Transversity from Single-Hadron TSSAs and Dihadron Fragmentation Theory Developments

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Transverse Polarization Phenomenon

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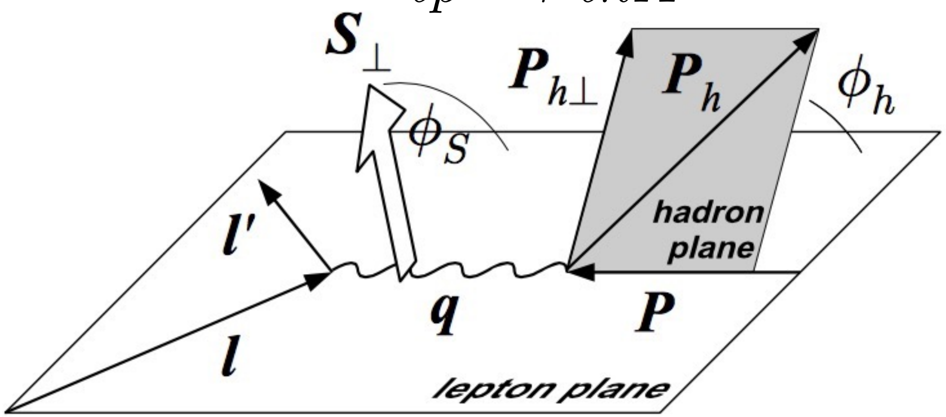
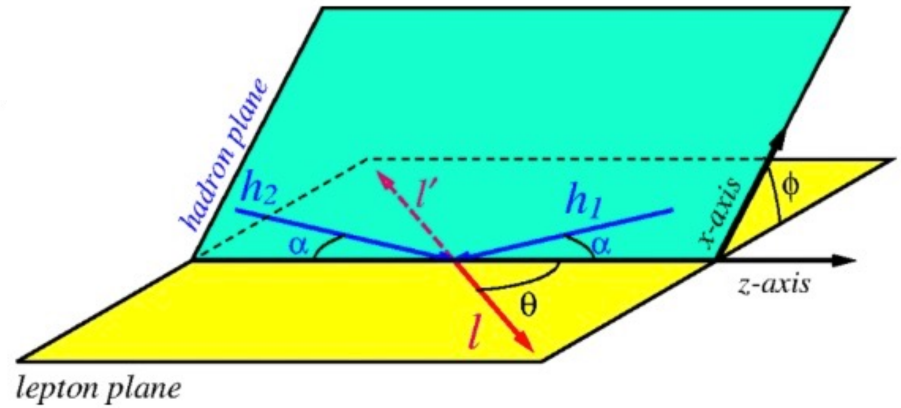
Background:
TSSAs for Single-Hadron Fragmentation

$$\ell p^{\uparrow} \to \ell h X$$



$$\{\pi, p\} p^{\uparrow} \to \{\ell^+\ell^-, W^{\pm}, Z\} X$$



$$F_{UT}^{\sin(\phi_h - \phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M}\, \boldsymbol{f_{1T}^{\perp}} D_1\right]$$

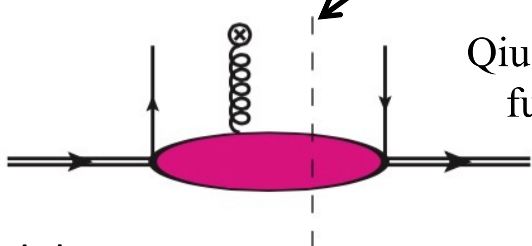
$$F_{TU}^{\sin\phi} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_{aT}}{M_a}\, \boldsymbol{f_{1T}^{\perp}}\, \bar{f}_1\right]$$

TMD/Collins-Soper-Sterman (CSS) Evolution

OPE

Sudakov exponentials (gluon radiation)

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)} \sim \boldsymbol{F_{FT}(x, x; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T, Q)\right]$$



Qiu-Sterman function

$$g_{f_{1T}^{\perp}}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

(Aybat, et al. (2012); Bury, et al. (2021); Echevarria, et al. (2014, 2021))

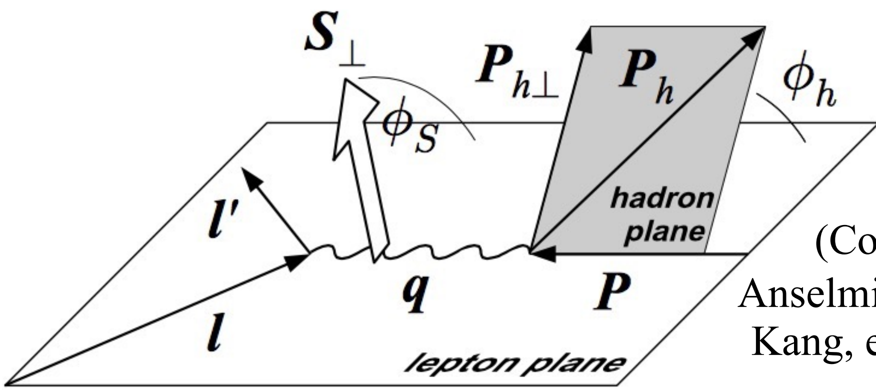
Parton model

$$\pi F_{FT}(x, x) = \int d^2\vec{k}_T\, \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x, k_T^2) \equiv f_{1T}^{\perp(1)}(x)$$

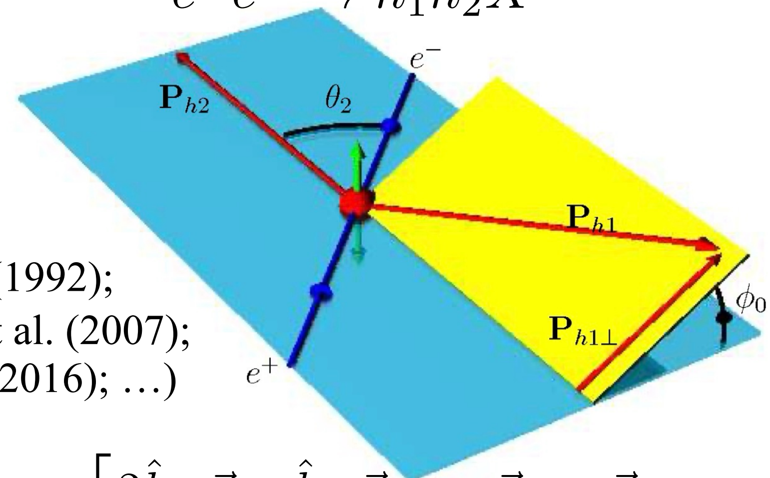
(Boer, Mulders, Pijlman (2003), see also del Rio, et al. (2024))

$\ell N^{\uparrow} \to \ell h X$

$e^+e^- \to h_1 h_2 X$



(Collins (1992); Anselmino, et al. (2007); Kang, et al. (2016); …)



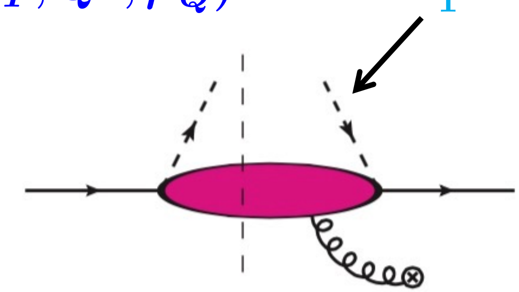
$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\boldsymbol{H_1^\perp}\right] \qquad F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

TMD/Collins-Soper-Sterman (CSS) Evolution

OPE — Sudakov exponentials (gluon radiation)

$$\boldsymbol{\tilde{h}_1(x, b_T; Q^2, \mu_Q)} \sim \boldsymbol{h_1(x; \mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{h_1}(b_T, Q)\right]$$

$$\boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q)} \sim \boldsymbol{H_1^{\perp(1)}(z; \mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q)\right]$$



qgq fragmentation function

Parton model

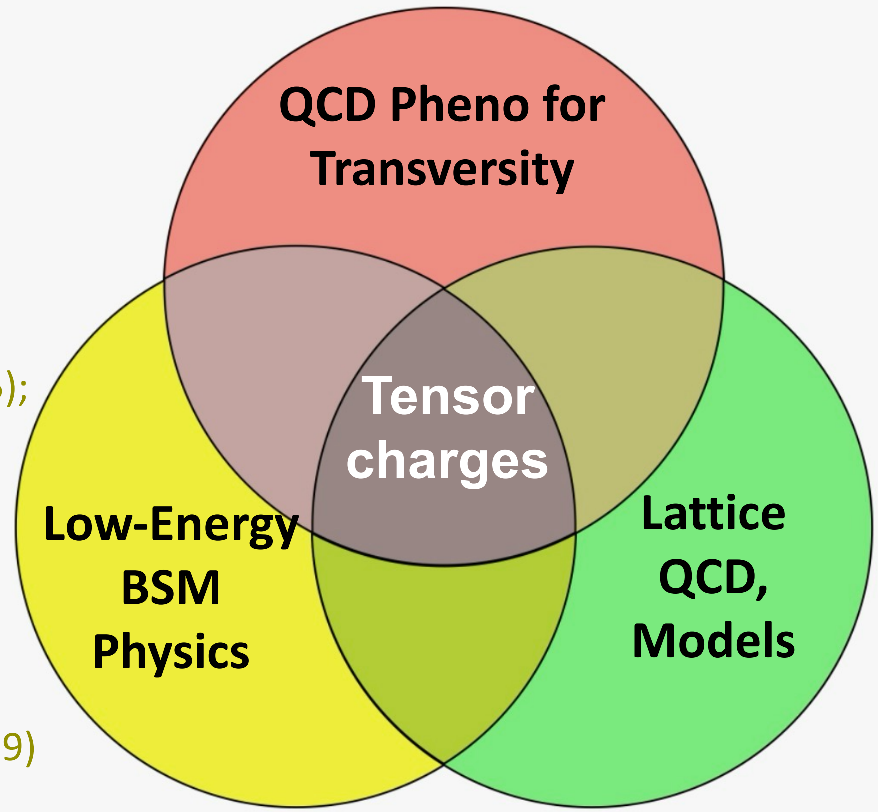
$$h_1(x) = \int d^2\vec{k}_T\, h_1(x, \vec{k}_T^2)$$

$$H_1^{\perp(1)}(z) = z^2\int d^2\vec{p}_\perp \frac{p_\perp^2}{2M_h^2} H_1^\perp(z, z^2 p_\perp^2)$$

$$\delta q \equiv \int_0^1 dx\,[h_1^q(x) - h_1^{\bar{q}}(x)] \qquad g_T \equiv \delta u - \delta d$$

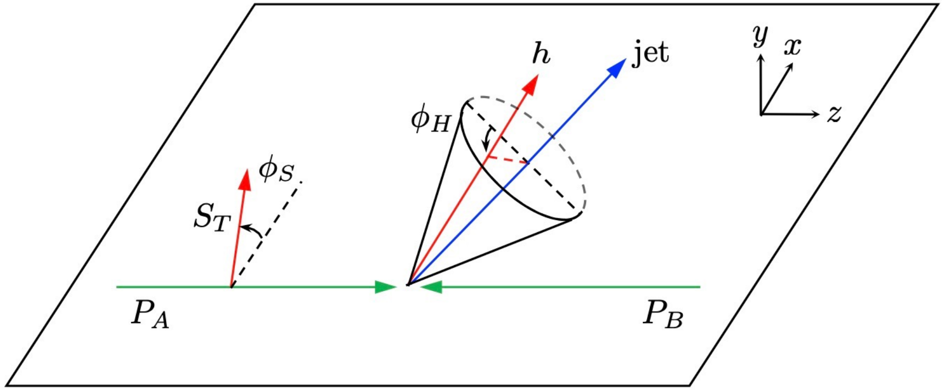
Anselmino, et al. (2007, 2009, 2013, 2015);
Goldstein, et al. (2014); Kang, et al. (2016); Radici, et al. (2013, 2015, 2018);
Benel, et al. (2020); D'Alesio, et al. (2020); Cammarota, et al. (2020);
Gamberg, et al. (2022); Cocuzza, et al. (2024)

Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, et al. (2006);
Cirigliano, et al. (2013);
Courtoy, et al. (2015);
Yamanaka, et al. (2017);
Liu, et al. (2018);
Gonzalez-Alonso, et al. (2019)



He, Ji (1995);
Barone, et al. (1997);
Schweitzer, et al. (2001);
Gamberg, Goldstein (2001);
Pasquini, et al. (2005);
Wakamatsu (2007);
Lorce (2009);
Gupta, et al. (2018);
Yamanaka, et al. (2018);
Hasan, et al. (2019);
Alexandrou, et al. (2019, 2023);
Yamanaka, et al. (2013);
Pitschmann, et al. (2015);
Xu, et al. (2015);
Wang, et al. (2018);
Liu, et al. (2019);
Gao, et al. (2023)

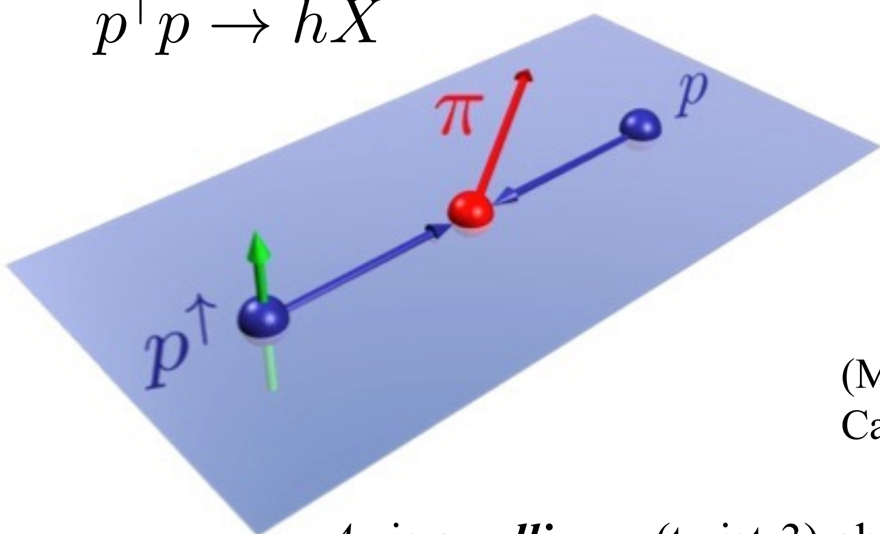
$$p^{\uparrow}p \to (h\,jet)\,X$$



(Yuan (2008); D'Alesio, Murgia, Pisano (2017); Kang, Prokudin, Ringer, Yuan (2017), …)

$$F_{UT}^{\sin(\phi_S-\phi_H)} \sim H^{\rm Collins}_{ab\to c}(\hat{s},\hat{t},\hat{u}) \otimes \boldsymbol{h_1^a(x_1)} \otimes f_1^b(x_2) \otimes (j_\perp/(z_h M_h))\boldsymbol{H_1^{\perp h/c}(z_h, j_\perp^2)}$$

$$p^{\uparrow} p \to hX$$



$$d\Delta\sigma(S_T) \sim \underbrace{H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1}_{\text{Qiu-Sterman term}}$$

$$+ \underbrace{H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \tilde{\boldsymbol{H}}\right)}_{\text{Fragmentation term}}$$

(Metz, DP (2012); Kanazawa, et al. (2014); Cammarota, et al. (2020); Gamberg, et al. (2017, 2022))

A_N is a ***collinear*** (twist-3) observable

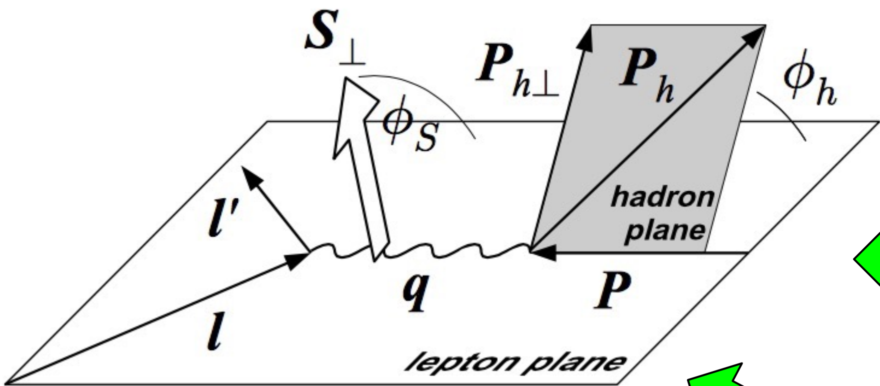
Updated QCD Global Analysis of TSSAs for Single-Hadron Fragmentation

Gamberg, Malda, Miller, DP, Prokudin, Sato, Phys. Rev. D **106**, 034014 (2022)

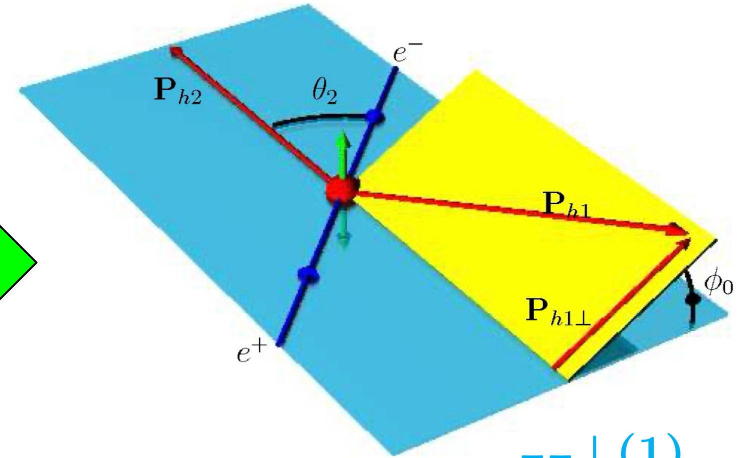
User-friendly jupyter notebook to calculate functions and asymmetries:
https://colab.research.google.com/github/pitonyak25/jam3d_dev_lib/blob/main/JAM3D_Library.ipynb

LHAPDF tables available (thanks to C. Cocuzza):
https://github.com/pitonyak25/jam3d_dev_lib/tree/main/LHAPDF_tables

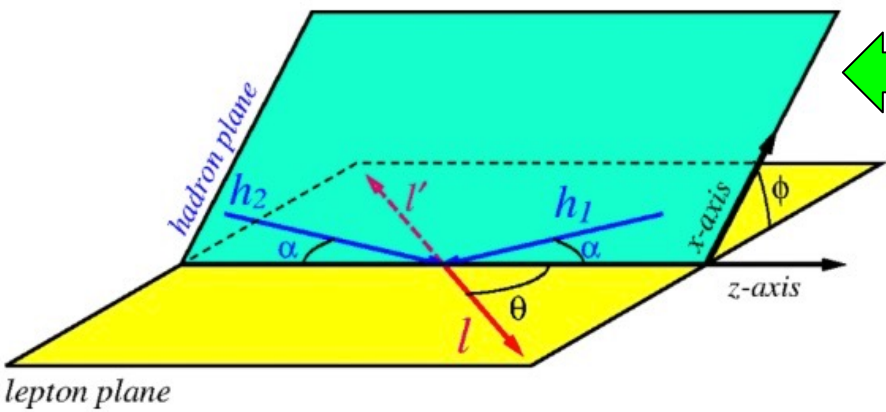
Jefferson Lab Angular Momentum Collaboration



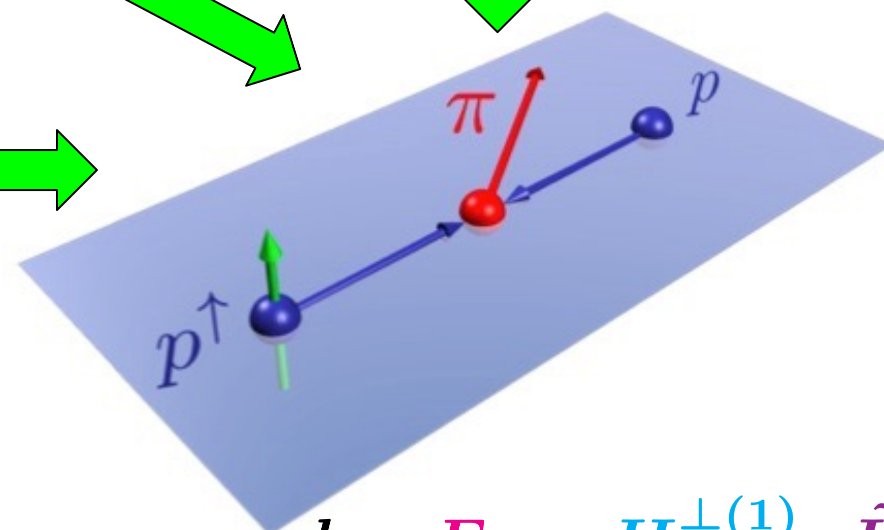
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

- Analyze TSSAs in SIDIS, Drell-Yan, e^+e^- annihilation, and proton-proton collisions and extract

$$h_1(x),\ F_{FT}(x,x),\ H_1^{\perp(1)}(z),\ \tilde{H}(z)$$

along with the relevant transverse momentum widths for the Sivers, transversity, and Collins functions: $\langle k_T^2\rangle_{f_{1T}^\perp}, \langle k_T^2\rangle_{h_1}, \langle p_\perp^2\rangle_{H_1^\perp}^{fav}, \langle p_\perp^2\rangle_{H_1^\perp}^{unf}$

- We use a Gaussian ansatz: $F^q(x,k_T^2) \sim F^q(x)e^{-k_T^2/\langle k_T^2\rangle}$ where

$$F^q(x) = \frac{N_q\, x^{a_q}(1-x)^{b_q}(1+\gamma_q\, x^{\alpha_q}(1-x)^{\beta_q})}{\mathrm{B}[a_q+2, b_q+1] + \gamma_q \mathrm{B}[a_q+\alpha_q+2, b_q+\beta_q+1]}$$

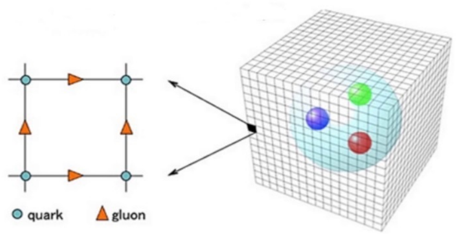
NB. $\{\gamma, \alpha, \beta\}$ only used for $H_1^{\perp(1)}(x)$, $b_u = b_d$ for $h_1(x), f_{1T}^{\perp(1)}(x)$

- DGLAP-type evolution for the collinear functions analogous to Duke & Owens (1984): double-log Q^2-dependent term explicitly added to the parameters

- Additional data/constraints included in the fit compared to 2020:
  - Collins and Sivers effects (3D-binned) SIDIS data from HERMES (2020)
  - $A_{UT}^{\sin\phi_S}$ data (*x* and *z* projections only) from HERMES (2020)

$$\int d^2\vec{P}_{hT}\, F_{UT}^{\sin\phi_S} = -\frac{x}{z}\sum_q e_q^2\, \frac{2M_h}{Q}\, h_1^{q/N}(x)\, \tilde{H}^{h/q}(z)$$

  - Lattice data on g_T at the physical pion mass from ETMC (Alexandrou, et al. (2019))

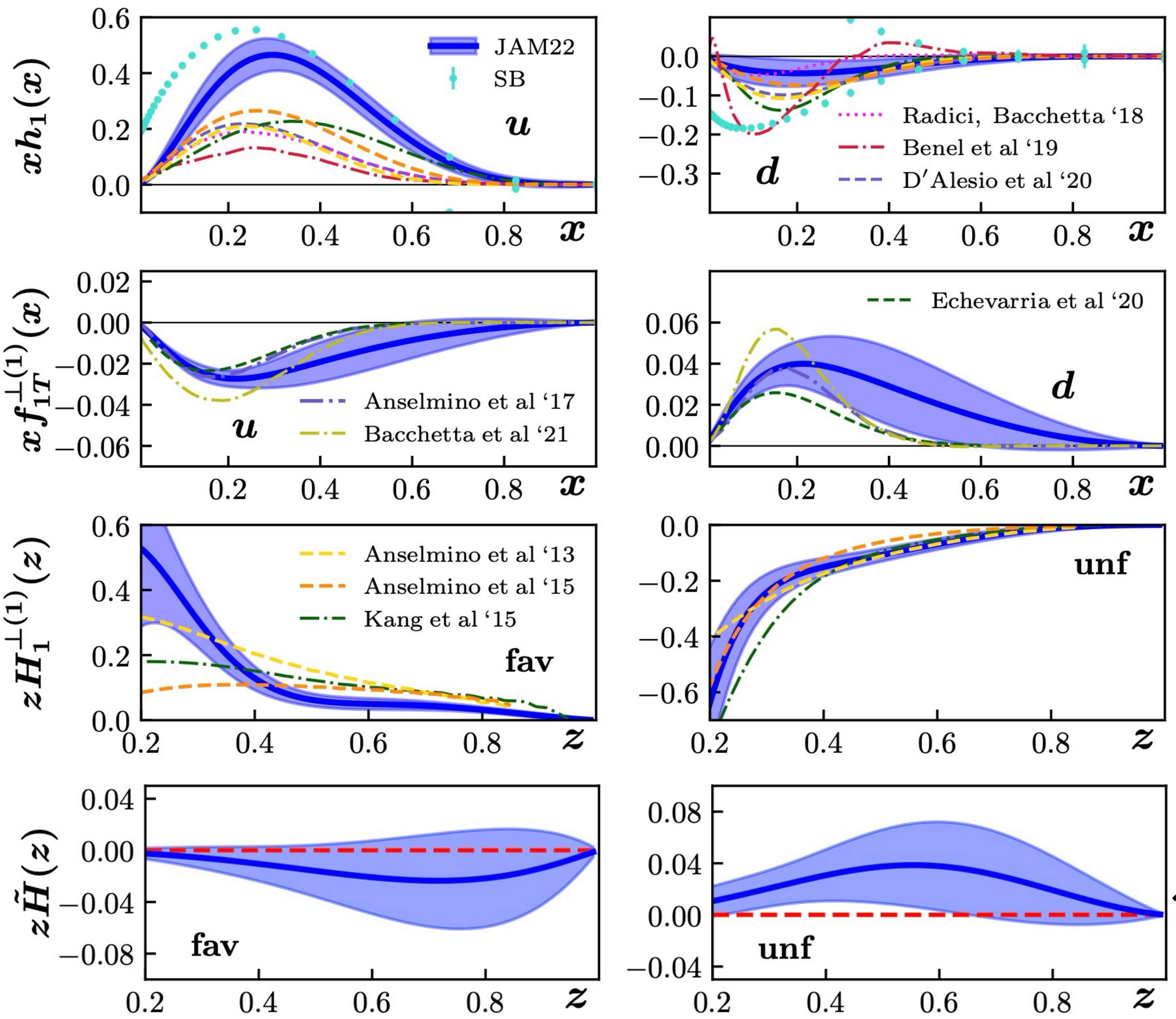


  - Imposing the Soffer bound on transversity: $|h_1^q(x)| \leq \frac{1}{2}(f_1^q(x) + g_1^q(x))$

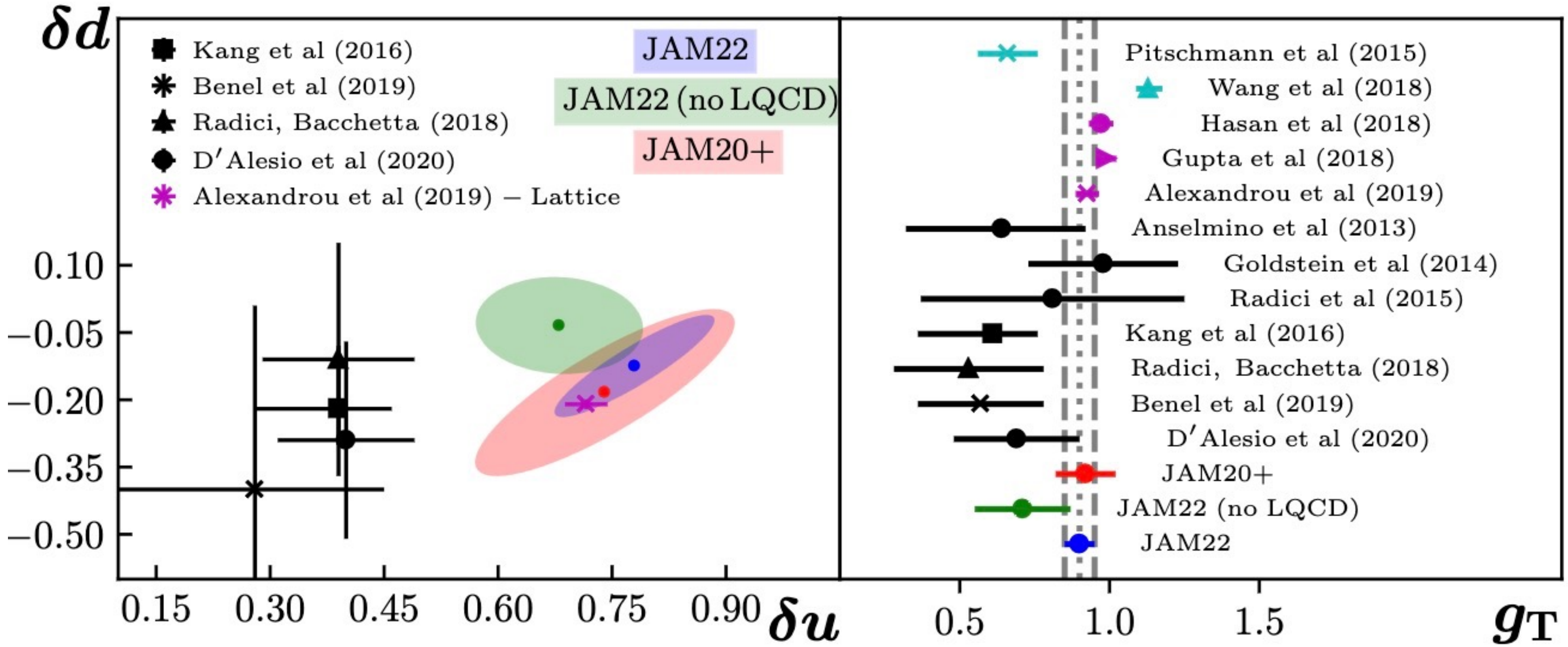
Generate "data" (central value and 1-σ uncertainty) using recent simultaneous fit of f_1 and g_1 from Cocuzza, et al. (2022) and add to the χ^2 if SB is violated by more than the uncertainty in the data

$$\chi^2/N_{\text{pts.}} = 647/634 = 1.02$$

Observable	Reactions	Non-Perturbative Function(s)	χ^2/npts
$A_{UT}^{\sin(\phi_h-\phi_S)}$	$e+(p,d)^\uparrow \to e+(\pi^+,\pi^-,\pi^0)+X$	$f_{1T}^\perp(x,\vec{k}_T^2)$	$182.9/166 = 1.10$
$A_{UT}^{\sin(\phi_h+\phi_S)}$	$e+(p,d)^\uparrow \to e+(\pi^+,\pi^-,\pi^0)+X$	$h_1(x,\vec{k}_T^2), H_1^\perp(z,z^2\vec{p}_T^2)$	$181.0/166 = 1.09$
${}^*A_{UT}^{\sin\phi_S}$	$e+p^\uparrow \to e+(\pi^+,\pi^-,\pi^0)+X$	$h_1(x), \tilde{H}(z)$	$18.6/36 = 0.52$
$A_{UC/UL}$	$e^+ + e^- \to \pi^+\pi^-(UC,UL)+X$	$H_1^\perp(z,z^2\vec{p}_T^2)$	$154.9/176 = 0.88$
$A_{T,\mu^+\mu^-}^{\sin\phi_S}$	$\pi^- + p^\uparrow \to \mu^+\mu^- + X$	$f_{1T}^\perp(x,\vec{k}_T^2)$	$6.92/12 = 0.58$
$A_N^{W/Z}$	$p^\uparrow + p \to (W^+,W^-,Z)+X$	$f_{1T}^\perp(x,\vec{k}_T^2)$	$30.8/17 = 1.81$
A_N^π	$p^\uparrow + p \to (\pi^+,\pi^-,\pi^0)+X$	$h_1(x), F_{FT}(x,x) = \frac{1}{\pi} f_{1T}^{\perp(1)}(x), H_1^{\perp(1)}(z), \tilde{H}(z)$	$70.4/60 = 1.17$
Lattice g_T	——	$h_1(x)$	$1.82/1 = 1.82$



First direct information from experiment on $\tilde{H}(z)$



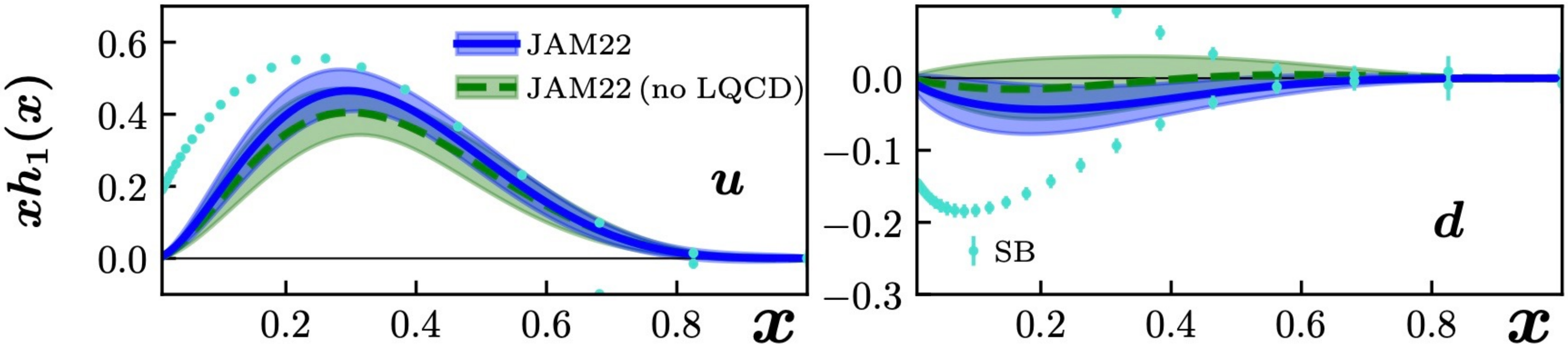
- TMD that only include e^+e^- and SIDIS Collins effect data (e.g., Kang, et al. (2016), D'Alesio, et al. (2020)) and dihadron analyses (e.g., Radici, Bacchetta (2018); Benel, Courtoy, Ferro-Hernandez (2019)), are generally below the lattice values for g_T and δu
- Note that one initially finds JAM3D-22 has more tension with lattice, but this does *not* imply phenomenology and lattice are incompatible – one can only fully answer this by including lattice data in the analysis (use it as a prior)
- **Once g_T is included (as a Bayesian prior), we find the non-perturbative functions can accommodate it *and still describe the experimental data well***

- Slight update (JAM3D-22*) relevant for comparison with the JAMDiFF analysis:
  - include transversity antiquarks with $\bar{u} = -\bar{d}$ (from large-N_c limit (Pobylitsa (2003)))
  - $\delta u, \delta d$ from ETMC and PNDME are both included in the with lattice fit (rather than just g_T from ETMC)
  - incorporate constraint on the "a" parameter from the small-x asymptotic behavior of transversity (Kovchegov, Sievert (2019))

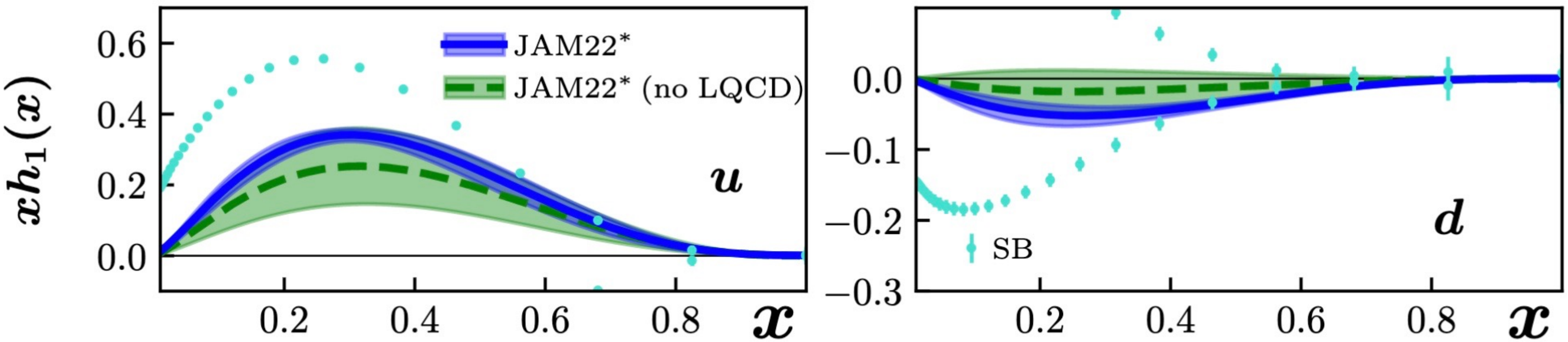
$$a \xrightarrow{x \to 0} 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \quad \Longrightarrow \quad a = 0.250 \pm 0.125$$

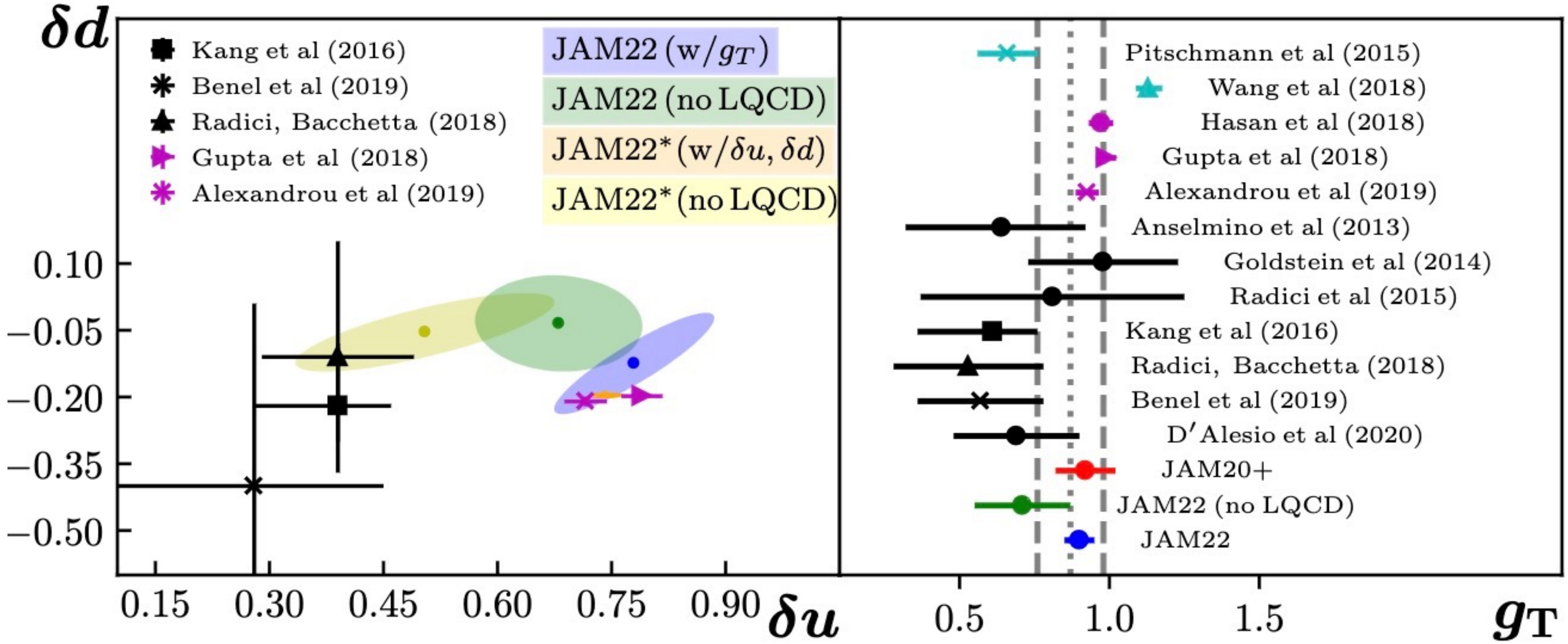
50% uncertainty due to unaccounted for $1/N_c$ and NLO corrections

JAM3D-22



JAM3D-22*

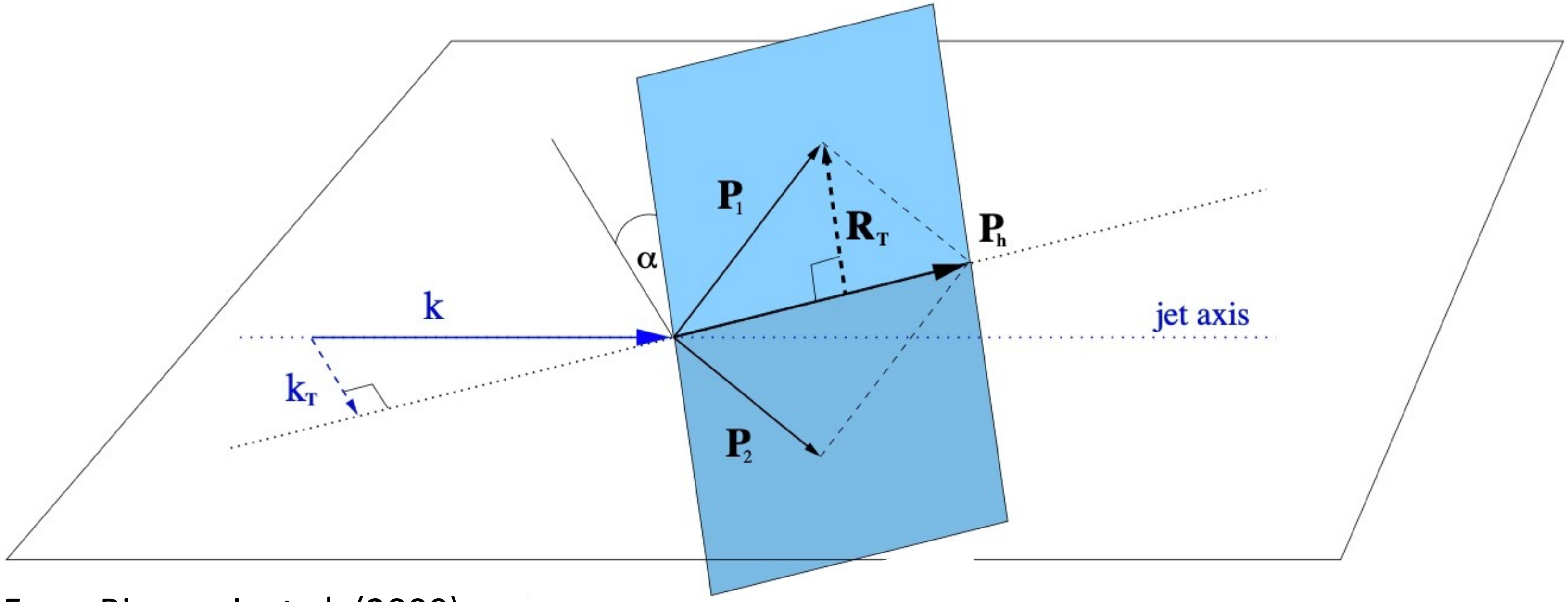




NB: **The experimental data is still described very well even when including $\boldsymbol{\delta u, \delta d}$ from LQCD as a Bayesian prior in the fit**

New Dihadron Fragmentation Theory Developments

DP, Cocuzza, Metz, Prokudin, Sato, Phys. Rev. Lett. **132**, 011902 (2024)



From Bianconi, et al. (2000)

Bianconi, et al. (2000); Bacchetta, Radici (2003, 2004), …

$$P_h = P_1 + P_2 \qquad R = (P_1 - P_2)/2 \qquad z = z_1 + z_2 \qquad \zeta = (z_1 - z_2)/z$$

$$P_1 = \left(\frac{M_1^2 + \vec{R}_T^2}{(1+\zeta)P_h^-}, \frac{1+\zeta}{2} P_h^-, \vec{R}_T \right) \qquad P_2 = \left(\frac{M_2^2 + \vec{R}_T^2}{(1-\zeta)P_h^-}, \frac{1-\zeta}{2} P_h^-, -\vec{R}_T \right)$$

$$\vec{R}_T^2 = \frac{1-\zeta^2}{4} M_h^2 - \frac{1-\zeta}{2} M_1^2 - \frac{1+\zeta}{2} M_2^2$$

$$\Delta_{\alpha\beta}^{h_1 h_2/i}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) = \frac{1}{N_i}\sum_X\!\!\!\!\!\!\int \frac{d\xi^+ d^2\vec{\xi}_\perp}{(2\pi)^3}\, e^{ik\cdot\xi}\, \mathcal{O}_{\alpha\beta}^{h_1 h_2/i}(\xi)\Big|_{\xi^-=0}$$

<u>quark fragmentation</u> ($N_i = N_c$)

$$\begin{aligned}\mathcal{O}_{\alpha\beta}^{h_1 h_2/q}(\xi) =& \langle 0|\mathcal{W}(\infty,\xi)\psi_{q,\alpha}(\xi^+, 0^-, \vec{\xi}_\perp)|P_1, P_2; X\rangle \\ & \times \langle P_1, P_2; X|\bar{\psi}_{q,\beta}(0^+, 0^-, \vec{0}_\perp)\mathcal{W}(0,\infty)|0\rangle\end{aligned}$$

<u>gluon fragmentation</u> ($N_i = N_c^2 - 1$)

$$\begin{aligned}\mathcal{O}_{\alpha\beta}^{h_1 h_2/g}(\xi) =& \langle 0|\mathcal{W}^{ba}(\infty,\xi)F^a_{+\alpha}(\xi^+, 0^-, \vec{\xi}_\perp)|P_1, P_2; X\rangle \\ & \times \langle P_1, P_2; X|F^c_{+\beta}(0^+, 0^-, \vec{0}_\perp)\mathcal{W}^{cb}(0,\infty)|0\rangle\end{aligned}$$

NB: we will focus on quark fragmentation, but similar results hold for gluon fragmentation

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\right] = D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

NEW definition of dihadron FFs

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\right] = D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

This ***prefactor is key to the number density interpretation*** of dihadron FFs (see also Majumder, Wang (2004))

$$\frac{d^3\vec{P}_1}{(2\pi)^3 2P_1^0}\,\frac{d^3\vec{P}_2}{(2\pi)^3 2P_2^0} = \frac{dz_1 d^2\vec{P}_{1\perp} dz_2 d^2\vec{P}_{2\perp}}{4(2\pi)^3(2\pi)^3 z_1 z_2}$$

$$\hat{N}_{h_j} \equiv \int \frac{dP_j^- d^2\vec{P}_{j\perp}}{(2\pi)^3\, 2P_j^-}\,\hat{a}_{h_j}^\dagger \hat{a}_{h_j} = \int \frac{dz_j d^2\vec{P}_{j\perp}}{(2\pi)^3\, 2z_j}\,\hat{a}_{h_j}^\dagger \hat{a}_{h_j}$$ (*j* = 1 or 2)

$$\hat{P}^\mu \equiv \sum_h \int \frac{dP^- d^2\vec{P}_\perp}{(2\pi)^3\, 2P^-}\,\hat{a}_h^\dagger P^\mu \hat{a}_h = \sum_h \int \frac{dz d^2\vec{P}_\perp}{(2\pi)^3\, 2z}\,\hat{a}_h^\dagger P^\mu \hat{a}_h$$

$$\frac{\mathbf{1}}{\mathbf{64\pi^3 z_1 z_2}}\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\right] = D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\vdots$$

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2 \int_0^{1-z_2} dz_1 \int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$= \langle \mathcal{N}(\mathcal{N}-1)\rangle$$

Expectation value for the total number of *hadron pairs* produced when the parton fragments

Note: Recent papers by Collins, Rogers (2024) and Rogers, Courtoy (2024) do *not* actually put into question our results regarding our DiFF definition being a number density.

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\right] = D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\vdots$$

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2 \int_0^{1-z_2} dz_1 \int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$= \langle \mathcal{N}(\mathcal{N}-1)\rangle$$

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Expectation value for the total number of *hadron pairs* produced when the parton fragments

If a prefactor of 1/(4z) = 1/(4(z_1+z_2)) is used, then sum rule proofs and deriving the expected parton model results for cross sections are not possible.

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\Big] = D_1^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\gamma_5\Big] = -\frac{\epsilon_\perp^{ij}R_\perp^i P_{h\perp}^j}{zM_h^2}\,G_1^{\perp\, h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})i\sigma^{i-}\gamma_5\Big] = -\frac{\epsilon_\perp^{ij}R_\perp^j}{M_h}H_1^{\sphericalangle'\, h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) + \frac{\epsilon_\perp^{ij}P_{h\perp}^j}{zM_h}H_1^{\perp'\, h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

***NB*: number density interpretation holds not only for unpolarized quarks (γ^- projection) but also for longitudinally ($\gamma^-\gamma^5$ projection) and transversely ($i\sigma^{i-}\gamma^5$ projection) polarized quarks**

Number sum rule

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2\int_0^{1-z_2} dz_1\int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\, D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$
$$= \langle \mathcal{N}(\mathcal{N}-1)\rangle$$

$$\Longrightarrow \boldsymbol{D_1^{h_1 h_2/i}(w, x, \vec{Y}^2, \vec{Z}^2, \vec{Y}\cdot\vec{Z})} \equiv \mathcal{J}\cdot D_1^{h_1 h_2/i}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

is a number density

Jacobian for the variable transformation

Number sum rule

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2\int_0^{1-z_2} dz_1\int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\,D_1^{h_1 h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = \langle\mathcal{N}(\mathcal{N}-1)\rangle$$

$$\Longrightarrow \boldsymbol{D_1^{h_1h_2/i}(w,x,\vec{Y}^2,\vec{Z}^2,\vec{Y}\cdot\vec{Z})} \equiv \mathcal{J}\cdot D_1^{h_1h_2/i}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

is a number density

Jacobian for the variable transformation

Using our new definition, DiFFs can now be interpreted as densities in any momentum variables of choice for the number of hadron pairs ($h_1 h_2$) fragmenting from the parton

Number sum rule

$$\sum_{h_1}\sum_{h_2}\int_0^1 dz_2\int_0^{1-z_2} dz_1\int d^2\vec{P}_{1\perp}\int d^2\vec{P}_{2\perp}\,D_1^{h_1 h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = \langle\mathcal{N}(\mathcal{N}-1)\rangle$$

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is a number density

Jacobian for the variable transformation

Momentum sum rule

$$\sum_{h_1}\int_0^{1-z_2} dz_1\int d^2\vec{P}_{1\perp}\,z_1\,D_1^{h_1 h_2/i}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) = (1-z_2)\,D_1^{h_2/i}(z_2,\vec{P}_{2\perp}^2)$$

NB: $D_1^{h_1 h_2/i}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})/D_1^{h_2/i}(z_2,\vec{P}_{2\perp}^2)$ is a conditional number density in the momentum (z_1, $P_{1\perp}$) for h_1 fragmenting from *i* given h_2 has fragmented from *i* with momentum (z_2, $P_{2\perp}$)

- Connection to phenomenology - work in a frame where the dihadron has no transverse momentum

$$P_h = P_1 + P_2 \qquad R = (P_1 - P_2)/2 \qquad z = z_1 + z_2 \qquad \zeta = (z_1 - z_2)/z$$

$$(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) \longrightarrow (z, \zeta, \vec{k}_T, \vec{R}_T) : \quad \mathcal{J} = z^3/2$$

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is a number density in $(z, \zeta, \vec{k}_T, \vec{R}_T)$

- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually $\boldsymbol{\zeta}$) is integrated out

$$\frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T \,\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\Big] = D_1^{h_1h_2/q}(z,\zeta,\vec{R}_T^2)$$

$$\frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T \,\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})i\sigma^{i-}\gamma_5\Big] = -\frac{\epsilon_T^{ij}R_T^j}{M_h}H_1^{\sphericalangle\, h_1h_2/q}(z,\zeta,\vec{R}_T^2)$$

are number densities in $(z,\zeta,\vec{R}_T)$

chiral-odd "interference" FF (IFF) that can couple to transversity

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are number densities in $(z,\zeta,\vec{R}_T)$

chiral-odd "interference" FF (IFF) that can couple to transversity

NB: Experiments report measurements in terms of M_h

$$\vec{R}_T^2 = \frac{1-\zeta^2}{4}M_h^2 - \frac{1-\zeta}{2}M_1^2 - \frac{1+\zeta}{2}M_2^2$$

One *cannot* simply replace the R_T dependence in the DiFF with an M_h and still maintain a number density interpretation in M_h

$$(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp}) \longrightarrow (z,\zeta,\vec{k}_T,M_h,\phi_{R_T}): \quad \mathcal{J} = z^3(1-\zeta^2)/8$$

- Experimental measurements are sensitive to the so-called "extended" DiFFs where k_T (and usually $\boldsymbol{\zeta}$) is integrated out

$$D_1^{h_1 h_2/i}(z, M_h) \equiv \frac{\pi}{2} M_h \int_{-1}^{1} d\zeta \,(1-\zeta^2) D_1^{h_1 h_2/i}(z, \zeta, \vec{R}_T^2)$$

is a number density in (z, M_h)

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$e^+e^- \to (h_1 h_2)\, X$

$$\frac{d\sigma}{dz\, dM_h} = \sum_q \frac{4\pi N_c \alpha_{\rm em}^2}{3Q^2}\, e_q^2\, D_1^{h_1 h_2/q}(z, M_h)$$

$e^+e^- \to h\, X$

$$\frac{d\sigma}{dz} = \sum_q \hat{\sigma}_0^q\, D_1^{h/q}(z)$$

total partonic cross section for $e^+e^- \to \gamma \to q\bar{q} \equiv \hat{\sigma}_0^q$

NB: also checked it works for gluon DiFF using $e^+e^- \to H \to gg$

This is exactly the structure *dσ* should have if *D_1* has a number density interpretation (alternative definitions would introduce additional factors that don't give the expected parton model result)

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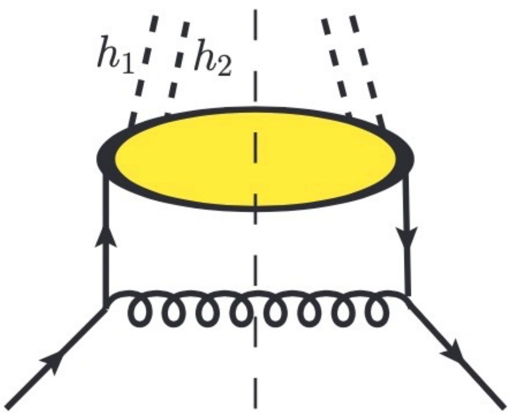
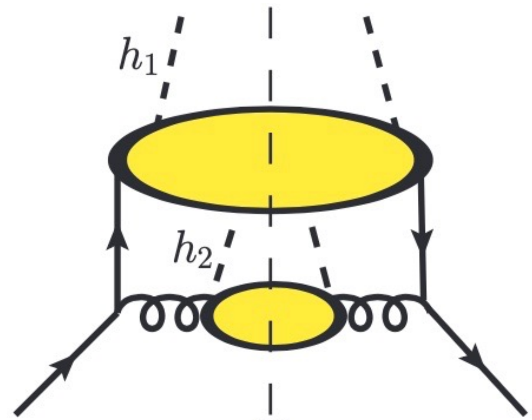
- Evolution equations for extended DiFFs



$$D_1^{h_1 h_2/i} \to D_1^{h_1 h_2/j}$$

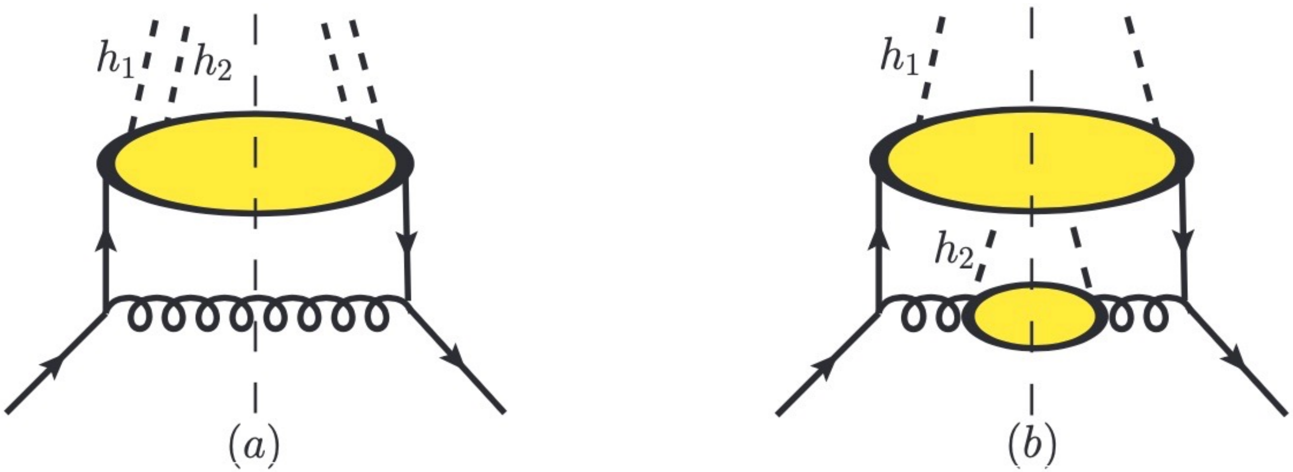
"Homogeneous term"



$$D_1^{h_1 h_2/i} \to D_1^{h_1/j} \otimes D_1^{h_2/k}$$

"Inhomogeneous term"

- Evolution equations for extended DiFFs



$$D_1^{h_1 h_2/i} \to D_1^{h_1 h_2/j}$$

"Homogeneous term"

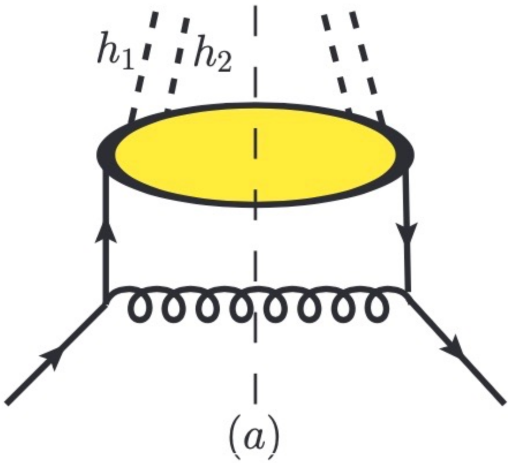
$$D_1^{h_1 h_2/i} \to D_1^{h_1/j} \otimes D_1^{h_2/k}$$

"Inhomogeneous term"

$$D_1^{h_1 h_2/q,(\mathrm{b})}(z_1, z_2, \vec{R}_T^2; \mu) = \frac{1}{\vec{R}_T^2} \frac{C_F \alpha_s}{2\pi^2} \mu^{2\epsilon} \int_{z_1}^{1-z_2} \frac{dw}{w(1-w)} D_1^{h_1/q}(z_1/w) D_1^{h_2/g}(z_2/(1-w)) \frac{1+w^2}{1-w}$$

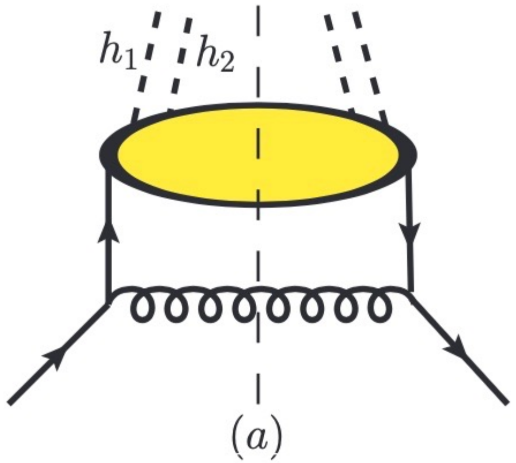
The inhomogeneous terms are *not* UV divergent at $\mathcal{O}(\alpha_s)$ when one keeps the dependence on R_T (see also Ceccopieri, et al. (2007))

- Evolution equations for extended DiFFs



$$D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2) = \frac{z}{32\pi^3(1-\zeta^2)} \int d^2\vec{k}_T \, \mathrm{Tr}\Big[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\Big]$$

Evolution equations for extended DiFFs



$$D_1^{h_1 h_2/q}(z,\zeta,\vec{R}_T^2) = \frac{z}{32\pi^3(1-\zeta^2)}\int d^2\vec{k}_T\,\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\right]$$

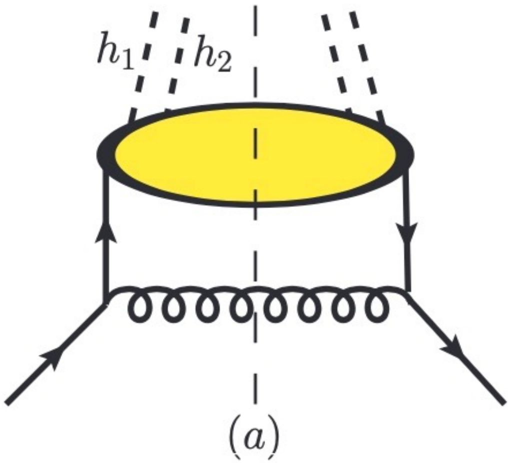
ζ dependence is not altered by evolution

$$D_1^{h/q}(z) = \frac{z}{4}\int d^2\vec{k}_T\,\Delta^{h/q}(z,\vec{k}_T)$$

Evolution is independent of the target (in the case of PDFs) or final (in the case of FFs) state (Collins (2011))

➡ The evolution equations of the (extended) DiFFs are the same as single-hadron collinear FFs

- Evolution equations for extended DiFFs



$$\frac{\partial \mathcal{D}^{h_1 h_2/i}(z,\zeta,\vec{R}_T^2;\mu)}{\partial \ln \mu^2} = \sum_{i'} \int_z^1 \frac{dw}{w} \mathcal{D}^{h_1 h_2/i'}\Big(\frac{z}{w},\zeta,\vec{R}_T^2;\mu\Big) P_{i\to i'}(w)$$

where $\mathcal{D} = D_1$ or $H_1^{\sphericalangle}$

- D_1: use unpolarized time-like splitting kernels
- $H_1^{\sphericalangle}$: use transversely polarized splitting kernels

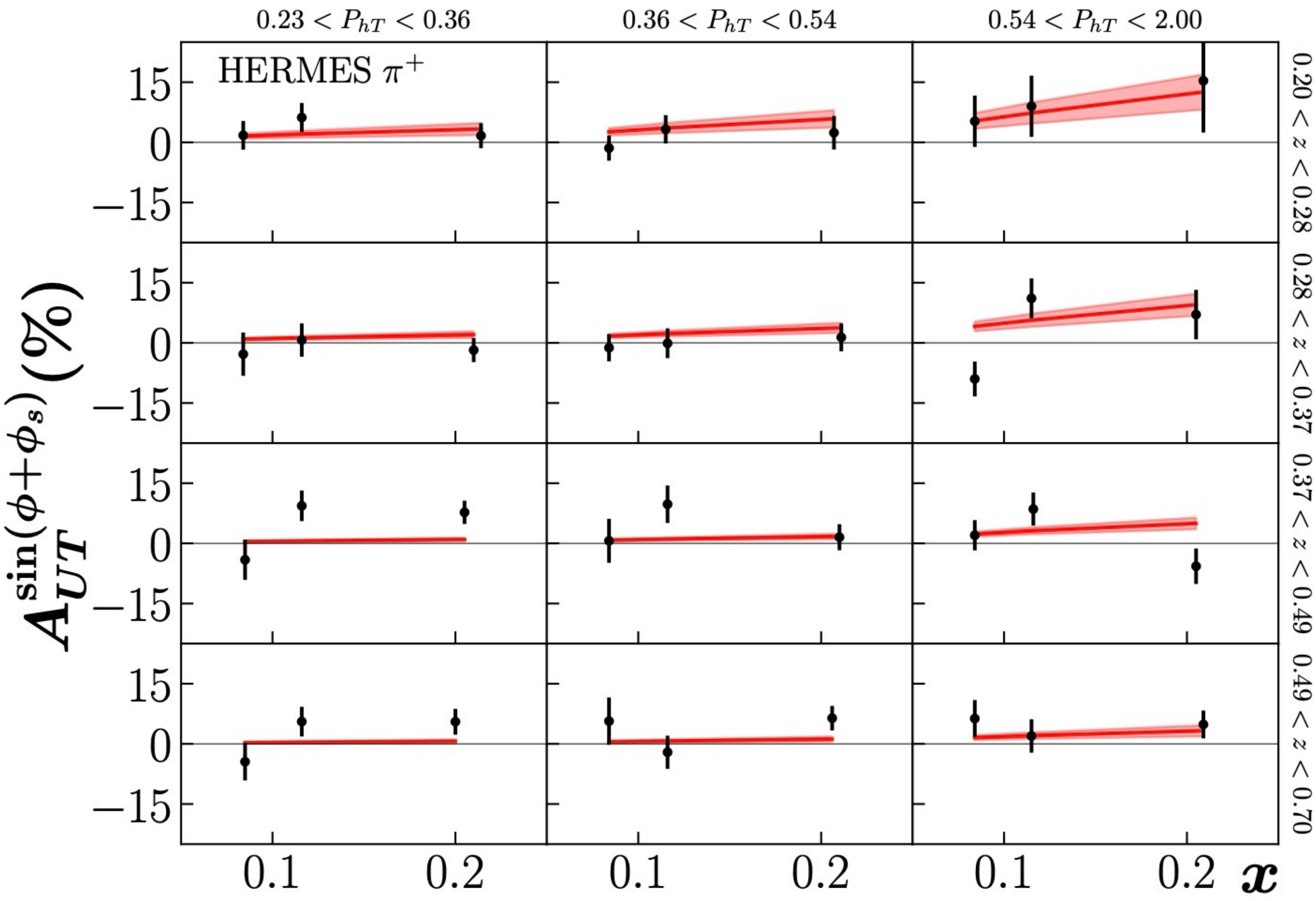
Summary

- We have updated our JAM3D-20 analysis using new data from HERMES (3D-binned Collins and Sivers effects and $A_{UT}^{\sin\phi_S}$) as well as constraints from lattice QCD (tensor charge g_T) and the Soffer bound on transversity.

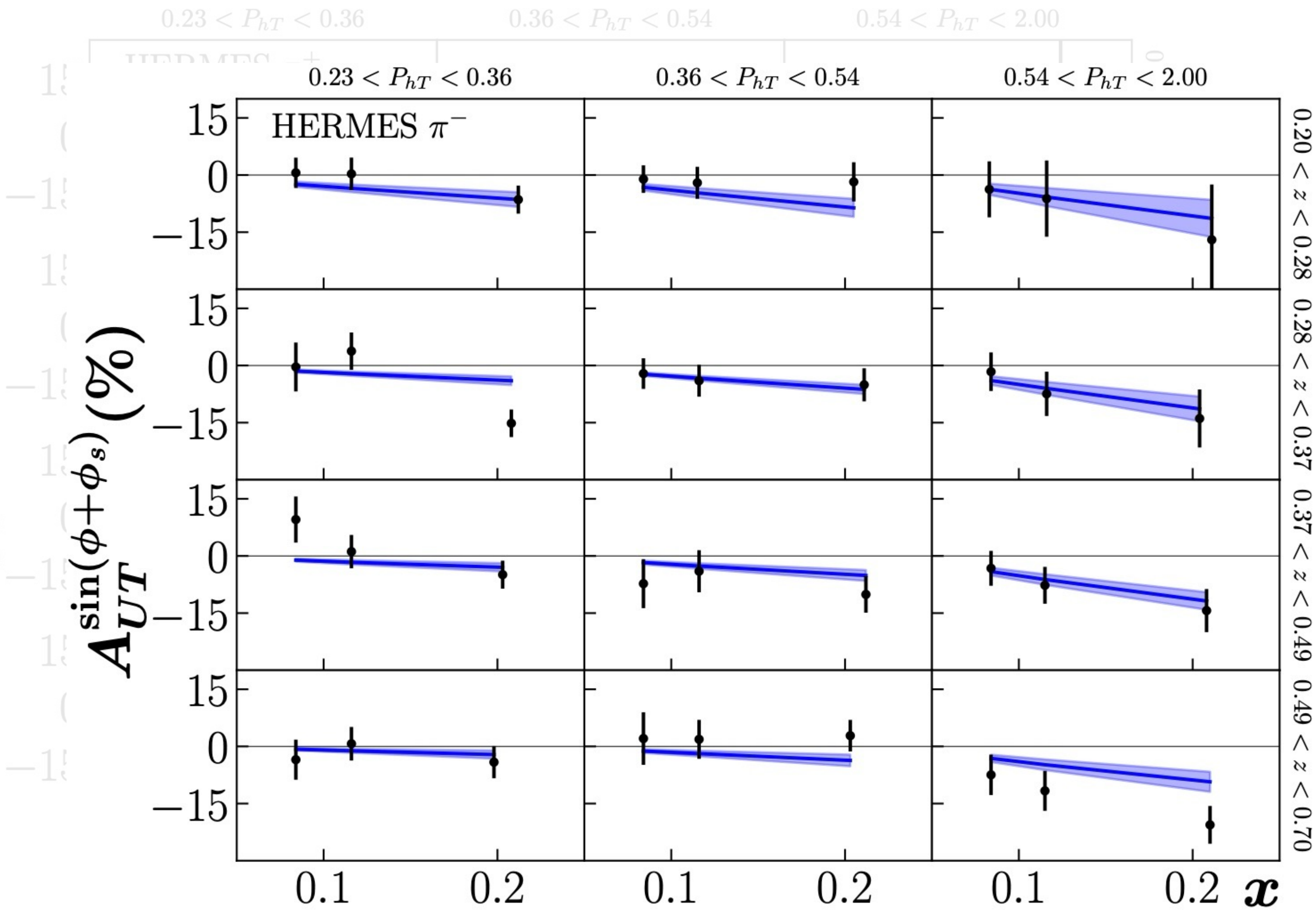
- Our JAM3D-22 results show it is still possible to accommodate these data/constraints and describe all TSSAs. The newly extracted transversity function and associated tensor charges are much more precise. We also have the first direct information from experiment on $\tilde{H}(z)$.

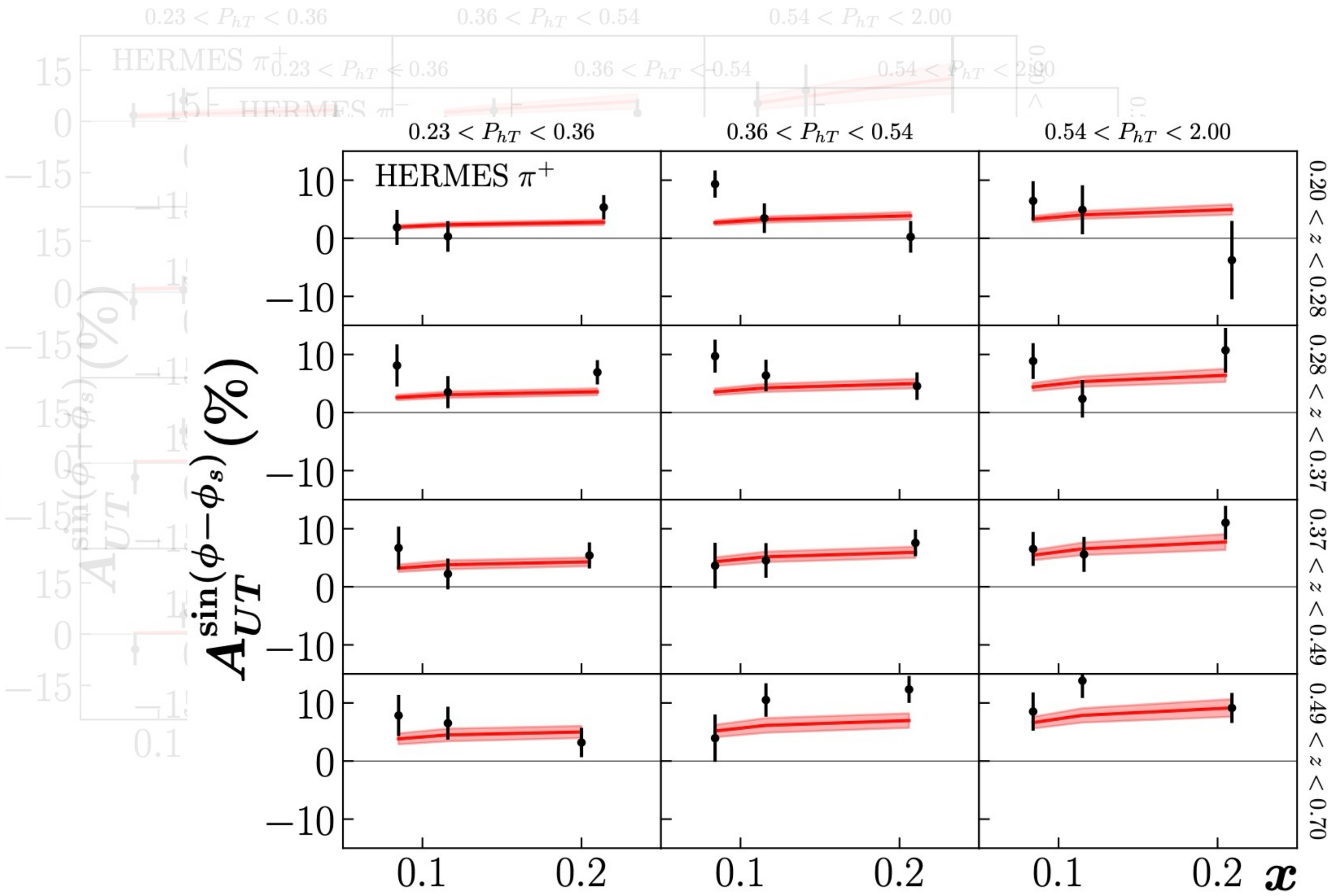
- We have introduced a new definition of dihadron fragmentation functions that is consistent with a number density interpretation, giving these functions a clear physical meaning, and derived their associated evolution equations.

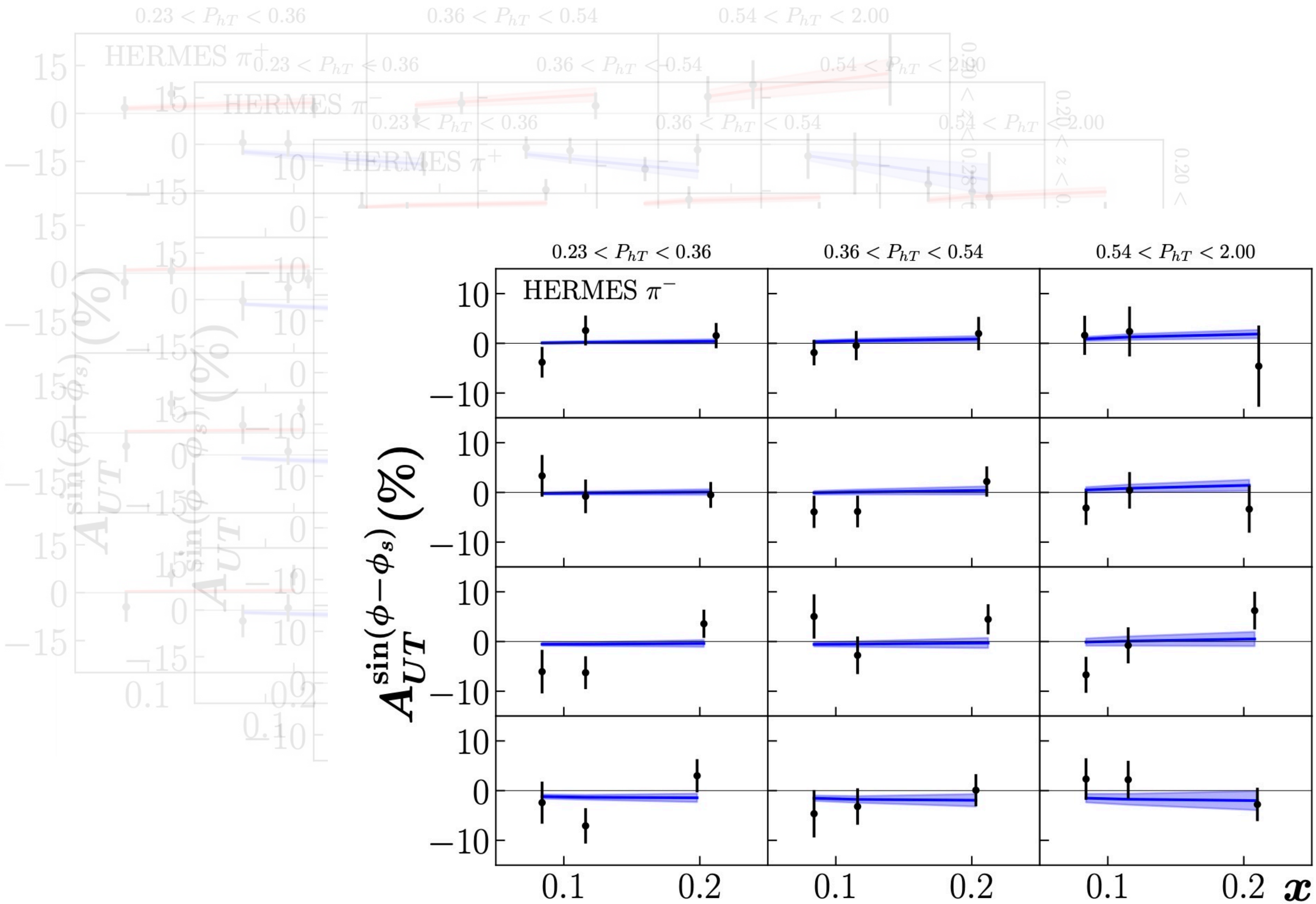
Backup Slides

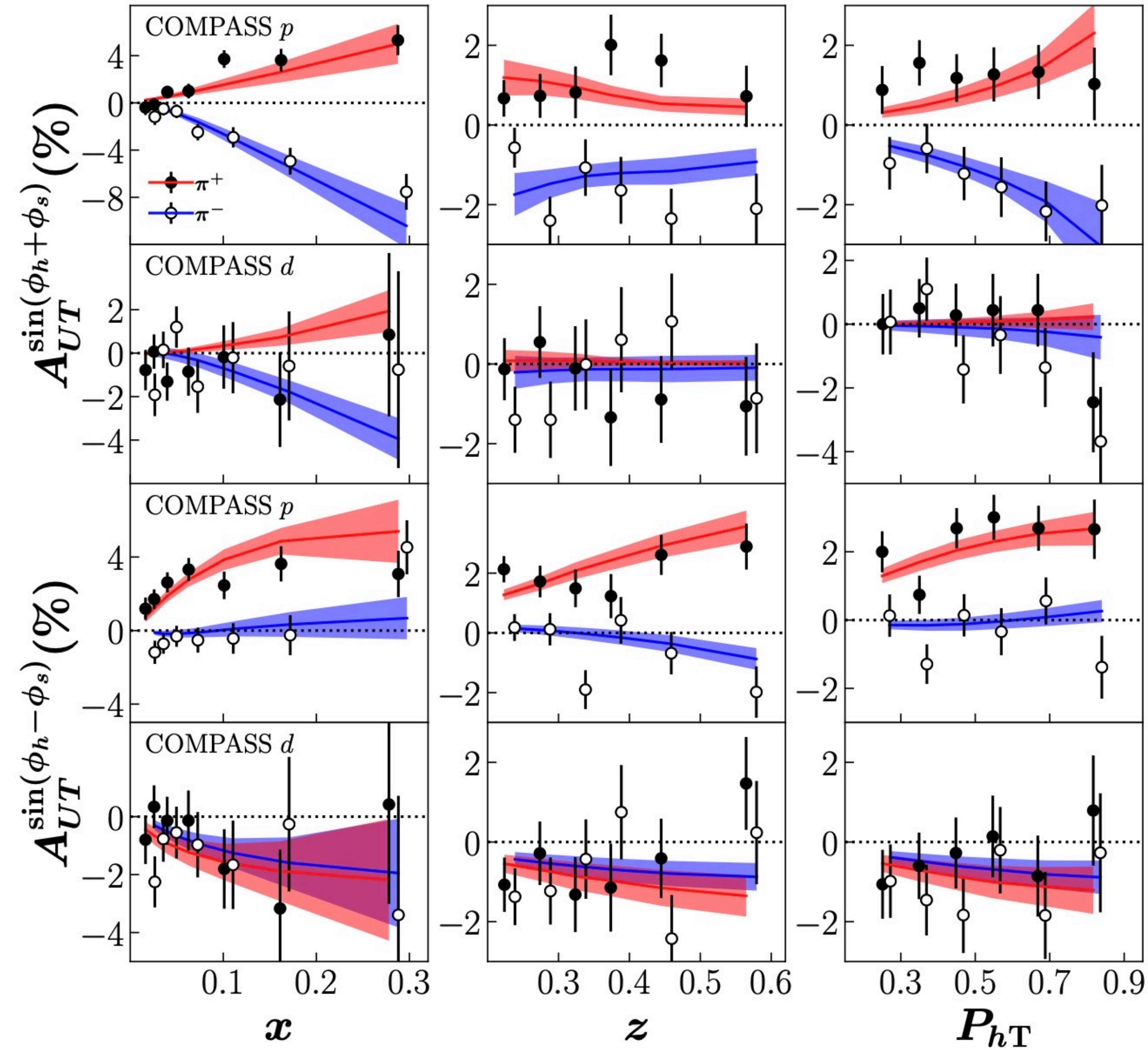


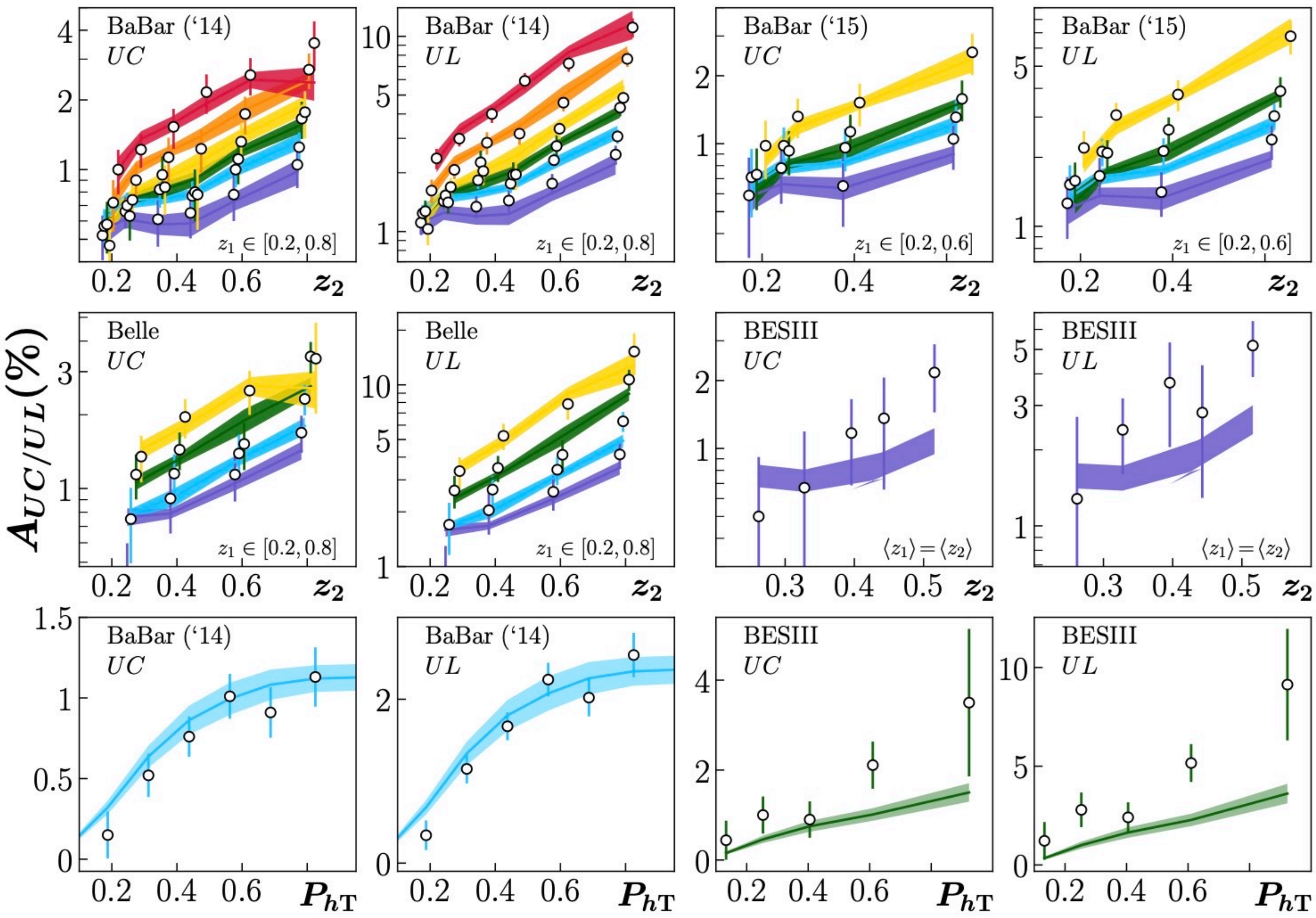
0.23 < PhT < 0.36
0.36 < PhT < 0.54
0.54 < PhT < 2.00
HERMES π+
0.20 < z < 0.28
0.28 < z < 0.37
0.37 < z < 0.49
0.49 < z < 0.70
A_UT^sin(φ+φs) (%)
15
0
−15
0.1
0.2
x

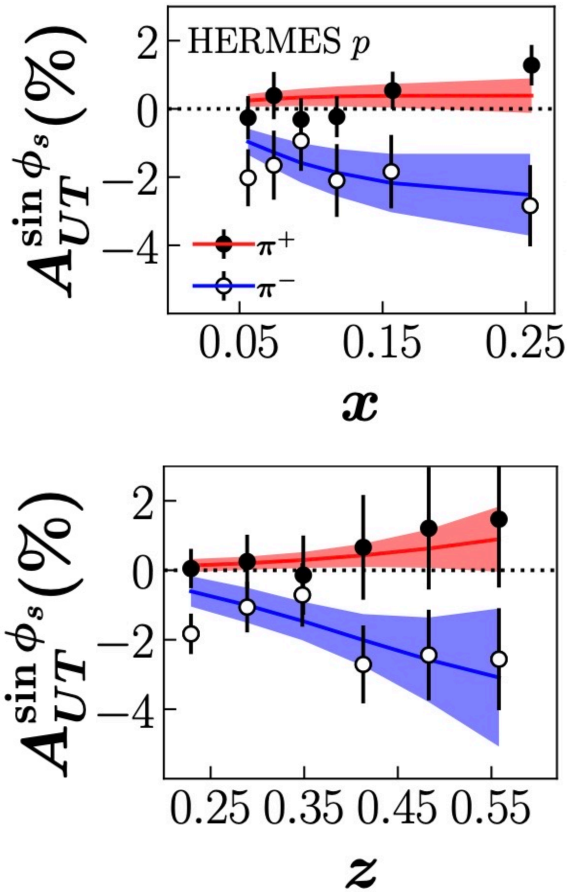

0.23 < P_{hT} < 0.36
0.36 < P_{hT} < 0.54
0.54 < P_{hT} < 2.00
HERMES π^-
$A_{UT}^{\sin(\phi+\phi_s)}$ (%)
0.20 < z < 0.28
0.28 < z < 0.37
0.37 < z < 0.49
0.49 < z < 0.70
15
0
−15
0.1
0.2
x

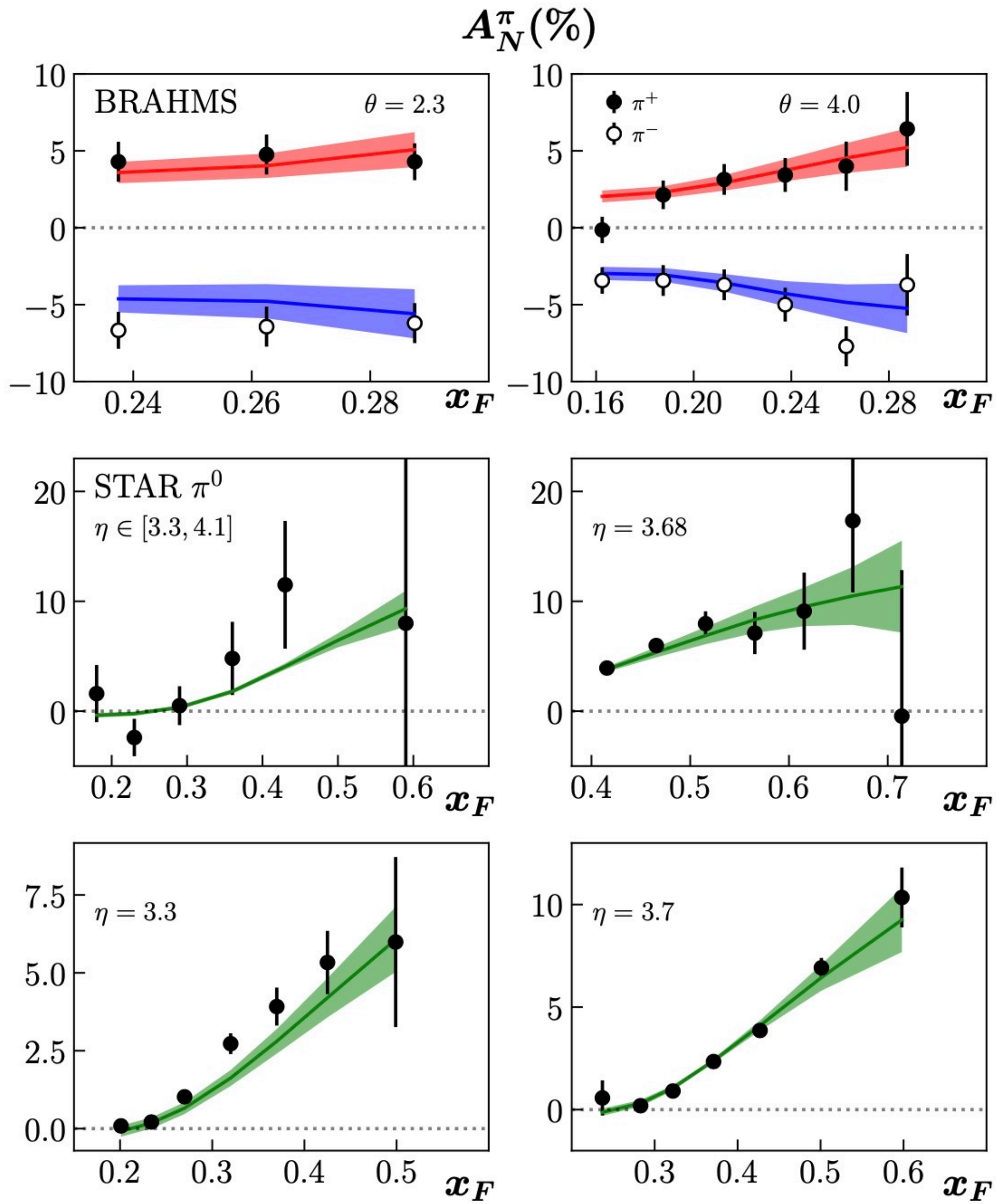

0.23 < P_{hT} < 0.36
0.36 < P_{hT} < 0.54
0.54 < P_{hT} < 2.00
HERMES π^+
$A_{UT}^{\sin(\phi-\phi_s)}$ (%)
0.20 < z < 0.28
0.28 < z < 0.37
0.37 < z < 0.49
0.49 < z < 0.70
10
0
−10
0.1
0.2
x

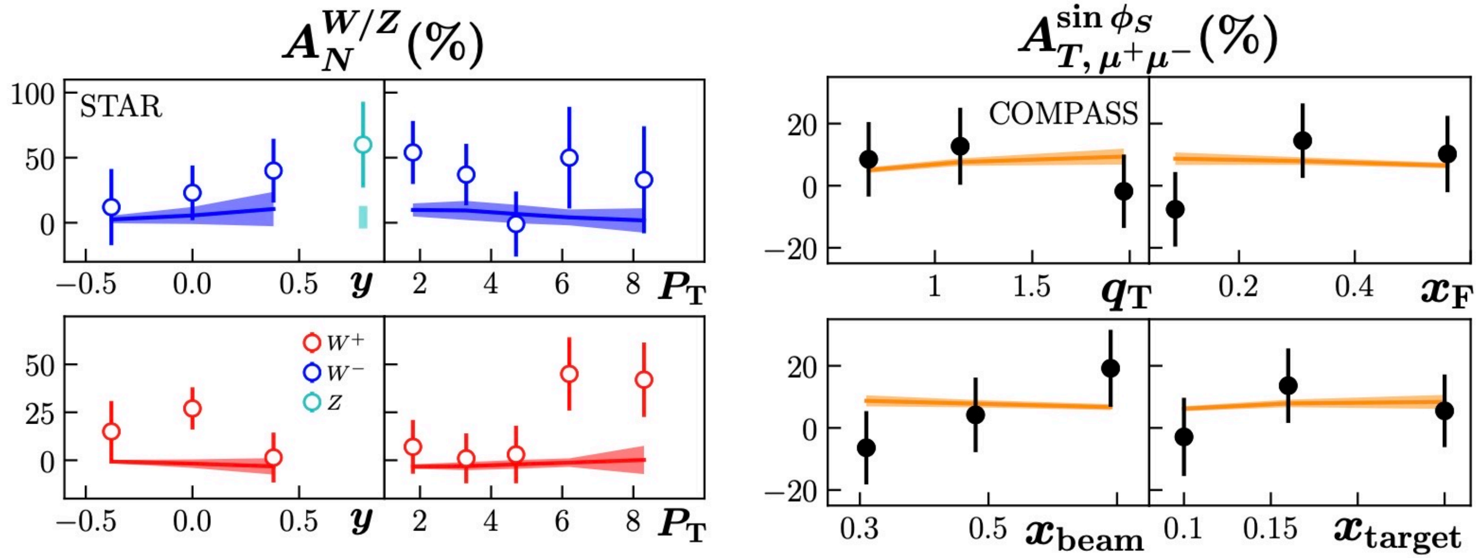

0.23 < P_{hT} < 0.36
0.36 < P_{hT} < 0.54
0.54 < P_{hT} < 2.00
HERMES π^-
$A_{UT}^{\sin(\phi-\phi_s)}$ (%)
0.20 < z < 0.28
0.28 < z < 0.37
0.37 < z < 0.49
0.49 < z < 0.70
x


COMPASS p
COMPASS d
COMPASS p
COMPASS d
π^+
π^-
$A_{UT}^{\sin(\phi_h+\phi_s)}$ (%)
$A_{UT}^{\sin(\phi_h-\phi_s)}$ (%)
x
z
P_{hT}


$A_{UC/UL}(\%)$
BaBar ('14) UC
$z_1 \in [0.2, 0.8]$
BaBar ('14) UL
$z_1 \in [0.2, 0.8]$
BaBar ('15) UC
$z_1 \in [0.2, 0.6]$
BaBar ('15) UL
$z_1 \in [0.2, 0.6]$
Belle UC
$z_1 \in [0.2, 0.8]$
Belle UL
$z_1 \in [0.2, 0.8]$
BESIII UC
$\langle z_1 \rangle = \langle z_2 \rangle$
BESIII UL
$\langle z_1 \rangle = \langle z_2 \rangle$
BaBar ('14) UC
BaBar ('14) UL
BESIII UC
BESIII UL
z_2
P_{hT}


HERMES p
$A_{UT}^{\sin\phi_s}(\%)$
π^+
π^-
2
0
−2
−4
0.05
0.15
0.25
x
$A_{UT}^{\sin\phi_s}(\%)$
0.25
0.35
0.45
0.55
z


$A_N^\pi(\%)$
BRAHMS
$\theta = 2.3$
$\theta = 4.0$
π^+
π^-
10
5
0
−5
−10
0.24
0.26
0.28
0.16
0.20
0.24
0.28
x_F
STAR π^0
$\eta \in [3.3, 4.1]$
$\eta = 3.68$
20
10
0
0.2
0.3
0.4
0.5
0.6
0.7
$\eta = 3.3$
$\eta = 3.7$
7.5
5.0
2.5
0.0


$A_N^{W/Z}$ (%)
STAR
W^+
W^-
Z
y
P_T
$A_{T,\mu^+\mu^-}^{\sin\phi_S}$ (%)
COMPASS
q_T
x_F
x_{beam}
x_{target}

- Response to comment by Rogers and Courtoy - arXiv:2404.02281
  - The potential issue about the violation of the number sum rule equally applies to single-hadron FFs. Nevertheless, the universally accepted number density interpretation of $D_1(z)$ (Collins and Soper (1982)) is not called into question.

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- "*Note that changes in variables here do not undermine the number density interpretation. If two different variable choices are related by a simple Jacobian J,*

$$\frac{d\hat{N}_h}{d\Phi} = J\frac{d\hat{N}_h}{d\Phi'}$$

then $d\hat{N}_h/d\Phi$ and $d\hat{N}_h/d\Phi'$ *both have equally valid number density interpretations in terms of their respective phase spaces,* $d\Phi$ or $d\Phi'$"

… We agree, as we already made this statement in our paper (see main talk slides), *but it is necessary to first show explicitly that one has defined a function that is a number density* (either for the variable set Φ or Φ')

- Response to comment by Rogers and Courtoy - arXiv:2404.02281 (continued)
  - *"At lowest order in perturbation theory, the Jacobian factor can simply be absorbed into the overall hard factor to maintain consistency with a factorization formula."*

$\ldots\ \tilde{D}_1^{h_1 h_2/q}(z,\zeta,\vec{R}_T^2)$ – different definition of the DiFF that is also *supposed to be a number density in* $(z,\zeta,\vec{R}_T)$

$$\Longrightarrow\ \frac{d\sigma}{dz\,d\zeta\,d^2\vec{R}_T} = \sum_q \boxed{(K\,\hat{\sigma}^q)}\,\tilde{D}_1^{h_1 h_2/q}(z,\zeta,\vec{R}_T^2)$$

NOT the total partonic cross section for $e^+e^- \to \gamma \to q\bar{q}$

$\Longrightarrow\ \tilde{D}_1^{h_1 h_2/q}(z,\zeta,\vec{R}_T^2)$ *cannot be a number density in* $(z,\zeta,\vec{R}_T)$