

Transversity from Dihadron Transverse-Spin Observables

Nobuo Sato

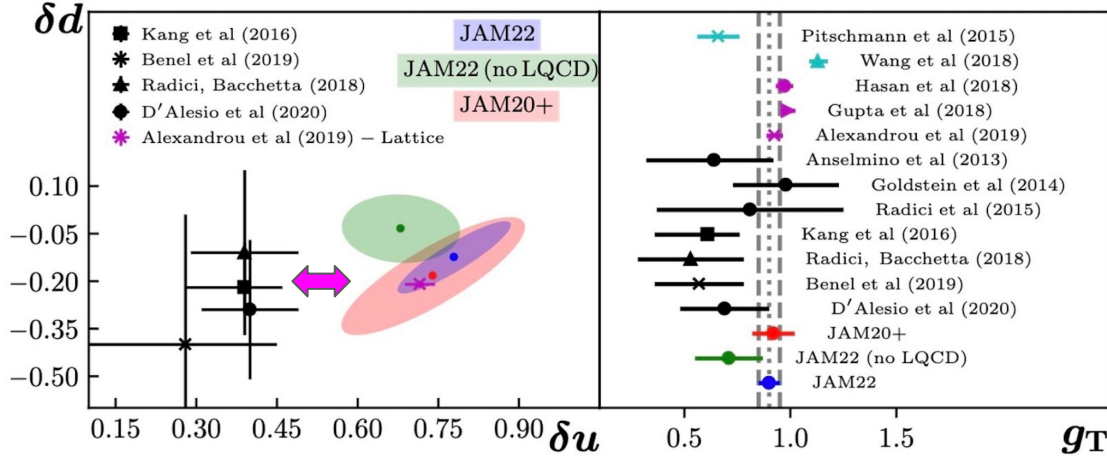
Pitonyak, Cocuzza, Metz, Prokudin, NS, '24 (PRL)
Cocuzza, Metz, Pitonyak, Prokudin, NS, Seidl '24 (PRL)
Cocuzza, Metz, Pitonyak, Prokudin, NS, Seidl '24 (PRD)



Transversity,
Trieste IT Jun 3 2024

Jefferson Lab

Motivations



- TMD+CT3 pheno in tension with other analyzes (δu)
- Radici, Bacchetta, and Benel, Courtoy, Ferro-Hernandez used collinear di-hadron observables to extract tensor charges
- **New fresh look at collinear di-hadron pheno**

Observable	Reactions	Non-Perturbative Function(s)	χ^2/npts	Exp. Refs.
$A_{UT}^{\sin(\phi_h - \phi_S)}$	$e + (p, d)^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	182.9/166 = 1.10	[22, 24, 27]
$A_{UT}^{\sin(\phi_h + \phi_S)}$	$e + (p, d)^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$h_1(x, \vec{k}_T^2), H_1^\perp(z, z^2 \vec{p}_T^2)$	181.0/166 = 1.09	[22, 24, 27]
$*A_{UT}^{\sin \phi_S}$	$e + p^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$h_1(x), \tilde{H}(z)$	18.6/36 = 0.52	[22, 24, 27]
$A_{UC/UL}$	$e^+ + e^- \rightarrow \pi^+ \pi^- (UC, UL) + X$	$H_1^\perp(z, z^2 \vec{p}_T^2)$	154.9/176 = 0.88	[29–32]
$A_{T, \mu^+ \mu^-}^{\sin \phi_S}$	$\pi^- + p^\uparrow \rightarrow \mu^+ \mu^- + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	6.92/12 = 0.58	[34]
$A_N^{W/Z}$	$p^\uparrow + p \rightarrow (W^+, W^-, Z) + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	30.8/17 = 1.81	[35]
A_N^π	$p^\uparrow + p \rightarrow (\pi^+, \pi^-, \pi^0) + X$	$h_1(x), F_{FT}(x, x) = \frac{1}{\pi} f_{1T}^{\perp(1)}(x), H_1^{\perp(1)}(z), \tilde{H}(z)$	70.4/60 = 1.17	[7, 9, 10, 13]
Lattice g_T	—	$h_1(x)$	1.82/1 = 1.82	[89]

Analysis setup

Collaboration	References	Observable	Process	Nonperturbative function(s)
Belle	[64]	$d\sigma/dz dM_h$	$e^+e^- \rightarrow (\pi^+\pi^-)X$	D_1
Belle	[112]	$A^{e^+e^-}$	$e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)X$	D_1, H_1^{\triangleleft}
HERMES	[118]	A_{UT}^{SIDIS}	$ep^\uparrow \rightarrow e'(\pi^+\pi^-)X$	$D_1, H_1^{\triangleleft}, h_1$
COMPASS	[117]	A_{UT}^{SIDIS}	$\mu\{p, D\}^\uparrow \rightarrow \mu'(\pi^+\pi^-)X$	$D_1, H_1^{\triangleleft}, h_1$
STAR	[97,121]	A_{UT}^{pp}	$p^\uparrow p \rightarrow (\pi^+\pi^-)X$	$D_1, H_1^{\triangleleft}, h_1$
ETMC	[77]	$\delta u, \delta d$	LQCD	h_1
PNDME	[71]	$\delta u, \delta d$	LQCD	h_1

$$\frac{d\sigma}{dz dM_h} = \frac{4\pi N_c \alpha_{\text{em}}^2}{3s} \sum_q \bar{e}_q^2 D_1^q(z, M_h)$$

$h_1(x; \mu^2)$ Transversity (TPDF)

$$A^{e^+e^-}(z, M_h, \bar{z}, \bar{M}_h) = \frac{\sin^2 \theta \sum_q e_q^2 H_1^{\triangleleft, q}(z, M_h) H_1^{\triangleleft, \bar{q}}(\bar{z}, \bar{M}_h)}{(1 + \cos^2 \theta) \sum_q e_q^2 D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \bar{M}_h)}$$

$H_1^{\triangleleft}(z, M_h; \mu^2)$ Interference FF (IFF)

$D_1(z, M_h; \mu^2)$ Dihadron FF (DiFFs)

$$A_{UT}^{\text{SIDIS}} = c(y) \frac{\sum_q e_q^2 h_1^q(x) H_1^{\triangleleft, q}(z, M_h)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h)}$$

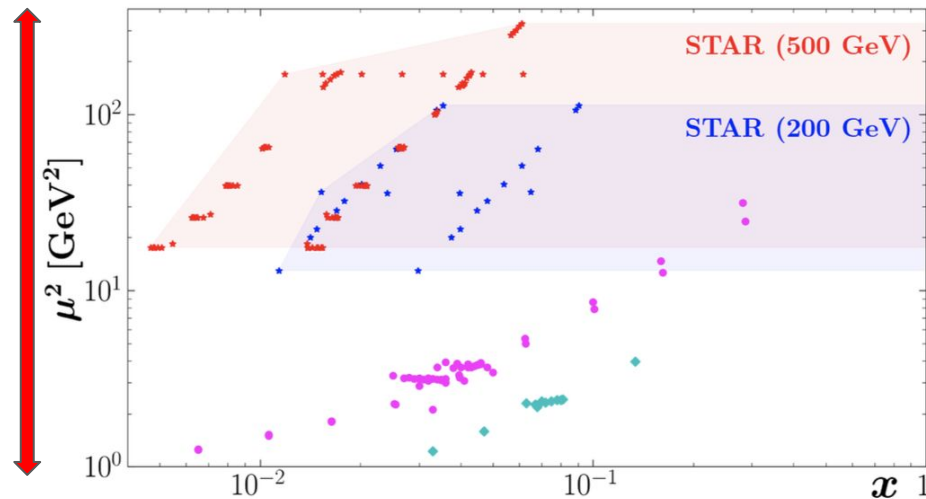
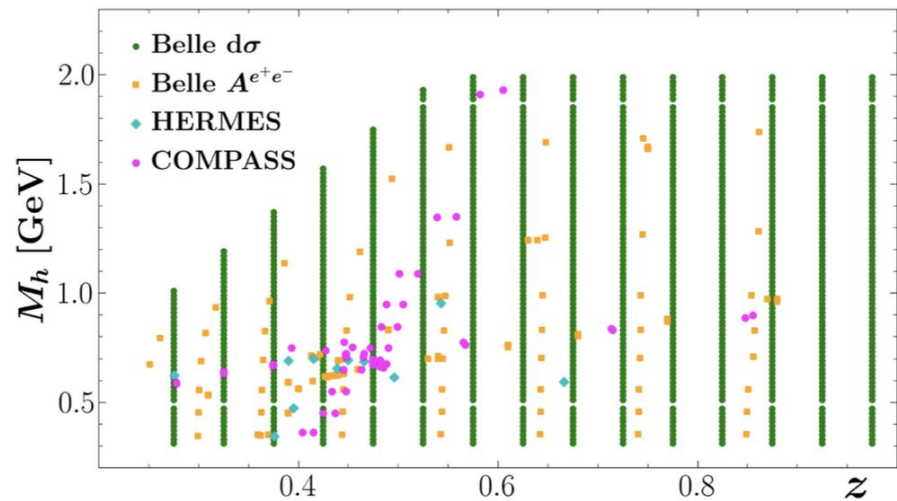
$$A_{UT}^{pp} = \frac{2P_{hT} \sum_i \sum_{a,b,c,d} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 \frac{dx_b}{z} h_1^a(x_a) f_1^b(x_b) \frac{d\Delta\hat{\sigma}_{a^\uparrow b \rightarrow c^\uparrow d}}{d\hat{t}} H_1^{\triangleleft, c}(z, M_h)}{2P_{hT} \sum_i \sum_{a,b,c,d} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 \frac{dx_b}{z} f_1^a(x_a) f_1^b(x_b) \frac{d\hat{\sigma}_{ab \rightarrow cd}}{d\hat{t}} D_1^c(z, M_h)}$$

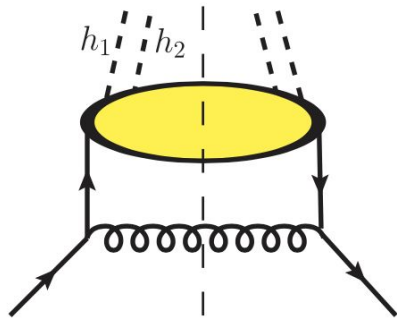
$$h_1(x; \mu^2)$$

$$H_1^{\triangleleft}(z, M_h; \mu^2)$$

$$D_1(z, M_h; \mu^2)$$

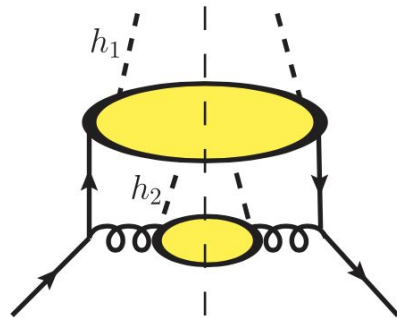
Need to
address scale
dependence





(a)

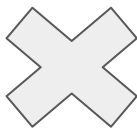
homogeneous



(b)

inhomogeneous

- We work at LO in pQCD hence we only consider homogeneous evolution
- The scale dependence obeys standard timelike DGLAP evolution
- DGLAP only changes the z dependence



$$\frac{\partial D_1^{h_1 h_2 / i}(z, M_h; \mu^2)}{\partial \mu^2} = \sum_j \int_z^1 \frac{dw}{w} D_1^{h_1 h_2 / i} \left(\frac{z}{w}, M_h; \mu^2 \right) P_{i \rightarrow j}(w)$$

Same for $H_1^{\leftarrow}(z, M_h; \mu^2)$ with transversely polarized splitting kernels

Model assumptions in TPDF $h_1(x; \mu_0^2)$

- Traditional parametrization

$$h_1^i(x) = \frac{N^i}{\mathcal{M}^i} x^{\alpha^i} (1-x)^{\beta^i} (1 + \gamma^i \sqrt{x} + \delta^i x)$$

- Reconstructed flavors

$$h_1^{u_v} \quad h_1^{d_v} \quad h_1^{\bar{u}_v} \quad h_1^{\bar{d}_v}$$

- Flavor assumptions (due to lack of observables)

$$h_1^{\bar{u}} = -h_1^{\bar{d}} \quad \text{Expectations from large } N_c \text{ limit}$$

- Impose Soffer bounds

$$|h_1^i(x; \mu)| \leq \frac{1}{2} [f_1^i(x; \mu) + g_1^i(x; \mu)]$$

- Impose small-x constraints (Kovchegov, Sievert '19)

$$\alpha^i \xrightarrow{x \rightarrow 0} 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} = 0.170 \pm 0.085$$

Added 50% conservative uncertainties

Model assumptions in DiFFs

$$D_1^{\pi^+ \pi^- / i}(z, M_h; \mu_0^2)$$

- Mh grid based parametrization

$$D_1^i(z, \mathbf{M}_h^{i,j}) = \sum_{k=1,2,3} \frac{N_{jk}^i}{\mathcal{M}_{jk}^i} z^{\alpha_{jk}^i} (1-z)^{\beta_{jk}^i}$$

- Reconstructed flavors

$$D_1^u \quad D_1^s \quad D_1^c \quad D_1^b \quad D_1^g$$

- Flavor assumptions

$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$

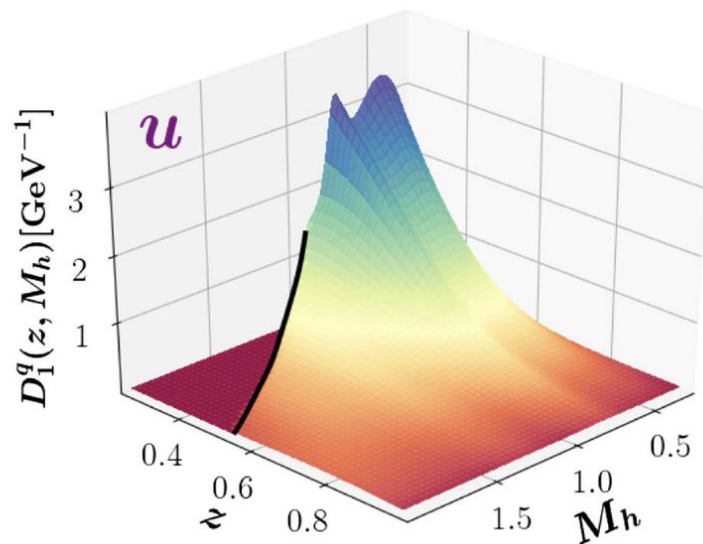
$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

- Flavor separation using Pythia 6&8

$$\frac{\sigma^{q=s,c,b}}{\sigma^{\text{tot}}}$$



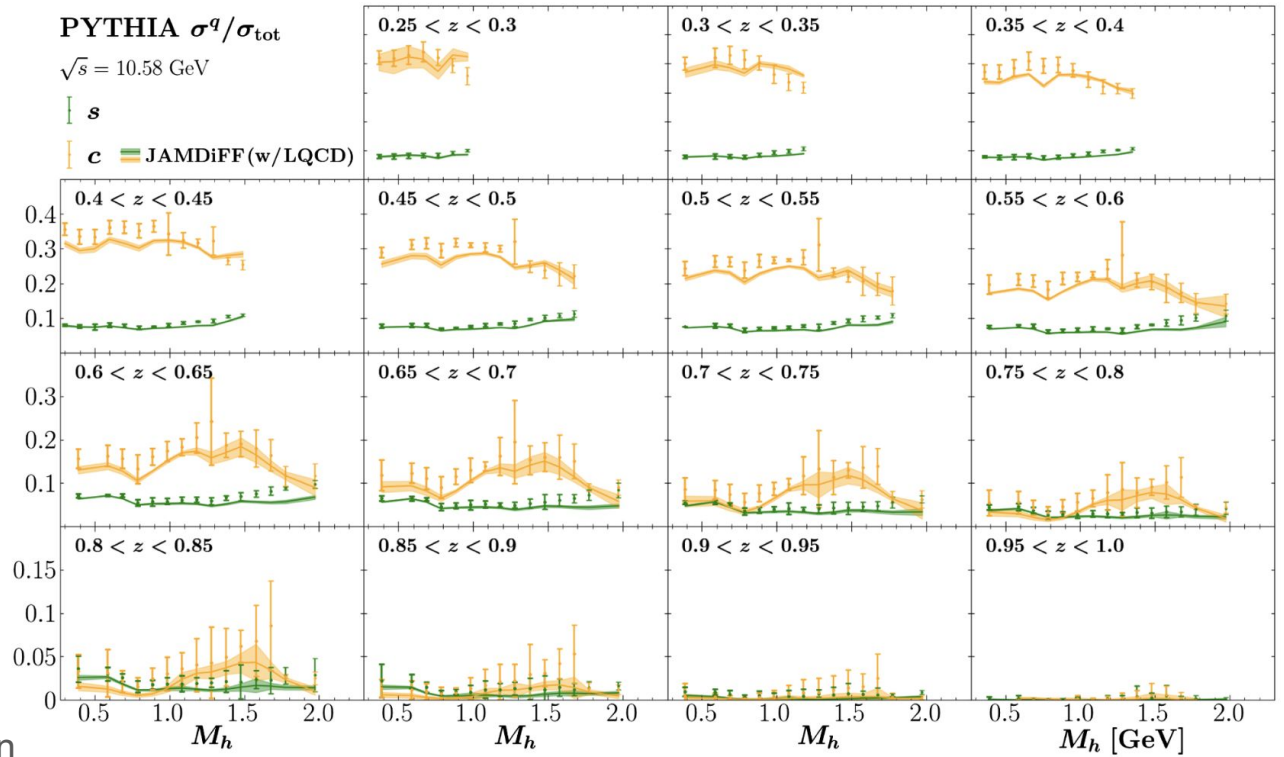
Generate Pythia data and assign uncertainties from various tunes



- Impose positivity

$$D_1^i(z, M_h; \mu) > 0,$$

- Error bars on Pythia stemming from different tunes.
- Simulated Pythia data at different energies to constrain the gluon DiFF Q=10.58 - 91.19 GeV



Model assumptions in IFFs

$$H_1^{\triangleleft, \pi^+ \pi^- / i}(z, M_h; \mu_0^2)$$

- M_h grid based parametrization

$$H_1^{\triangleleft, u}(z, M_h^{u, j}) = \sum_{k=1,2} \frac{N_{jk}^u}{\mathcal{M}_{jk}^u} z^{\alpha_{jk}^u} (1-z)^{\beta_{jk}^u}$$

- Reconstructed flavors

$$H_1^{\triangleleft, u}$$

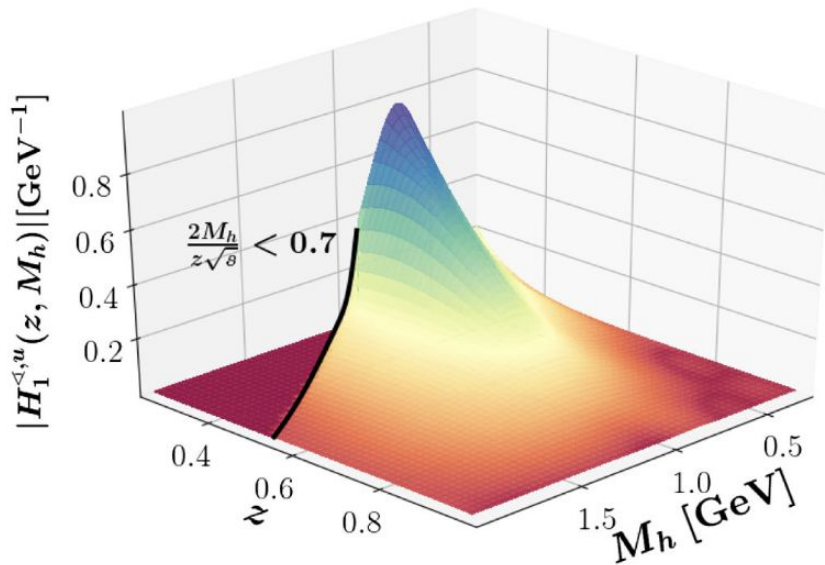
- Flavor assumptions

$$H_1^{\triangleleft, u} = -H_1^{\triangleleft, d} = -H_1^{\triangleleft, \bar{u}} = H_1^{\triangleleft, \bar{d}},$$

$$H_1^{\triangleleft, s} = H_1^{\triangleleft, \bar{s}} = H_1^{\triangleleft, c} = H_1^{\triangleleft, \bar{c}} = H_1^{\triangleleft, b} = H_1^{\triangleleft, \bar{b}} = 0.$$

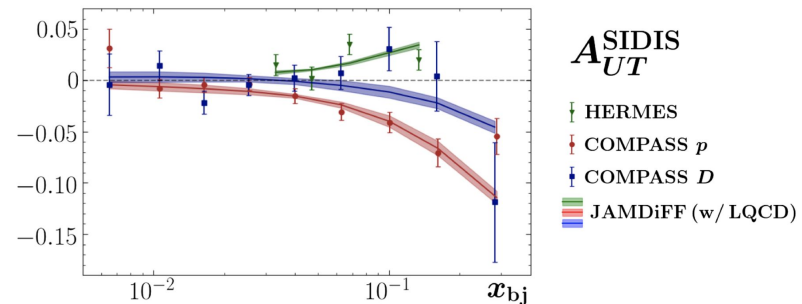
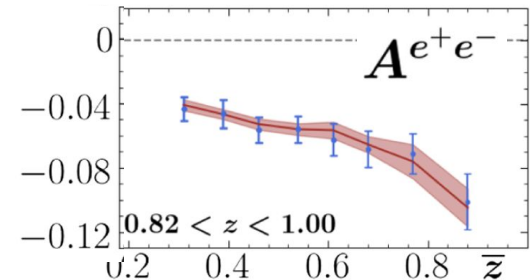
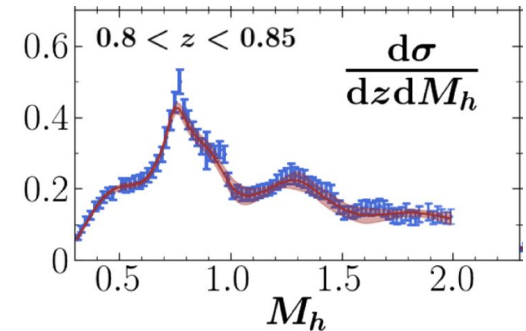
- Impose positivity bounds

$$|H_1^{\triangleleft, i}(z, M_h; \mu)| < D_1^i(z, M_h; \mu)$$

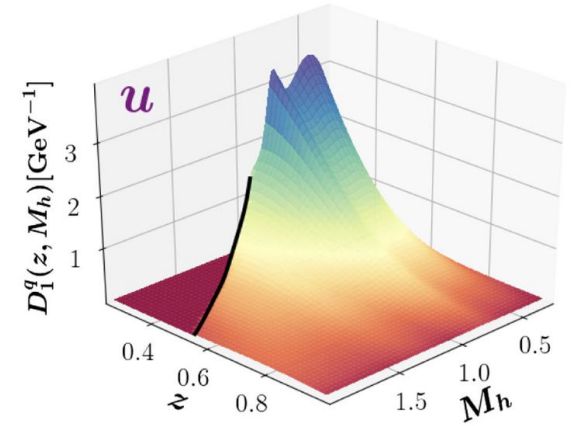
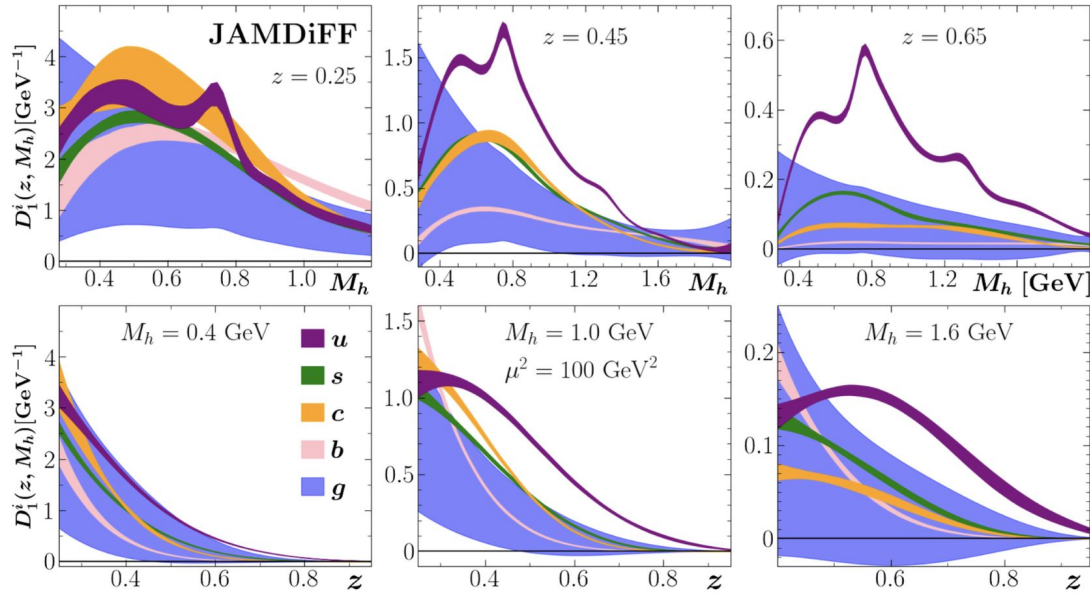


The results

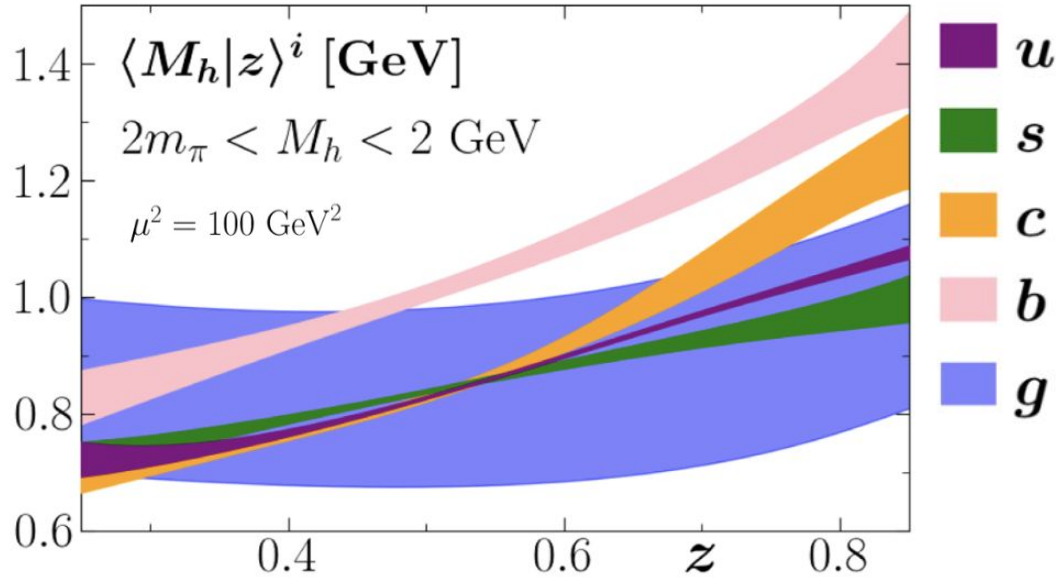
Experiment	Binning	N_{dat}	χ^2_{red}		
			JAMDiFF		
			(w/ LQCD)	(no LQCD)	(SIDIS only)
Belle (cross section) [64]	z, M_h	1094	1.01	1.01	1.01
	z, M_h	55	1.27	1.24	1.28
Belle (Artru-Collins) [112]	M_h, \bar{M}_h	64	0.60	0.60	0.60
	z, \bar{z}	64	0.42	0.42	0.41
HERMES [118]	x_{bj}	4	1.77	1.70	1.67
	M_h	4	0.41	0.42	0.47
	z	4	1.20	1.17	1.13
COMPASS (p) [117]	x_{bj}	9	1.98	0.65	0.59
	M_h	10	0.92	0.94	0.93
	z	7	0.77	0.60	0.63
COMPASS (D) [117]	x_{bj}	9	1.37	1.42	1.22
	M_h	10	0.45	0.37	0.38
	z	7	0.50	0.46	0.46
STAR [121]	$M_h, \eta < 0$	5	2.57	2.56	
	$M_h, \eta > 0$	5	1.34	1.55	
	$\sqrt{s} = 200 \text{ GeV}$	5	0.98	1.00	
	$R < 0.3$	5	1.73	1.74	
	η	4	0.52	1.46	
STAR [97]	$M_h, \eta < 0$	32	1.30	1.10	
	$M_h, \eta > 0$	32	0.81	0.78	
	$\sqrt{s} = 500 \text{ GeV}$	35	1.09	1.07	
	$R < 0.7$	7	2.97	1.83	
ETMC δu [77]		1	0.71		
ETMC δd [77]		1	1.02		
PNDME δu [71]		1	8.68		
PNDME δd [71]		1	0.04		
Total $\chi^2_{\text{red}} (N_{\text{dat}})$			1.01 (1475)	0.98 (1471)	0.96 (1341)



Reconstructed DiFFs



- More di-hadron pairs are produced at small z
- Clear reconstruction of the resonances
- DiFFs trend towards zero as we approach the physical threshold of $2m_{\pi}$
- DiFFs goes to zero at large M_h as wide angle radiation is suppressed
- The gluon DiFF is not fully consistent with zero and more realistic than what was done in the past.



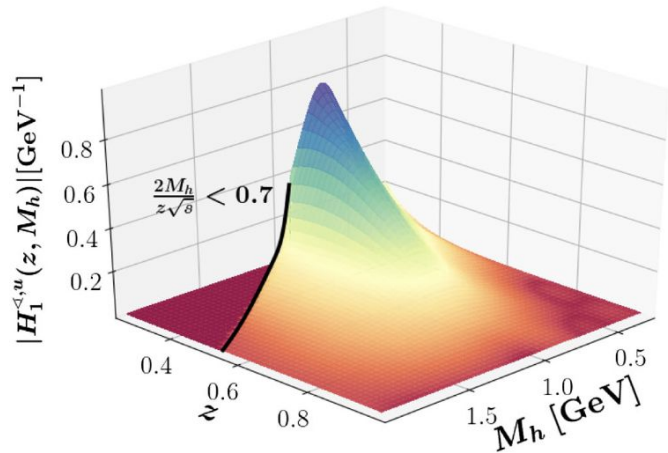
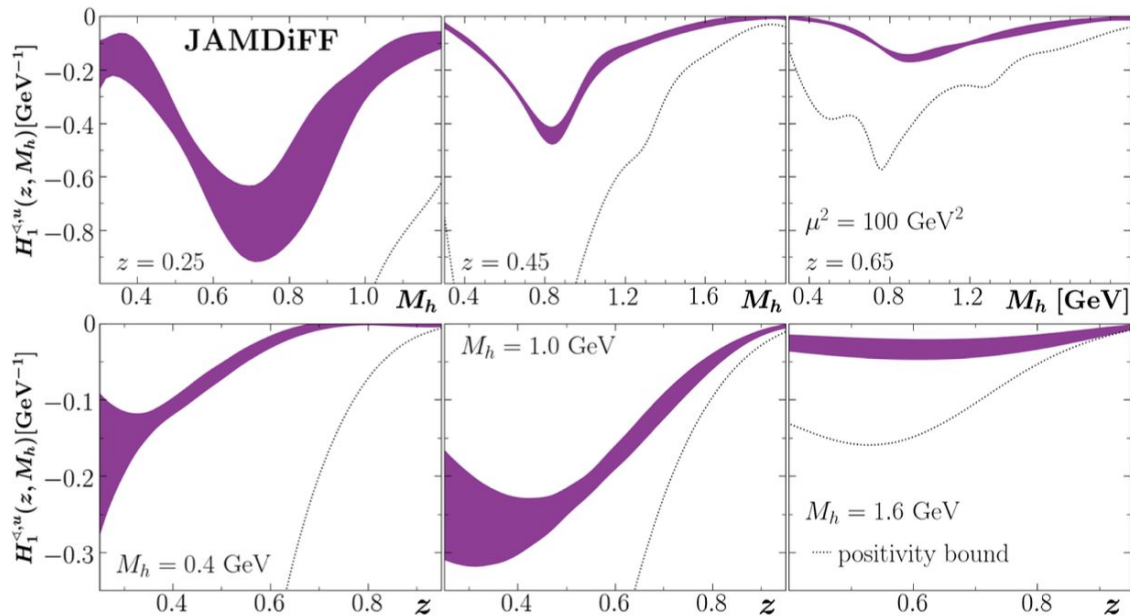
$$z = z_1 + z_2$$

$$M_h^2 = (P_{\pi^+} + P_{\pi^-})^2$$

$$\langle M_h | z \rangle^i = \frac{\int dM_h M_h D_1^i(z, M_h)}{\int dM_h D_1^i(z, M_h)}$$

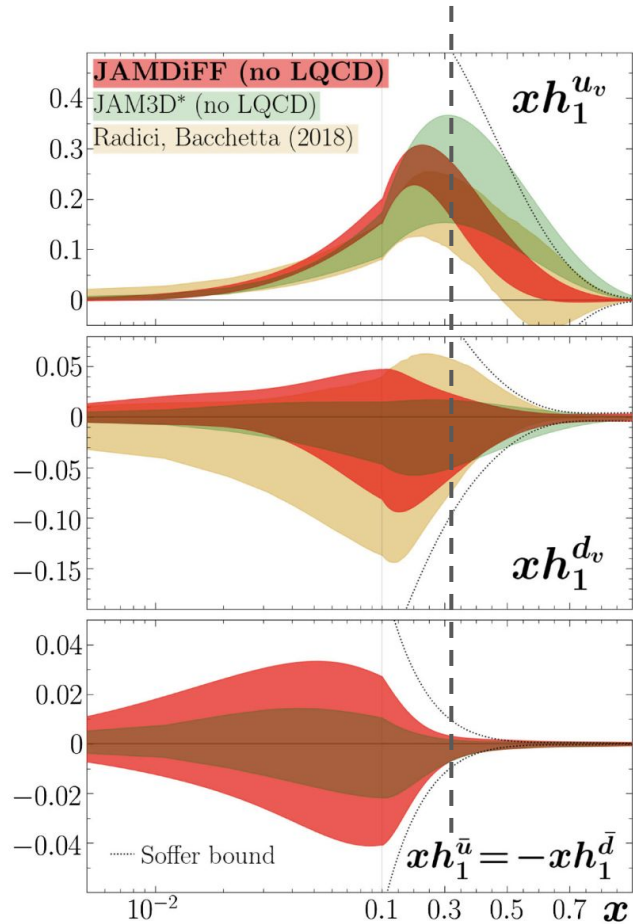
- As expected, the invariant mass of the di-hadron pairs grows as z grows. Quark mass dependence becomes more important at large z
- At small z , large production of di-hadron pairs occurs, and quark masses does not influence the values of $\langle M_h | z \rangle$
- Exception is the b -quark that due to its large mass generates larger M_h values

Reconstructed IFFs

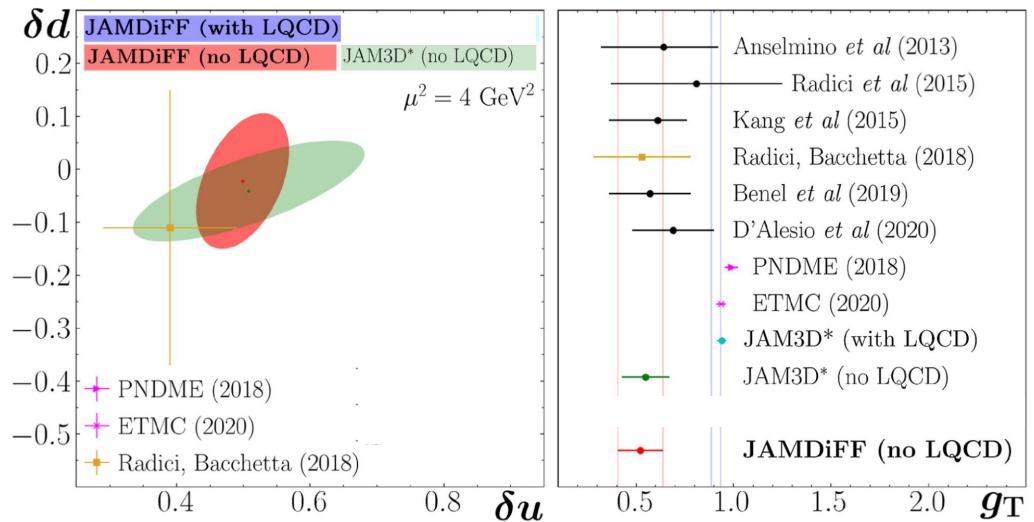


- Magnitude of IFF is proportional to DiFFs
- Marginal impact by positivity constraints
- The sign is not determined by the experimental data.

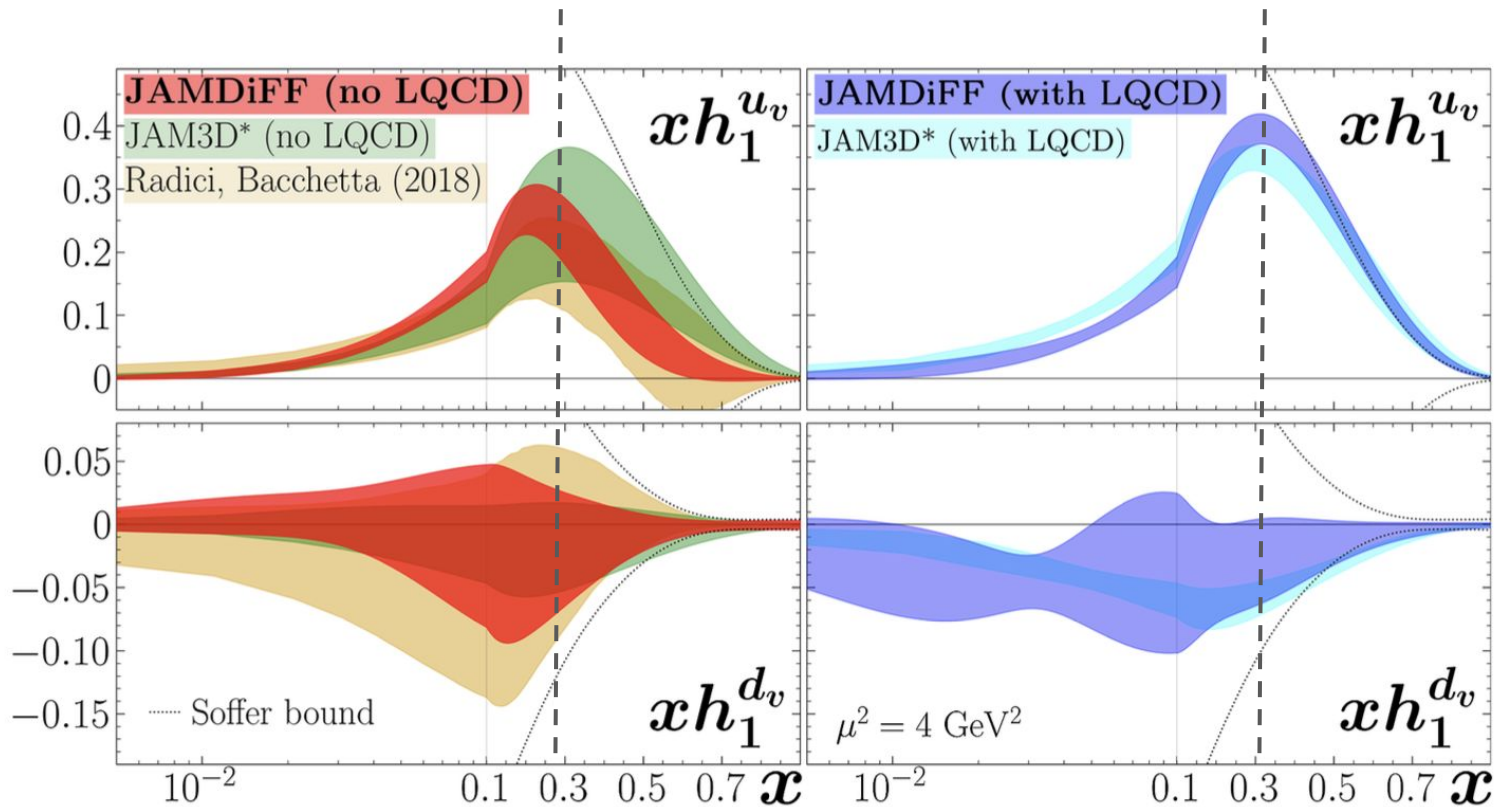
Reconstructed TPDF



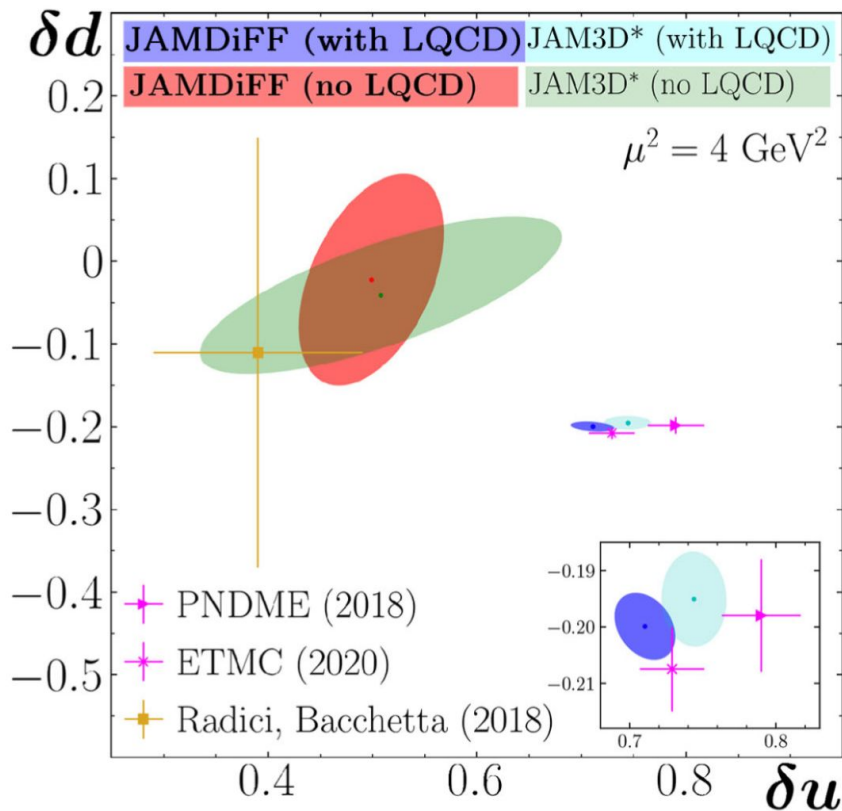
- Point-by-point in x constraints from data ends around $x \sim 0.3$.
- Below $x < 0.3$, the reconstructed transversity PDFs are consistent with Radici et al and TMD+CT3 (JAM3D) results.
- Beyond $x > 0.3$, only the u quark PDF has non-vanishing signal with some differences wrt JAM3D*
- JAM3D* includes antiquarks and small x constraints



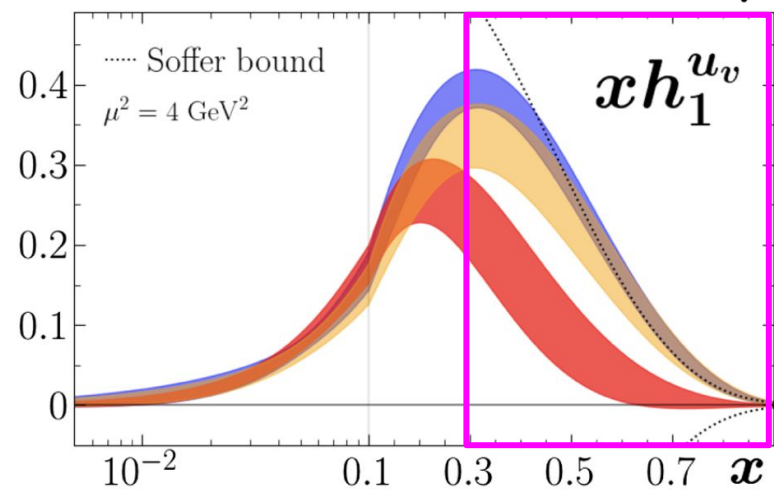
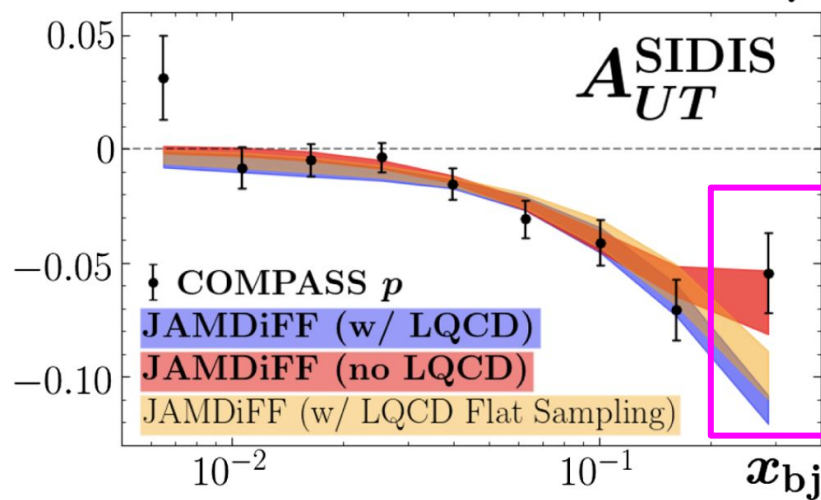
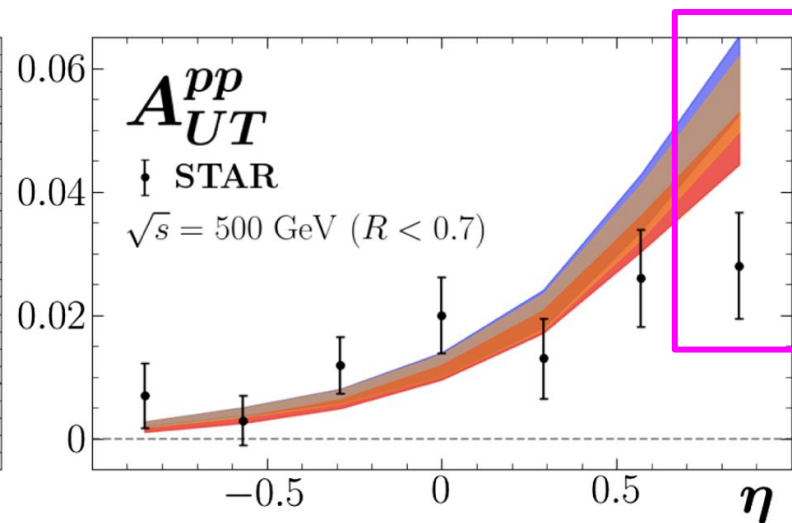
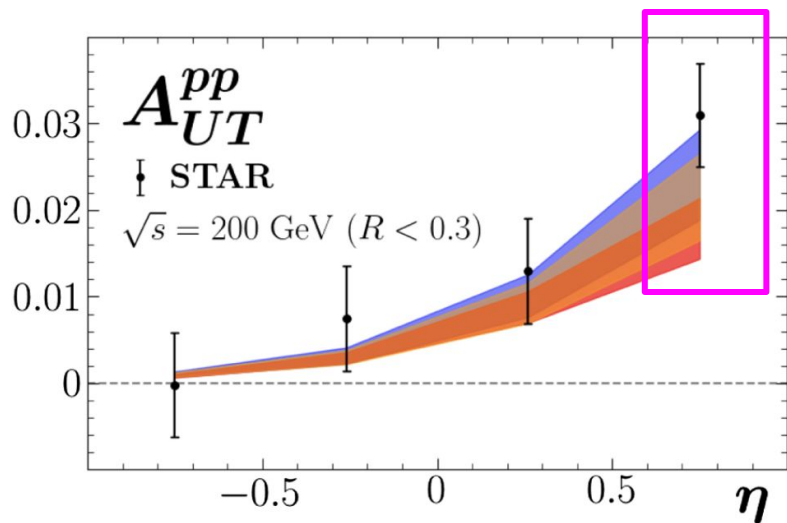
Reconstructed TPDF with LQCD



Reconstructed TPDF with LQCD



Experiment	χ^2_{red}	
	JAMDiFF (w/ LQCD)	JAMDiFF (no LQCD)
Belle (cross section) [64]	1.01	1.01
Belle (Artru-Collins) [112]	1.27	1.24
	0.60	0.60
HERMES [118]	0.42	0.42
	1.77	1.70
COMPASS (p) [117]	0.41	0.42
	1.20	1.17
COMPASS (D) [117]	1.98	0.65
	0.92	0.94
STAR [121]	0.77	0.60
	1.37	1.42
COMPASS (D) [117]	0.45	0.37
	0.50	0.46
STAR [121]	2.57	2.56
	1.34	1.55
$\sqrt{s} = 200 \text{ GeV}$ $R < 0.3$	0.98	1.00
	1.73	1.74
STAR [97]	0.52	1.46
	1.30	1.10
$\sqrt{s} = 500 \text{ GeV}$ $R < 0.7$	0.81	0.78
	1.09	1.07
ETMC δu [77]	2.97	1.83
ETMC δd [77]	0.71	
PNDME δu [71]	1.02	
PNDME δd [71]	8.68	
PNDME δd [71]	0.04	
Total χ^2_{red} (N_{dat})	1.01 (1475)	0.98 (1471)



Summary & outlook

- At present there is no significant tension between LQCD and experimental reconstruction of nucleon tensor charges
- Different reconstructions of tensor charges are mostly driven by large x data
- More high x data is needed to reach accurate reconstruction of TPDF above $x > 0.3$
- Inclusion of LQCD calculations as priors are very informative/useful in QCD phenomenology
- **The JAMDiFF results and JAM3D* results are very similar and one can perform a combined analysis (TMD+CT3 & DiFF) -> indicates possible universal nature of all SSAs and nucleon tensor charges**

