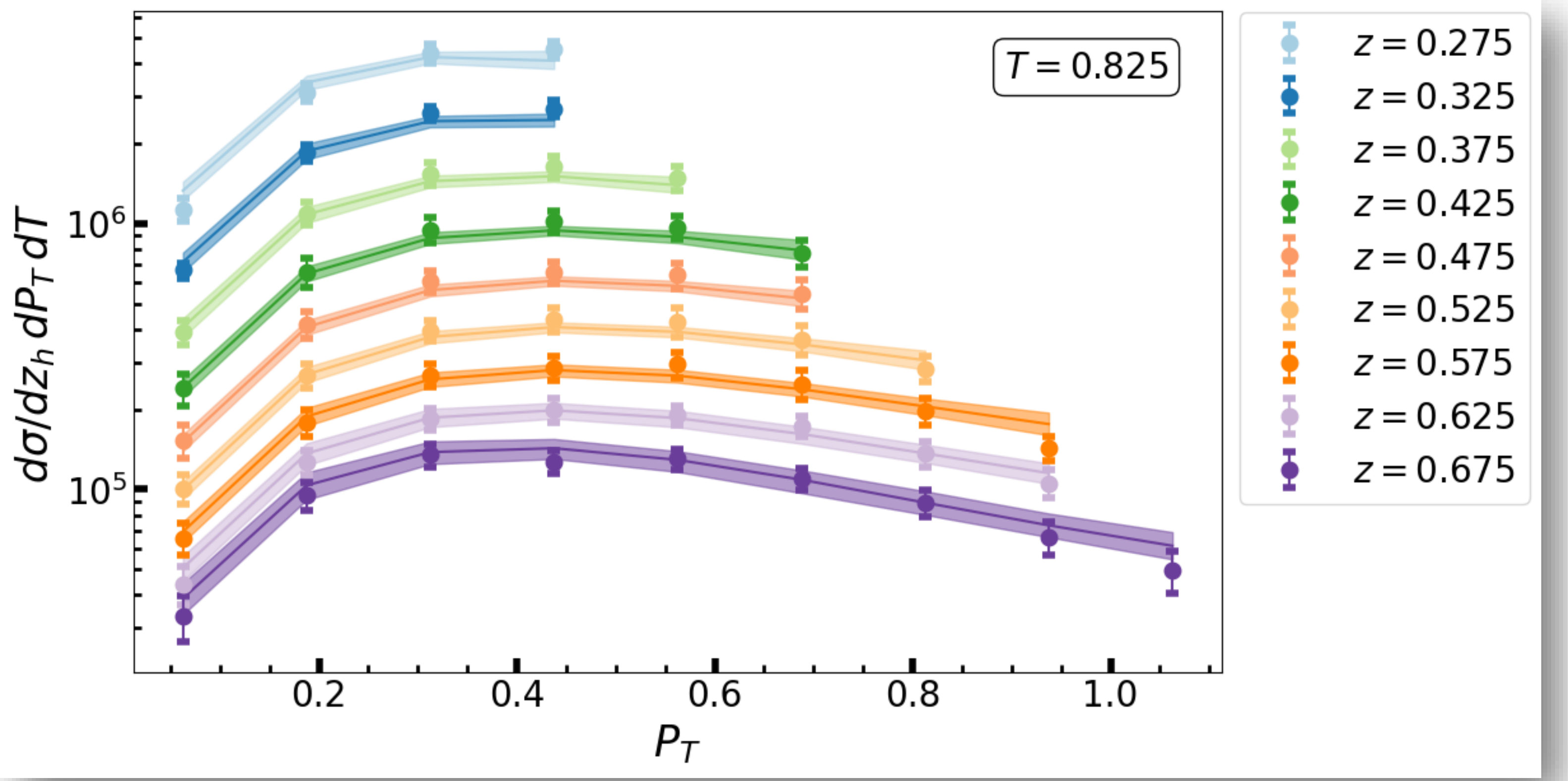
Where $p = p(R, W)$ and $M = M(R, W)$

$$R(z) = 1 - \alpha\, \frac{f(z)}{f(z_0)} \text{ with } f(z) = z\,(1-z)^{\frac{1-z_0}{z_0}},$$

$$W(z) = \frac{m_\pi}{R(z)}.$$

Preliminary step: TMD FF functional form

χ^2/d.o.f.	0.6183
z_0	$0.5521^{+0.0415}_{-0.0398}$
α	$0.3644^{+0.0250}_{-0.0282}$
g_0	$0.2943^{+0.0329}_{-0.0261}$
β	$4.7100^{+1.9856}_{-1.9856}$



The aim is to test the validity of the functional forms chosen for g_K and M_D

The value of the free parameters should not be taken at face value, especially those associated to g_K

Final fit: inclusion of NP thrust effects

Several recipes available.
We choose the simplest: minimal approach

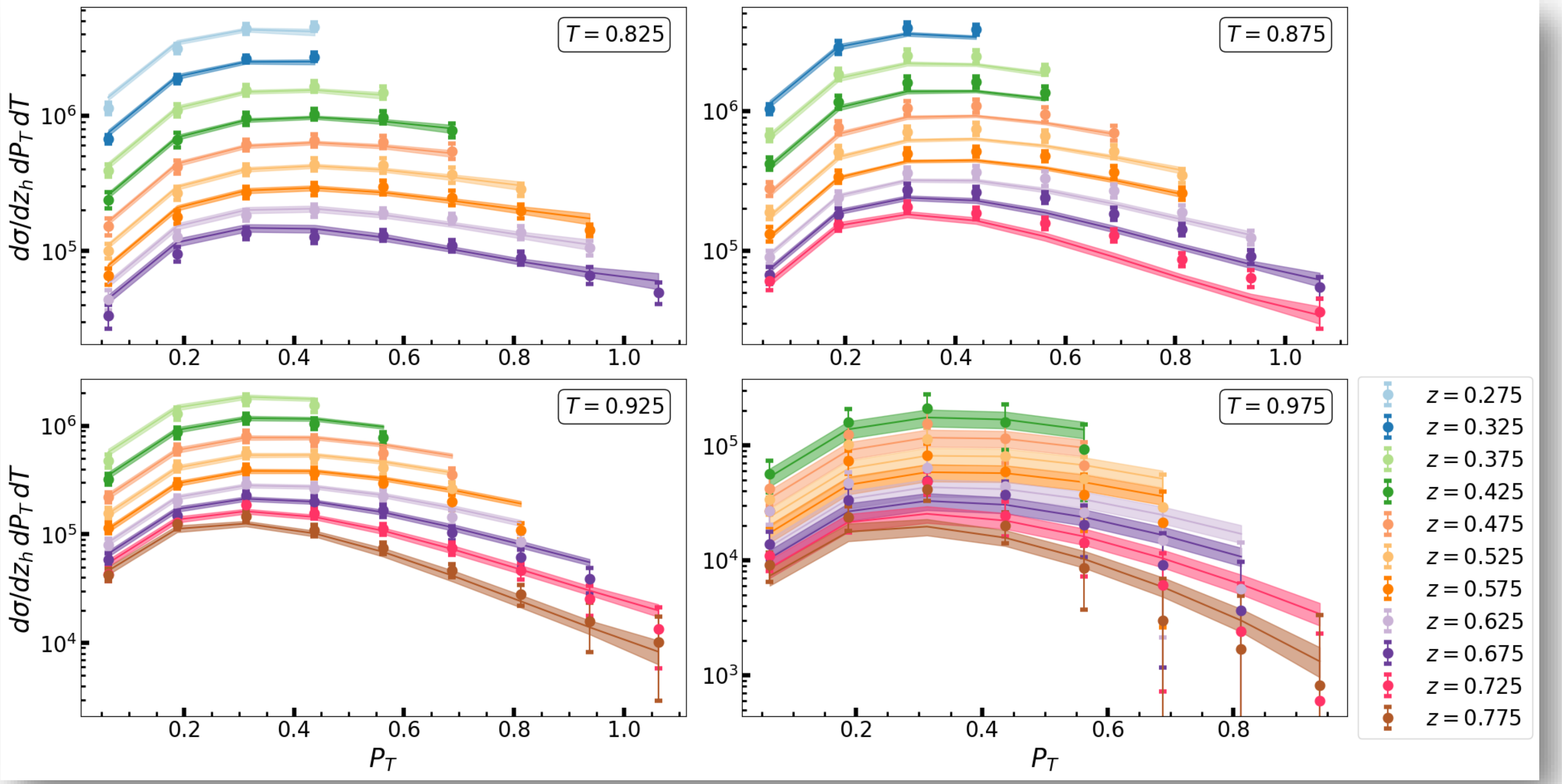
$$\frac{d\sigma}{dz\, dT\, d^2\vec{P}_T} = \left.\frac{d\sigma^{\text{pert.}}}{dz\, dT\, d^2\vec{P}_T}\right|_{T-T_0} f_{\text{NP}}(T)$$

With:
$f_{NP} = \tanh\left(\rho\,(1-T)\right)^2$

Case (B)

$g_K^B(b_T) = g_0 \tanh\left(\beta^2\, b_T^\star\, b_T\right)$

χ^2/d.o.f.	1.3421
z_0	$0.5334^{+0.0192}_{-0.0189}$
α	$0.3394^{+0.0127}_{-0.0134}$
g_0	$0.1205^{+0.0305}_{-0.0367}$
β	$2.0610^{+2.1042}_{-0.5193}$
T_0	$0.0467^{+0.0117}_{-0.0077}$
ρ	$8.1643^{+0.3053}_{-0.3011}$



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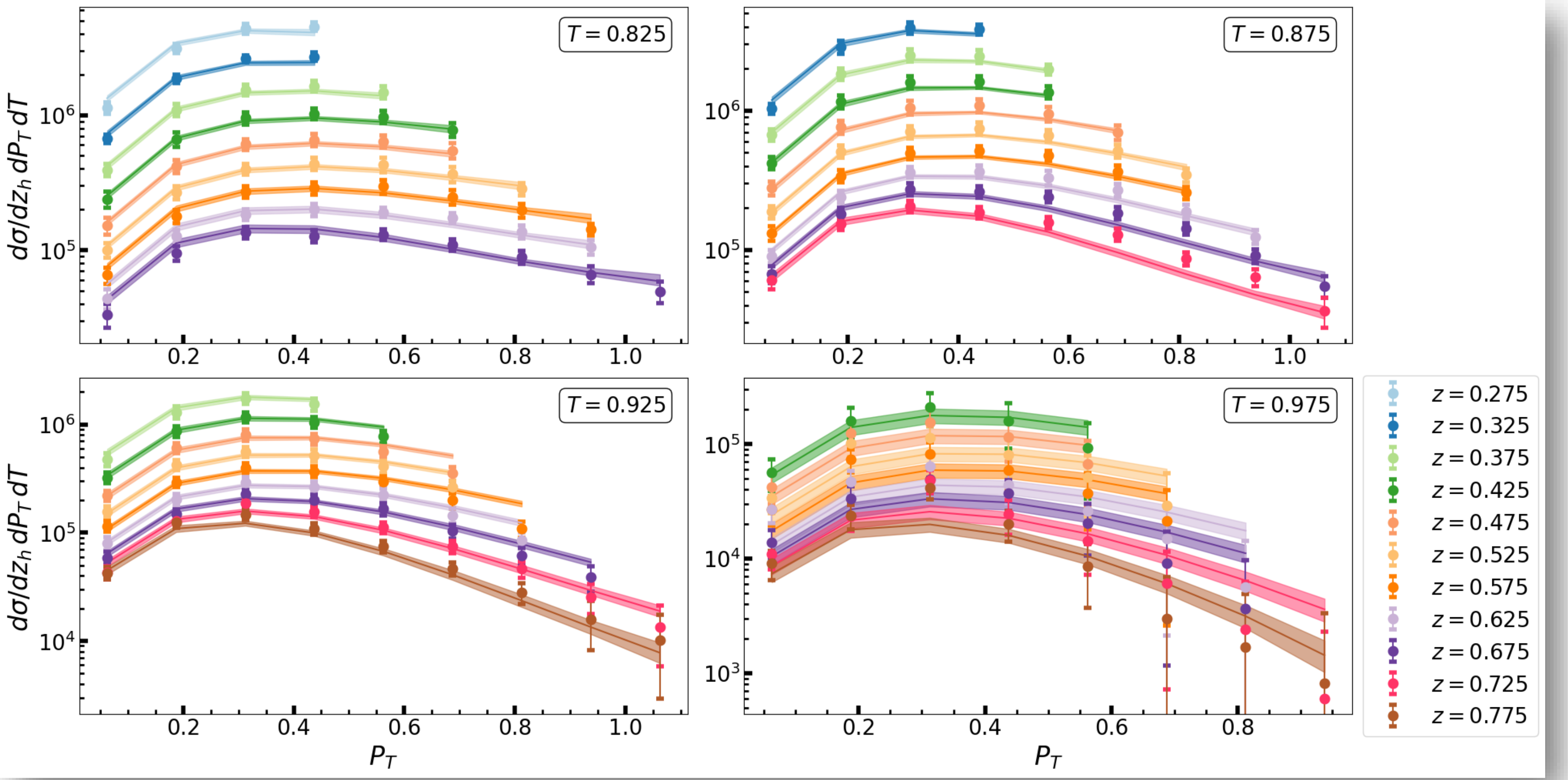
With:
$$f_{NP} = \tanh\left(\rho\,(1-T)\right)^2$$

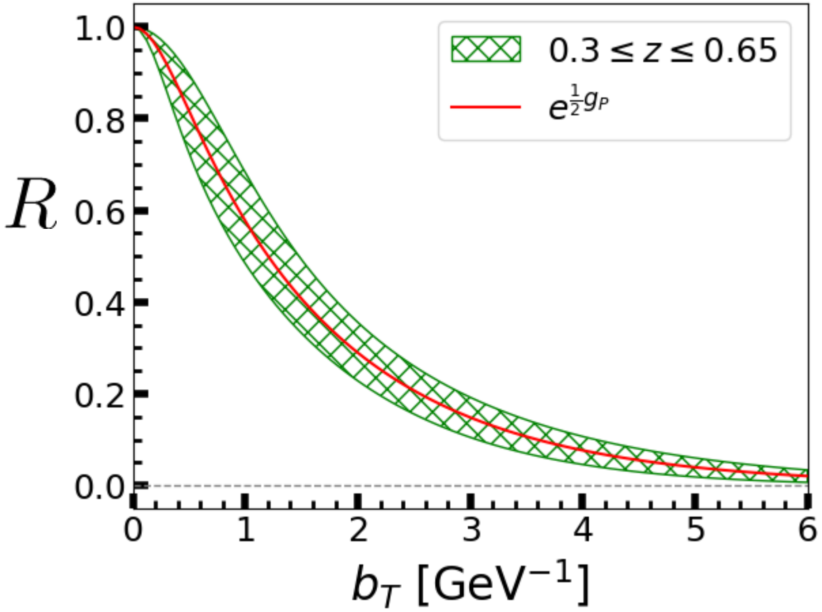
Case (A)

$$g_K^A(b_T) = g_0 \tanh\left(\beta^2 \frac{b_T^2}{b_{MAX}^2}\right)$$

χ^2/d.o.f.	1.0749
z_0	$0.5335^{+0.0194}_{-0.0180}$
α	$0.3403^{+0.0114}_{-0.0122}$
g_0	$0.1044^{+0.0446}_{-0.0742}$
β	$1.6765^{+0.8150}_{-0.8150}$
T_0	$0.0617^{+0.0295}_{-0.0134}$
ρ	$7.7205^{+0.2834}_{-0.2099}$



Spread of error bands can be used to constraint the "extraction" of g_P



Asymptotically linear

$$g_P = -\frac{b_T^2}{\alpha\sqrt{1+\frac{b_T^2}{\beta^2}}}$$

$\alpha = 0.48 \pm 0.15\ \mathrm{GeV}^{-2}$

$\beta = 0.65 \pm 0.21\ \mathrm{GeV}^{-1}$

Asymptotically sub-linear

$$g_P = -\alpha \log\left(1 + \beta\, b_T^2\right)$$

$\alpha = 2.51 \pm 0.21$

$\beta = 0.43 \pm 0.07\ \mathrm{GeV}^2$