NNLO global analysis of helicity PDFs

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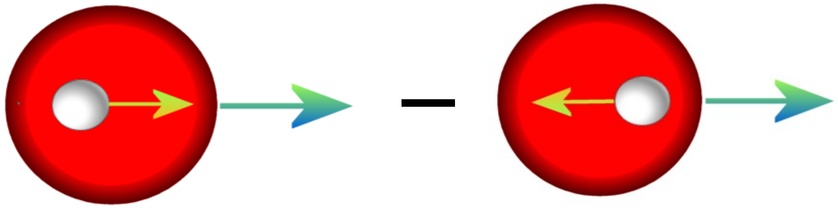
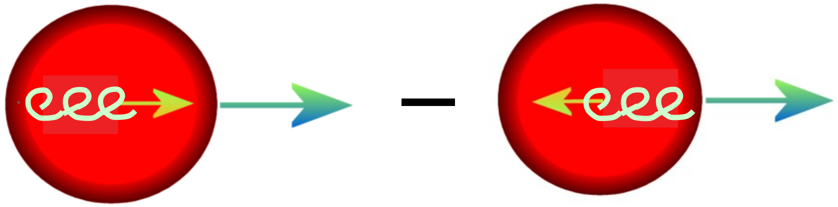
HRT

$$\Delta q(x) =$$



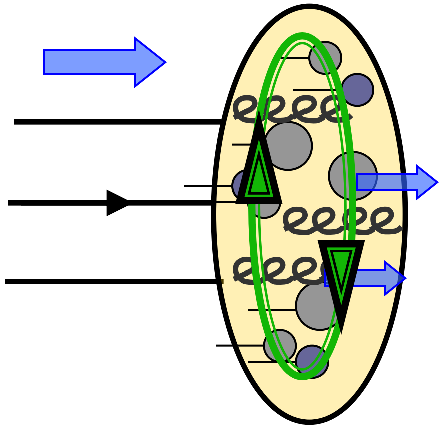
$$\Delta g(x) =$$



- in QCD: operator definition
  → dependence on “resolution” scale μ

proton spin:

Jaffe, Manohar; Chen et al;
Wakamatsu; Hatta; …



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta\Sigma = \int_0^1 dx\,\left[\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right](x) \quad \ll 1$$

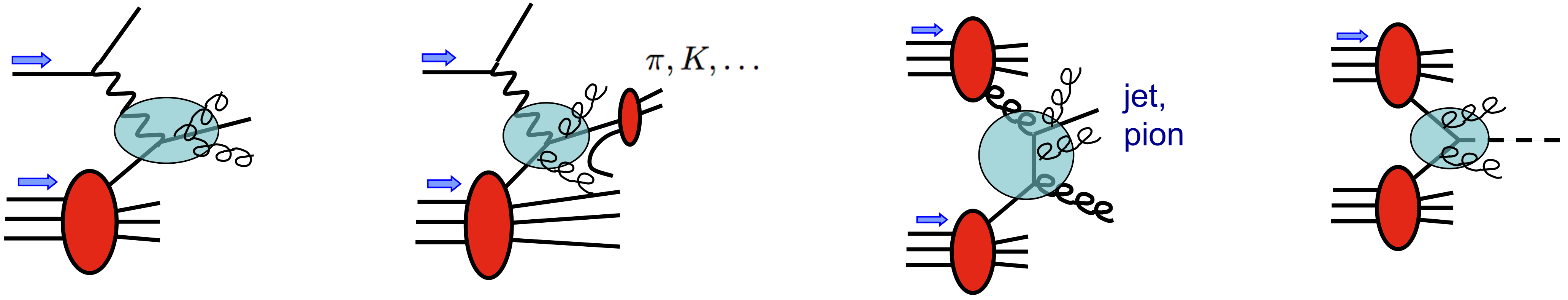
$$\Delta G = \int_0^1 dx\,\Delta g(x)$$

Outline:

- Helicity PDFs at NNLO: framework
- Results
- Concluding remarks

Thanks to my collaborators
I. Borsa, D. de Florian, R. Sassot, M. Stratmann

Helicity PDFs at NNLO: framework



DIS SIDIS pp high-p_T W bosons

- Partonic hard scattering: $\Delta\hat{\sigma}_{ab} = \Delta\hat{\sigma}_{ab}^{\mathrm{LO}} + \frac{\alpha_s}{\pi}\Delta\hat{\sigma}_{ab}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta\hat{\sigma}_{ab}^{\mathrm{NNLO}} + \ldots$

- PDF evolution: $\Delta\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi}\Delta P_{ij}^{\mathrm{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^3 \Delta P_{ij}^{\mathrm{NNLO}} + \ldots$

First NLO "global" analysis of nucleon helicity (GRSV, 1996)

PHYSICAL REVIEW D VOLUME 53, NUMBER 9 1 MAY 1996

Next-to-leading-order radiative parton model analysis of polarized deep inelastic lepton-nucleon scattering

M. Glück, E. Reya, and M. Stratmann
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(Received 28 August 1995; revised manuscript received 10 January 1996)

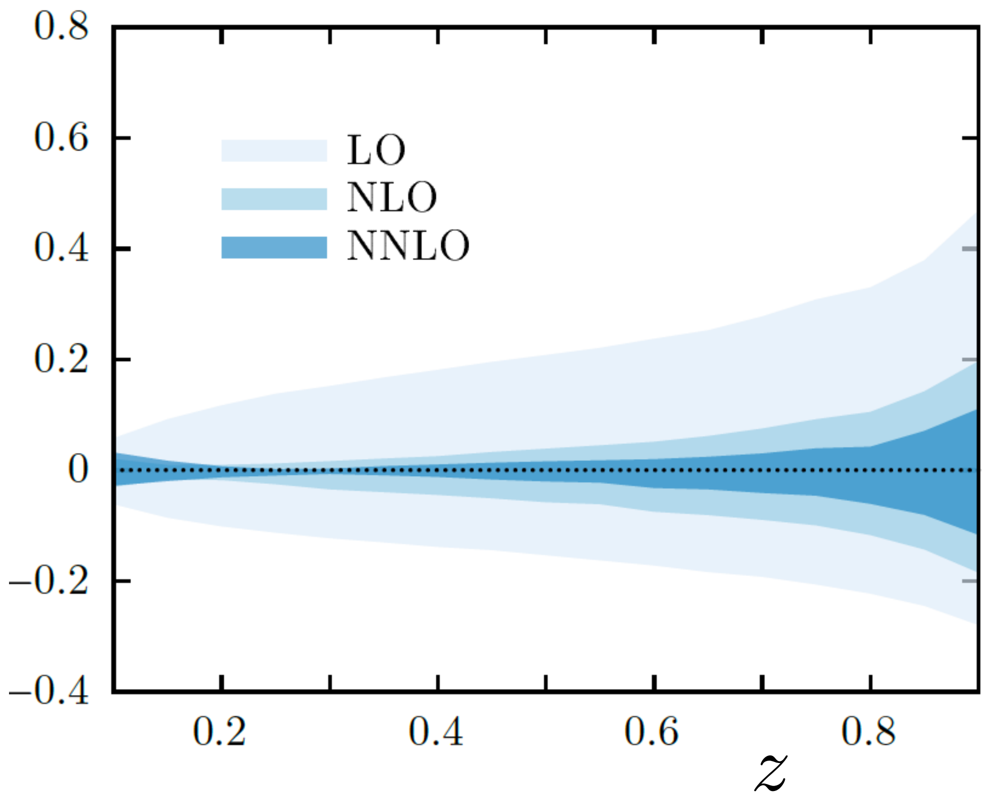
A next-to-leading-order QCD analysis of spin asymmetries and structure functions in polarized deep inelastic lepton-nucleon scattering is presented within the framework of the radiative parton model. A consistent NLO formulation of the Q^2 evolution of polarized parton distributions yields two sets of plausible NLO spin-dependent parton distributions in the conventional $\overline{\text{MS}}$ factorization scheme. They respect the fundamental positivity constraints down to the low resolution scale $Q^2=\mu_{\text{NLO}}^2=0.34$ GeV^2. The Q^2 dependence of the spin asymmetries $A_1^{p,n,d}(x,Q^2)$ is similar to the leading-order (LO) one in the range $1\leqslant Q^2\leqslant 20$ GeV^2 and is

Why NNLO ?

- need per cent accuracy for EIC and JLab (cf. LHC experience)
- reduce theory uncertainty

$$\frac{\sigma(\mu) - \sigma(Q)}{\sigma(Q)}$$

$$Q/2 \le \mu_{R,F} \le 2Q$$



$Q^2 > 5\,\mathrm{GeV}^2$

SIDIS@EIC

Abele, De Florian, WV

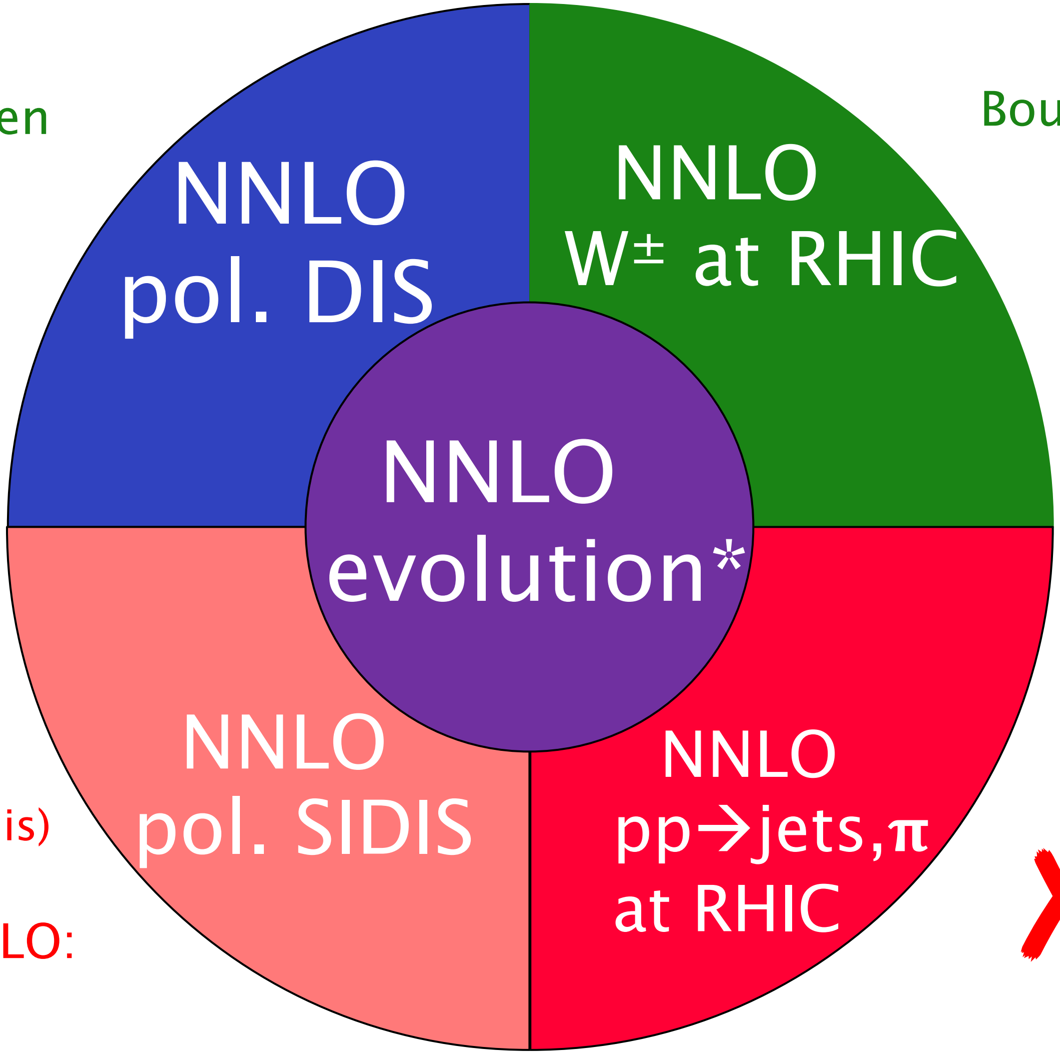
- progress in lattice computations

hard scattering: $$\Delta\hat{\sigma}_{ab} = \Delta\hat{\sigma}_{ab}^{\rm LO} + \frac{\alpha_s}{\pi}\Delta\hat{\sigma}_{ab}^{\rm NLO} + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta\hat{\sigma}_{ab}^{\rm NNLO} + \dots$$

evolution: $$\Delta\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi}\Delta P_{ij}^{\rm LO} + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{\rm NLO} + \left(\frac{\alpha_s}{2\pi}\right)^3 \Delta P_{ij}^{\rm NNLO} + \dots$$

Zijlstra, van Neerven 1994

Boughezal, Li, Petriello 2021



* Moch, Vogt, Vermaseren
Blümlein, Marquard, Schneider, Schönwald
QCD Pegasus: A. Vogt

Bonino, Gehrmann, Löchner, Schönwald, Stagnitto 2024
Goyal, Lee, Moch, Pathak, Rana, Ravindran 2024
(not suitable yet for glob. analysis)

✗ use soft-gluon approx.

soft-gluon approximate NNLO:

Anderle, Ringer, WV 2012
Abele, de Florian, WV 2021

NNLO helicity PDFs:

- DIS–only analysis   Taghavi-Shahri et al., 2016

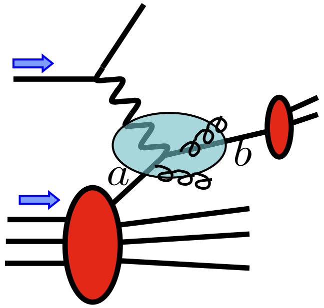
- DIS and (approximate) SIDIS   MAP: Bertone, Chiefa, Nocera 2024

- Fully global analysis with approximate NNLO for SIDIS and pp

  BDSSV: Borsa, de Florian, Sassot,Stratmann, WV (to appear)

$$\Delta\hat{\sigma}_{qq}^{\mathrm{N}^k\mathrm{LO}}(\hat{x},\hat{z}) \sim \alpha_s^k \Big[\delta(1-\hat{x})\left(\frac{\ln^{2k-1}(1-\hat{z})}{1-\hat{z}}\right)_+ + \delta(1-\hat{z})\left(\frac{\ln^{2k-1}(1-\hat{x})}{1-\hat{x}}\right)_+ + \frac{1}{(1-\hat{x})_+}\left(\frac{\ln^{2k-2}(1-\hat{z})}{1-\hat{z}}\right)_+ + \frac{1}{(1-\hat{z})_+}\left(\frac{\ln^{2k-2}(1-\hat{x})}{1-\hat{x}}\right)_+ + \dots \Big]$$

$$\hat{x} = \frac{Q^2}{2p_a \cdot q} \qquad \hat{z} = \frac{p_a \cdot p_b}{p_a \cdot q}$$



- logs can be resummed to all orders: threshold resummation

Anderle, Ringer, WV
Abele, de Florian, WV

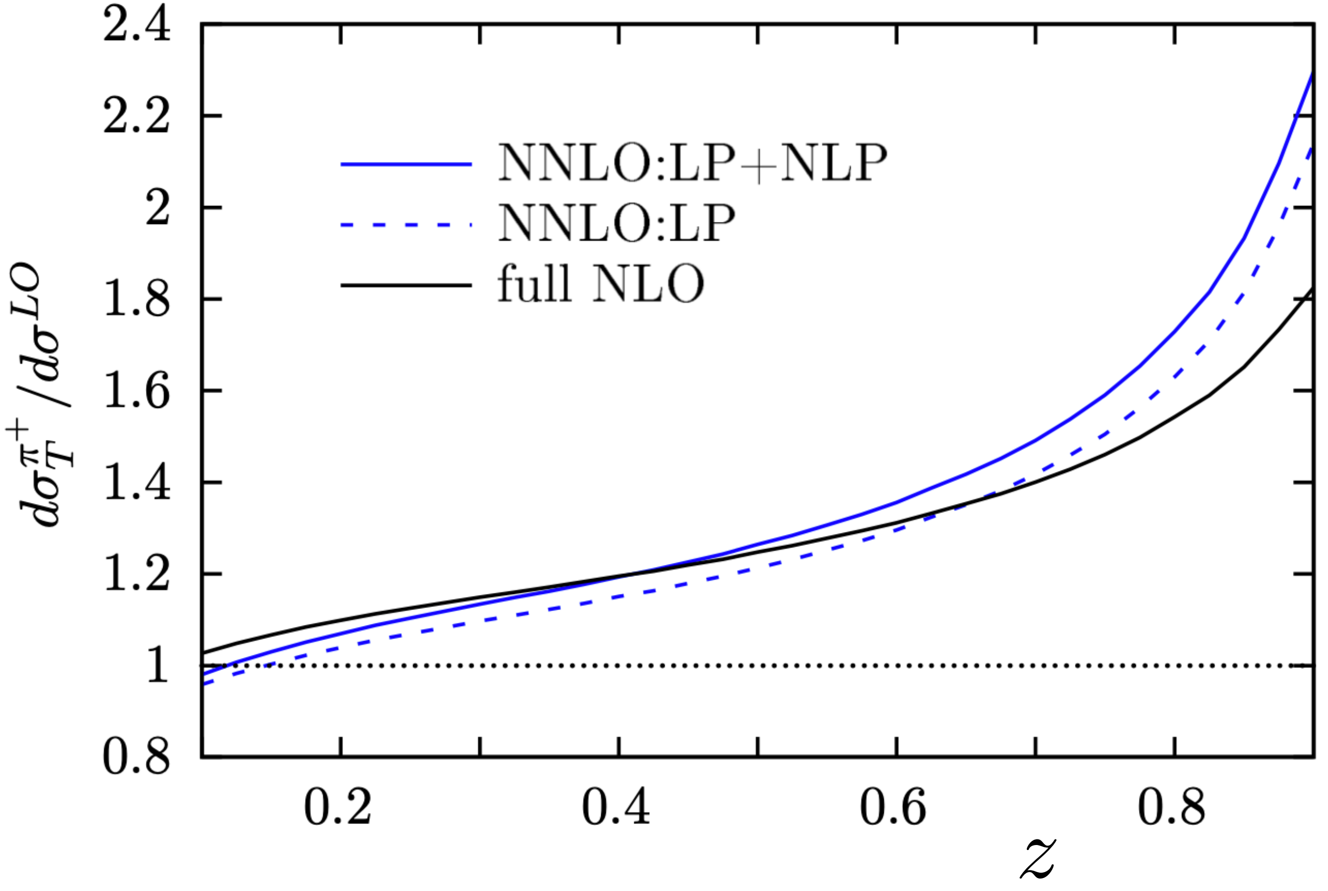
Fixed Order →

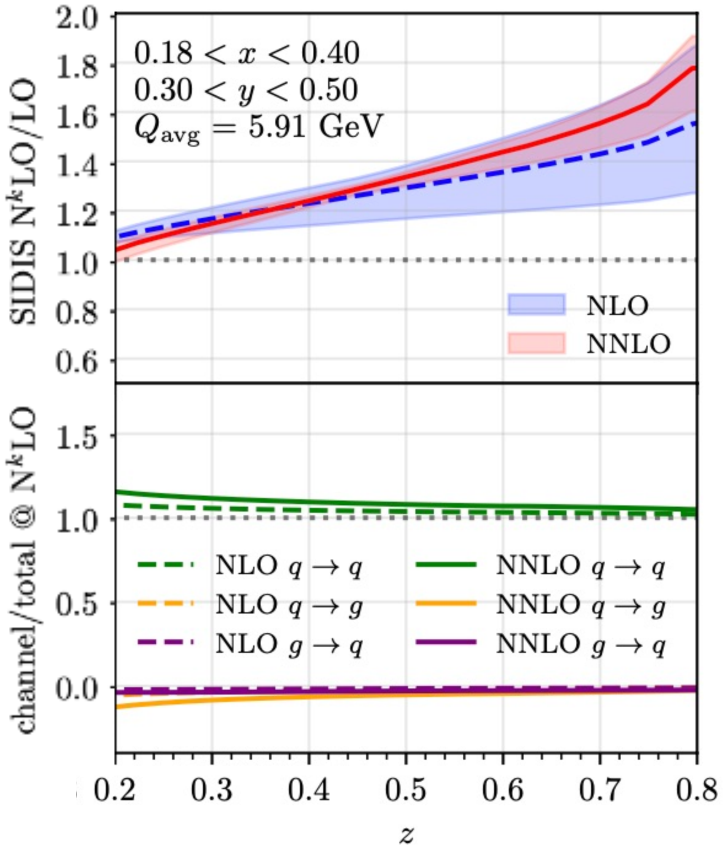
Resummation ↓

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	
N^kLO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

LL NLL NNLL

$\mu\, p \to \mu\, \pi^+\, X$ COMPASS



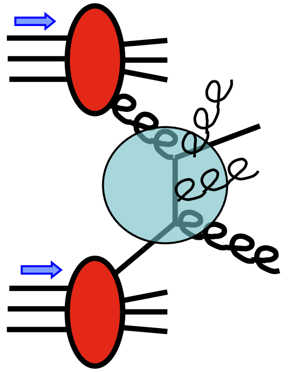


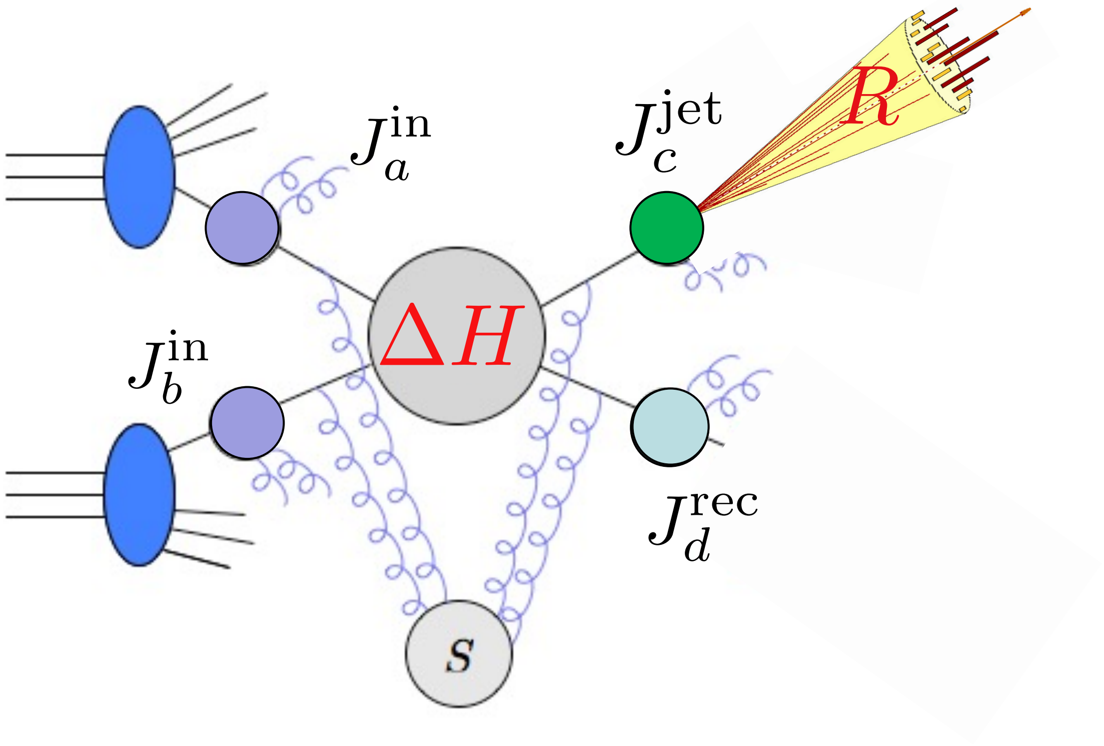
Full NNLO unpolarized:

Bonino, Gehrmann, Stagnitto

Goyal, Moch, Pathak, Rana, Ravindran

Approximate NNLO corrections for pp → jet+X at RHIC:





Kidonakis, Oderda, Sterman
de Florian, WV
Hinderer, Ringer, Sterman, WV, ….

threshold logs $\ln\left(1-\frac{s_{\text{rad}}}{s}\right)$

$$\Delta\hat{\sigma}^{ab\to cd} \sim J_a^{\text{in}} \times J_b^{\text{in}} \times J_c^{\text{jet}} \times J_d^{\text{rec}} \times \text{Tr}\left[\Delta H\, \mathcal{S}^\dagger S \mathcal{S}\right]_{ab\to cd}$$

Global NNLO analysis: results

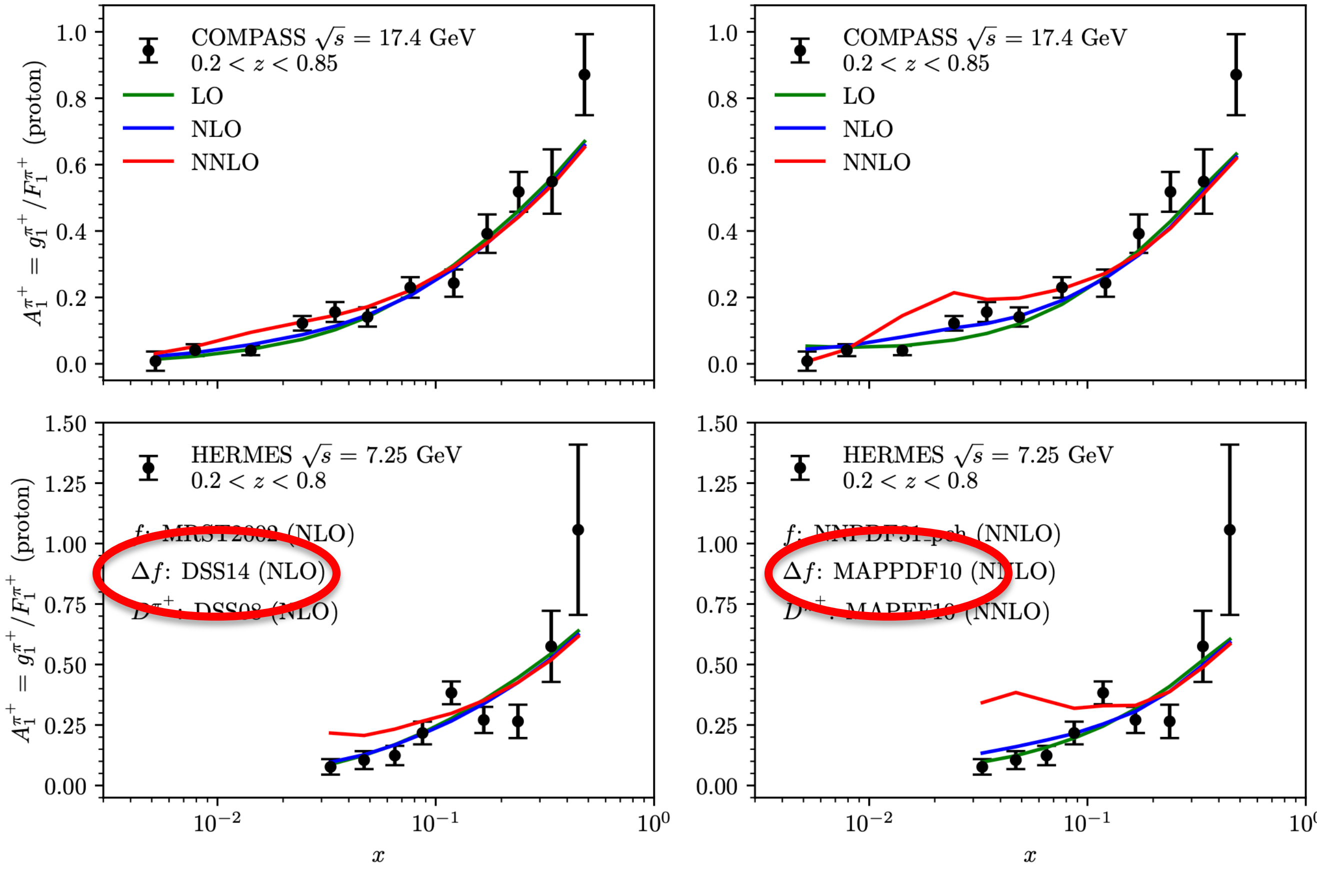
Parameters & data selection:

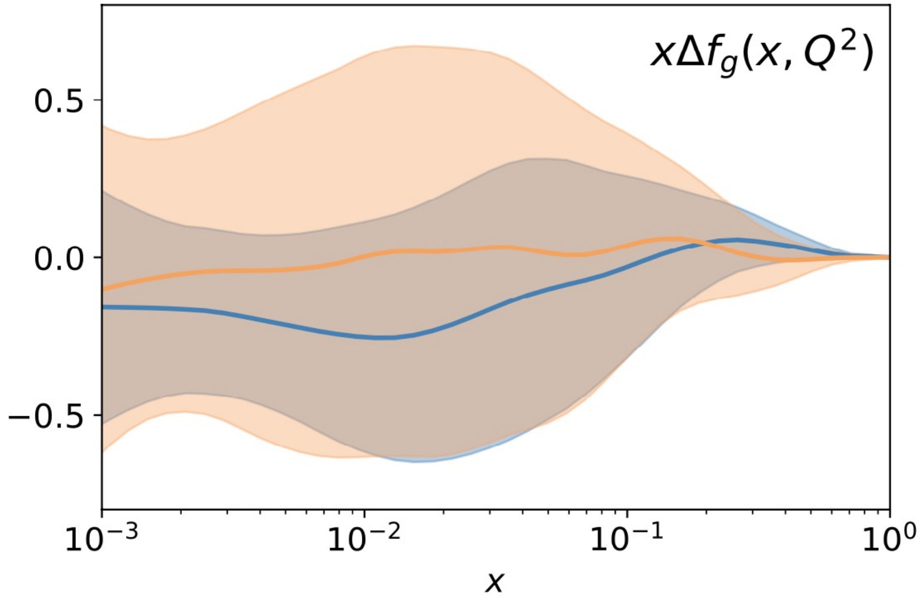
- unpol. PDFs: MSHT20-NNLO
- unpol. FFs: Borsa-de Florian-Sassot-Stratmann NLO 2022,2024
  (BDSSV NNLO fit does not have pp → hadron data)
- data:

DIS:	EMC,SMC,E142,E143,E154,E155, HERMES, COMPASS, HALL-A,CLAS (p,n,d,He)	**378**	
SIDIS:	HERMES, COMPASS (p-$\pi^{\pm}$,d-$\pi^{\pm}$)	**277**	however…
PP-JETS:	STAR run 5,6,9,12,13,15 $(\sqrt{s} = 200, 510\,GeV)$ (no dijets yet)	**91**	
PP-$\pi^0/\pi^{\pm}$:	PHENIX, STAR	**82**	
PP $W^{\pm}$:	PHENIX, STAR	**22**	
Total:		**850**	

- interesting feature for SIDIS:

Bonino et al.





MAP: Bertone, Chiefa, Nocera

- "bump" not present in threshold approximation for SIDIS

  → breakdown of approximation?

- in any case, lower-x SIDIS data involve FFs (and unpol. PDFs) outside regimes where they have been determined (and don't work so well)

- we find:

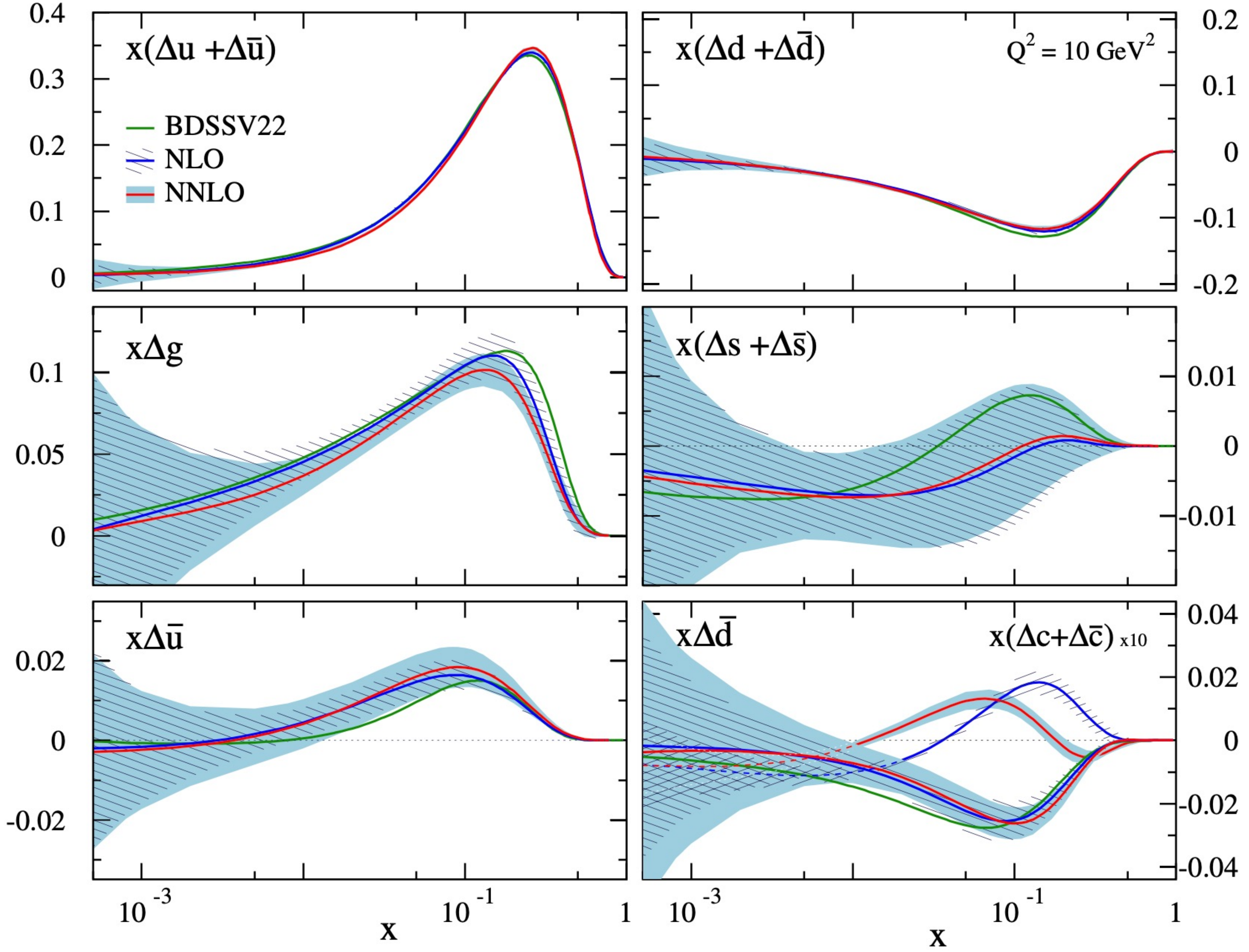
	NLO	NNLO
DIS	304.72	308.74
all SIDIS	276.08	322.46
all pp	199.72	193.91

- introducing cut x>0.12 (and $p_T^h > 2$ GeV):

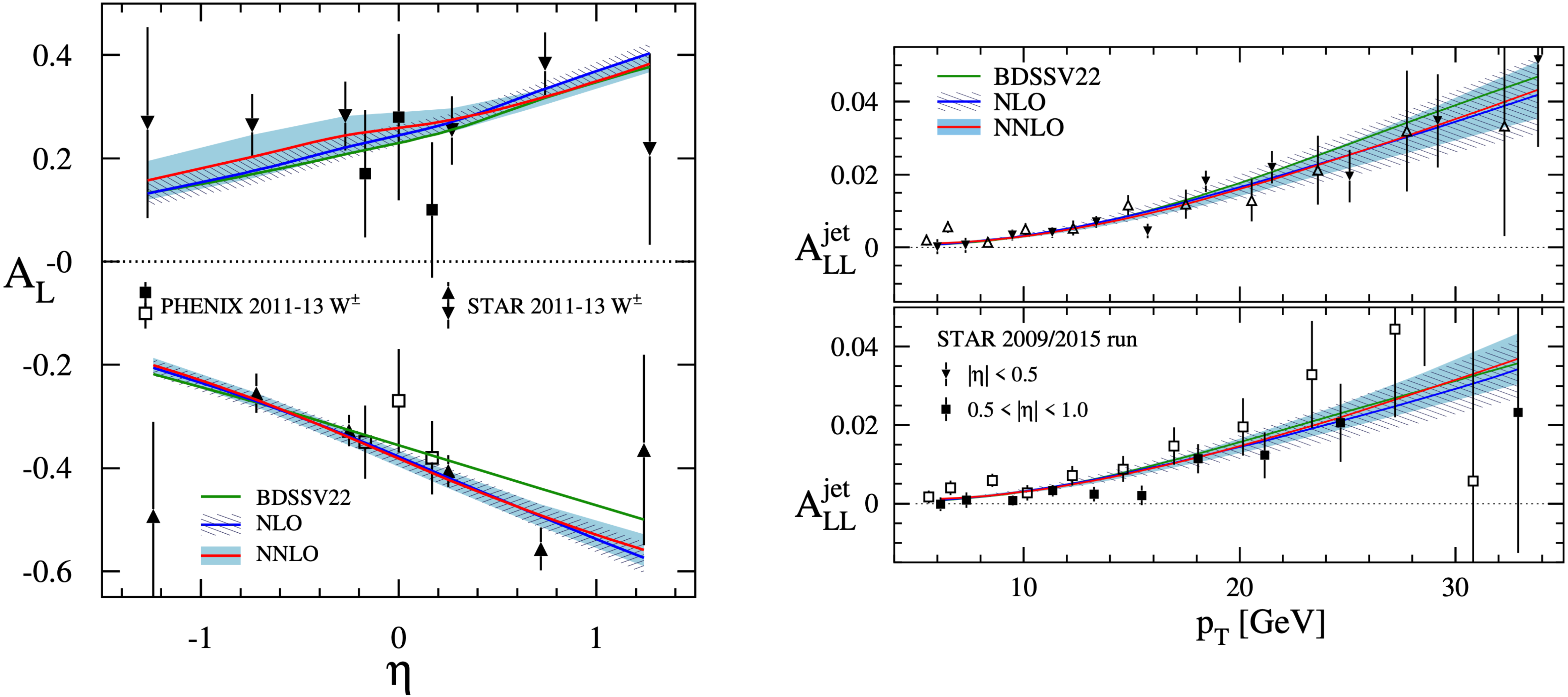
implemented in our analysis to be conservative

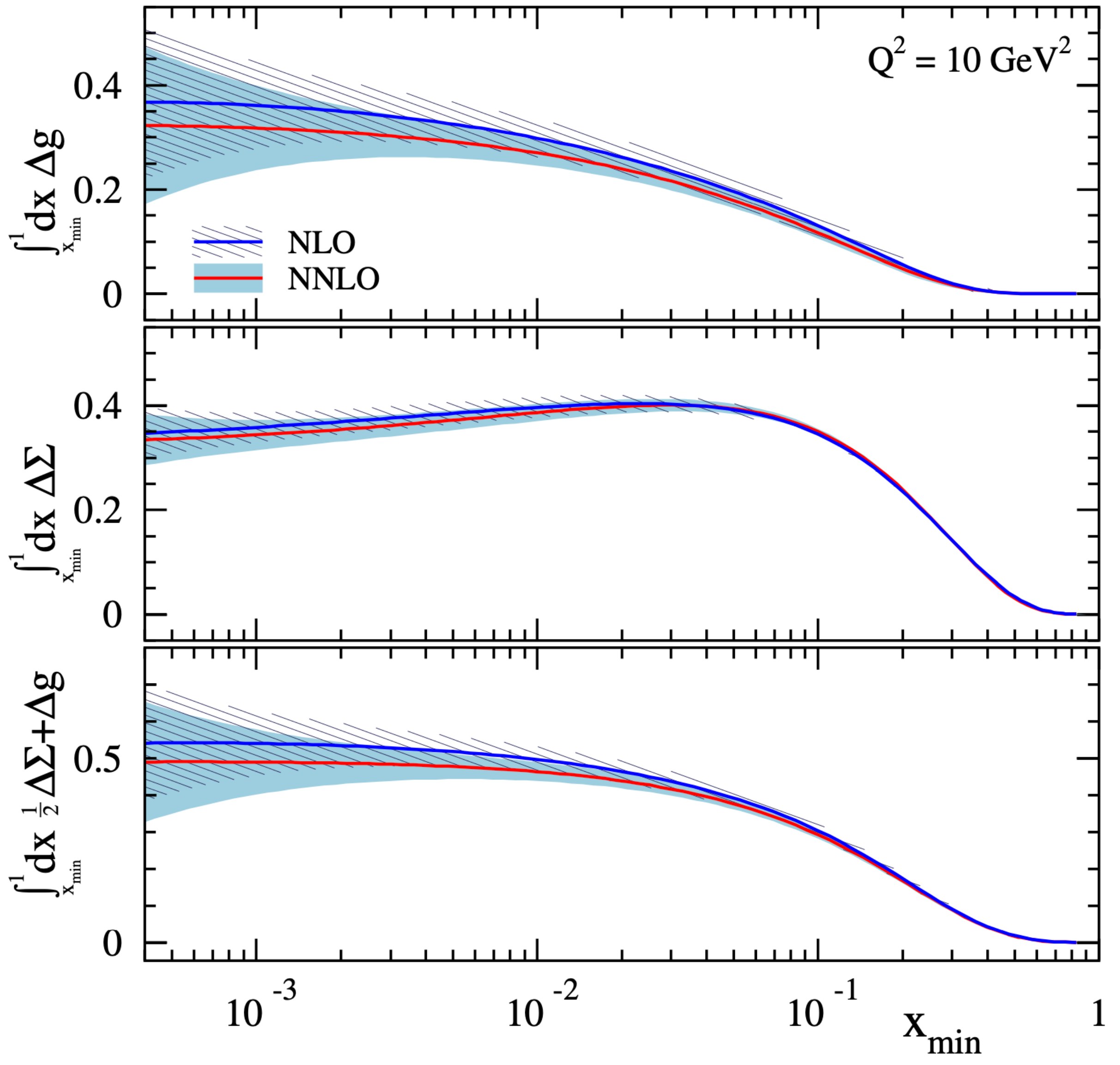
	NLO	NNLO
DIS	302.76	294.48
all SIDIS	126.62	124.42
all pp	196.93	189.90

- similar observation: FF fits and MAP analysis



x(Δu +Δū)
BDSSV22
NLO
NNLO
x(Δd +Δd̄)
Q^2 = 10 GeV^2
xΔg
x(Δs +Δs̄)
xΔū
xΔd̄
x(Δc+Δc̄) x10
x


0.4
0.2
-0
-0.2
-0.4
-0.6
-1
0
1
η
A_L
PHENIX 2011-13 $W^{\pm}$
STAR 2011-13 $W^{\pm}$
BDSSV22
NLO
NNLO
0.04
0.02
0
10
20
30
p_T [GeV]
A_{LL}^{jet}
STAR 2009/2015 run
|η| < 0.5
0.5 < |η| < 1.0



Q^2 = 10 GeV^2
NLO
NNLO
∫_{x_min}^1 dx Δg
∫_{x_min}^1 dx ΔΣ
∫_{x_min}^1 dx ½ΔΣ+Δg
0
0.2
0.4
0.5
10^-3
10^-2
10^-1
1
x_min

Concluding remarks:

- qualitative step forward: first global NNLO analysis of helicity PDFs
- pQCD analysis “in good shape”
- further improvements will come: full SIDIS, improved NNLO for high-p_T
- SIDIS crisis?
- numerous outstanding issues: low-x, power corrections, synergies with lattice