university of groningen / faculty of science and engineering / van swinderen institute for particle physics and gravity

Gluon TMDs & quarkonia

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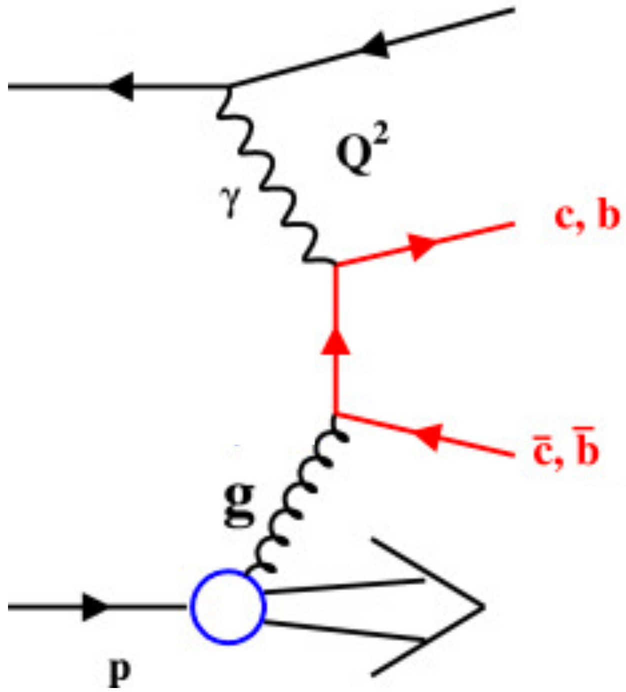
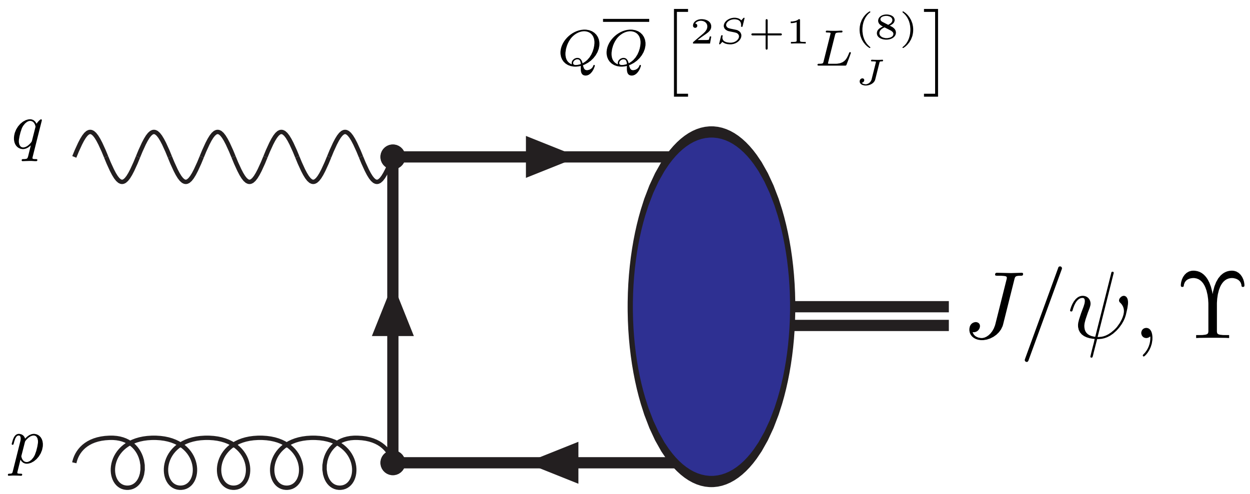
Overview

- Gluon TMDs
- Quarkonium production in pp collisions
- Quarkonium production in ep collisions
- Shape functions
- Gluon GTMDs from coherent diffractive quarkonium production

Gluon TMDs

Probing gluon TMDs

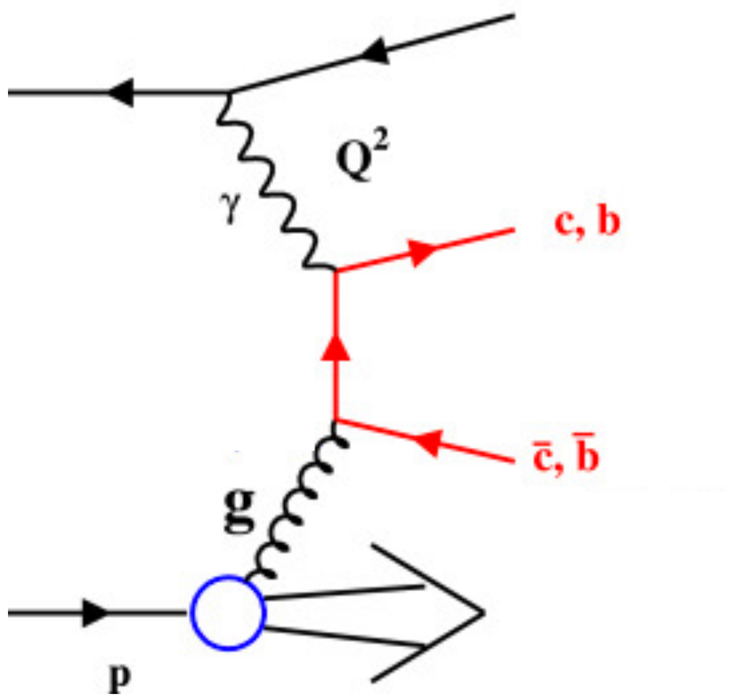


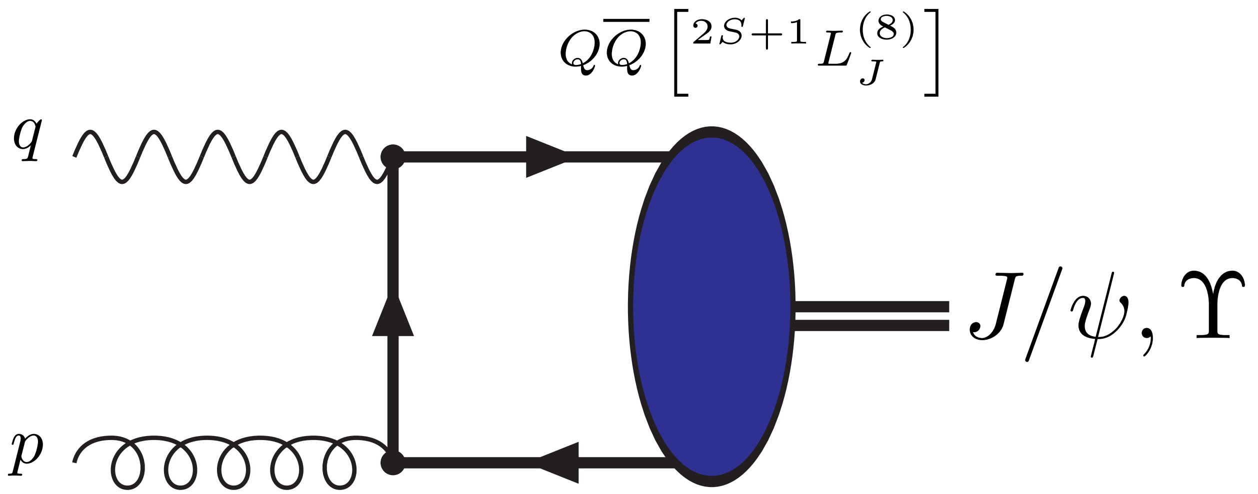


$$ep \to e' Q\bar{Q} X \qquad\qquad e\,p \to e'\,\mathcal{Q}\,X$$

Open heavy quark pair production and quarkonium production are among the simplest processes that are sensitive to the transverse momentum of gluons

Probing gluon TMDs





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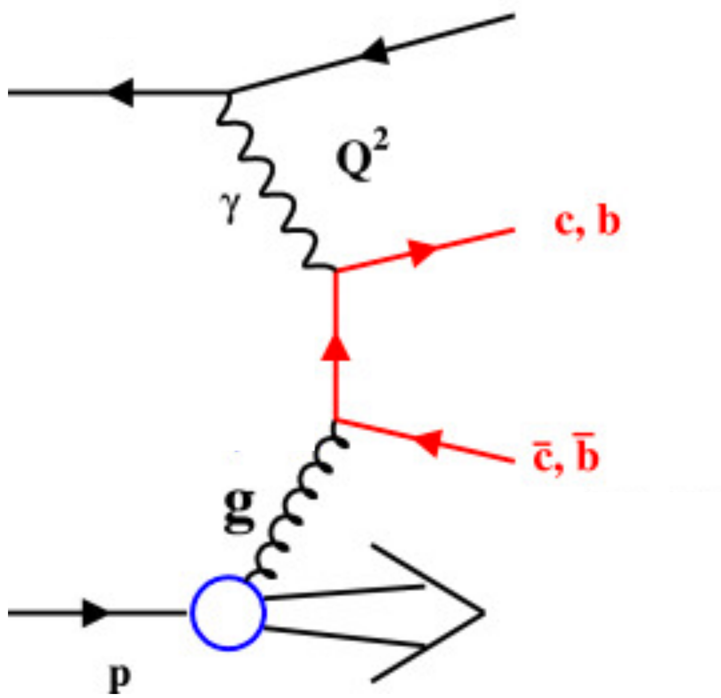

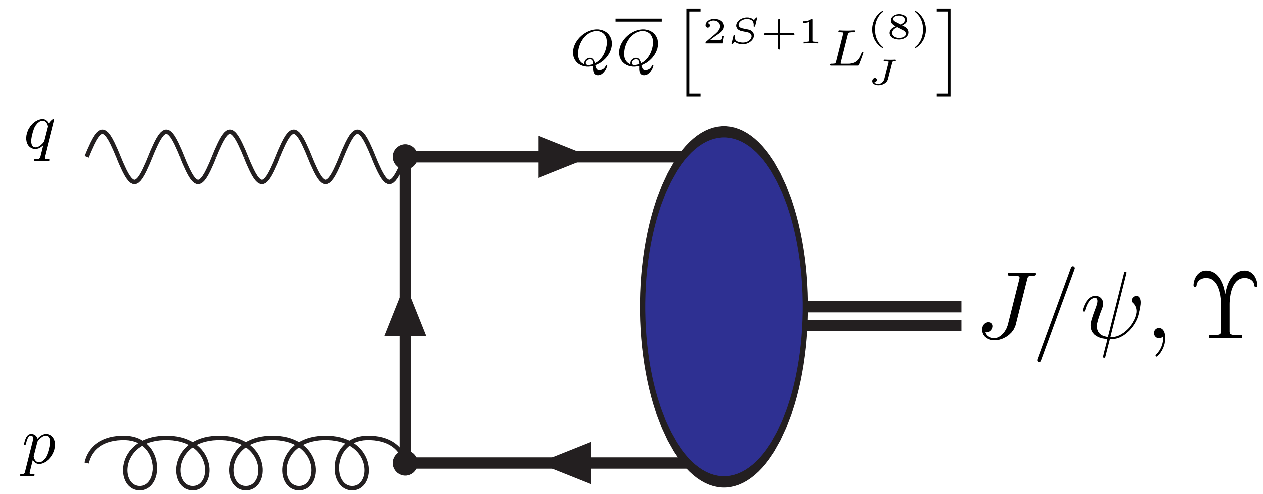

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Jet pair production a good option also, especially at small x, where gluons dominate

Nuclei can also help boost the gluon density, but not for the polarized case

Gluons TMDs

Gluon TMD correlator: $\Gamma_g^{\mu\nu}(x, p_T) \propto \langle P|F^{+\nu}(0)\,\mathcal{U}\,F^{+\mu}(\xi^-, \xi_T)\,\mathcal{U}'|P\rangle$

transverse momentum dependent (TMD)

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For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu}\, f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\boldsymbol{p}_T^2}{2M_p^2} \right) h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) \right\}$$

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unpolarized gluon TMD

linearly polarized gluon TMD

Mulders, Rodrigues '01

Gluons inside *unpolarized* protons can be polarized!

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unpolarized gluon TMD

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For transversely polarized protons:

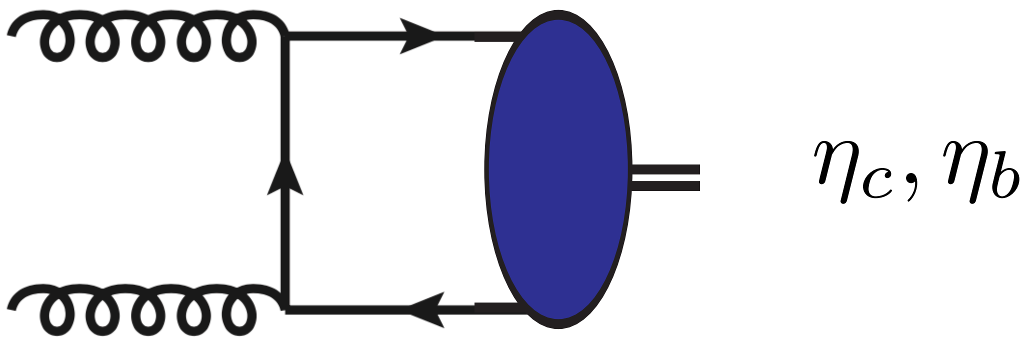
gluon Sivers TMD

$$\Gamma_T^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2}\left\{ g_T^{\mu\nu} \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M_p} f_{1T}^{\perp g}(x, \boldsymbol{p}_T^2) + \ldots \right\}$$

pp collisions

$\eta_{c,b}$ production

$pp \rightarrow \eta_Q X$



For C-even (pseudo-)scalar quarkonium production gg→CS is leading contribution

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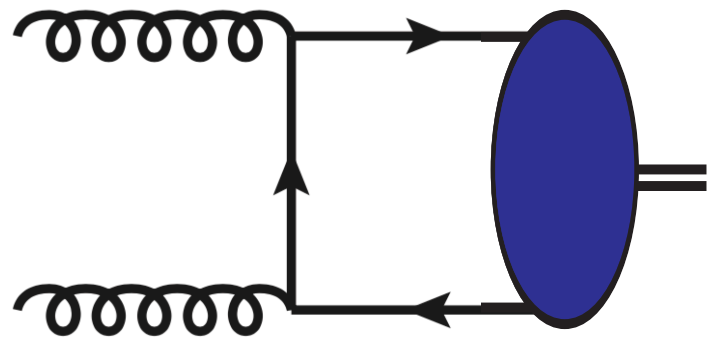

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In LO NRQCD the differential cross section in pp (and pA) is: DB, Pisano, 2012

$$\frac{d\sigma(\eta_Q)}{dy\, d^2\boldsymbol{q}_T} = \frac{2}{9}\frac{\pi^3\alpha_s^2}{M^3\, s}\,\langle 0|\mathcal{O}_1^{\eta_Q}({}^1S_0)|0\rangle\, \mathcal{C}\left[f_1^g\, f_1^g\right]\,\left[1 - R(\boldsymbol{q}_T^2)\right]$$

Contribution from the linearly polarized gluon TMD:

$$R(\boldsymbol{q}_T^2) \equiv \frac{\mathcal{C}[w h_1^{\perp g} h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]} \qquad w = \frac{1}{2M_h^4}\left[(\boldsymbol{p}_{aT}\cdot\boldsymbol{p}_{bT})^2 - \frac{1}{2}\boldsymbol{p}_{aT}^2\boldsymbol{p}_{bT}^2\right]$$

$$\mathcal{C}[w f f] \equiv \int d^2\boldsymbol{p}_{aT}\int d^2\boldsymbol{p}_{bT}\,\delta^2(\boldsymbol{p}_{aT}+\boldsymbol{p}_{bT}-\boldsymbol{q}_T)\, w(\boldsymbol{p}_{aT},\boldsymbol{p}_{bT}) f(x_a,\boldsymbol{p}_{aT}^2) f(x_b,\boldsymbol{p}_{bT}^2)$$

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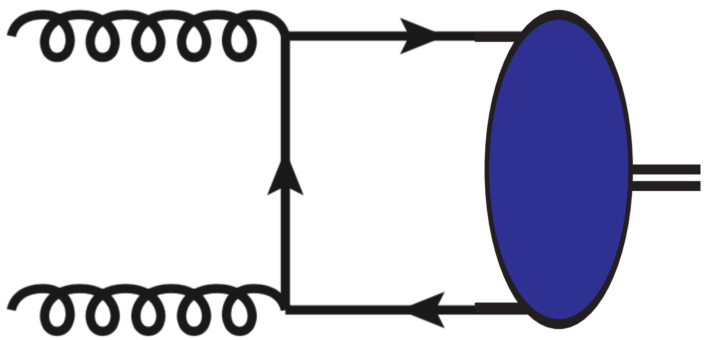

η_c, η_b

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$v_c^2 \approx 0.3$
$v_b^2 \approx 0.1$

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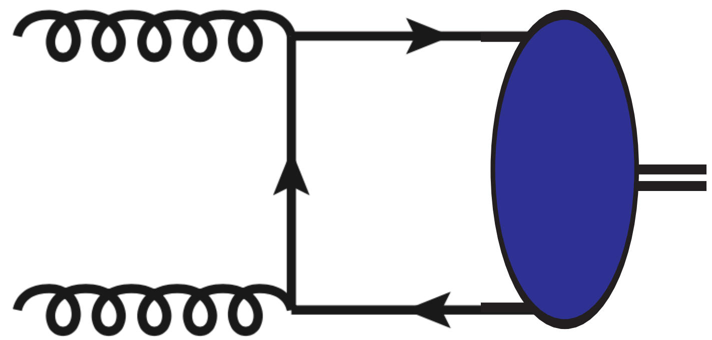

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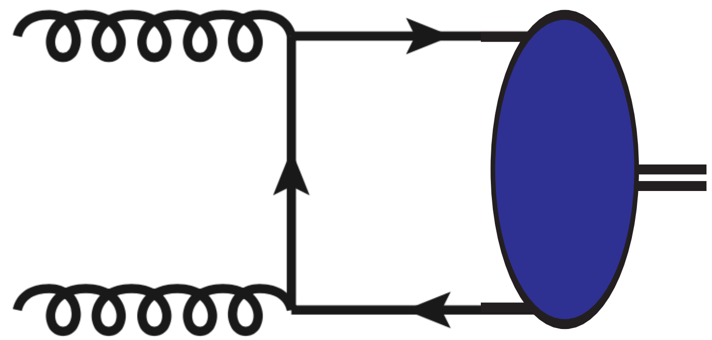
CO contributes at order same order in v, but suppressed by $\mathcal{O}\left(\frac{v^0}{2N_c}\right) \approx 0.17$

$\chi_{c,b}$ production

$pp \to \chi_Q X$



$\chi_{c0}\,,\,\chi_{b0}$

In LO NRQCD the differential cross sections in pp and pA are: DB, Pisano, 2012

$$\frac{d\sigma(\eta_Q)}{dy\, d^2\boldsymbol{q}_T} = \frac{2}{9}\frac{\pi^3\alpha_s^2}{M^3\, s}\langle 0|\mathcal{O}_1^{\eta_Q}({}^1S_0)|0\rangle\, \mathcal{C}\left[f_1^g\, f_1^g\right]\, \left[1 - R(\boldsymbol{q}_T^2)\right]$$

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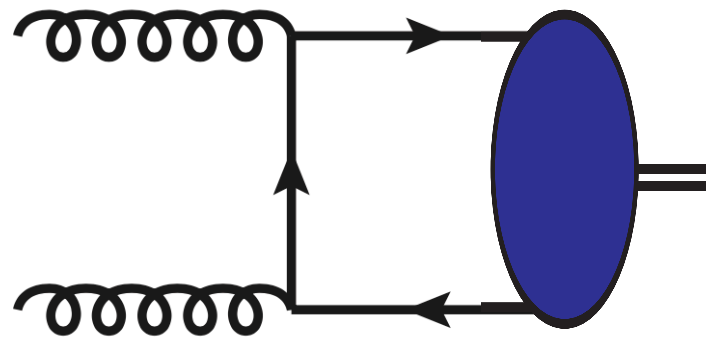

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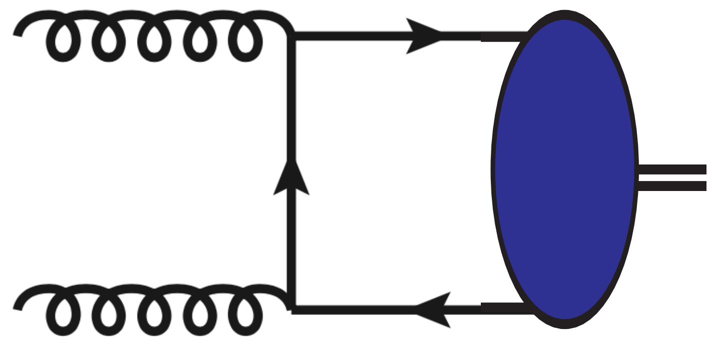

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χ_{QJ} LDMEs are order v² w.r.t. η_Q

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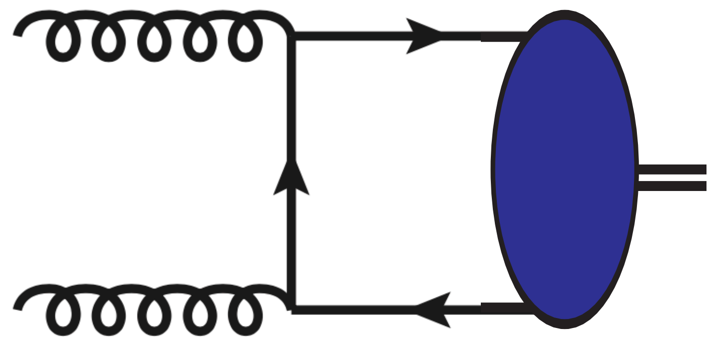

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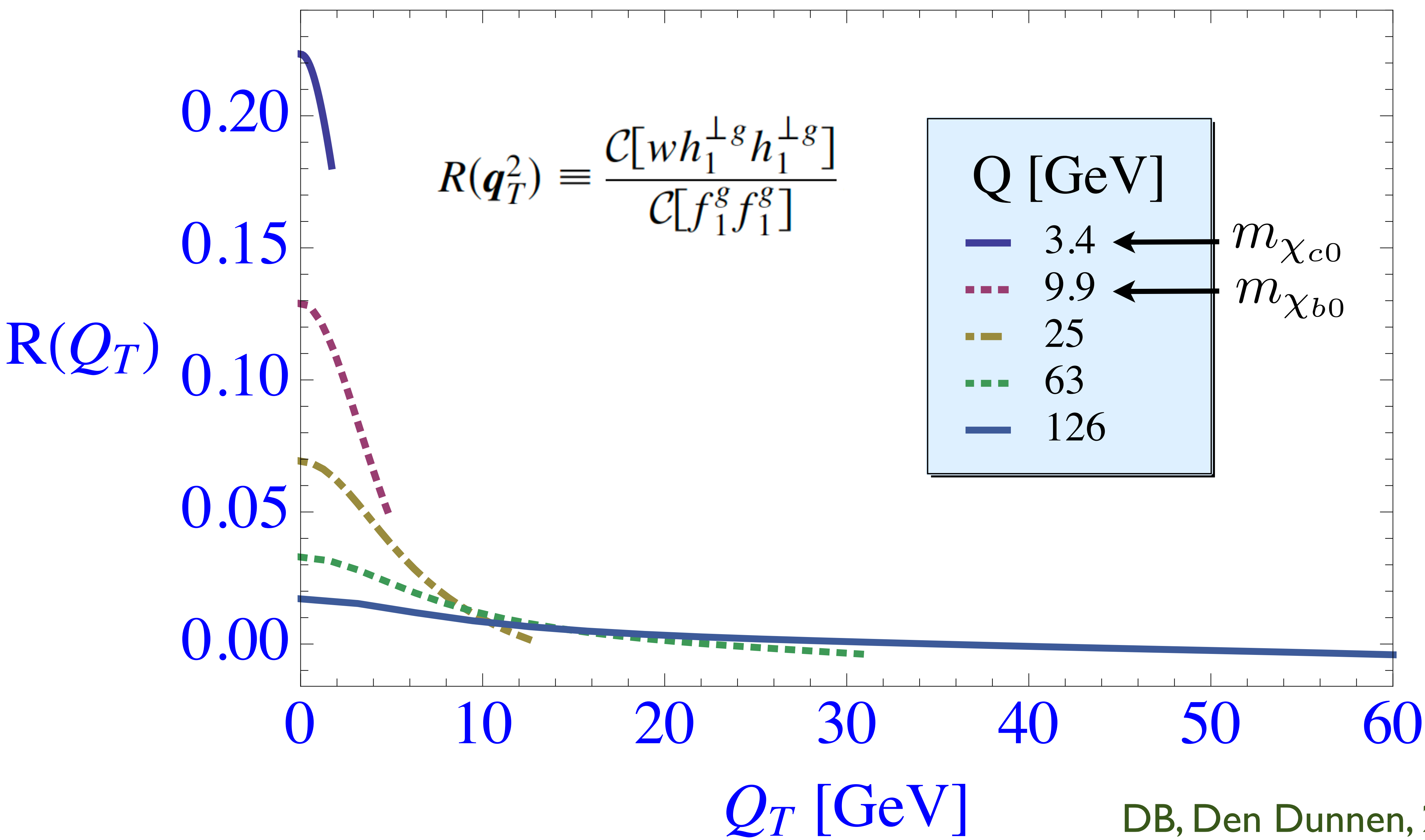
$$R(\boldsymbol{q}_T^2) \equiv \frac{\mathcal{C}[w\,h_1^{\perp g}\,h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]}$$

For χ_{Q1} there is no contribution due to Landau-Yang theorem

Effect of TMD evolution

Comparing $pp \to H\,X$, where $H = \chi_{c0}$, χ_{b0} or Higgs allows to test TMD evolution

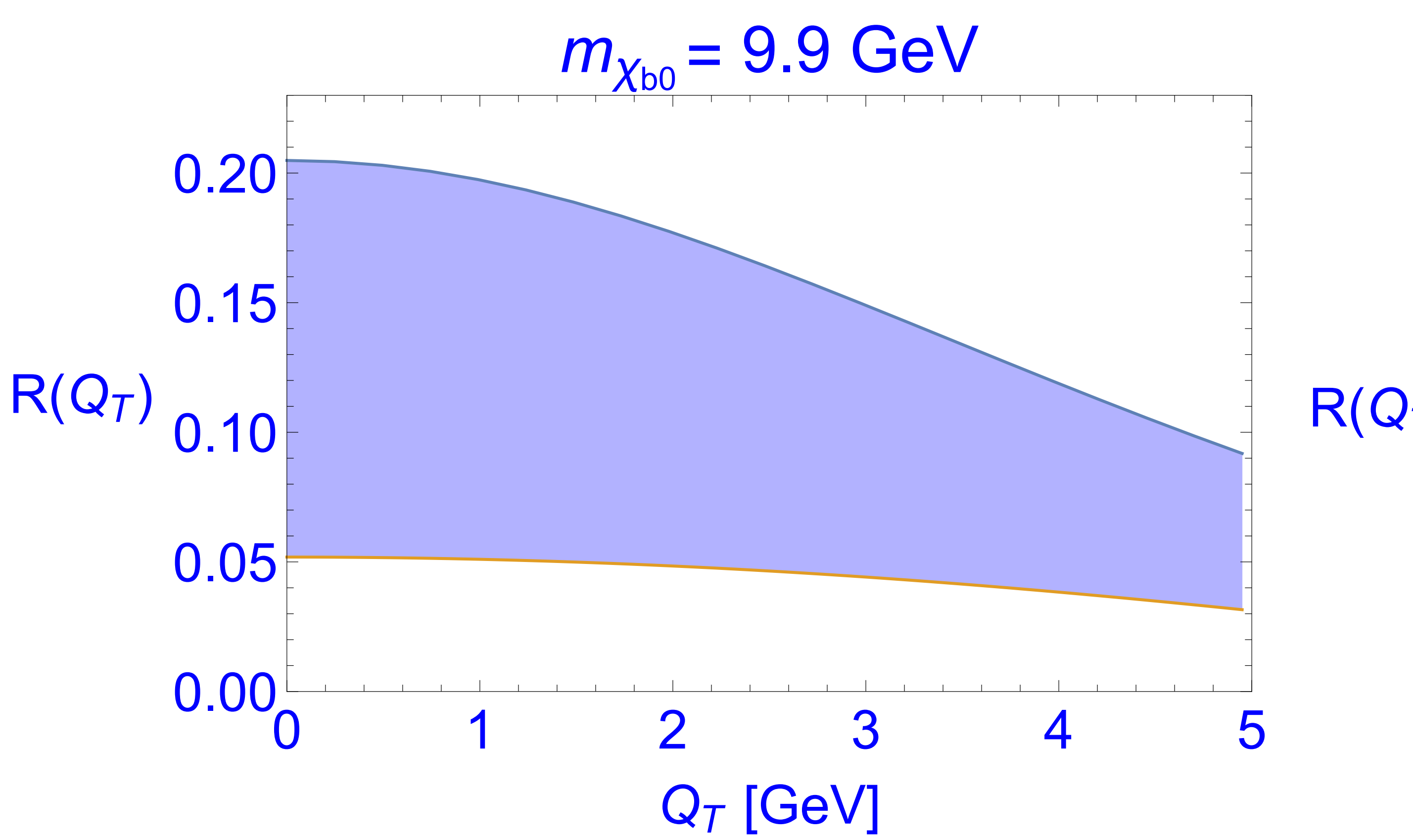
The relative contribution from linearly polarized gluons w.r.t. unpolarized gluons decreases with increasing mass of the produced state (which sets the hard scale):



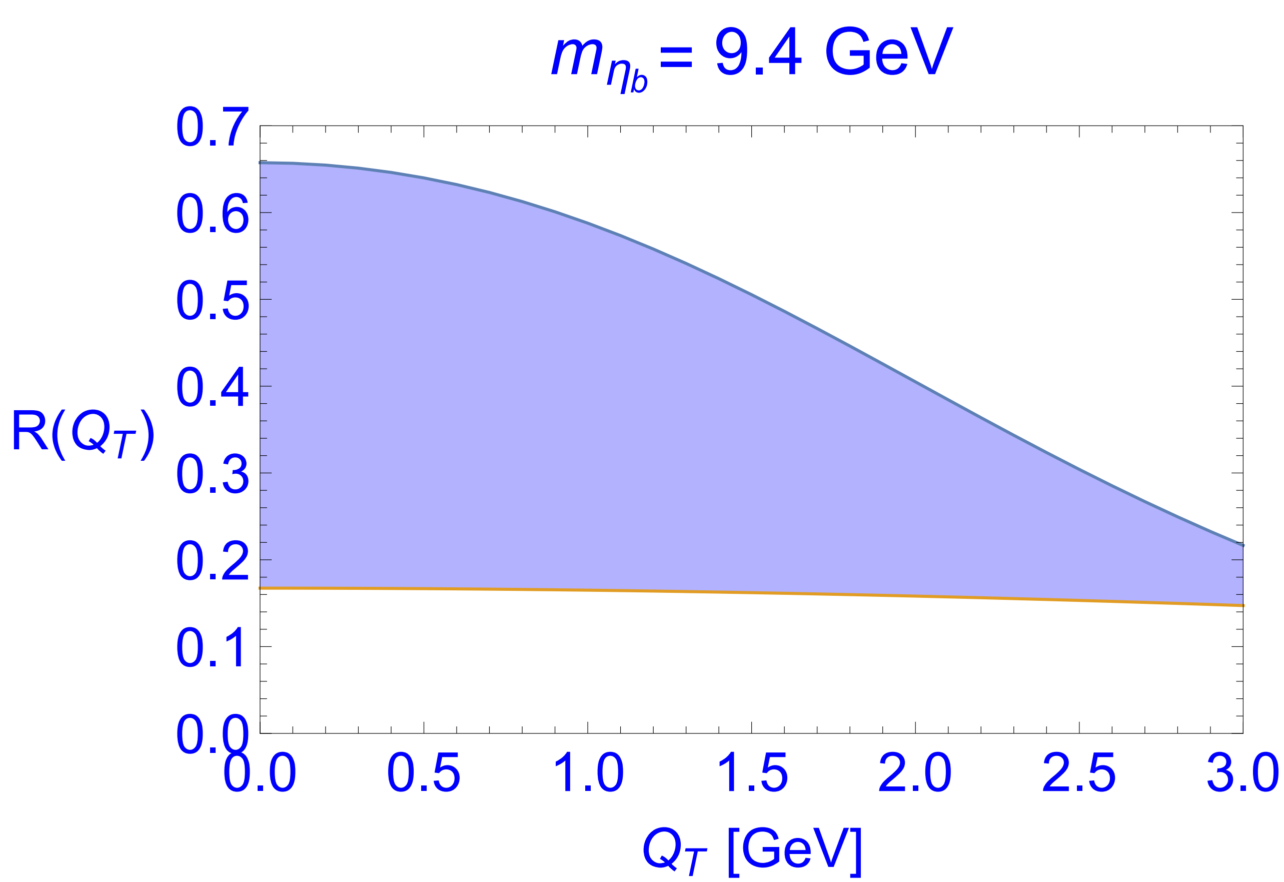
DB, Den Dunnen, 2014

Bottomonium production in pp

The range of predictions for C-even (pseudo-)scalar bottomonium production:



DB, Den Dunnen, 2014



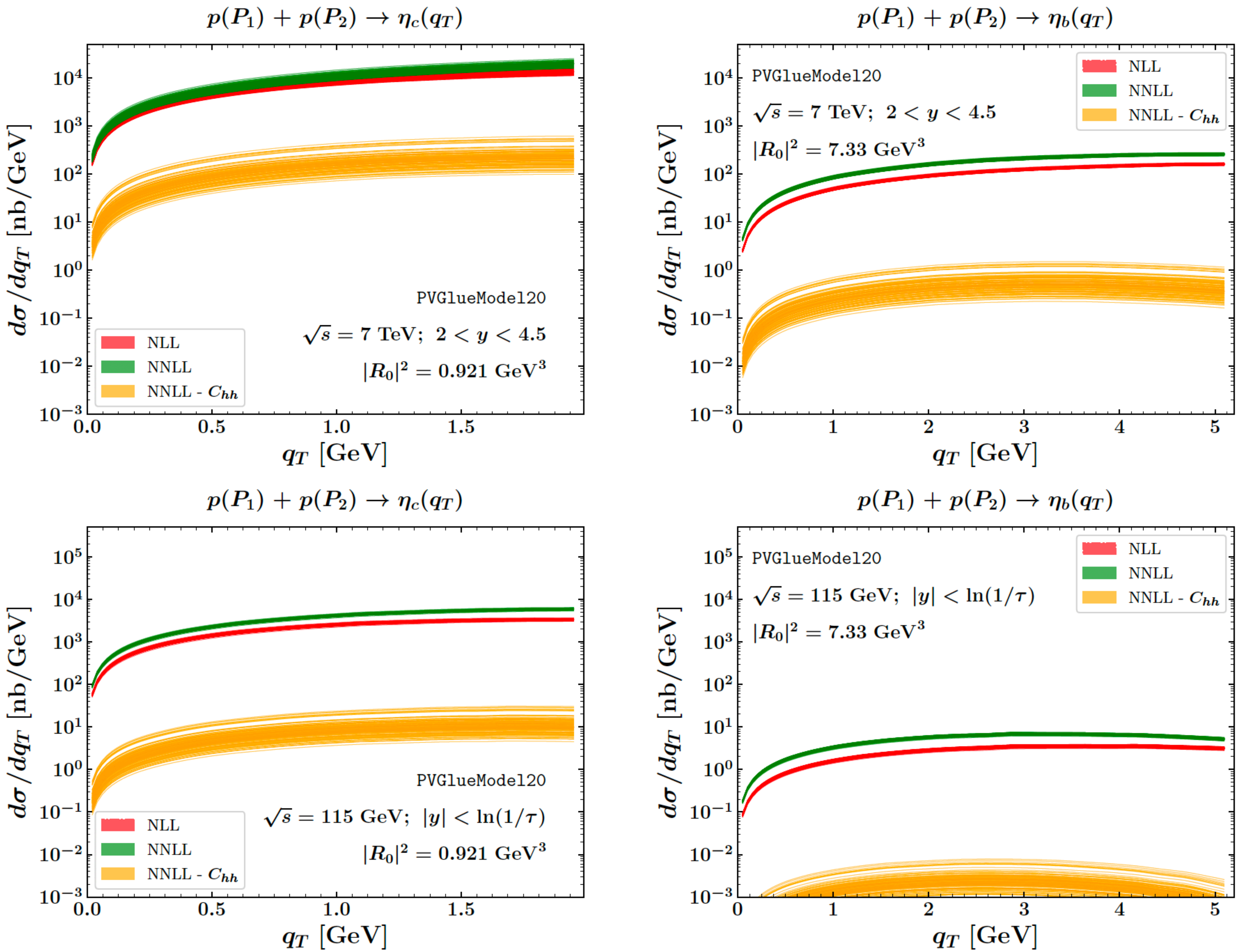
Echevarria, Kasemets, Mulders, Pisano, 2015

Variation of nonperturbative input for the TMDs and treatment of the very small b region (b<1/Q)

Variation of the nonperturbative Sudakov factor and the renormalization scale

Very large theoretical uncertainties, even more so for charmonium production, but contribution of 20% or more can be expected

$pp \to \eta_{c,b}\, X$



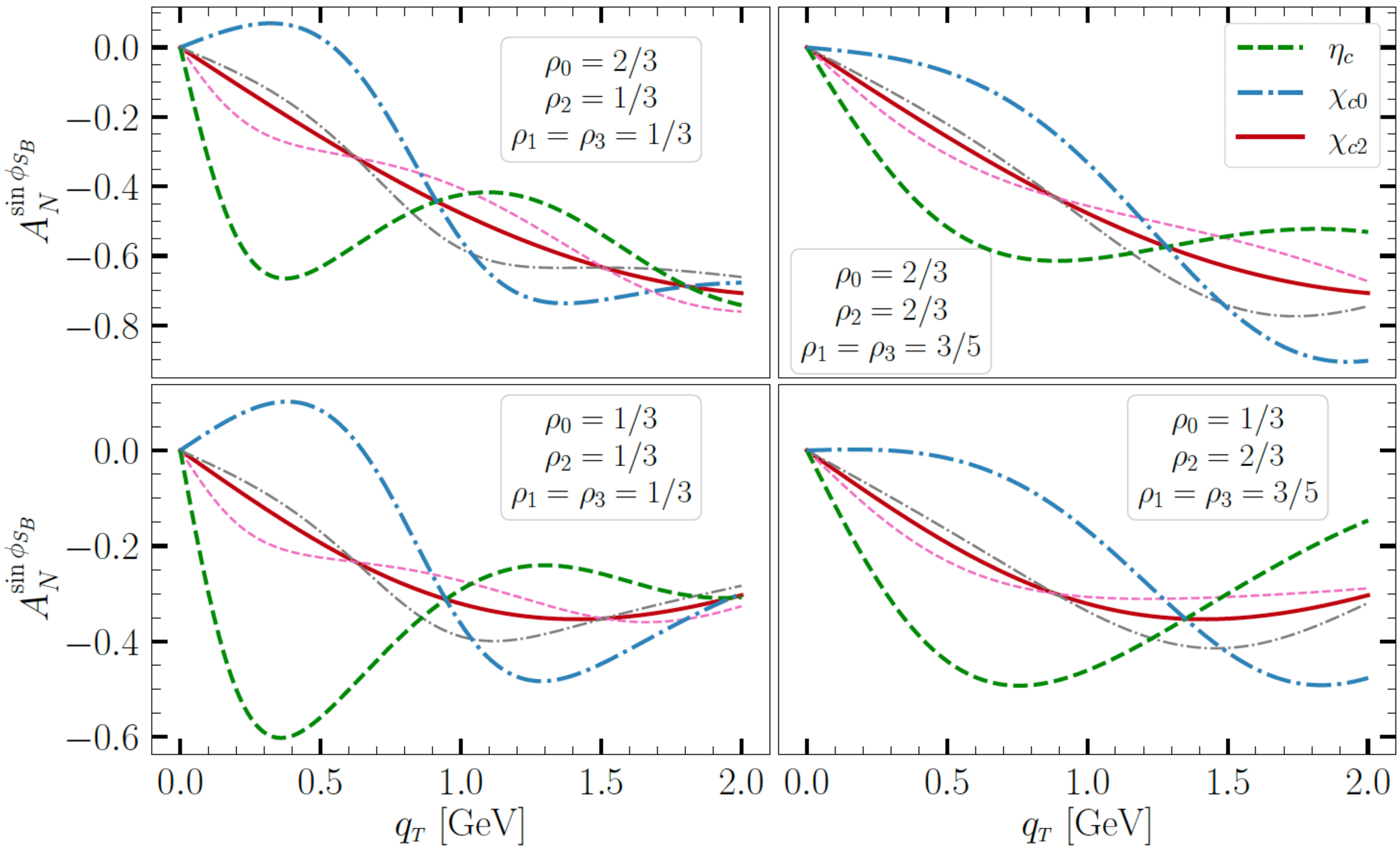
Bacchetta, Celiberto, Radici, Signori, 2022

$p^{\uparrow}p \to \eta_{c,b}\, X$

$$F_{TU}^{\eta_Q,\sin\phi_{S_A}} = H^{\eta_Q}\Big(\mathcal{C}\big[w_{TU}^{f}\, f_1^{g} f_{1T}^{\perp g}\big] - \mathcal{C}\big[w_{TU}^{h}\, h_1^{\perp g} h_1^{g}\big] + \mathcal{C}\big[w_{TU}^{h^\perp}\, h_1^{\perp g} h_{1T}^{\perp g}\big]\Big)\,\langle 0|\mathcal{O}_1^{\eta_Q}({}^1S_0)|0\rangle\,,$$

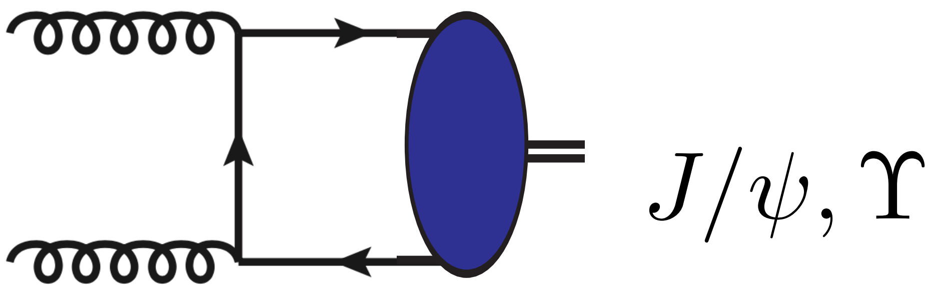
$$F_{TU}^{\chi_{Q0},\sin\phi_{S_A}} = H^{\chi_{Q0}}\Big(\mathcal{C}\big[w_{TU}^{f}\, f_1^{g} f_{1T}^{\perp g}\big] + \mathcal{C}\big[w_{TU}^{h}\, h_1^{\perp g} h_1^{g}\big] - \mathcal{C}\big[w_{TU}^{h^\perp}\, h_1^{\perp g} h_{1T}^{\perp g}\big]\Big)\,\langle 0|\mathcal{O}_1^{\chi_{Q0}}({}^3P_0)|0\rangle\,,$$

$$F_{TU}^{\chi_{Q2},\sin\phi_{S_A}} = H^{\chi_{Q2}}\,\mathcal{C}\big[w_{TU}^{f}\, f_1^{g} f_{1T}^{\perp g}\big]\,\langle 0|\mathcal{O}_1^{\chi_{Q2}}({}^3P_2)|0\rangle\,,$$



Kato, Maxia, Pisano, 2024

J/ψ (associated) production



CS not allowed by Landau-Yang theorem
CO complicated link structure & possibly factorization breaking

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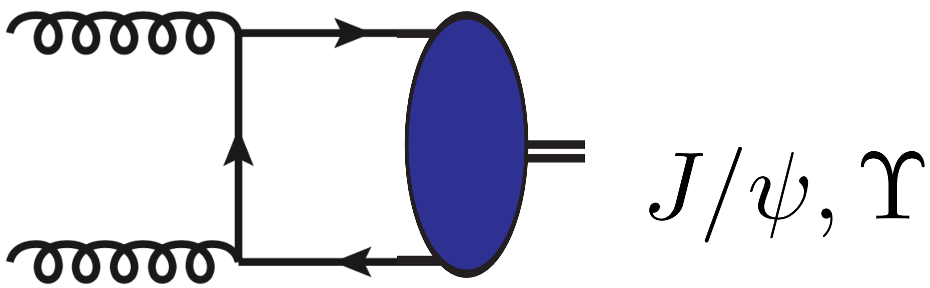

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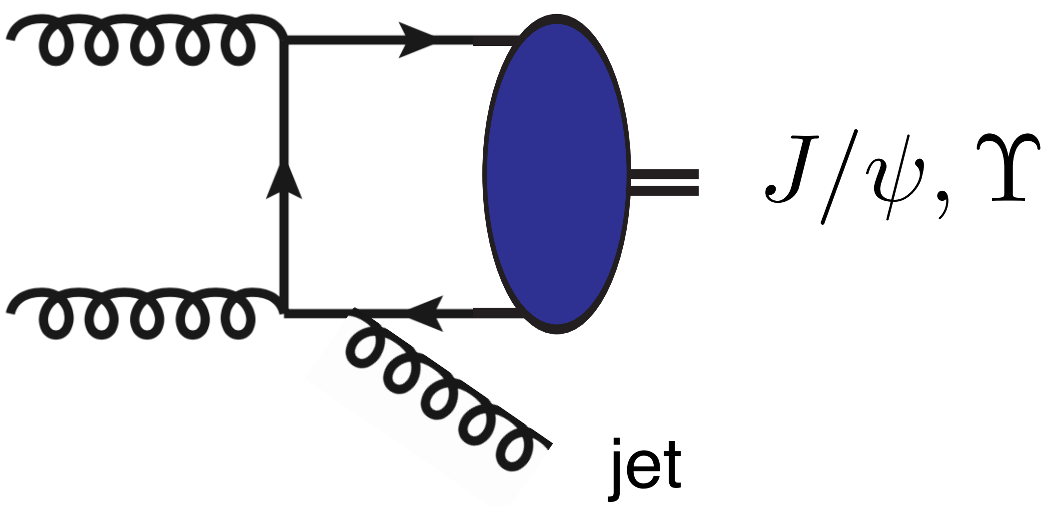

CS allowed, nevertheless complicated link structure & possibly factorization breaking

Berger, Qiu, Wang, 2005;
Kang, Ma, Qiu, Sterman, 2014;
D'Alesio, Murgia, Pisano, Rajesh, 2019;
…

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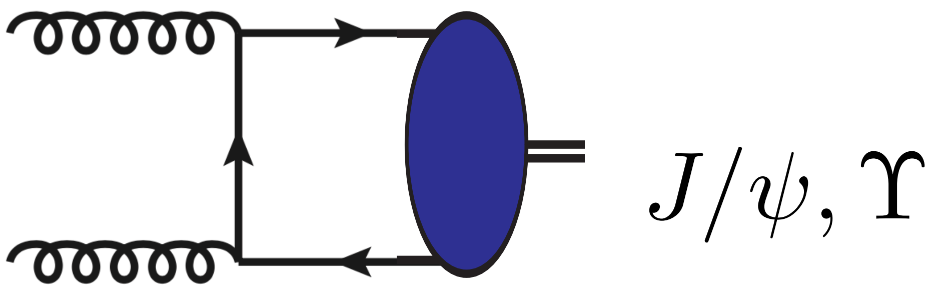

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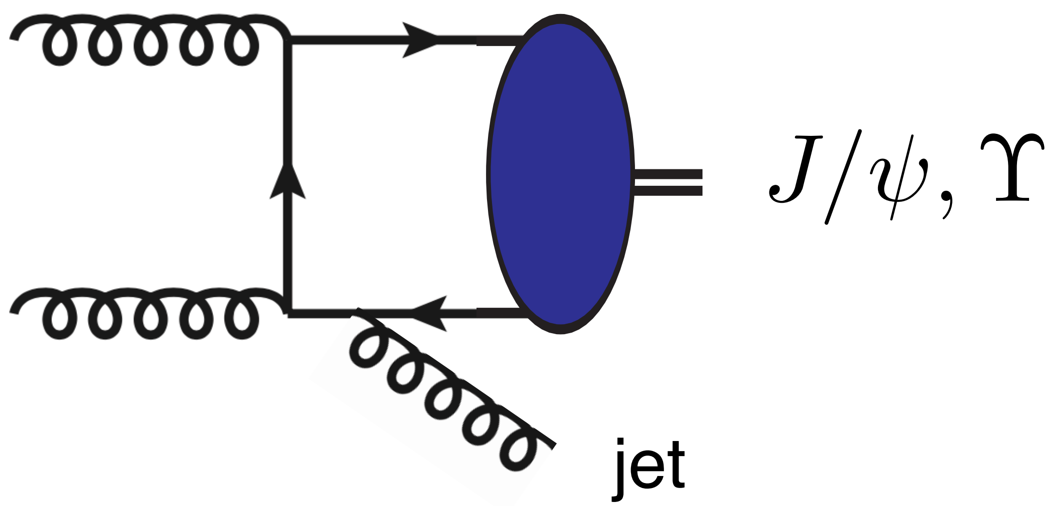

CS allowed, nevertheless complicated link structure & possibly factorization breaking

Berger, Qiu, Wang, 2005;
Kang, Ma, Qiu, Sterman, 2014;
D'Alesio, Murgia, Pisano, Rajesh, 2019;
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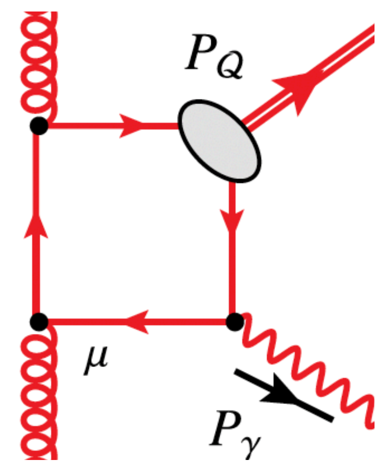
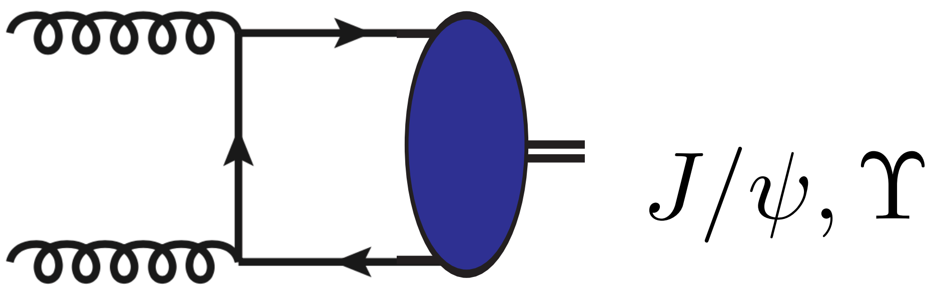


Associated production with a photon is fine

J/ψ (associated) production



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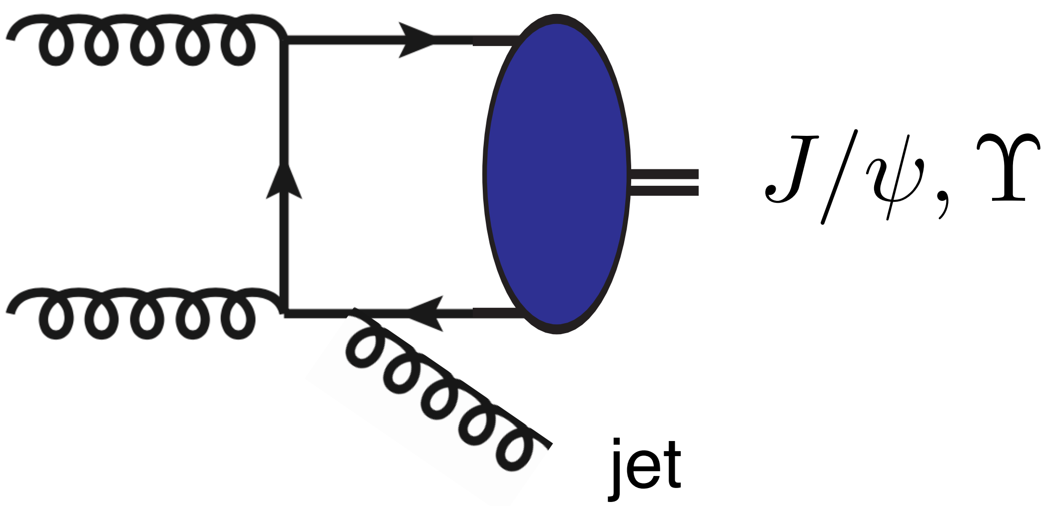

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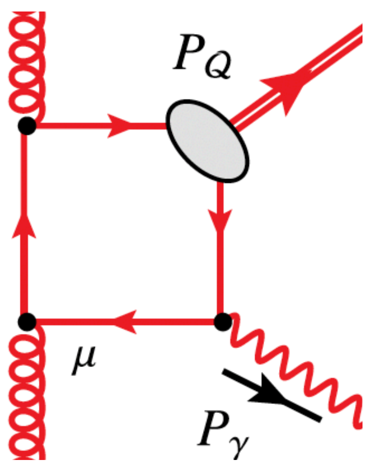
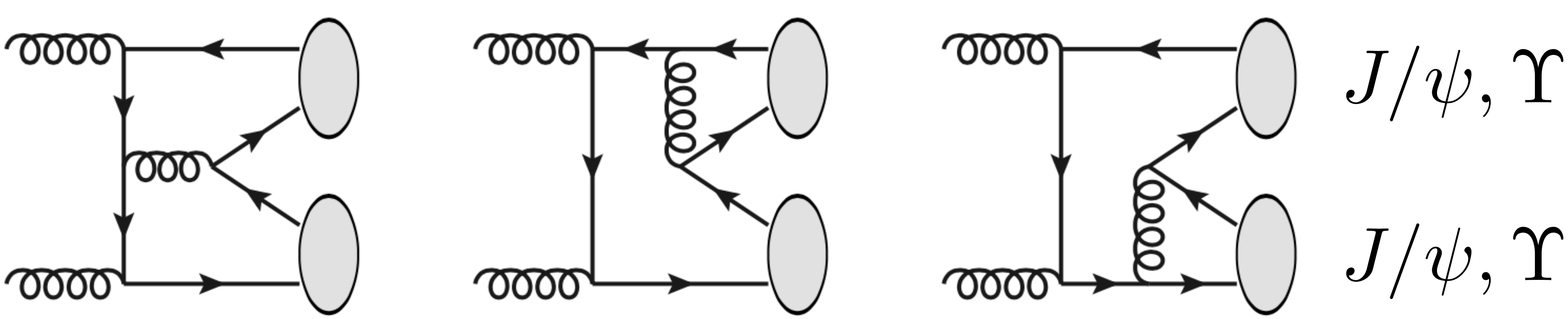


Associated production with a photon is fine

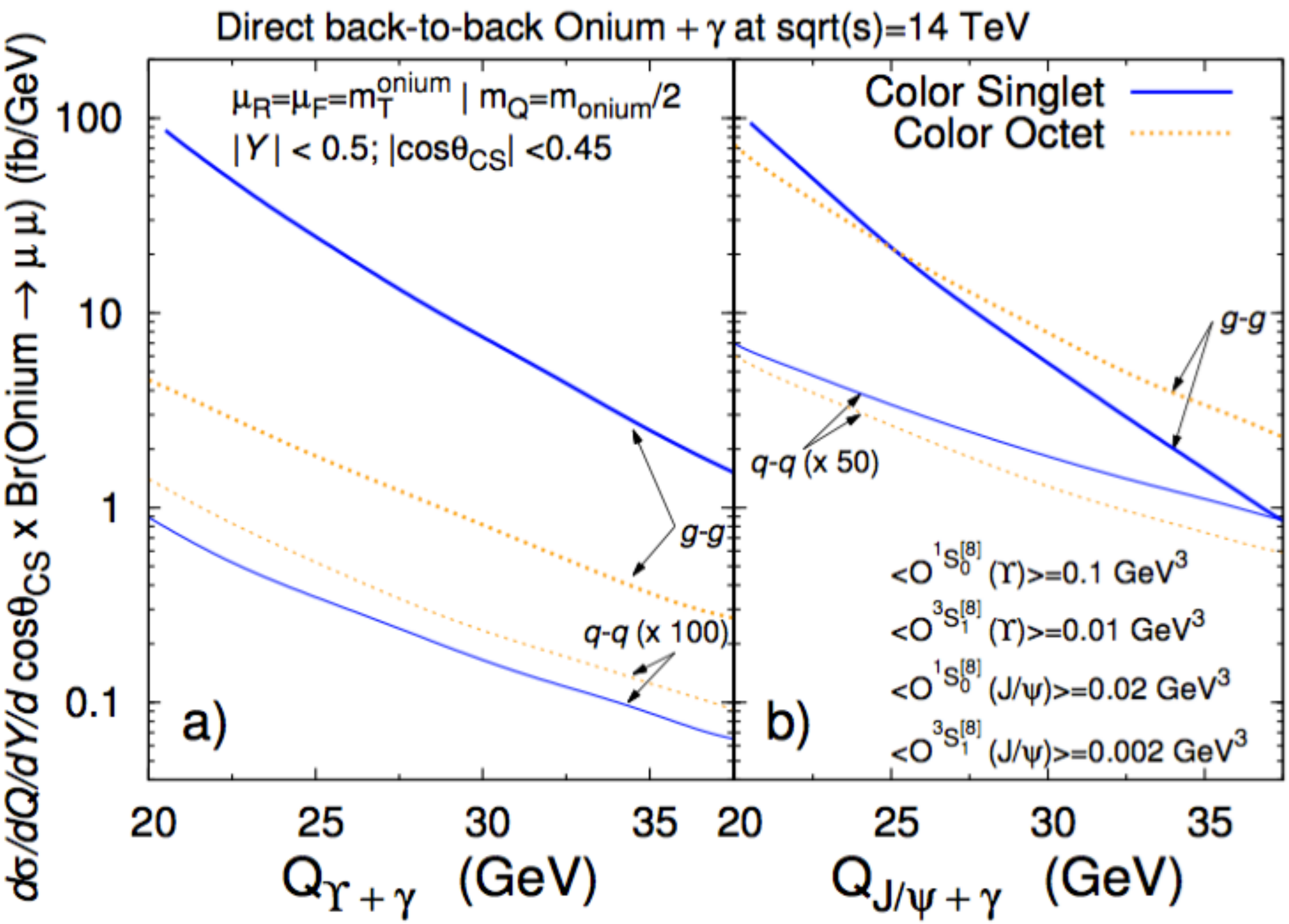
Double J/ψ production too:



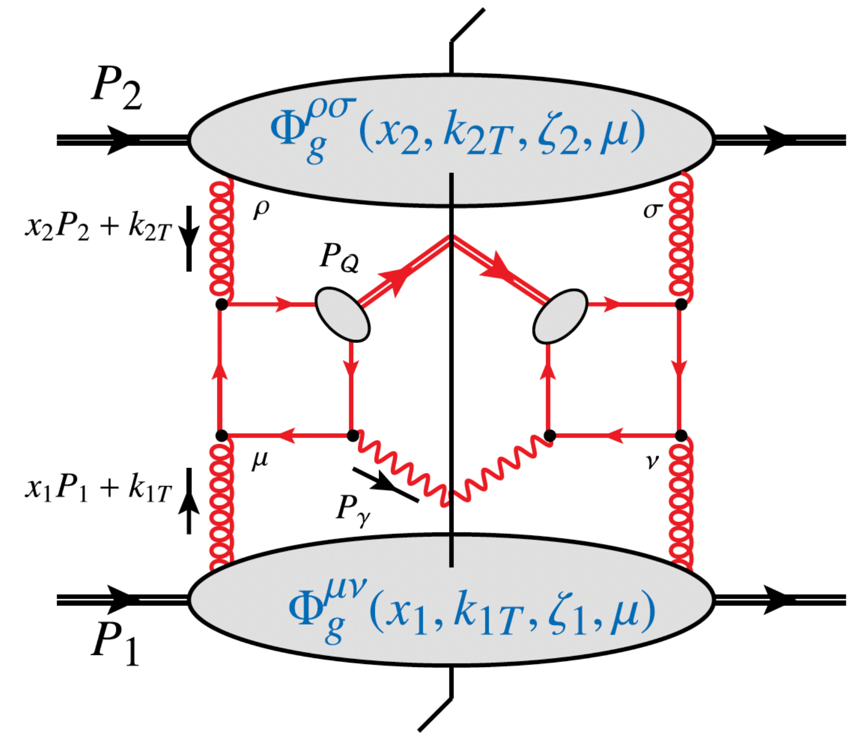
CS–CS » CO–CO

Associated J/ψ production

$p\,p \to \mathcal{Q}\,\gamma\,X$ could be a good process to extract $f_1^g(x, p_T^2)$ at LHC

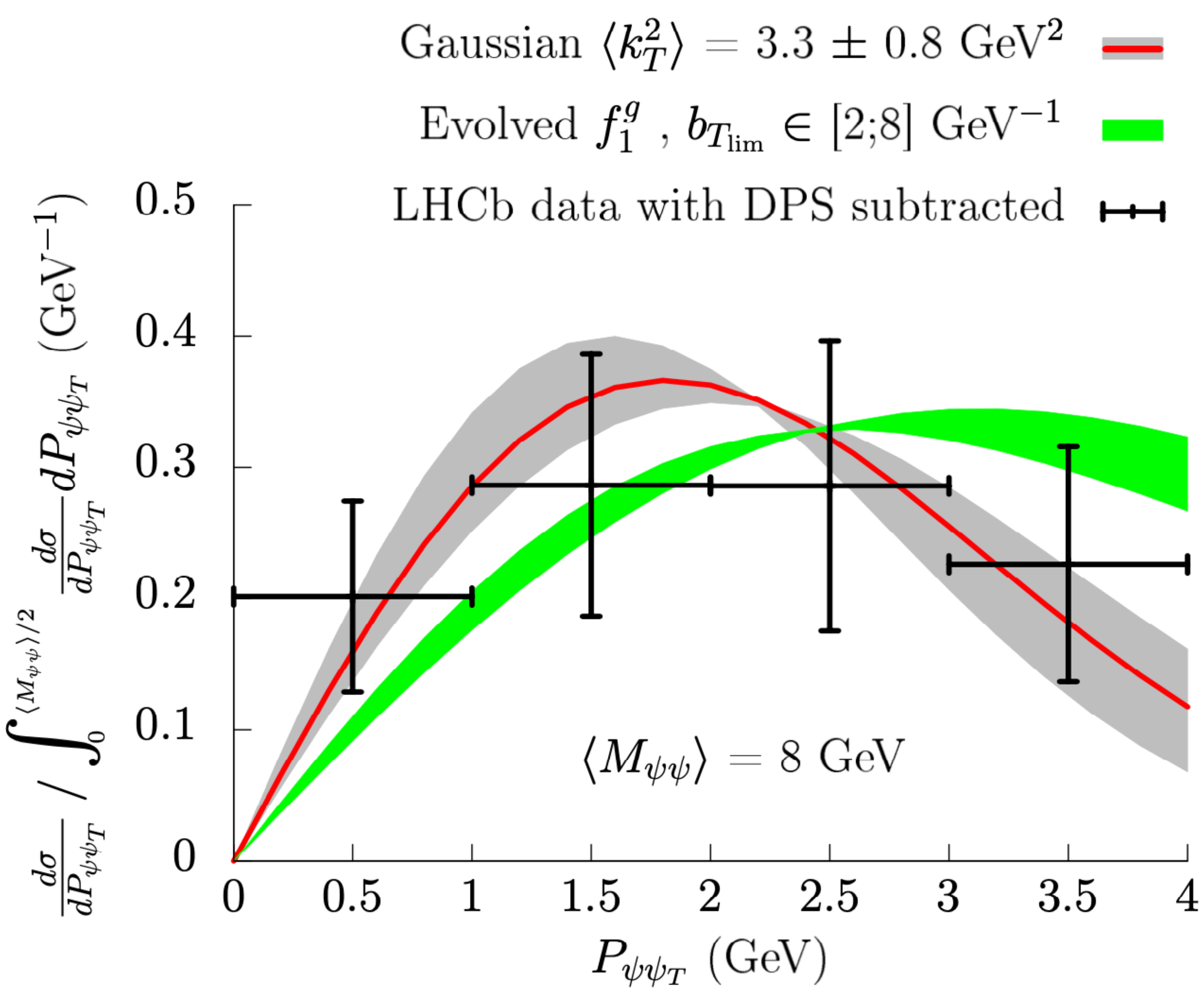


Den Dunnen, Lansberg, Pisano, Schlegel, 2014

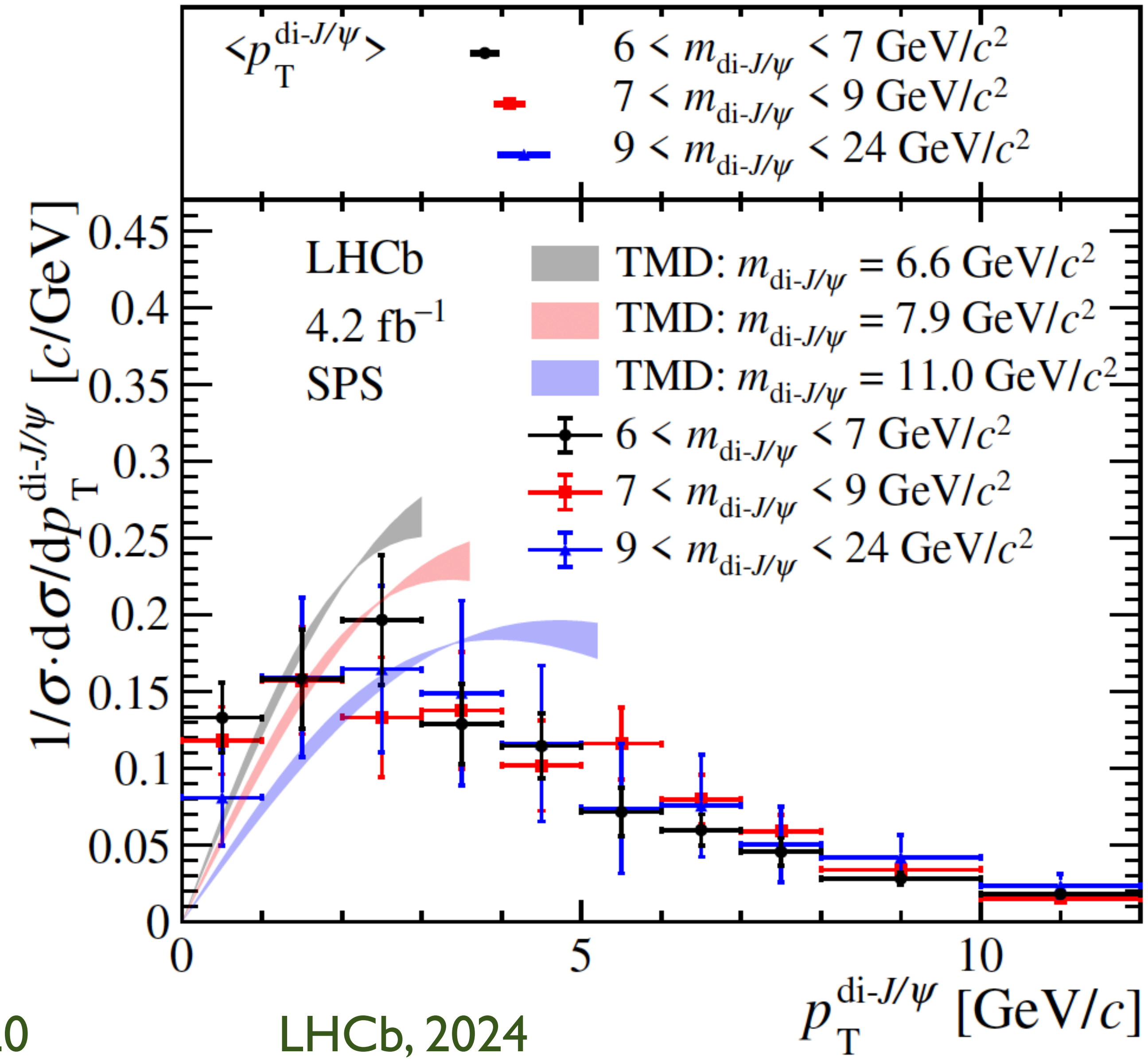


The CS contribution dominates in ϒ+γ production and for lower invariant mass of the pair also in J/ψ+γ production

J/ψ pair production

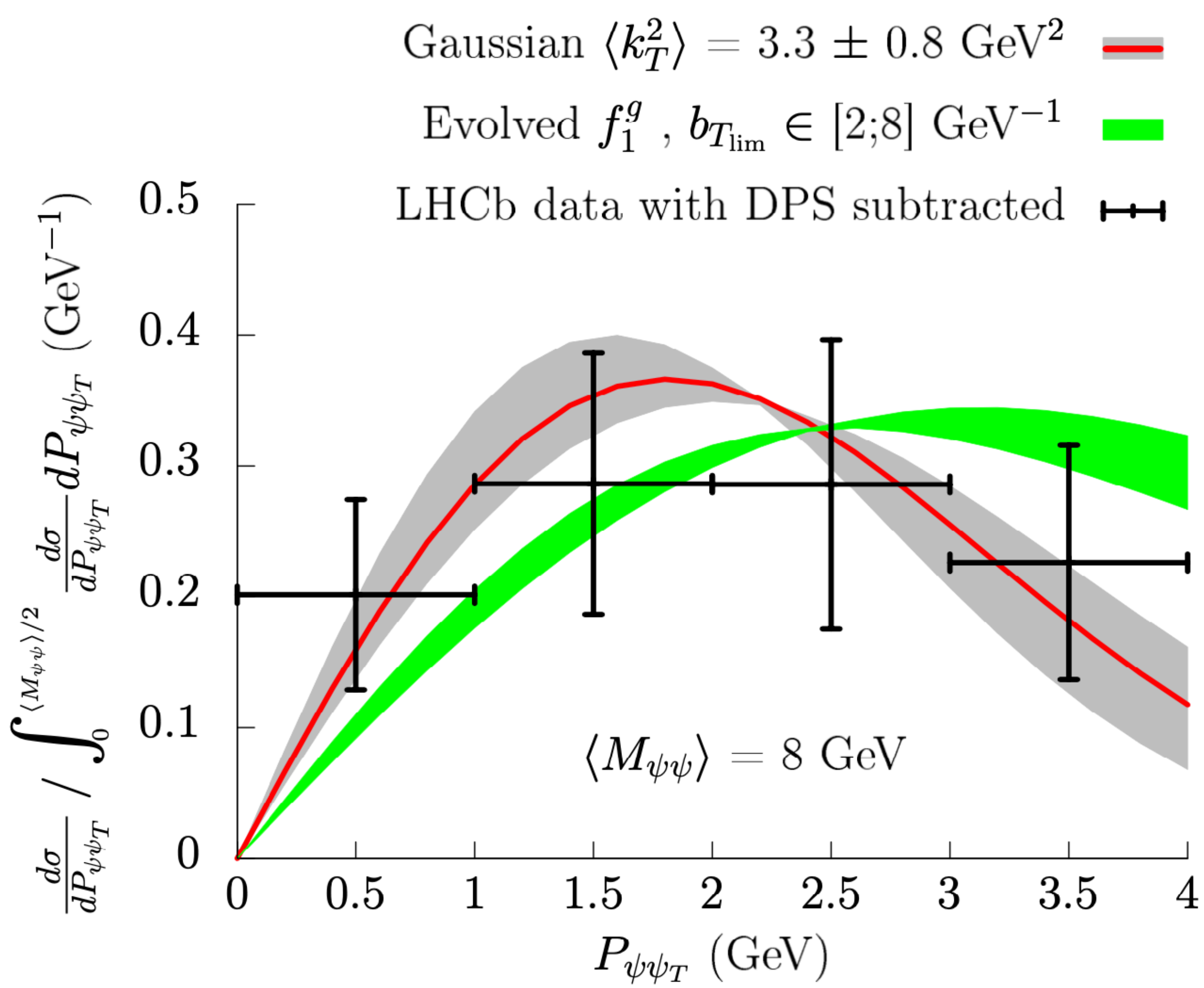


Scarpa, DB, Echevarria, Lansberg, Pisano, Schlegel, 2020

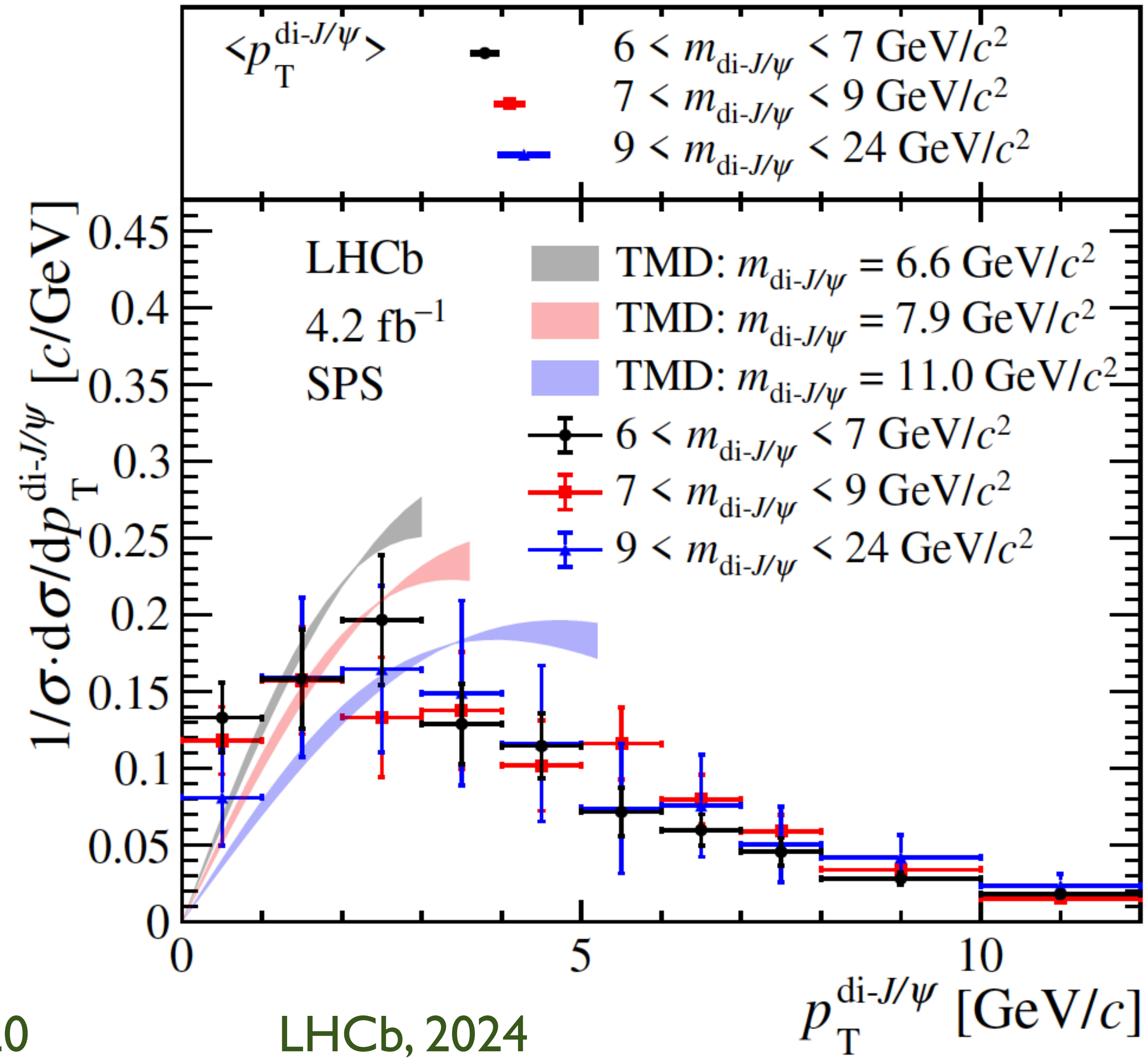


LHCb, 2024

J/ψ pair production



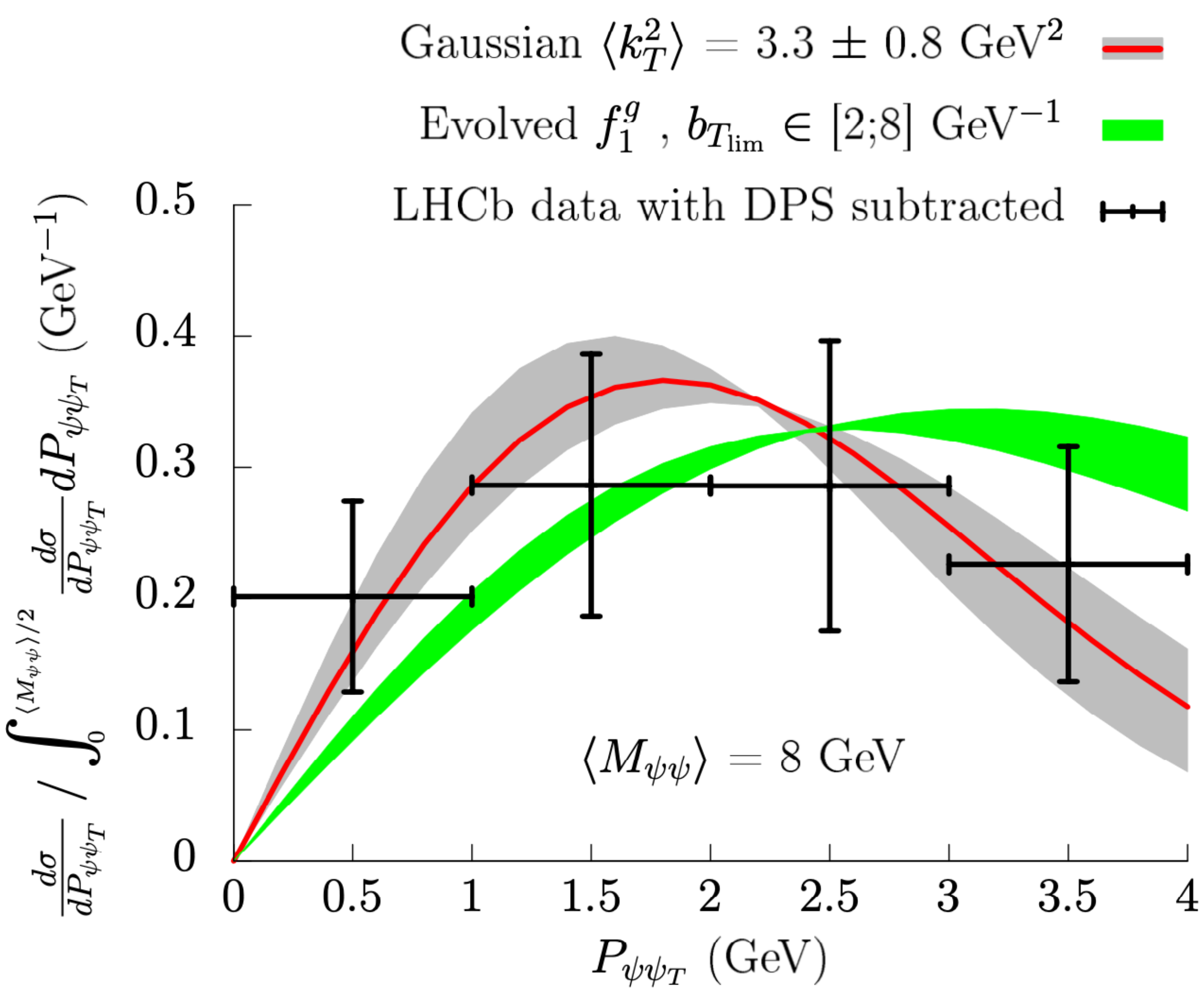
Scarpa, DB, Echevarria, Lansberg, Pisano, Schlegel, 2020



LHCb, 2024

J/ψ pair invariant mass allows to study TMD evolution

J/ψ pair production



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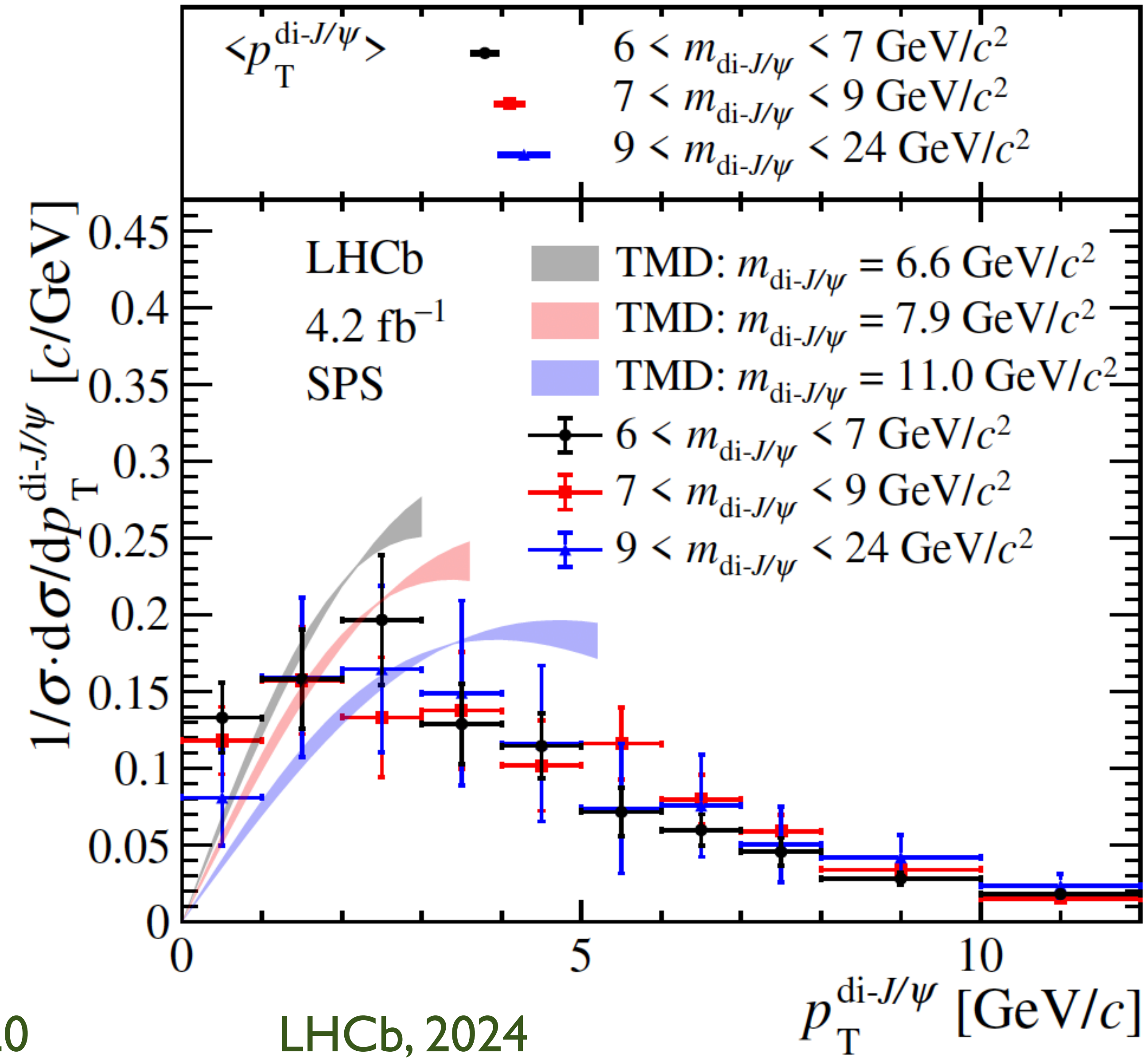


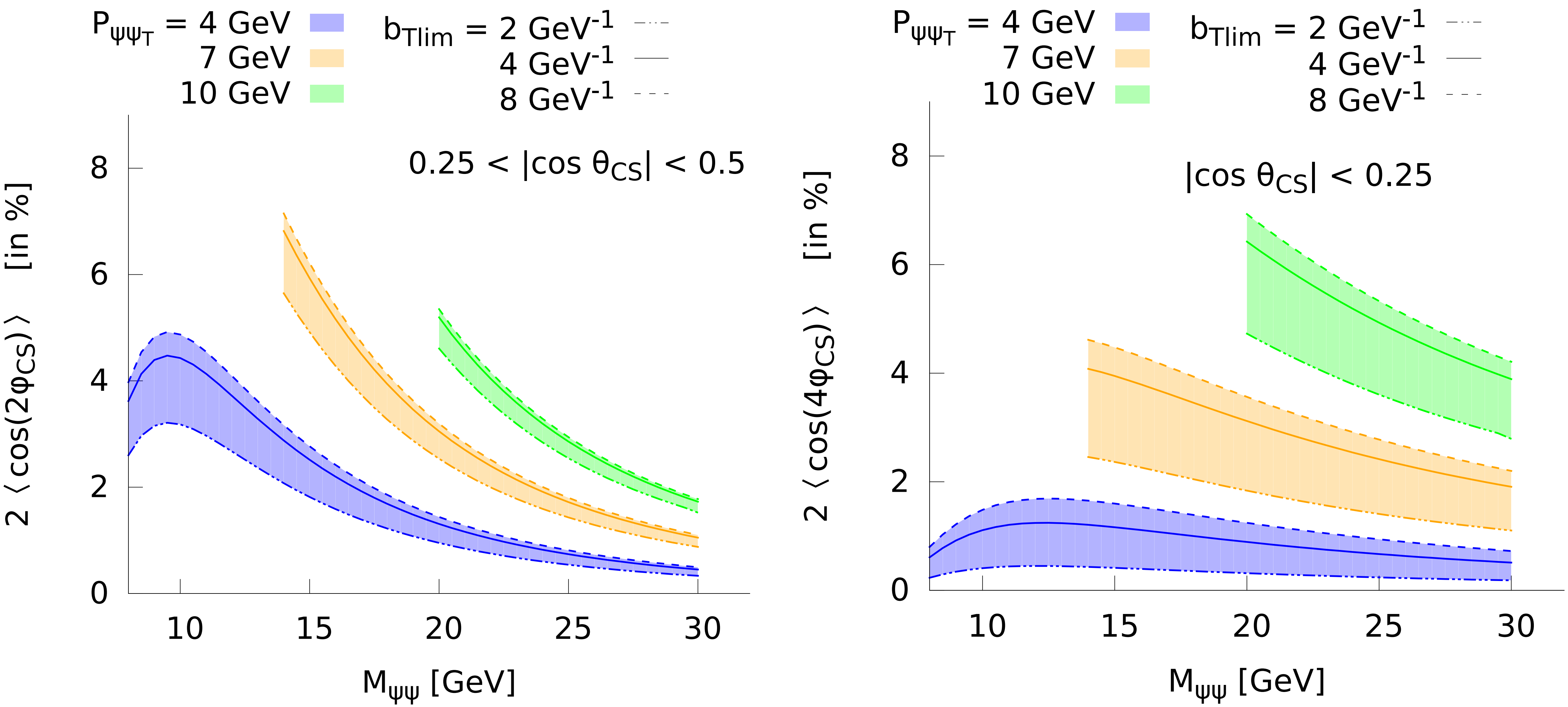
LHCb, 2024

J/ψ pair invariant mass allows to study TMD evolution

The shape of this normalized (DPS subtracted) distribution and its scale evolution is not fully described by the TMD description (also not within uncertainties from nonperturbative physics) [soon to be updated]

Linear gluon polarization in di-J/Ψ production

$h_1^{\perp g}$ can be probed through angular modulations in $p\,p \to J/\psi\, J/\psi\, X$



Estimated to lead to 1-5% level azimuthal modulations at LHC (incl. TMD evolution)

Scarpa, DB, Echevarria, Lansberg, Pisano, Schlegel, 2019

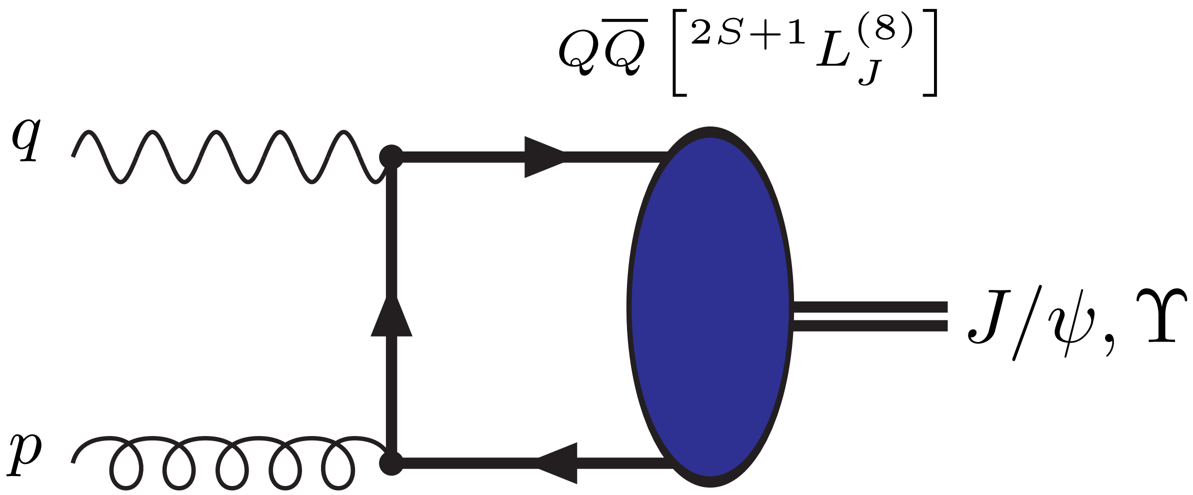
CO contributions estimated to be below the percent level, except at large Δy

ep collisions

Quarkonium production in ep

$e\, p \to e'\, \mathcal{Q}\, X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson

Mukherjee, Rajesh, 2017; Sun, Zhang, 2017; Bacchetta, DB, Pisano, Taels, 2018; Kishore, Mukherjee, 2018; Kishore, Mukherjee, Siddiqah, 2021; …



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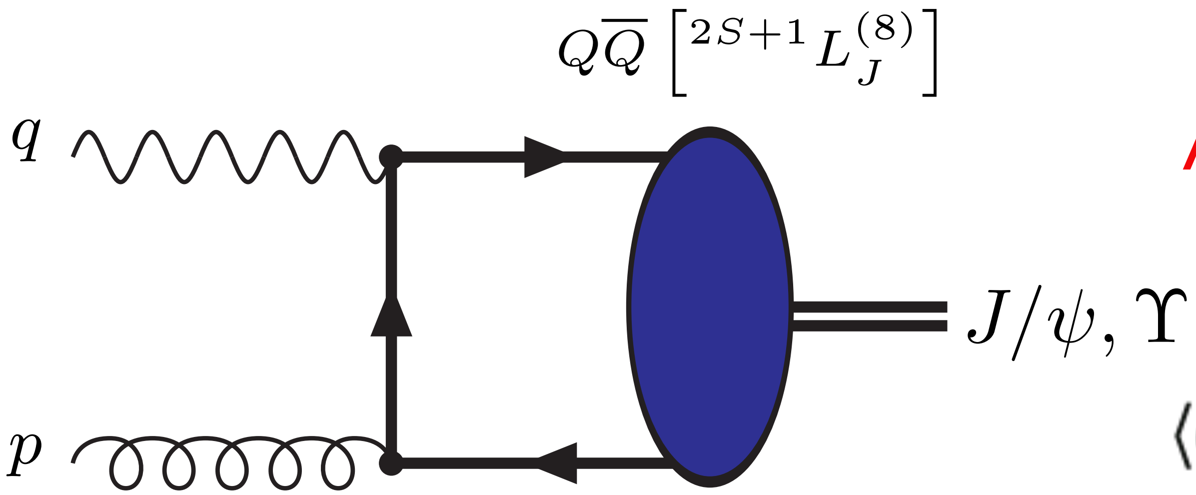

A $\cos(2\phi_T)$ asymmetry probes $h_1^{\perp\, g}$

$$\langle \cos 2\phi_T \rangle = \frac{(1-y)\,\mathcal{B}_T^{\gamma^* g \to \mathcal{Q}}}{[1+(1-y)^2]\mathcal{A}_{U+L}^{\gamma^* g \to \mathcal{Q}} - y^2 \mathcal{A}_L^{\gamma^* g \to \mathcal{Q}}} \times \frac{\boldsymbol{q}_T^2}{2M_p^2} \frac{h_1^{\perp g}(x, \boldsymbol{q}_T^2)}{f_1^g(x, \boldsymbol{q}_T^2)}.$$

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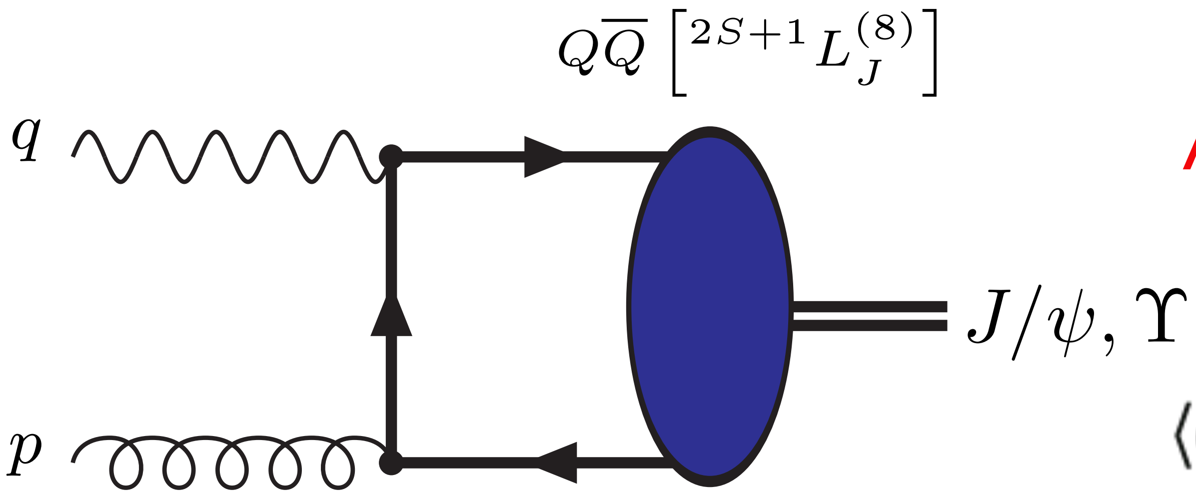

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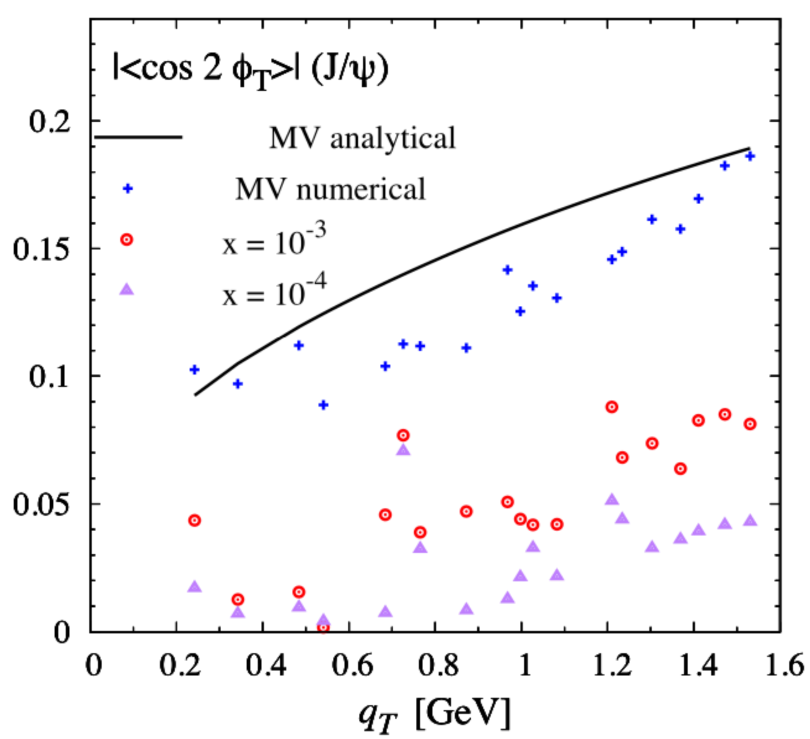
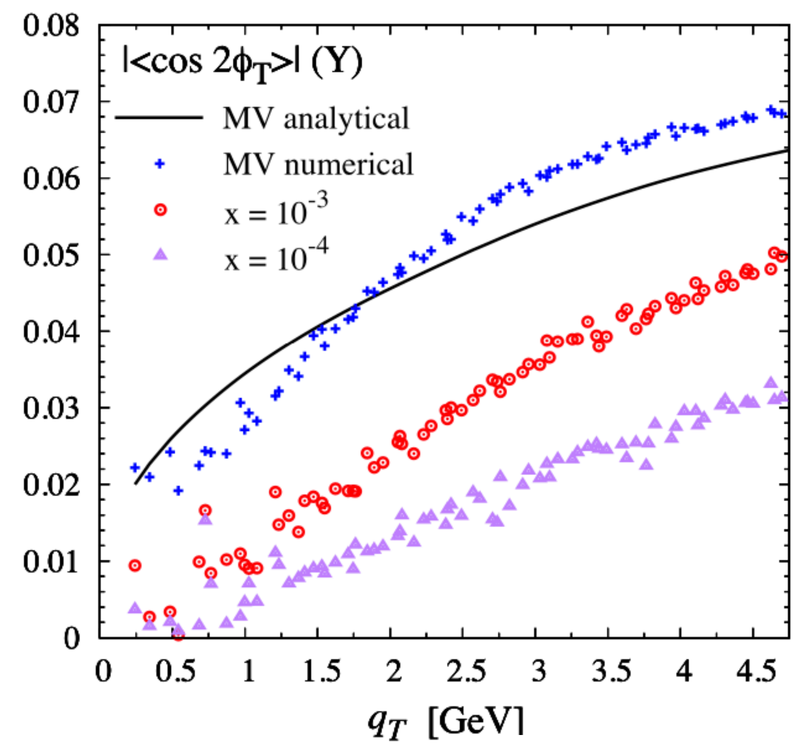
In LO NRQCD the prefactor of the asymmetry depends on y, Q, M_Q and on two quite uncertain Color Octet (CO) Long Distance Matrix Elements (LDMEs)

CS is α_s suppressed, but v^3 enhanced, higher minimum Q^2 and z cuts suppress it

Bacchetta, DB, Pisano, Taels, 2018

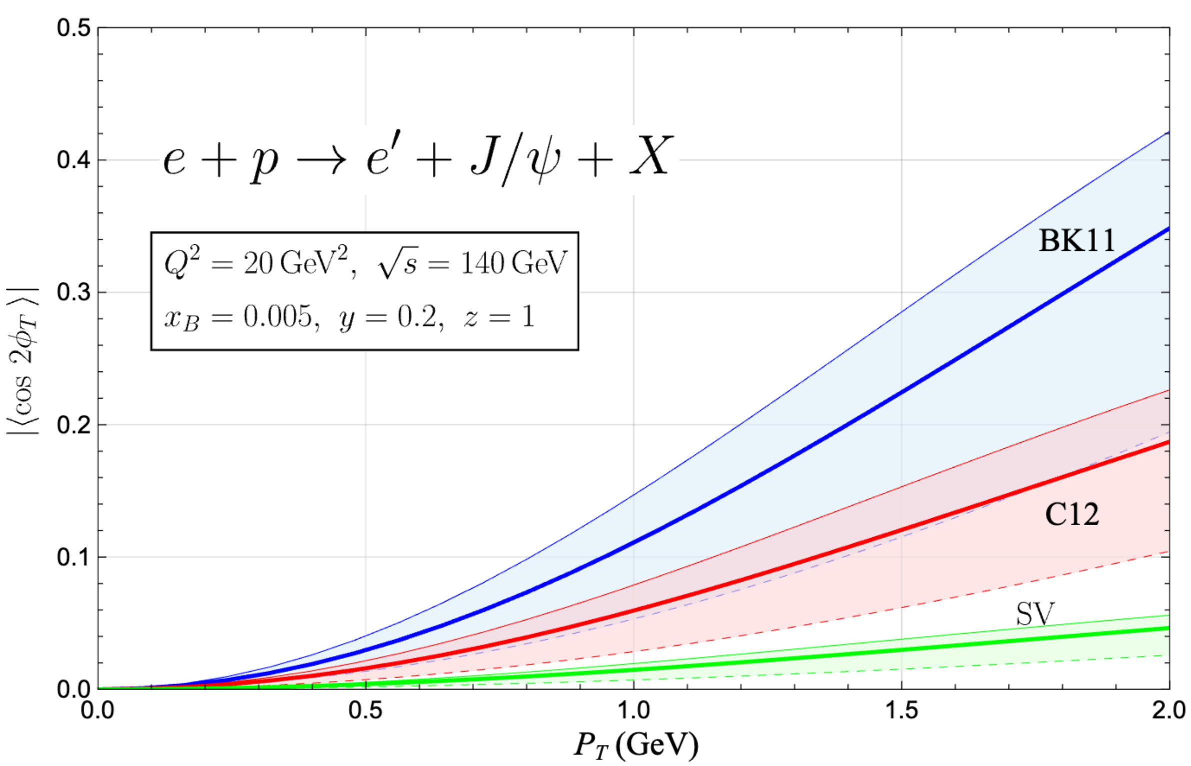
cos 2ϕ_T asymmetry - predictions



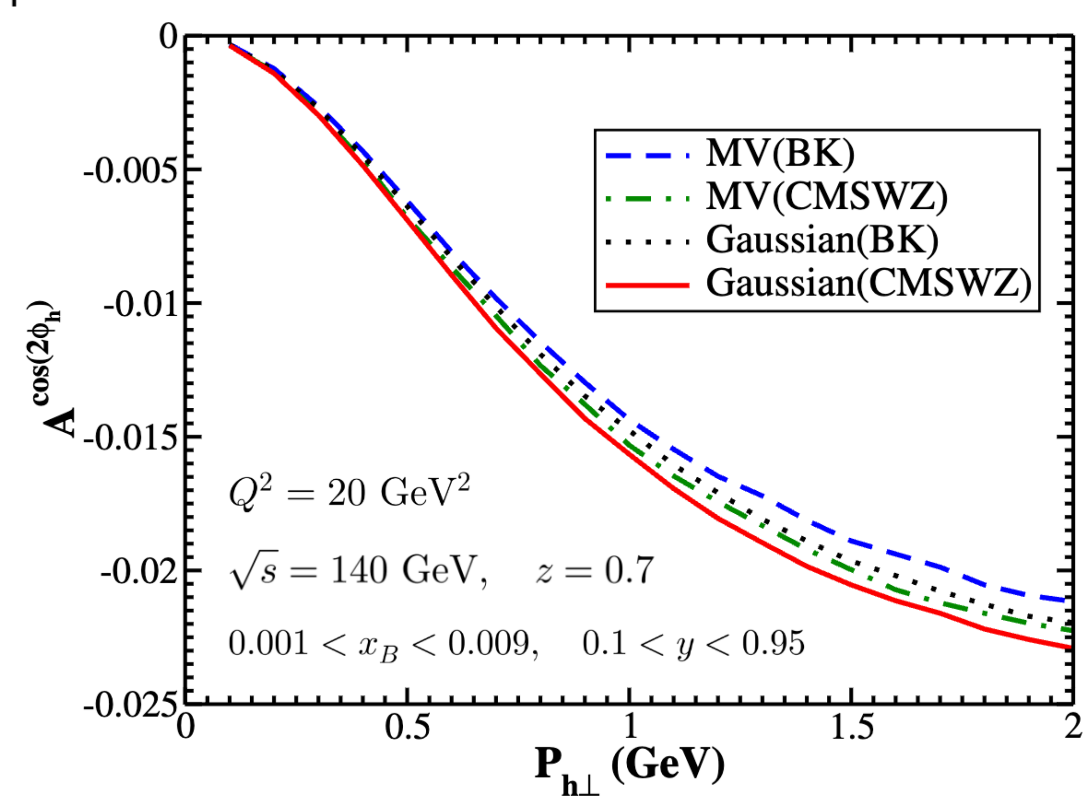


Asymmetries for $Q=M_Q$ & y=0.1 in the small-x MV model, including nonlinear evolution (numerical implementation on a 2D lattice)

Bacchetta, DB, Pisano, Taels, 2018



Bor, DB, 2022



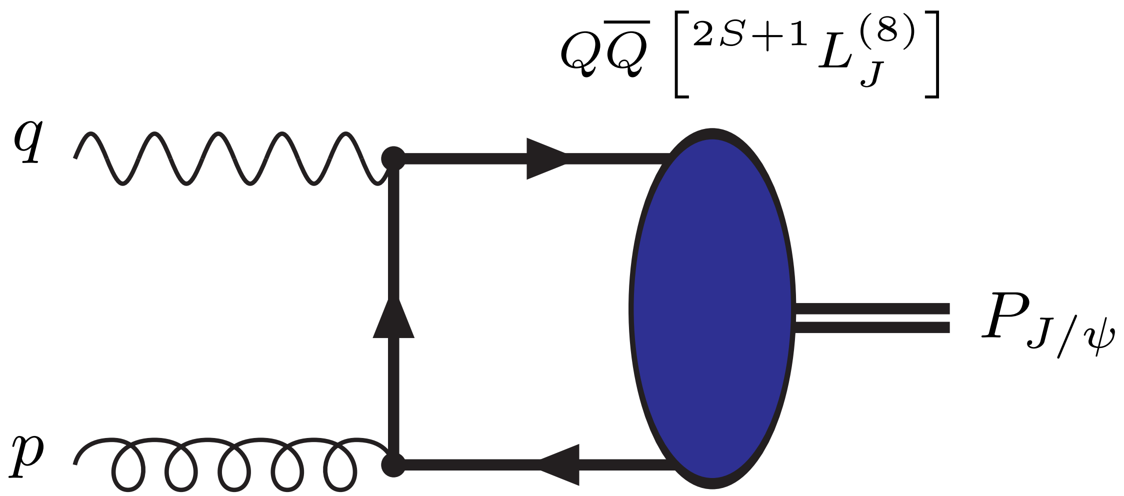
Kishore, Mukherjee, Pawar, Siddiqah, 2022

Despite the large uncertainties sizable cos2ϕ_T asymmetries are possible

Quarkonium production in ep

$e\,p^{\uparrow} \to e'\,\mathcal{Q}\,X$ with $\mathcal{Q}$ either a J/ψ or a Υ meson

Godbole, Misra, Mukherjee, Rawoot, 2012/3; Godbole, Kaushik, Misra, Rawoot, 2015; Mukherjee, Rajesh, 2017; Rajesh, Kishore, Mukherjee, 2018; …



Using LO NRQCD the Sivers asymmetry is:

$$A^{\sin(\phi_S-\phi_T)} = \frac{|\boldsymbol{q}_T|}{M_p}\,\frac{f_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}{f_1^g(x,\boldsymbol{q}_T^2)}$$

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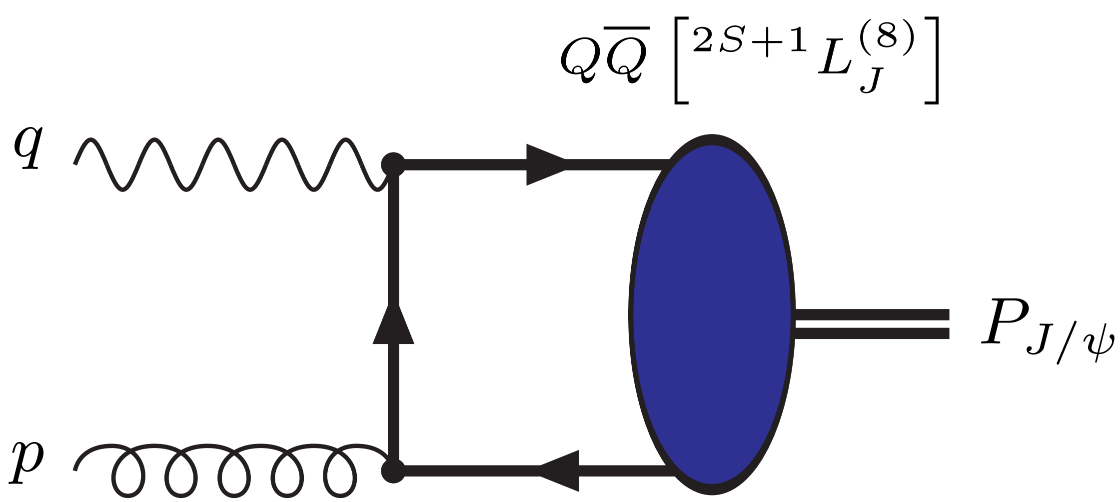

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$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S+\phi_T)}} = \frac{\boldsymbol{q}_T^2}{M_p^2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_1^g(x,\boldsymbol{q}_T^2)}$$

$$\frac{A^{\cos 2\phi_T}}{A^{\sin(\phi_S-3\phi_T)}} = -\frac{1}{2}\,\frac{h_1^{\perp\,g}(x,\boldsymbol{q}_T^2)}{h_{1T}^{\perp\,g}(x,\boldsymbol{q}_T^2)}$$

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CO NRQCD LDMEs cancel out in ratios of asymmetries at LO

Bacchetta, DB, Pisano, Taels, 2018

Quarkonium production in ep

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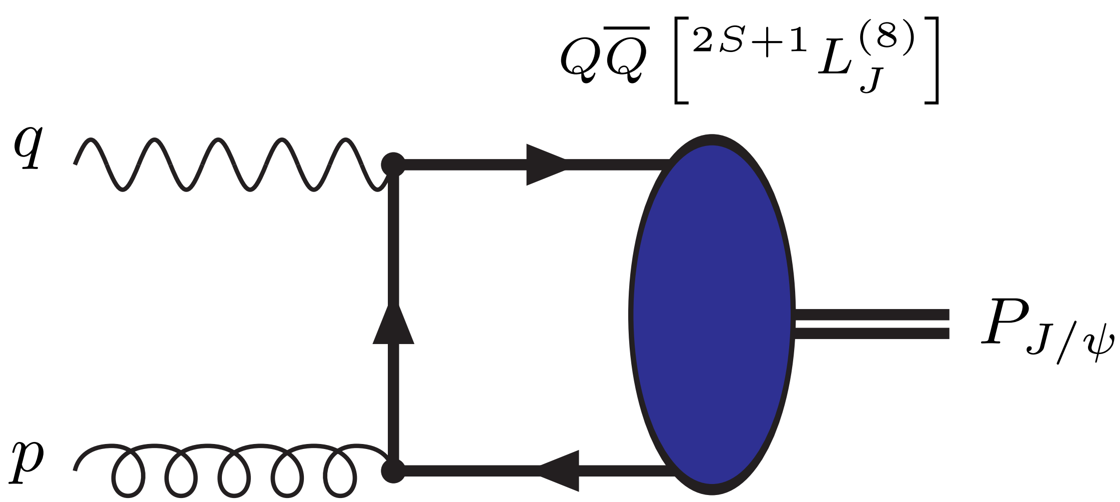

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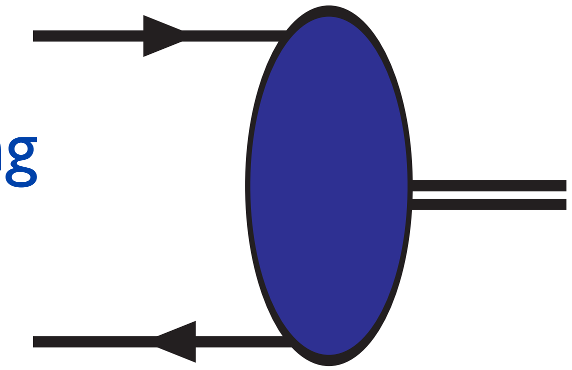
Higher order corrections and shape functions will complicate this simple picture

Shape functions

Effect of smearing



In reality the process of $Q\bar{Q} \rightarrow J/\psi$ involves some k_T-smearing

TMD factorization requires inclusion of shape function Δ

Echevarria, 2019; Fleming, Makris & Mehen, 2019

$$\mathcal{F}_{UU,T} = \frac{2\pi^2\alpha_s e_c^2}{M_\psi(M_\psi^2+Q^2)}\left[\langle 0|\mathcal{O}(^1S_0^{[8]})|0\rangle + 4\,\frac{(7M_\psi^4+2M_\psi^2Q^2+3Q^4)}{M_\psi^2(M_\psi^2+Q^2)^2}\langle 0|\mathcal{O}(^3P_0^{[8]})|0\rangle\right]$$
$$\times\, f_1^g(x,p_T^2)\Big|_{\boldsymbol{p}_T=\boldsymbol{q}_T}$$

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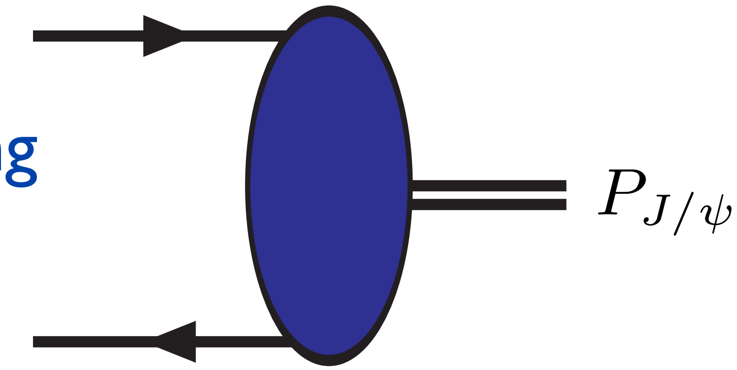

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LO NRQCD: $\Delta^{[n]}(\boldsymbol{k}_T^2; \mu^2) = \langle 0|\mathcal{O}(n)\,|0\rangle\, \delta^2(\boldsymbol{k}_T)$

↓

$$\int \mathrm{d}^2\boldsymbol{p}_T \int \mathrm{d}^2\boldsymbol{k}_T\, \delta^2(\boldsymbol{q}_T - \boldsymbol{p}_T - \boldsymbol{k}_T)\, f_1^g(x, \boldsymbol{p}_T^2; \mu^2)\, \Delta^{[n]}(\boldsymbol{k}_T^2, \mu^2)$$

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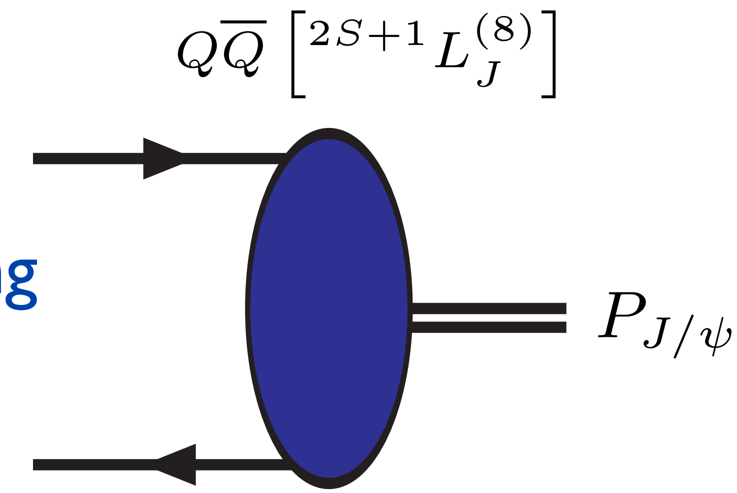

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Perturbative tail:

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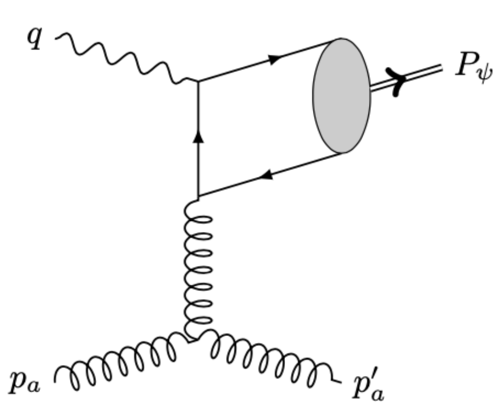
DB, D'Alesio, Murgia, Pisano, Taels, 2020



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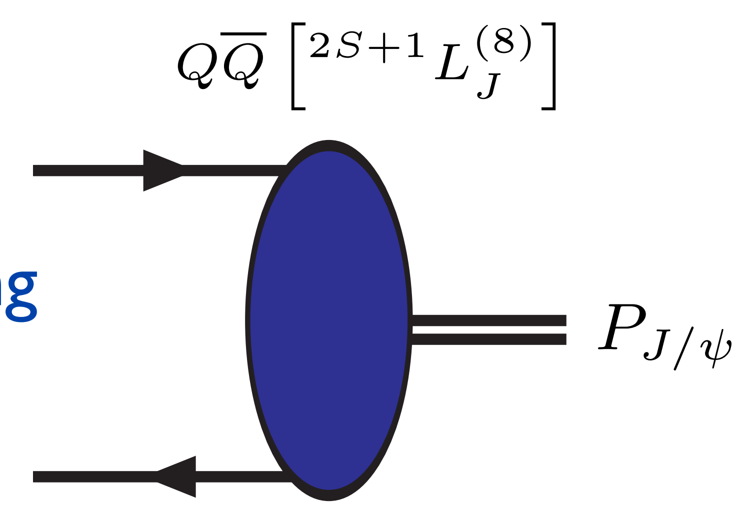

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$$\mu_H^2 \equiv \widetilde{Q}^2 = M_\psi^2 + Q^2$$

Perturbative tail:

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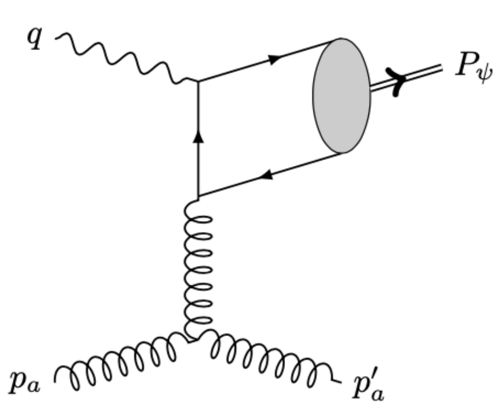
DB, D'Alesio, Murgia, Pisano, Taels, 2020

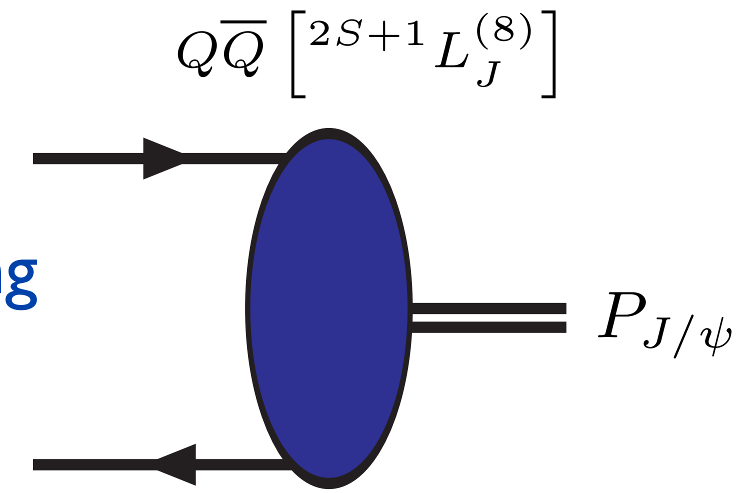
Correcting for unexpected non-analytic behavior:

$$\Delta^{[n]}(z, \boldsymbol{k}_T^2; \widetilde{Q}^2) = -\frac{\alpha_s}{2\pi^2 \boldsymbol{k}_T^2} C_A \left(1 + \log \frac{M_\psi^2}{M_\psi^2 + Q^2}\right) \langle \mathcal{O}[n]\rangle \delta(1-z).$$



DB, Bor, Maxia, Pisano, Yuan, 2023

Effect of smearing



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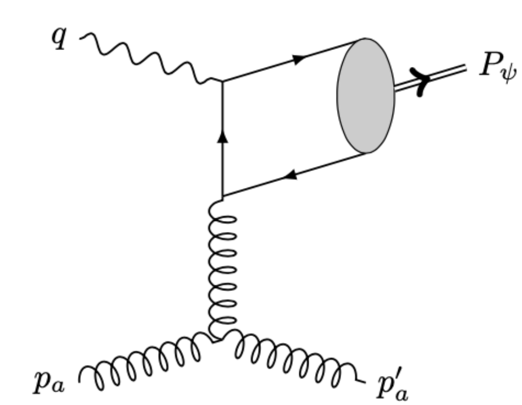
DB, D'Alesio, Murgia, Pisano, Taels, 2020

Correcting for unexpected non-analytic behavior:

$$\Delta^{[n]}(z,\boldsymbol{k}_T^2;\widetilde{Q}^2) = -\frac{\alpha_s}{2\pi^2\boldsymbol{k}_T^2}C_A\left(1+\log\frac{M_\psi^2}{M_\psi^2+Q^2}\right)\langle\mathcal{O}[n]\rangle\delta(1-z).$$

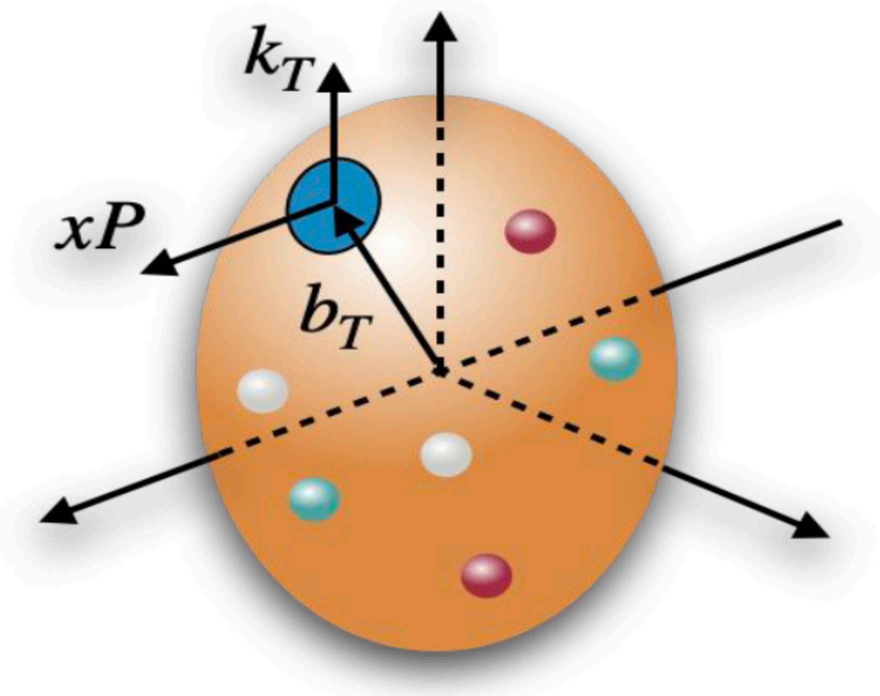


DB, Bor, Maxia, Pisano, Yuan, 2023

Process dependence!

Gluon GTMDs
&
quarkonia

3D momentum and spatial distributions



TMDs - 3D momentum structure (x & k_T)
GPDs - 3D spatial structure (ξ & t or z & b_T)

GTMDs - combined 5D (or 6D) structure

This talk only zero skewness (ξ=0) and mostly small x

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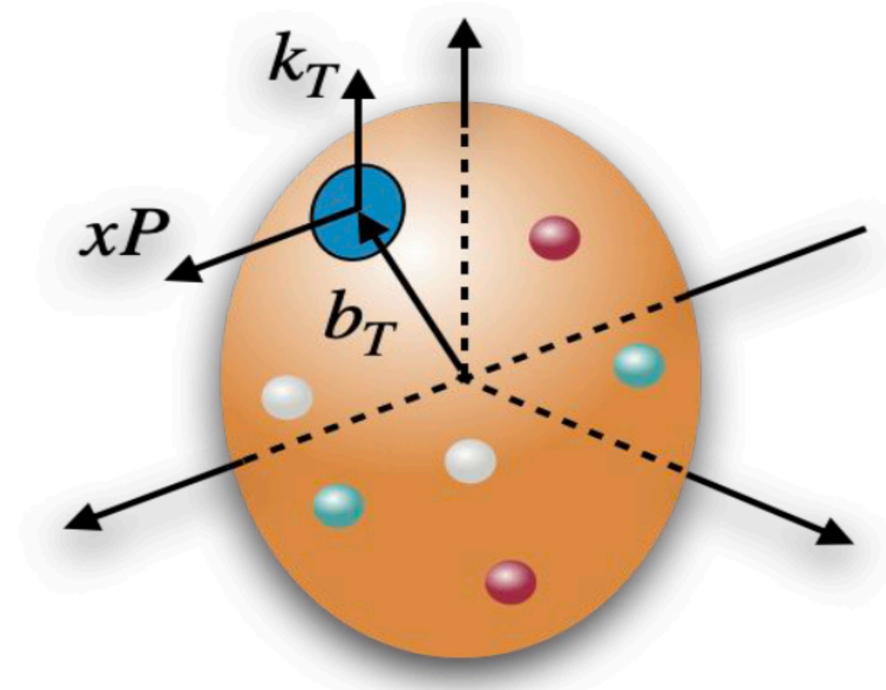
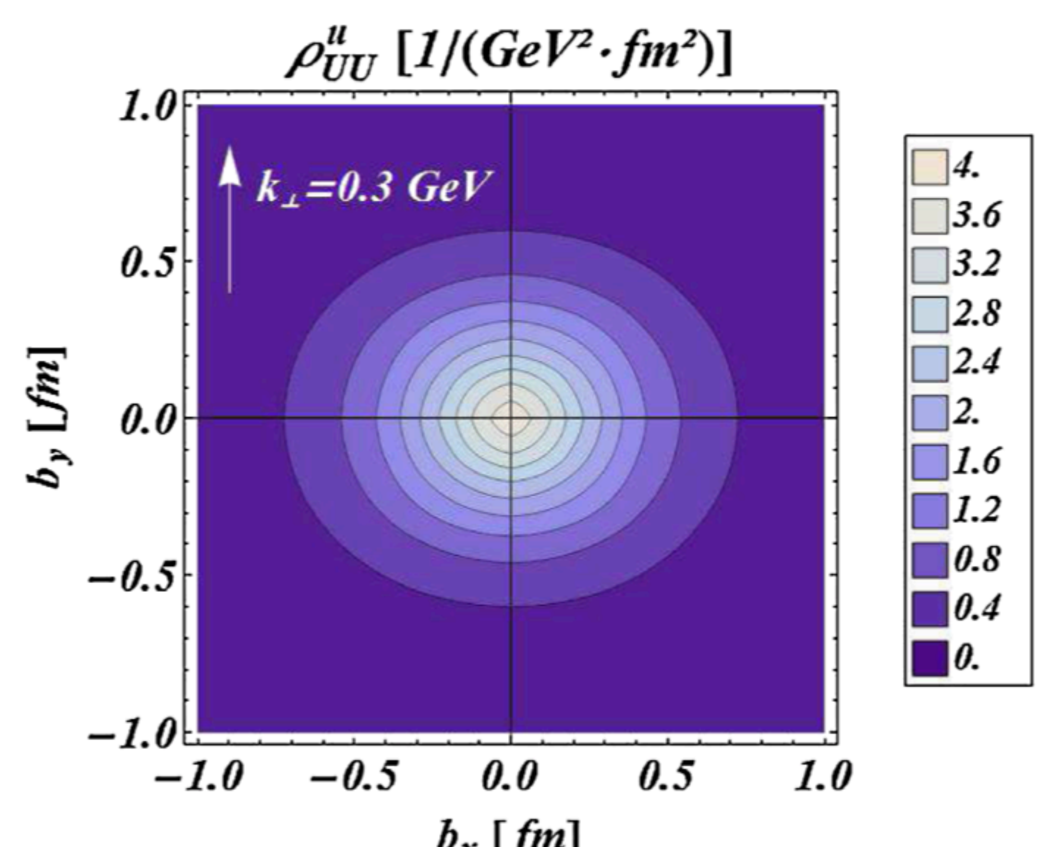
Teaches us about orbital angular momentum

Lorce, Pasquini, 2011; Hatta, 2011; …





GTMDs - 5D parton distributions

Off-forward distributions, like GPDs, give access to the transverse spatial distributions; here the proton stays intact but gets a momentum kick

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GTMDs can be seen as:

- off-forward TMDs
- transverse momentum dependent GPDs
- Fourier transforms of Wigner distributions

$$G(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) \stackrel{FT}{\longleftrightarrow} W(x, \boldsymbol{k}_T, \boldsymbol{b}_T)$$

Meißner, Metz, Schlegel, 2009 Ji, 2003; Belitsky, Ji & Yuan, 2004

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GTMDs combine all properties of TMDs and GPDs, such as process dependence & nontrivial impact parameter dependence

Gluon GTMDs for unpolarized protons

For unpolarized protons there are 2 (real valued) gluon TMDs:

$$\Gamma_U^{ij}(x, \boldsymbol{k}_T) \;=\; \frac{x}{2}\left[-\,g_T^{ij}\,f_1(x, \boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\,h_1^{\perp}(x, \boldsymbol{k}_T^2)\right]$$

$$a_T^{ij} \equiv a_T^i a_T^j - \frac{1}{2} a_T^2 \delta_T^{ij}$$

Mulders, Rodrigues, 2001

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For GTMDs one has one more vector so more anisotropic terms can arise

For unpolarized protons there are 4 (complex valued) gluon GTMDs

$$G^{[U,U']\,ij}(x, \boldsymbol{k}_T, \boldsymbol{\Delta}_T) = x\left(\delta_T^{ij}\,\mathcal{F}_1 + \frac{k_T^{ij}}{M^2}\,\mathcal{F}_2 + \frac{\Delta_T^{ij}}{M^2}\,\mathcal{F}_3 + \frac{k_T^{[i}\Delta_T^{j]}}{M^2}\,\mathcal{F}_4\right)$$

DB, van Daal, Mulders, Petreska, 2018

Lorce, Pasquini, 2013; More, Mukherjee, Nair, 2018

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DB, van Daal, Mulders, Petreska, 2018

Lorce, Pasquini, 2013; More, Mukherjee, Nair, 2018

Like for TMDs gauge links [U,U'] will matter for GTMDs: [+,+] or [+,−]

Dipole gluon GTMD

In the x→0 the [+,−] gluon GTMD becomes a correlator of a single Wilson loop:

$$G^{[+,-]\,ij}(\boldsymbol{k},\boldsymbol{\Delta}) = \frac{2N_c}{\alpha_s}\left[\frac{1}{2}\left(\boldsymbol{k}^2 - \frac{\boldsymbol{\Delta}^2}{4}\right)\delta_T^{ij} + k_T^{ij} - \frac{\Delta_T^{ij}}{4} - \frac{k_T^{[i}\Delta_T^{j]}}{2}\right] G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta})$$

$$G^{[\square]}(\boldsymbol{k},\boldsymbol{\Delta}) \equiv \int \frac{d^2\boldsymbol{x}\, d^2\boldsymbol{y}}{(2\pi)^4}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})+i\boldsymbol{\Delta}\cdot\frac{\boldsymbol{x}+\boldsymbol{y}}{2}}\, \frac{\langle p'|\, S^{[\square]}(\boldsymbol{x},\boldsymbol{y})\,|p\rangle\big|_{\mathrm{LF}}}{\langle P|P\rangle},$$

$$S^{[\square]}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) \equiv \frac{1}{N_c}\mathrm{Tr}\left[U^{[\square]}(\boldsymbol{y}_\perp,\boldsymbol{x}_\perp)\right] \qquad U^{[\square]}(y,x) = U^{[+]}_{[x,y]}U^{[-]}_{[y,x]}$$

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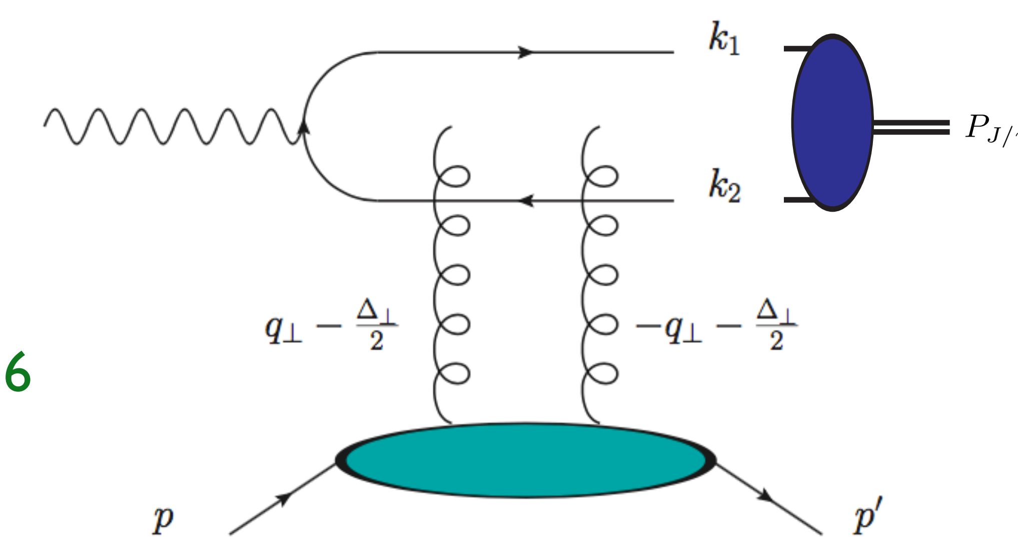
DB, van Daal, Mulders, Petreska, 2018

It can be probed in exclusive coherent diffractive dijet or J/ψ production

Altinoluk, Armesto, Beuf, Rezaeian, 2016; Hatta, Xiao, Yuan, 2016

Kowalski, Teaney, 2003; Kowalski, Motyka, Watt, 2006; …



$$S^{[\square]}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) \to 1 - S^{[\square]}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) \qquad G^{[\square]} \to \mathcal{F}^{[\square]}$$

Diffractive J/ψ production

The transverse momentum dependence of the GTMD is probed indirectly

$$\mathcal{A}_{T,L} = \frac{\pi i}{2N_c} \int_0^1 dz \int d^2 r_\perp \left(\Psi_V^* \Psi_\gamma\right)_{T,L} (r_\perp, z) \int d^2 q_\perp J_0 \left(|q_\perp + \delta_\perp| r_\perp\right) \mathcal{F}_0^{[\Box]}(x, q_\perp, \Delta_\perp)$$

$$\delta_\perp = (\frac{1}{2} - z)\Delta_\perp$$

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$$J_0\left(|q_\perp + \delta_\perp| r_\perp\right) \approx 1 - \frac{(q_\perp + \delta_\perp)^2 r_\perp^2}{4}$$

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MV-like model

We consider the MV-like model:

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2}\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\, e^{-\epsilon_r r_\perp^2} \left[1 - \exp\left(-\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2\Lambda^2} + e\right]\right)\right]$$

Similar to Hagiwara, Hatta, Pasechnik, Tasevsky & Teryaev, 2017; Salazar, Schenke, 2019

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χ sets the normalization of Q_s and is x dependent (of GBW form)

$$\chi(x) = \bar{\chi}\left(\frac{x_0}{x}\right)^\lambda \qquad x_0 = 3\times 10^{-4} \quad \lambda = 0.29$$

Q_s is proportional to the proton (Gaussian) or nuclear (Woods-Saxon) profile

For details see DB, Setyadi, 2023

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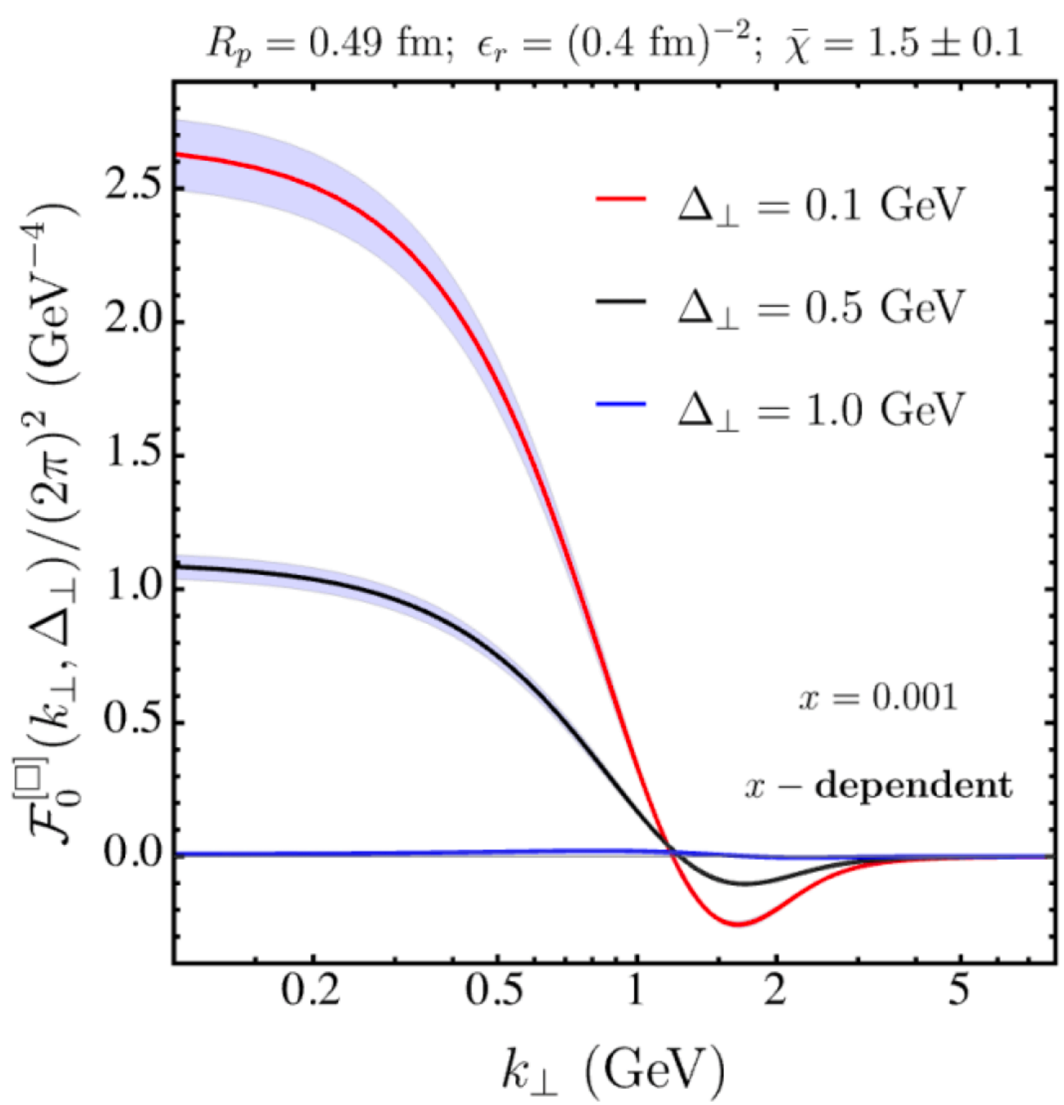
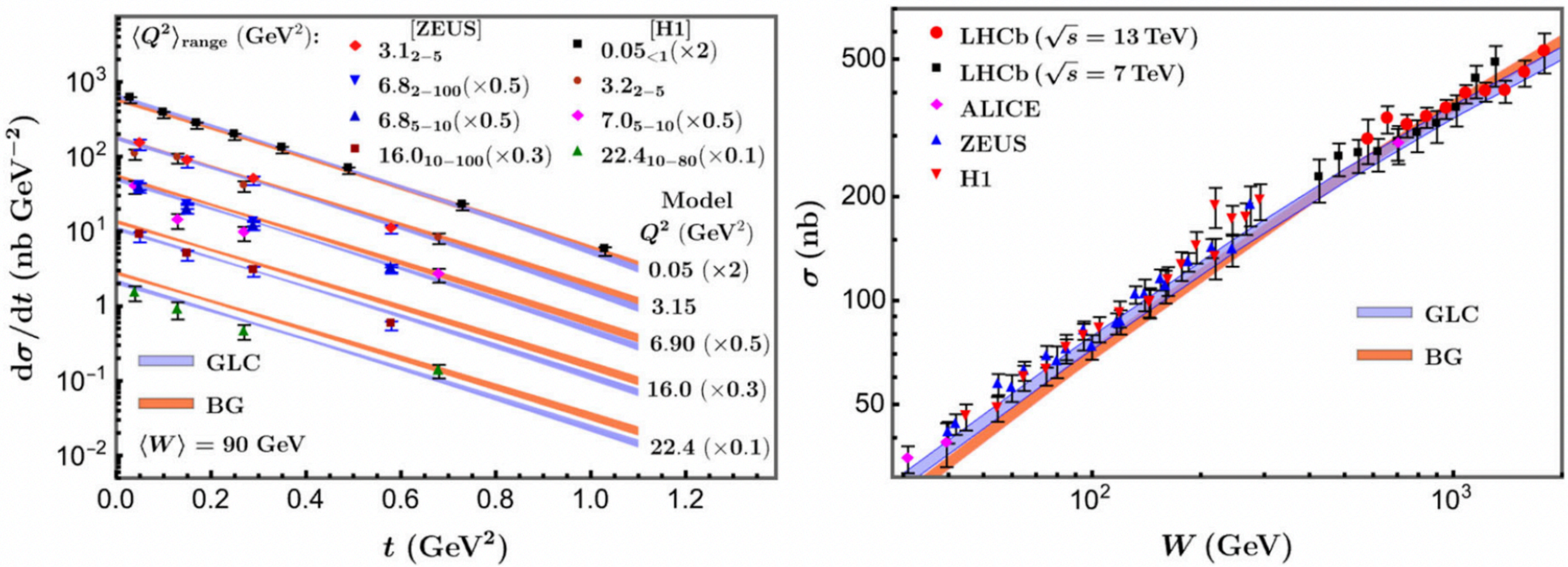
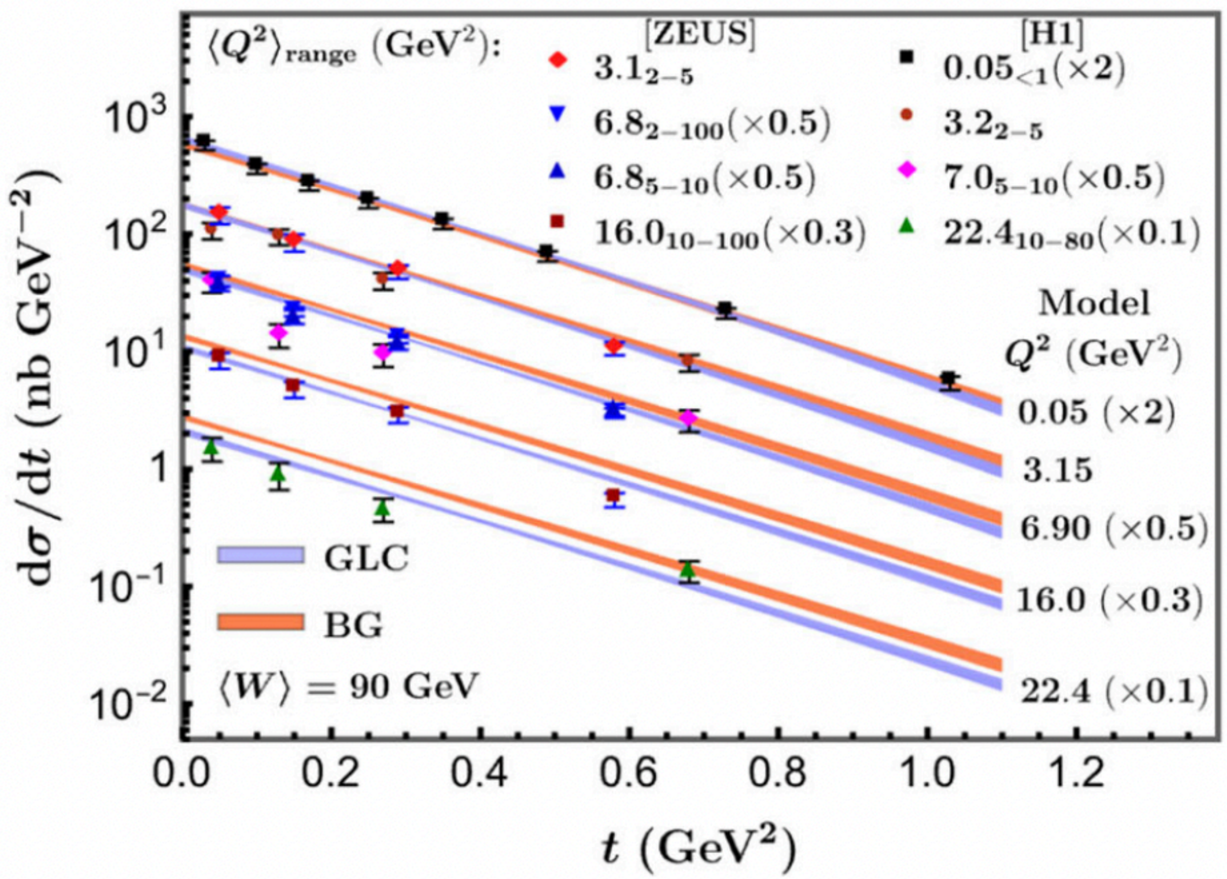
Dominant contribution from:

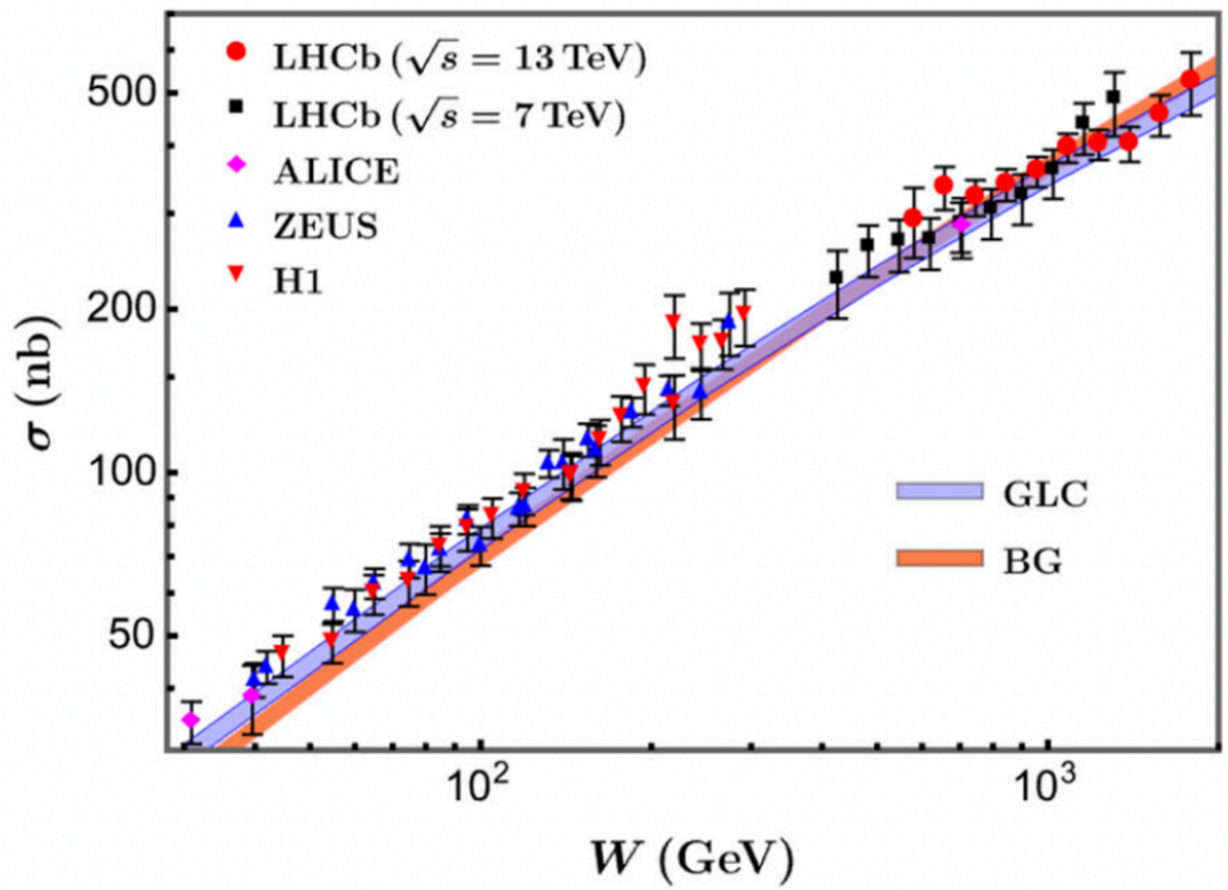
$$\Delta_\perp \ll K_\perp \text{ or } M_V$$





Diffractive J/ψ production data of HERA (H1 & ZEUS) can be reasonably well described





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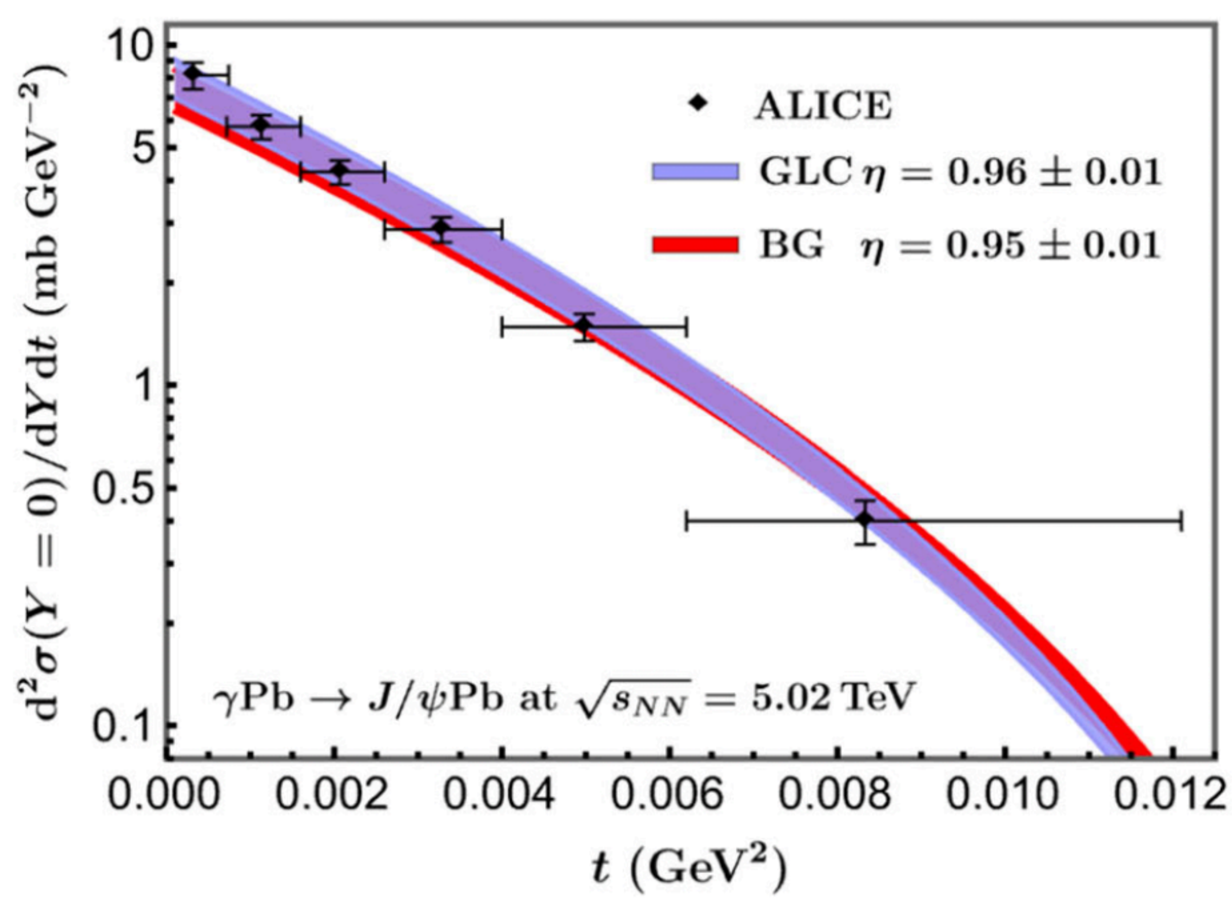


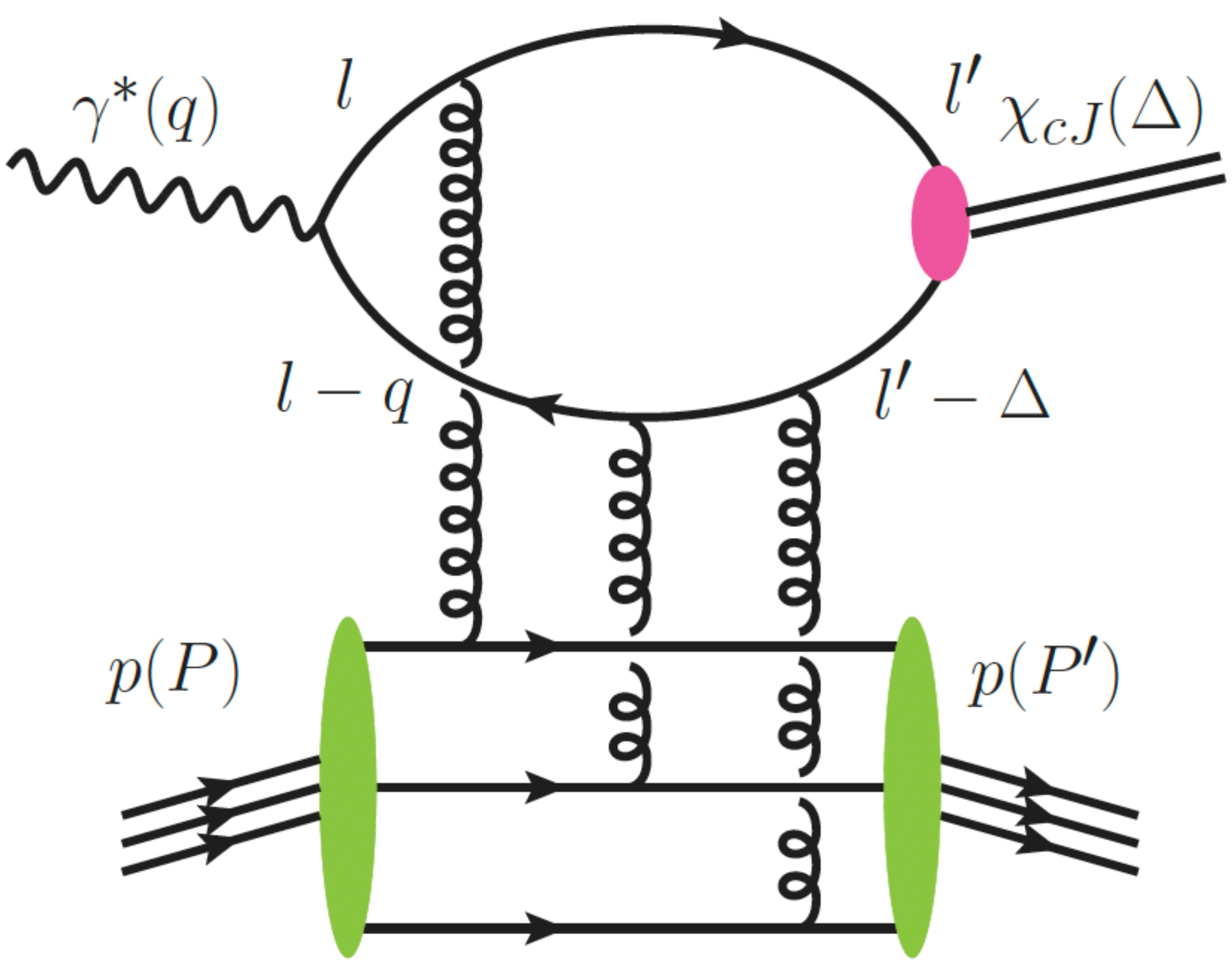
Description of ALICE UPC data qualitatively fine with an A dependence somewhat smaller than $A^{1/3}$, but this is dependent on the profile functions

DB, Setyadi, 2023

Exclusive χ_c production at EIC

In a similar way the odderon can be probed

$$\mathcal{O}(\boldsymbol{x}, \boldsymbol{y}) \equiv \frac{1}{2iN_c} \mathrm{Tr}\left(U^{[\Box]} - U^{[\Box]\dagger}\right)$$



C-even final state requires
C-odd t-channel exchange

Constructive interference
with photon t-channel exchange
for |t| ~ 1 GeV^2

Benić, Dumitru, Kaushik, Motyka, Stebel, 2024

Conclusions

Conclusions

- Inclusive quarkonium, quarkonium plus photon, and di-quarkonium production processes in both pp and ep collisions are well suited to study gluon TMDs

- Predictions are still quite uncertain because of many factors: unknown nonperturbative Sudakov factors, scale uncertainty, poorly known LDMEs or more generally unknown shape functions, process dependence, …

- The linear polarization of gluons inside unpolarized hadrons is expected to lead to sizable cos2ϕ & cos4ϕ asymmetries

- Gluon GTMDs can be studied using quarkonium production in coherent diffractive processes in ep collisions, but also in UPC of heavy ions

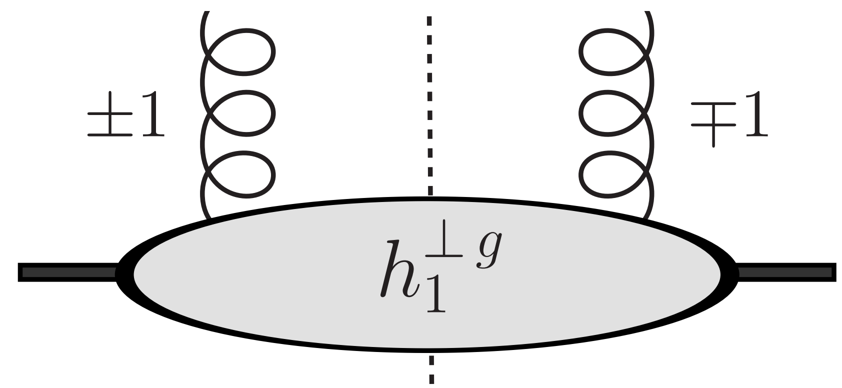
Back-up slides

Gluon polarization inside unpolarized protons

$$\Gamma_U^{\mu\nu}(x,\boldsymbol{p}_T) = \frac{x}{2}\left\{ -g_T^{\mu\nu}\, f_1^g(x,\boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu}\frac{\boldsymbol{p}_T^2}{2M_p^2}\right) h_1^{\perp\, g}(x,\boldsymbol{p}_T^2)\right\}$$

Linearly polarized gluons can exist in **unpolarized** hadrons

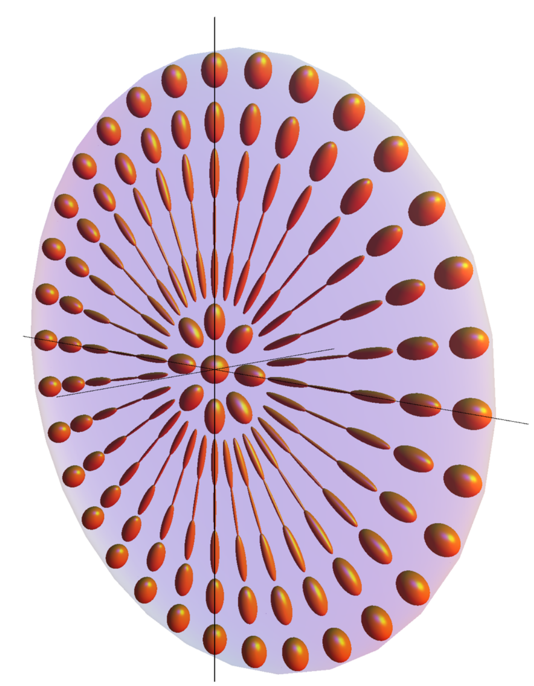
[Mulders, Rodrigues, 2001]



an interference between ±1 helicity gluon states

It requires nonzero transverse momentum

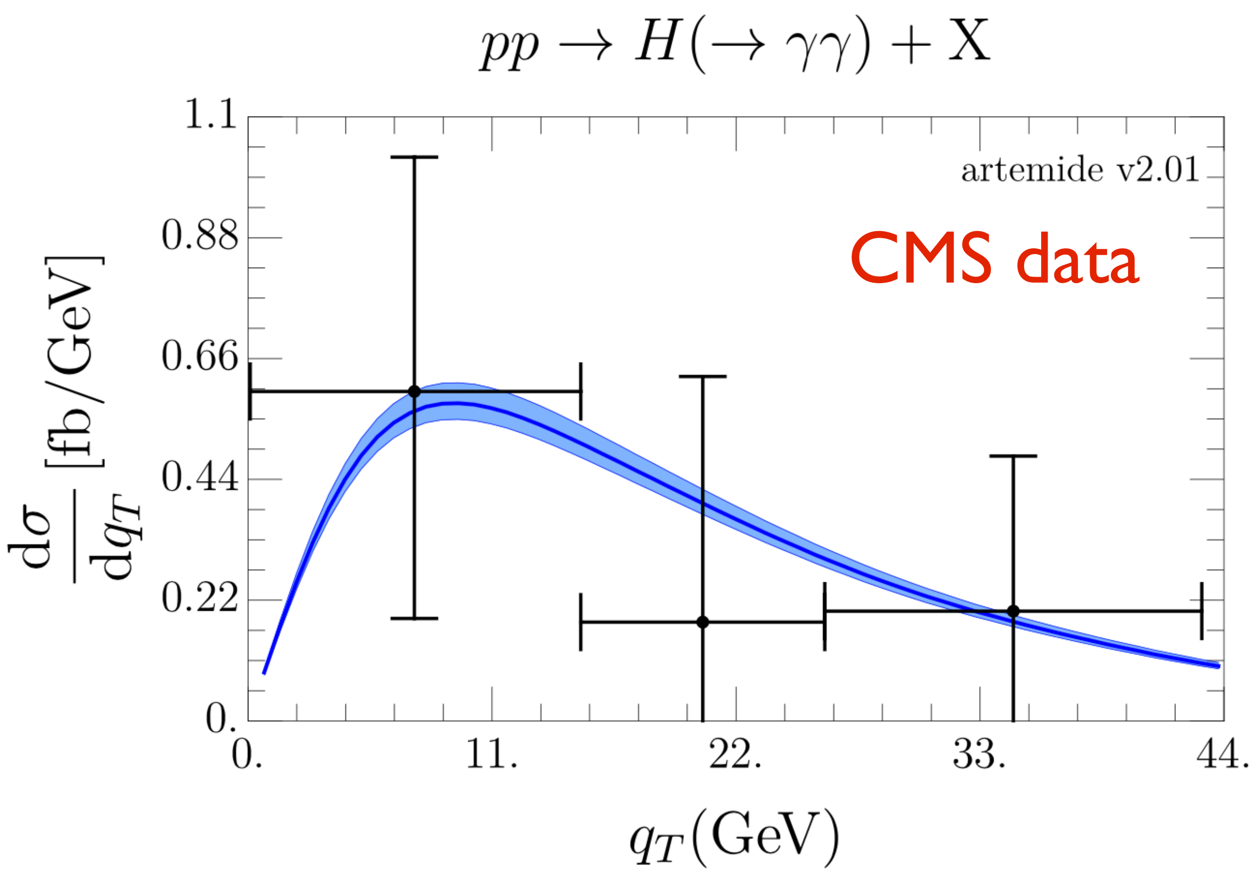
For $h_1^{\perp\, g} > 0$ gluons are linearly polarized along k_T, with a $cos\ 2\phi$ distribution around it, where $\phi = \angle(k_T, \varepsilon_T)$



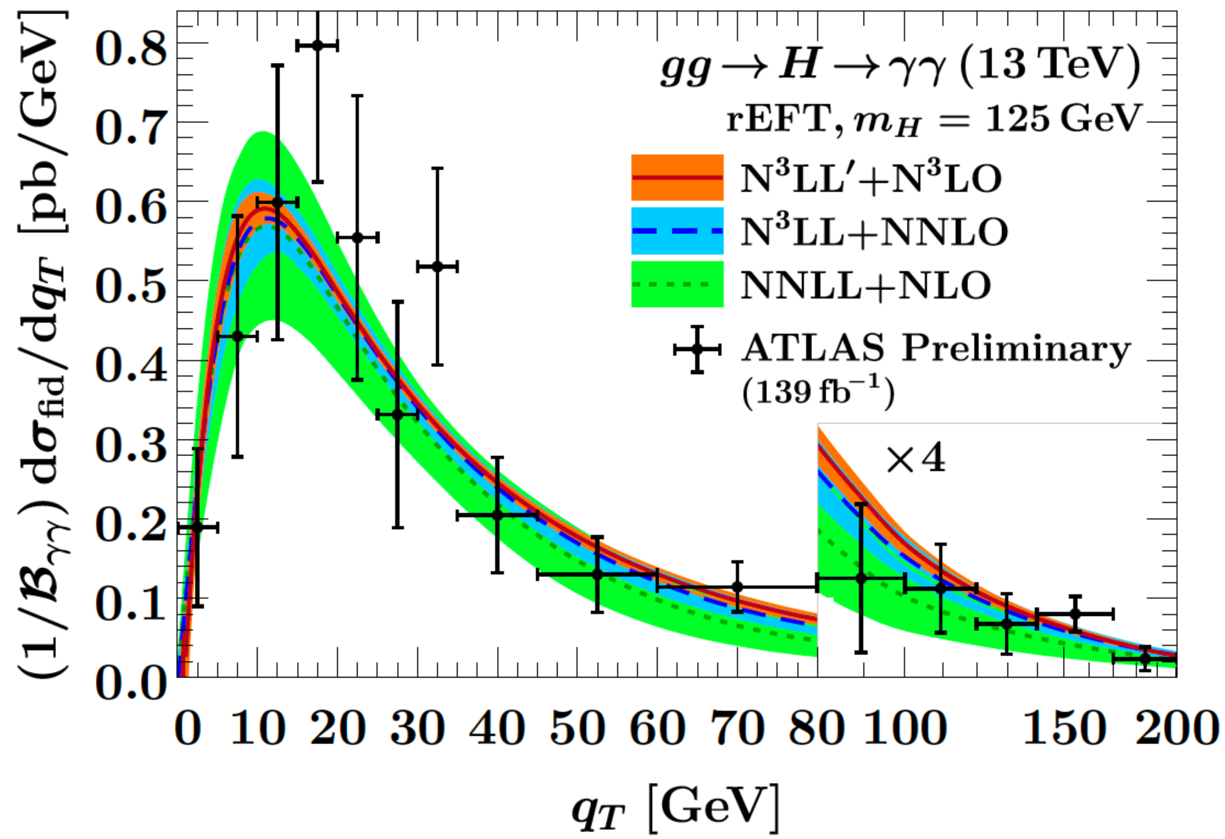
The linearly polarized gluons can affect the transverse momentum spectrum

Gluon TMDs in Higgs production

Higgs p_T distribution



Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, 2019

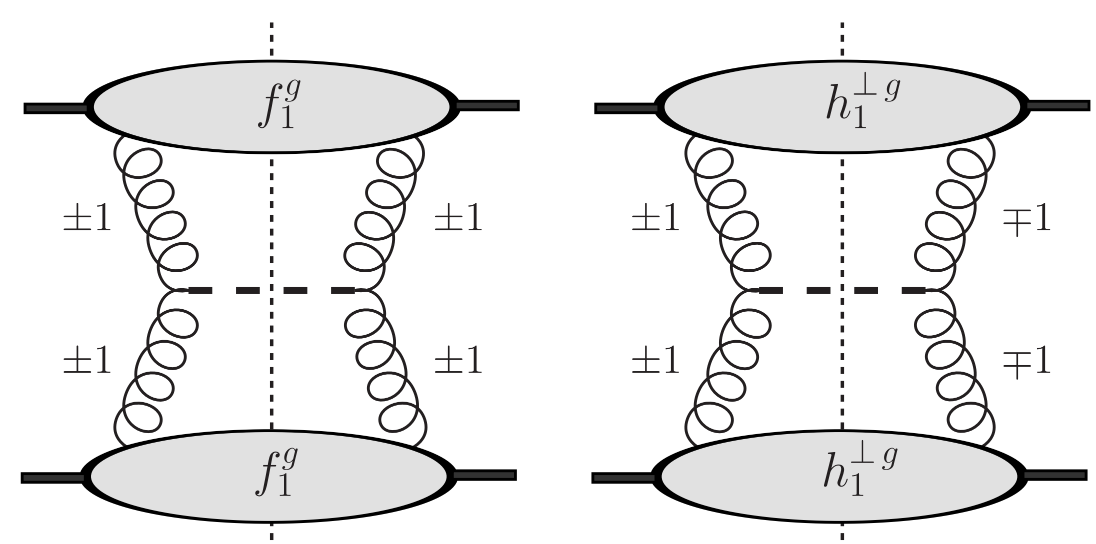


Billis, Dehnadi, Ebert, Michel, Tackmann, 2021

Probes mostly the unpolarized gluon distribution, perturbative description quite good (5-10% uncertainty at "low" p_T < 10 GeV)

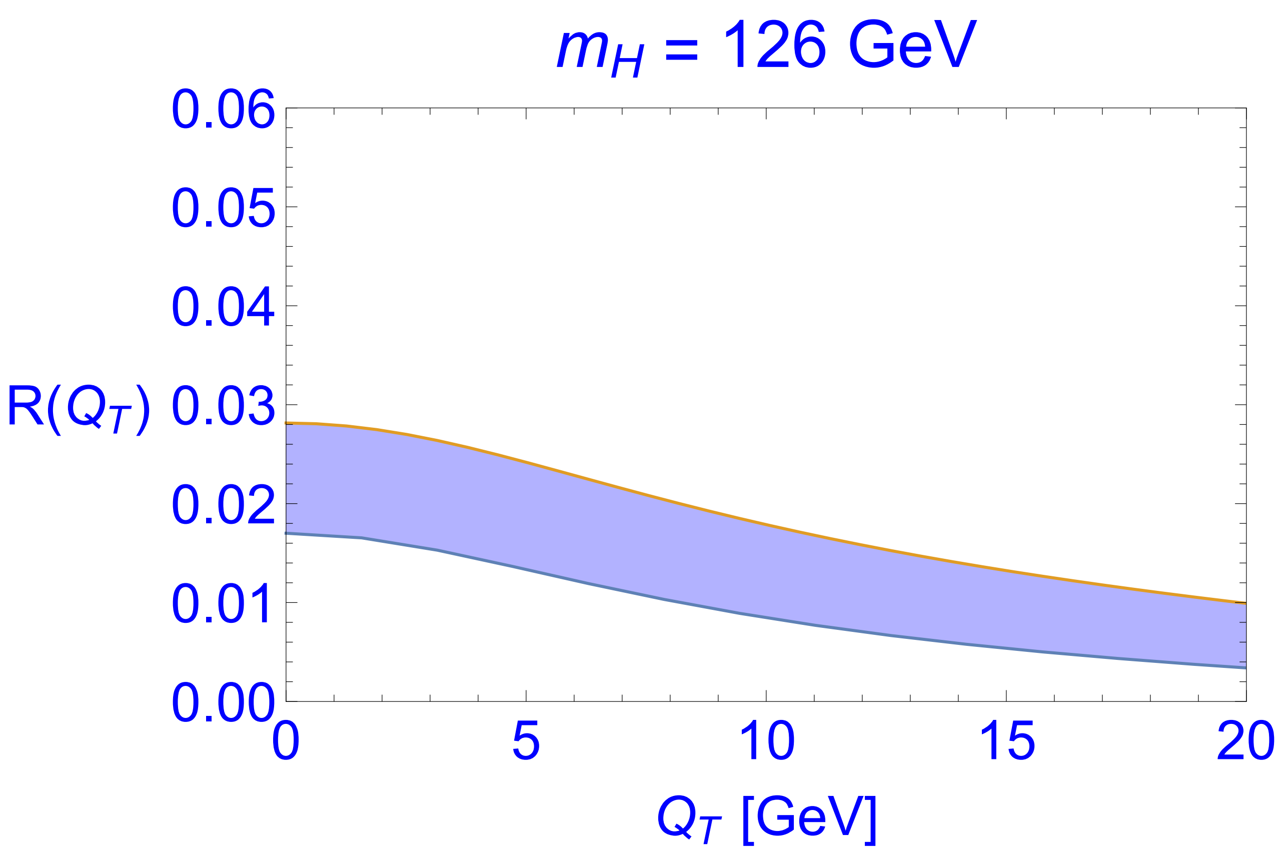
There is a contribution from linearly polarized gluons in Higgs production

Catani, Grazzini, 2010; Sun, Xiao, Yuan, 2011;
DB, Den Dunnen, Pisano, Schlegel, Vogelsang, 2012

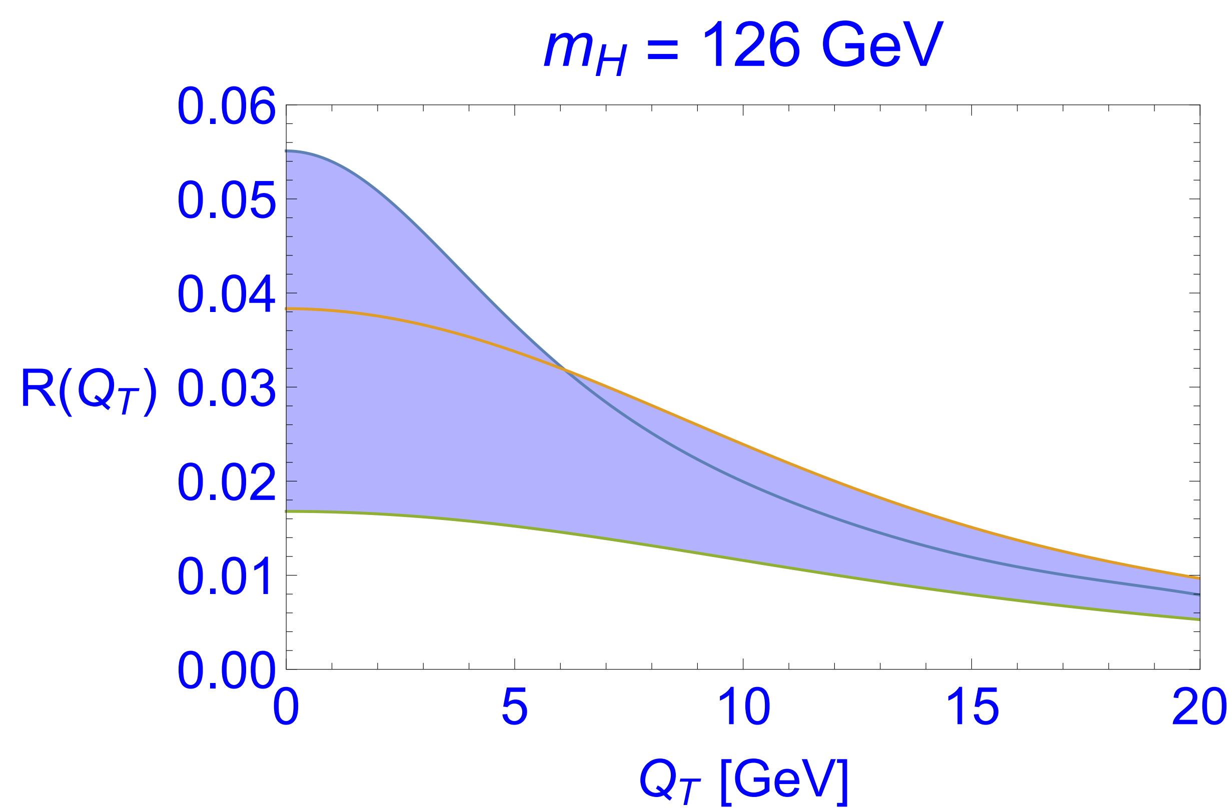


Gluon TMDs in Higgs production

Contribution from the linearly polarized gluon TMD: $\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H h_1^{\perp g} h_1^{\perp g}]}{\mathcal{C}[f_1^g f_1^g]}$



DB, Den Dunnen, 2014



Echevarria, Kasemets, Mulders, Pisano, 2015

Left: variation of the nonperturbative input and of the large Q_T behavior

Right: variation of the nonperturbative input and the renormalization scale

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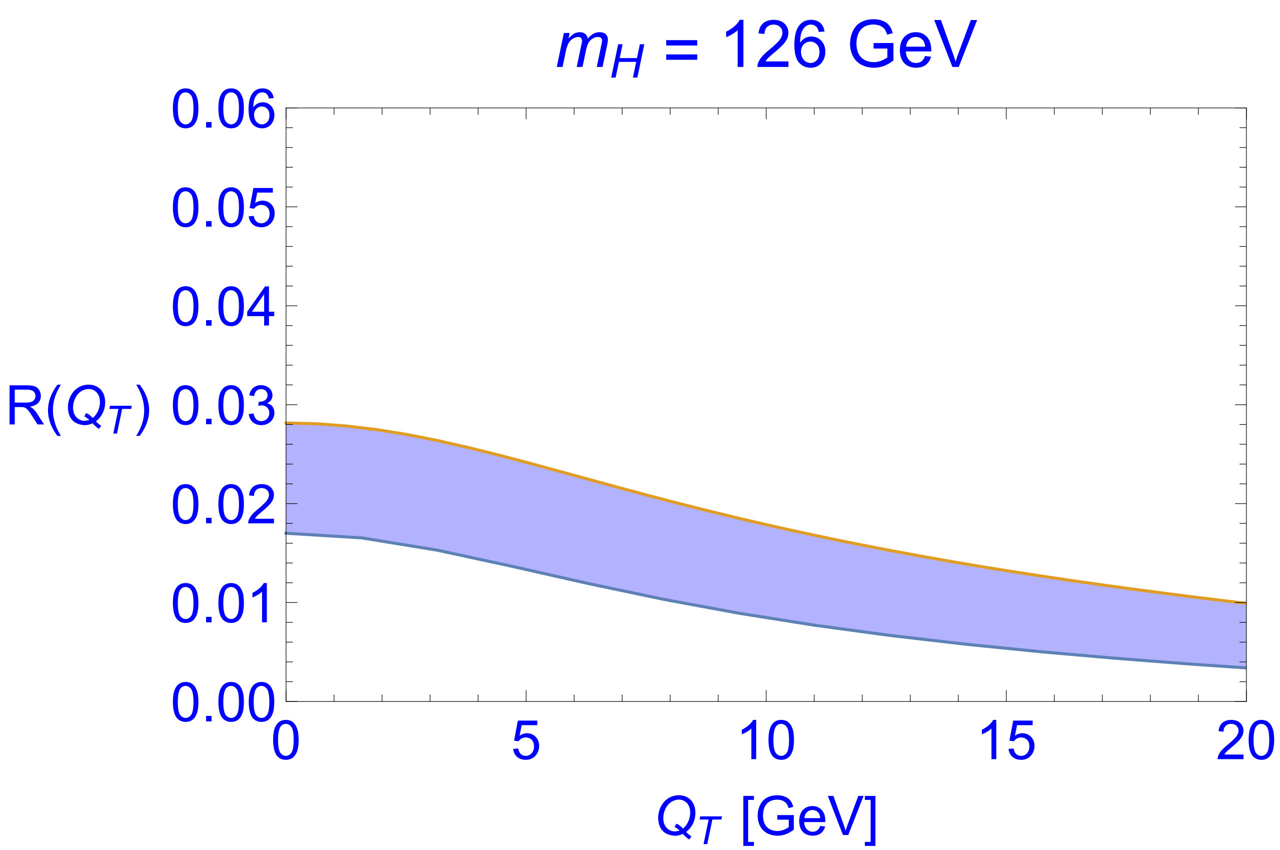

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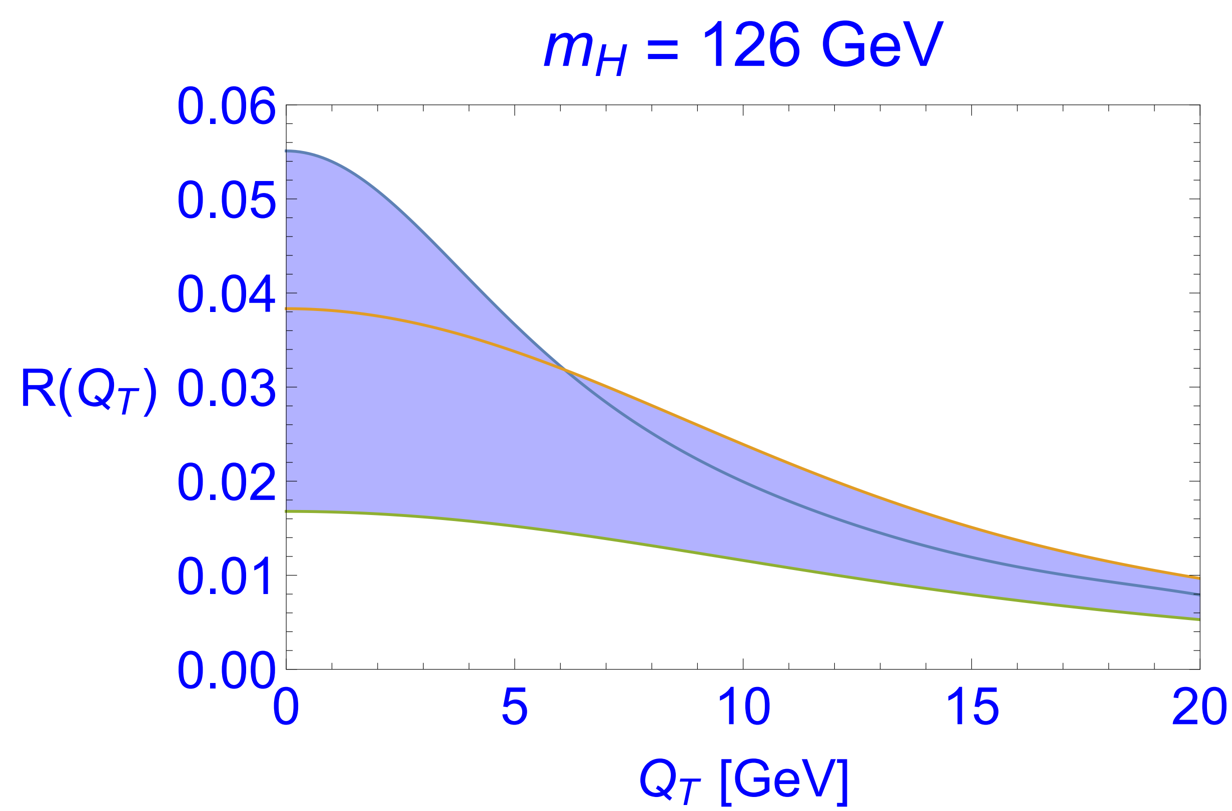

Echevarria, Kasemets, Mulders, Pisano, 2015

Left: variation of the nonperturbative input and of the large Q_T behavior

Right: variation of the nonperturbative input and the renormalization scale

- effect of linear gluon polarization in Higgs production on the order of 2-5%
- extraction of $h_1^{\perp g}$ from Higgs production may be too challenging