TMD EXTRACTIONS AND MOMENTS

Ignazio Scimemi for Transversity 2024, June 6th, Trieste

V. Moos, I. S., A. Vladimirov, P. Zurita, arXiv:2305.07473 [hep-ph], O. del Rio, A. Prokudin, I.S., A. Vladimirov e-Print: 2402.01836

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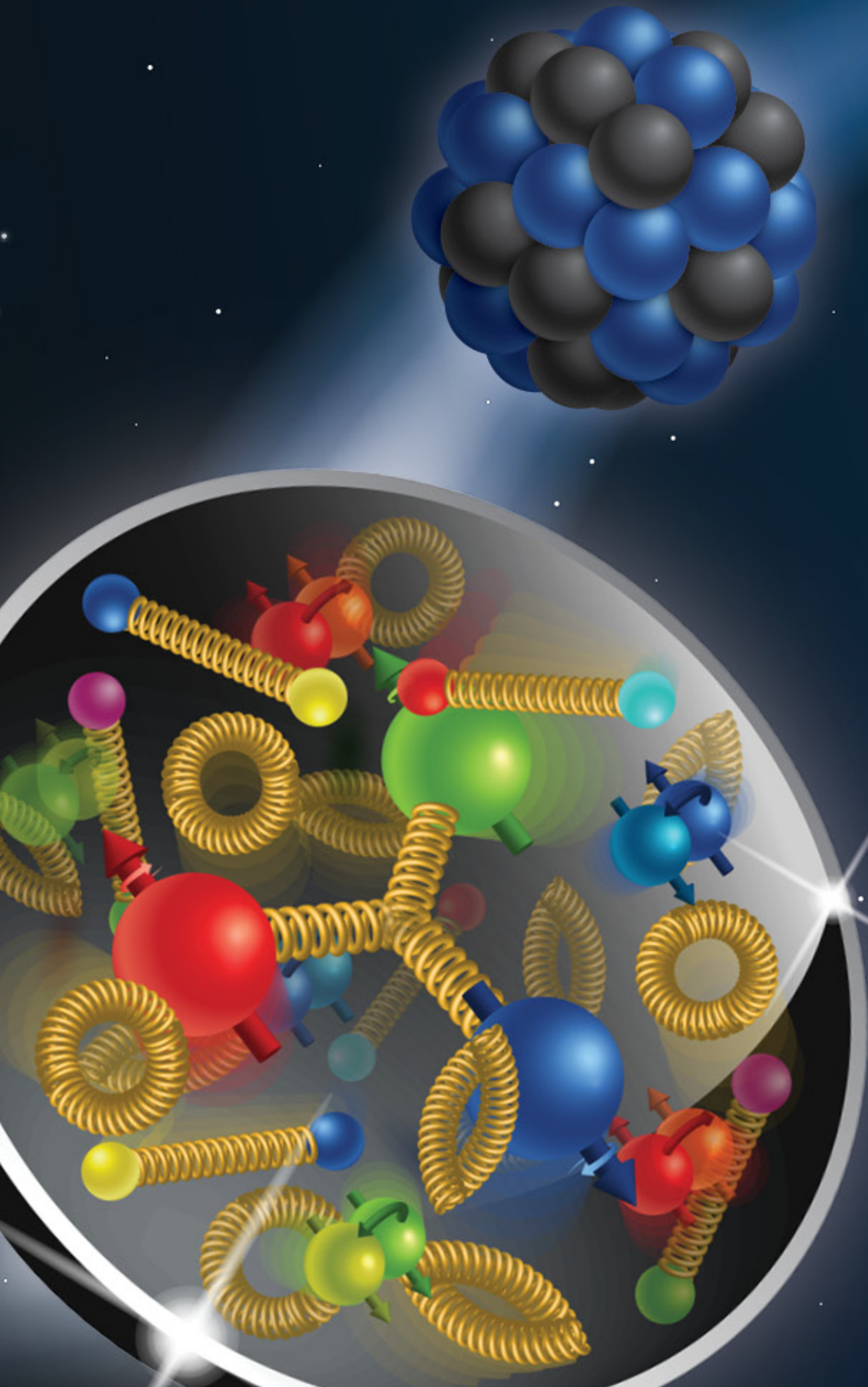


ATOM STRUCTURE IN XXTH CENTURY⇒QM

PROTON STRUCTURE IN XXITH CENTURY⇒
QCD SOLID STATE...

HADRON DYNAMICS NEEDS PRECISE/SPECIFIC DISTRIBUTIONS:
PDF, FF, **TMD,** GPD, GTMD,..
WIGNER DISTRIBUTIONS

ANSA.it

ANSA Latina

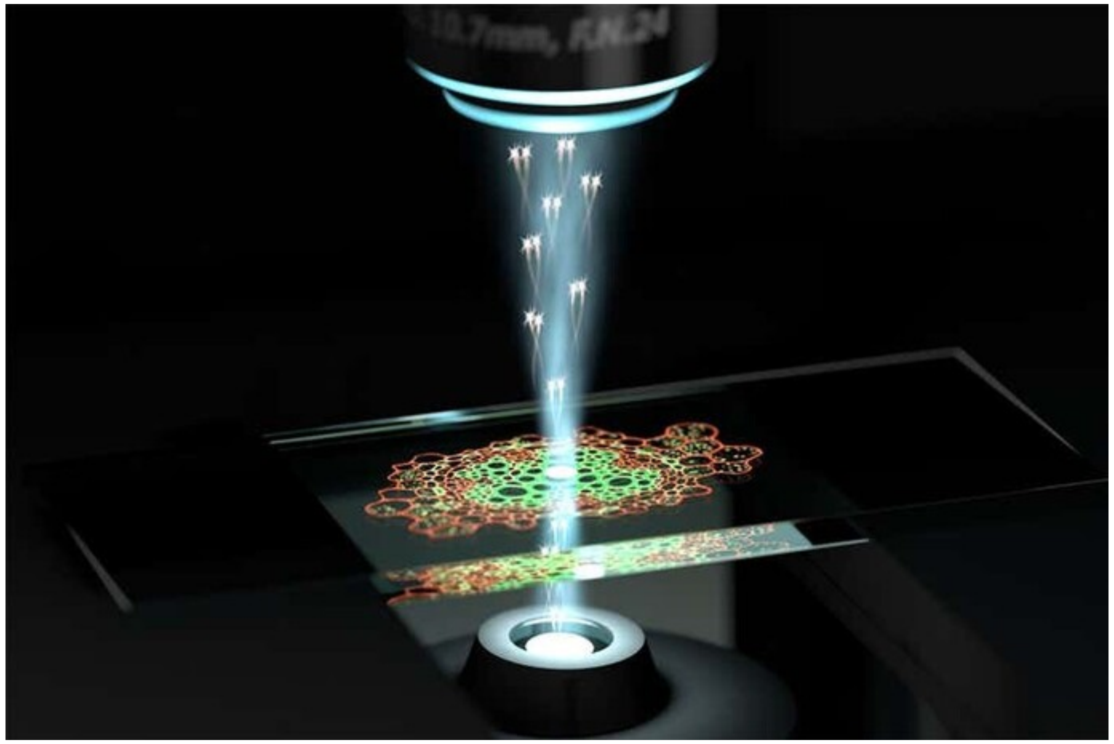
Un proyecto que permite soñar con un microscopio subnuclear

Físico italiano investiga movimiento de los quarks dentro de los protones.

MADRID 28 NOV -, 28 noviembre 2023, 12:29

Redaccion ANSA

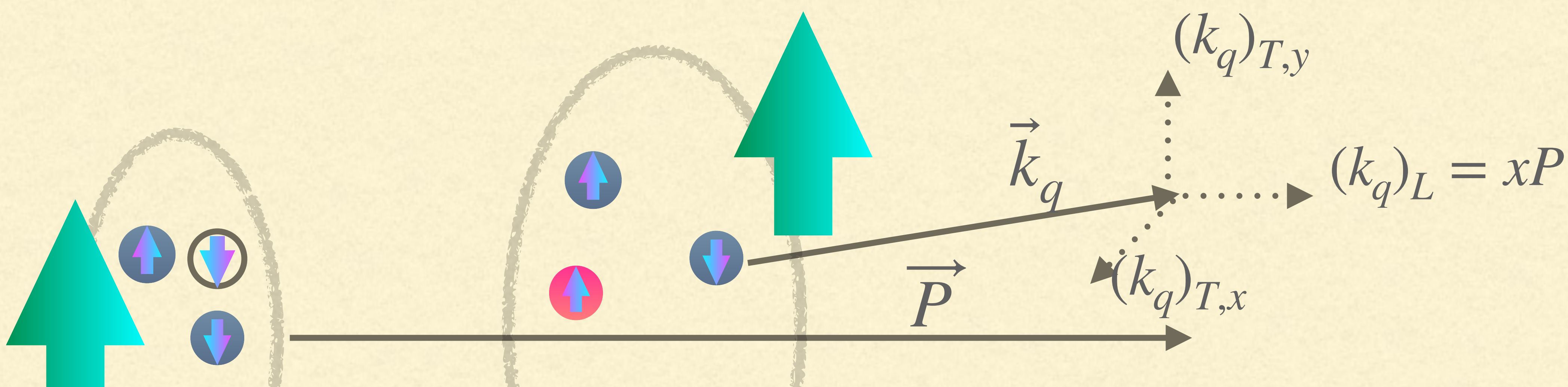
Compartir



↑ Un microscopio cuántico. Se necesitaría uno subatómico para poder ver dentro de los quarks. -

(ANSA) MADRID - Así como gracias a la mecánica cuántica se descubrió cómo los electrones se mueven dentro del átomo, "en un futuro, quizás podría haber un microscopio subnuclear capaz de detectar los movimientos de los quarks dentro de los protones", dice a

HADRON STRUCTURES



PDF: $f_{q/H}(x;\mu)$

TMDPDF: $f_{1,q/H}(x,k_T;\mu,\zeta)$

Quark Polarization | Time-reversal flip | Gluon Polarization

Nucleon Polarization

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_{1T}, h_{1T}^{\perp}$

Nucleon Polarization

GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

FACTORIZATION FORMULA

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \sum_{f_1,f_2} \int \frac{d^2\boldsymbol{b}}{4\pi} e^{i(\boldsymbol{b}\cdot\boldsymbol{q}_T)} H_{f_1 f_2}(Q,Q)\{R[\boldsymbol{b};(Q,Q^2)]\}^2 F_{f_1\leftarrow h_1}(x_1,\boldsymbol{b}) F_{f_2\leftarrow h_2}(x_2,\boldsymbol{b})$$

- Its range of applicability is provided by $\delta = \dfrac{q_T}{Q} \ll 1$, fixed-$q_T$, $\delta \sim 0.25$
- We have a non-perturbative evolution kernel, R[], (whose perturbative part is known at N3LO!!). We can work with different schemes (CSS, ζ-prescription).
- We have a re-factorization of TMD at large transverse momentum in Wilson coefficients (now at N3LO!!) and PDF (now used at NNLO!!, but N3LO on the way)
- PDF are just part of a model . Very useful but also problematic: PDF bias M. Bury, F. Hautmann, S. Leal-Gomez, I. S., A. Vladimirov, PZ, JHEP 10 (2022) 118

$$F_{f\leftarrow h}(x,b) = \sum_{f'} f^f_{NP}(x,b) \int_x^1 \frac{dy}{y} C_{f\leftarrow f'}(y, \mathbf{L}_{\mu_{\rm OPE}}, a_s(\mu_{\rm OPE}))\, f_{f\leftarrow h}(x/y, \mu_{\rm OPE})$$

ART23

PUBLIC CODE **ARTEMIDE,**
HTTPS://GITHUB.COM/VLADIMIROVALEXEY/ARTEMIDE-PUBLIC

V. Moos, I. S., A. Vladimirov, P. Zurita, arXiv:2305.07473 [hep-ph]

- TMD flavor dependence included
- All the latest LHC datasets!
- W-boson production! (only Tevatron, $m_T > 50$ GeV)
- Increased perturbative accuracy! (N^4LL: highest QCD perturbative precision in a non-perturbative extraction)
- Includes collinear PDF uncertainties!
- A full new fit to Drell-Yan data (627 points)

EVOLUTION KERNEL

We understand that both perturbative and non-perturbative elements should be combined.

$$\mathscr{D}(b,\mu) = \mathscr{D}_{\text{small}-\text{b}}(b^*,\mu^*) + \int_{\mu^*}^{\mu} \frac{d\mu'}{\mu'}\Gamma_{\text{cusp}}(\mu') + \mathscr{D}_{\text{NP}}(b),$$

$$\mathscr{D}_{\text{NP}}(b) = bb^* \left[c_0 + c_1 \ln\left(\frac{b^*}{B_{\text{NP}}}\right)\right]$$

$$b^*(b) = \frac{b}{\sqrt{1+\frac{\vec{b}^2}{B_{\text{NP}}^2}}}, \qquad \mu^*(b) = \frac{2e^{-\gamma_E}}{b^*(b)},$$

OPTIMAL TMD

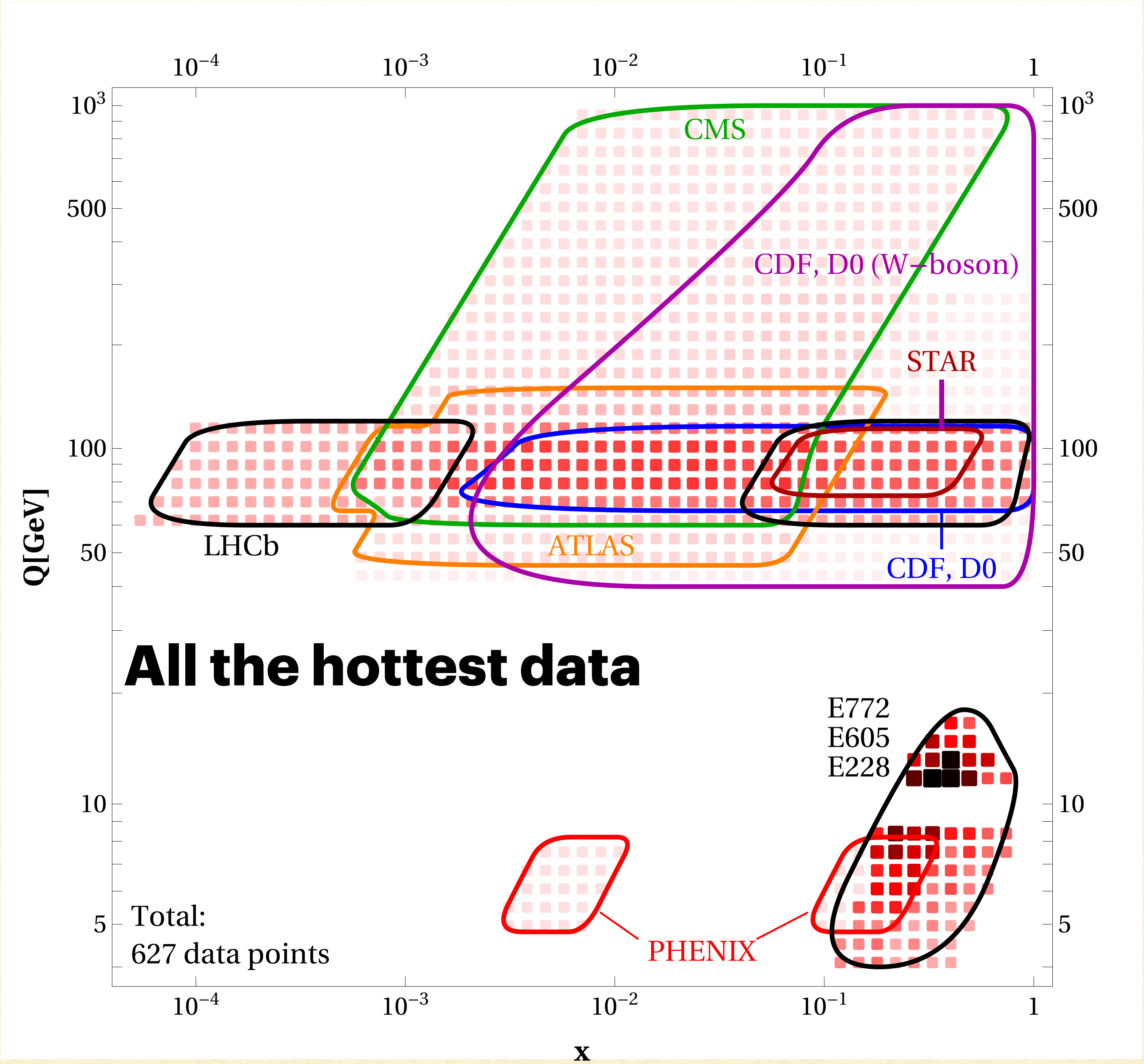
The ζ-prescription

$$\begin{cases} \Gamma_{\text{cusp}}(\mu)\ln\left(\dfrac{\mu^2}{\zeta_\mu(b)}\right) - \gamma_V(\mu) = 2\mathscr{D}(b,\mu)\dfrac{d\ln\zeta_\mu(b)}{d\ln\mu^2}\,. \\ \mathscr{D}(\mu_0, b) = 0, \qquad \gamma_F(\mu_0,\zeta_0) = 0. \end{cases}$$

$$f_{1,q\leftarrow h}(x,b) \equiv f_{1,q\leftarrow h}(x,b,\mu,\zeta_\mu)$$ Scale independence

$$f(x,b;\mu,Q^2) = \left(\frac{Q^2}{\zeta_\mu(b)}\right)^{-\mathscr{D}(b,\mu)} f(x,b)$$ Evolution decoupling

ART23



627 data points

New in!

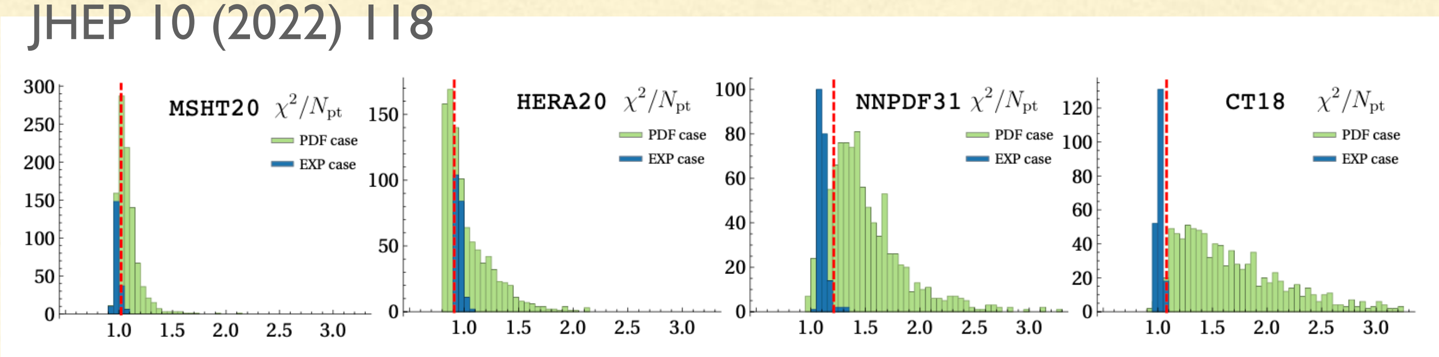
- **PHENIX**: DY data at $\sqrt{s} = 200$ GeV
- **STAR**: Z/γ-boson production at $\sqrt{s} = 510$ GeV (preliminary).
- **CMS** and **LHCb**: y-differential Z-boson production at $\sqrt{s} = 13$ TeV.
- **ATLAS**: high precision differential Z-boson cross-section.
- **CMS**: high-Q neutral-boson production.
- **Tevatron**: W-boson production.

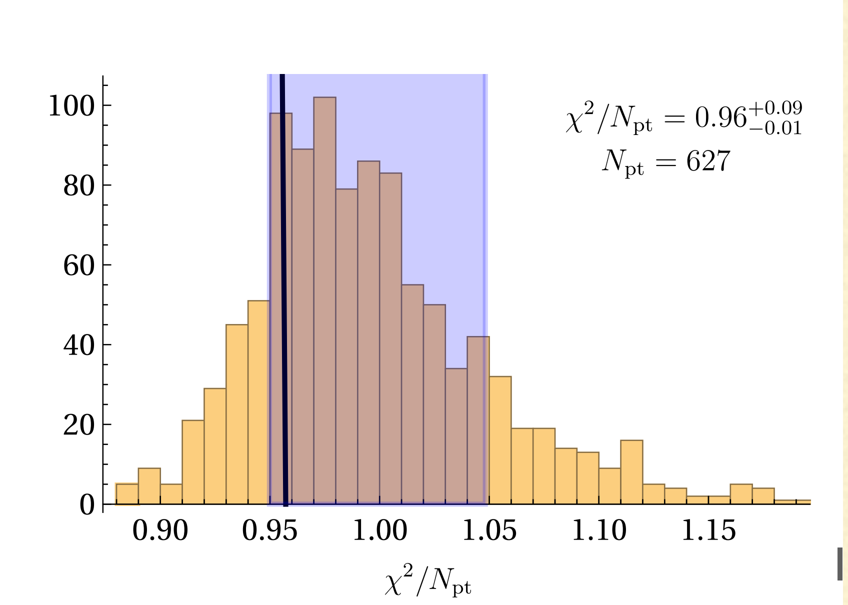
ART23

$f = u, \bar{u}, d, \bar{d}, sea$

- Parameterization: $f^f_{NP}(x, b) = 1/\cosh[(\lambda^f_1(1-x)+\lambda^f_2 x)b]$,
- In total, 13 parameters
- Reference PDFs: MSHT20
- Fitting procedure: construct simultaneous replicas of the **data** **AND** the **PDFs**. Then fit.

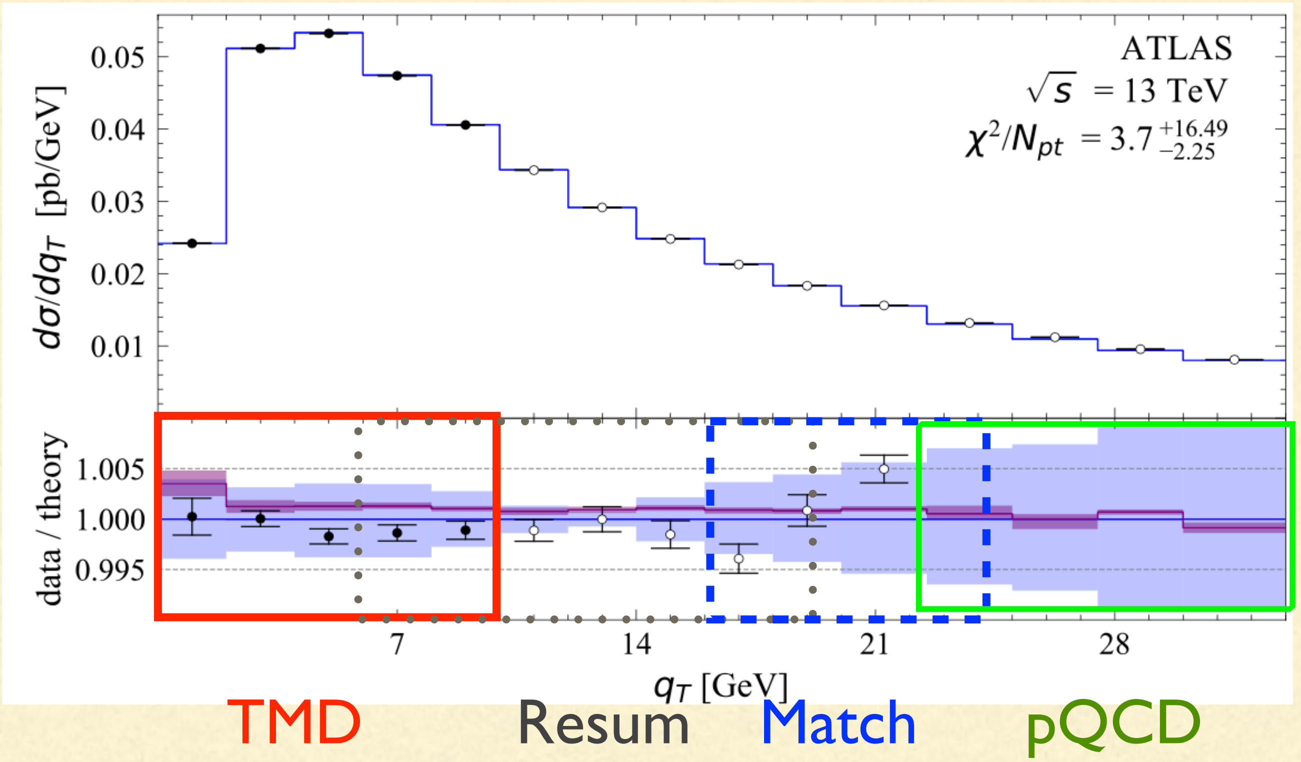
JHEP 10 (2022) 118



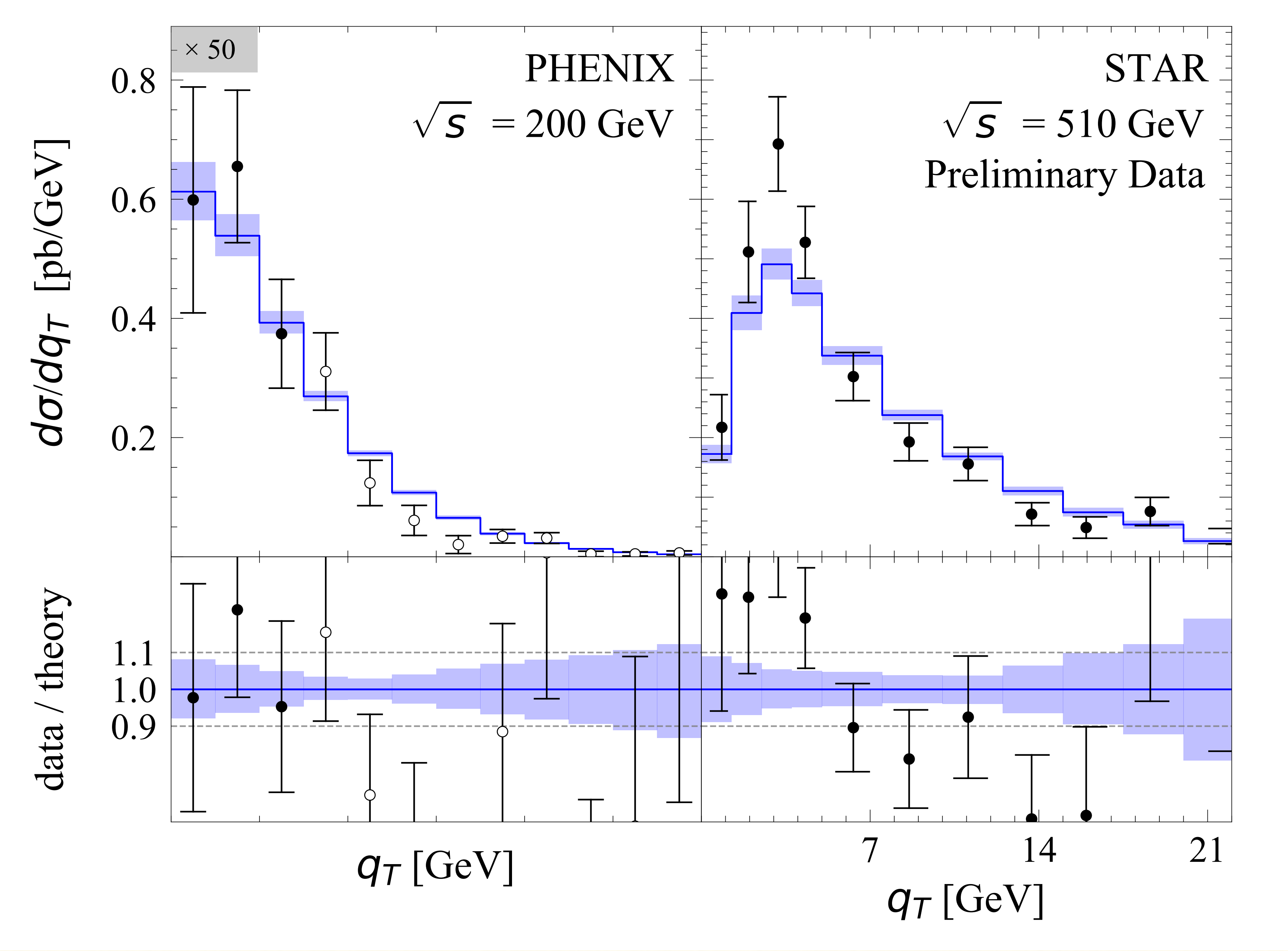


ART23: RESULTS

- $\chi^2/N_{pt} = 0.93$ (0.957 for the mean prediction), 68%CI (0.950, 1.048)
- Higher data precision plays a key role here.
- Realistic uncertainty bands than in SV19. Main error from PDF.
- Future: per mille precision with Power Corr. and different fit

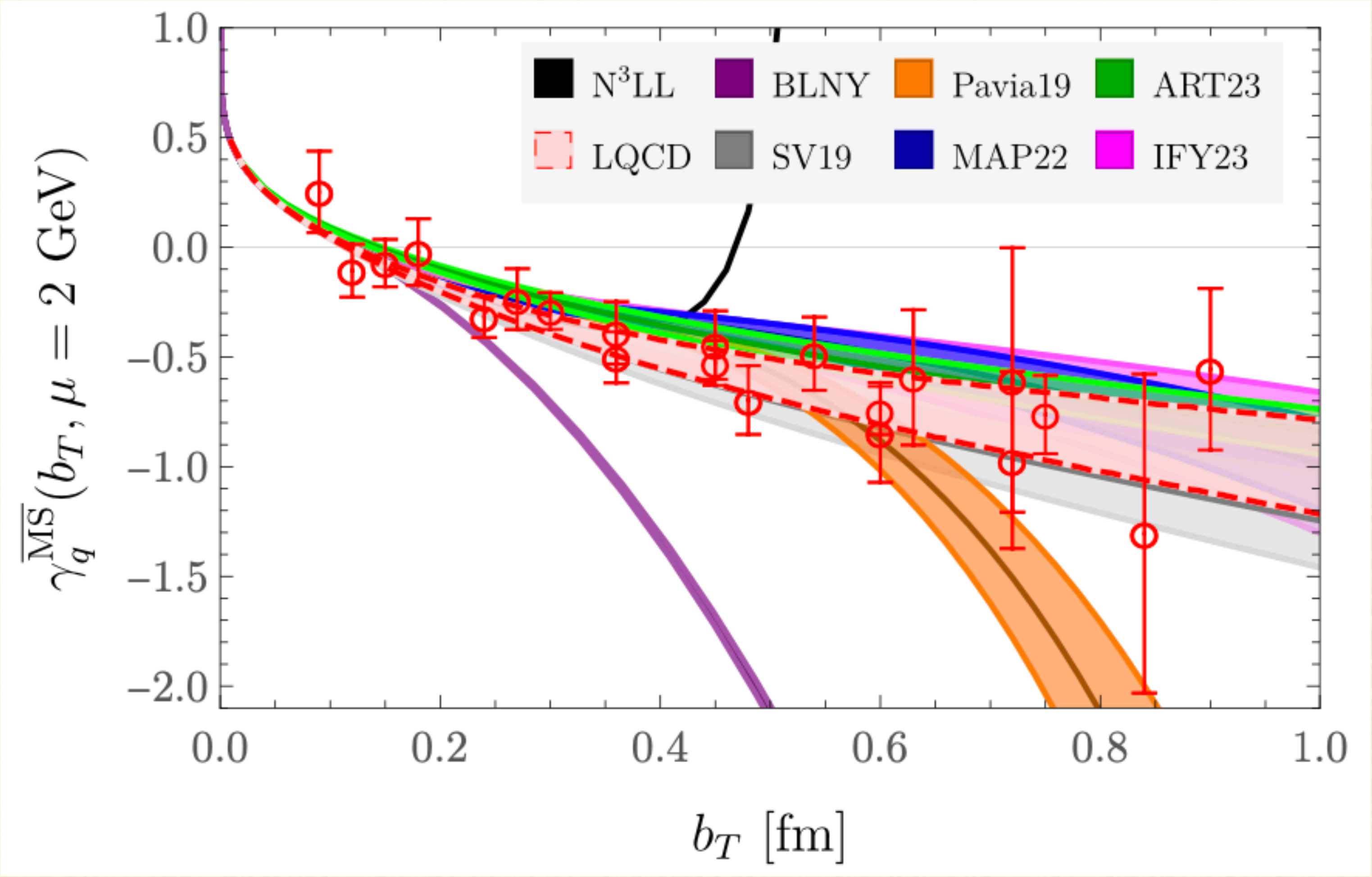


STAR



× 50
PHENIX
$\sqrt{s}$ = 200 GeV
STAR
$\sqrt{s}$ = 510 GeV
Preliminary Data
$d\sigma/dq_T$ [pb/GeV]
0.8
0.6
0.4
0.2
data / theory
1.1
1.0
0.9
q_T [GeV]
7
14
21
q_T [GeV]

ART23: LATTICE COMPARISON



Artur Avkhadiev,1 Phiala E. Shanahan,1 Michael L. Wagman,2 and Yong Zhao: arXive2402.06725

Results in detail

dataset	N_{pt}	χ^2_D/N_{pt}	$\chi^2_\lambda/N_{\text{pt}}$	χ^2/N_{pt}
CDF (run1)	33	0.51	0.16	$0.67^{+0.05}_{-0.03}$
CDF (run2)	45	1.58	0.11	$1.59^{+0.26}_{-0.14}$
CDF (W-boson)	6	0.33	0.00	$0.33^{+0.01}_{-0.01}$
D0 (run1)	16	0.69	0.00	$0.69^{+0.08}_{-0.03}$
D0 (run2)	13	2.16	0.16	$2.32^{+0.40}_{-0.32}$
D0 (W-boson)	7	2.39	0.00	$2.39^{+0.20}_{-0.18}$
ATLAS (8TeV, $Q \sim M_Z$)	30	1.60	0.49	$2.09^{+1.09}_{-0.35}$
ATLAS (8TeV)	14	1.11	0.11	$1.22^{+0.47}_{-0.21}$
ATLAS (13 TeV)	5	1.94	1.75	$3.70^{+16.5}_{-2.24}$
CMS (7TeV)	8	1.30	0.00	$1.30^{+0.03}_{-0.01}$
CMS (8TeV)	8	0.79	0.00	$0.78^{+0.02}_{-0.01}$
CMS (13 TeV, $Q \sim M_Z$)	64	0.63	0.24	$0.86^{+0.23}_{-0.11}$
CMS (13 TeV, $Q > M_Z$)	33	0.73	0.12	$0.92^{+0.40}_{-0.15}$
LHCb (7 TeV)	10	1.21	0.56	$1.77^{+0.53}_{-0.31}$
LHCb (8 TeV)	9	0.77	0.78	$1.55^{+0.94}_{-0.50}$
LHCb (13 TeV)	49	1.07	0.10	$1.18^{+0.25}_{-0.01}$
PHENIX	3	0.29	0.12	$0.42^{+0.15}_{-0.10}$
STAR	11	1.91	0.28	$2.19^{+0.51}_{-0.31}$
E288 (200)	43	0.31	0.07	$0.38^{+0.12}_{-0.05}$
E288 (300)	53	0.36	0.07	$0.43^{+0.08}_{-0.04}$
E288 (400)	79	0.37	0.05	$0.48^{+0.11}_{-0.03}$
E772	35	0.87	0.21	$1.08^{+0.08}_{-0.05}$
E605	53	0.18	0.21	$0.39^{+0.03}_{-0.00}$
Total	627	0.79	0.17	$0.96^{+0.09}_{-0.01}$

DY+SIDIS: ART24

We did a fit in SV19. No clear problem encountered in the fit.

We are providing a new version (work in progress).

Up to now the fitting confirms SV19.

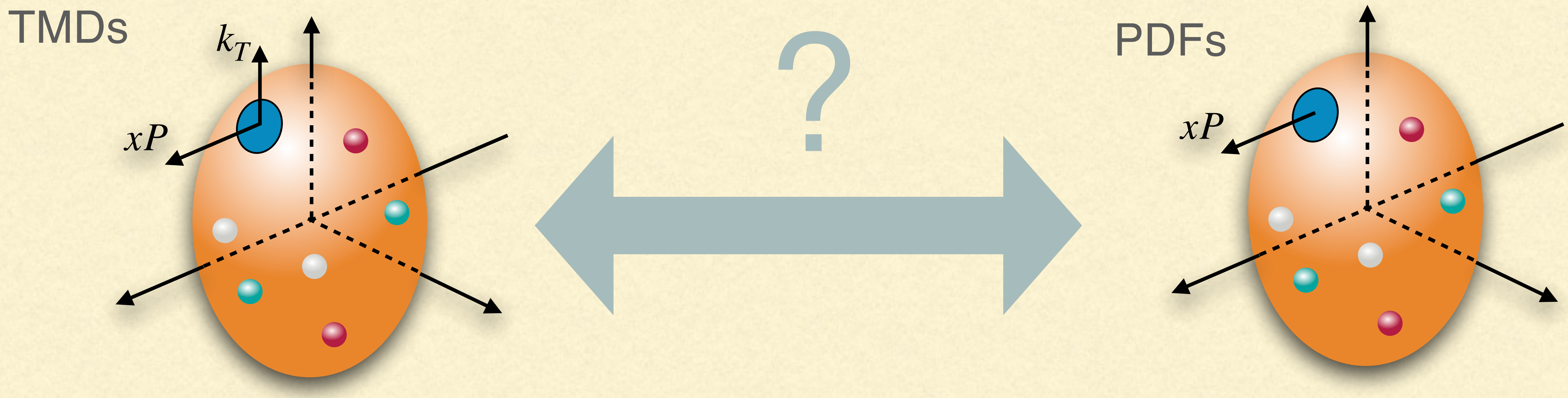
However our understanding of the result is getting different.

Both MAPS and SV19 and current fit show that there are unsolved theoretical problems which can be hidden by the fits:

Power corrections should be put under control in SIDIS data.

WHAT IS THE RELATIONSHIP?

Oscar del Rio, Alexei Prokudin, I.S., Alexey Vladimirov e-Print: 2402.01836 (2024)



In principle TMDs are related to PDFs upon integration out the transverse momentum, but what about renormalization scale?

Evolution

DGLAP EQUATIONS

Integro-differential equations

Non diagonal in flavor space

$$\mu^2 \frac{d}{d\mu^2} f_q(x,\mu) = \sum_{f'} \int_x^1 \frac{dy}{y} P_{q\to q'}(y) f_{q'}\left(\frac{x}{y},\mu\right)$$

μ= *UV renormalization scale*

COLLINS-SOPER EQUATIONS

Double scale differential equations

Diagonal in flavor space

$$\frac{d\ln\tilde{F}(x,b_T,\mu,\zeta)}{d\ln\mu} = \gamma_F(\mu)$$

$$\frac{\partial\ln\tilde{F}(x,b_T,\mu,\zeta)}{\partial\ln\sqrt{\zeta}} = \tilde{K}(b_T,\mu)$$

$$\frac{d\tilde{K}(b_T,\mu)}{d\ln\mu} = -\gamma_K(\mu)$$

ζ= *Collins-Soper parameter*

Collins-Soper kernel $\tilde{K}$ is specific for TMDs

TRANSVERSE MOMENTUM MOMENTS

O. del Rio, A. Prokudin, I.S., A. Vladimirov e-Print: 2402.01836

- TMMs are weighted integrals with an upper cut-off

$$\mathcal{M}^{[\Gamma]}_{\nu_1 \ldots \nu_n}(x,\mu) \equiv \int^{\mu} d^2\vec{k}_T\, \vec{k}_{T\nu_1} \ldots \vec{k}_{T\nu_n} F^{[\Gamma]}(x,k_T)$$

for TMDs in the ζ-prescription which has no scale dependence

$$\mathcal{M}^{*\,[\Gamma]}_{\nu_1 \ldots \nu_n}(x,\mu) \equiv \int^{\mu} d^2\vec{k}_T\, \vec{k}_{T\nu_1} \ldots \vec{k}_{T\nu_n} F^{[\Gamma]}(x,k_T;\mu,\mu^2)$$

for TMDs in the general prescription *For 0-moment: M. Ebert, J. Michel, I. Stewart, Z. Sun, JHEP 07 (2022) 129*

- The upper cut-off becomes the scale at which the collinear functions are evaluated
- TMMs obey DGLAP equations
- We provide a definition for all moments

TMDS IN b-SPACE AND $\mathcal{G}$ OPERATION

TMDs in b space are parametrized as

$$\tilde{F}^{[\gamma^+]}(x,b) = \tilde{f}_1(x,b) + i\epsilon_T^{\mu\nu} b_\mu s_{T\nu} M \tilde{f}_{1T}^{\perp}(x,b),$$

$$\tilde{F}^{[\gamma^+\gamma^5]}(x,b) = \lambda \tilde{g}_1(x,b) + i(b\cdot s_T) M \tilde{g}_{1T}^{\perp}(x,b),$$

$$\tilde{F}^{[i\sigma^{\alpha+}\gamma^5]}(x,b) = s_T^{\alpha} \tilde{h}_1(x,b) - i\lambda b^{\alpha} M \tilde{h}_{1L}^{\perp}(x,b)$$
$$+ i\epsilon_T^{\alpha\mu} b_\mu M \tilde{h}_1^{\perp}(x,b) + \frac{M^2}{4}(g_T^{\alpha\mu}\mathbf{b}^2 + 2b^\alpha b^\mu) s_{T\mu} \tilde{h}_{1T}^{\perp}(x,b)$$

TMDS IN b-SPACE AND $\mathcal{G}$ OPERATION

Fourier transformation: angular integrations are trivial

D. Boer, L. Gamberg, B. Musch, and A. Prokudin, JHEP 10, 021 (2011)

$$\widetilde{F}^{(n)}(x, b_T; \mu, \zeta) \equiv n! \left(\frac{-1}{M^2 b} \partial_b \right)^n \widetilde{F}(x, b; \mu, \zeta) = \frac{2\pi\, n!}{(bM)^n} \int_0^\infty dk_T\, k_T \left(\frac{k_T}{M} \right)^n J_n(bk_T)\, F(x, k_T; \mu, \zeta)$$

$$\widetilde{f}_1(x,b) \equiv \widetilde{f}_1^{(0)}(x,b), \qquad f_{1T}^{\perp}(x,b) \equiv \widetilde{f}_{1T}^{\perp(1)}(x,b),$$

$$\widetilde{g}_1(x,b) \equiv \widetilde{g}_1^{\perp(0)}(x,b), \qquad \widetilde{g}_{1T}^{\perp}(x,b) \equiv \widetilde{g}_{1T}^{\perp(1)}(x,b),$$

$$\widetilde{h}_1(x,b) \equiv \widetilde{h}_1^{\perp(0)}(x,b), \qquad \widetilde{h}_{1L}^{\perp}(x,b) \equiv \widetilde{h}_{1L}^{\perp(1)}(x,b),$$

$$\widetilde{h}_1^{\perp}(x,b) \equiv \widetilde{h}_1^{\perp(1)}(x,b), \qquad \widetilde{h}_{1T}^{\perp}(x,b) \equiv \widetilde{h}_{1T}^{\perp(2)}(x,b).$$

The superscript (m) determines the large k_T asymptotic $\quad f(x, k_T) \propto \frac{M^{2m}}{(k_T^2)^{m+1}}$

TMDS IN b-SPACE AND $\mathcal{G}$ OPERATION

- Operation $\mathcal{G}$ is defined as

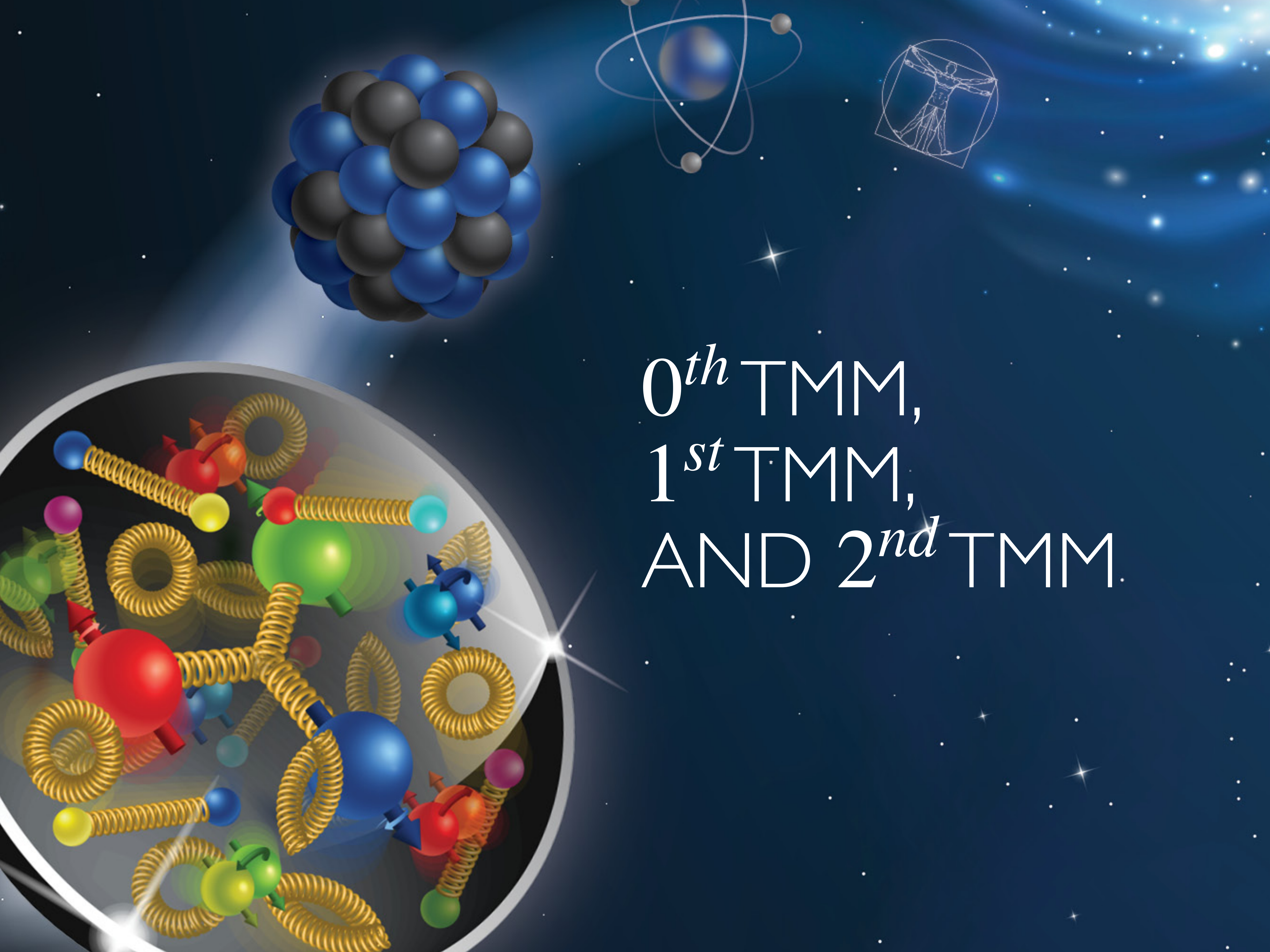
$$\mathcal{G}_{n,m}[f](x,\mu) \;=\; \int^{\mu} d^2\boldsymbol{k}_T \left(\frac{\boldsymbol{k}_T^2}{2M^2}\right)^n f(x,k_T)$$

- Without cut-off it corresponds to the conventional n^{th} moment of TMD, m is the corresponding superscript of the TMD $\tilde{f}$

- Its properties: $n=m$ logarithmic divergence, $n=m+l$ power divergence in μ

$$\mathcal{G}_{m,m}[f](x,\mu) \;\propto\; \log(\mu)\ ,$$
$$\mathcal{G}_{m+l,m}[f](x,\mu) \;\propto\; \mu^{2l} \text{ for } m+l \geq 0$$

- The logarithmic divergence for $n=m$ is the UV divergence that corresponds to the divergence of the collinear functions



0^{th} TMM,
1^{st} TMM,
AND 2^{nd} TMM

ZEROTH TMM

- The 0^{th} TMM is

$$
\begin{aligned}
\mathcal{M}^{[\gamma^+]}(x,\mu) = \int^{\mu} d^2\boldsymbol{k}_T F^{[\gamma^+]}(x,k_T) &= \int^{\mu} d^2\boldsymbol{k}_T f_1(x,k_T), \\
\mathcal{M}^{[\gamma^+\gamma_5]}(x,\mu) = \int^{\mu} d^2\boldsymbol{k}_T F^{[\gamma^+\gamma^5]}(x,k_T) &= \lambda \int^{\mu} d^2\boldsymbol{k}_T g_1(x,k_T), \\
\mathcal{M}^{[i\sigma^{\alpha+}\gamma^5]}(x,\mu) = \int^{\mu} d^2\boldsymbol{k}_T F^{[i\sigma^{\alpha+}\gamma^5]}(x,k_T) &= s_T^{\alpha} \int^{\mu} d^2\boldsymbol{k}_T h_1(x,k_T) \\
&\quad - \int^{\mu} d^2\boldsymbol{k}_T \frac{\boldsymbol{k}_T^2}{M^2}\left(\frac{g_T^{\alpha\mu}}{2} + \frac{k_T^{\alpha}k_T^{\mu}}{\boldsymbol{k}_T^2}\right) s_{T\mu} h_{1T}^{\perp}(x,k_T),
\end{aligned}
$$

$\propto \mu^{-2}$ so we drop it

ZEROTH TMM

- In practice we obtain PDF in a certain (TMD) scheme

$$\begin{aligned}\mathcal{M}^{[\gamma^+]}(x,\mu) &= \mathcal{G}_0[f_1](x,\mu),\\ \mathcal{M}^{[\gamma^+\gamma^5]}(x,\mu) &= s_L\mathcal{G}_0[g_1](x,\mu),\\ \mathcal{M}^{[i\sigma^{\alpha+}\gamma^5]}(x,\mu) &= s_T^\alpha\mathcal{G}_0[h_1](x,\mu),\end{aligned}$$

$\Rightarrow$

$$\begin{aligned}\mathcal{G}_0[f_1](x,\mu) &= q^{(\mathrm{TMD})}(x,\mu)+\mathcal{O}(\mu^{-2}),\\ \mathcal{G}_0[g_1](x,\mu) &= \Delta q^{(\mathrm{TMD})}(x,\mu)+\mathcal{O}(\mu^{-2}),\\ \mathcal{G}_0[h_1](x,\mu) &= \delta q^{(\mathrm{TMD})}(x,\mu)+\mathcal{O}(\mu^{-2}).\end{aligned}$$

Using Wilson coefficients of small-b and large-μ asymptotic expansion of Hankel transform one obtains

$$\mathcal{G}_0[F](x,\mu) = \Big\{\mathbf{1}+\alpha_s C_1+\alpha_s^2 C_2 +\alpha_s^3\Big[\frac{2\zeta_3}{3}\left(P_1\otimes P_1\otimes P_1-3\beta_0 P_1\otimes P_1+2\beta_0^2 P_1\right)+C_3\Big]+\mathcal{O}(\alpha_s^4)\Big\}\otimes f(x,\mu)+\mathcal{O}(\mu^{-2}),$$

R. Wong, Computers & Mathematics with Applications 3, 271 (1977).
R. F. MacKinnon, Mathematics of Computation 26, 515 (1972).

ZEROTH TMM

- All scales in theTMD are set to μ and we have a DGLAP equation

$$\mu^2 \frac{d}{d\mu^2} f^{(\mathrm{TMD})}(x,\mu) = P' \otimes f^{(\mathrm{TMD})}(x,\mu).$$

- Therefore it is the same as PDFs but computed in a different scheme.
- The difference in splitting functions is of order α_s^2 and it is calculable

$$P' - P = -\alpha_s^2 \beta_0 C_1 - \alpha_s^3 \left(2\beta_0 C_2 - \beta_0 C_1 \otimes C_1 + \beta_1 C_1\right) + \mathcal{O}(\alpha_s^4).$$

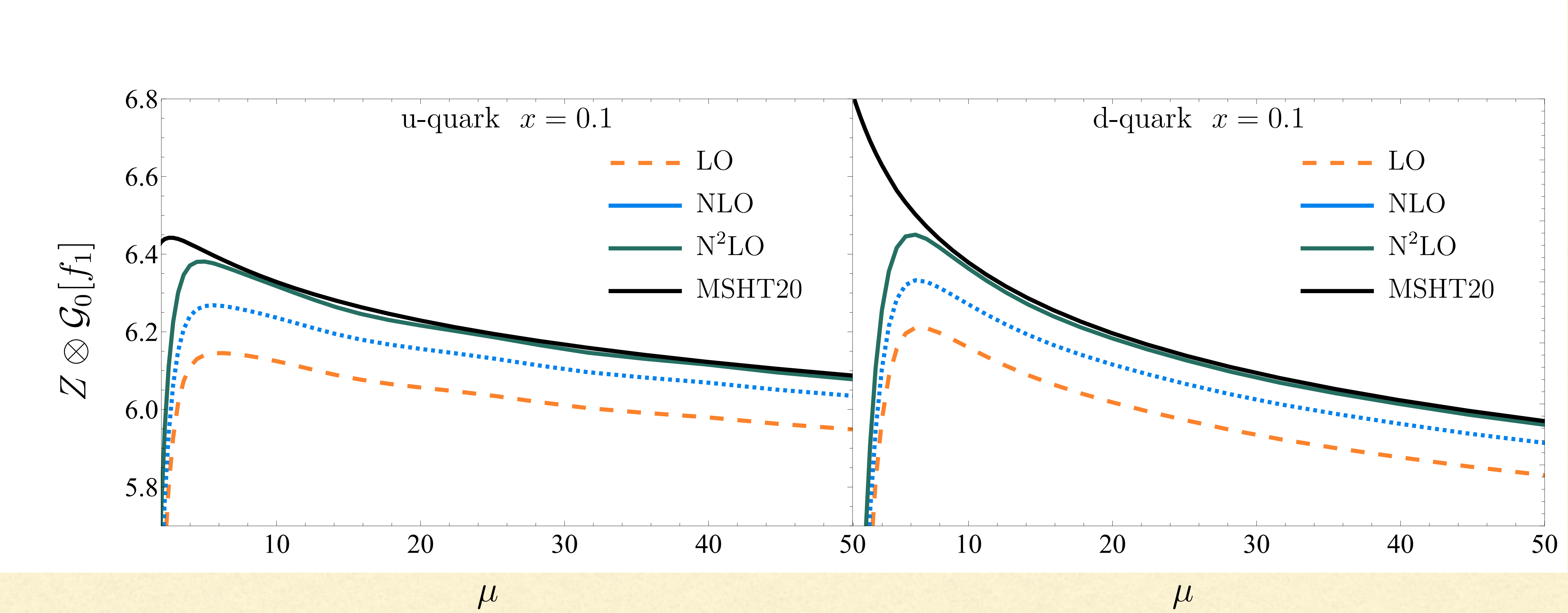
ZEROTH TMM

- We call this scheme TMD-scheme and the coefficient to transform to $\overline{MS}$ scheme reads

$$f_f^{(\overline{\text{MS}})}(x,\mu) \;=\; \sum_{f'}\int_x^1 \frac{dy}{y} Z_{f\leftarrow f'}^{\overline{\text{MS}}/\text{TMD}}(y,\mu) f_{f'}^{(\text{TMD})}\left(\frac{x}{y},\mu\right)$$

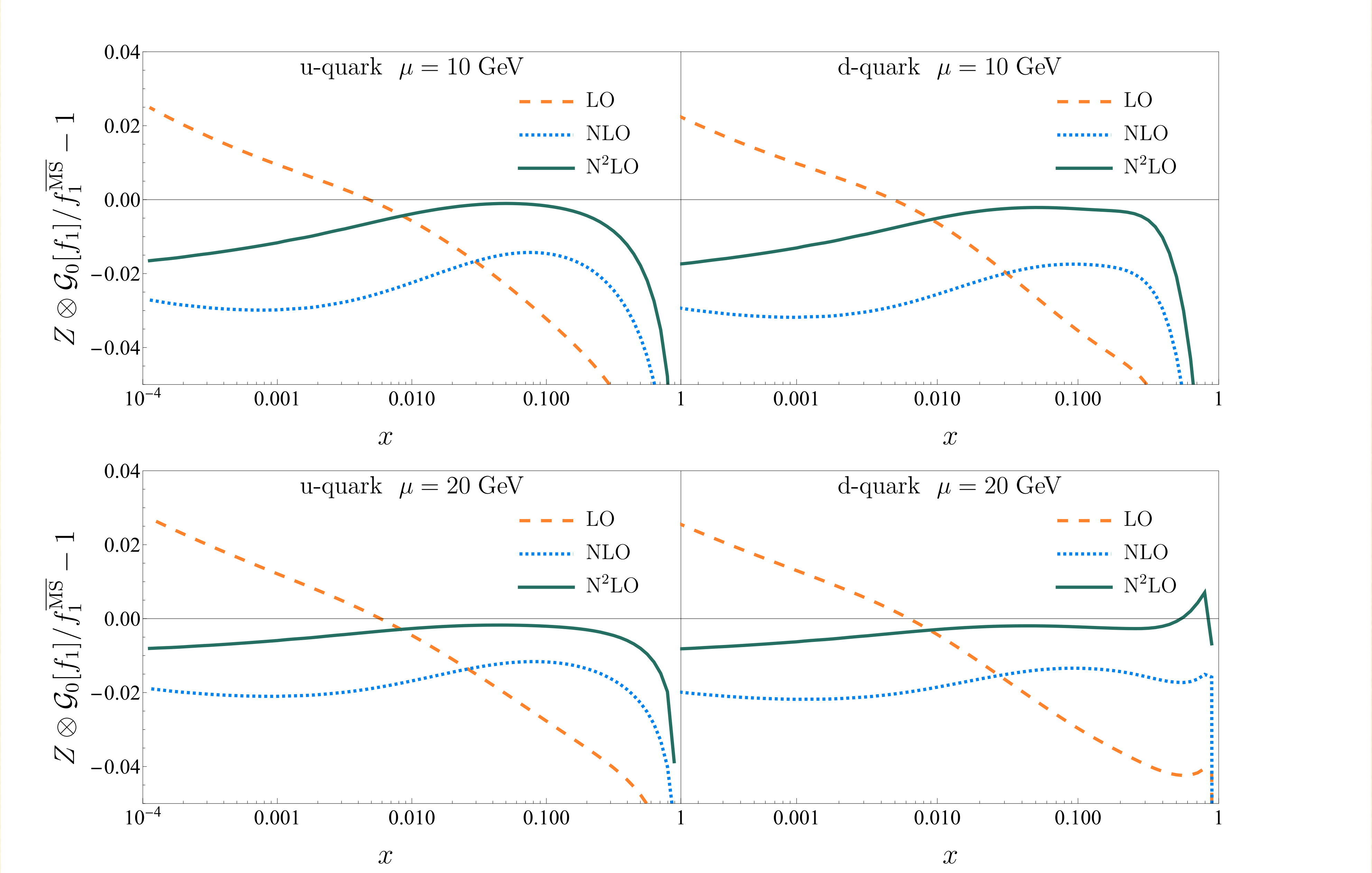
$$\begin{aligned}Z^{\overline{\text{MS}}/\text{TMD}} &= \mathbf{1} - \alpha_s C_1 - \alpha_s^2\left(C_2 - C_1\otimes C_1\right)\\ &-\alpha_s^3\left[C_3 + C_1\otimes C_1\otimes C_1 - C_1\otimes C_2 - C_2\otimes C_1 + \frac{2\zeta_3}{3}P_1\otimes(P_1-\beta_0\cdot\mathbf{1})\otimes(P_1-2\beta_0\cdot\mathbf{1})\right] + \mathcal{O}(\alpha_s^4)\end{aligned}$$

ZEROTH TMM



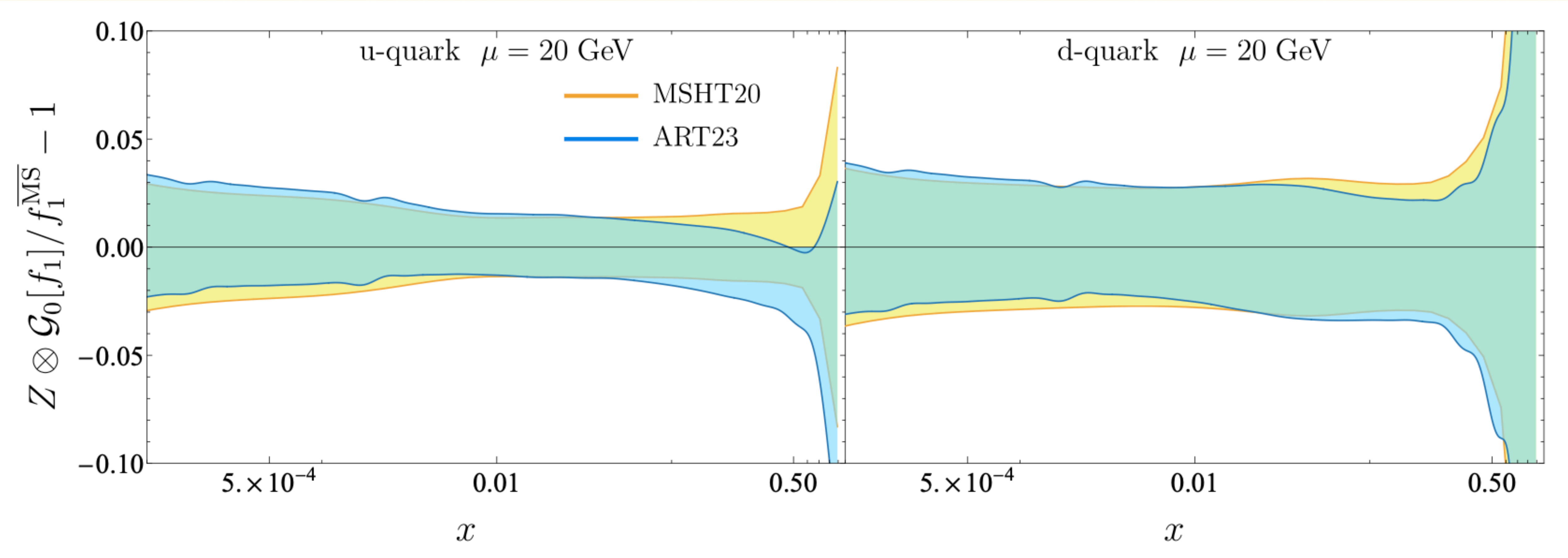
- Above $\mu \geq 5$ GeV the correspondence is quite precise

ZEROTH TMM



- ➤ TMDs are from ART 23 extraction *V. Moos, I. Scimemi, A. Vladimirov, and P. Zurita, (2023), arXiv:2305.07473*

ZEROTH TMM:
FROM PDF TO TMD TO PDF



- We can reproduce the errors: a very nice consistency check.

FIRST TMM

- The 1^{st} TMM is related the small-b power expansion of a TMD

$$\mathcal{G}_1[f_{1T}^{\perp}](x,\mu) = \pm\frac{\pi}{2}T^{(\mathrm{TMD})}(-x,0,x;\mu)+\mathcal{O}(\mu^{-2}),$$

$$\mathcal{G}_1[g_{1T}^{\perp}](x,\mu) = \frac{x}{2}\int_x^1\frac{dy}{y}\Delta q^{(\mathrm{TMD})}(y,\mu)+x\int_{-1}^1 dy_1dy_2dy_3\delta(y_1+y_2+y_3)\int_0^1 d\alpha\delta(x-\alpha y_3)\Big[\frac{\Delta T^{(\mathrm{TMD})}(y_{123};\mu)}{y_2^2}+\frac{T^{(\mathrm{TMD})}(y_{123};\mu)-\Delta T^{(\mathrm{TMD})}(y_{123};\mu)}{2y_2y_3}\Big]+\mathcal{O}(\mu^{-2}),$$

$$\mathcal{G}_1[h_{1L}^{\perp}](x,\mu) = -\frac{x^2}{2}\int_x^1\frac{dy}{y}\delta q^{(\mathrm{TMD})}(y,\mu) -x\int_{-1}^1 dy_1dy_2dy_3\delta(y_1+y_2+y_3)\int_0^1 d\alpha\alpha\delta(x-\alpha y_3)H^{(\mathrm{TMD})}(y_{123};\mu)\frac{y_3-y_2}{y_2^2y_3}+\mathcal{O}(\mu^{-2}),$$

$$\mathcal{G}_1[h_1^{\perp}](x,\mu) = \mp\frac{\pi}{2}E^{(\mathrm{TMD})}(-x,0,x;\mu)+\mathcal{O}(\mu^{-2}),$$

I. S., A. Vladimirov, Eur. Phys. J. C 78, 802 (2018), F. Rein, S. Rodini, A. Schäfer, and A. Vladimirov, JHEP 01, 116 (2023)

- The evolution is of the correct DGLAP-type...

  ...With a difference at NLO

$$\mu^2\frac{d}{d\mu^2}\mathcal{G}_1[F](x,\mu) = R_t\otimes P_t'\otimes t+\mathcal{O}(\alpha_s^2)$$

$$P_t'-P_t = \mathcal{O}(\alpha_s^2)$$

QIU-STERMAN FUNCTIONS

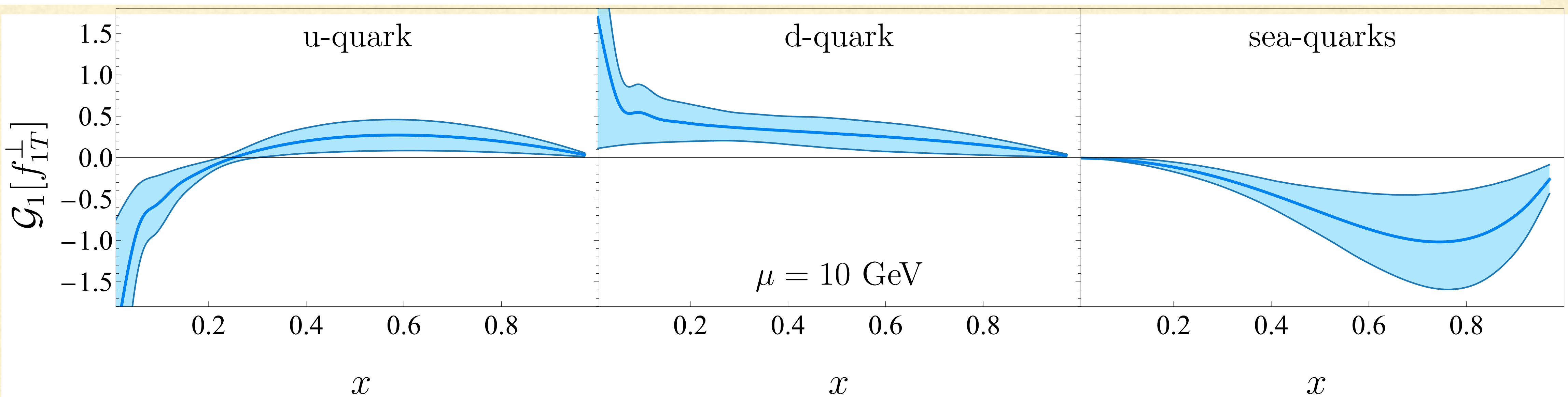
Oscar del Rio, Alexei Prokudin, Ignazio Scimemi, Alexey Vladimirov e-Print: 2402.01836 (2024)

- Even though it is not possible to relate the 1^{st} TMM of the Sivers functions to the full twist-3 functions with 3 variables $T(x_1, x_2, x_3)$, it is related to Qiu-Sterman functions $T(-x, 0, x; \mu)$

$$\langle \boldsymbol{k}^{u}_{T,1} \rangle = -0.011^{+0.011}_{-0.023}\ \text{GeV}, \quad \langle \boldsymbol{k}^{d}_{T,1} \rangle = 0.17^{+0.21}_{-0.17}\ \text{GeV}, \quad \langle \boldsymbol{k}^{sea}_{T,1} \rangle = -0.26^{+0.26}_{-0.32}\ \text{GeV}$$

$$\langle \boldsymbol{k}^{g}_{T,1} \rangle \simeq 0.14^{+0.31}_{-0.14}\ \text{GeV}$$

potentially sizable gluon Sivers function



Using M. Bury, A. Prokudin, A. Vladimirov, Phys.Rev.Lett. 126 (2021)

SECOND TMM

The 2nd moment is power divergent

$$\mathcal{M}^{[\gamma^+]}_{\mu\nu,\text{div}}(x,\mu) = -g_{T,\mu\nu}M^2\mathcal{G}_{1,0}[f_1],$$

$$\mathcal{M}^{[\gamma^+\gamma^5]}_{\mu\nu,\text{div}}(x,\mu) = -\lambda g_{T,\mu\nu}M^2\mathcal{G}_{1,0}[g_1],$$

$$\mathcal{M}^{[i\sigma^{\alpha+}\gamma^5]}_{\mu\nu,\text{div}}(x,\mu) = s_{T,\alpha}g_{T,\mu\nu}M^2\mathcal{G}_{1,0}[h_1] + (g_{T,\mu\alpha}s_{T,\nu} + g_{T,\nu\alpha}s_{T,\mu} - g_{T,\mu\nu}s_{T,\alpha})\frac{M^2}{2}\mathcal{G}_2[h^\perp_{1T}]$$

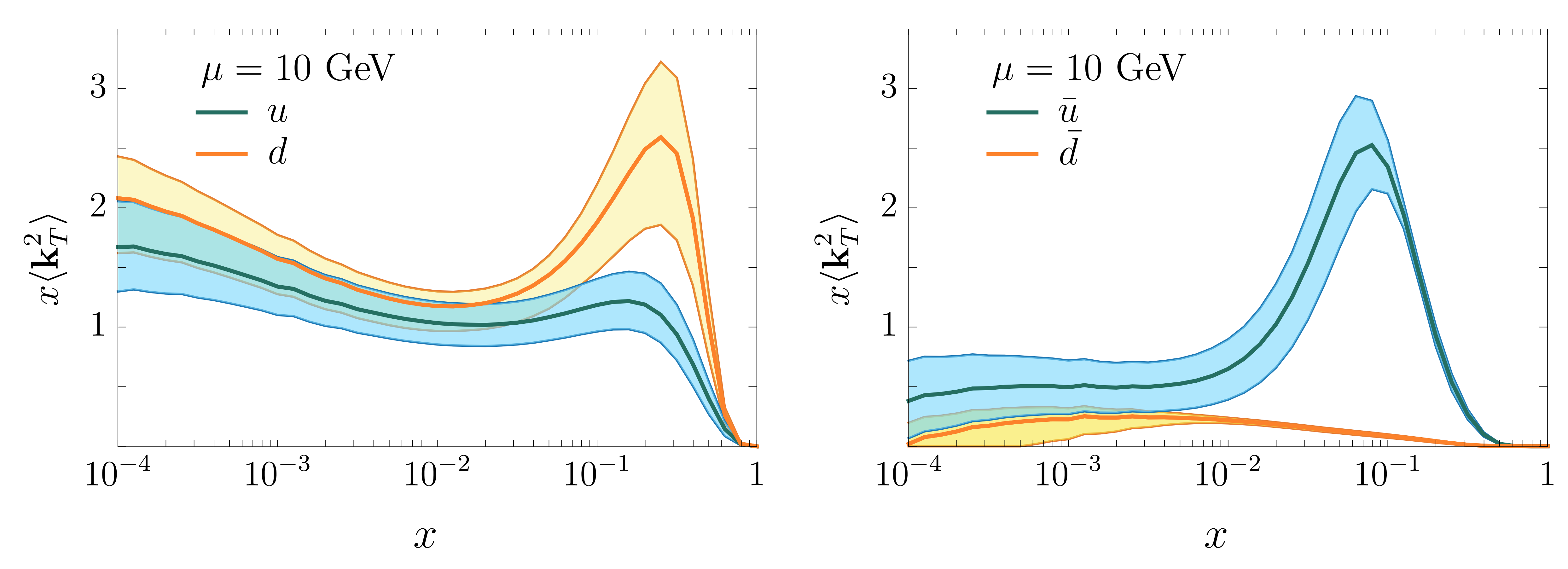
The asymptotic power divergence part is computed analytically ...

$$\mathcal{G}_{n+1,n}[F](x,\mu) = \frac{\mu^2}{2M^2}\text{AS}[\mathcal{G}_{n+1,n}[F]](x,\mu) + \overline{\mathcal{G}}_{n+1,n}[F](x,\mu),$$

... the width of TMDs

$$\langle \boldsymbol{k}_T^2 \rangle = -g_T^{\mu\nu}\mathcal{M}^{[\gamma^+]}_{\mu\nu} = 2M^2\overline{\mathcal{G}}_{1,0}[f_1]$$

SECOND TMM



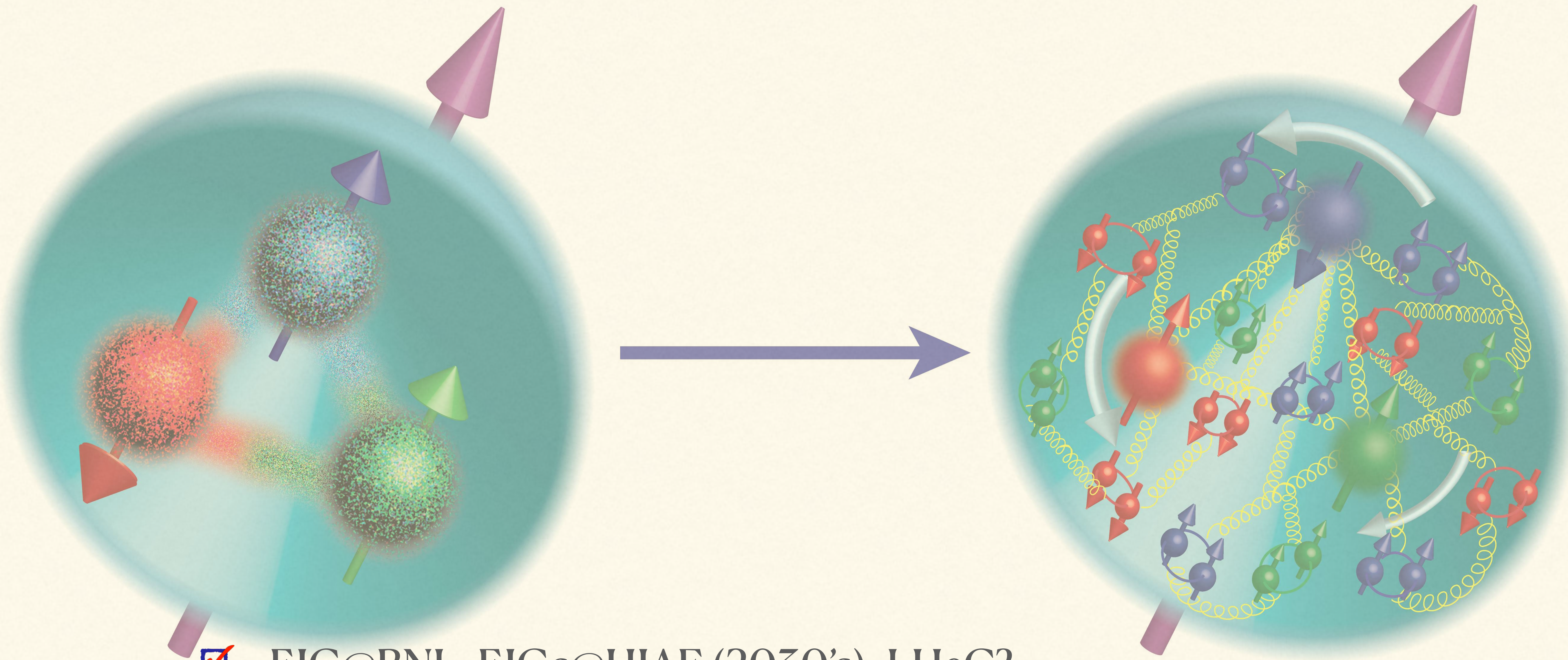
$$\langle x\vec{k}_T^2\rangle_u = 0.52 \pm 0.12 \text{ GeV}^2, \qquad \langle x\vec{k}_T^2\rangle_d = 1.10 \pm 0.28 \text{ GeV}^2,$$

$$\langle x\vec{k}_T^2\rangle_{\bar{u}} = 0.42 \pm 0.06 \text{ GeV}^2, \qquad \langle x\vec{k}_T^2\rangle_{\bar{d}} = 0.024 \pm 0.004 \text{ GeV}^2 .$$

CONCLUSIONS: SPIN UP!!

- ☑ ART23 reaches N4LL (caveat PDF), flavor dependence of TMD included, latest DY data, complete evaluation of errors (PDF errors!!)
- ☑ TMM: a robust relations of the 3D and 1D nucleon structures are established, very precious definitions, especially for polarized measurements.
- ☑ TMMs are weighted integrals of TMDs with an upper cut-off, they obey DGLAP (type) equations. As result of integrations we obtain collinear functions in a particular TMD-scheme that is related to $\bar{MS}$-scheme by a calculable factor
- ☑ The usage of TMMs will be useful in the future theoretical and phenomenological studies, as well as in lattice QCD studies
- ☑ SIDIS at low energy needs understanding Power Corrections

CONCLUSIONS: FUTURE!!



- ☑ EIC@BNL, EICc@HIAF (2030's), LHeC?
- ☑ All LHC labs+LHC initiatives (SMOG at LHCb, LHCSpin, etc.)
- ☑ Belle and Belle II

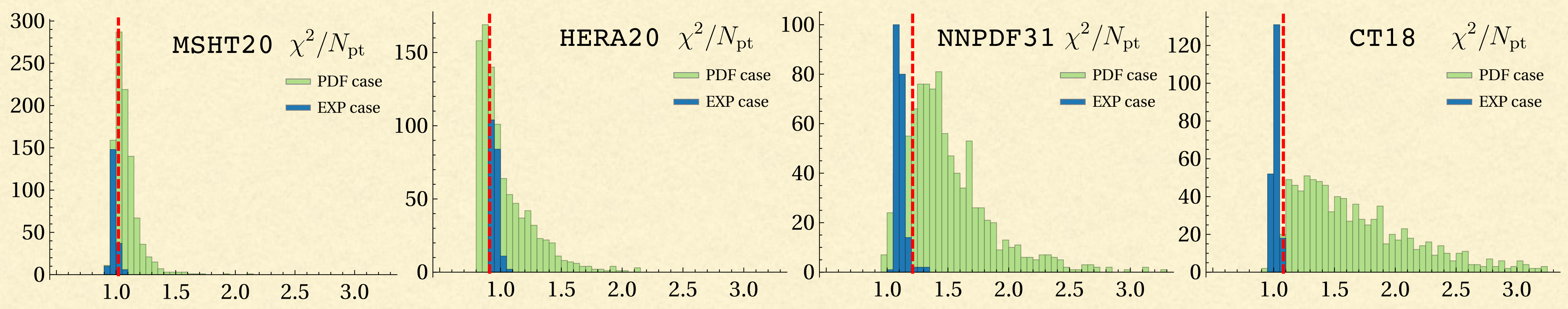
BACK UP SLIDES

PDF Bias

M. Bury, F. Hautmann, S. Leal-Gomez, I. S., A. Vladimirov, P.Zurita, JHEP 10 (2022) 118

- We simplify models but with flavor separation to mitigate PDF bias

$$f_{NP}^{f}(x,b) = \exp\left(-\frac{\lambda_1^f(1-x)+\lambda_2^f x}{\sqrt{1+\lambda_0 x^2 \mathbf{b}^2}}\mathbf{b}^2\right) \qquad f = u, \bar{u}, d, \bar{d}, sea$$



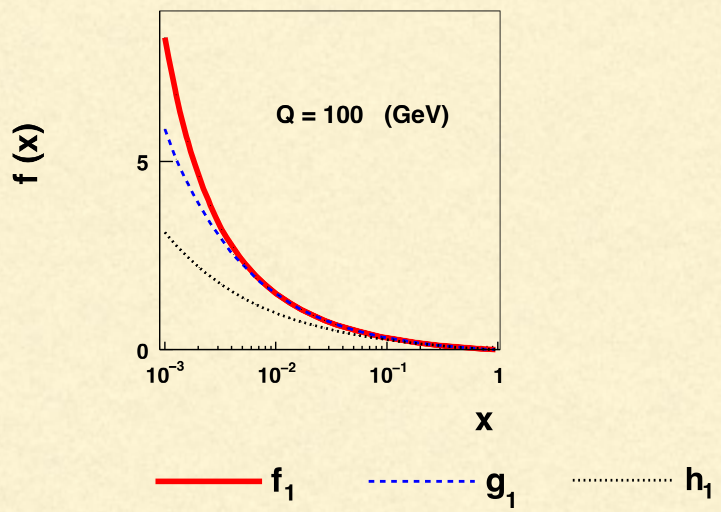
- ALL PDF DISTRIBUTIONS HAVE SIMILAR χ^2
- THE SPREAD OF χ^2 OF PDF REPLICA IS HIGHLY REDUCED
- FINAL χ^2: MSHT20 (1.12), HERA20 (0.91), NNPDF31(1.21), CT18 (1.08)

PREVIOUS WORKS

- Numerical study

  A. Bacchetta, A. Prokudin, Nucl.Phys.B 875 (2013) 536-551

$$f^q(x;\mu,\zeta_F) \equiv 2\pi \int_0^{\mu} k_T dk_T f_{q/P}(x,k_T;\mu,\zeta_F)$$

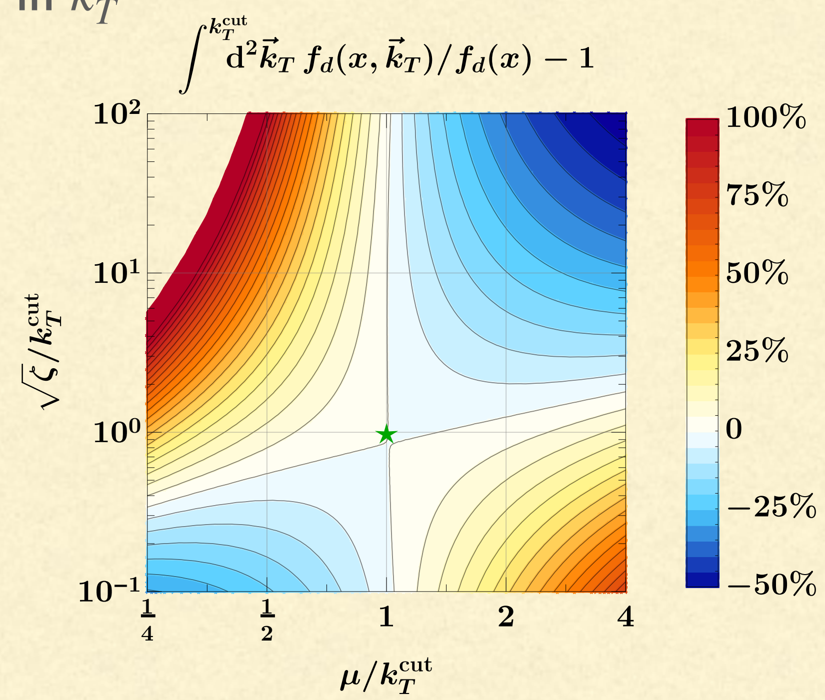


- Proposed for polarized TMDs

  L. Gamberg, A. Metz, D. Pitonyak, A. Prokudin Phys.Lett.B 781 (2018) 443-454

$$\int d^2\mathbf{k}_T \frac{k_T^2}{2M_P^2} f_{1T}^{\perp j}(x,k_T;Q^2,\mu_Q;C_5) \equiv f_{1T}^{\perp(1)j}(x;Q^2,\mu_Q;C_5)$$

b_{min} instead of a cut in k_T



- Studied in great deal of details in

  M. A. Ebert, J. K. L. Michel, I. W. Stewart and Z. Sun, JHEP 07 (2022) 129
  J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers Phys.Rev.D 107 (2023) 9, 094029

$$\int_{k_T \le k_T^{\rm cut}} d^2\mathbf{k}_T\, f_{i/p}\big(x,\mathbf{k}_T,\mu = k_T^{\rm cut},\sqrt{\zeta} = k_T^{\rm cut}\big) \simeq f_i(x,\mu = k_T^{\rm cut})$$

ZEROTH TMM

- If TMDs are defined in a general scheme (TMD2-scheme), the same conclusions are valid, all scales should be defined by the cut-off

M. A. Ebert, J. K. L. Michel, I. W. Stewart and Z. Sun, JHEP 07 (2022) 129

$$\mu = \mu_{\rm OPE} = \mu_{\rm TMD} = \sqrt{\zeta}$$

$$\mu^2 \frac{d}{d\mu^2} f^{(\rm TMD2)}(x,\mu) = \overline{P} \otimes f^{(\rm TMD2)}(x,\mu)$$

$$\overline{P} - P = -\alpha_s^2 \beta_0 \overline{C}_1 - \alpha_s^3 \left[2\beta_0 \overline{C}_2 - \beta_0 \overline{C}_1 \otimes \overline{C}_1 + \beta_1 \overline{C}_1 - 2\zeta_3 \Gamma_0 \beta_0 \left(P_1 + \left(\frac{\gamma_1}{2} - \frac{2\beta_0}{3} \right) \cdot \mathbf{1} \right) \right] + \mathcal{O}(\alpha_s^4)$$

FIRST TMM

- The 1^{st} TMM is

$$\mathcal{M}_\mu^{[\gamma^+]}(x,\mu) = \int^\mu d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu} F^{[\gamma^+]}(x,k_T) \;=\; -\int^\mu d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu} \epsilon_T^{\rho\nu} \frac{k_{T\rho} s_{T\nu}}{M} f_{1T}^\perp(x,k_T),$$

$$\mathcal{M}_\mu^{[\gamma^+\gamma_5]}(x,\mu) = \int^\mu d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu} F^{[\gamma^+\gamma^5]}(x,k_T) \;=\; -\int^\mu d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu} \frac{(k_T\cdot s_T)}{M} g_{1T}^\perp(x,k_T),$$

$$\mathcal{M}_\mu^{[i\sigma^{\alpha+}\gamma^5]}(x,\mu) = \int^\mu d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu} F^{[i\sigma^{\alpha+}\gamma^5]}(x,k_T) \;=\; \int^\mu d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu} \frac{\lambda k_T^\alpha}{M} h_{1L}^\perp(x,k_T) - \int^\mu d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu} \frac{\epsilon_T^{\alpha\rho} k_{T\rho}}{M} h_1^\perp(x,k_T).$$

- And using $\mathcal{G}$ symbol

$$\mathcal{M}_\mu^{[\gamma^+]}(x,\mu) \;=\; -\epsilon_{T,\mu\nu} s_T^\nu M \mathcal{G}_1[f_{1T}^\perp](x,\mu),$$

$$\mathcal{M}_\mu^{[\gamma^+\gamma^5]}(x,\mu) \;=\; -s_{T\mu} M \mathcal{G}_1[g_{1T}^\perp](x,\mu),$$

$$\mathcal{M}_\mu^{[i\sigma^{\alpha+}\gamma^5]}(x,\mu) \;=\; -\lambda g_{T,\mu\alpha} M \mathcal{G}_1[h_{1L}^\perp](x,\mu) - \epsilon_{T,\mu\alpha} M \mathcal{G}_1[h_1^\perp](x,\mu).$$

- It is related to collinear twist-3 PDFs projected onto Qiu-Sterman type functions $x_1, x_2, x_3 \to x$ with the projection operator $R_t = \pi\delta(x_2)\delta(x_1+x_2+x_3)\delta(x_3-x)$

SECOND TMM

Oscar del Rio, Alexei Prokudin, Ignazio Scimemi, Alexey Vladimirov e-Print: 2402.01836 (2024)

$$
\begin{aligned}
\mathcal{M}_{\mu\nu}^{[\gamma^+]}(x,\mu) = \int^{\mu} d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu}\boldsymbol{k}_{T\nu} F^{[\gamma^+]}(x,k_T) \;&=\; \int^{\mu} d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu}\boldsymbol{k}_{T\nu} f_1(x,k_T),\\
\mathcal{M}_{\mu\nu}^{[\gamma^+\gamma_5]}(x,\mu) = \int^{\mu} d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu}\boldsymbol{k}_{T\nu} F^{[\gamma^+\gamma^5]}(x,k_T) \;&=\; \lambda \int^{\mu} d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu}\boldsymbol{k}_{T\nu} g_1(x,k_T),\\
\mathcal{M}_{\mu\nu}^{[i\sigma^{\alpha+}\gamma^5]}(x,\mu) = \int^{\mu} d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu}\boldsymbol{k}_{T\nu} F^{[i\sigma^{\alpha+}\gamma^5]}(x,k_T) \;&=\; s_T^{\alpha} \int^{\mu} d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu}\boldsymbol{k}_{T\nu} h_1(x,k_T)\\
&\quad - \int^{\mu} d^2\boldsymbol{k}_T \boldsymbol{k}_{T\mu}\boldsymbol{k}_{T\nu} \frac{\boldsymbol{k}_T^2}{M^2}\left(\frac{g_T^{\alpha\rho}}{2} + \frac{k_T^{\alpha}k_T^{\rho}}{\boldsymbol{k}_T^2}\right) s_{T\rho} h_{1T}^{\perp}(x,k_T)
\end{aligned}
$$

QIU-STERMAN FUNCTIONS

- Burkardt sum rule: $$\sum_{f=q,\bar{q},g} \int_0^1 dx \mathcal{M}_{\nu,f}^{[\gamma^+]}(x,\mu) = \sum_{f=q,\bar{q},g} \langle \boldsymbol{k}_{T,\nu}^f \rangle = 0$$

$$\langle \boldsymbol{k}_{T,1}^u \rangle = -0.011^{+0.011}_{-0.023} \text{ GeV}, \qquad \langle \boldsymbol{k}_{T,1}^d \rangle = 0.17^{+0.21}_{-0.17} \text{ GeV}, \qquad \langle \boldsymbol{k}_{T,1}^{sea} \rangle = -0.26^{+0.26}_{-0.32} \text{ GeV}$$

$$\langle \boldsymbol{k}_{T,1}^g \rangle \simeq 0.14^{+0.31}_{-0.14} \text{ GeV}$$

potentially sizable gluon Sivers function