June 3, 2024
Trieste

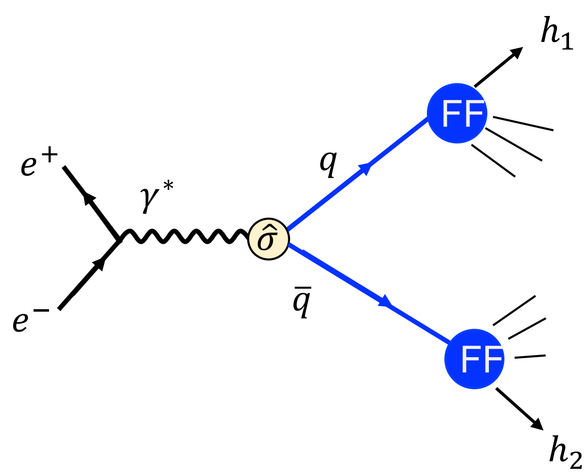
New MC results on polarized quark fragmentation: Collins asymmetries in e^+e^-

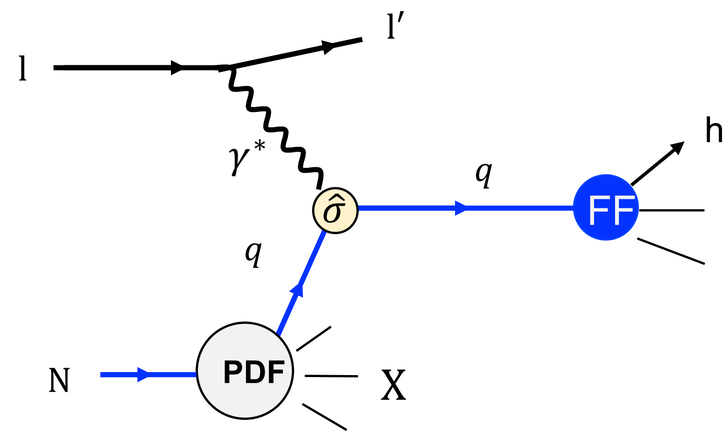
Albi Kerbizi
University of Trieste and INFN Trieste

in collaboration with Xavier Artru, Leif Lönnblad and Anna Martin

POLFRAG

Studying hadronization





- ❑ e^+e^- annihilation to hadrons
  access to FFs x FFs

- ❑ Semi Inclusive DIS
  access to PDFs x FFs

Possibilities for studying hadronization

- ❑ Phenomenological fits — parametrize FFs (and PDFs), extract from data
  e.g. extraction of Collins FF (or IFF) and transversity

- ❑ Modeling — analytic calculations, or
  Monte Carlo event generators
  develop a model, implement in a program, compare the results with the data, make predictions

Modeling hadronization: the string+$^3P_0$ model

- We have developed a model for the simulation of the fragmentation polarized quarks
  string+$^3P_0$ model: adds spin to the Lund string model

AK, Artru, Belghobsi, Bradamante, Martin, PRD 97, 074010 (2018)	2018	PS mesons
AK, Artru, Belghobsi, Martin, PRD 100, 014003 (2019)	2019	PS mesons
AK, Artru, Martin, PRD 104, 114038 (2021)	2021	PS mesons + VM

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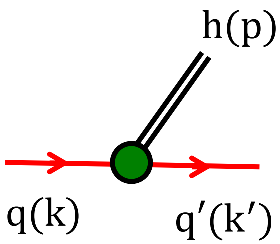


$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$
$Z_+ = p^+/k^+$

Quark splitting described by a 2x2 splitting amplitude

$$T_{q',h,q} \propto \left[F^{\mathrm{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \; \Gamma_{h,s_h}$$

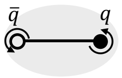
$^3P_0$ mechanism
μ complex mass paramter

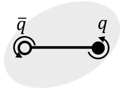
Coupling
e.g.
$\Gamma_{h=PS} = \sigma_z$

$\mathrm{Im}(\mu) \rightarrow$ T spin effects (Collins, dihadron)
$\mathrm{Re}(\mu) \rightarrow$ L spin effects ($G_1^{\perp}$..)

Coupling of quarks to h

PS mesons	no free parameters
Vector Mesons	f_L fraction of L polarized mesons
	θ_{LT} oblique polarization (L/T interference)





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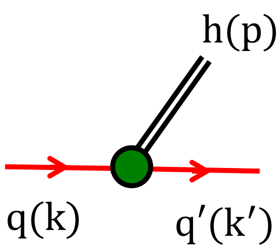

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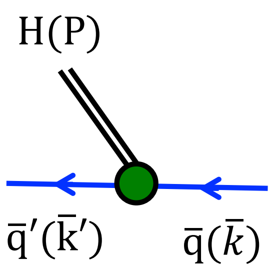

$$\bar{\mathbf{k}}_T = \mathbf{P}_T + \bar{\mathbf{k}}'_T$$
$$Z_- = P^-/\bar{k}^-$$

For anti-quark splitting

$$\{q, h, q'\} \to \{\bar{q}, H, \bar{q}'\},\;\; Z_+ \to Z_-,\;\; \{\mathbf{k}_T, \mathbf{p}_T, \mathbf{k}'_T\} \to \{\bar{\mathbf{k}}_T, \mathbf{P}_T, \bar{\mathbf{k}}'_T\}$$

AK, Artru, PRD 109 (2024) 5, 054029

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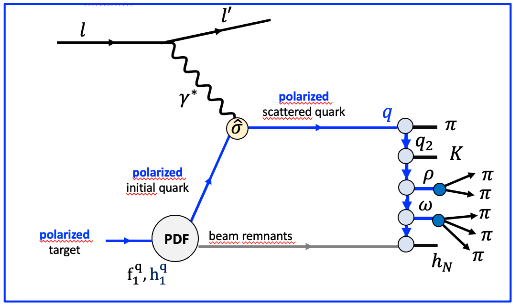
- Applied to
  - SIDIS: polarized fragmentation quarks of struck quarks
    polarization of remnant neglected
    implemented in Pythia via StringSpinner (public)

    AK, L. Lönnblad, CPC **272** (2022) 108234;
    CPC **292** (2023) 108886



→ promising description of transverse-spin asymmetry data
*see most recent version including PS + VM production CPC **292** (2023) 108886*

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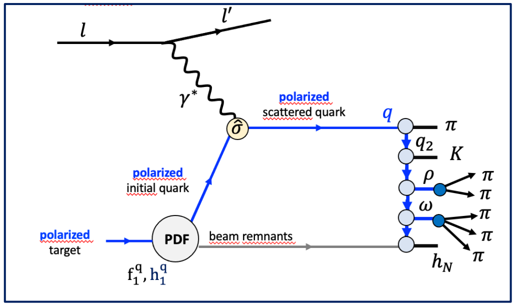

→ promising description of transverse-spin asymmetry data
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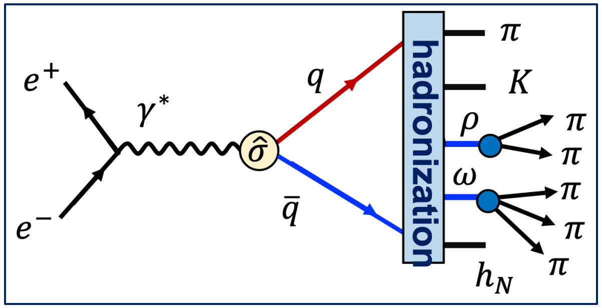
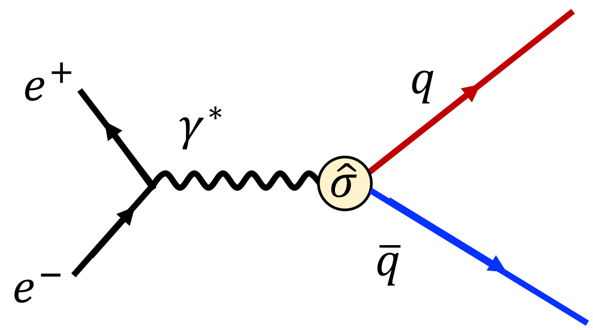
- e^+e^- annihilation to hadrons → this talk
hadronize $q\bar{q}$ using the string+$^3P_0$ model accounting for
correlated spin states of q and $\bar{q}$
quantum mechanical spin-correlations in fragmentation
AK, X. Artru, PRD 109 (2024) 5, 054029



Application of the string+$^3P_0$ model to e^+e^- annihilation

AK, X. Artru, PRD 109 (2024) 5, 054029

Recursive recipe for $\mathrm{e^+e^-}$



Steps:

1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

Recursive recipe for e^+e^-

Steps:

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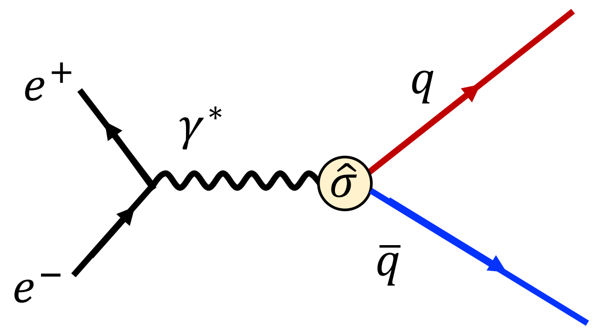
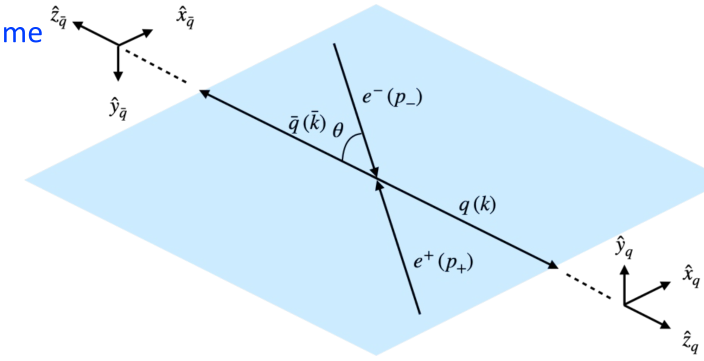


[AK, X. Artru, PRD 109 (2024) 5, 054029]

Set up the scattering $e^+e^- \rightarrow q\bar{q}$ in the c.m.s

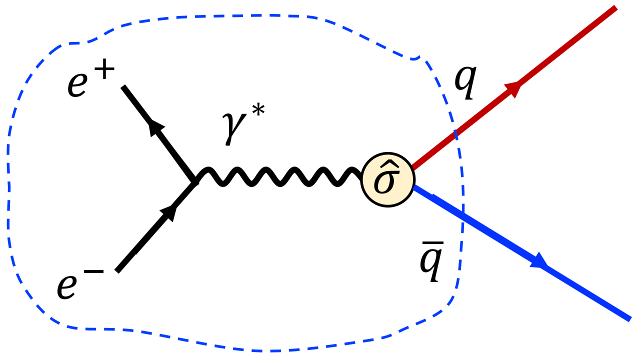
generate the quark flavors and kinematics using differential cross section

antiquark helicity frame (AHF)



quark helicity frame (QHF)

Recursive recipe for e^+e^-



Steps:
1. Hard scattering
2. **Joint spin density matrix**
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- ❑ Set up the joint spin density matrix of the $q\bar{q}$ pair

$$\rho(q,\bar{q}) = C^{q\bar{q}}_{\alpha\beta} \quad \sigma^{\alpha}_{q} \otimes \sigma^{\beta}_{\bar{q}}$$

correlation coefficients

Pauli matrices along QHF and AHF

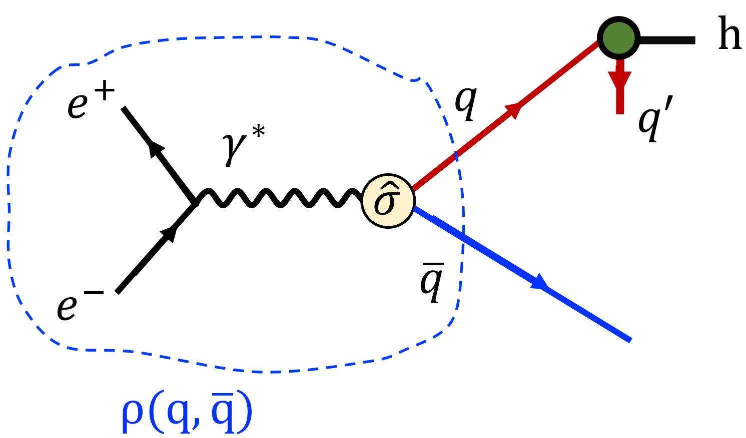
$\alpha = 0, x_q, y_q, z_q$

$\beta = 0, x_{\bar{q}}, y_{\bar{q}}, z_{\bar{q}}$

For γ^* exchange

$$\rho(q,\bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma^z_q \otimes \sigma^z_{\bar{q}} + \frac{\sin^2\theta}{1+\cos^2\theta}\left[\sigma^x_q \otimes \sigma^x_{\bar{q}} + \sigma^y_q \otimes \sigma^y_{\bar{q}}\right]$$

Recursive recipe for e^+e^-



Steps:
1. Hard scattering
2. Joint spin density matrix
3. **Hadron emission from *q***
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Emit the first hadron using the splitting function (emission probability density)

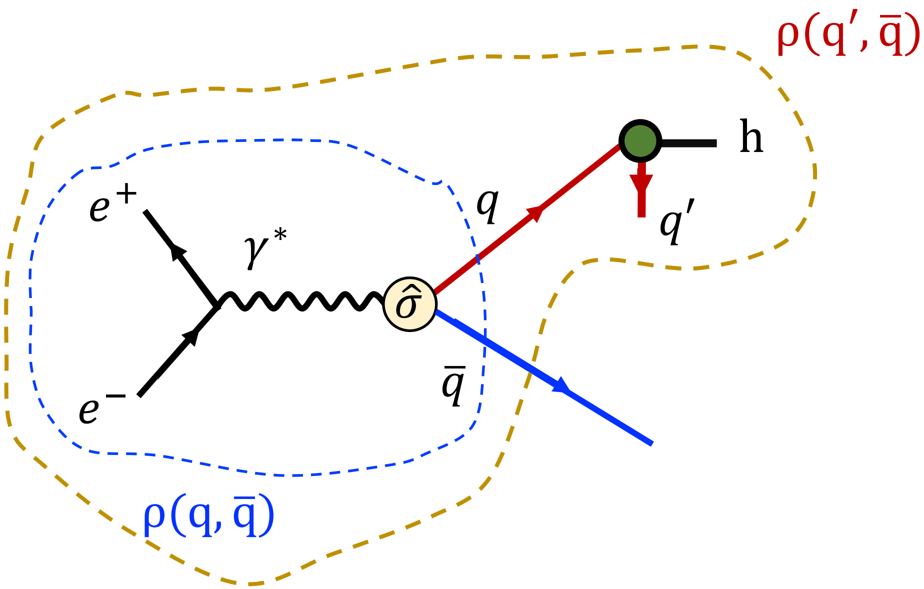
$$\frac{dP(q \to h + q'; q\bar{q})}{dZ_+ Z_+^{-1} d^2 p_T} = \mathrm{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^\dagger_{q',h,q} = F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}})$$

$$\mathbf{T}_{q',h,q} \equiv T_{q',h,q} \otimes 1_{\bar{q}}$$

in the QHF

- VM emission → backup

Recursive recipe for e^+e^-



Steps:
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4. **Update density matrix**
5. Hadron emission from $\bar{q}$
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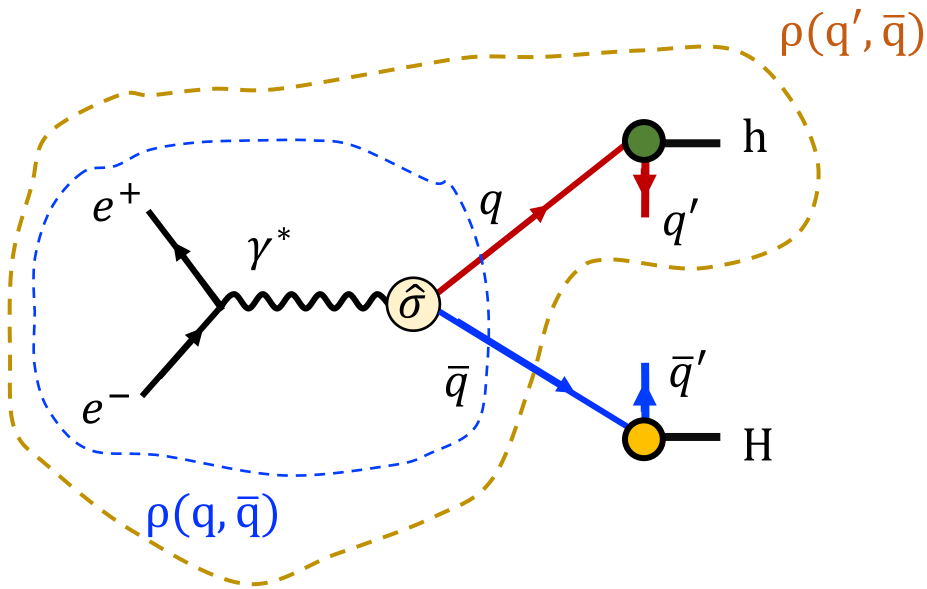
[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Evaluate the spin density matrix $\rho(q'\bar{q})$

$$\rho(q', \bar{q}) = \mathbf{T}_{q',h,q} \;\; \rho(q, \bar{q}) \;\; \mathbf{T}^{\dagger}_{q',h,q}$$

includes the information on the emission of h

Recursive recipe for e^+e^-



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5. **Hadron emission from $\bar{q}$**
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Emit a hadron from the $\bar{q}$ side using the splitting function

$$\frac{dP(\bar{q} \to H + \bar{q}'; q'\bar{q})}{dZ_- Z_-^{-1} d^2P_T} = \mathrm{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}',H,\bar{q}}\, \rho(q',\bar{q})\, \mathbf{T}^{\dagger}_{\bar{q}',H,\bar{q}} = F_{\bar{q}',H,\bar{q}}(Z_-, P_T; \bar{k}_T, C^{q'\bar{q}})$$

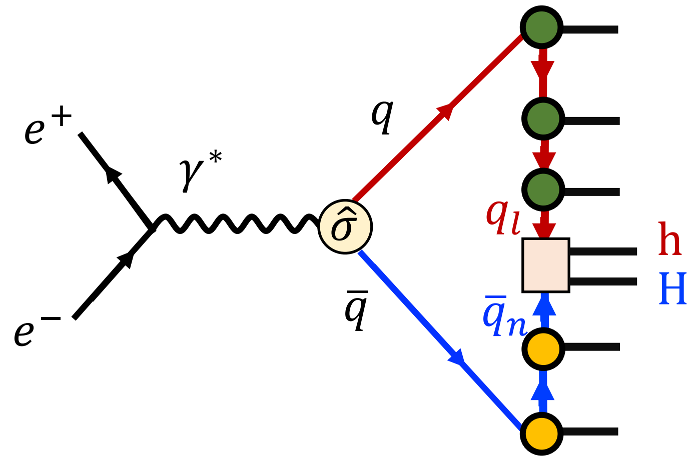
Depend on the azimuthal angle h

Expressed in the AHF

conditional probability of emitting H, having emitted h
→ correlations between the transverse momenta

[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

Recursive recipe for e^+e^-



Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. **Exit condition**

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Iterate until the exit condition is called and the last quark pair is hadronized
  more details in PRD 109 (2024) 5, 054029

Simulations of e^+e^- annihilation with spin effects

now possible with Pythia 8.3 + StringSpinner

- $\sqrt{s} = 10.6\ \mathrm{GeV}$, γ^* exchange, quarks produced u, d, s
  consistent with BELLE and BABAR data

- Free parameters

spin-less hadronization	as in standard Pythia 8.3
complex mass μ	as in AK, Lonnblad, CPC 292 (2023) 108886
$f_L = 0.12$	~ T pol. VMs
$\theta_{LT} = -0.65$	interference between T and L pol. of VMs

  found to give a satisfactory agreement with e^+e^- data,
  ok also for SIDIS

- **Compare with Collins asymmetries**

The Collins asymmetries in e^+e^- for back-to-back h_1h_2

- **Thrust axis method**

Boer et al., NPB504, 345 (1997).
Boer, NPB, 806:23–67, 2009
Anselmino et al., PRD 92, 114023 (2015)
D'Alesio et al., JHEP 10 (2021) 078
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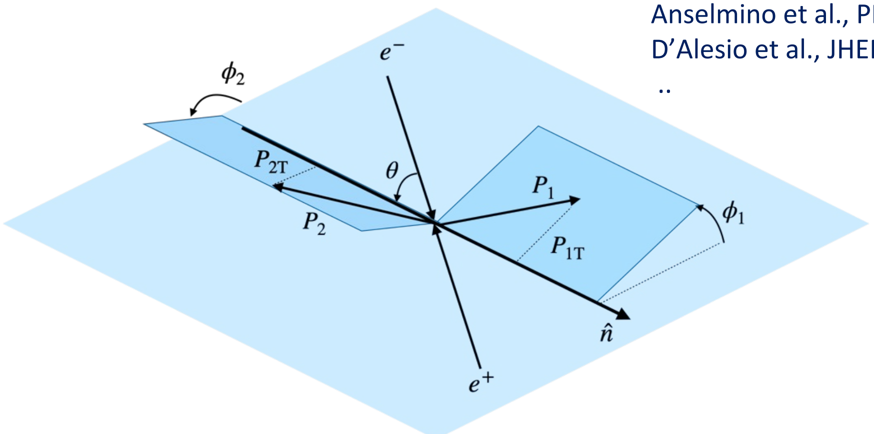

$$N_{h_1h_2} \propto 1 + \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle} A_{12} \cos(\phi_1 + \phi_2)$$

Collins asymmetry

$$A_{12} = \frac{\sum_q e_q^2 \; H_{1q}^{\perp h_1} \; H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2 \; D_{1q}^{h_1} \; D_{1\bar{q}}^{h_2}}$$

- Measured asymmetry

$$A_{12}^{UL(UC)} \simeq A_{12}^{U} - A_{12}^{L(C)}$$

U unlike sign pair e.g. $\pi^+\pi^- + \pi^-\pi^+$
L like sign pair e.g. $\pi^+\pi^+ + \pi^-\pi^-$
C charged pair e.g. $\pi^+\pi^- + \pi^-\pi^+ + \pi^+\pi^+ + \pi^-\pi^-$

The Collins asymmetries in e^+e^- for back-to-back h_1h_2

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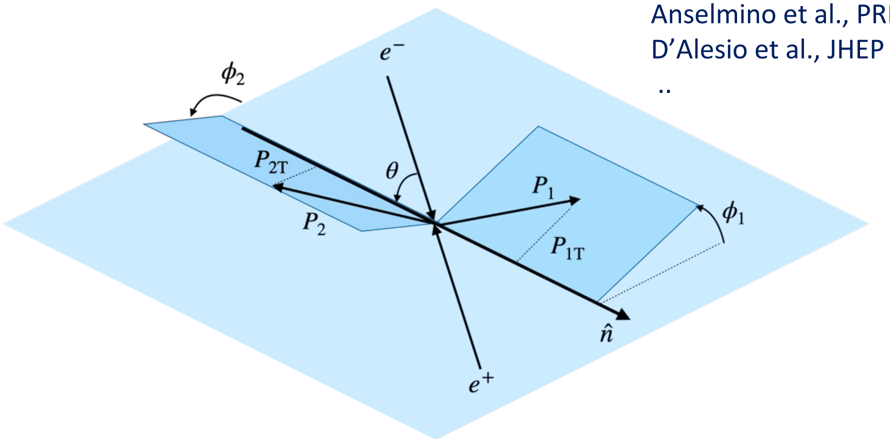
❑ Thrust axis method



$$N_{h_1h_2} \propto 1 + \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle} A_{12} \cos(\phi_1 + \phi_2)$$

Collins asymmetry

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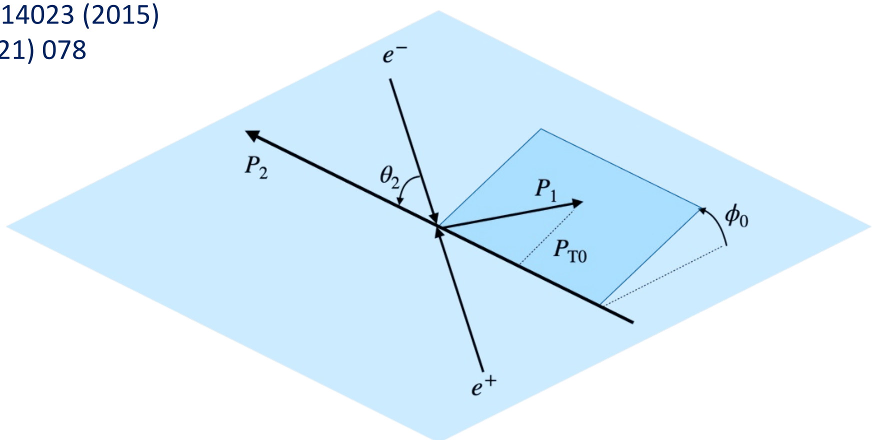
❑ Hadronic plane method



$$N_{h_1h_2} \propto 1 + \frac{\langle \sin^2\theta_2 \rangle}{\langle 1+\cos^2\theta_2 \rangle} A_0 \cos(2\phi_0)$$

Collins asymmetry

$$A_0 = \frac{\sum_q e_q^2 \; w H_{1q}^{\perp h_1} \otimes H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2 \; D_{1q}^{h_1} \otimes D_{1\bar{q}}^{h_2}}$$

❑ Measured asymmetry

$$A_{12}^{UL(UC)} \simeq A_{12}^{U} - A_{12}^{L(C)}$$

❑ Measured asymmetry

$$A_{0}^{UL(UC)} \simeq A_{0}^{U} - A_{0}^{L(C)}$$

U unlike sign pair e.g. $\pi^+\pi^- + \pi^-\pi^+$
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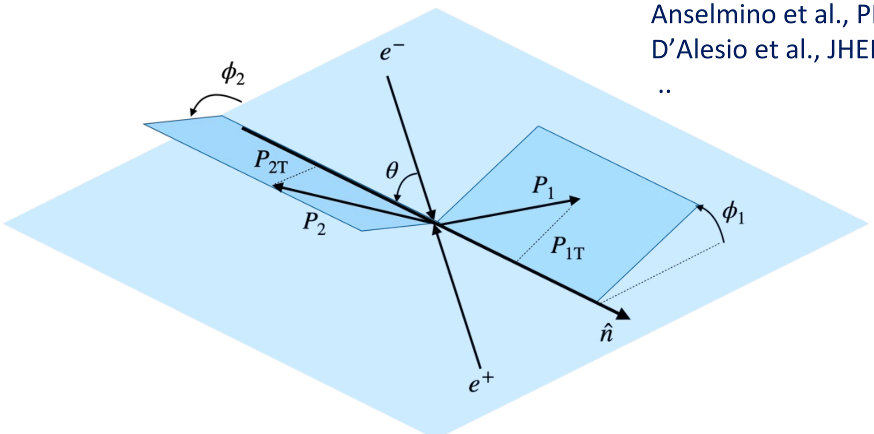

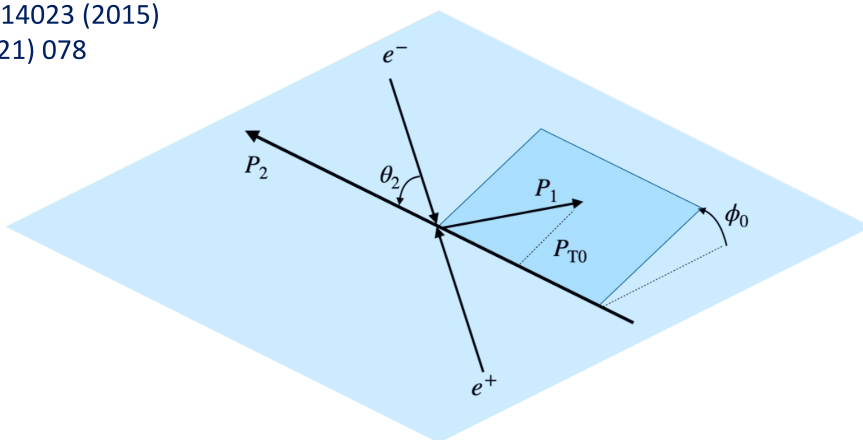

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Measured by	BELLE	2006 ($\pi\pi$), 2008 ($\pi\pi$), 2019 ($\pi\pi, \pi^0\pi, \eta\pi$)
	BABAR	2014 ($\pi\pi$), 2015 ($\pi\pi, \pi K, KK$)
	BESIII	2016 ($\pi\pi$)

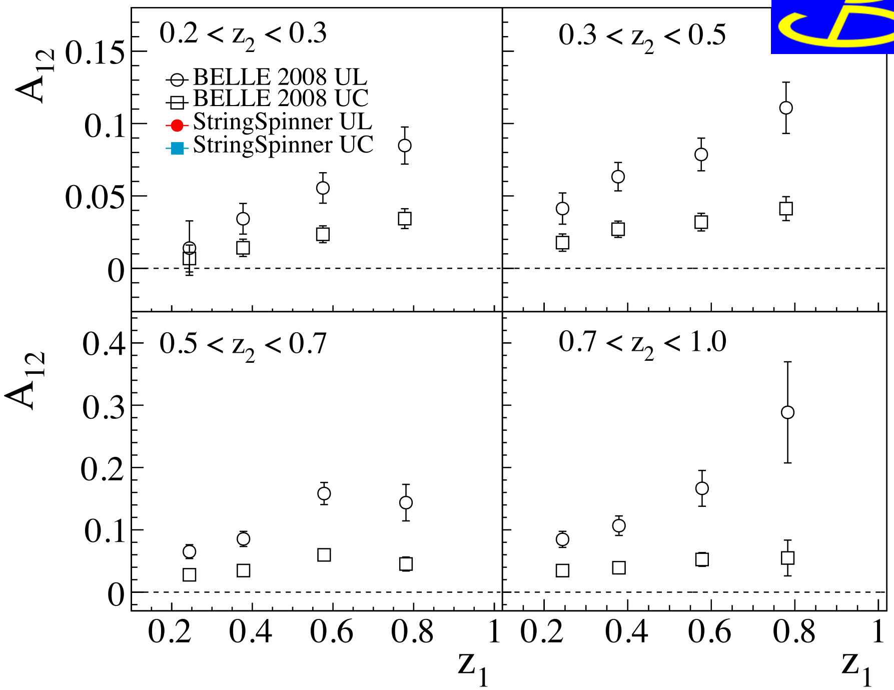
Used in many phenomenological fits to extract h_1 and H_1, here only the model results

Comparison with the A_{12} asymmetry

AK, L. Lönnblad, A. Martin, in preparation

A_{12} asymmetry for charged π π pairs

PRD 78, 032011 (2008)



$$z_i = \frac{2E_{h_i}}{\sqrt{s}}$$

Belle asymmetries corrected for thrust smearing

Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\ \text{GeV}$$

A_{12} asymmetry for charged π π pairs

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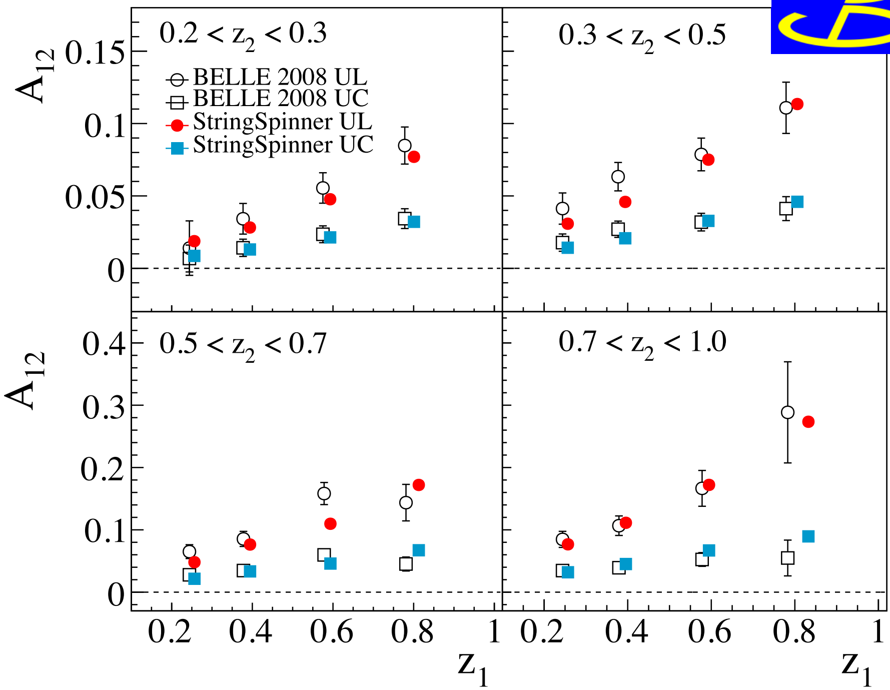

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StringSpinner reproduces trend and size

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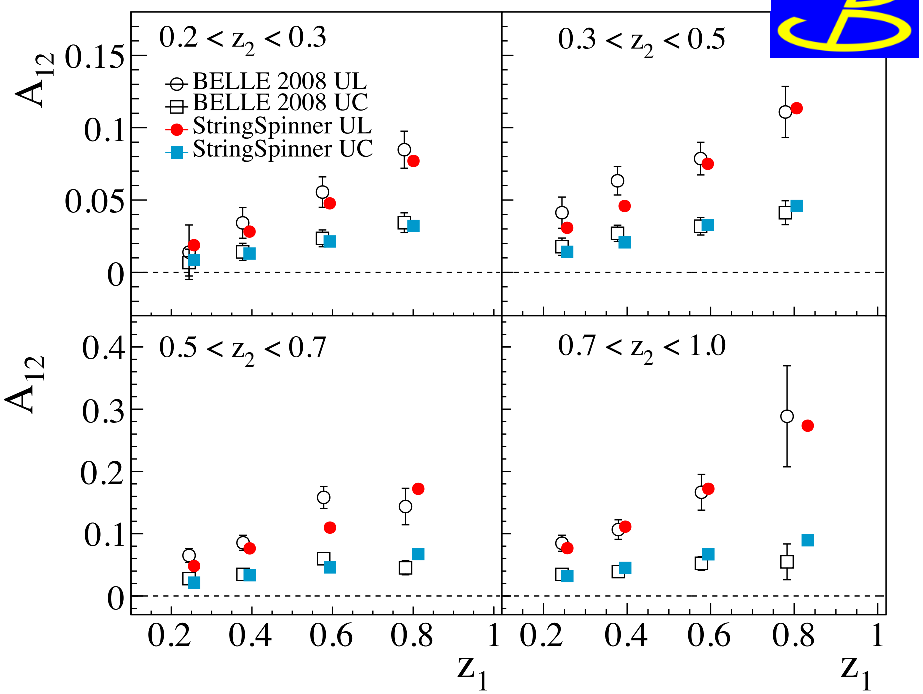

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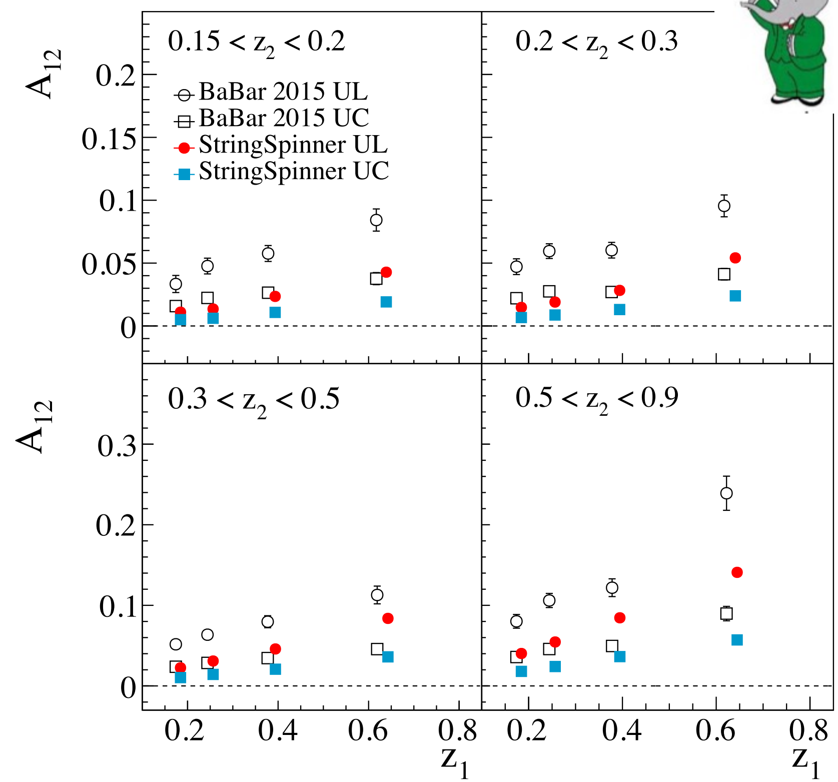
PRD 92, 111101(R) (2015)



BaBar asymmetries corrected for thrust smearing

Cuts:

$$T > 0.8, z > 0.15, Q_T < 3.5 \text{ GeV}, \alpha_O < \pi/4$$

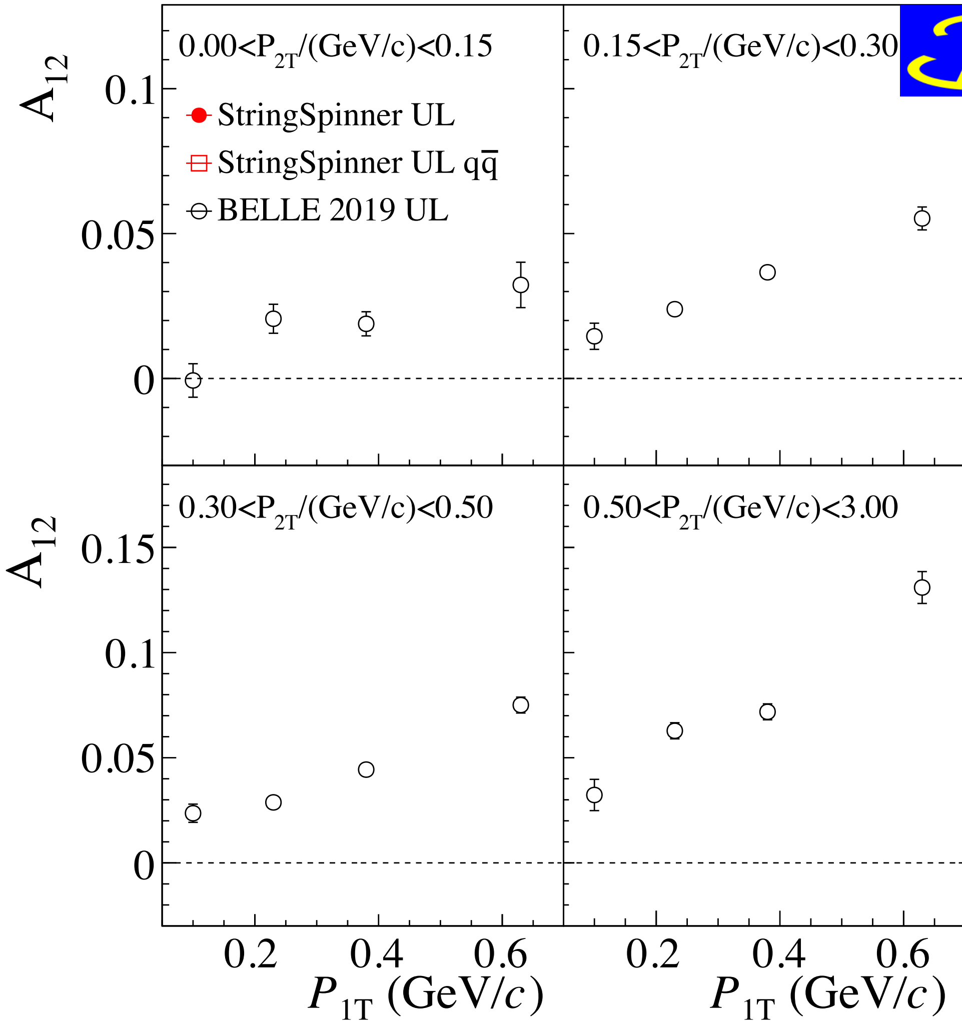
StringSpinner lower than BABAR

difference between BABAR and BELLE known

PRD 90, 052003 (2014)

A_{12}^{UL} asymmetry for charged $\pi\,\pi$ pairs

$P_{T1} \times P_{T2}$ - dependence w.r.t thrust



PRD 100, 092008 (2019)

Asymmetries using thrust axis,
not corrected for thrust smearing

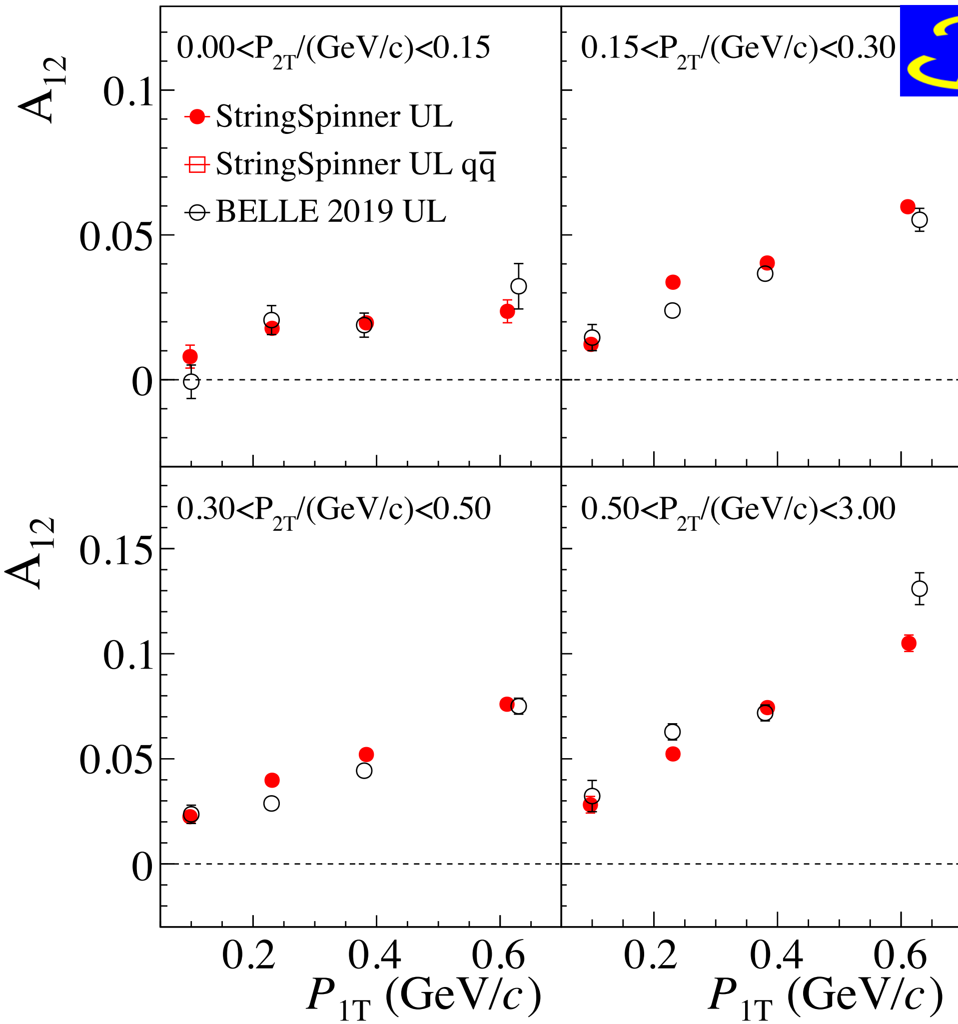
$T > 0.8$

$z > 0.2, P_T < 3.0$ GeV/c

$\alpha_0 < 0.3$ rad

A_{12}^{UL} asymmetry for charged $\pi\ \pi$ pairs

$P_{T1} \times P_{T2}$ - dependence w.r.t thrust



PRD 100, 092008 (2019)

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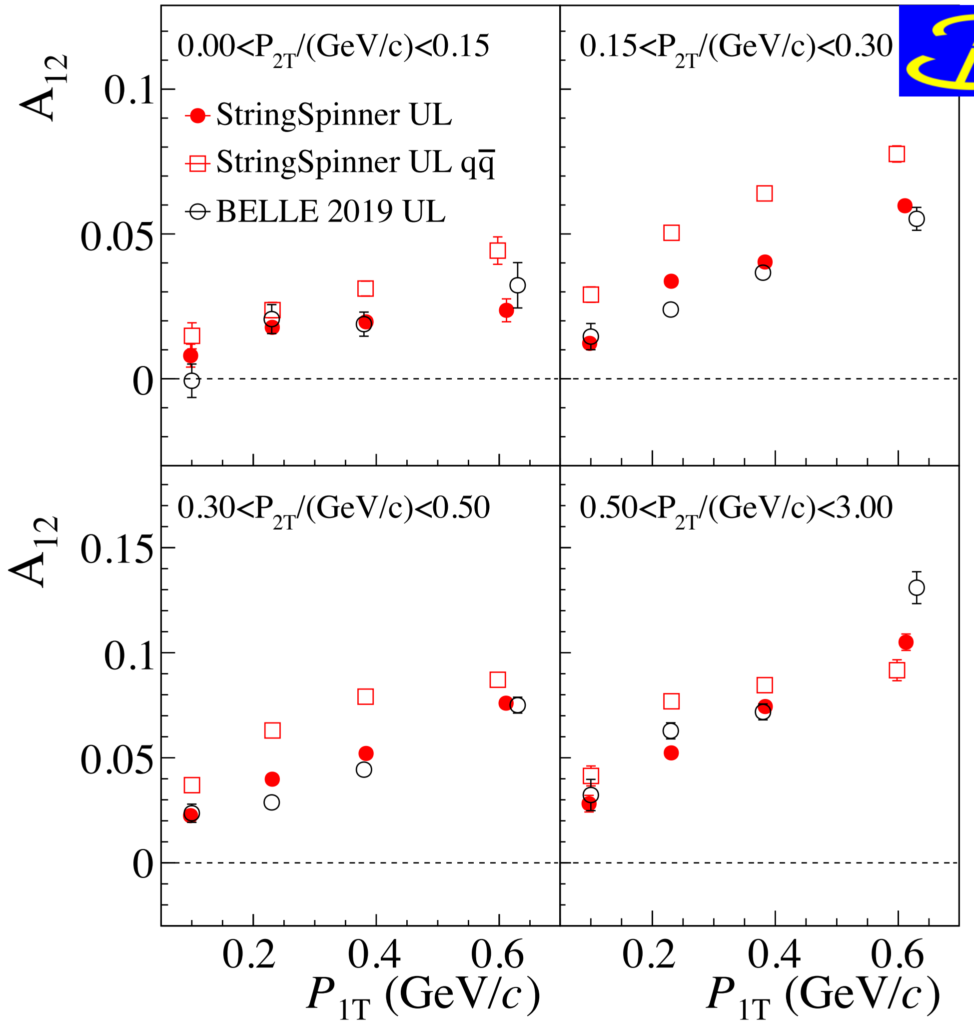
StringSpinner reproduces the nearly linear trend observed by BELLE

A_{12}^{UL} asymmetry for charged $\pi\ \pi$ pairs
$P_{T1} \times P_{T2}$ - dependence w.r.t thrust



PRD 100, 092008 (2019)

Asymmetries using thrust axis,
not corrected for thrust smearing

$T > 0.8$

$z > 0.2, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$ rad

StringSpinner reproduces the nearly linear trend observed by BELLE

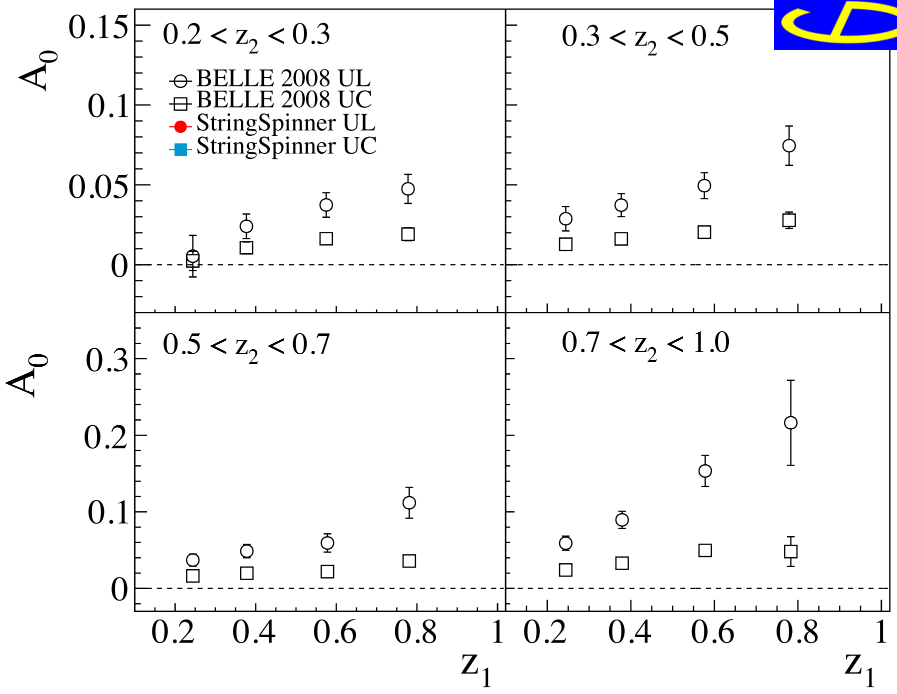
Linear trend shows up as an effect of the misalignment between thrust and $q\bar{q}$ axis
strong effect by thrust

Comparison with the A_0 asymmetry

AK, L. Lönnblad, A. Martin, in preparation

A_0 asymmetry for $\pi\,\pi$ pairs:

PRD 78, 032011 (2008)

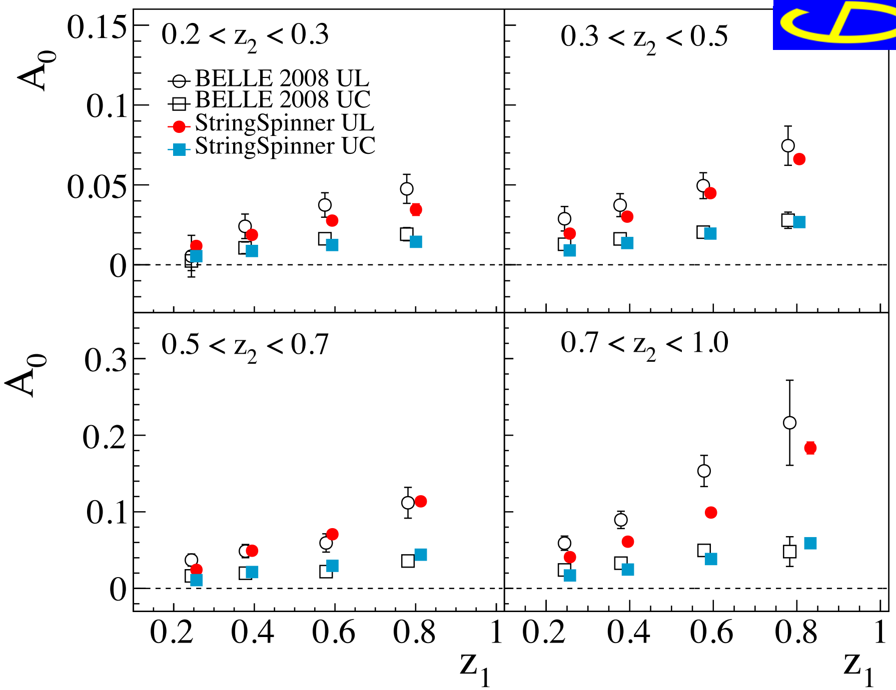


Cuts:

$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$

A_0 asymmetry for π π pairs:

PRD 78, 032011 (2008)



Cuts:

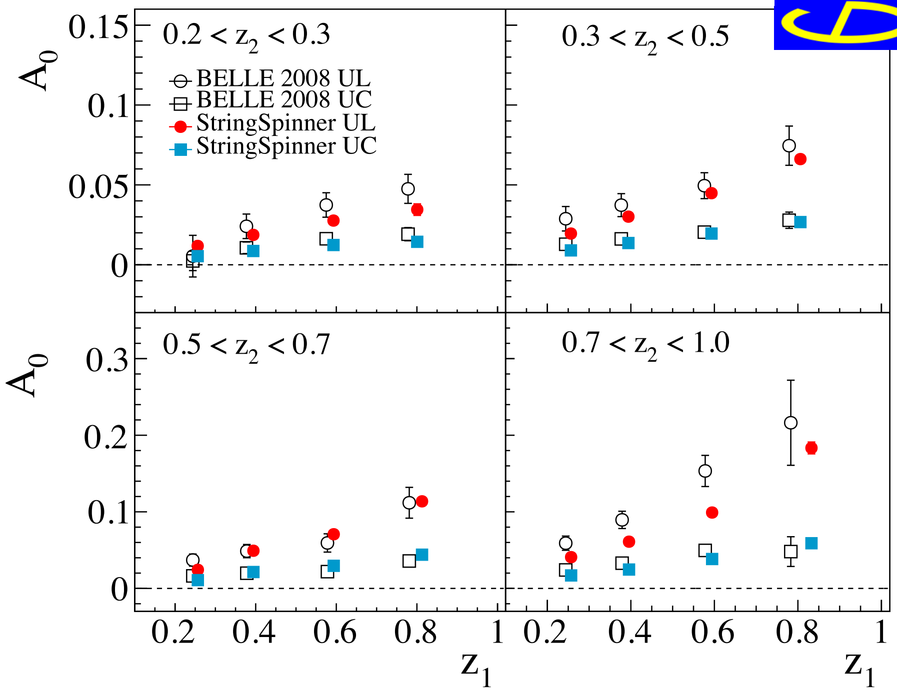
$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

Trend reproduced by string+$^3P_0$

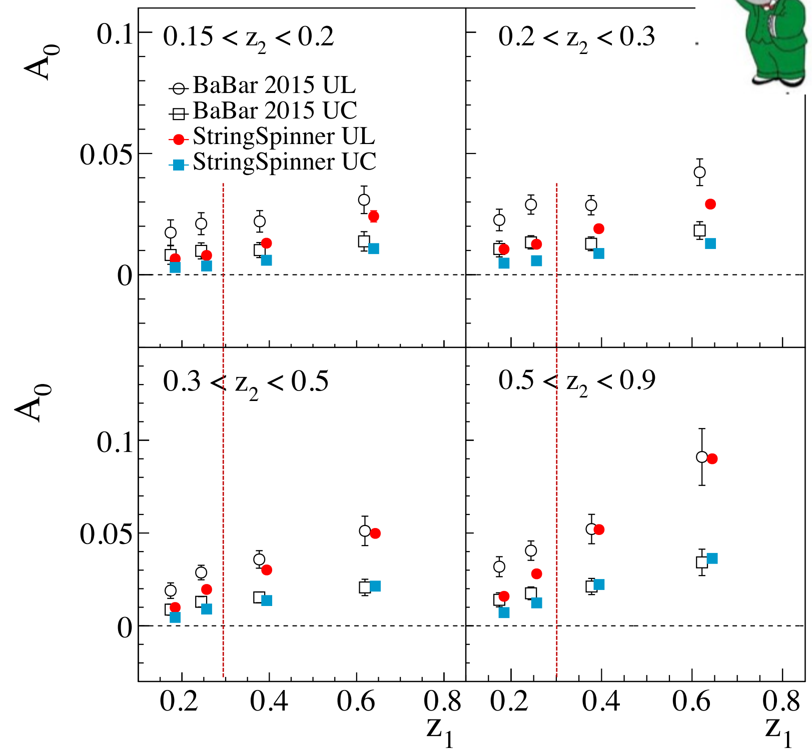
somewhat lower values in the last z_2 bin

A_0 asymmetry for π π pairs:

PRD 78, 032011 (2008)



PRD 92, 111101(R) (2015)



Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

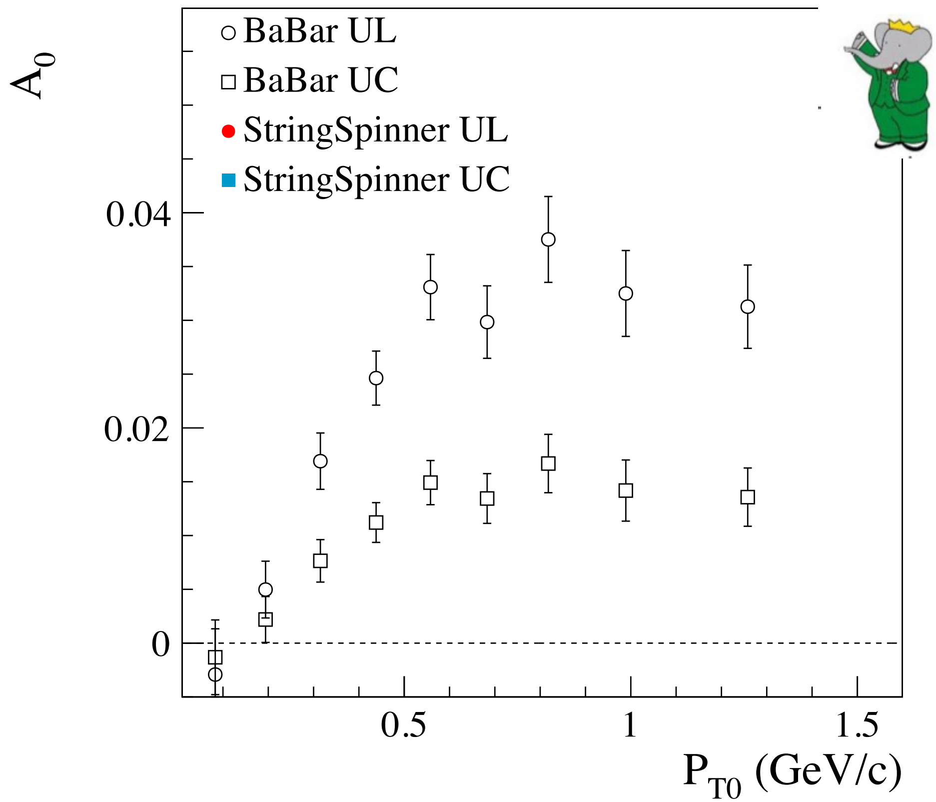
Trend reproduced by string+$^3P_0$

somewhat lower values in the last z_2 bin

BABAR asymmetries higher for z_1<0.3 z_2<0.3

A_0 asymmetry for π π pairs: P_{T0} dependence

PRD 90, 052003 (2014)

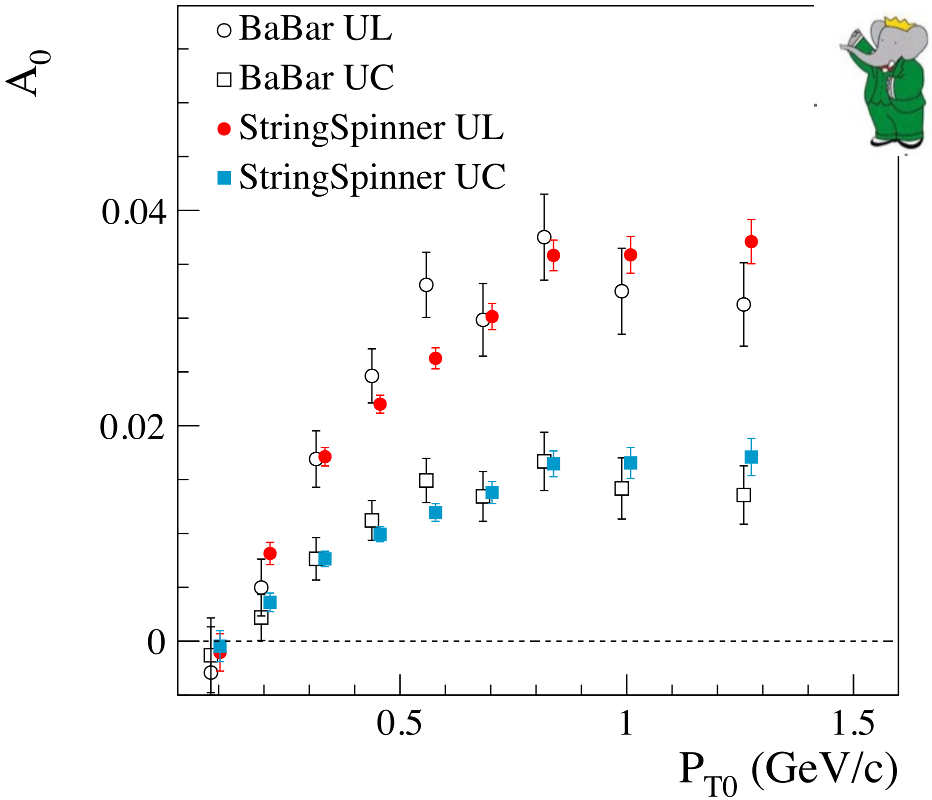


Cuts:

$T > 0.8, z > 0.15, Q_T < 3.5\text{GeV}$

A_0 asymmetry for π π pairs: P_{T0} dependence

PRD 90, 052003 (2014)



Cuts:
$T > 0.8, z > 0.15, Q_T < 3.5\text{GeV}$

Transverse-momentum dependence reproduced by string+$^3P_0$!

Conclusions

- The string+$^3P_0$ model is applied to the fragmentation of a string stretched between quarks with entangled spin states

  these kind of strings are produced also in pp collisions..

- Implemented in Pythia 8.3 for e^+e^- using StringSpinner

- Encouraging results on Collins asymmetries in e^+e^-

  study of Artru-Collins asymmetries ongoing

- More developments of the string+$^3P_0$ model foreseen

  baryon production

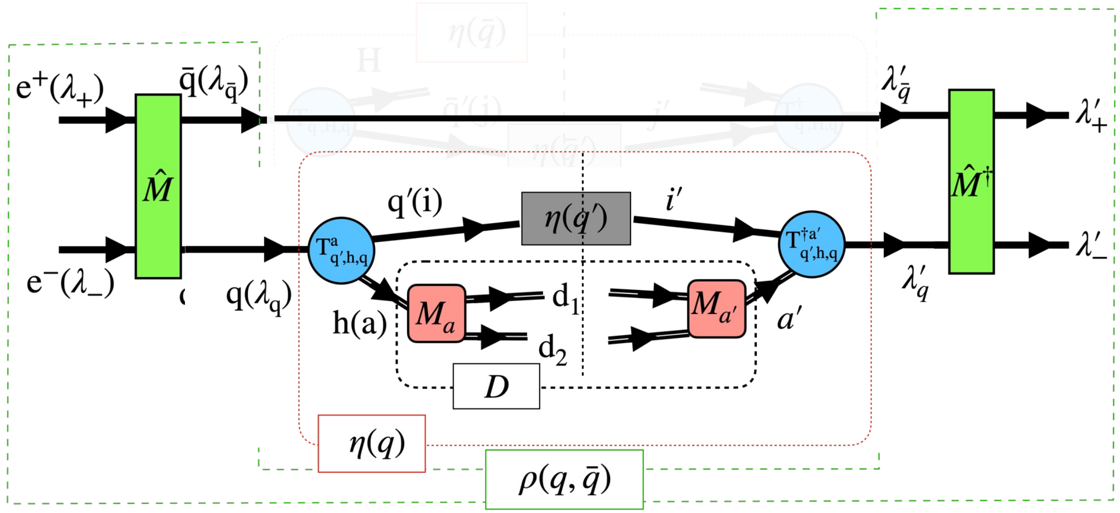
  connection to a parton shower

  …

Still a long way to an event generator fully implementing spin effects..
but several steps already done!

Backup

The recursive recipe for simulating e^+e^- annihilation: VM emission



For a vector meson h=VM

$$\rightarrow \eta(q) = \mathbf{T}^{a'\dagger}_{q',h=VM,q}\ \eta(q')\ \mathbf{T}^{a}_{q',h=VM,q} D_{a'a},\ \eta(q') = 1_{q'} \text{ and } \eta(\bar{q}) = 1_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$)

$$\frac{dP(q \rightarrow h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2p_T} = \mathrm{Tr}_{q'\bar{q}}\ \mathbf{T}^{a}_{q',h,q}\ \rho(q,\bar{q})\ \mathbf{T}^{a\dagger}_{q',h,q} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$$

ii) Calculate the spin density matrix of h=VM, and decay the meson

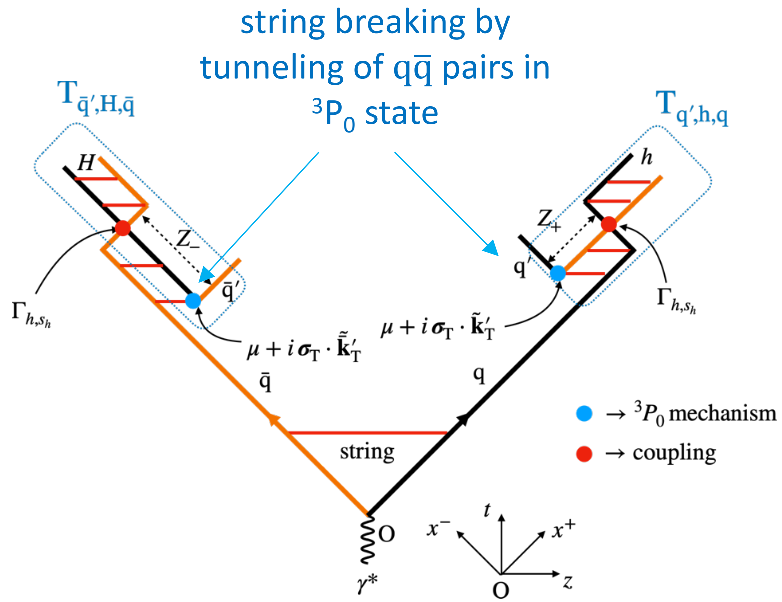
$$\rho_{aa'}(h) = \mathrm{Tr}_{q'\bar{q}}\ \mathbf{T}^{a}_{q',h,q}\ \rho(q,\bar{q})\ \mathbf{T}^{a'\dagger}_{q',h,q}$$

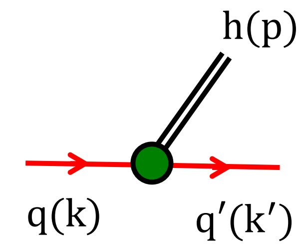
iii) Decay the meson $p \rightarrow p_1\, p_2..$

$$dN(p_1, p_2..)/d\Omega \propto M^{a}_{dec.}(p \rightarrow p_1 p_2..)\ \rho_{aa'}(h) M^{\dagger a'}_{dec.}(p \rightarrow p_1 p_2..)$$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, ..) = M^{\dagger a'}_{dec.}(p \rightarrow p_1 p_2..)\ M^{a}_{dec.}(p \rightarrow p_1 p_2..)$

The hadronization model: **string+$^3P_0$**





quark splitting $q \to h + q'$

Relevant variables:
$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$
$Z_+ = p^+/k^+$
$\varepsilon_h^2 = M^2 + p_T^2$

Transverse vectors defined w.r.t. string axis

Quark splitting amplitude in the string+$^3P_0$ model

$$T_{q',h,q} \propto C_{q',h,q}\ D_h(M^2) \left(\frac{1-Z_+}{\varepsilon_h^2}\right)^{\frac{a}{2}} \exp\left[-\frac{b_L \varepsilon_h^2}{2Z_+}\right] N_a^{-\frac{1}{2}}(\varepsilon_h^2)\ e^{-\frac{b_T k'^2_T}{2}} \quad [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \quad \Gamma_{h,s_h}$$

flavor — mass — longitudinal momentum — transverse momentum (w.r.t string axis) — $^3P_0$ mechanism [μ complex mass paramter] — Coupling e.g. $\Gamma_{h=PS} = \sigma_z$

Free param. Lund

Free param. string+$^3P_0$

AK, Artru, Martin, PRD 104, 114038 (2021)

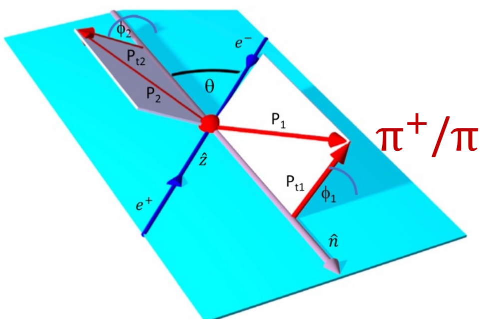
Steps for the extraction of Collins asymmetries

Example of $e^+e^- \to \pi^+\pi^- X$

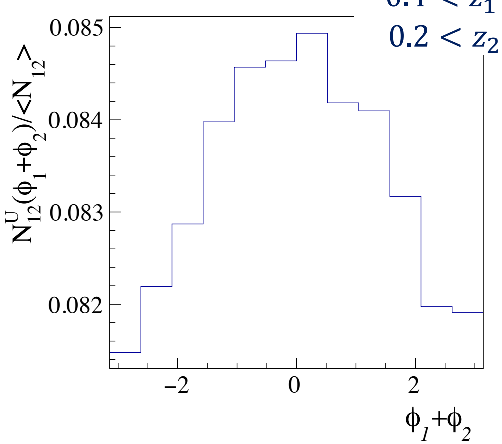
i) Evaluate normalized yields for $\pi^\pm - \pi^\mp$ "Unlike pairs"

$$R^U_{12} = \frac{N^U_{12}(\phi_1 + \phi_2)}{\langle N_{12}\rangle}$$

π^-/π^+



π^+/π^-

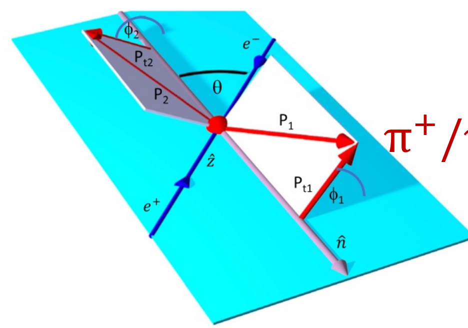


$0.4 < z_1 < 0.5$
$0.2 < z_2 < 1.0$

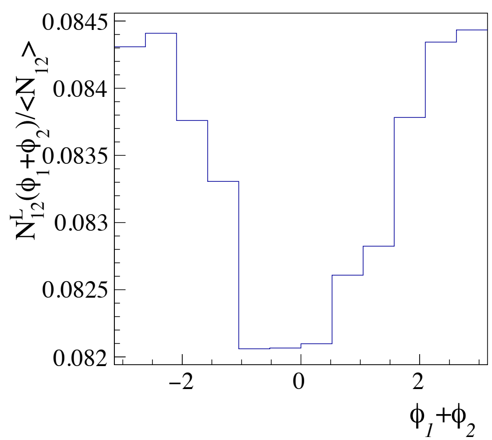
ii) Evaluate normalized yields for $\pi^\pm - \pi^\pm$ "Like pairs"

$$R^L_{12} = \frac{N^L_{12}(\phi_1 + \phi_2)}{\langle N_{12}\rangle}$$

π^+/π^-



π^+/π^-

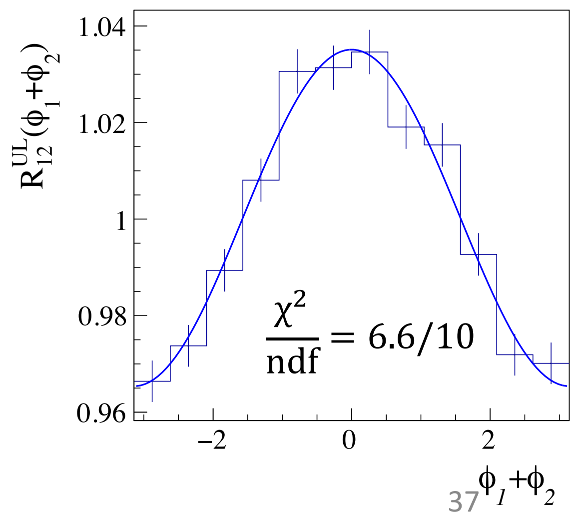


ii) Evaluate the ratio $\frac{R^U_{12}}{R^L_{12}}$ and fit the asymmetry

$$R^{UL}_{12} = \frac{R^U_{12}}{R^L_{12}} \approx 1 + A^{UL}_{12}\cos(\phi_1 + \phi_2)$$

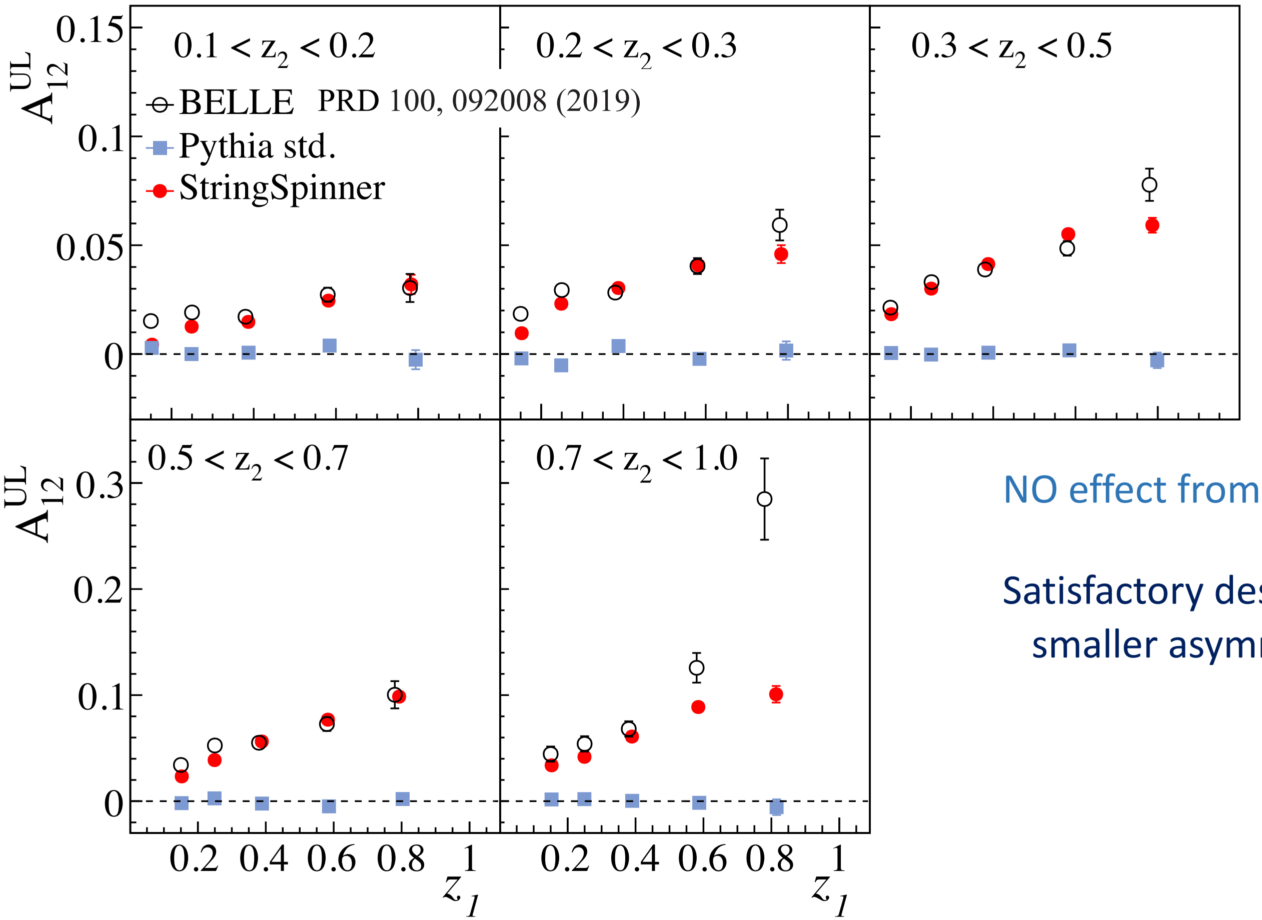
Fit function

$$f(\phi_1 + \phi_2) = p_0 + p_1\cos(\phi_1 + \phi_2)$$



A_{12}^{UL} asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

$z_1 \times z_2$ - dependence



Asymmetries w.r.t thrust axis (not $q\bar{q}$ axis)

$T > 0.8$

$z > 0.1, P_T < 3.0$ GeV/c

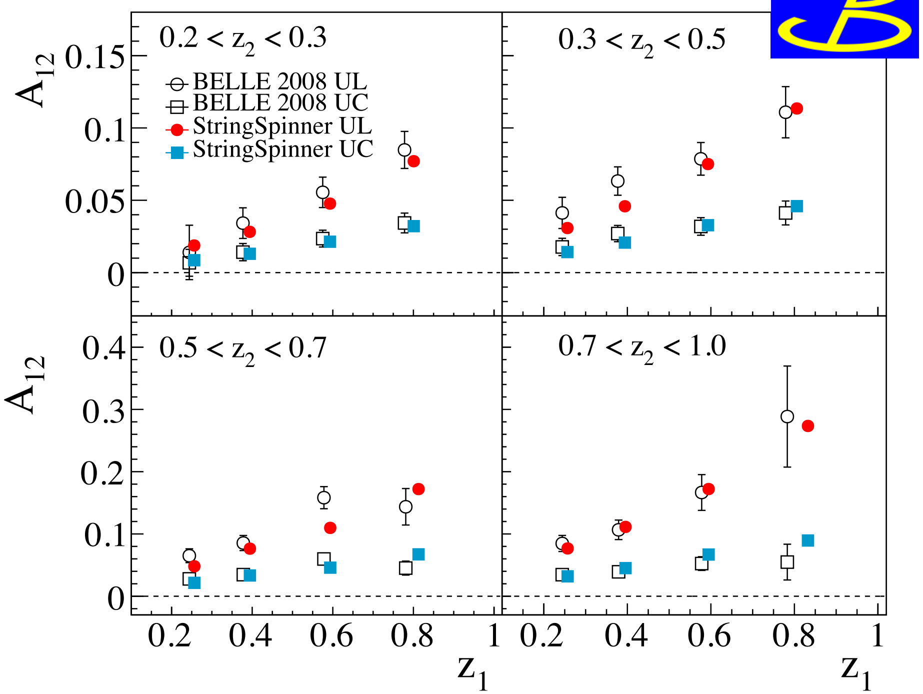
$\alpha_O < 0.3$

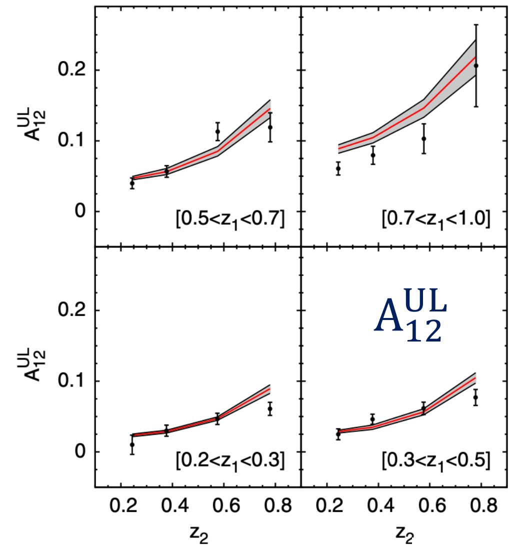
NO effect from Pythia

Satisfactory description from StringSpinner
smaller asymmetries at large z

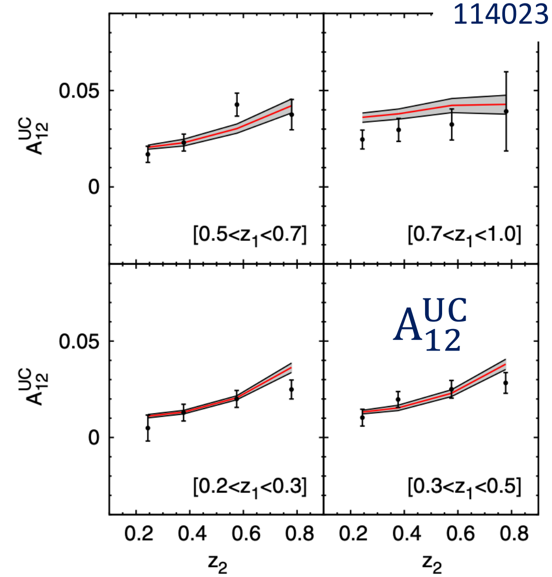
A_{12} asymmetry for charged π π pairs

PRD 78, 032011 (2008)





Anselmino et al., PRD 92, 114023 (2015)



Belle asymmetries corrected for thrust smearing

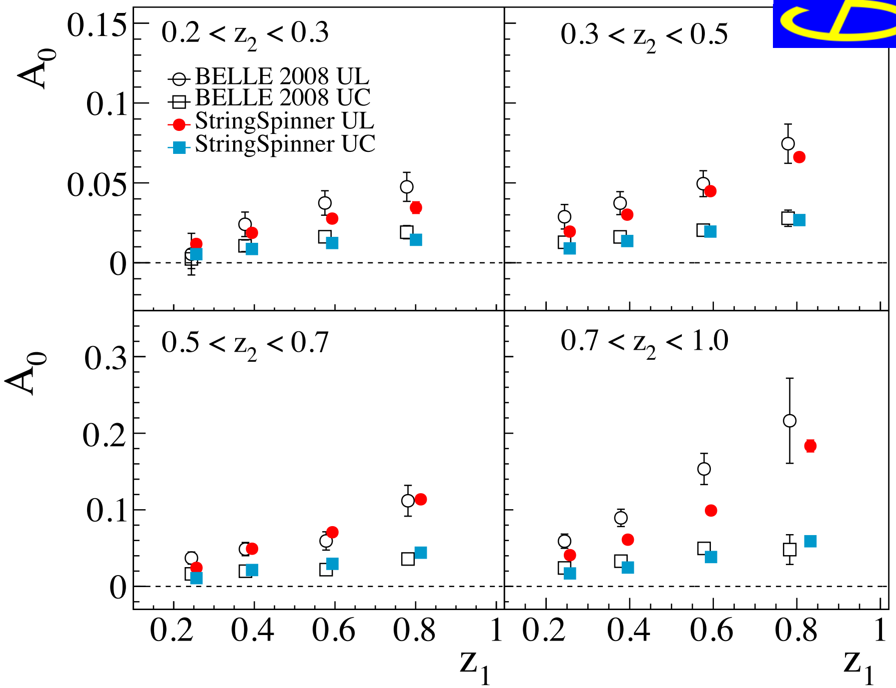
Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5 \text{ GeV}$$

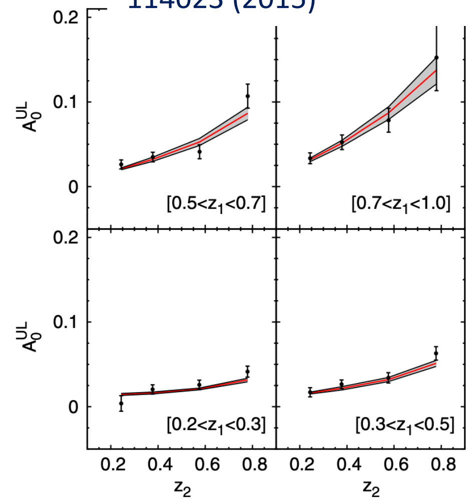
StringSpinner reproduces trend and size

A_0 asymmetry for charged pions

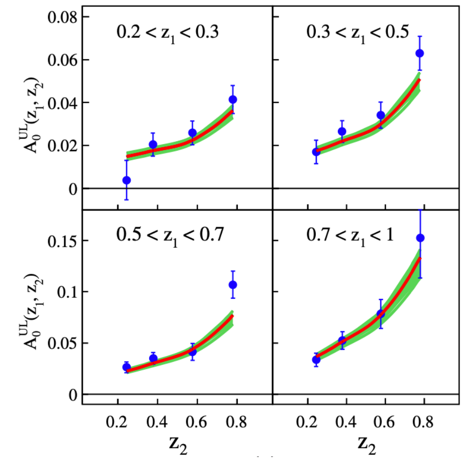
PRD 78, 032011 (2008)

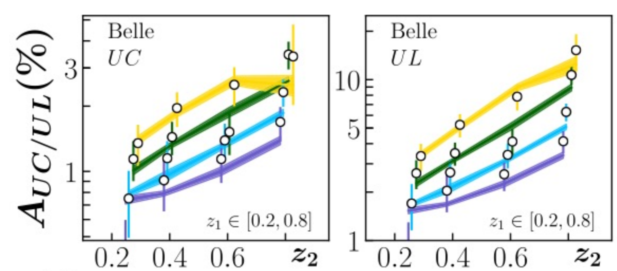


Anselmino et al., PRD 92, 114023 (2015)



Kang et al., PRD 93, 014009 (2016)





JAM, PRD 102, 054002 (2020)

Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

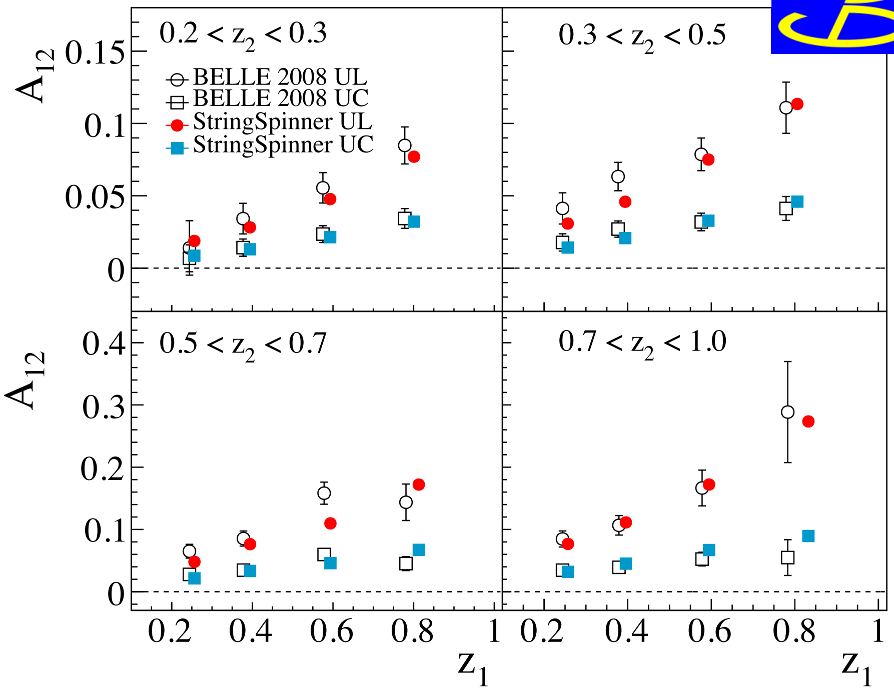
Trend reproduced by string+$^3P_0$

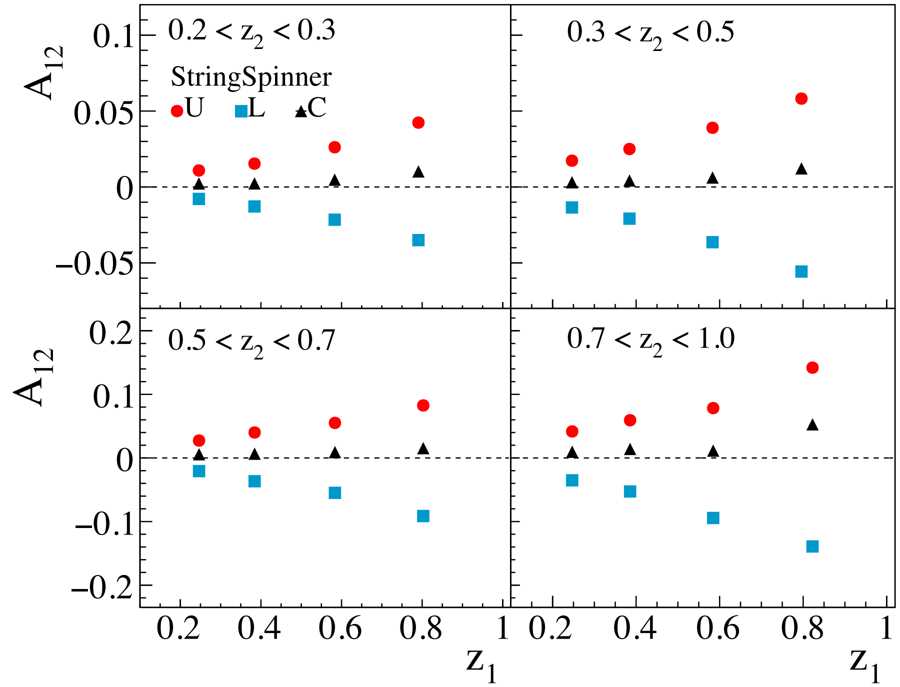
somewhat lower values in the last z_2 bin

A_0 asymmetry essential observable included in phenomenological fits

A_{12} asymmetry for charged π π pairs

PRD 78, 032011 (2008)





Belle asymmetries corrected for thrust smearing

Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5 \text{ GeV}$$

StringSpinner reproduces trend and size

Opposite sign for A_{12}^{U} and A_{12}^{L}, as expected

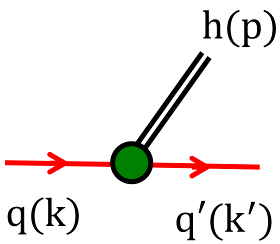
$A_{12}^{C} \simeq 0 \rightarrow$ explains $A_{12}^{UC} < A_{12}^{UL}$

$$A_{12}^{UL(UC)} \simeq A_{12}^{U} - A_{12}^{L(C)}$$

Modeling spin-dependent hadronization: the string+$^3P_0$ model

- ❑ Extension of the Lund string fragmentation model to include the quark spin
- ❑ Basic quantity – quark (and antiquark) splitting amplitude

AK, Artru, Martin, PRD 104, 114038 (2021)



$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$

$Z_+ = p^+/k^+$

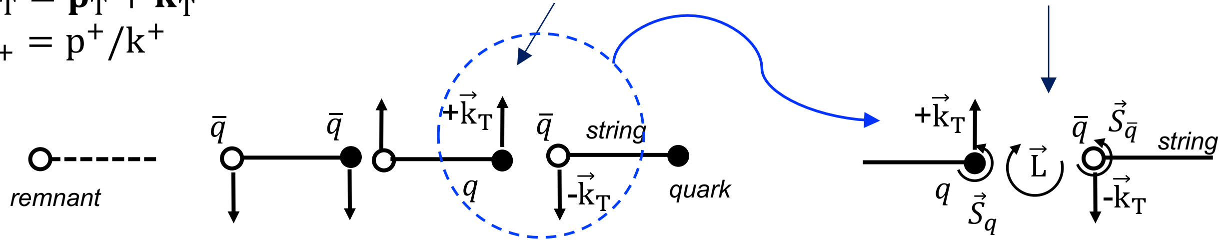
PS meson→ σ_z

$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \; [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \;\; \Gamma_{h,s_h}$$

Lund splitting fcnt.

$^3P_0$ wave function

coupling



VM with pol. $\vec{V}$

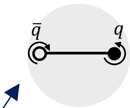
$G_L V_L^* \, 1 + G_T \, \vec{\sigma}_T \cdot \vec{V}_T^* \sigma_z$

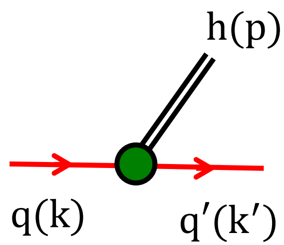
Modeling spin-dependent hadronization: the string+$^3P_0$ model

- Extension of the Lund string fragmentation model to include the quark spin
- Basic quantity – quark (and antiquark) splitting amplitude

AK, Artru, Martin, PRD 104, 114038 (2021)

PS meson→ σ_z





$$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$$
$$Z_+ = p^+/k^+$$

$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \; \Gamma_{h,s_h}$$

Lund splitting fcnt.

$^3P_0$ wave function

coupling

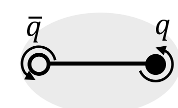
Free parameters:

as in Lund Model

complex «mass»

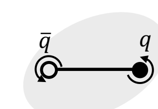
$\mathrm{Im}(\mu)$ responsible for transverse spin effects, e.g. Collins effect

$$f_L = \frac{|G_L/G_T|^2}{2 + |G_L/G_T|^2}$$



fraction of long. pol. VMs

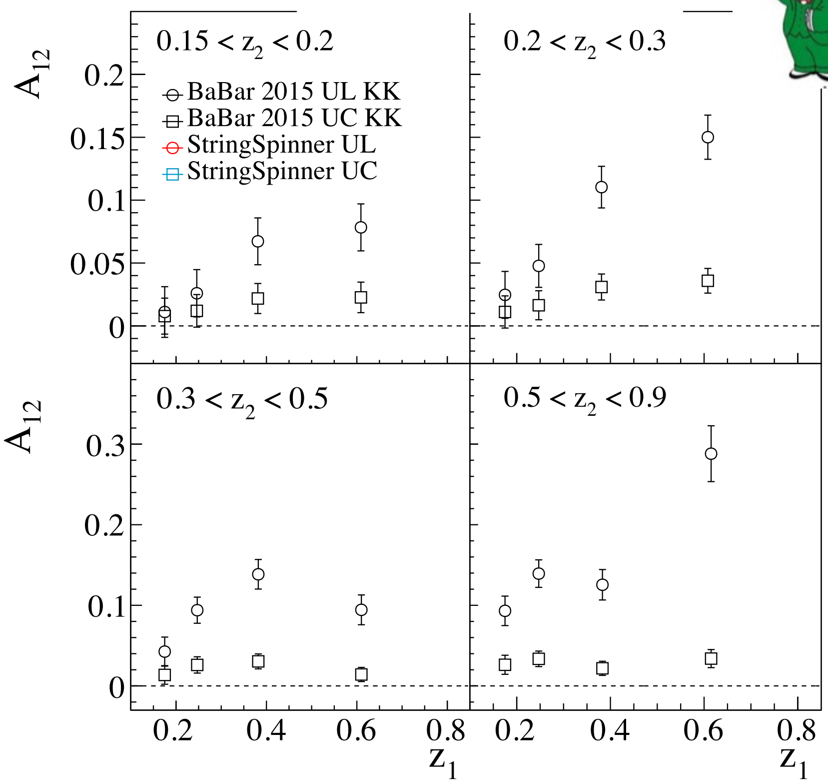
$$\theta_{LT} = \arg\left(\frac{G_L}{G_T}\right)$$



oblique pol. (LT interference)

A_{12} asymmetry for charged KK pairs

PRD 92, 111101(R) (2015)



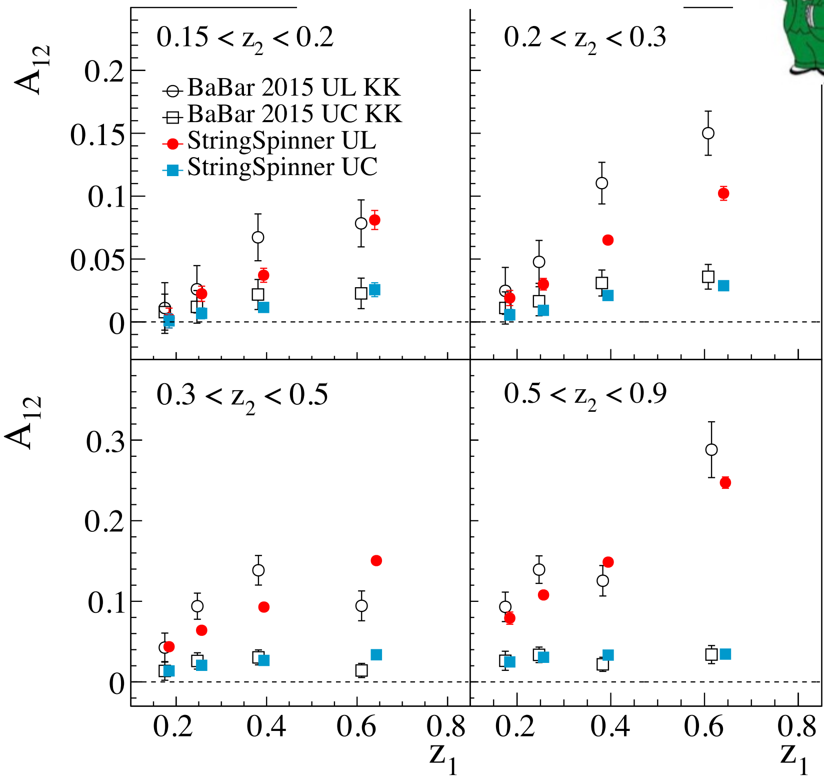
Corrected for thrust smearing

Cuts

$T > 0.8, z > 0.15, Q_T < 3.5$ GeV$, \alpha_O < \pi/4$

A_{12} asymmetry for charged KK pairs

PRD 92, 111101(R) (2015)



Corrected for thrust smearing

Cuts

$T > 0.8, z > 0.15, Q_T < 3.5\ \mathrm{GeV}, \alpha_O < \pi/4$

A_{12}^{UC} much smaller than A_{12}^{UL} at large z reproduced by string+$^3P_0$