Nucleon Transversity from lattice QCD

University of Cyprus

Constantia Alexandrou

The Cyprus Institute

Extended Twisted Mass Collaboration

AQTIVATE

European Joint Doctorate, grant agreement No. 101072344

RESEARCH & INNOVATION FOUNDATION

EXCELLENCE/0421/0043

Outline

- Introduction
  - State-of-the-art lattice QCD simulations
- 3D structure of the nucleon
  - First and second Mellin moments
    - ➡ Charges & from factors
    - ➡ Transverse densities, Phys.Rev.D 107 (2023) 5, 054504
  - Direct computation of parton distributions (PDFs and GPDs)
- Conclusions

Quantum ChromoDynamics (QCD)

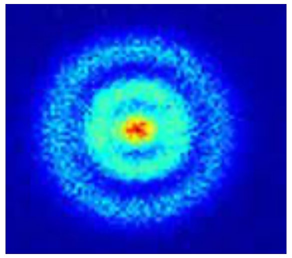
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

Fritzsch, Gell-Mann and Leutwyler, Phys. Lett. 47B (1973) 365

- Unique properties:
  - ★Confinement
  - ★Asymptotic freedom
  - ★Mass generation via interaction

QED

Quantum theory of the electromagnetic force mediated by exchange of photons



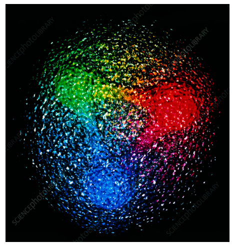
Hydrogen atom

$$m_{\text{Hydrogen}} = \underbrace{0.51\ \text{MeV}}_{m_{e^-}} + \underbrace{938.29\ \text{MeV}}_{m_{p^+}} - \underbrace{13.6\ \text{eV}}_{\text{E}_{\text{binding}}}$$

A. Stodolna et al., PRL 110 (2013) 213001

QCD

Quantum theory of the strong force mediated by exchange of gluons



Artist impression

Proton

$$m_p = \underbrace{2.3\ \text{MeV}}_{2\times m_u} + \underbrace{4.7\ \text{MeV}}_{m_d} + \underbrace{929\ \text{MeV}}_{\text{E}_{\text{binding}}}$$

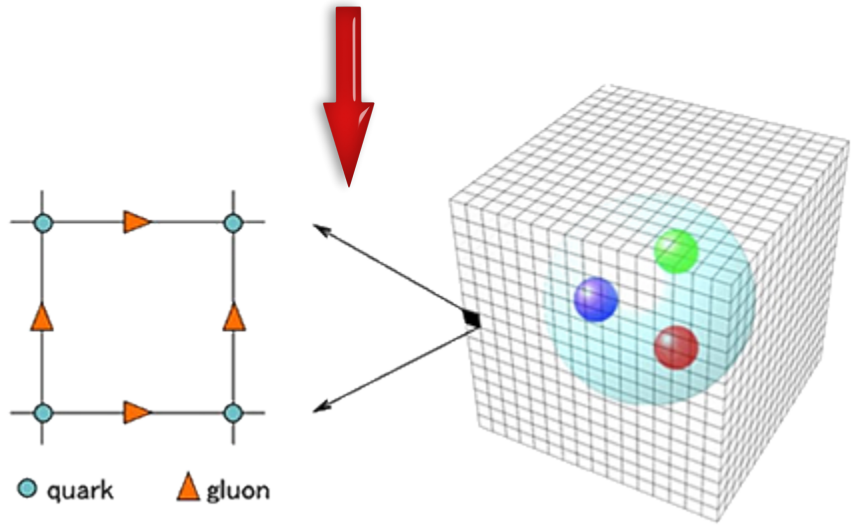
99% of proton mass from interaction!

Lattice QCD provides an *ab initio* method to study a wide class of strong interaction phenomena

- Lattice QCD uses directly $\mathcal{L}_{QCD}$ or the action $S_{QCD} = \int d^4x\, \mathcal{L}_{QCD}$

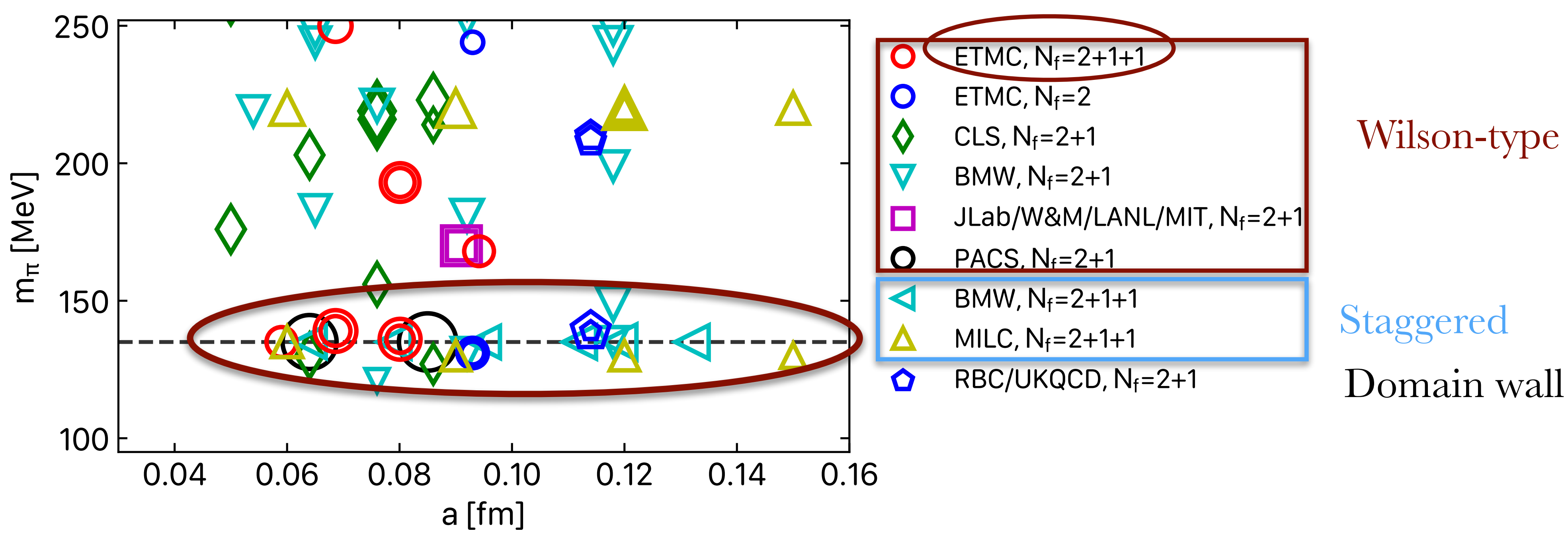
Simulations of lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



1. Simulation of gauge ensembles {U}:

$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



ETMC: S. Bacchio, J. Finkenrath, R. Frezzotti, B. Kostrzewa, C. Urbach

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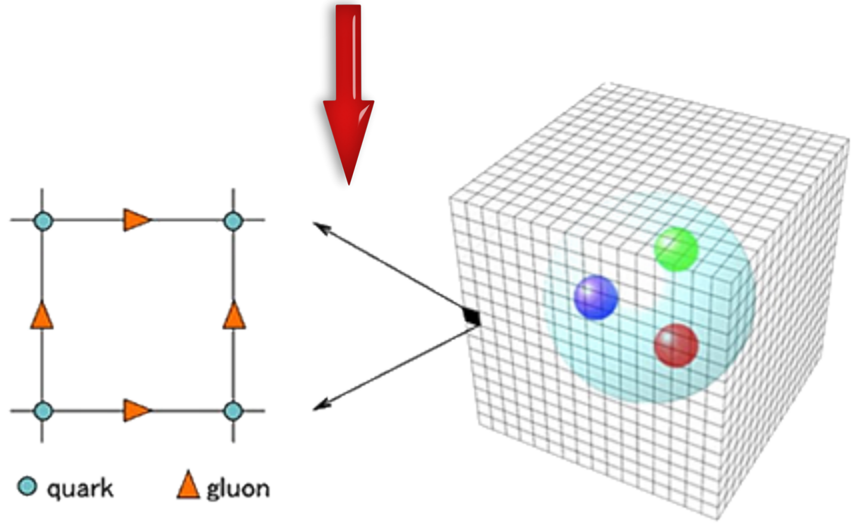

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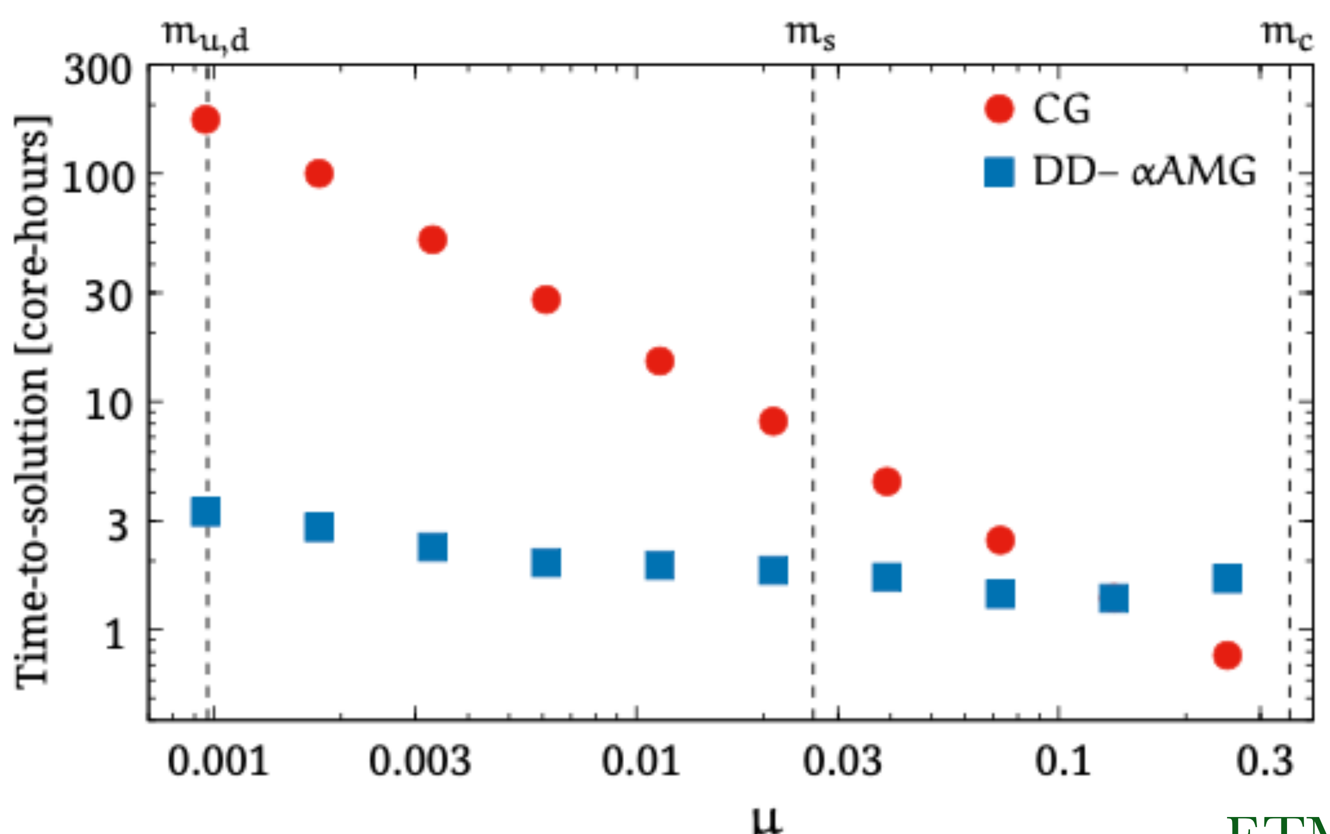
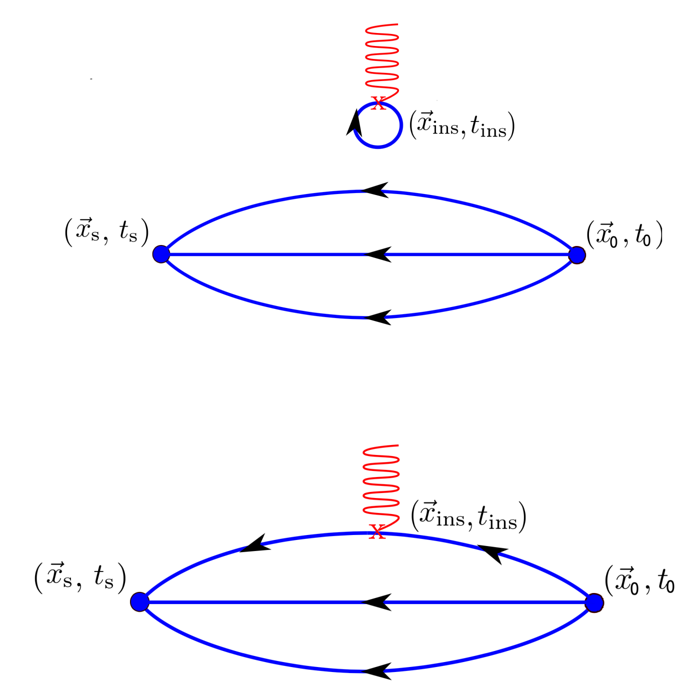
2. Quark propagators: inverse of Dirac matrix $D_f[U]$: Multi-grid solvers



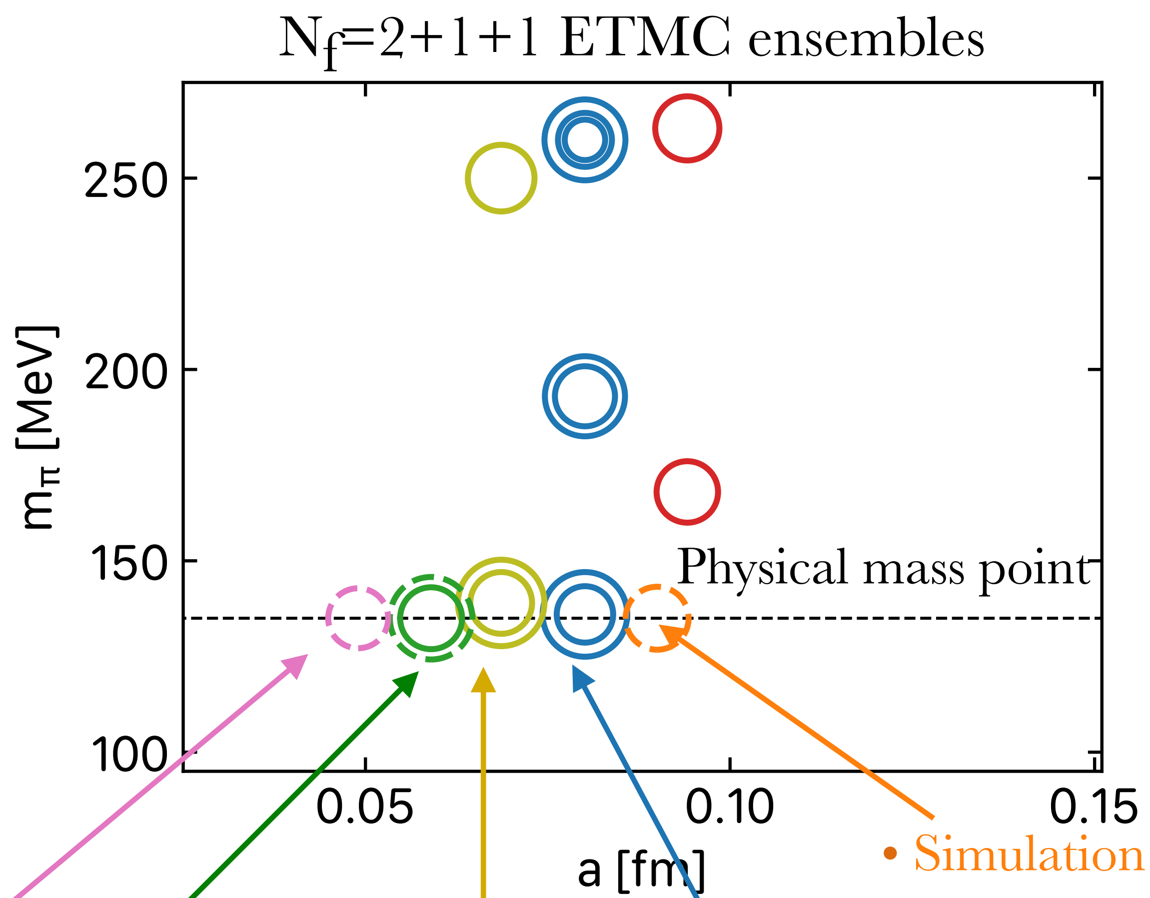


ETMC: S. Bacchio, J. Finkenrath

Gauge ensembles generated by ETMC

5 ensembles completed and 3 under production at physical pion mass

- 5 lattice spacings 0.05<a<0.1 fm —> take continuum limit **directly at the physical point** avoiding chiral extrapolation removing a major systematic error in the baryon sector
- 2 volumes at a=0.08 fm, 0.07 fm and 0.06 fm of Lm_π~3.6 (5.1 fm) and Lm_π ~5.4 (7.7 fm) completed



- Simulation ongoing 48^3 x 96 a~0.09 fm

- Analysis completed for 64^3x128 a~0.08 fm
- Analysis ongoing for 96^3x192 a~0.08 fm

- Simulation ongoing for 112^3x224, a~0.05 fm

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C. A. *et al.* (ETMC) Phys. Rev. D98 (2018) 054518

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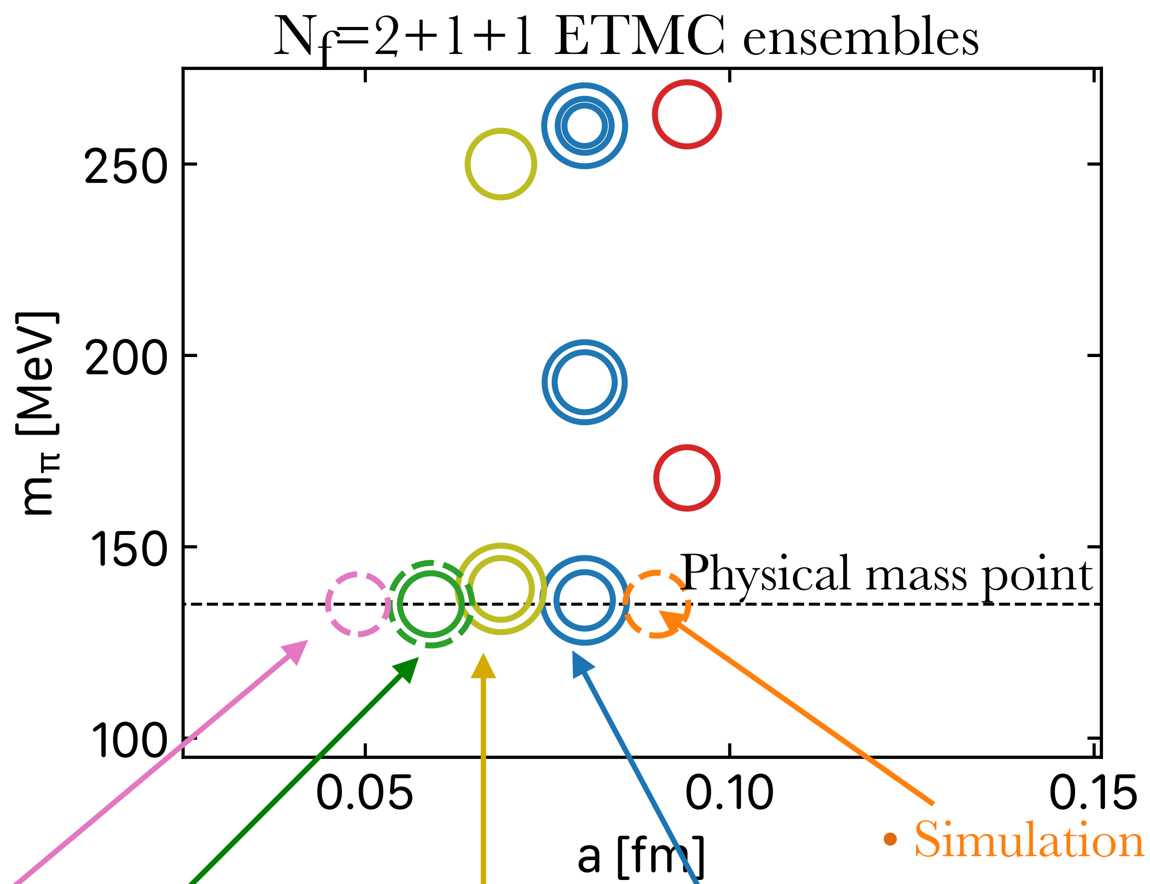

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C. A. *et al.* (ETMC) Phys. Rev. D98 (2018) 054518

Results in this talk from the analysis of 3 physical mass point ensembles

- B-ensemble: 64³ x 128, a~0.08 fm
- C-ensemble: 80³x160, a~0.07 fm
- D-ensemble:96³x192, a~0.06 fm

EuroHPC Joint Undertaking

Computational resources

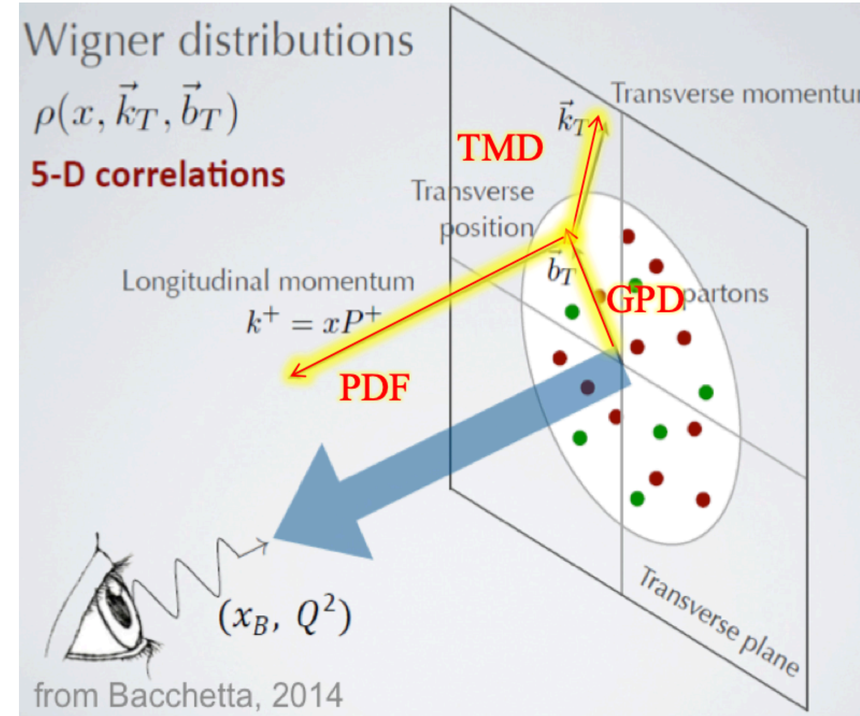
PRACE

Extended Twisted Mass Collaboration

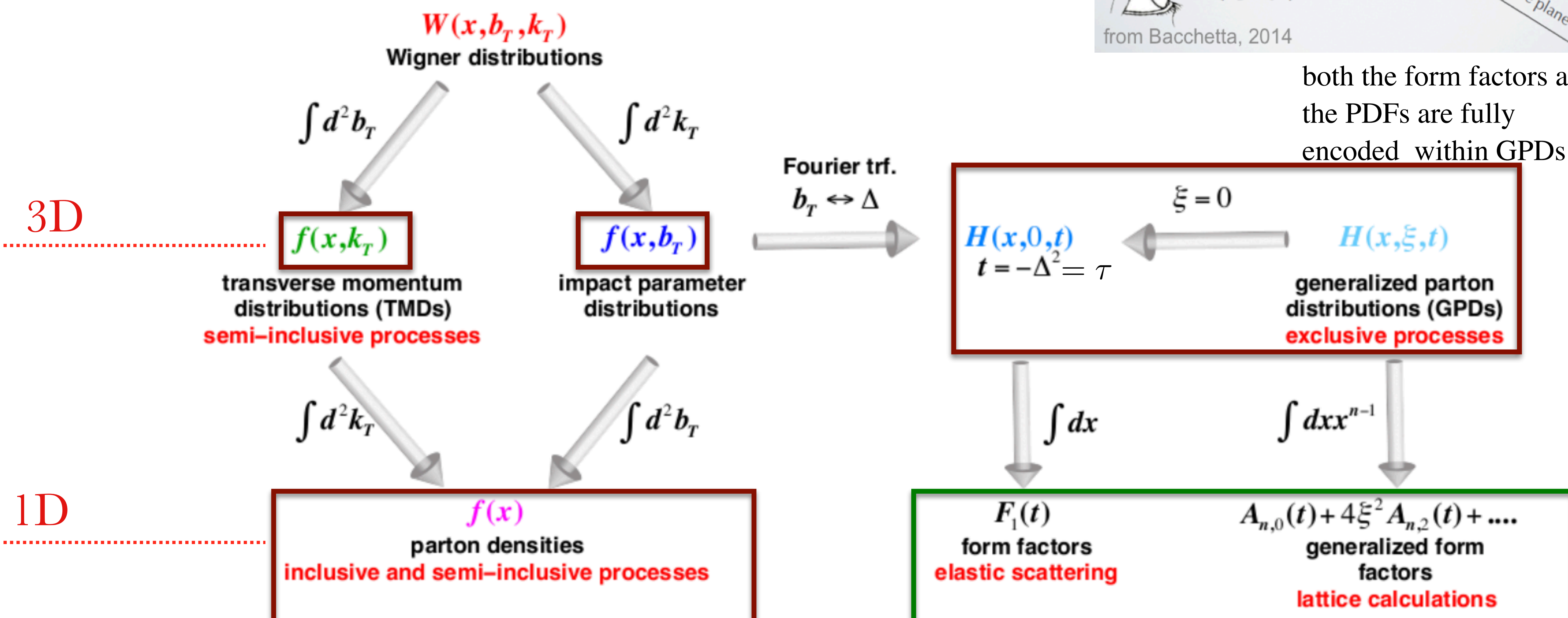
3D structure of hadrons

- The 3D-structure of the nucleon is a major part of on-going experiments and of the future EIC

- Lattice QCD can contribute towards this goal - many recent developments to compute Mellin moments but also directly parton distributions



both the form factors and the PDFs are fully encoded within GPDs



Studies in lattice QCD since the 1980s

EIC white paper, arXiv:1212.1701

Computation of Mellin moments of GPDs

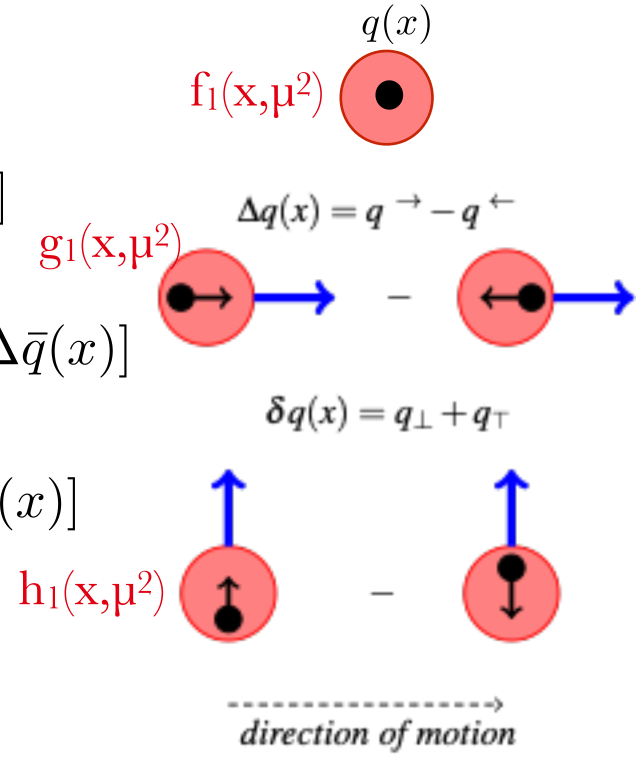
- Light-cone matrix elements cannot be computed using a Euclidean lattice formulation of QCD
- Expansion of light-cone operator leads to a tower of local twist-2 operators —> connected to moments that can be computed in lattice QCD

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar\psi\gamma^{\{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi \quad \overset{unpolarized}{\rightarrow} \quad \langle x^n\rangle_q = \int_0^1 dx\, x^n \left[q(x) - (-1)^n \bar q(x)\right]$$

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar\psi\gamma_5\gamma^{\{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi \quad \overset{helicity}{\rightarrow} \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx\, x^n \left[\Delta q(x) + (-1)^n \Delta\bar q(x)\right]$$

$$\mathcal{O}_T^{\rho\mu_1\cdots\mu_n} = \bar\psi\sigma^{\rho\{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi \quad \overset{transversity}{\rightarrow} \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx\, x^n \left[\delta q(x) - (-1)^n \delta\bar q(x)\right]$$

$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_\top + q_\perp$$



Twist-2 PDFs

Ph. Hagler, Phys. Rept. 490 (2010) 49

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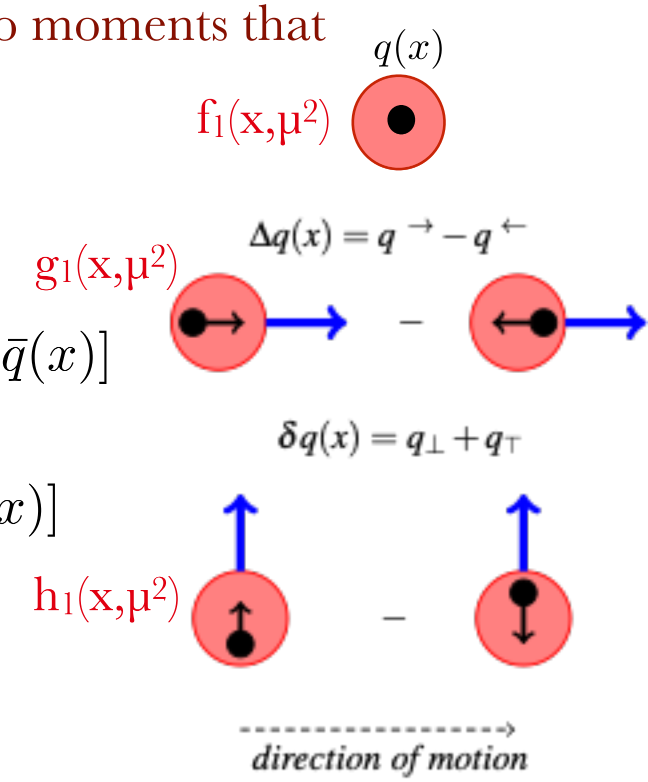

Twist-2 PDFs

- For off-diagonal matrix elements we obtain moments of GPDs or the generalised form factors (GFFs) e.g unpolarized

Ph. Hagler, Phys. Rept. 490 (2010) 49

$$\int_{-1}^1 dx\, x^{n-1}H(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1}\left[(2\xi)^i A_{ni}(\tau) + \mathrm{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

$$\int_{-1}^1 dx\, x^{n-1}E(x,\xi,\tau) = \sum_{i=0,2,\cdots}^{n-1}\left[(2\xi)^i B_{ni}(\tau) - \mathrm{mod}(n,2)(2\xi)^n C_{n0}(\tau)\right]$$

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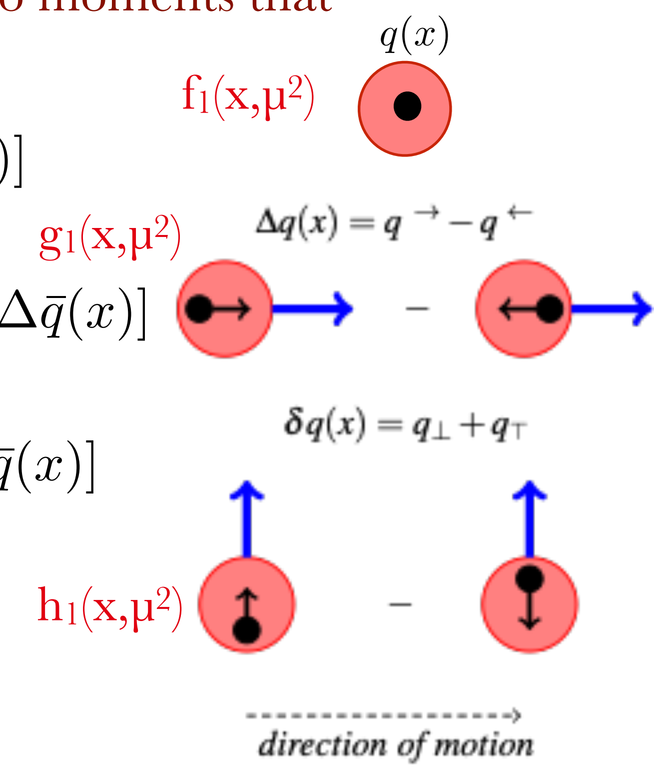

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Ph. Hagler, Phys. Rept. 490 (2010) 49

Special cases: n=1,2 for the nucleon

- n=1: $\tau=0$ —> charges g_V, g_A, g_T

  $\tau \neq 0$ —> form factors: $A_{10}(\tau) = F_1(\tau)$, $B_{10}(\tau) = F_2(\tau)$, $\tilde A_{10}(\tau) = G_A(\tau)$, $\tilde B_{10}(\tau) = G_p(\tau)$
- n=2: generalised form factors: $A_{20}(\tau)$, $B_{20}(\tau)$, $C_{20}(\tau)$, $\tilde A_{20}(\tau)$, $\tilde B_{20}(\tau)$

$$\langle x\rangle_q = A_{20}(0), \quad \langle x\rangle_{\Delta q} = \tilde A_{20}(0), \quad \langle x\rangle_{\delta q} = A_{20}^T(0) \quad \text{and } J_q = \tfrac{1}{2}[A_{20}(0) + B_{20}(0)] = \tfrac{1}{2}\Delta\Sigma_q + L_q$$

- Spin and momentum sums: $\sum_q\left[\frac{1}{2}\Delta\Sigma_q + L_q\right] + J_g = \frac{1}{2}$, $\quad \sum_q\langle x\rangle_q + \langle x\rangle_g = 1$

Mellin moments - precision era of lattice QCD

First Mellin moments

- Moments for small n are readily accessible on the lattice from matrix elements of local operators
- Computation of the low Mellin moments has a long history, G. Martinelli and Ch. Sachradja Phys. Lett. B217 (1989) 319
- Only recently we have results directly at the physical point (i.e. simulations with $m_\pi \sim 135$ +/-10 MeV)

Nucleon isovector charges

$$g_V = \langle 1 \rangle_{u-d}$$
$$g_A = \langle 1 \rangle_{\Delta u - \Delta d}$$
$$g_T = \langle 1 \rangle_{\delta u - \delta d}$$

- $g_V = 1$
- $g_A = 1.2764 \pm 0.0006$ reproduce
- $g_T = 0.53 \pm 0.25$ M. Radici and A. Bacchetta. PRL 120 (2018) 192001

Determine for each quark flavour

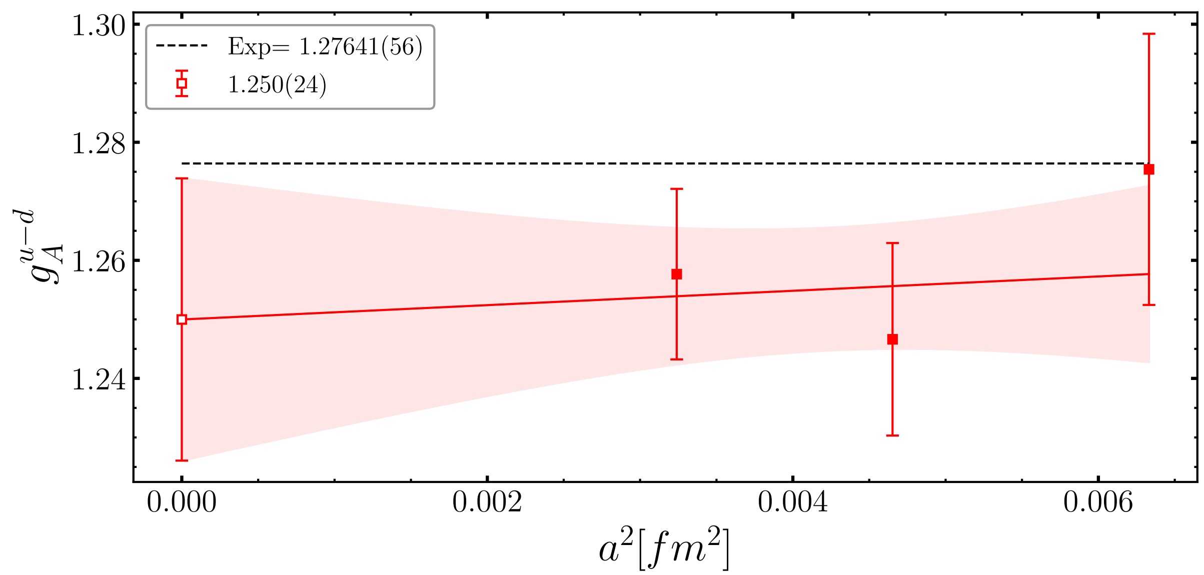
- e.g. $\Delta\Sigma_{q^+} = g_A^q$

$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx \left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$

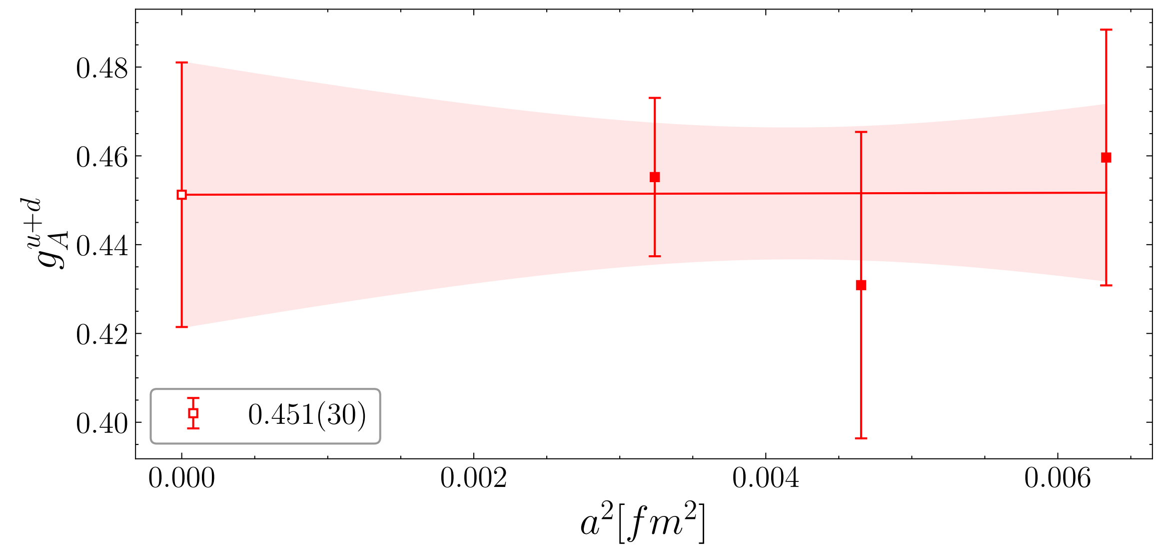
Axial charges

- Axial charges extracted directly from the forward matrix element

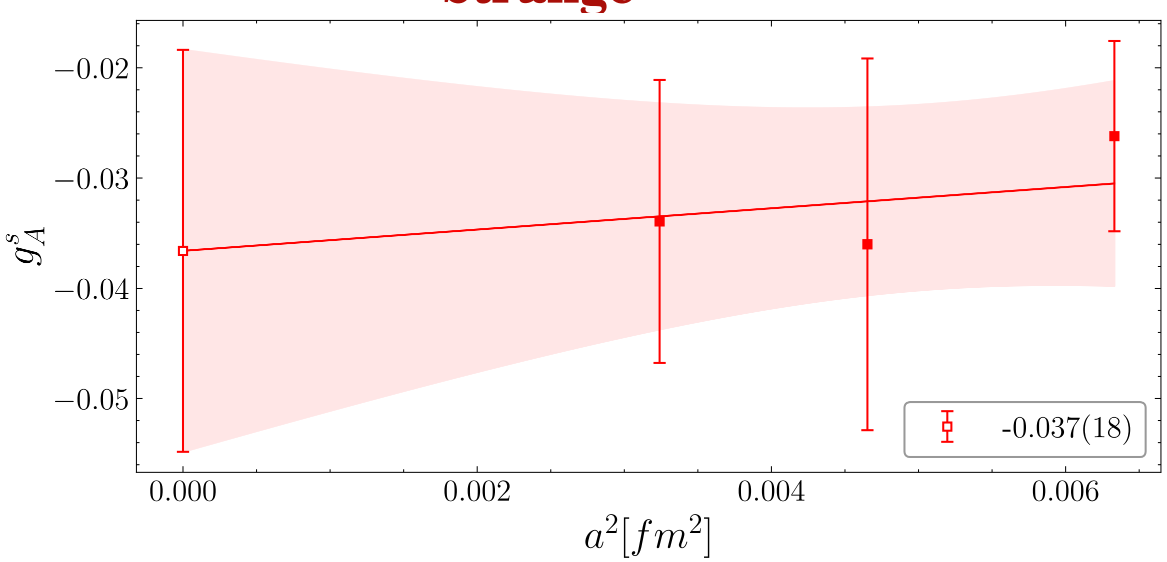
Isovector



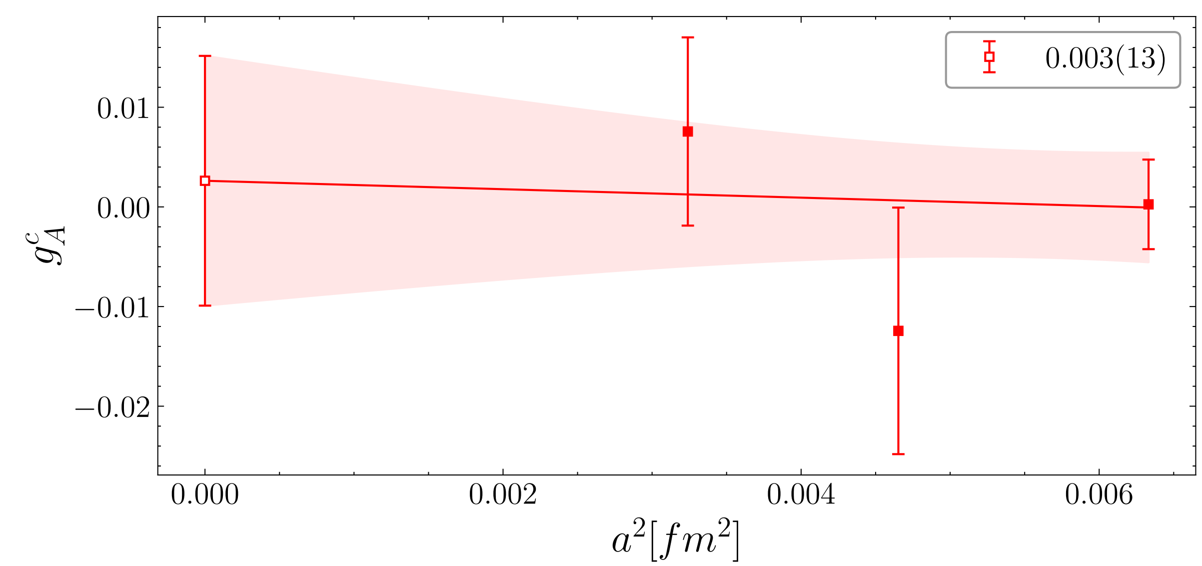
Isoscalar including disconnected



Strange

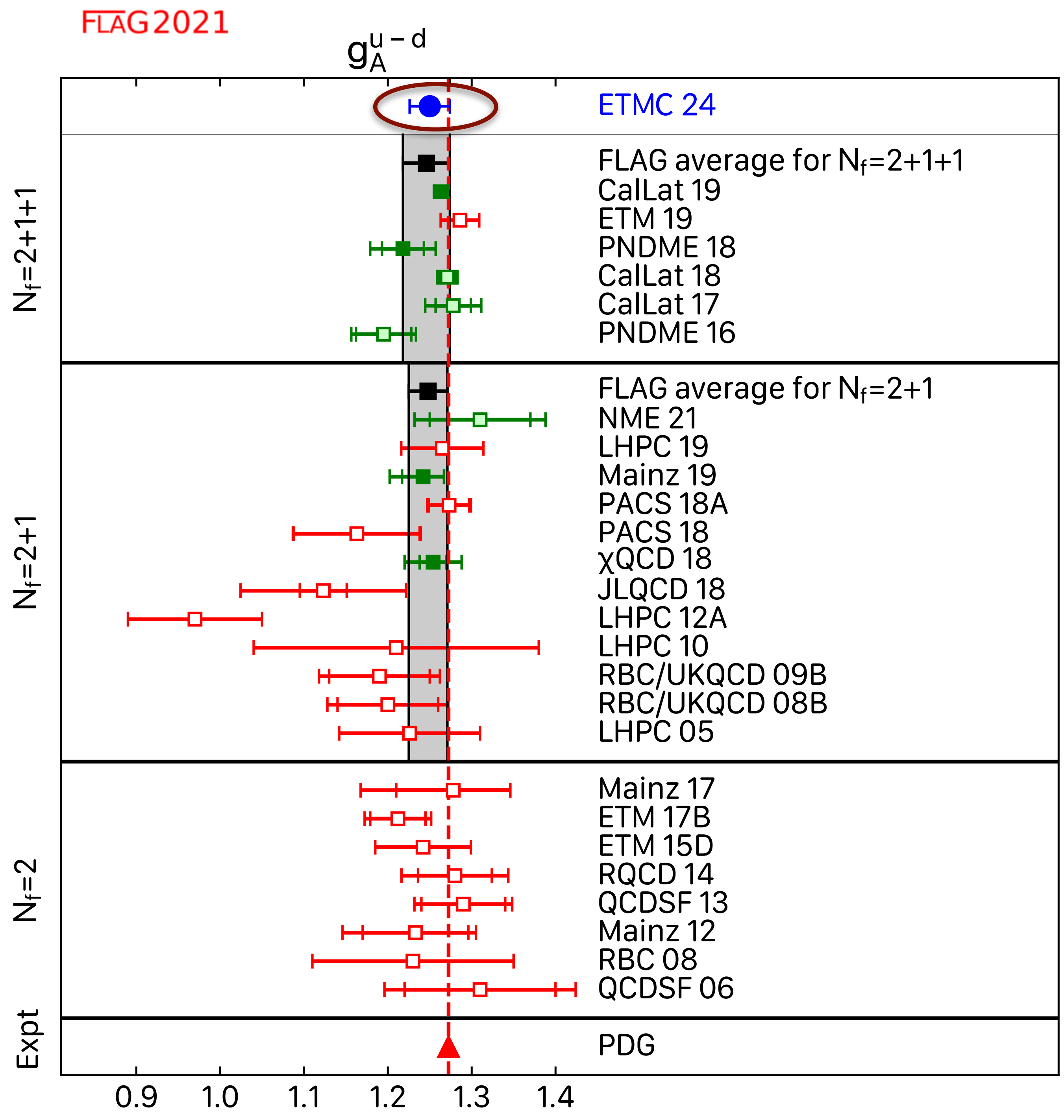


Charm



- Non-zero strangeness, upper limit on charmness of 0.013
- With our two additional lattice spacings we expect more stability in the results and reduced errors at the continuum limit

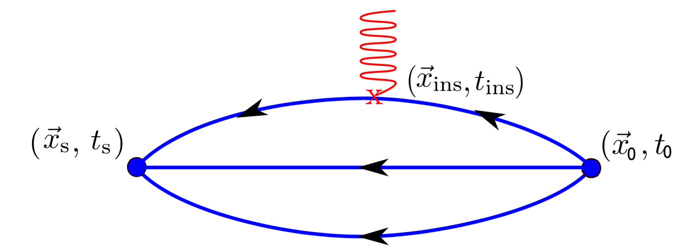
Nucleon isovector (u-d) axial charge

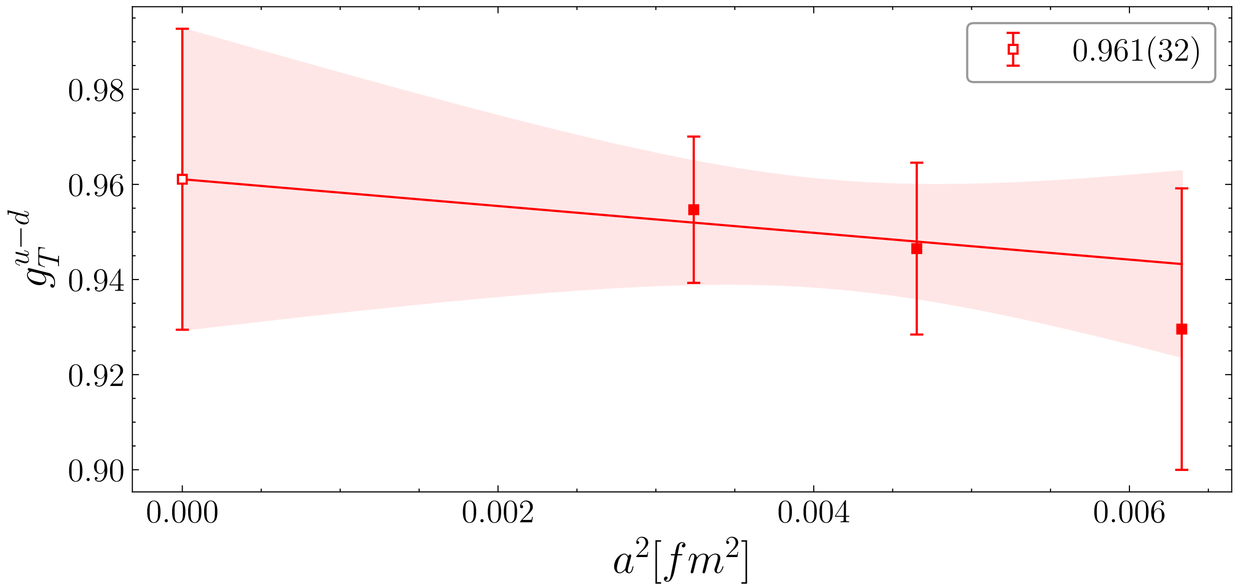


Lattice QCD results on g_A consistent with experimental value

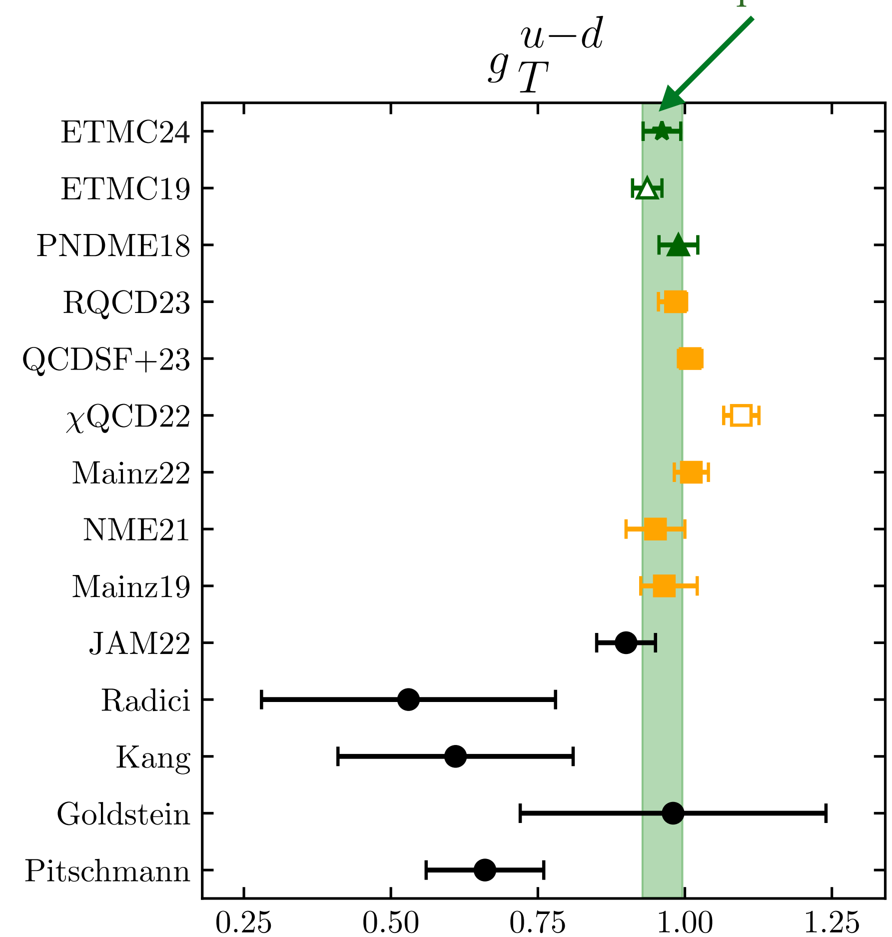
Nucleon isovector (u-d) tensor charge

- Only connected contributions

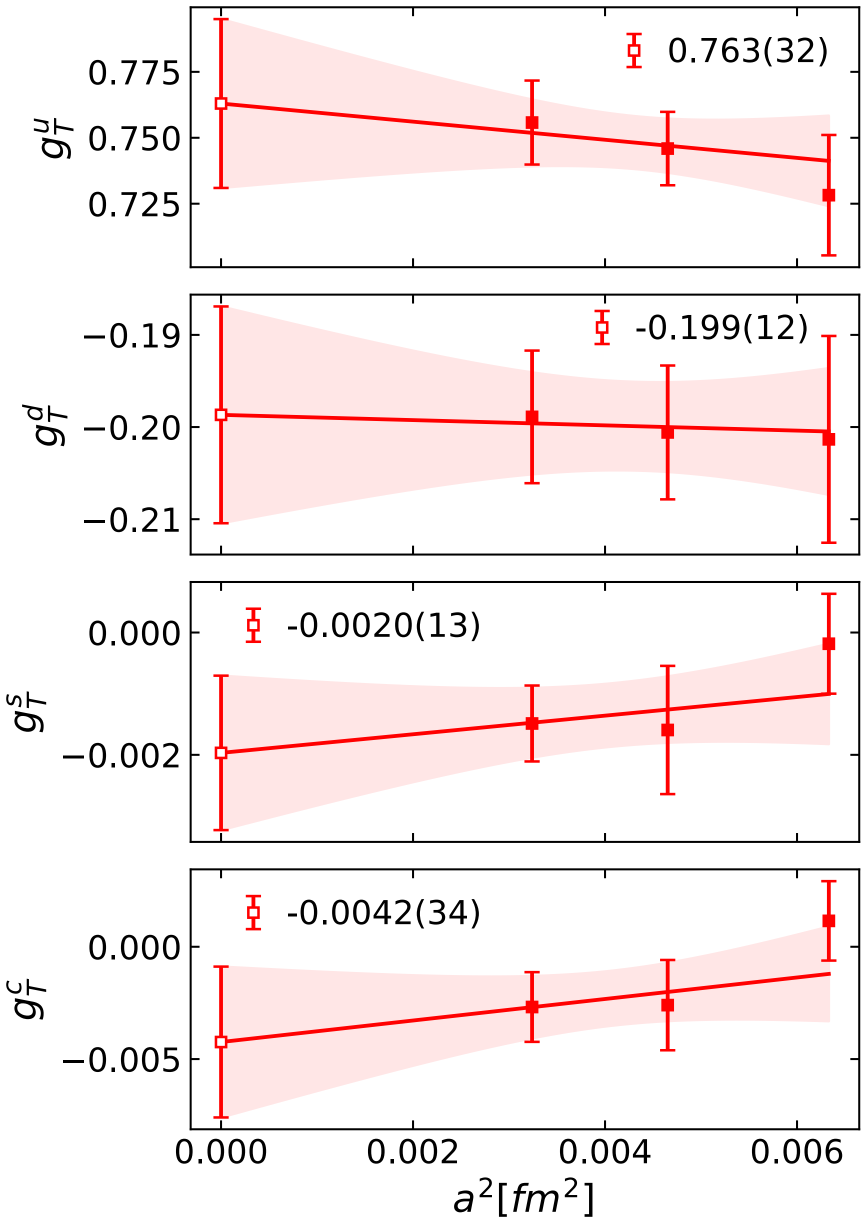


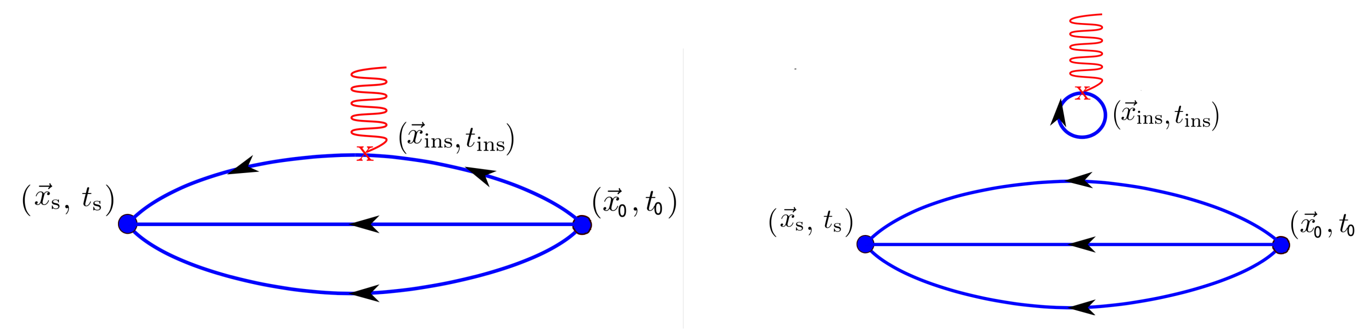


Continuum limit directly at physical pion mass



Flavor diagonal tensor charge

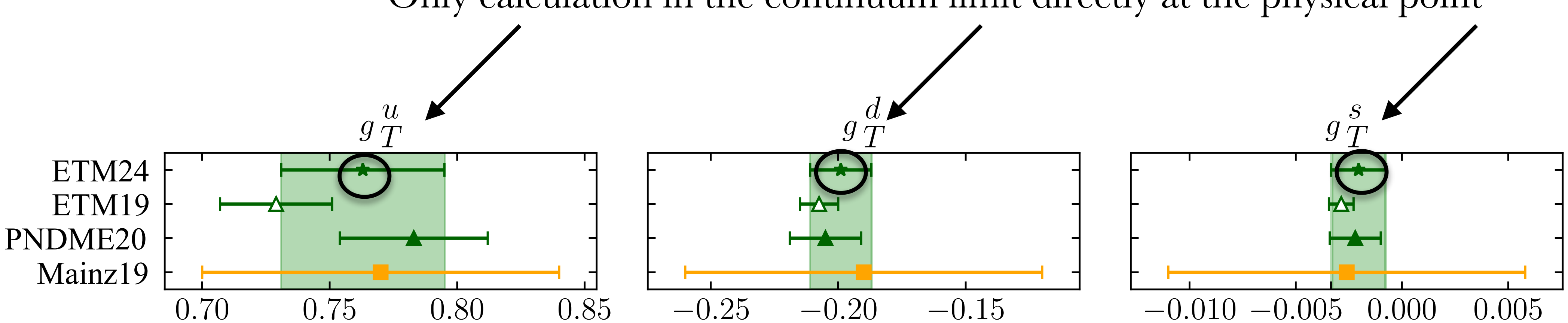




- Evaluate both connected and disconnected contributions
- Obtain flavor diagonal tensor charge for the first time in the continuum using only physical point ensembles - input for phenomenology
- JAM3D-22: g_T^u=0.78(11) and g_T^d=-0.12(11), arXiv:2306.12998

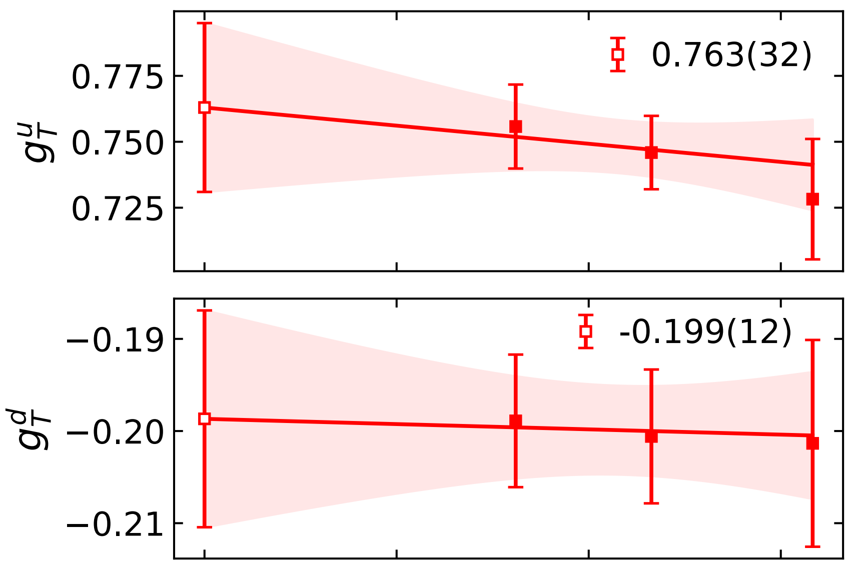
Thanks to Daniel Pitonyak

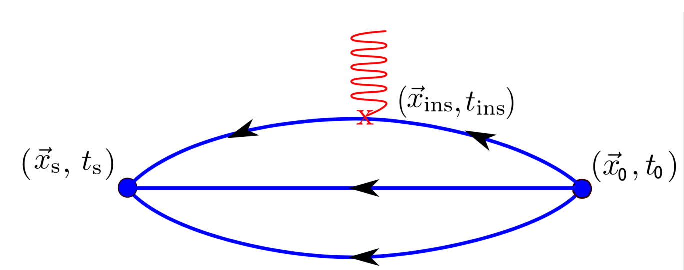
Only calculation in the continuum limit directly at the physical point

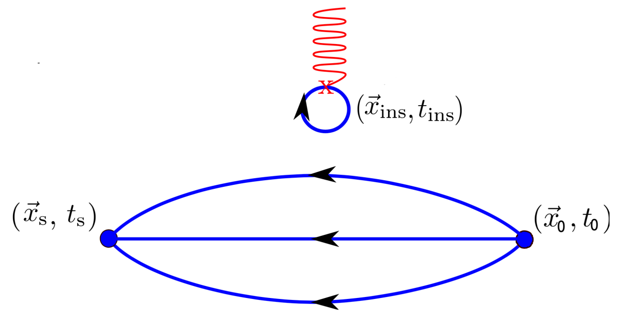


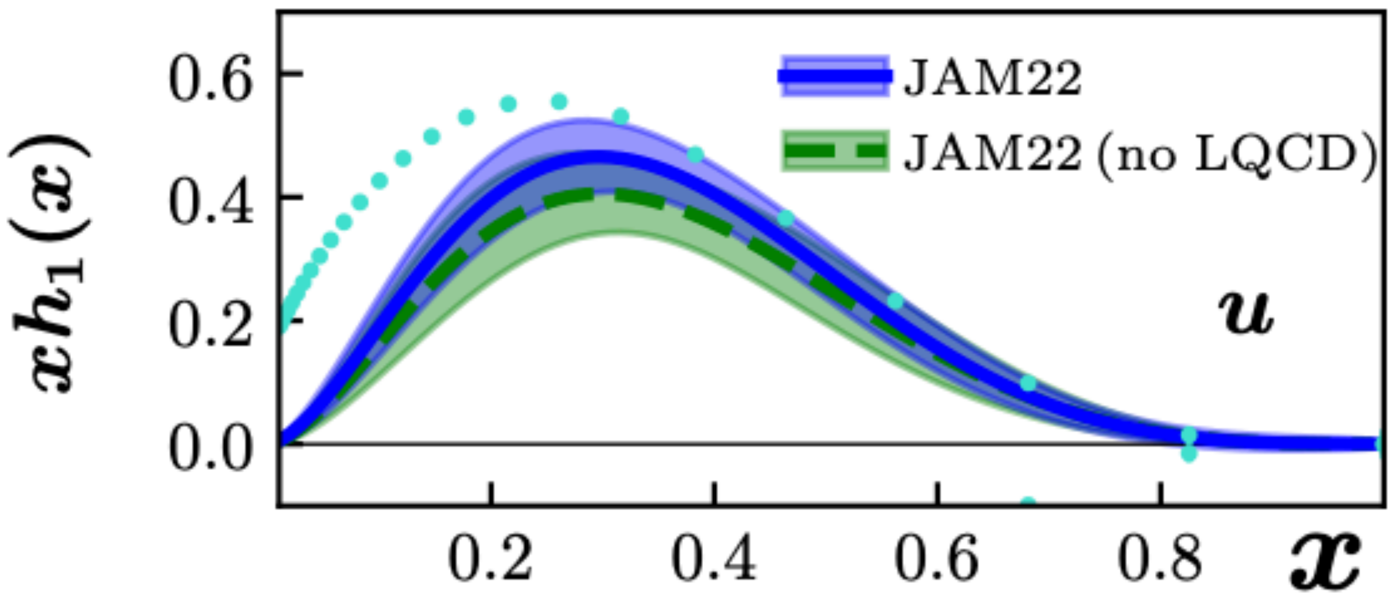
Precision era of lattice QCD for first Mellin moments including flavor diagonal

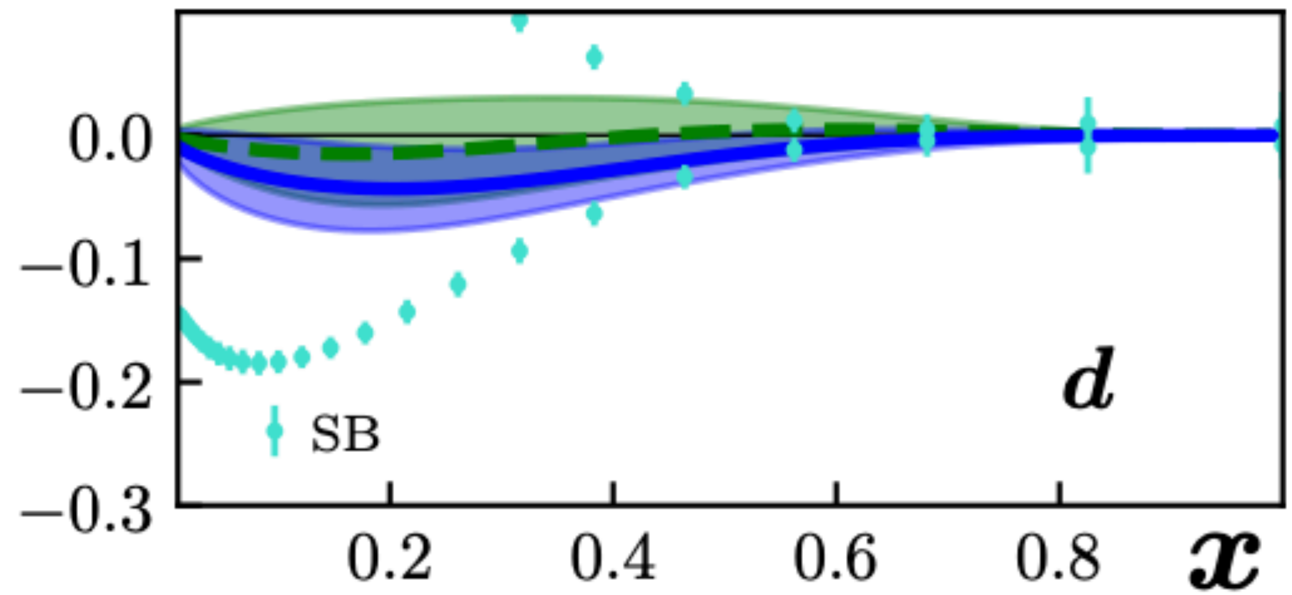
Flavor diagonal tensor charge







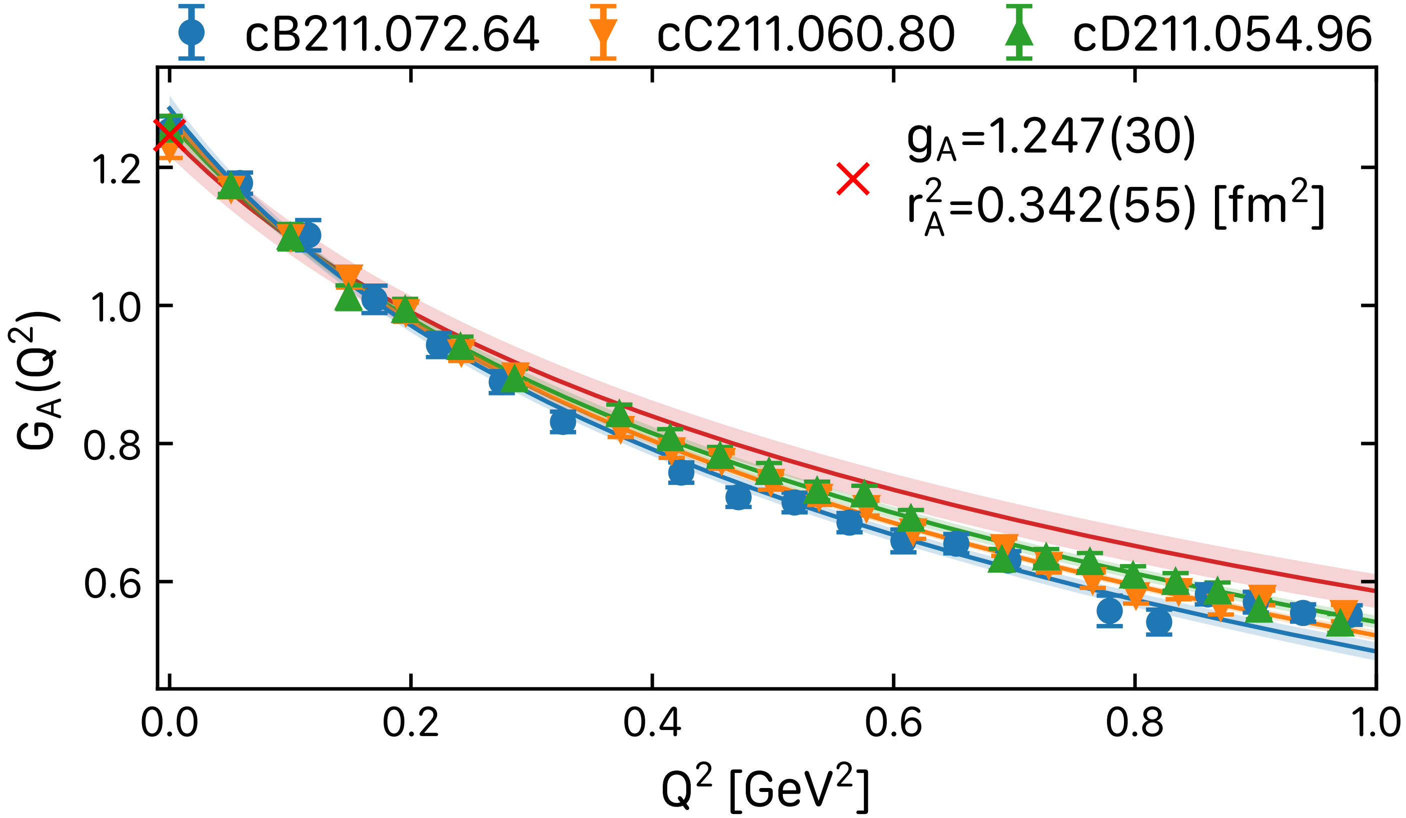


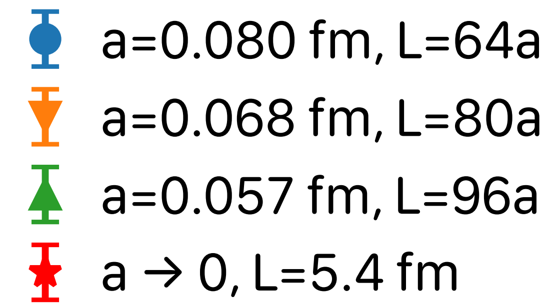


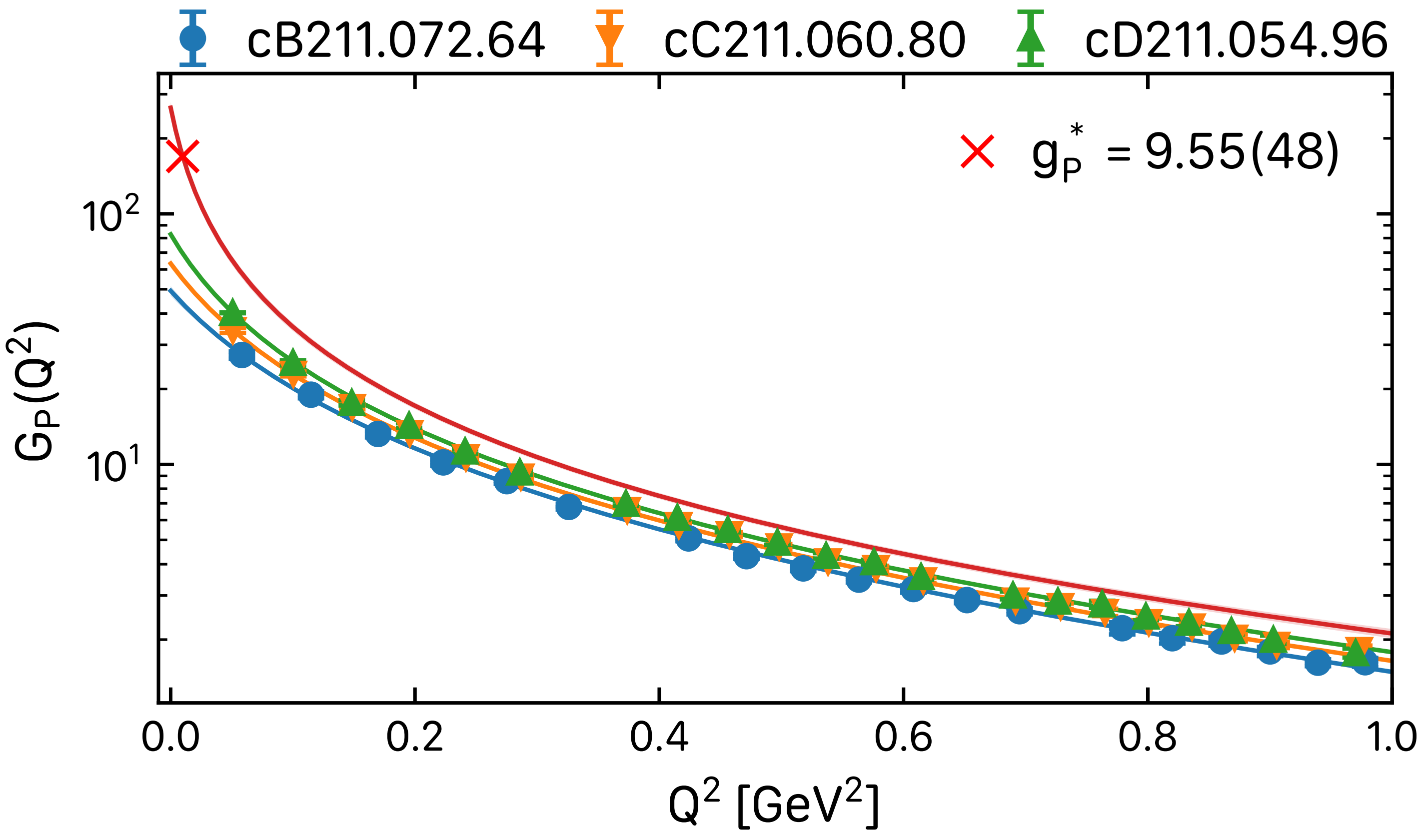
❋Precision results on the isovector tensor charge - input for phenomenology e.g. JAM3D-22 analysis

L. Gamberg et al. (JAM) Phys.Rev.D 106 (2022) 3, 034014, arXiv:2205.00999

Axial form factors







- Provide input for neutrino experiments

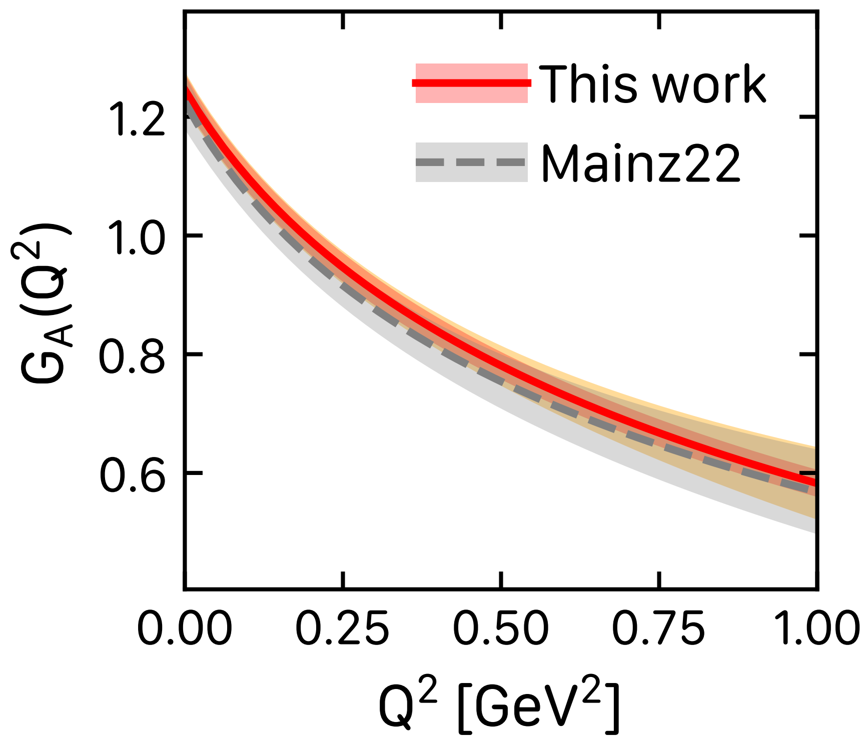
$$g_P^* \equiv \frac{m_\mu}{2m_N} G_P(0.88\, m_\mu^2)$$

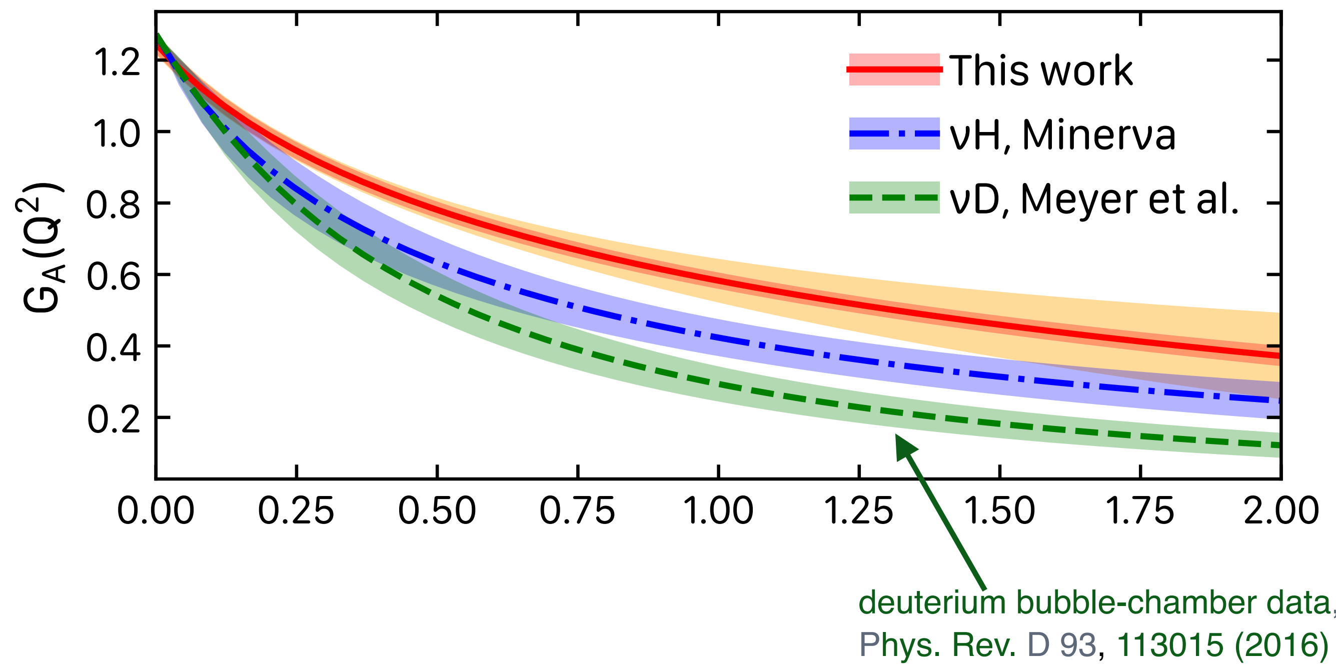
- Pseudoscalar form factor G_5 also computed and shows similar behavior to G_P —> check PCAC and Goldberger-Treiman relation

- Dipole and z-expansion fits, various ranges ⟶ model average using AIC

C.A., S. Bacchio, M. Constantinou, J. Finkenrath, R. Frezzotti, B. Kostrzewa, G. Koutsou, G. Spanoudes, C. Urbach, arXiv:2309.05774

Recent results on $G_A(Q^2)$ and $G_P(Q^2)$





❋Lattice QCD results closer to the new Minerva antineutrino-hydrogen data

T. Cai *et al.*, Nature 614, 48 (2023)

❋Agreement between our results and those of Mainz

D. Djukanovic et al. PRD 106, 074503 (2022), arXiv: 2207.03440

Second Mellin moments

- Quark unpolarised moment: $\mathcal{O}^{\mu\nu,f} = \bar{\psi}_f \gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}} \psi_f$

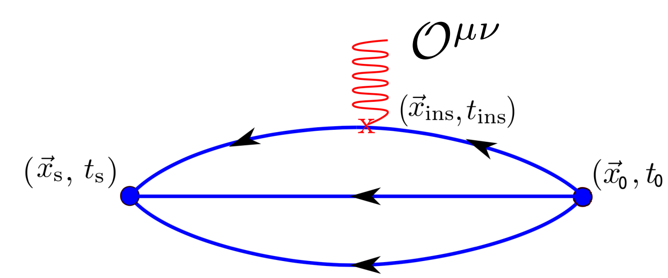
- Gluon unpolarised moment: $\mathcal{O}^{\mu\nu,g} = F^{\{\mu\rho} F_\rho^{\nu\}}$

Field strength tensor

$$\langle N(p',s')|\mathcal{O}^{\mu\nu,f}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^f(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^f(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^f(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

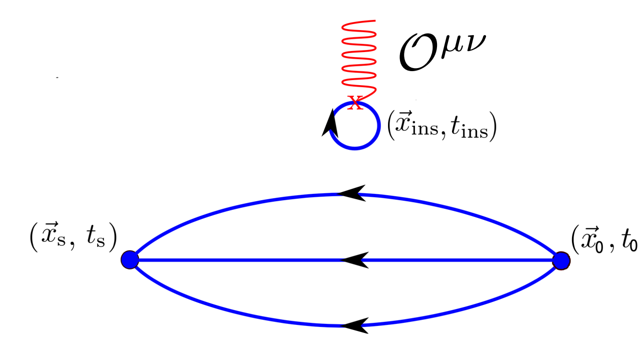
$$\langle x\rangle_{q_f} = A_{20}^f(0) \qquad J_{q_f} = \frac{1}{2}\left[A_{20}^f(0) + B_{20}^f(0)\right]$$

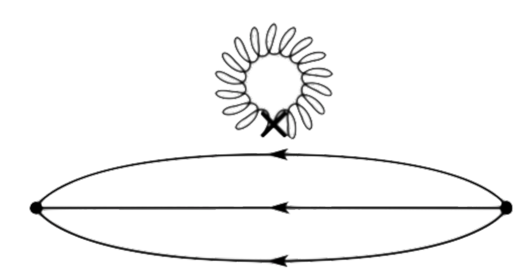
Momentum fraction carried by quark - best measured



- Equivalent expression for gluon

$$\langle x\rangle_g = A_{20}^g(0) \qquad J_g = \frac{1}{2}\left[A_{20}^g(0) + B_{20}^g(0)\right]$$





➡ Momentum sum: $\sum_f \langle x\rangle_{q_f} + \langle x\rangle_g = 1$

➡ Spin sum: $\sum_f \left[\frac{1}{2}\Delta\Sigma_{q_f} + L_{q_f}\right] + J_g = \frac{1}{2}$

$\mathrm{J_q}$

- Matrix elements of helicity and transversity one derivative operators yield: $\langle x\rangle_{\Delta q_f}$, $\langle x\rangle_{\delta q_f}$

Transversity moments

❋First Mellin moment of transversity GPD

$$\langle N(p',s')|\bar{\psi}_f \sigma^{\mu\nu}\psi_f|N(p,s)\rangle = \bar{u}_N(p',s')\left[\sigma^{\mu\nu}\, A^f_{T10}(q^2) + i\frac{\gamma^{[\mu}q^{\nu]}}{2m_N}\; B^f_{T10}(q^2) + \frac{P^{[\mu}q^{\nu]}}{m_N^2}\,\widetilde{A}^f_{T10}(q^2)\right] u_N(p,s)$$

$$g^f_T = A^f_{T10}(0)$$

❋Second Mellin moment of transversity GPD: $\mathcal{O}_T^{\mu\nu\rho,f} = \bar{\psi}_f \sigma^{[\mu\{\nu]}\overleftrightarrow{D}^{\rho\}}\psi_f$

$$\langle N(p',s')|\mathcal{O}_T^{\mu\nu\rho,f}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\Big[A^f_{T20}(q^2)\, i\sigma^{[\mu\{\nu]}P^{\rho\}} + \tilde{A}^f_{T20}(q^2)\,\frac{P^{[\mu}q^{\{\nu]}P^{\rho\}}}{m_N^2} +$$
$$B^f_{T20}(q^2)\,\frac{\gamma^{[\mu}q^{\{\nu]}P^{\rho\}}}{2m_N} + \tilde{B}^f_{T20}(q^2)\,\frac{\gamma^{[\mu}P^{\{\nu]}q^{\rho\}}}{m_N}\Big]u_N(p,s)$$

$$\langle x\rangle_{\delta q_f} = A^f_{T20}(0)$$

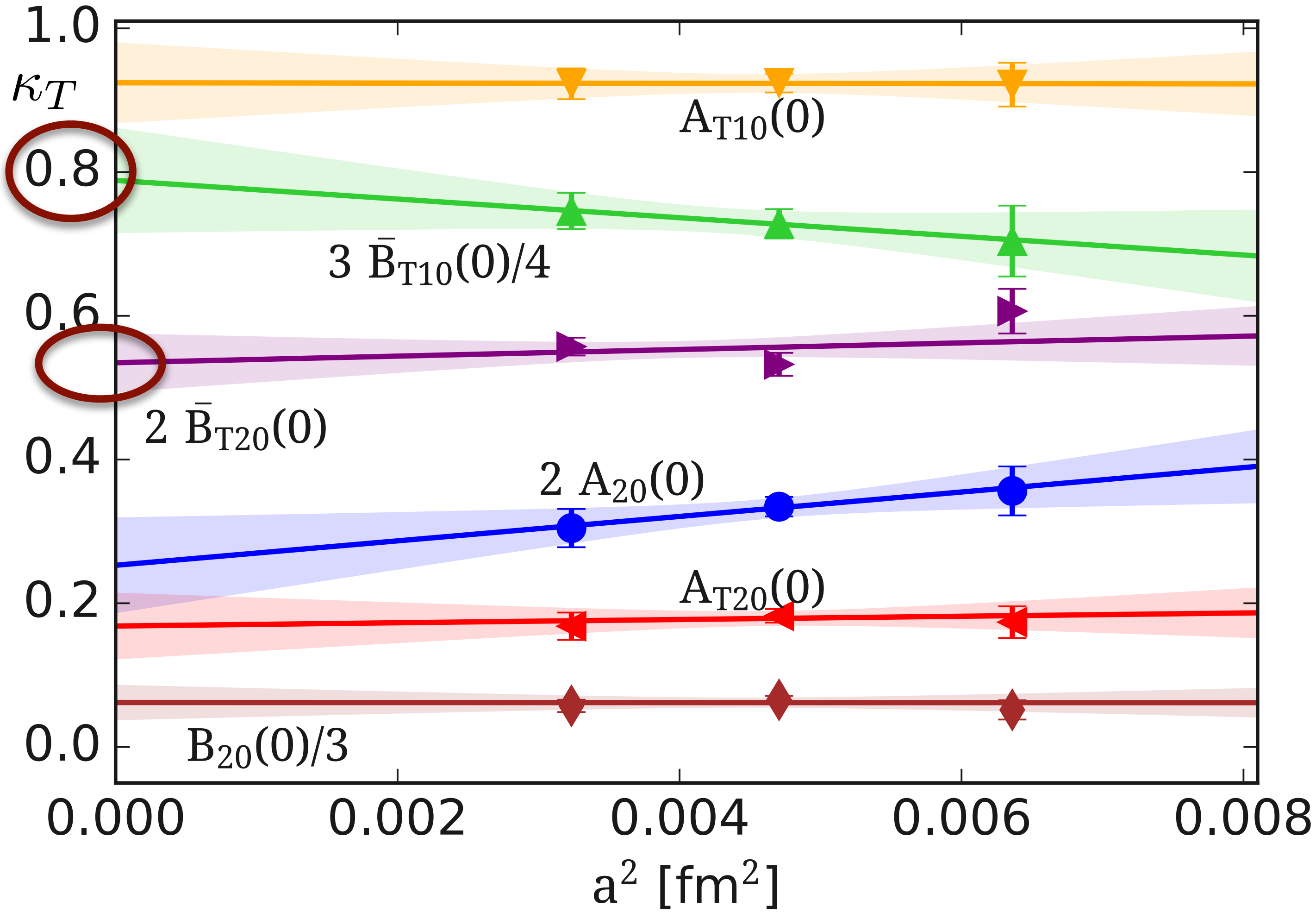
$$A_{Tn0}(q^2) = \int_{-1}^{1} dx\, x^{n-1} H_T(x,0,q^2),$$
$$B_{Tn0}(q^2) = \int_{-1}^{1} dx\, x^{n-1} E_T(x,0,q^2),$$
$$\widetilde{A}_{Tn0}(q^2) = \int_{-1}^{1} dx\, x^{n-1} \widetilde{H}_T(x,0,q^2),$$
$$\widetilde{B}_{Tn0}(q^2) = \int_{-1}^{1} dx\, x^{n-1} \widetilde{E}_T(x,0,q^2),$$

M. Diehl and Ph. Hägler, Eur. Phys. J. C 44, 87 (2005), hep-ph/0504175.

Transversity GPDs can be written in terms of the combination: $E_T + 2\widetilde{H}_T$

Continuum limit of isovector generalised form factors

- Continuum extrapolate in a²
- All results are in the $\overline{\text{MS}}$ scheme at 2 GeV



$$\bar{B}_{T10} = B_{T10} + 2\widetilde{A}_{T10}$$

- The u-d anomalous tensor magnetic moment

$$\kappa_T = \bar{B}_{T10}(0) = 1.051(94)$$

$$E'_T + 2H'_T \leftrightarrow -h_1^{\perp}$$

—> non-zero Boer-Mulders function

- Momentum fraction

$$\langle x \rangle_{u-d} = A_{20}(0) = 0.126(32)$$

- u-d total angular momentum

$$J_{u-d} = [A_{20}(0) + B_{20}(0)] = 0.156(46)$$

- Second transversity moment

$$\langle x \rangle_{\delta u - \delta d} = A_{T20}(0) = 0.168(44)$$

- Second transversity moment of

$$\widetilde{B}_{T20}(0) = 0.267(19)$$

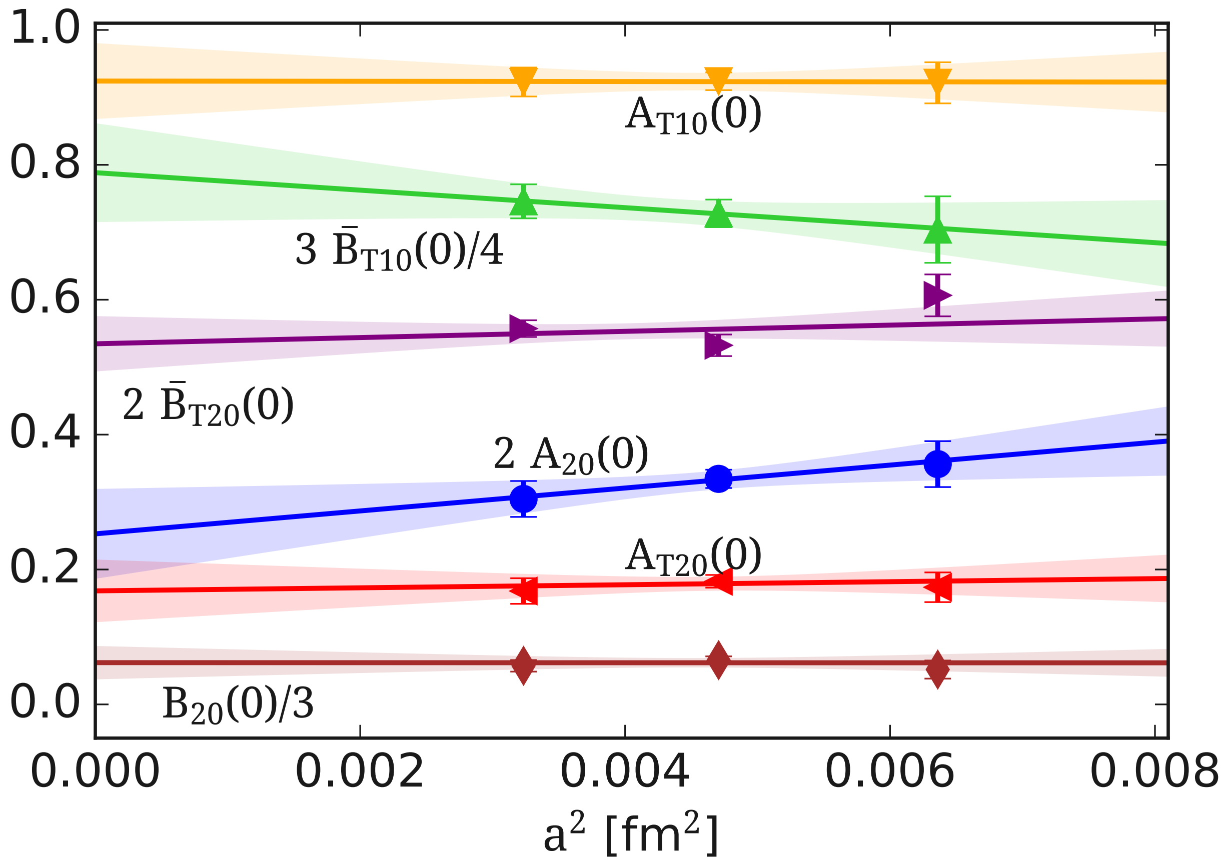
Its value not known in phenomenology

C.A. *et al* (ETMC) Phys.Rev.D 107 (2023) 5, 054504, arXiv: 2202.09871

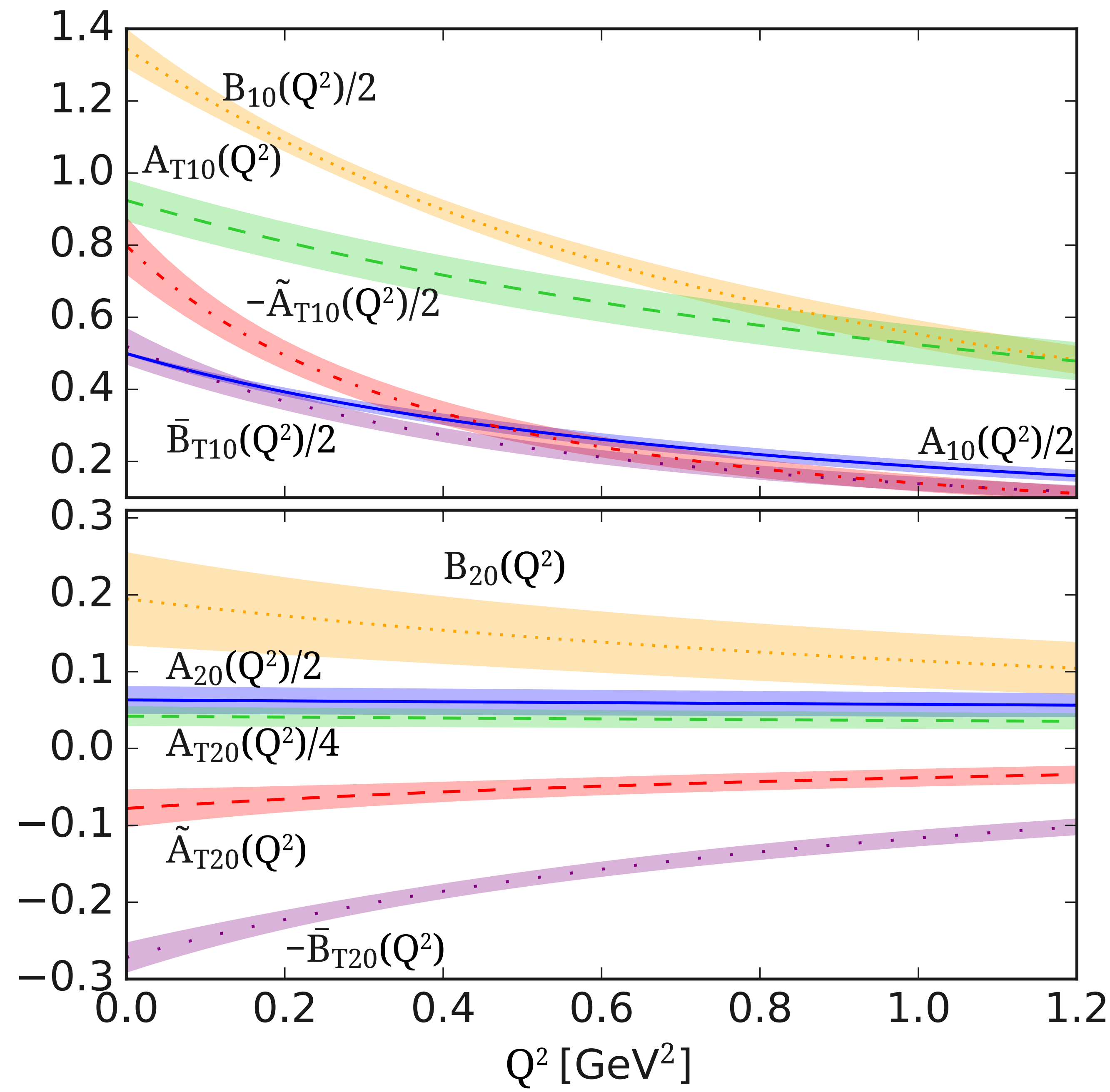
Continuum limit of generalised form factors

- Continuum extrapolate in a^2
- Fit the Q^2-dependence to $F(Q^2) = \frac{F(0)}{(1+Q^2/m^2)^p}$ and compute Fourier transform

p accounts for correct behavior for large Q^2 and small b_T



$$\tilde{B}_{T10} = B_{T10} + 2\tilde{A}_{T10}$$



C.A. *et al* (ETMC) Phys.Rev.D 107 (2023) 5, 054504, arXiv: 2202.09871

Spin densities in the transverse plane

M. Diehl and Ph. Haggler, Eur. Phys. J. C 44, 87 (2005), hep-ph/0504175.

- Examine transversity at zero skewness in impact parameter space

- Compute isovector generalised form factors and Fourier transform to impact parameter space

$$\rho(x, \mathbf{b}_\perp, \mathbf{s}_\perp, \mathbf{S}_\perp) = \frac{1}{2}\Big[H(x, b_\perp^2) + \frac{\mathbf{b}_\perp^j \epsilon^{ji}}{m_N}\left(\mathbf{S}_\perp^i E'(x, b_\perp^2) + \mathbf{s}_\perp^i \bar{E}'_T(x, b_\perp^2)\right) + \mathbf{s}_\perp^i \mathbf{S}_\perp^i \left(H_T(x, b_\perp^2) - \frac{\Delta_{b_\perp} \tilde{H}_T(x, b_\perp^2)}{4m_N^2}\right)$$
$$+ \mathbf{s}_\perp^i (2\mathbf{b}_\perp^i \mathbf{b}_\perp^j - \delta^{ij} b_\perp^2) \mathbf{S}_\perp^j \frac{\tilde{H}''_T(x, b_\perp^2)}{m_N^2}\Big]$$

x : longitudinal momentum fraction
$\mathbf{s}_\perp$: transverse quark spin
$\mathbf{S}_\perp$: transverse nucleon spin
$\mathbf{b}_\perp$: transverse impact parameter

$$F' \equiv \frac{\partial}{\partial b_\perp^2} F, \quad \Delta_{b_\perp} F \equiv 4 \frac{\partial}{\partial b_\perp^2}\left(b_\perp^2 \frac{\partial}{\partial b_\perp^2}\right) F$$

- Take moments

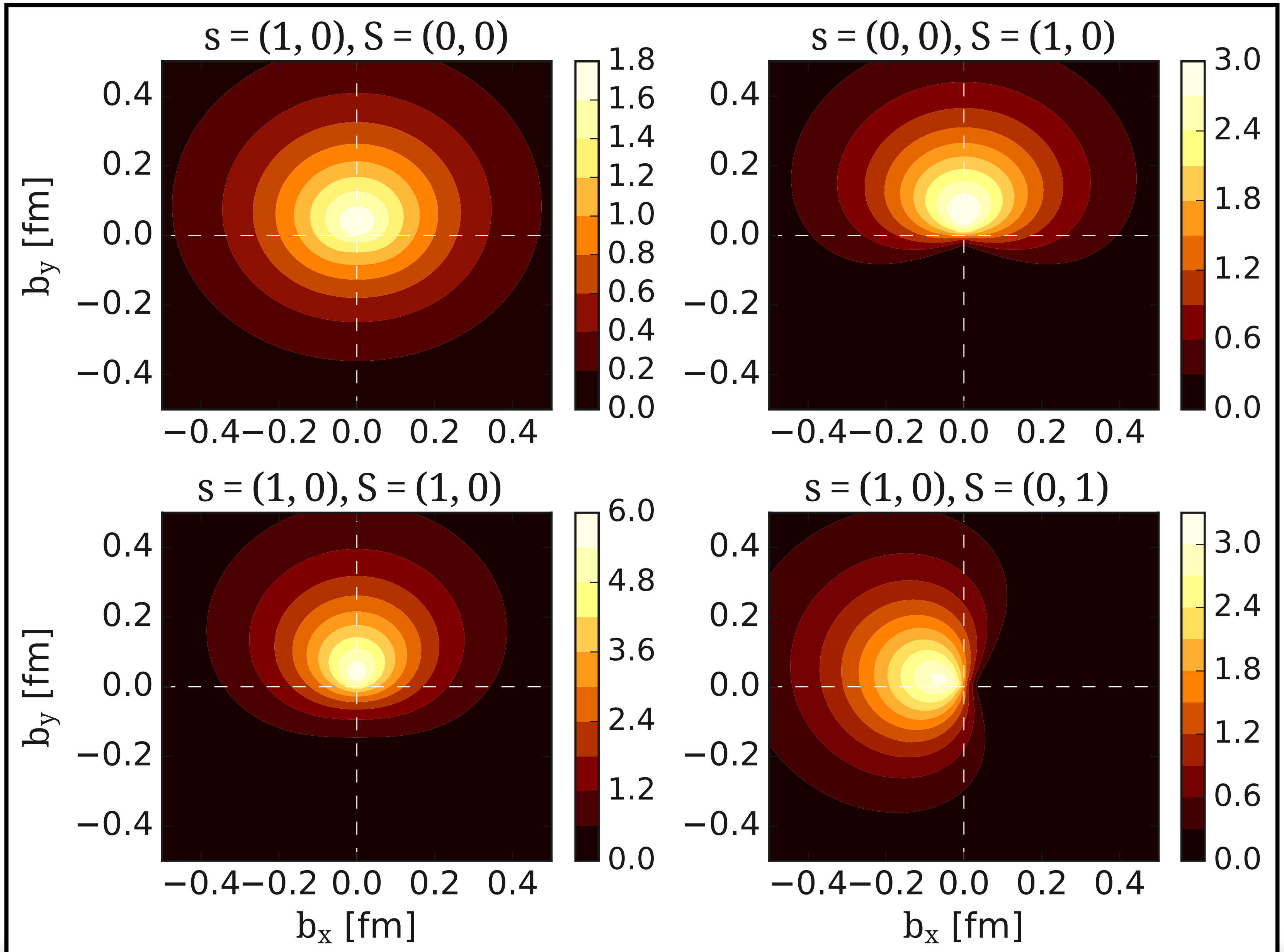
$$\langle x^{n-1} \rangle_\rho (\mathbf{b}_\perp, \mathbf{s}_\perp, \mathbf{S}_\perp) \equiv \int_{-1}^{1} dx \; x^{n-1} \rho(x, \mathbf{b}_\perp, \mathbf{s}_\perp, \mathbf{S}_\perp)$$

Transverse density distribution for n=1

- Contours of the the probability density for the **first moment** as a function of b_x and b_y

transversely polarized quarks in an unpolarized nucleon

unpolarized quarks in a transversely polarized nucleon



Large effect due to B_{10} & $\tilde{B}_{T10}(0)$

Related to Sivers asymmetry, M. Burkardt (2004)

transversely polarized quarks in a transversely polarized nucleon

transversely polarized quarks in a perpendicularly polarized nucleon

- Qualitative behavior similar to that found in M. Diehl and Ph. Hägler, Eur. Phys. J. C 44, 87 (2005), hep-ph/0504175

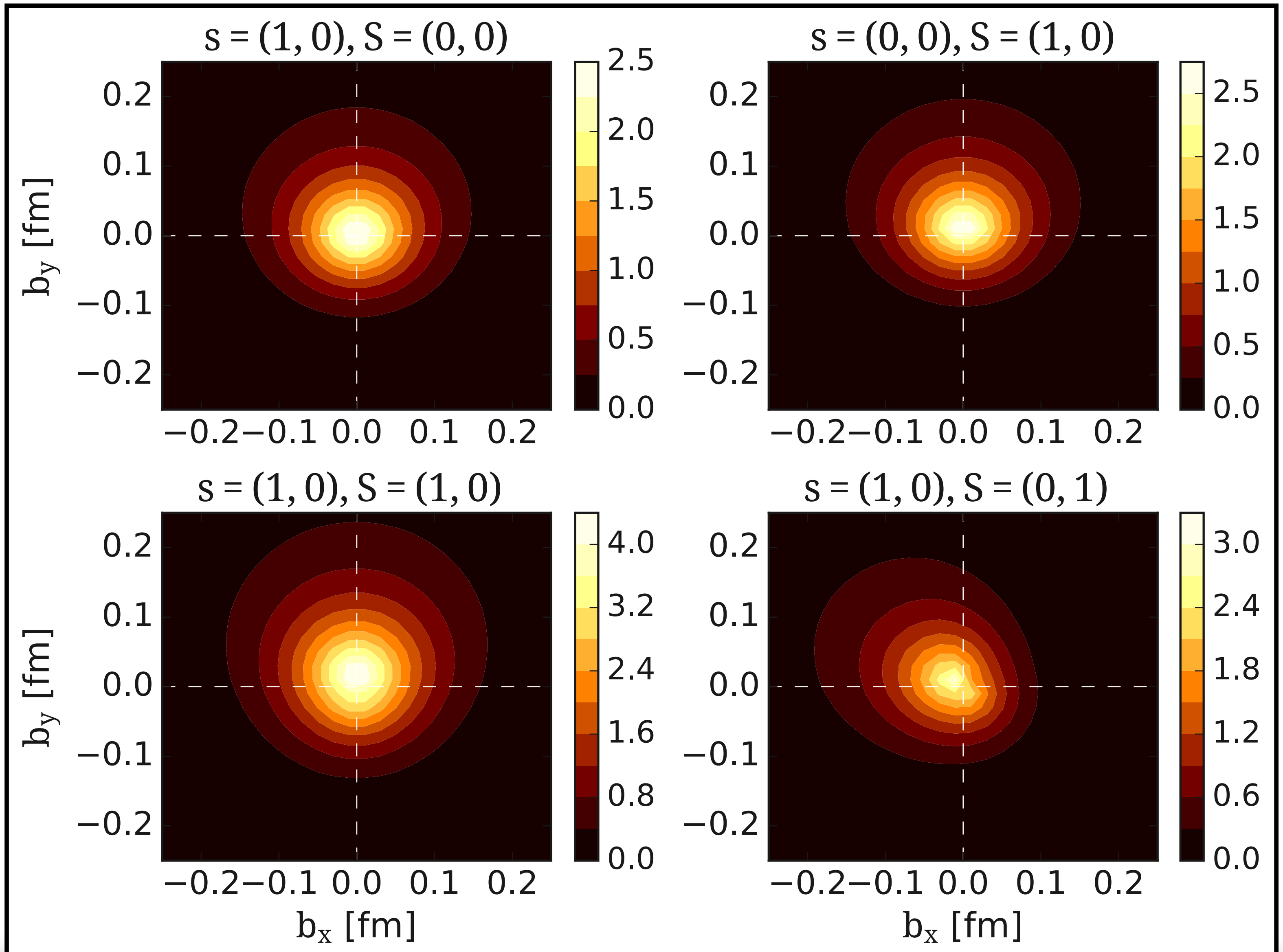
C.A. *et al.* (ETMC, *Phys.Rev.D* 107 (2023) 5, 054504, arXiv: 2202.09871

Transverse density distribution n=2

❋ Contours of the the probability density for the **second moment** as a function of b_x and b_y

transversely polarized quarks in an unpolarized nucleon

unpolarized quarks in a transversely polarized nucleon



transversely polarized quarks in a transversely polarized nucleon

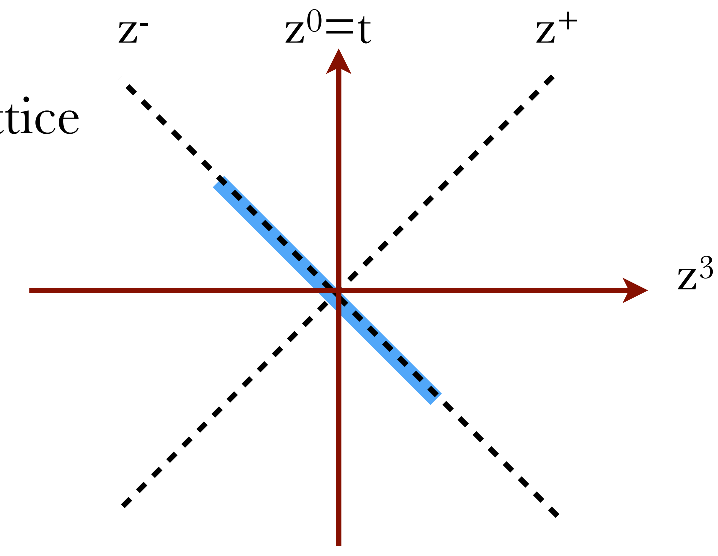
transversely polarized quarks in a perpendicularly polarized nucleon

Distortion is milder than for n=1 due to the milder dependence of $A_{20}(t)$ compared to $A_{10}(t)$

New era of direct computation of x-dependencne of parton distributions

Large momentum effective theory(LaMET)

- PDFs light-cone correlation matrix elements - cannot be computed on a Euclidean lattice



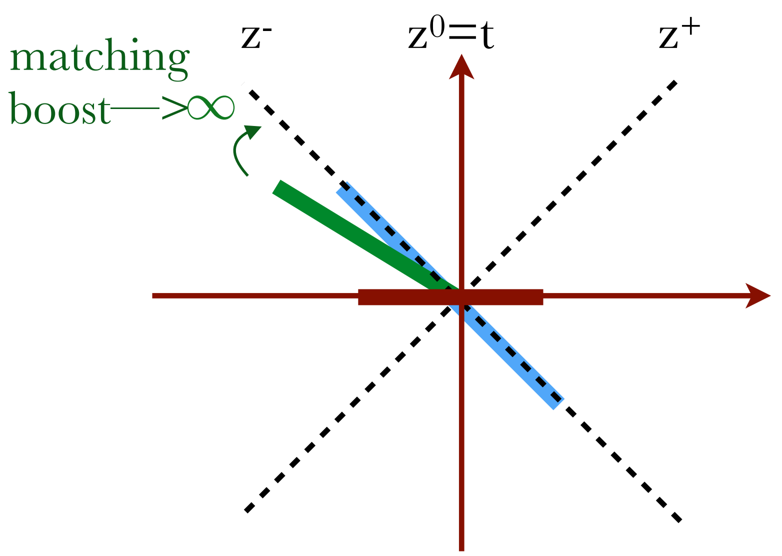
$$F_\Gamma(x) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle N(p)|\bar{\psi}(-z/2)\Gamma W(-z/2,z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$

- Define spatial correlators e.g. along z^3 and boost nucleon state to large momentum —> quasi PDFs (have same IR behaviour)

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

- Match to the infinite momentum frame using the matching kernel computed in perturbation theory (possible due to asymptotic freedom of QCD)

- Allow momentum transfer —> generalised parton distributions



Direct computation of PDFs

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle |_\mu$$

Renormalise non-perturbatively, $Z(z,\mu)$
Need to eliminate both UV and exponential divergences

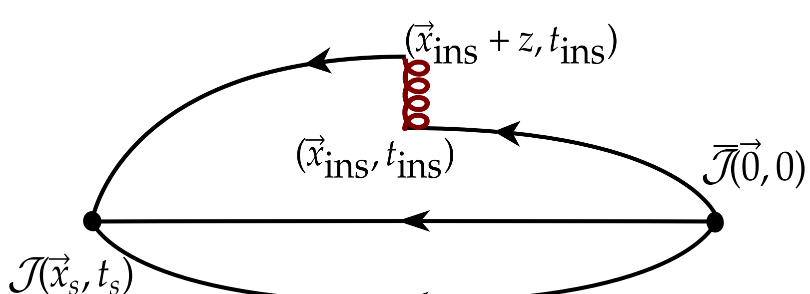
- Match using LaMET

Perturbative kernel

$$\tilde{F}_\Gamma(x, P^z, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

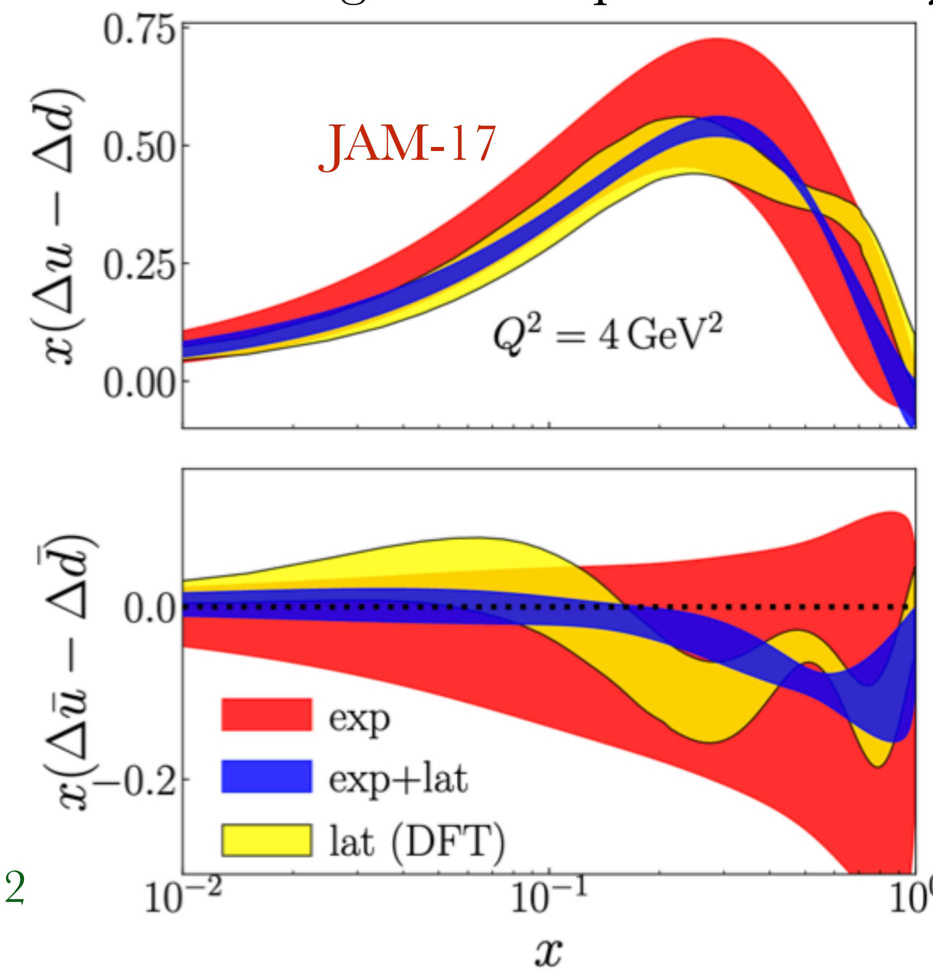
Isovector (u-d)



$$\Gamma = \begin{array}{ll} \gamma_0 & \text{unpolarised} \\ \gamma_5\gamma_3 & \text{helicity} \\ \sigma_{3i}, i = 1, 2 & \text{transversity} \end{array}$$

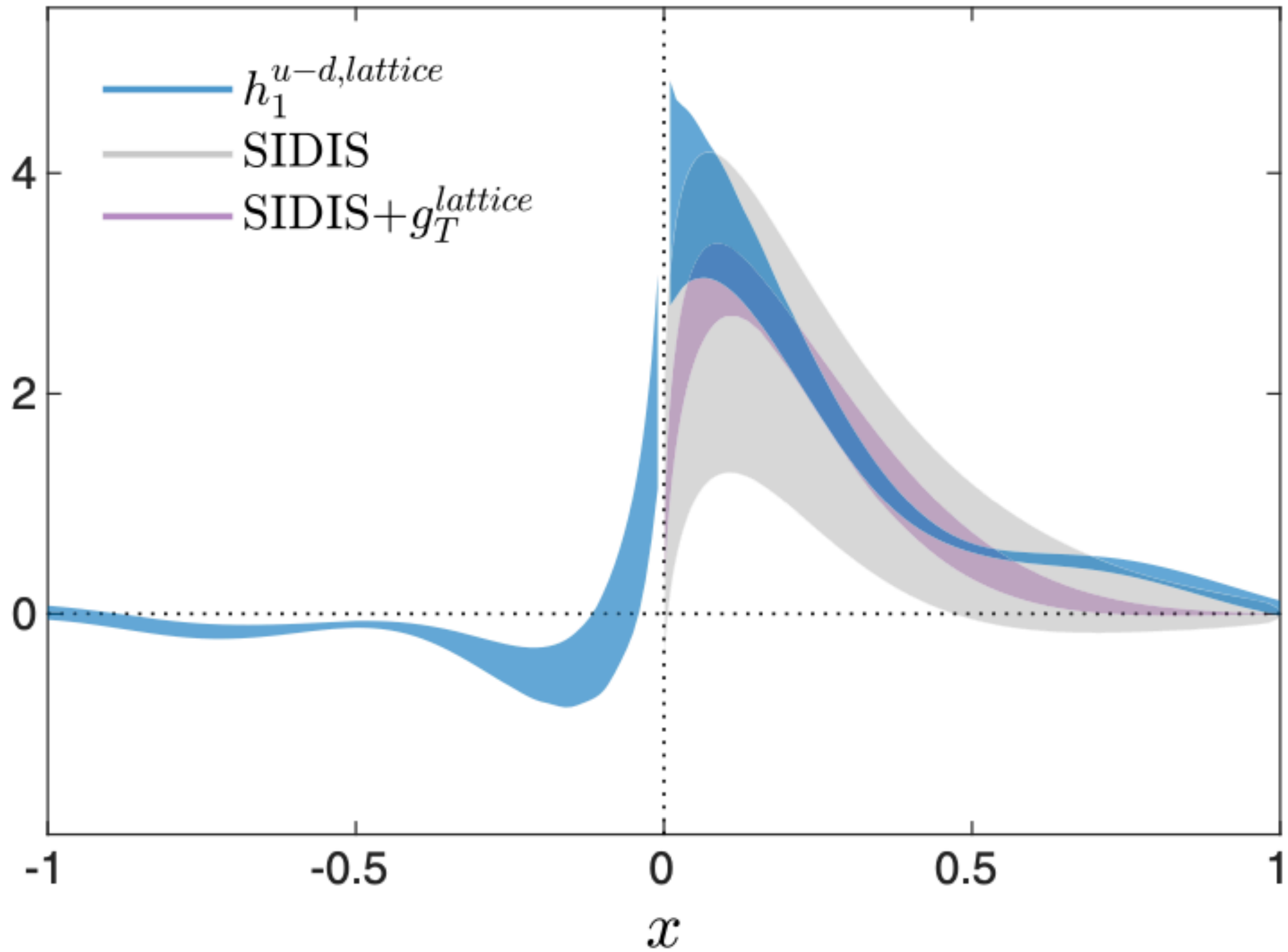
C.A. et al. (ETMC) Phys. Rev. Lett. **121**, 112001 (2018) , 1807.00232

Combining lattice input for helicity



Transversity PDF

❋Isovector transversely at physical pion mass and a=0.093 fm in the $\overline{\text{MS}}$ scheme at 2 GeV



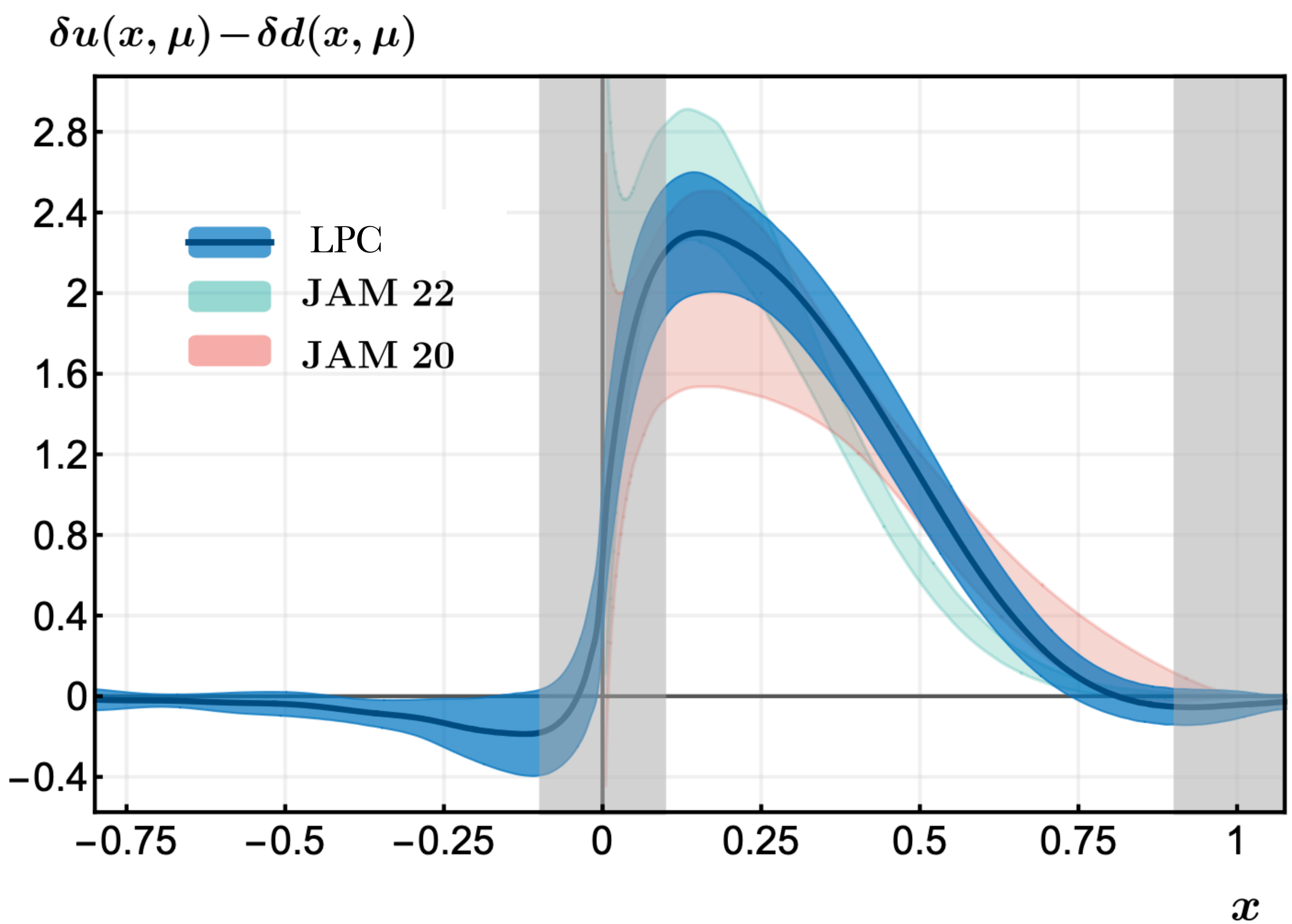
C.A. *et al.* (ETMC) Phys. Rev. D 98 (2018) 091503(R), arXiv:1807.00232

Transversity PDFs

- Lattice Parton Collaboration (LPC) analysed ensembles with 4 lattice spacings a={0.098, 0.085,0.064,0.049} fm , pion masses ranging from 220 to 350 and momentum boosts up to 2.8 GeV

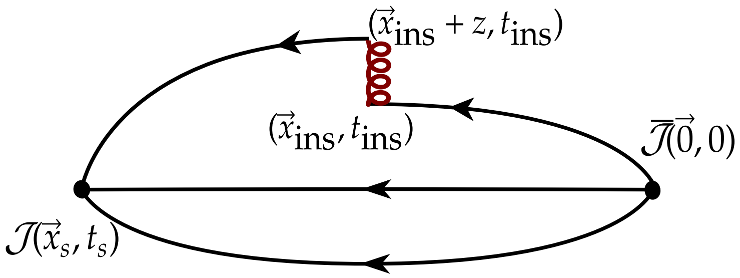
- Renormalization is done using a hybrid scheme separating the short and long distances

- Continuum, chiral extrapolations and large momentum limit are done simultaneously

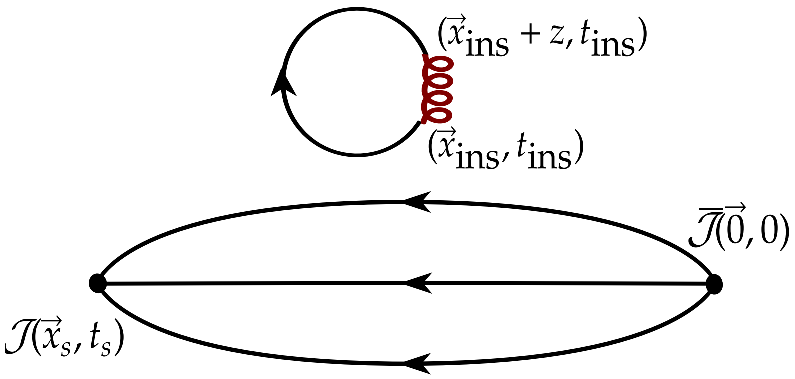


F. Yao et al. (LPC) Phys. Rev. Lett. 131 (2023) 26, 261901, 2208.08008

Isoscalar and strange PDFs



Connected isoscalar: compute like isovector



$$\mathcal{L}(t_{\text{ins}}, z) = \sum_{\vec{x}_{\text{ins}}} \text{Tr}\left[D_q^{-1}(x_{\text{ins}}; x_{\text{ins}}+z)\gamma^3\gamma^5 W(x_{\text{ins}}, x_{\text{ins}}+z)\right]$$

Two studies on disconnected with heavier than physical pion mass:

- Mixed action - clover valence on staggered sea, m_π=310 and m_π=690 MeV, only strange

R. Zhang, H.W. Lin, B. Yoon (2020), 2005.011

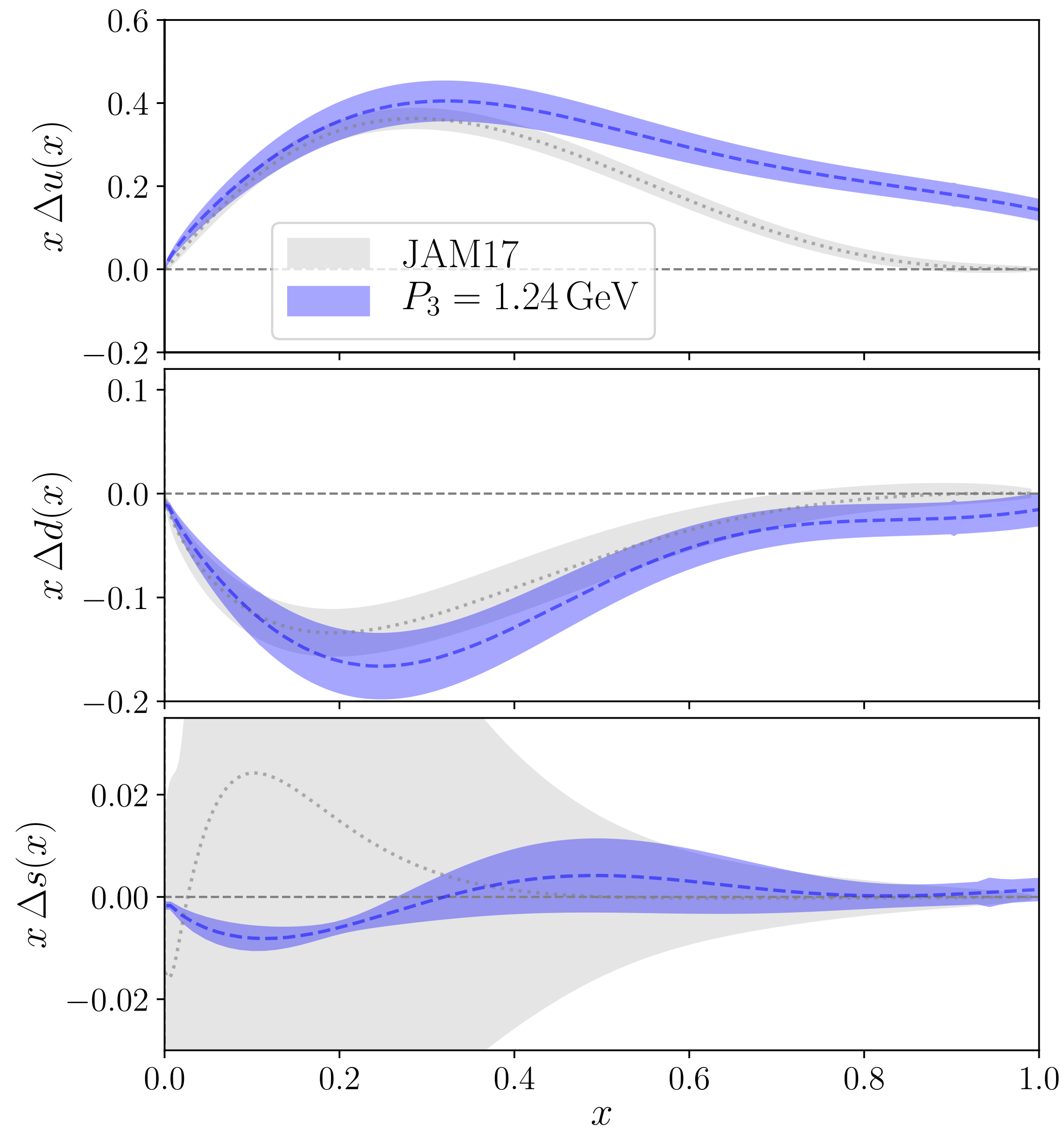
- Twisted mass fermions

$32^3 \times 64$	a=0.0938(3)(2) fm	$m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV	$m_\pi L \approx 4.0$

C. A., M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Manigrasso (2020), 2009.13061

Helicity distributions

$32^3 \times 64$ $L = 3.0$ fm	a=0.0938(3)(2) fm $m_\pi \approx 260$ MeV	$m_N = 1.050(8)$ GeV $m_\pi L \approx 4.0$



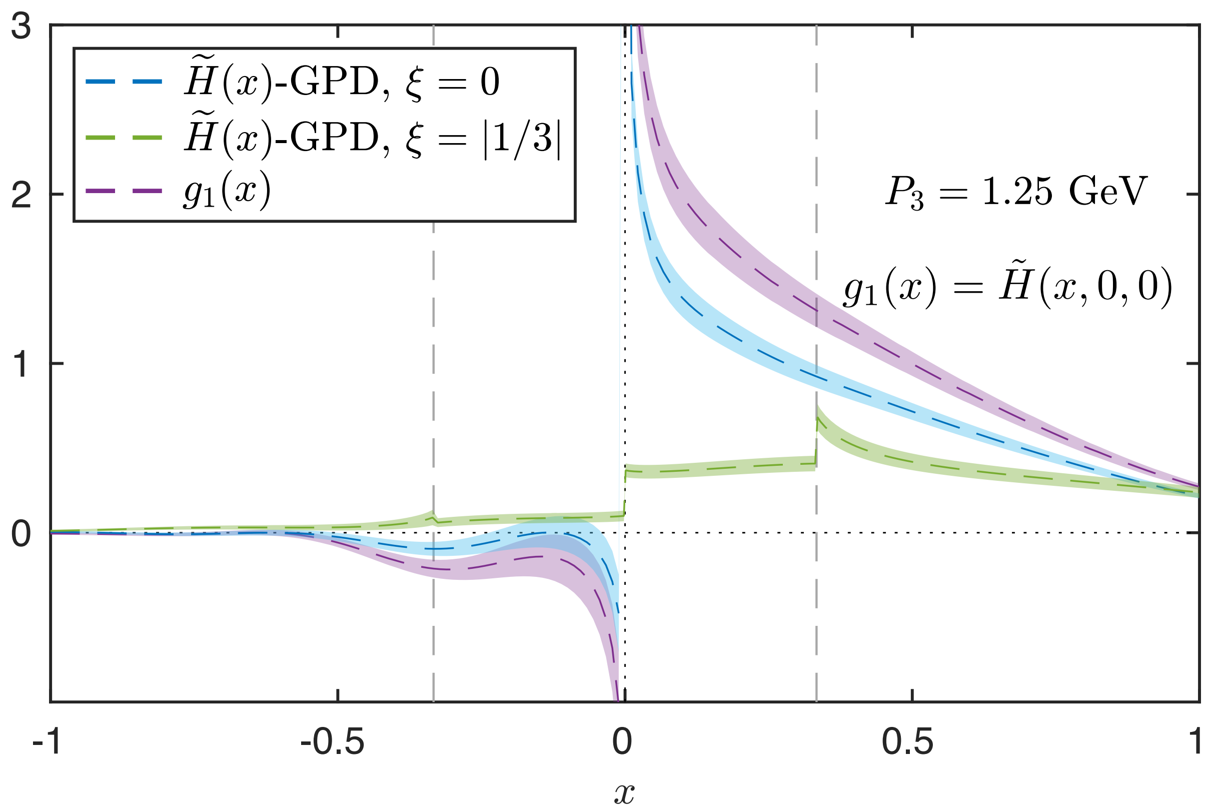
C. A., M. Constantinou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. 126 (2021) 10, 102003, arXiv:2009.1306

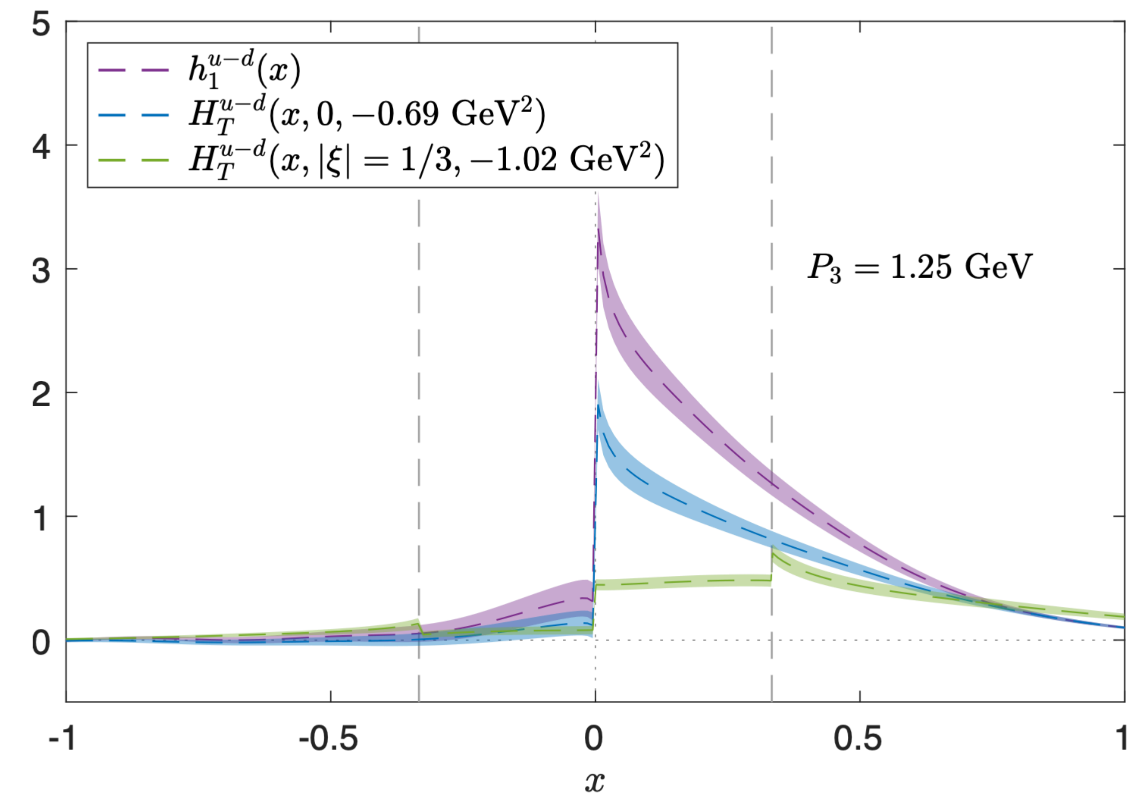
C.A., G. Iannelli, K. Jansen, F. Manigrasso, Phys. Rev. D 102 (2020) 9, 094508, arXiv:2007.13800

Helicity & transversity GPDs

$32^3 \times 64$ $L = 3.0$ fm	a=0.0938(3)(2) fm $\quad m_N = 1.050(8)$ GeV $m_\pi \approx 260$ MeV $\quad m_\pi L \approx 4.0$

Q^2=0.69 GeV^2





C. A. *et al.* (ETMC) Phys. Rev. Lett. 125 (2020) 262001,2008.10573
C.A. *et al.*(ETMC) , Phys.Rev.D 105 (2022) 3, 034501, 2108.10789

New developments of expressing GPDs in terms of Lorentz invariant amplitudes allows easier access to a range of momentum transfers in lattice QCD calculations

S. Bhattacharya *et al.* Phys. Rev. D 106 (2022) 114512, 2209.05373 for unpolarized
S. Bhattacharya *et al.* Phys. Rev. D 109 (2024) 034508, 2310.13114 for helicity

Towards TMD PDFs in lattice QCD

X. Ji, *et al.* Phys. Rev. D 99 (2019) 114006, 1801.05930

M. A. Ebert, I. W. Stewart, Y. Zhao, Phys.Rev.D 99 (2019) 3, 034505, 1811.00026; JHEP 09 (2019) 037, 1901.03685; JHEP 03 (2020) 099 ,1910.08569

- Quasi-TMDs formulated in the LaMET approach

- First results obtained for the unpolarised nucleon TMD PDF by the Lattice Parton Collaboration (LPC)

X. Ji *et al.* (LPC) 2211.02340

$$f^{\rm TMD}(x,\vec{b}_T,\mu,\zeta) = H\left(\frac{\zeta_z}{\mu^2}\right) e^{-\ln\left(\frac{\zeta_z}{\zeta}\right)K(b_T,\mu)}\, \tilde{f}(x,\vec{b}_T,\mu,\zeta_z)\sqrt{S_r(b_T,\mu)} + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\zeta_z},\frac{M^2}{(P^z)^2},\frac{1}{b_T^2\zeta_z}\right)$$

perturbative matching kernel

Collins-Soper kernel, which is non-perturbative for $q_T \sim 1/b_T \sim \Lambda_{\rm QCD}$

Rapidity independent reduced soft function

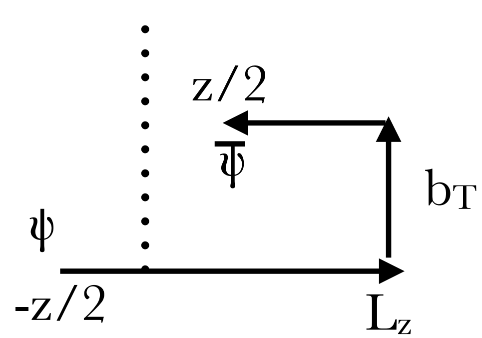
- $\zeta_z = (2xP^z)^2$ is the Collins-Soper scale of the quasi-TMD

- Quasi-TMD PDF is given as $\tilde{f}^{\rm TMD}(x,\vec{b}_T,\mu,\zeta_z) = \int \frac{dz}{2\pi}\, e^{-iz\zeta_z}\, \frac{P^z}{E_{\vec{P}}} B_\Gamma(z,\vec{b}_T,\mu,P^z)$

Renormalised beam function obtained from the bare

$$\tilde{B}_{0,\Gamma}(z,\vec{b}_T,L,P^z;1/a) = \langle N(P^z)|\bar{\psi}(z/2,\vec{0}_T)\Gamma\mathcal{W}(z,\vec{b}_T,L\hat{z})q(-z/2,\vec{b}_T)|N(P^z)\rangle$$

$$\mathcal{W}(z,\vec{b}_T,L\hat{z}) =$$

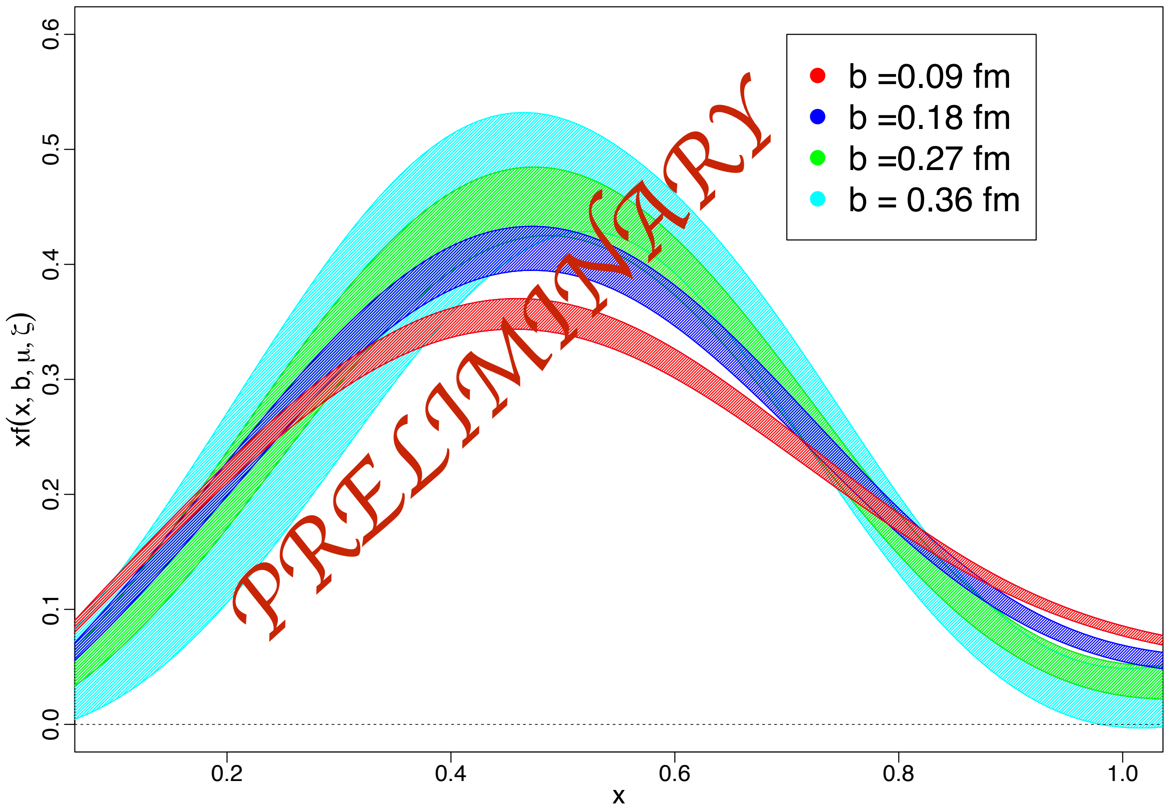


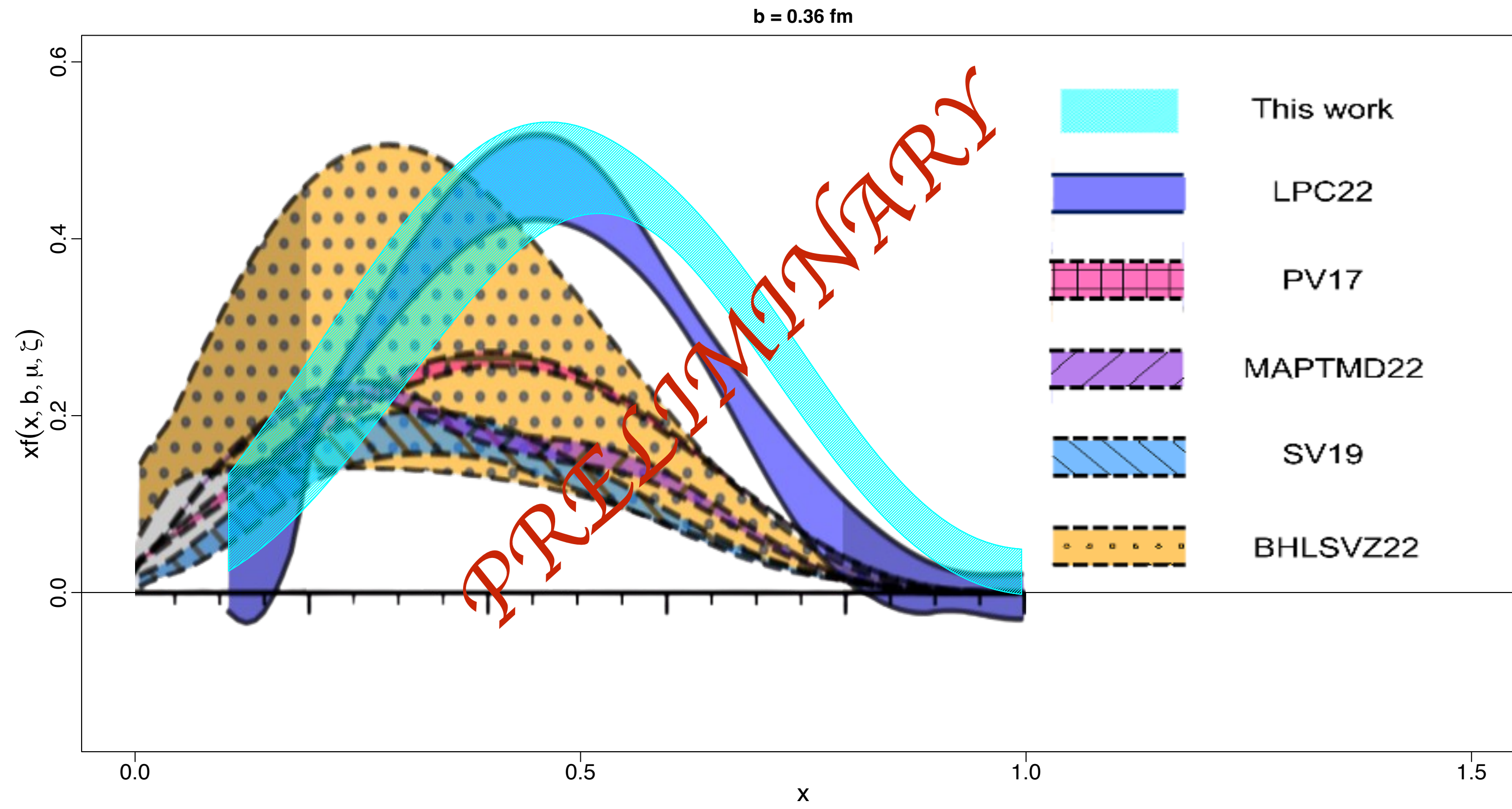
Nucleon unpolarised isovector TMD PDF

- LPC published the first results modelling the momentum dependence and taking the chiral and continuum limits

Jin-Chen He et al. (LPC) arXiv:2211.02340

- ETMC has preliminary results at one lattice spacing (0.093 fm) and heavier than physical pion mass (350 MeV), renormalised with the ratio scheme





Conclusions

- **Precision era of lattice QCD**: Moments of PFDs can be extracted precisely - we can extract a lot of interesting physics and also reconstruct the PDFs
- Results on isovector and gluon PDFs using simulations with physical pion mass using various approaches (quasi-distributions, pseudo-distributions, current-current correlates, etc)
- Calculations of GPDs using a suitable for lattice frame and extraction of Lorentz invariant amplitudes
- The calculation of sea quark contributions is feasible providing valuable input e.g. the strange helicity
- Exploratory studies of TMDs

    ✦ Way forward: continuum limit, larger boosts, volume effects