



Updates on transversity extractions

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Transversity 2024 Università di Trieste June 3rd, 2024

U. D'Alesio, CF, A. Prokudin, PLB 803 (2020) 135347 M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712

Introduction - Transversity function

- collinear transversity function $h_1^q(x)$ describes the collinear structure of spin- $\frac{1}{2}$ hadrons at leading twist
- chiral-odd quantity \Rightarrow not accessible in inclusive DIS
- extracted in SIDIS together with Collins FF (TMD framework) or in two-hadron production with dihadron FF (collinear pQCD)
- Soffer Bound [J. Soffer, PRL74 (1995) 1292–1294]

$$|h_1^q(x, Q^2)| \le \frac{1}{2} \left[f_{q/p}(x, Q^2) + g_{1L}^q(x, Q^2) \right] \equiv SB^q(x, Q^2)$$

• bound preserved by Q^2 evolution up to NLO in QCD

[V. Barone, PLB 409 (1997) 499-502; W. Vogelsang, PRD 57 (1998) 1886-1894]



Introduction - tensor charges (I)

• quarks contribute to nucleon tensor charge via the first Mellin moment of the non-singlet quark combination, δq

$$\delta q = \int_0^1 \left[h_1^q(x) - h_1^{\overline{q}}(x) \right] dx$$

• isovector combination of tensor charges, g_T

$$g_T = \delta u - \delta d$$

- δq and g_T computed in lattice QCD as a matrix element over 0 < x < 1, also estimated starting from phenomenological extractions
- g_T is related to BSM effects: a bridge between QCD phenomenology, lattice QCD and BSM physics



Introduction - tensor charges (II)



Adapted from Fig. 3 of [L. Gamberg et al., PRD 106 (2022) 3, 034014]

- different parametrizations for different phenomenological analyses
- experimental data not available for the full x-range
- lattice QCD estimates done with different settings, computed as matrix element over 0 < x < 1



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• adopting

$$h_1^q(x, Q_0^2) \propto SB^q(x, Q_0^2)$$

is very common in phenomenological fits, both in collinear QCD and TMD physics

[M. Anselmino et al., PRD 75 (2007) 054032 & PRD 92 (11) (2015) 114023] [A. Bacchetta, A. Courtoy, M. Radici, JHEP03 (2013) 119; M. Radici, A. Bacchetta, PRL120 (19) (2018) 192001]

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[J. Benel, A. Courtoy, R. Ferro-Hernandez, EPJC 80 (2020) 5, 465]



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 new approach: no automatic fulfillment of the SB in the parametrization, but application of the SB a posteriori

[U. D'Alesio, CF, A. Prokudin, PLB 803 (2020) 135347]



U. D'Alesio, CF, A. Prokudin, PLB 803 (2020) 135347

- global fit of TMD transversity and Collins functions from SIDIS and e^+e^- data
- $h_1^q(x, k_{\perp}^2)$ accessible through SIDIS azimuthal asymmetries:

$$A_{UT}^{\sin(\phi_h + \phi_S)} = \frac{2(1 - y)}{1 + (1 - y)^2} \frac{F_{UT}^{\sin(\phi_h + \phi_S)}}{F_{UU}}$$

where $F_{UU} = C[f_1D_1]$ and $F_{UT}^{\sin(\phi_h + \phi_S)} = C[h_1^q H_1^{\perp q}]$

- Collins function also accessible from $\cos(2\phi_0)$ modulation of $e^+e^- \rightarrow h_1h_2X$ cross sections via $A_0^{UL(C)} \propto C[H_1^{\perp \bar{q}}H_1^{\perp q}]$
- baseline fit: [M. Anselmino et al., PRD 92 (2015) 11, 114023]
- dataset:
 - (a) $A_{UT}^{\sin(\phi_h + \phi_S)}$ data from HERMES and COMPASS
 - (b) $A_0^{UL(C)}$ measurements from BELLE, BABAR and BESIII



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• Gaussian parametrization:

$$\begin{split} h_1^q(x,k_{\perp}^2) &= h_1^q(x) \frac{e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle} , \qquad h_1^q(x,Q_0^2) = \mathcal{N}_q^\tau(x) \, \text{SB}^q(x,Q_0^2) \\ \mathcal{N}_q^\tau(x) &= N_q^\tau x^\alpha (1-x)^\beta \, \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta} , \quad (q = u_v, \, d_v) \end{split}$$

upon constraining

$$|N_q^T| \leq 1$$

SB is automatically fulfilled



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 - remove potential bias in the parametrization
 - test if data compatible with SB
- two cases: "using SB" ($|N_q^T| \le 1$ a posteriori) or "no SB" ($|N_q^T| \le 1$)



$h_1^q \& H_1^{\perp q}$ global fit - results



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- shaded grey areas correspond to regions where data is not available
- almost same $\chi^2_{
 m dof} pprox$ 0.93
- out of 10⁵ MC sets produced for the "noSB" case, \approx 16% fulfill $|N_q^T| \leq 1 \Rightarrow$ sets for "using SB" case
- $h_1^{u_v}(x)$ does not change while relaxing the SB constraint, $h_1^{d_v}(x)$ apparently violates SB
- violation is less than 1σ statistically significant where data is available



$h_1^q \& H_1^{\perp q}$ global fit - tensor charges



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U. D'Alesio, F. Murgia PRD 70 (2004) 074009; M. Anselmino et al., PRD 73 (2006) 014020 L. Gamberg, Z.-B. Kang, PLB 696 (2011) 109; U. D'Alesio et al., PRD 96 (2017) 036011 ...

• complementary data are needed to reduce the extent to which extrapolation for δq is performed

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- A_N in $p^{\uparrow}p \rightarrow h X$:

$$A_{N} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\Delta\sigma}{2d\sigma} \simeq \frac{d\Delta\sigma_{\text{Siv}} + d\Delta\sigma_{\text{Col}}}{2d\sigma}$$

with

$$d\Delta\sigma_{\text{Col}} \propto \sum_{a,b,c,d} h_1^a(x_a, k_{\perp a}) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes d\Delta\sigma^{a^{\uparrow}b \to c^{\uparrow}d} \otimes H_1^{\perp c}(z, k_{\perp h})$$



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- apply Bayesian simultaneous reweighting on Sivers, transversity and Collins function extractions from SIDIS and e^+e^- data

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- The simultaneous reweighting is perfomed on A_N data:
 - BRAHMS for π^{\pm} production at $\sqrt{\mathrm{s}}=$ 200 GeV

allow for a direct flavor separation

- STAR for $\pi^{\rm 0}$ production at $\sqrt{\rm s}=$ 200 GeV
- latest STAR data for non-isolated $\pi^{0}{}^{\prime}{\rm s}$ at $\sqrt{{\rm s}}=$ 200 GeV and $\sqrt{{\rm s}}=$ 500 GeV

kinematics aligned with SIDIS and e^+e^-



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 $0.1 \lesssim x_F \lesssim 0.7$



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- median as cental value, 2σ CL asymmetric uncertainties



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- priors from updated fit of SIDIS and e^+e^- data, including latest HERMES data
- bands for updated fit based on a reduced sample (2000 MC sets) generated with a compression procedure
- SB applied a posteriori

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- uncertainty reduction up to 80 90% for h_1^q at large x
- dominant contribution to A_N from the Collins mechanism

not seen before SB application a posteriori



A_N simultaneous reweigthing - tensor charges

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 consistency of different h^q₁ extractions within different approaches exploiting a variety of experimental data



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- we proposed a new approach for the application of positivity bounds in phenomenological fits that allows to:
 - properly explore the parameter space
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Thank you





Fit results - Collins function

• parametrization:

$$\begin{split} H_1^{\perp q}(z,p_{\perp}^2) &= \mathcal{N}_q^{\mathsf{C}}(z) \frac{zm_h}{M_{\mathsf{C}}} \sqrt{2e} \, e^{-p_{\perp}^2/M_{\mathsf{C}}^2} \, \mathcal{D}_{h/q}(z,p_{\perp}^2) \,, \, (q = \mathsf{fav},\mathsf{unf}) \\ \mathcal{N}_{\mathsf{fav}}^{\mathsf{C}}(z) &= \mathcal{N}_{\mathsf{fav}}^{\mathsf{C}} z^{\gamma} (1-z)^{\delta} \frac{(\gamma+\delta)^{\gamma+\delta}}{\gamma^{\gamma} \delta^{\delta}} \,, \qquad \mathcal{N}_{\mathsf{unf}}^{\mathsf{C}}(z) = \mathcal{N}_{\mathsf{unf}}^{\mathsf{C}} \end{split}$$

 Collins function mostly constrained by e⁺e⁻ data essentially no change between "using SB" and "no SB" cases



$$H_{1}^{\perp(1)\,q}(z) = z^{2} \int d^{2}\boldsymbol{p}_{\perp} \frac{p_{\perp}^{2}}{2m_{h}^{2}} H_{1}^{\perp q}(z, z^{2}p_{\perp}^{2})$$
$$= \sqrt{\frac{e}{2}} \frac{1}{zm_{h}} \frac{M_{C}^{3} \langle p_{\perp}^{2} \rangle}{\left(\langle p_{\perp}^{2} \rangle + M_{C}^{2} \right)^{2}} \mathcal{N}_{q}^{C}(z) D_{h/q}(z)$$



A_N simultaneous reweigthing - priors - Collins

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A_N and transversity

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- $p^{\uparrow}p \rightarrow hX$ processes can be described within the GPM, where a factorized formulation in terms of TMDs is assumed as a starting point
- a color gauge invariant formulation of GPM (CGI-GPM) was developed, with inclusion of initial and final state interaction; process dependence of the Sivers function is recovered
- A_N in $p^{\uparrow}p \rightarrow h X$:

$$A_{\rm N} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\Delta\sigma}{2d\sigma} \simeq \frac{d\Delta\sigma_{\rm Siv} + d\Delta\sigma_{\rm Col}}{2d\sigma}$$

with

$$\begin{split} d\Delta\sigma_{\text{Siv}}^{\text{CGI-GPM}} &\propto \sum_{a,b,c,d} f_{1T}^{\perp a}(\mathbf{x}_{a},\mathbf{k}_{\perp a}) \otimes f_{b/p}(\mathbf{x}_{b},\mathbf{k}_{\perp b}) \otimes H_{ab \rightarrow cd}^{\text{Inc}} \otimes D_{h/c}(\mathbf{z},\mathbf{k}_{\perp h}) \\ d\Delta\sigma_{\text{Col}} &\propto \sum_{a,b,c,d} h_{1}^{a}(\mathbf{x}_{a},\mathbf{k}_{\perp a}) \otimes f_{b/p}(\mathbf{x}_{b},\mathbf{k}_{\perp b}) \otimes d\Delta\sigma^{a^{\uparrow}b \rightarrow c^{\uparrow}d} \otimes H_{1}^{\perp c}(\mathbf{z},\mathbf{k}_{\perp h}) \end{split}$$

and

$$d\sigma \propto \sum_{a,b,c,d} f_{a/p}(x_a,k_{\perp a}) \otimes f_{b/p}(x_b,k_{\perp b}) \otimes H^U_{ab \to cd} \otimes \mathsf{D}_{h/c}(z,k_{\perp h})$$

- GPM results (Sivers): $H_{ab \rightarrow cd}^{lnc} \rightarrow H_{ab \rightarrow cd}^{U}$
- gluon Sivers effect negligible in the region of moderate and forward rapidity



Results - BRAHMS

J. H. Lee, F. Videbæk, AIP Conf. Proc. 915, 533–538 (2007) M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



- reweighted curves with reduced uncertainties
- GPM describes these data better than CGI-GPM
- quality of description increases if data with $P_T < 1.5$ GeV (gray points) is not considered



Results - STAR (I)

B. I. Abelev et al., PRL 101, 222001 (2008); J. Adams et al., PRL 92 171801 (2004); L. Adamczyk et al., PRD 86 (2012) 051101 M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 854 (2024) 138712



- both GPM and CGI-GPM in qualitative agreement with the data
- reweighted bands able to describe data at moderate x_F
- shape better representing the steady increase of A_N at large x_F



Results - STAR (II)



STAR, $p^{\uparrow}p \rightarrow \pi^0 X$, 2.7 < η < 4.0



- data not showing the usual steady increase at large x_F
- reweighted curves describe the data
- if reweighting was performed on these data solely, bands would be flatter

