

# Transversely polarized $\Lambda$ production within a jet in unpolarized proton-proton collisions

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TRANSVERSITY 2024

MARCO ZACCHEDDU – JEFFERSON LAB

IN COLLABORATION WITH: U. D'ALELIO, L. GAMBERG & F. MURGIA

# Outline

- Motivations
- Hadron in jet in unpolarized proton-proton collisions  $pp \rightarrow h(\text{jet})X$
- Polarizing FF from Belle  $e^+e^-$
- Predictions and comparison against STAR data

# Motivations

Lambda Transverse Polarization: longstanding problem!

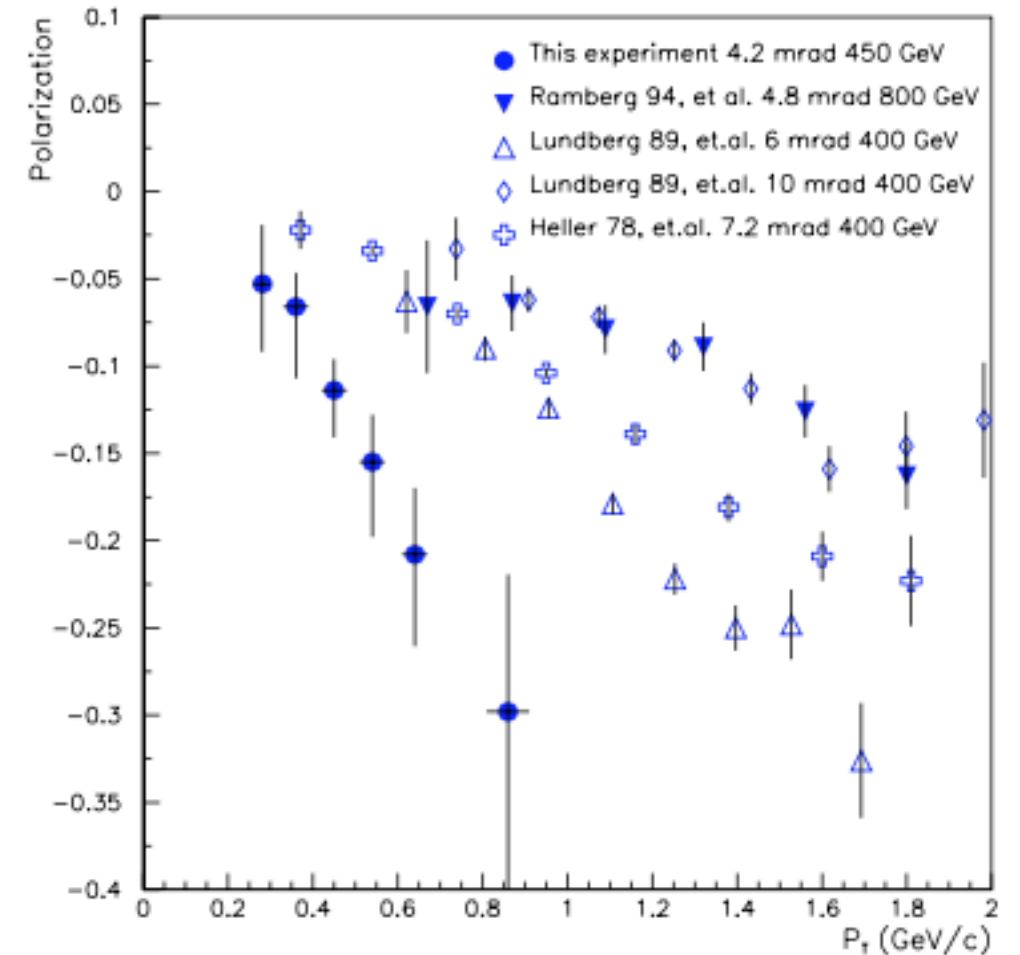
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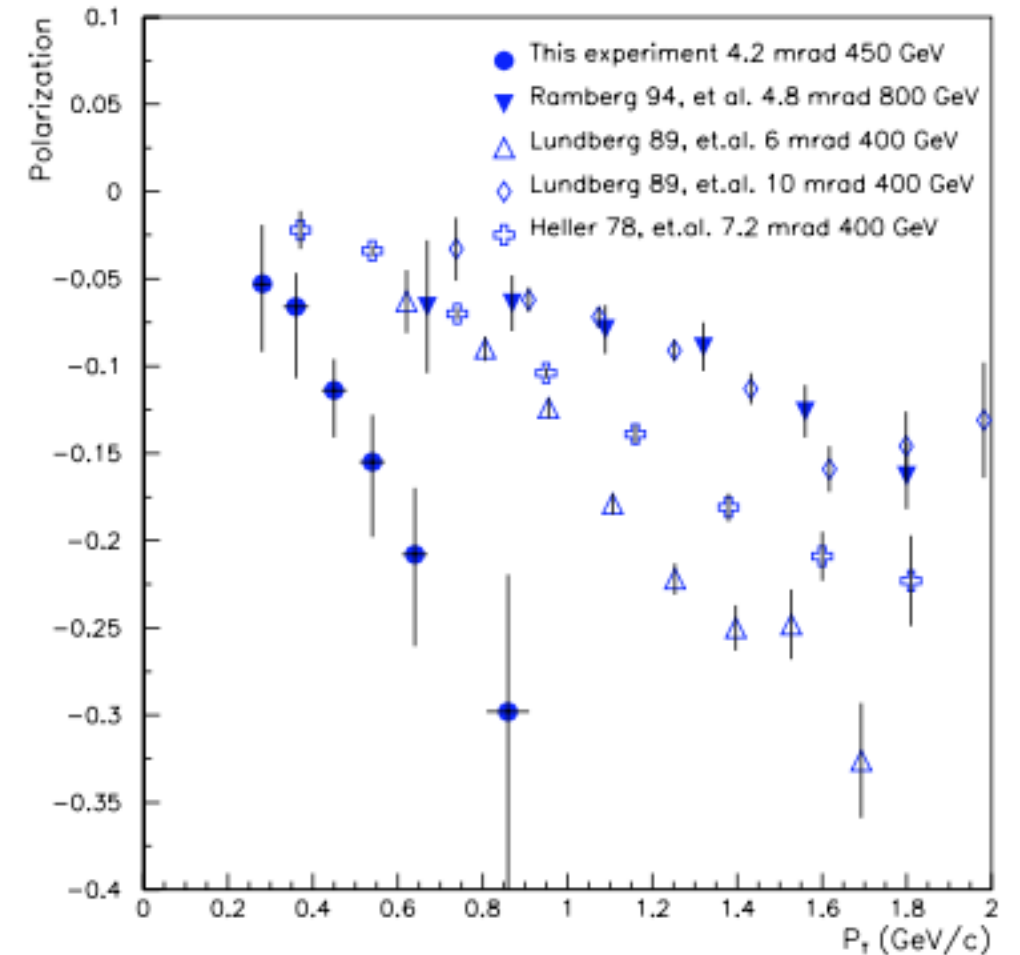
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$$P_T \simeq 1 - 2 \%$$

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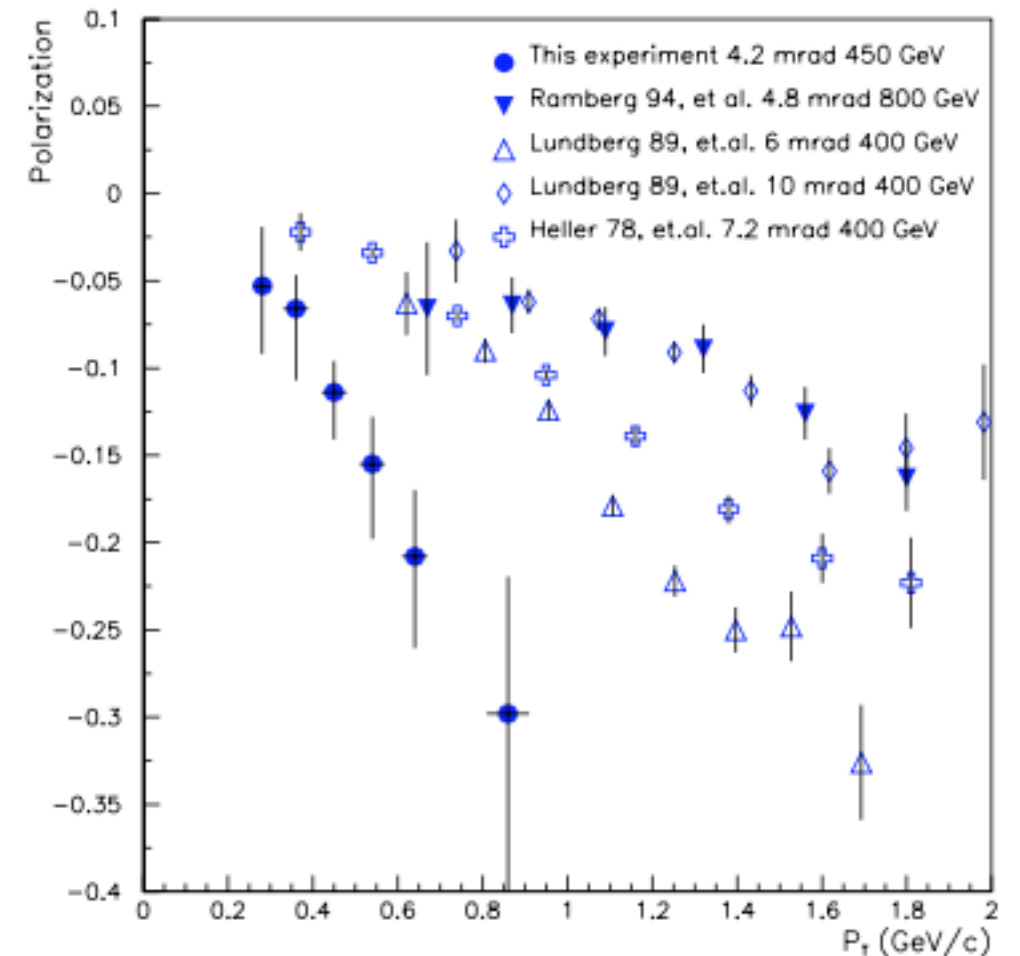
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Polarizing TMD-FF introduced

[P.J. Mulders and R.D. Tangerman, Nucl. Phys. B461, 197 (1996)]

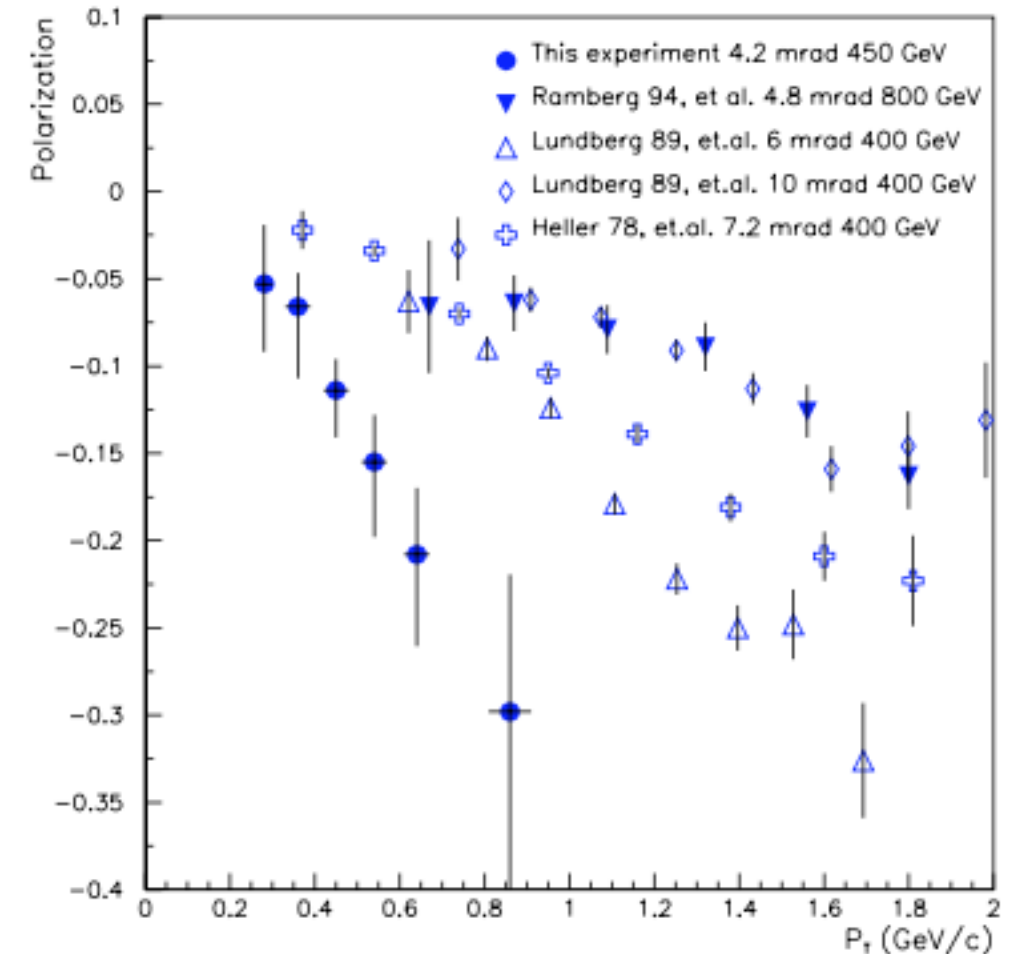
Studied phenomenologically in a simplified TMD approach

[M. Anselmino, D. Boer, U. D'Alesio, and F. Murgia. Phys. Rev. D 63. 054029 (2001)]

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# Motivations

Observation of Transverse  $\Lambda/\bar{\Lambda}$  Hyperon Polarization in  $e^+e^-$  Annihilation at Belle

- 2 data set @  $\sqrt{s} = 10.58$  GeV

[Y. Guan et al., Phys. Rev. Lett. 122. 042001 (2019)]

Double hadron production:

- $e^+e^- \rightarrow \Lambda\pi/K + X$

Single-inclusive hadron production:  $\rightarrow$  Factorization theorems

- $e^+e^- \rightarrow \Lambda(\text{jet}) + X$



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*Fixed scale:*

- **D'Alesio, Murgia, MZ; Phys.Rev.D 102 (2020) 5, 054001**
- Callos, Kang, Terry; Phys.Rev.D 102 (2020) 9, 096007
- Chen, Liang, Pan, Song, Wei; Phys.Lett.B 816 (2021) 136217  $\rightarrow$  *SU(2) isospin symmetry issue*

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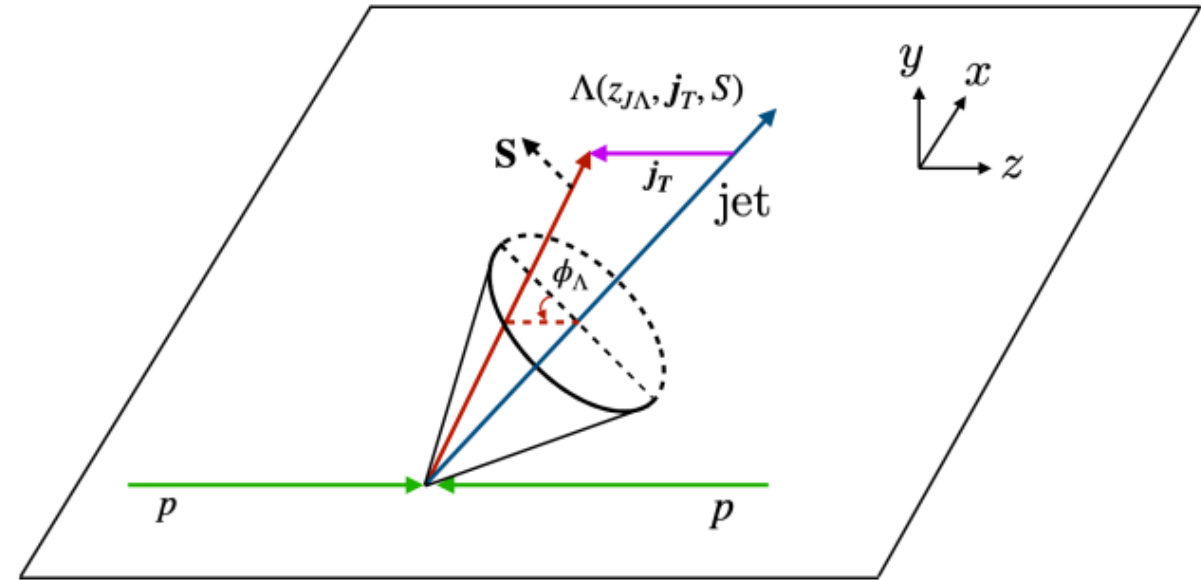
*CSS scheme:*

- **D'Alesio, Gamberg, Murgia, MZ; JHEP 12 (2022) , PRD 108 (2023),  $\rightarrow$  SU(2) isospin issue and predictions for SiDIS**
- Li, Wang, Yang, Lu '21
- Gamberg, Kang, Shao, Terry, Zhao ,21

# Unpolarized proton-proton collisions: $pp \rightarrow \Lambda (\text{jet}) X$

- Complementary to SiDIS and  $e^+e^-$
- 2 scales:
  - $p_{jT}$  T.M. of the jet
  - $p_{\perp\Lambda}$  Lambda T.M. w.r.t. the jet
- Transverse momentum effects considered only for FFs
- Different studies for hadron in jet production:
  - Collins effect in pp collisions [Yuan '08, D'Alesio, Murgia, Pisano '11,'17; Kang et al.'17]
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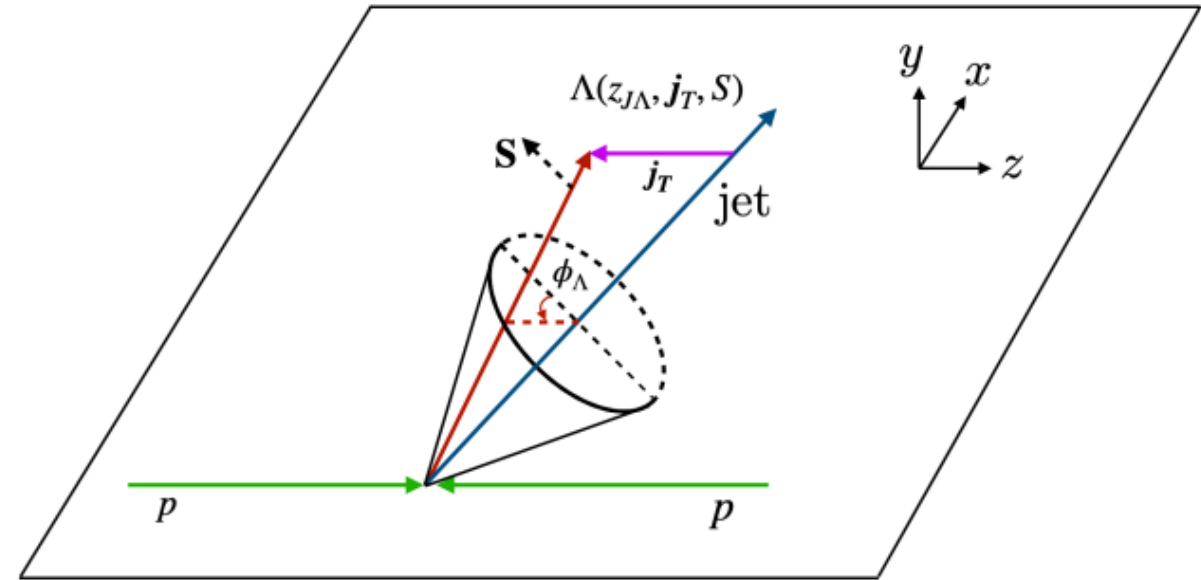
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**NEW DATA from STAR for  $\Lambda$  transverse polarization**  
**T. Gao @ SPIN2023: [arxiv/2402.01168](https://arxiv.org/abs/2402.01168)**

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$$P_T^\Lambda(p_j, \xi, p_{\perp\Lambda}) = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\Delta\sigma}{d\sigma_{\text{unp}}}$$

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- Factorization scale at LO:  $\mu = p_{jT}$
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Scaling variables

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Long. mom. fraction

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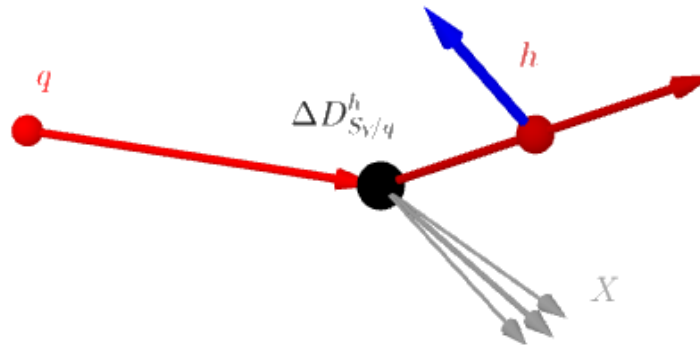
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Polarizing FF

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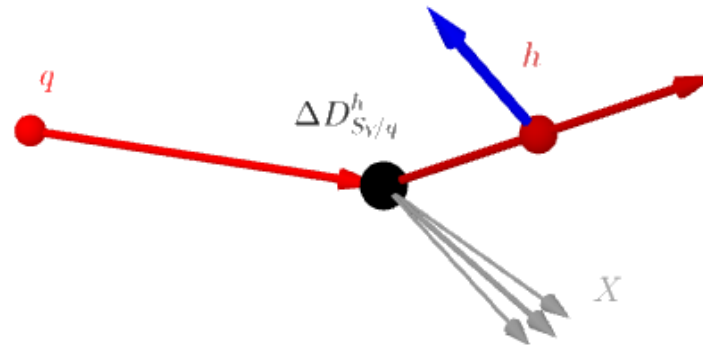
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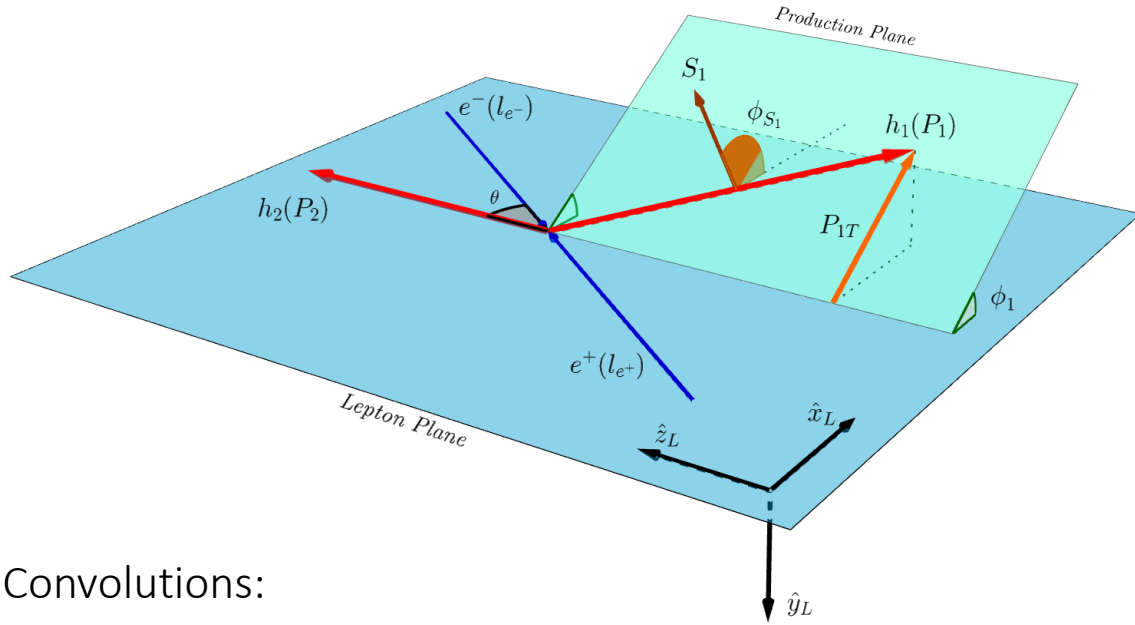
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→ Extracted from  $e^+e^-$  Belle data  
 D'Alesio, Gamberg, Murgia, MZ;  
*JHEP* 12 (2022) , *PRD* 108 (2023)

# Double hadron production in $e^+e^-$ processes



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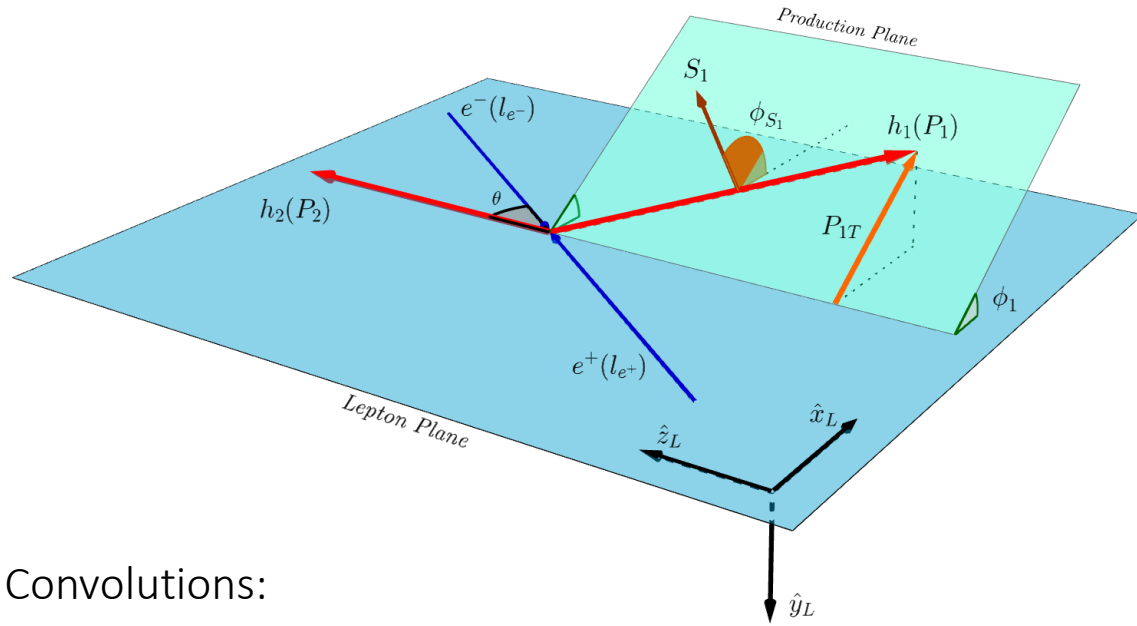
$$P_n^{h_1}(z_{h_1}, z_{h_2}) = \frac{M_1 \int dq_T q_T d\phi_1 \mathcal{B}_1 \left[ \tilde{D}_{1T}^{\perp(1)} \tilde{\bar{D}}_1 \right]}{\int dq_T q_T d\phi_1 \mathcal{B}_0 \left[ \tilde{D}_1 \tilde{\bar{D}}_1 \right]}$$

Convolutions:

$$\mathcal{B}_0 \left[ \tilde{D} \tilde{\bar{D}} \right] = \frac{1}{z_1^2 z_2^2} \sum_q e_q^2 \int \frac{db_T}{2\pi} b_T J_0(b_T q_T) d_{q/h_1}(z_1; \bar{\mu}_b) d_{\bar{q}/h_2}(z_2; \bar{\mu}_b) \\ \times M_{D_1}(b_c(b_T), z_1) M_{D_2}(b_c(b_T), z_2) e^{-g_K(b_c(b_T); b_{\max}) \ln \left( \frac{Q^2 z_1 z_2}{M_1 M_2} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

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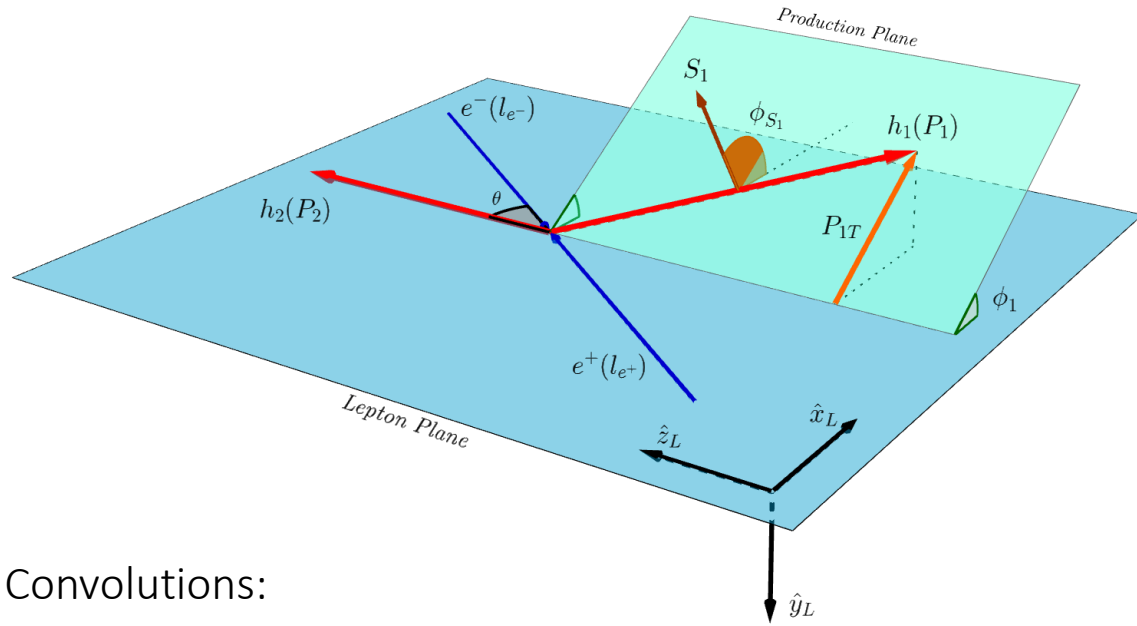
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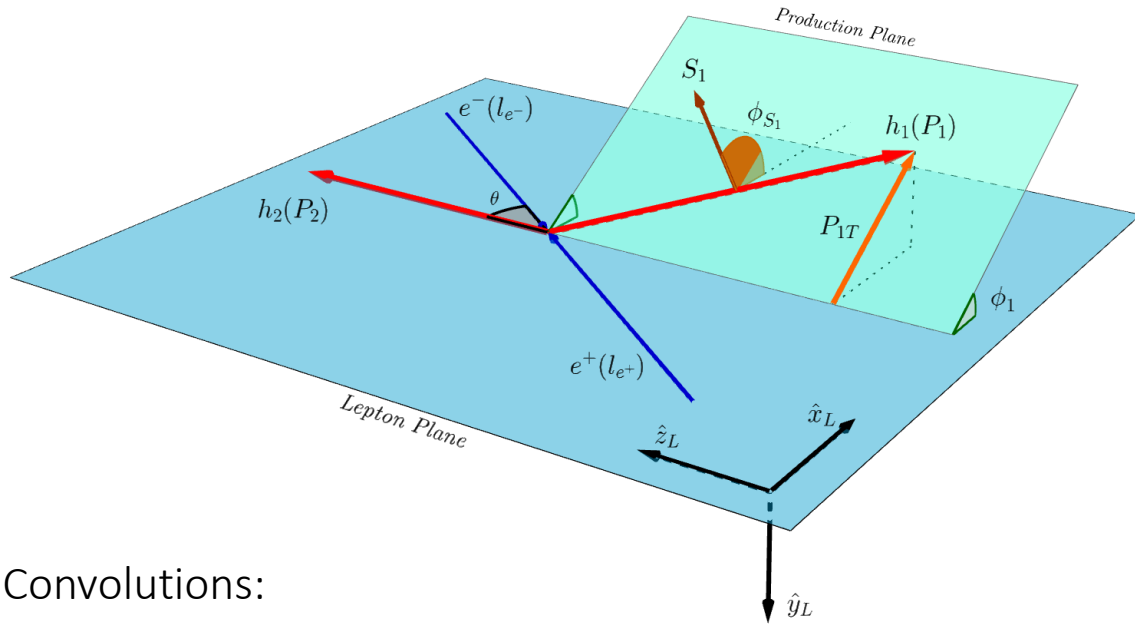
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Unpolarized FFs:  
DSS set for  $\pi/K$   
AKK set for  $\Lambda$

Non-perturbative functions from  
Bacchetta et al., *JHEP* 06 (2017) 081



# Double hadron production in $e^+e^-$ processes

Data selection:

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$$\mathcal{N}_q^p(z) = N_q z^{a_q} (1-z)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}$$

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Isospin symmetry

- No SU(2):  $N_u, N_d, N_s, N_{sea}$
- SU(2):  $N_u = N_d, N_{\bar{u}} = N_{\bar{d}}, N_s, N_{\bar{s}}$

See also:

Chen, Liang, Pan, Song, Wei;  
*Phys.Lett.B* 816 (2021) 136217

# Double hadron production in $e^+e^-$ processes

Scenarios considered:

1. No Charm, No SU(2) sym.  
pFFs for: *up, down strange and sea*;
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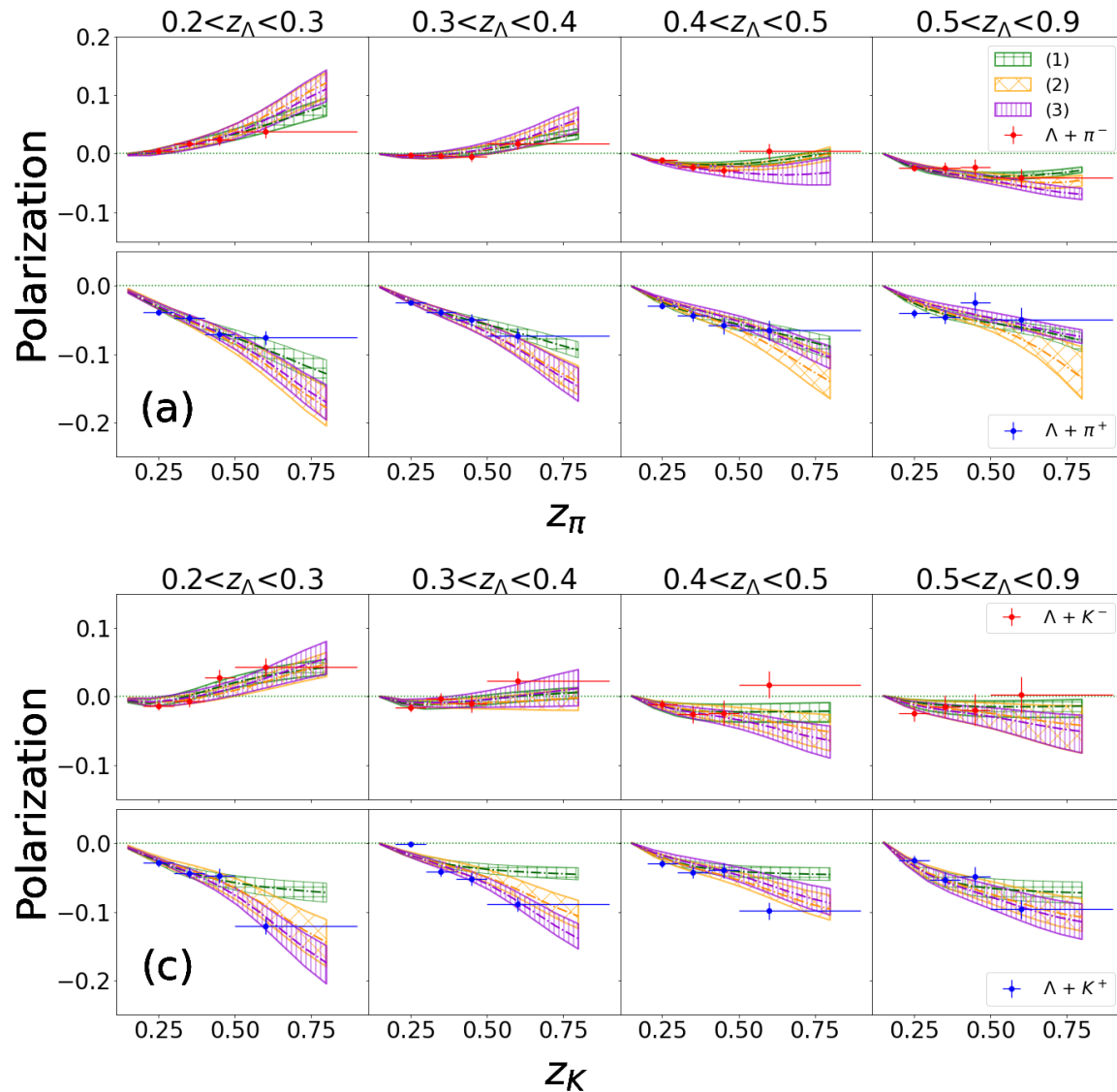
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$\chi_{dof}^2$
96 points
1,174
1,259
1,361

# Double hadron production in $e^+e^-$ processes

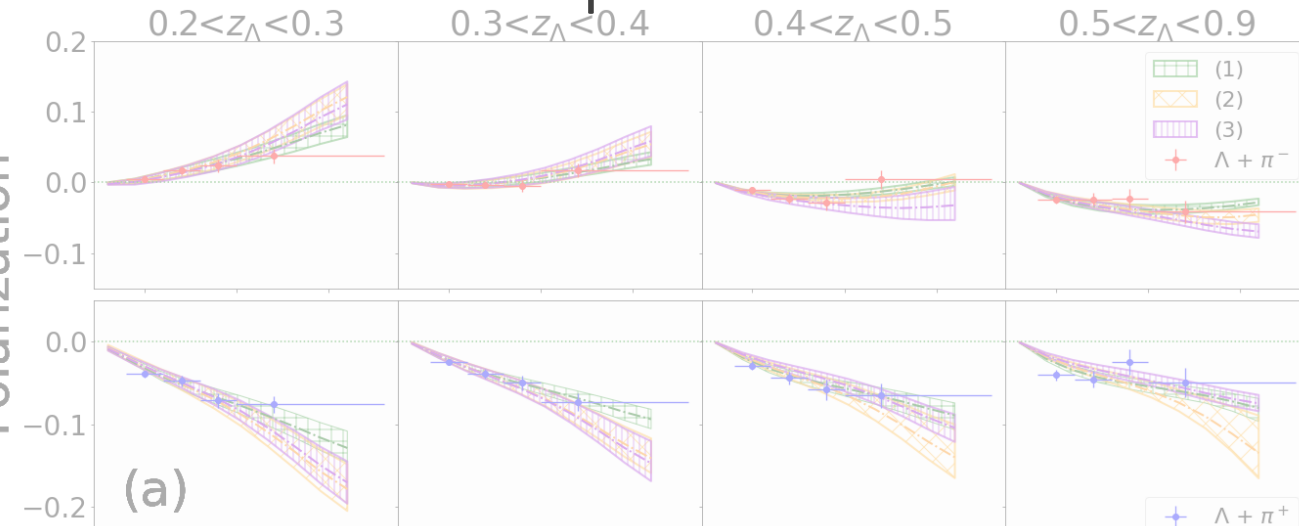


- All scenarios can describe  $\Lambda\pi^\pm, \bar{\Lambda}\pi^\pm, \Lambda K^-, \bar{\Lambda}K^+$  data;
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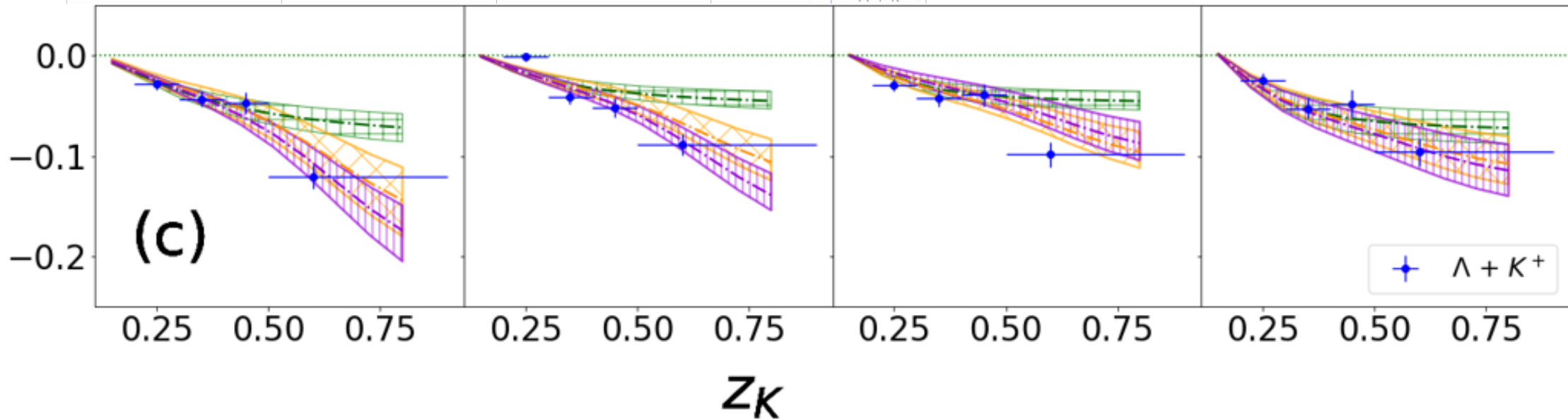


# Double hadron production in $e^+e^-$ processes

Polarization



(a)

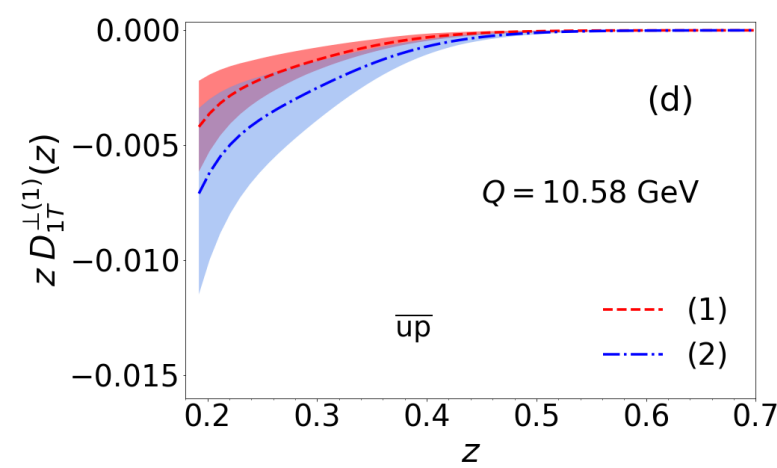
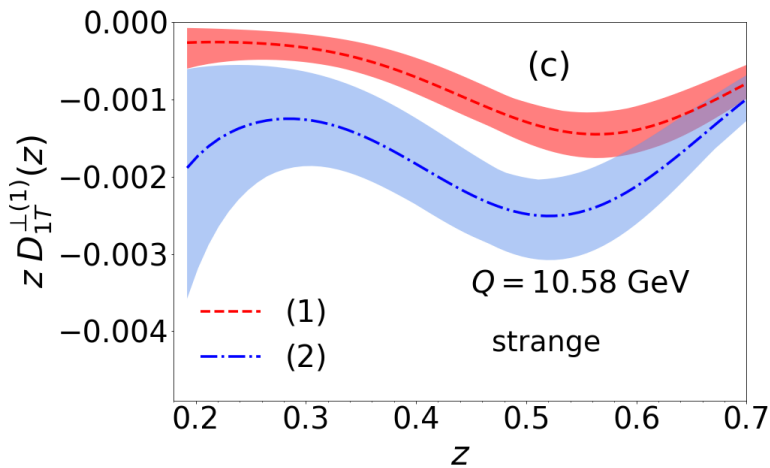
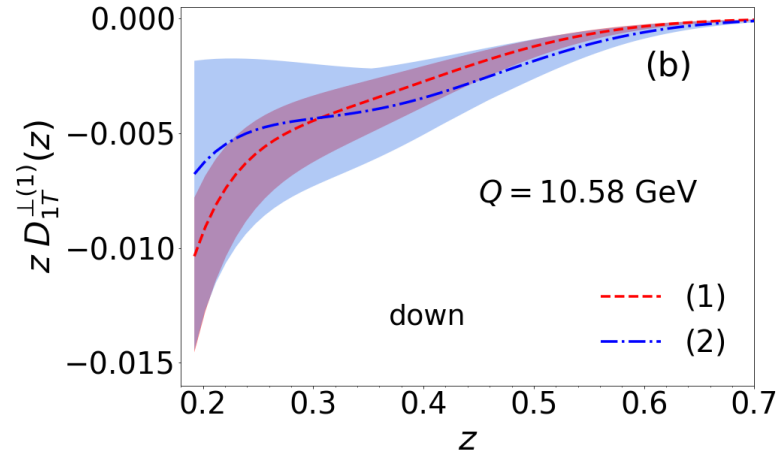
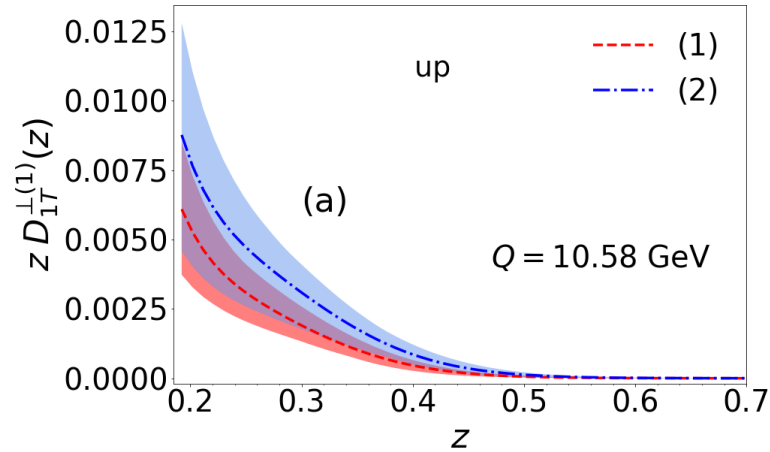


(c)

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- With the Charm contribution we obtain similar good fits and description (SC 2 and 3)

# Double hadron production in $e^+e^-$ processes

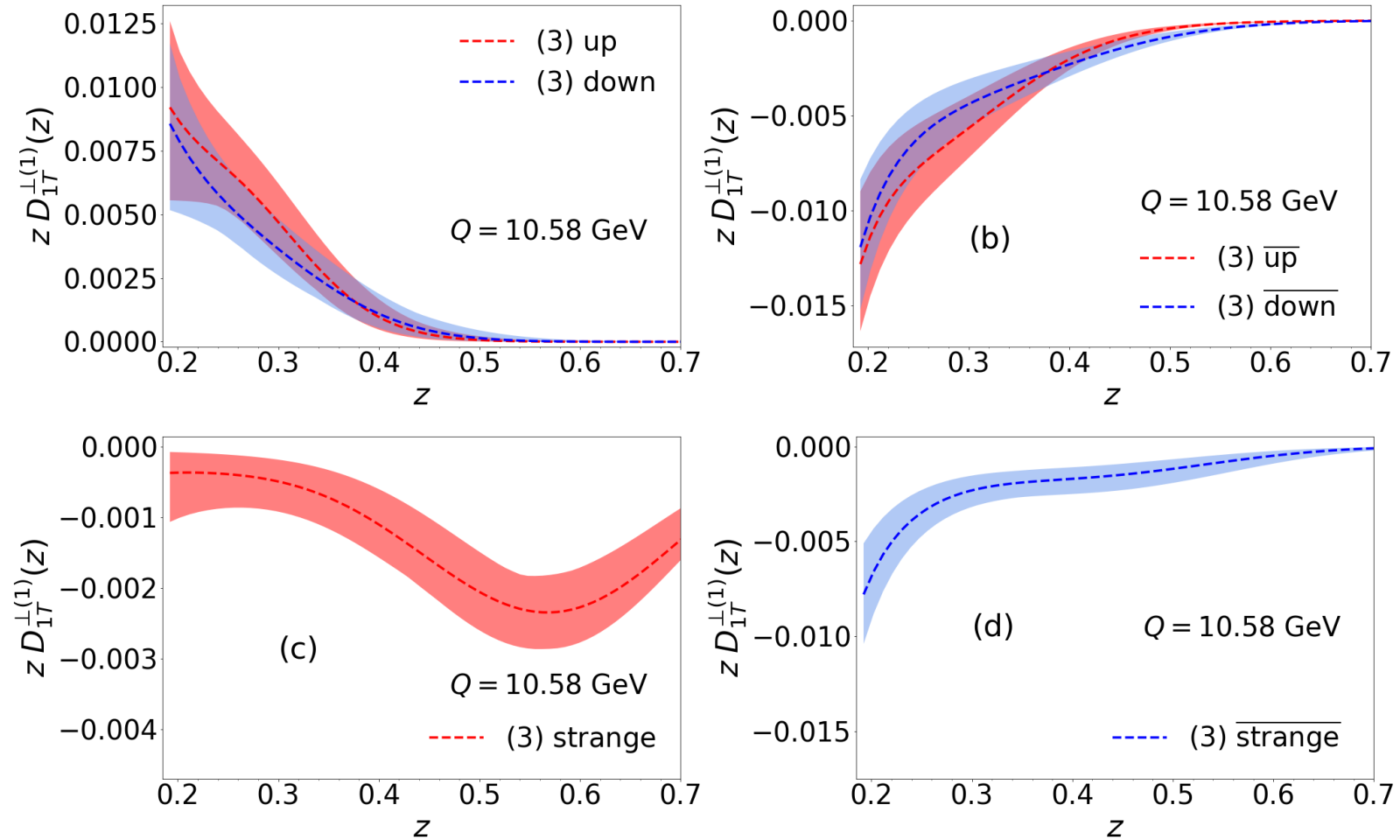
First moments: (1) & (2) scenarios



- pFFs are different in magnitude due to the charm contribution;
- $up$  pFF is positive;
- First moments are compatible, except for the strange f.m.
- Similar size for the Gaussian width.

# Double hadron production in $e^+e^-$ processes

First moments: (3) scenario



- $up/down$  pFFs are positive;
- $\overline{u\bar{p}}/\overline{d\bar{down}}$  pFFs are negative;
- $strange/\overline{strange}$  pFFs are negative;
- $up$  &  $\overline{strange}$  compatible with (1,2) scn.
- The negative sea contribution is larger in size;
- Similar size for the Gaussian width.

# Double hadron production in $e^+e^-$ processes

## Some remarks:

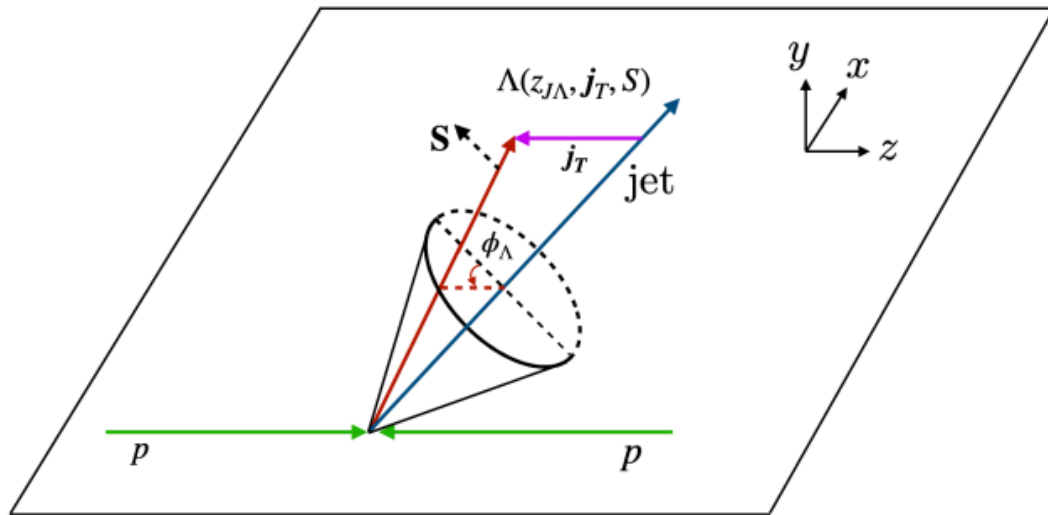
- Charm contribution in the unpolarized C.S. is necessary;  
Attempts were made to include this contribution also in the polarized c.s.
- We cannot distinguish between the (2) and (3) scenarios.  
If Normalization factors are free,  
*up* & *down* come out opposite, violating the SU(2) symmetry.

→ SiDIS and pp collisions

# Unpolarized proton-proton collisions: $pp \rightarrow \Lambda (\text{jet}) X$

STAR data: arxiv/2402.01168

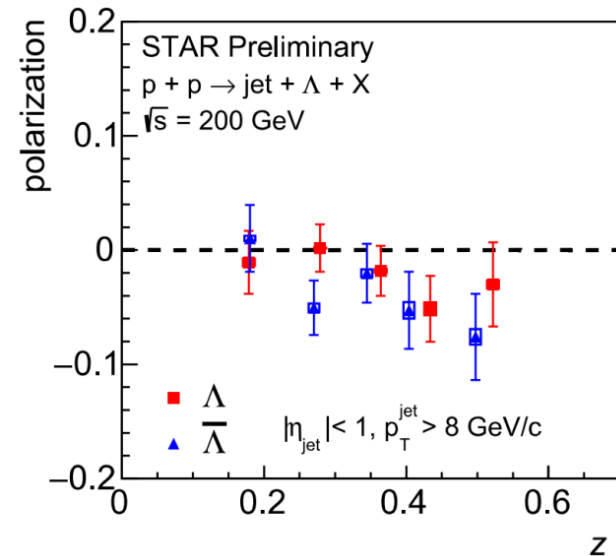
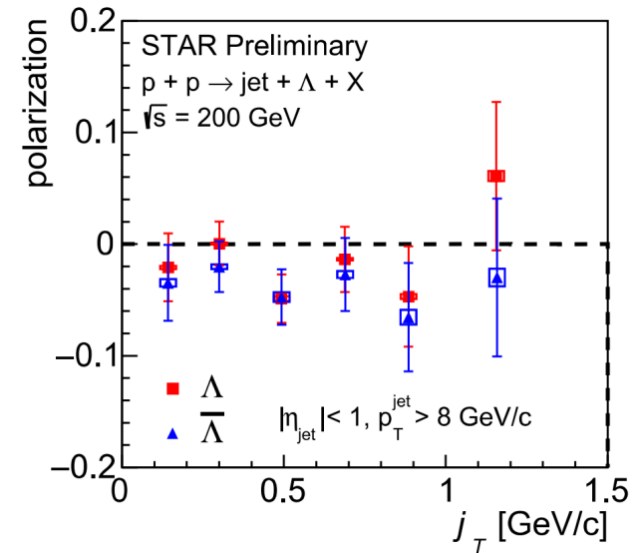
$$A(p_A) B(p_B) \rightarrow \text{jet}(p_j) \Lambda^\uparrow(p_\Lambda) X \quad \sqrt{s} = 200 \text{ GeV}$$



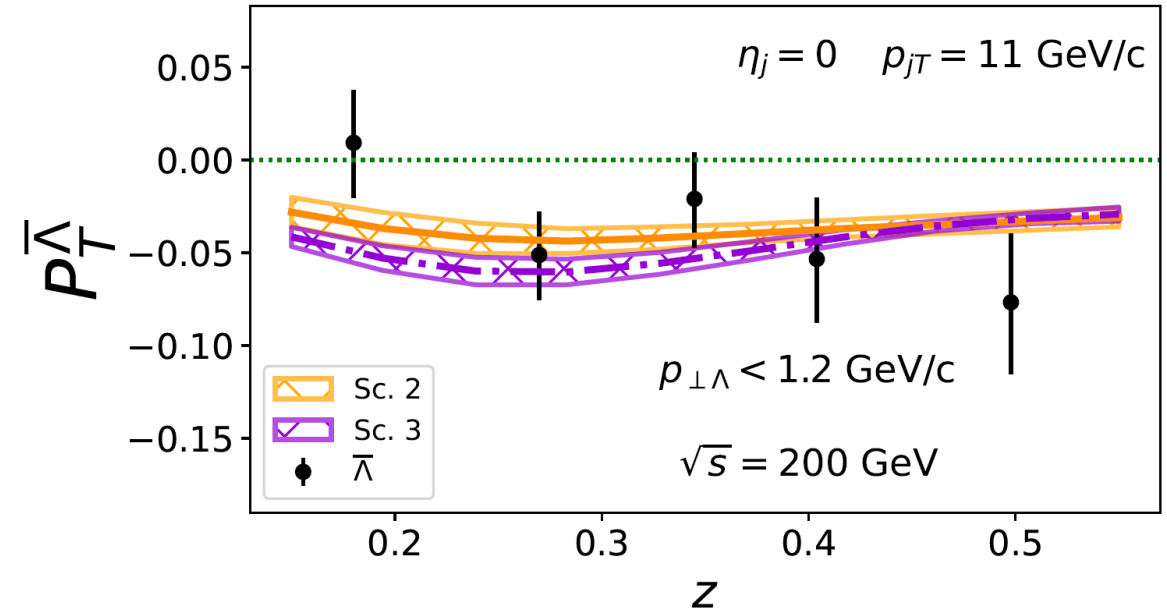
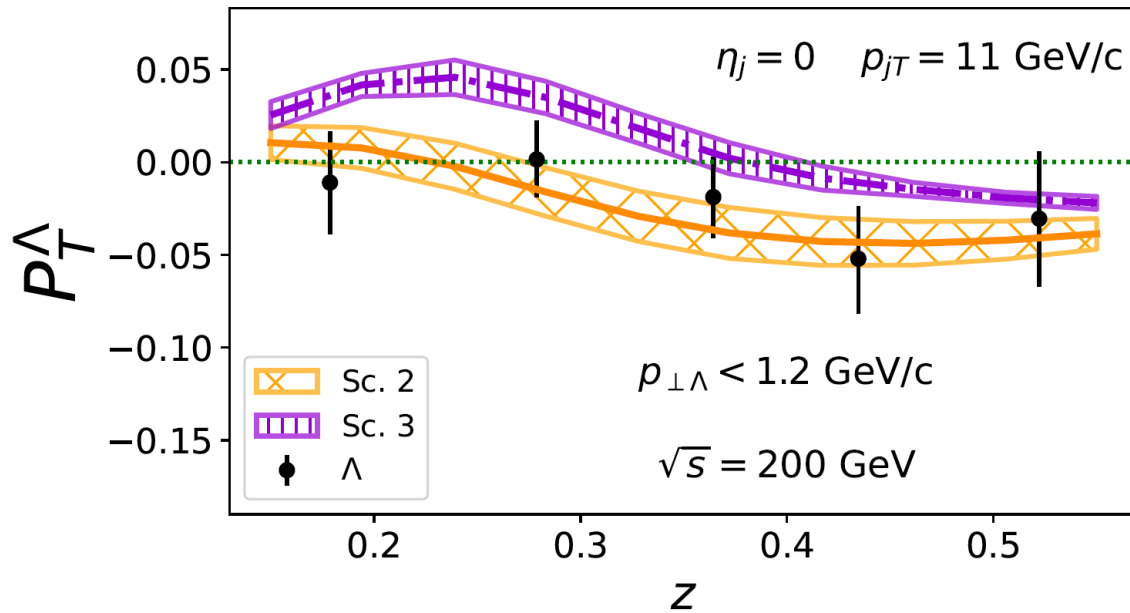
$$p_{\perp\Lambda} \leq 1.6 \text{ GeV}/c, \quad 0 \leq z \leq 1,$$

Kinematic cuts:  $8 \leq p_{jT} \leq 25 \text{ GeV}/c$  with  $\langle p_{jT} \rangle = 11 \text{ GeV}/c$ ,  
 $|\eta_j| \leq 1.0$ ,  $p_{T\Lambda} \leq 10 \text{ GeV}/c$ ,  $|\eta_\Lambda| \leq 1.5$

Anti- $k_T$  algorithm with cone radius  $R = 0.6$



# Unpolarized proton-proton collisions: $pp \rightarrow \Lambda (\text{jet}) X$



The behaviour in  $z$  is driven by the relative contribution of the polFFs:

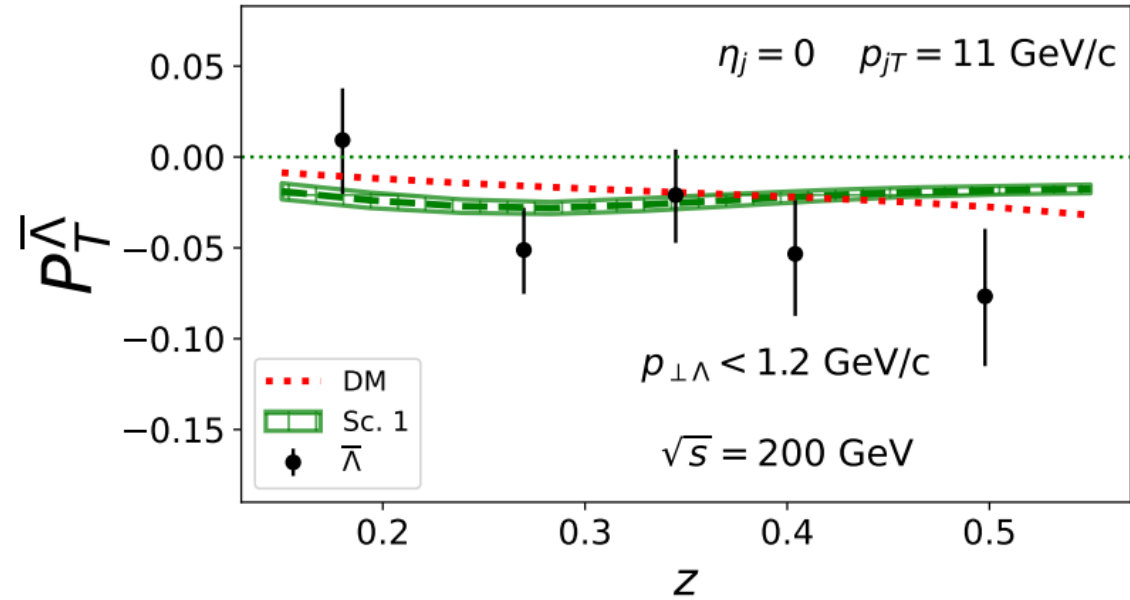
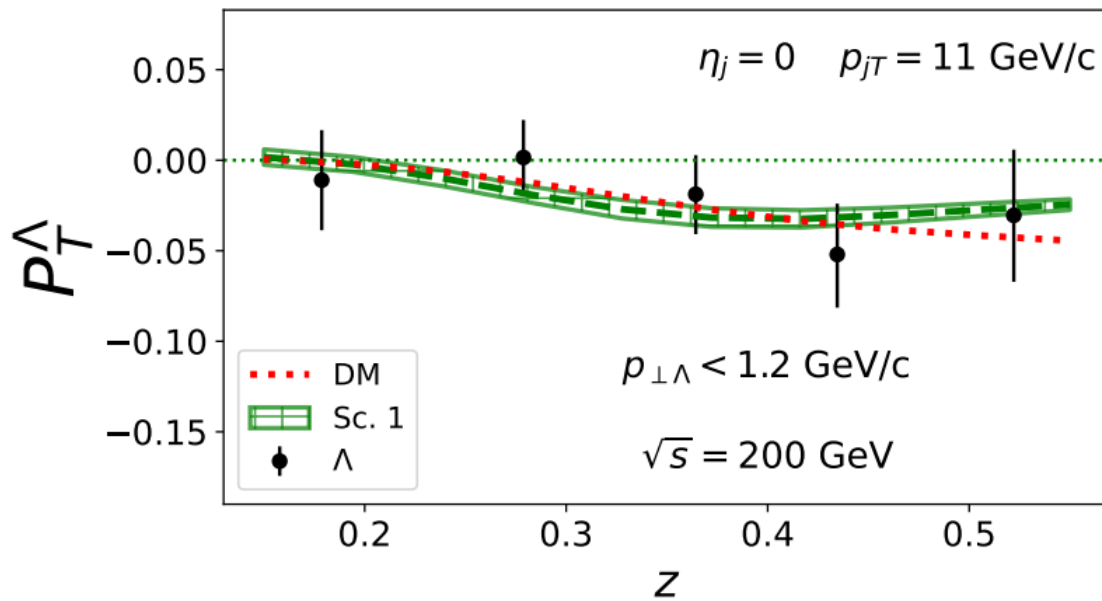
- In Sc. 1 and 2 only the up is positive  $\rightarrow$  leads to a negative value of the polarization

In Sc. 3 both up and down are positive  $\rightarrow$  leads to positive value of the polarization at small  $z$  and negative at intermediate values.

- Lambda-bar: the polarization is negative and is driven by the negative sign of the sea polFFs.

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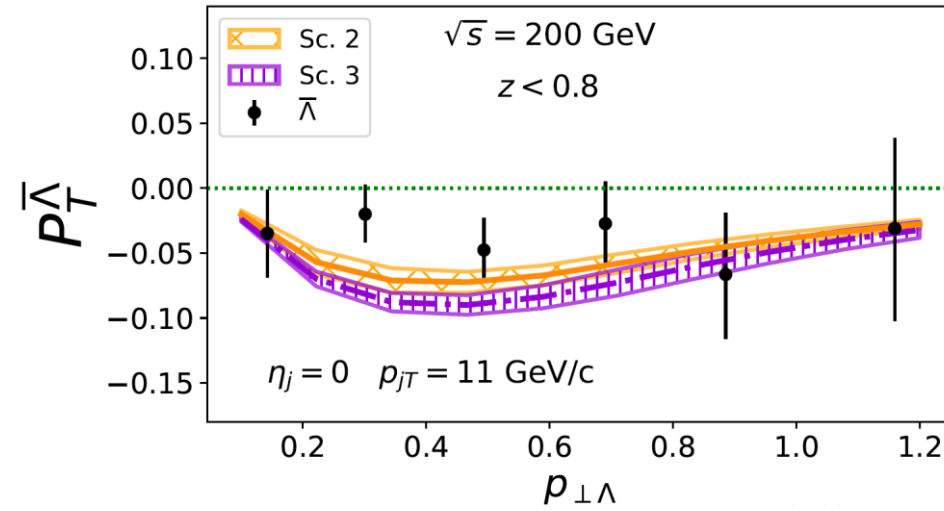
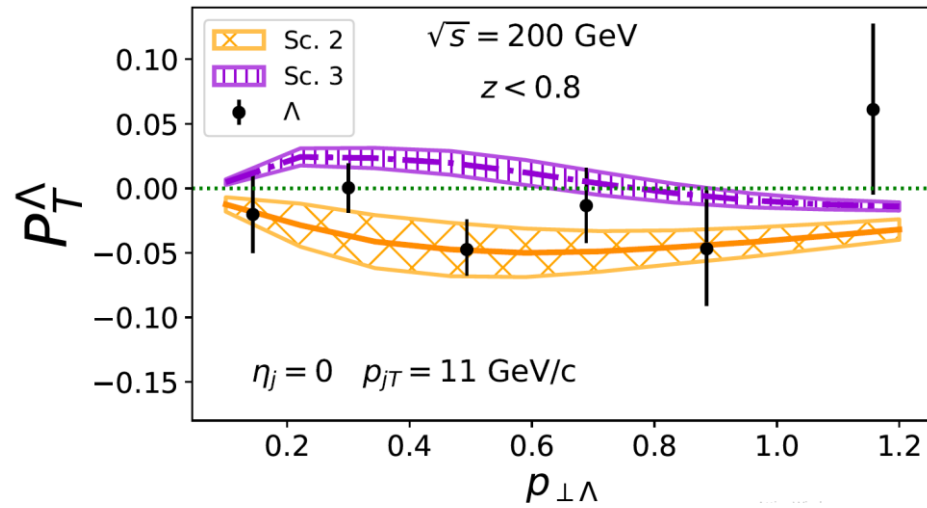
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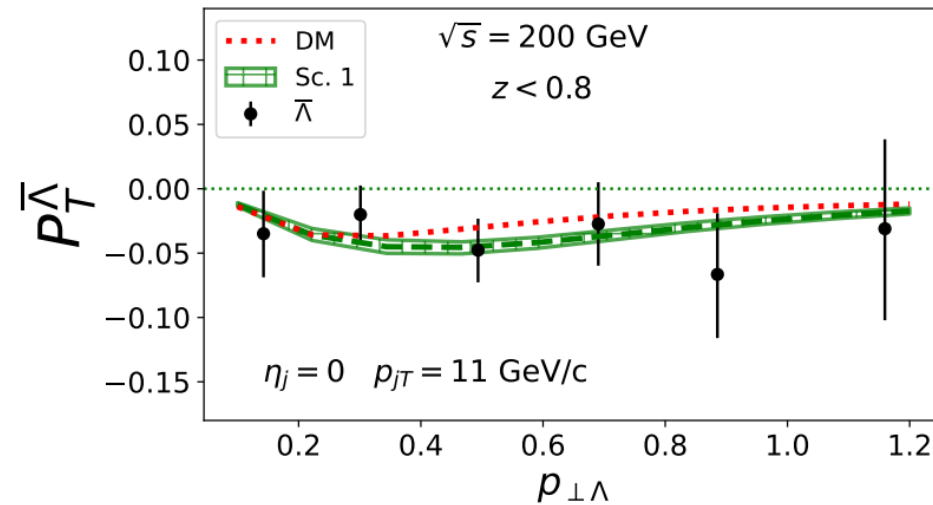
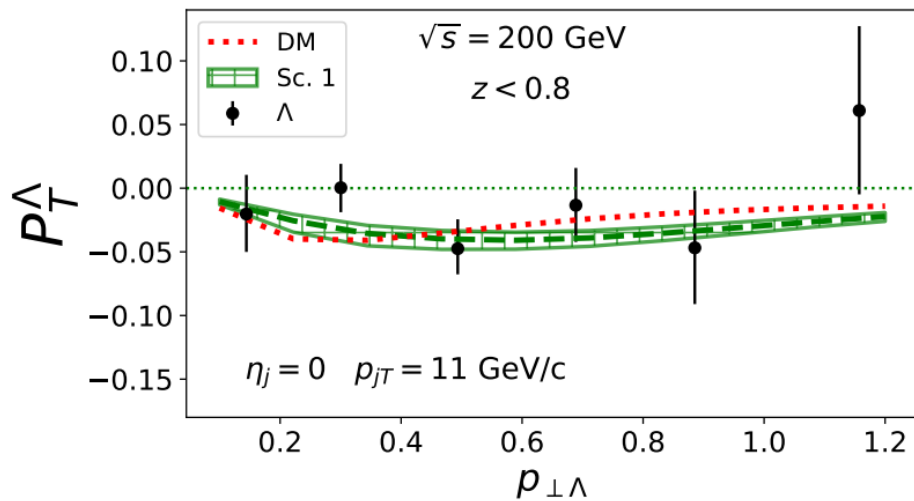
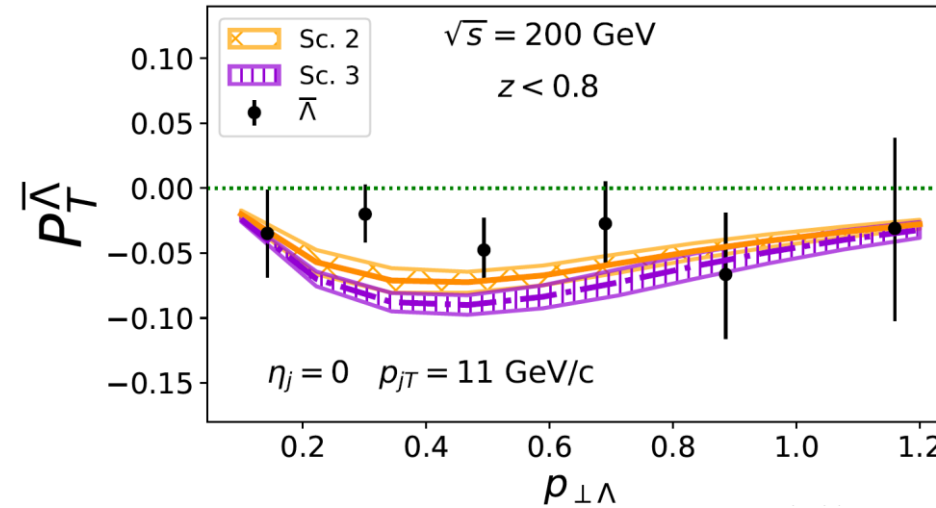
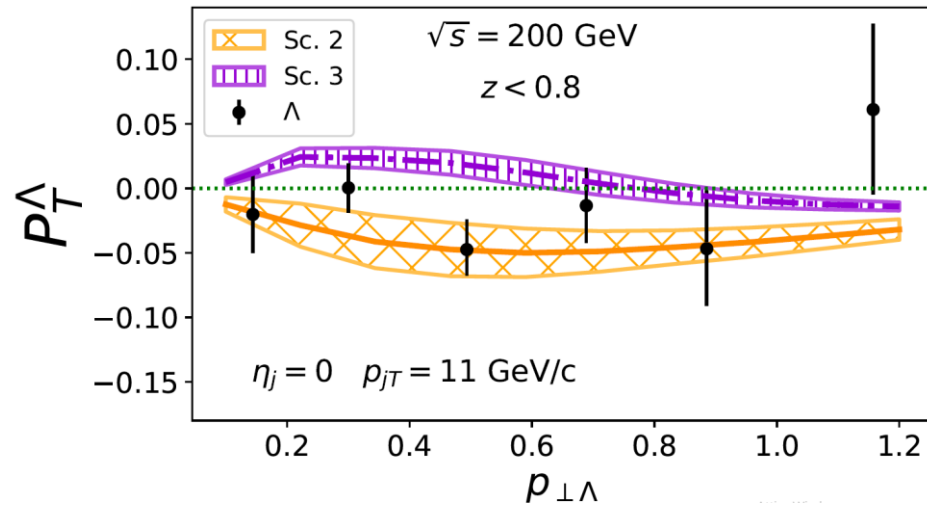
Similar comments as for the  $z$  behavior.





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## General remarks:

- Large experimental error bars: no strong conclusions
- General agreement with data in favor of the predicted universality of the  $\text{polFF}$
- Sc. 1 & Sc. 2 (no  $\text{SU}(2)$ ) a bit better than Sc. 3 at describing the data

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## First hint of the size of the Gluon polFF:

- UnpFF contribution to unpolarized cross sec. is about 50% ;
  - Since quark contribution to polarization is about 5-8% ;
- Gluon polFF can be only around 10% of its positivity bound

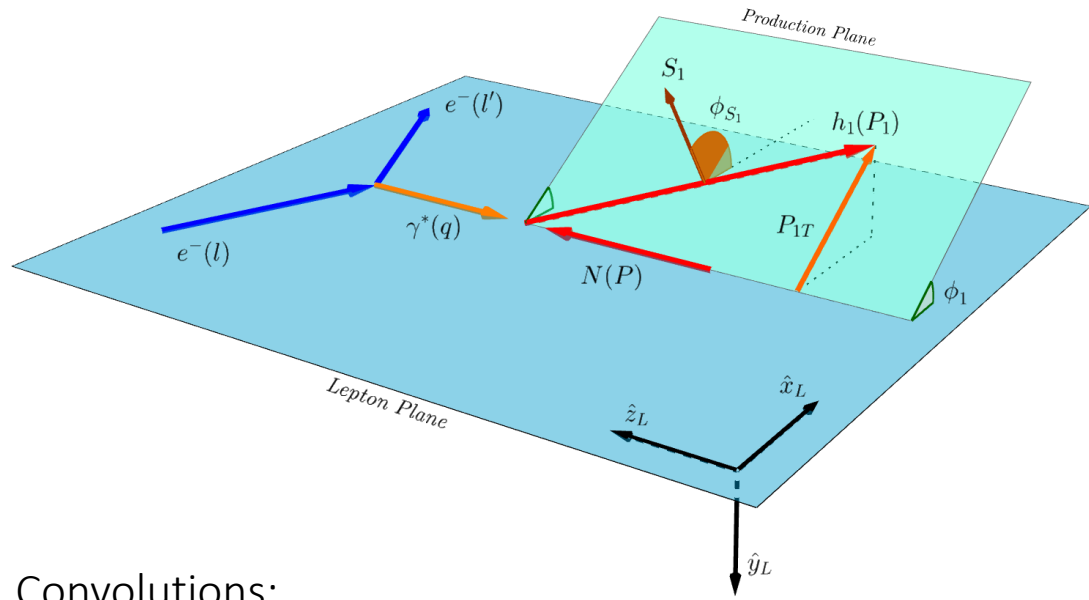
# Conclusions

- $\Lambda$  in jet pp collisions: tool to test universality of the polFF
- TMD effects only in the fragmentation mechanism
- Estimates based on fits of Belle data on transverse  $\Lambda$  polarization:
  - Good agreement with STAR pp data
  - Test of universality and SU(2) symmetry issue
- First hint of the role of the polFF for gluons



# Backup

# Semi-inclusive Deep Inelastic Scattering



Polarization:

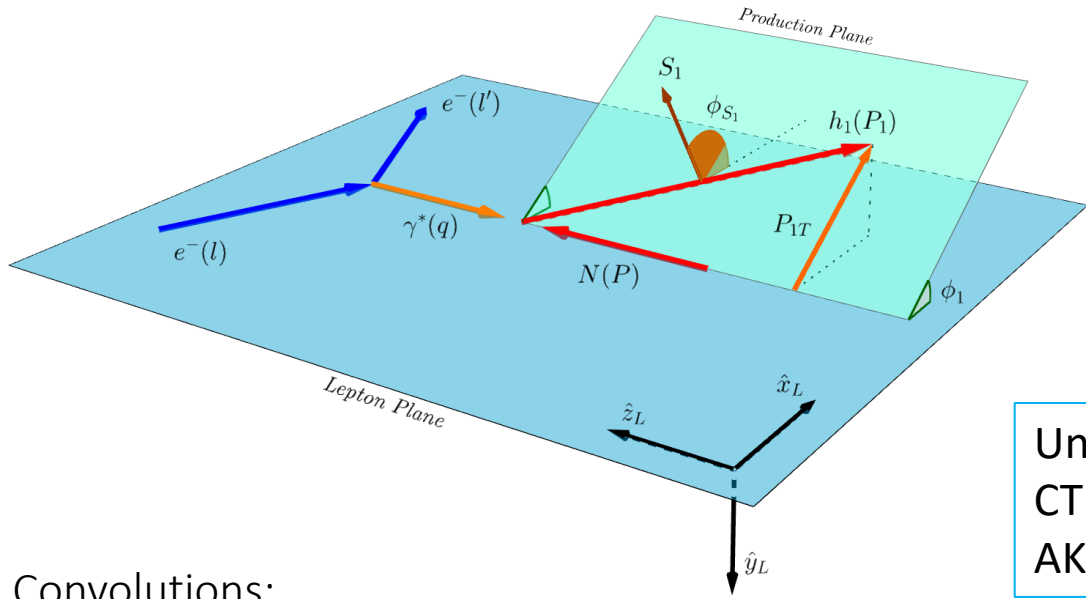
$$P_n^{h_1}(x_B, z_h) \equiv \frac{M_1 \int dq_T q_T d\phi_1 \mathcal{B}_1 \left[ \tilde{f}_1 \tilde{D}_{1T}^{\perp(1)} \right]}{\int dq_T q_T d\phi_1 \mathcal{B}_0 \left[ \tilde{f}_1 \tilde{D}_1 \right]}$$

Convolutions:

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AKK set for  $\Lambda$

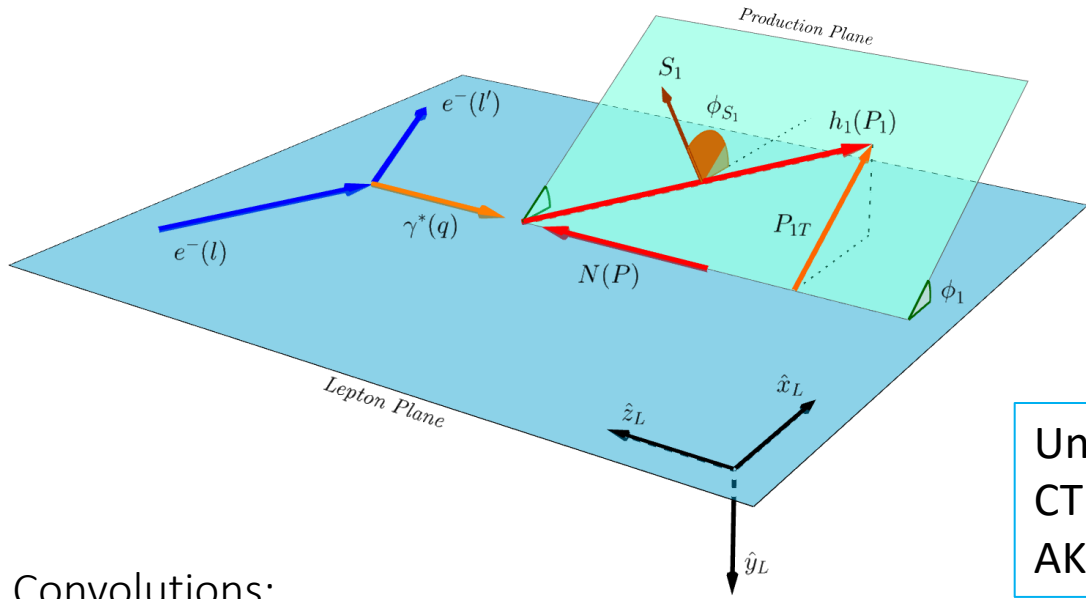
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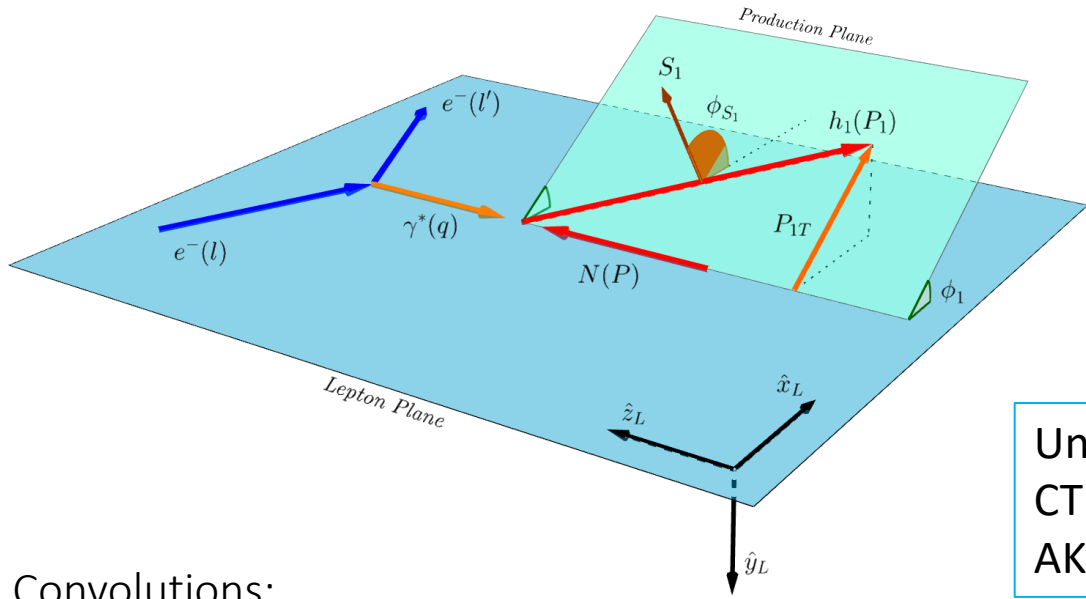
Non-perturbative functions from  
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Convolutions:

$$\mathcal{B}_0 \left[ \tilde{f}_1 \tilde{D}_1 \right] = \frac{1}{z^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T J_0(b_T q_T) f_{N/q}(x; \bar{\mu}_b) d_{q/h}(z; \bar{\mu}_b) \times M_{f_1}(b_c(b_T), x) M_{D_h}(b_c(b_T), z) e^{-g_K(b_c(b_T); b_{\max}) \ln \left( \frac{Q^2 z}{x M_P M_h} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

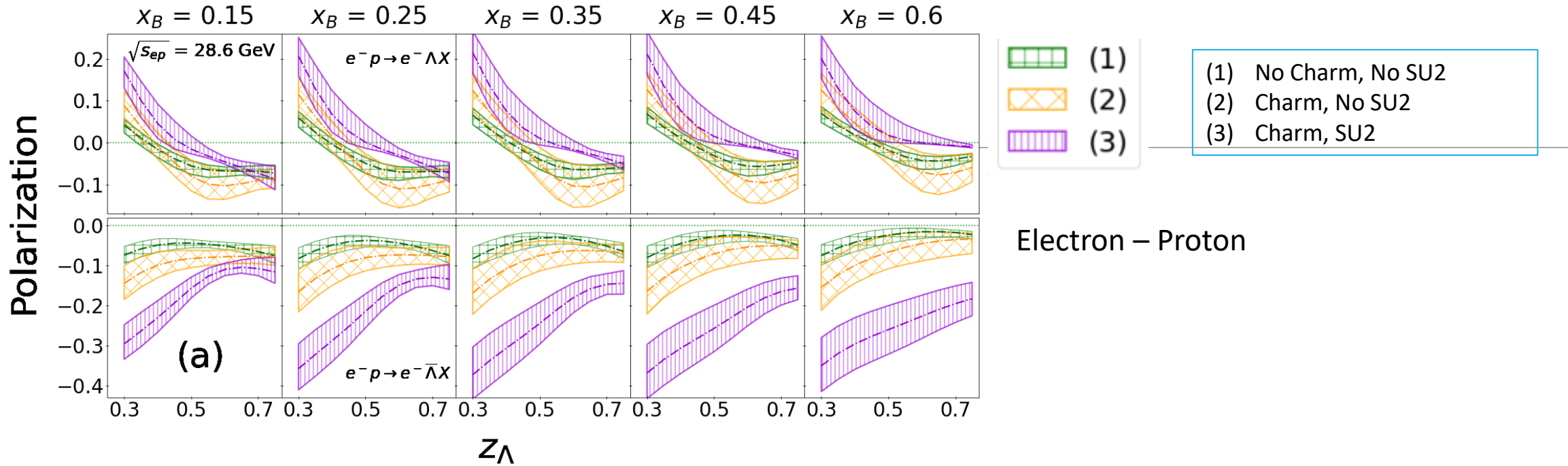
$$\mathcal{B}_1 \left[ \tilde{f}_1 \tilde{D}_{1T}^{\perp(1)} \right] = \frac{1}{z^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T q_T) f_{N/q}(x; \bar{\mu}_b) D_{1T,q}^{\perp(1)}(z; \bar{\mu}_b) \times M_{f_1}(b_c(b_T), x) M_{D_1}^{\perp}(b_c(b_T), z) e^{-g_K(b_c(b_T); b_{\max}) \ln \left( \frac{Q^2 z}{x M_P M_h} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

Predictions are given at different energies:

$E_N$ (GeV)	$E_{e^-}$ (GeV)	$\sqrt{s_{eN}}$ (GeV)
41	5	28.6
100	10	63.2

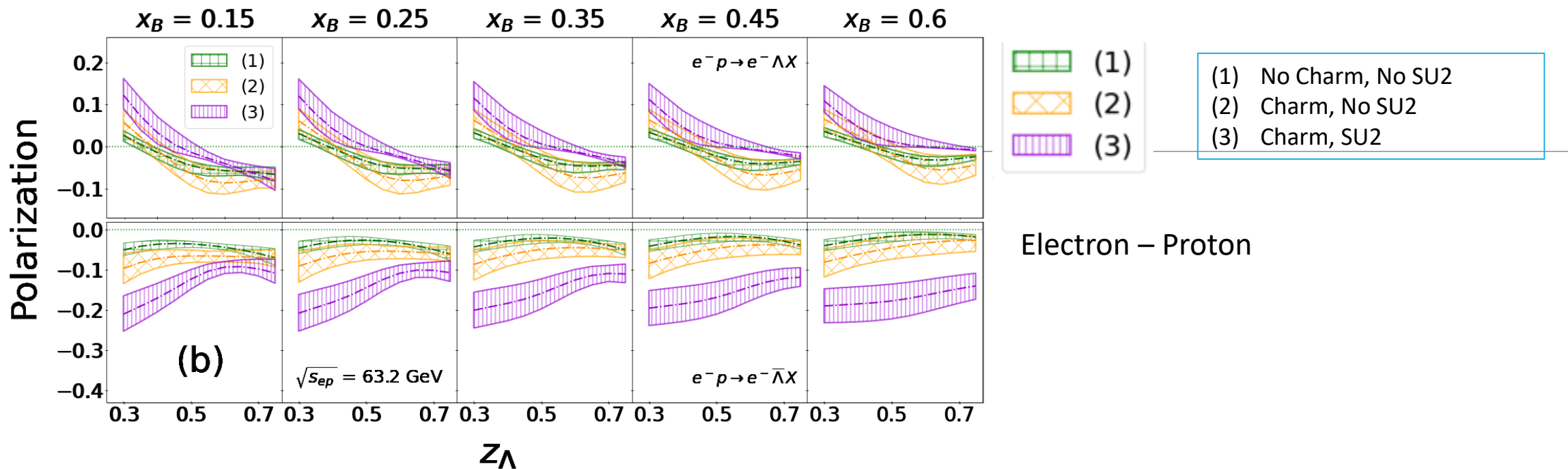
$$s = 4E_N E_e, \quad Q^2 = x_B y s$$

# Semi-inclusive Deep Inelastic Scattering



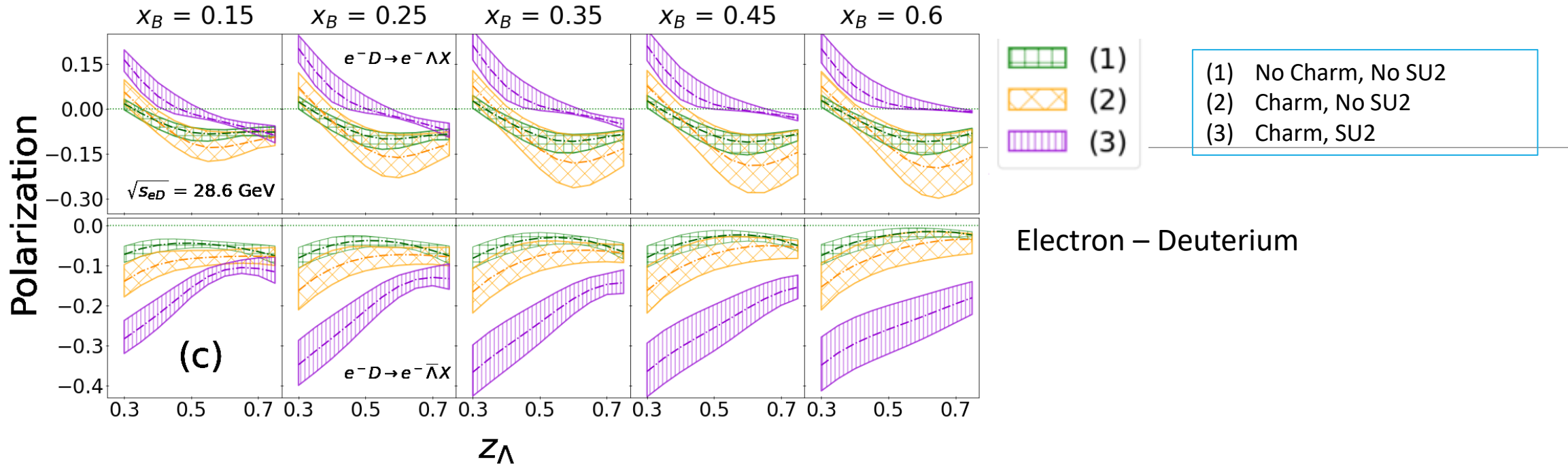
- (1) & (2) scenarios: polarization of similar size and behavior;
- (3) scenario: similar size;
- $\Lambda$  pol. decreases and becomes negative;
- $\bar{\Lambda}$  is always negative;
- $\bar{\Lambda}$  pol. similar or slightly greater size;
- $\bar{\Lambda}$  most significant difference;
- $\sqrt{s_{ep}}=28,6$  pol. has the same size, for greater values there is a general reduction as  $x_B$  grows.

# Semi-inclusive Deep Inelastic Scattering



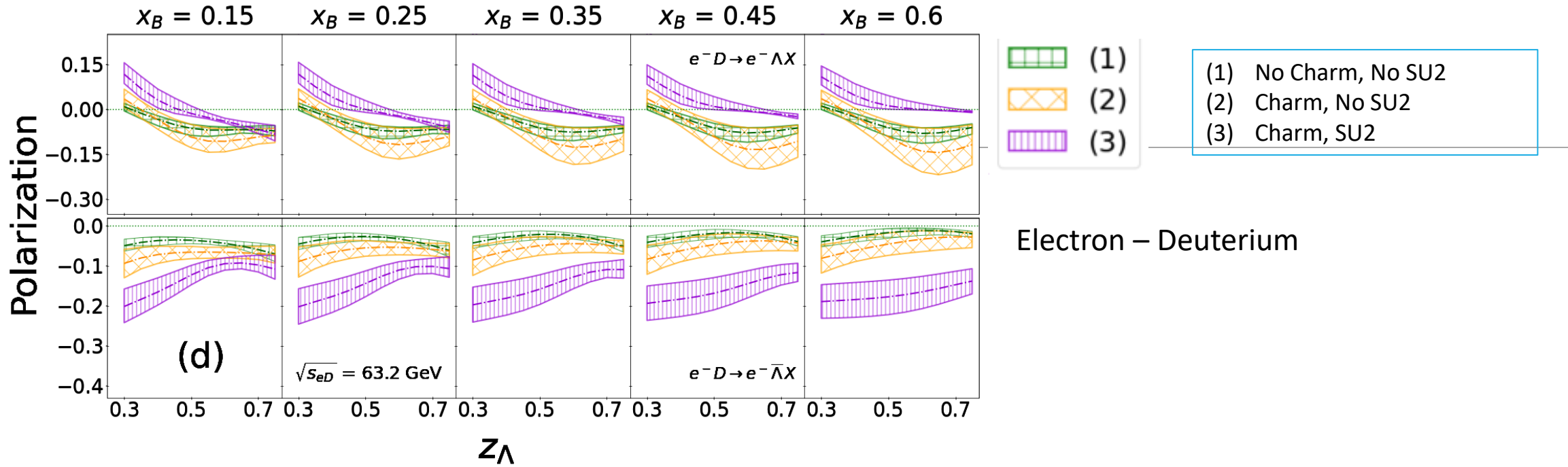
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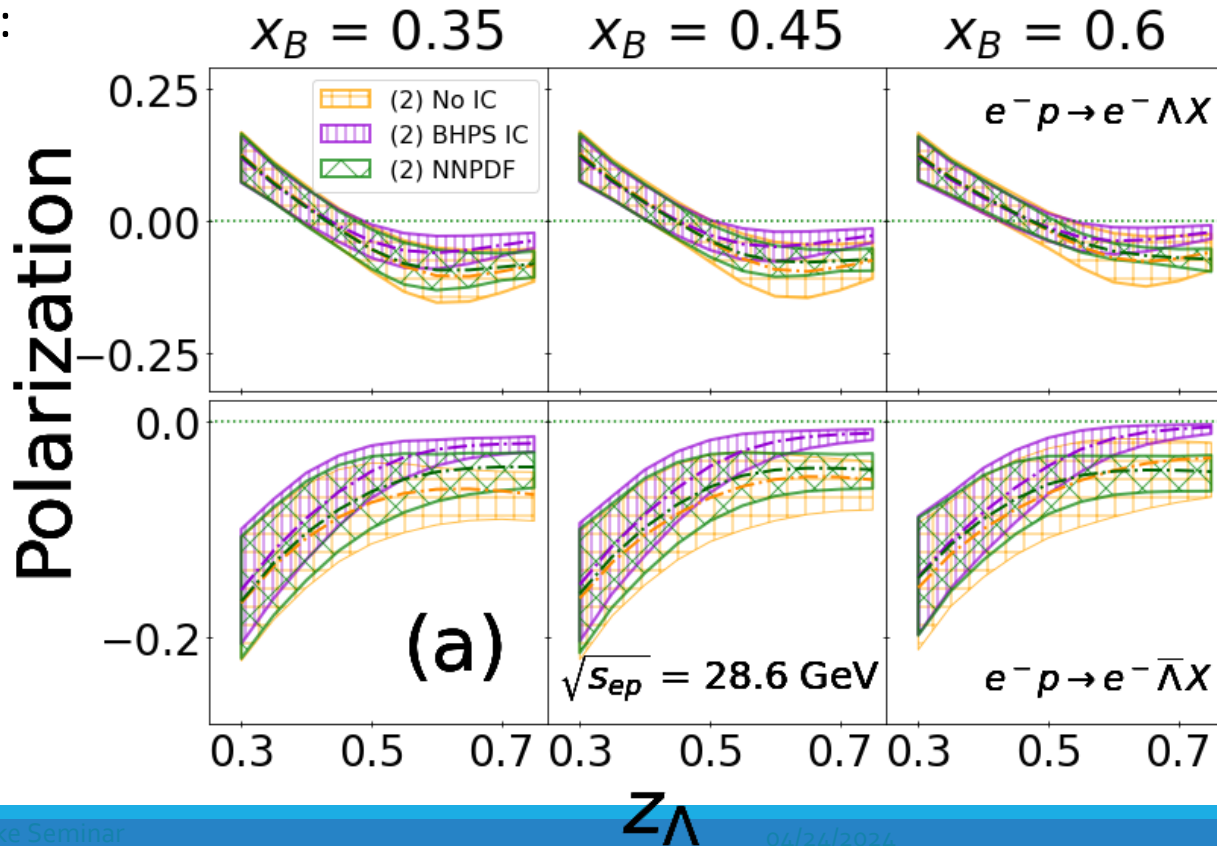
# Semi-inclusive Deep Inelastic Scattering

The charm contribution in the fragmentation process is relevant

Intrinsic Charm (IC) component in the proton:

- CT14nnloIC set with BHPS model [T.-J. Hou et al., *JHEP* 02 (2018) 059]
- NNPDF4.0nnlo set [NNPDF Coll., *Eur.Phys.J.C* 82 (2022) 5, 428]

(2) Scenario:



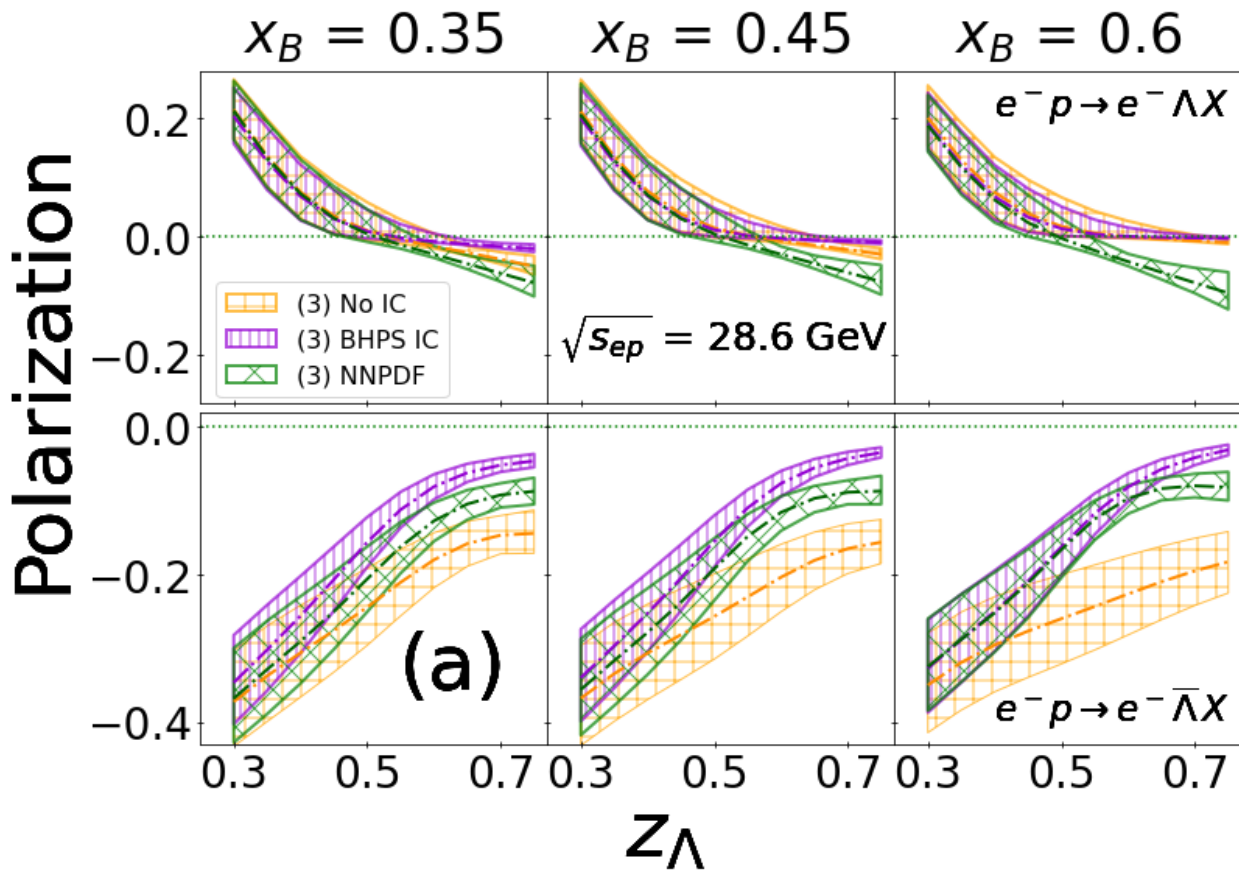
(2) Charm, No SU2

- BHPS and NNPDF: similar polarization of previous predictions
- Same behavior is present for greater values of the c.m. energy.



# Semi-inclusive Deep Inelastic Scattering

(3) Scenario:



(3) Charm, SU2

- Estimates vary significantly as  $x_B$  increases;
- $\bar{\Lambda}$  estimates with BHPS and NNPDF different from the previous ones;
- $\Lambda$  :decreases to zero
- $\Lambda$  : NNPDF become negative