Factorization at twist-3 for polarized SIDIS

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June 3, 2024

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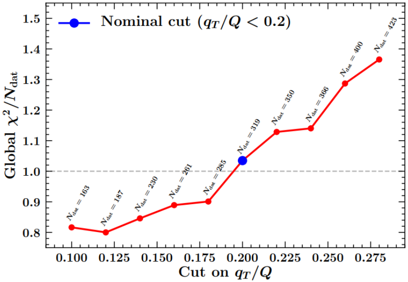
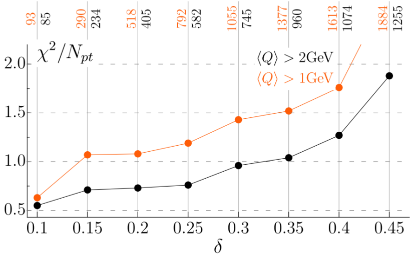
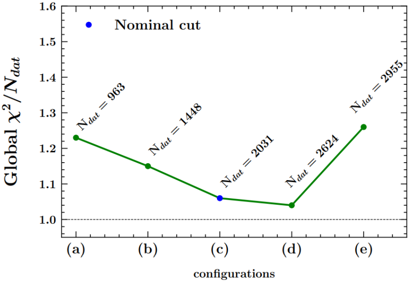
Introduction

Limitation of LP







$$\delta = \frac{|q_T|}{Q} \lesssim 0.25$$

Images courtesy of:

Scimemi, Vladimirov, JHEP 06 (2020) 137

MAP-coll., JHEP 10 (2022) 127

Bacchetta et al., JHEP 07 (2020) 117

Assumptions of LP TMD factorization

1. Massless hadrons ⇒ Lab Frame to Factorization Frame $P^2 = p_h^2 = 0$

$$P^\mu \sim P^+ \bar{n}^\mu \quad p_h^\nu \sim p_h^- n^\nu$$

2. Neglect $\frac{q_T}{Q}$ and $\frac{k_T}{Q}$ corrections

3. Neglect quark-gluon-quark and higher operators

Beyond the LP paradigm

1. Inclusion of kinematic corrections $\frac{k_T}{Q}$
   ✓Resummed to all powers in Vladimirov, JHEP 12 (2023) 008
2. $\frac{q_T}{Q}$ corrections $\Leftrightarrow$ singular behavior for $b \to 0$ in the hard part
3. $\frac{M}{Q}$ corrections, change in the factorization frame, very limited studies
4. Genuine corrections: inclusion of quark-gluon-quark operators

SIDIS beyond LP

~~'generic' twist-3~~

$\bar{\psi}(z_1 n + b)\gamma_T^{\mu}\psi(z_2 n)$

Via EOM

$\partial_T^{\mu}\bar{\psi}(z_1 n + b)\gamma^{+}\psi(z_2 n)$

and

$\bar{\psi}(z_1 n + b)\gamma^{+}F^{\mu +}\psi(z_2 n)$

QED gauge invariance

Boost invariance
$\Leftrightarrow$
rescaling invariance of rapidity scales

Zero longitudinal momentum contributions

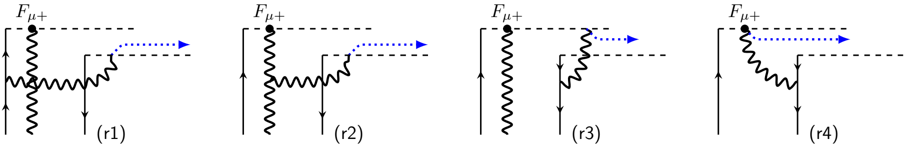
Important terms at zero gluon momentum

$$\widetilde{\Phi}_{\bar{q}q}^{[\Gamma_T]}(z_1, z_2, b) = -\frac{1}{2}\int_L^0 d\tau \frac{\partial}{\partial b^{\mu}}\left(\widetilde{\Phi}_{11}^{[\gamma^{\mu}\gamma^{+}\Gamma_T]}(z_1+\tau, z_2, b) - \widetilde{\Phi}_{11}^{[\Gamma_T\gamma^{+}\gamma^{\mu}]}(z_1, z_2+\tau, b)\right)$$
$$+\frac{i}{2}\int_L^0 d\tau \int_L^{\tau} d\sigma \left(\widetilde{\Phi}_{\mu,21}^{[\gamma^{\mu}\gamma^{+}\Gamma_T]}(z_1+\tau, z_1+\sigma, z_2, b) - \widetilde{\Phi}_{\mu,12}^{[\Gamma_T\gamma^{+}\gamma^{\mu}]}(z_1, z_2+\sigma, z_2+\tau, b)\right)$$

$$
\begin{aligned}
\widetilde{W}^{\mu\nu}_{\text{gNLP}} \;=\;& i\sum_{n,m}\Bigg\{\int_{-\infty}^{\infty} d\hat{u}_1 d\hat{u}_2 d\hat{u}_3\, \delta(\hat{u}_1+\hat{u}_2+\hat{u}_3)\delta(\tilde{x}-\hat{u}_3)\Bigg[\cdots + \\
& T^{\mu\nu\rho}_{-}(\bar{n},n)\left(\mathbb{C}_R(x,\hat{u}_2)\mathbf{\Phi}^{[\Gamma_n^+]}_{\rho,\oplus}\Delta^{[\Gamma_m^-]}_{11} + \pi\mathbb{C}_I(x,\hat{u}_2)\mathbf{\Phi}^{[\Gamma_n^+]}_{\rho,\ominus}\Delta^{[\Gamma_m^-]}_{11}\right)\Bigg] \\
+\;& \int_{-\infty}^{\infty}\frac{d\hat{w}_1 d\hat{w}_2 d\hat{w}_3}{|\hat{w}_1|}\,\delta\left(\frac{1}{\hat{w}_1}+\frac{1}{\hat{w}_2}+\frac{1}{\hat{w}_3}\right)\delta(\tilde{z}-\hat{w}_3)\Bigg[\cdots + \\
& T^{\mu\nu\rho}_{-}(\bar{n},n)\mathbb{C}_2(z,\hat{w}_2)\Phi^{[\Gamma_n^+]}_{11}\mathbf{\Delta}^{[\Gamma_m^-]}_{\rho,\oplus}\Bigg]\Bigg\}
\end{aligned}
$$

Rapidity evolution

Special rapidity divergences



$$\zeta \frac{\partial}{\partial \zeta} F(x_{123}, b, \zeta) = -\mathcal{D}(b, \mu) F(x_{123}, b, \zeta)$$

$$\left[\frac{\partial}{\partial b^\rho} - \frac{\partial_\rho \mathcal{D}}{2} \ln\left(\frac{\zeta}{\bar{\zeta}}\right) \right] \Phi_{11}(x, b, \mu, \zeta)$$

From factorization theorem $\zeta\bar{\zeta} = (Q^2 - q_T^2)^2$

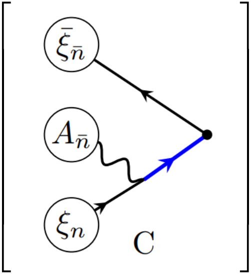
If only the partial derivative the re-scaling $\zeta \to \alpha\zeta \quad \bar{\zeta} \to \frac{\bar{\zeta}}{\alpha}$ would be broken:

$$\frac{\partial}{\partial\alpha}\left\{\Phi_{11}\left(\frac{\zeta}{\alpha}\right)\Delta_{11}(\alpha\bar{\zeta})\right\} = 0$$

$$\frac{\partial}{\partial\alpha}\left\{\left[\partial_T\Phi_{11}\left(\frac{\zeta}{\alpha}\right)\right]\Delta_{11}(\alpha\bar{\zeta})\right\} \propto \partial_\mu\mathcal{D}$$

Complex structure of NLP

Complex coefficient functions



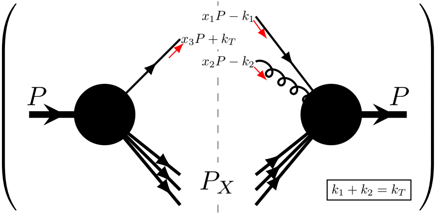
$$= -ig\bar{n}^{\mu}\bar{\xi}_{\bar{n}}(0)\int_{\pm\infty}^{0} d\sigma\gamma_{\nu}F^{\nu+}(\sigma n)\int_{\pm\infty}^{0} dz^{+}\xi_{n}(z^{+}\bar{n})$$

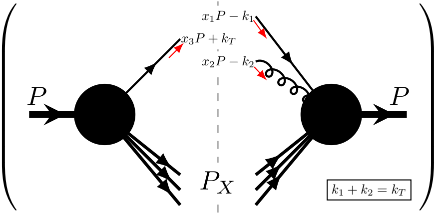
$$\propto \frac{1}{x_g \mp i0} = \frac{1}{(x_g)_+} \pm i\pi\delta(x_g) \equiv \mathbb{C}_R \pm i\pi\mathbb{C}_I$$

Convoluted with

$$\Phi_{21}(x_{1g3}, b) \sim \langle \bar{q}(x_1 n + b) g F^{\mu+}(x_g n + b) q(x_3 n)\rangle = \Phi_{\oplus}(x_{1g3}, b) - i\Phi_{\ominus}(x_{1g3}, b)$$

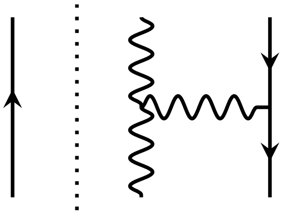
Complex TMDs

$\Phi_{\oplus}(x_1 < 0, x_2 < 0, x_3 > 0, k_T) \propto \mathrm{Re}$  ,

$\Phi_{\ominus}(x_1 < 0, x_2 < 0, x_3 > 0, k_T) \propto \mathrm{Im}$  ,

Complex TMDs

Do not decouple, evolution mix zero- and non-zero- gluon momenta



For fragmentation functions zero parton-momentum is forbidden $\Delta(z_1, 0, z_2) = 0$

$$\bullet = \oplus, \ominus$$

	U	L	T$_{J=0}$	T$_{J=1}$	T$_{J=2}$
U	$f_\bullet^\perp$	$g_\bullet^\perp$		$h_\bullet$	$h_\bullet^\perp$
L	$f_{\bullet L}^\perp$	$g_{\bullet L}^\perp$	$h_{\bullet L}$		$h_{\bullet L}^\perp$
T	$f_{\bullet T}, \quad f_{\bullet T}^\perp$	$g_{\bullet T}, \quad g_{\bullet T}^\perp$	$h_{\bullet T}^{D\perp}$	$h_{\bullet T}^{A\perp}$	$h_{\bullet T}^{S\perp}, \quad h_{\bullet T}^{T\perp}$

How important are these effects?

1. Difficult to estimate: suppressed as $\frac{M}{Q}$ but enhanced in Bessel convolution
2. limited information
3. Only 16 out of 32 distributions in NLO cross-section
4. Only 7 of the 16 have non-vanishing small-b matching at tree-level
5. 8 of the 16 have singular $\frac{1}{\boldsymbol{b}^2}$ matching at one-loop

Tree-level matching & Singular Matching

Only 7 of the 16 have non-vanishing small-b matching at tree-level

$$f_{\oplus,T} = T(x_{1,2,3}) \qquad g_{\ominus,T} = -\Delta T(x_{1,2,3})$$
$$f^{\perp}_{\oplus,L} = x_3 \mathcal{C}\left[u^2 T(y_{1,2,3})\right] \qquad g^{\perp}_{\ominus,L} = -x_3 \mathcal{C}\left[u^2 \Delta T(y_{1,2,3})\right]$$
$$h_{\oplus} = E(x_{1,2,3}) \qquad h_{\ominus,L} = -H(x_{1,2,3}) \qquad h^{D\perp}_{\ominus,T} = -x_3 \mathcal{C}\left[u H(y_{1,2,3})\right]$$

8 of the 16 have singular $\frac{1}{\boldsymbol{b}^2}$ matching at one-loop

$$f^{\perp}_{\oplus L}(x_1,x_2,x_3,b) = \frac{2a_s}{M^2 b^2}\Big[- C_F \frac{x_2}{x_1} g_1(-x_1)\left(\theta(x_2,x_3) - \theta(-x_2,-x_3)\right) + T_F \frac{x_1 - x_3}{x_2} f_g(-x_2)\left(\theta(x_1,x_3) - \theta(-x_1,-x_3)\right)\Big]$$

Tree-level matching

$F_{UU}^{\cos\phi}$	$\frac{4m_h}{Q}\mathcal{J}_1^{[I]}[h_\oplus H_1^\perp]$
$F_{LU}^{\sin\phi}$	$\frac{4m_h}{Q}\mathcal{J}_1^{[R]}[h_\oplus H_1^\perp]$
$F_{UL}^{\sin\phi} =$	$-\frac{4m_h}{Q}\mathcal{J}_1^{[R]}[h_{\ominus L} H_1^\perp] + \frac{2M}{Q}\mathcal{J}_1^{[I]}[D_1 f_{\oplus L}^\perp]$
$F_{LL}^{\cos\phi}$	$-\frac{2M}{Q}\mathcal{J}_1^{[R]}[D_1(f_{\oplus L}^\perp + g_{\ominus L}^\perp)] - \frac{4m_h}{Q}\mathcal{J}_1^{[I]}[h_{\ominus L} H_1^\perp]$
$F_{UT}^{\sin\phi_S}$	$\frac{2M}{Q}\mathcal{J}_0^{[R]}\left[M m_h b^2 H_1^\perp h_{\ominus T}^{D\perp}\right] - \frac{2M}{Q}\mathcal{J}_0^{[I]}\left[D_1 f_{\oplus T}\right]$
$F_{UT}^{\sin 2\phi-\phi_S}$	$-\frac{2m_h}{Q}\mathcal{J}_2^{[R]}[H_1^\perp h_{\ominus T}^{D\perp}]$
$F_{LT}^{\cos\phi_S}$	$\frac{2M}{Q}\mathcal{J}_0^{[R]}\left[D_1(f_{\oplus T} + g_{\ominus T})\right]$

Conclusions

1. LP factorization works well in his domain, can we extend its validity to use already available data?

2. Beyond LP, the structure of factorization is more complex: non-trivial interplay between n and $\bar{n}$ sectors

3. New NLP-specific phenomena:
    1. Complex TMDs: interference amplitudes rather than 'square' amplitudes
    2. Special rapidity divergences: canceled in between terms
    3. Restoration of boost invariance for kinematic NLP