Flavour physics phenomenology in view of future experiments

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Summary. — In this proceeding I will review the current advancements in flavour physics phenomenology, by focusing on key tree-level and loop-level processes in the quark sector. I will discuss semi-leptonic processes related to $b \to c$ quark transitions, unitarity in the first row of the CKM matrix, $b \to s$ quark transitions as well as rare kaon decays, showing in each case possible developments for the future.

1. – Introduction

Flavour physics represents one of the pillars of the Standard Model (SM) of particle physics and is a fundamental tool in the context of high-intensity physics. In complementarity with the high-energy frontier of direct searches for New Physics (NP), it allows to identify NP signals indirectly through a careful comparison between precise theoretical predictions and low-energy observables well-measured across various experimental facilities.

Flavour physics can also enable us to discover the underlying theoretical structure beyond the SM one. Important hints in this sense are given by the Yukawa terms present in the SM Lagrangian, which break the global $U(3)^5$ flavour symmetry of the SM gauge sector down to a smaller $U(1)_B \times U(1)_L$ group (corresponding to the accidental conservation of the baryon and of the lepton numbers). Focusing specifically on the quark sector for the case of this proceeding, the Yukawa terms can be directly related to the quark mass terms and to the Cabibbo-Kobayashi-Maskawa (CMK) matrix elements through bi-unitary transformations. Some fundamental questions (which do not find clear answers within the SM) concerning these quantities then follow:
1. Why there are three generations of quarks ?
2. What determines the hierarchical structure that we observe in the quark masses ?
3. Why is the CKM matrix very close to the unit matrix and is it really unitary ?
These questions can be further investigated by following two main paths, apparently independent but, in reality, deeply interconnected. The first one is the Unitarity Triangle Analysis (UTA), which exploits the property of unitarity of the CKM matrix by means of an useful parametrization like the Wolfeinstein one [1]. The sides, the angles and also

the area of the unitarity triangle are physical quantities at all the effects and can be determined from a combined analysis of the decays of hadrons and of meson-antimeson mixings. The second one is the study of rare decays, which are the manifestation of broken (accidental) symmetries and, thus, are the most apppealing places to look for possible NP effects. Here we will focus mainly on Flavour Changing Neutral Currents (FCNCs), featuring a well-known suppression within the SM in light of the Glashow-Iliopolous-Maiani (GIM) mechanism [2].

In order to follow this two-path strategy, in what follows I'll review the main problems affecting some of the most interesting channels for flavour physics phenomenology, giving some insights on possible developments for the future.

2. – Tree-level processes

2·1. *$b \to c$ quark transitions.* – $B \to D^{(*)}\ell\nu$ decays are very challenging processes since they are affected by two unsolved problems. On the one hand, a $\sim 3\sigma$ tension exists between the inclusive and the exclusive determinations of the CKM matrix element $|V_{cb}|$ [3]. On the other hand, there is an important discrepancy between the theoretical value and the measurements of the ratios $R(D^{(*)}) \equiv \Gamma(B \to D^{(*)}\tau\nu_\tau)/\Gamma(B \to D^{(*)}\ell\nu_\ell)$, which are a fundamental test of Lepton Flavour Universality (LFU). The HFLAV Collaboration [4] routinely computes the world averages of the available measurements of the $R(D^{(*)})$ ratios and of their SM theoretical predictions, which are found to be discrepant at the $\sim 3\sigma$ level. This tension is mainly fuelled by $B \to D^*\ell\nu$ channel.

A key role in the study of $B \to D^*\ell\nu$ decay is played by the hadronic Form Factors (FFs), which encode all the information about strong dynamics. At present, three different lattice QCD collaborations have determined these FFs at non-zero recoil [5, 6, 7]. The first global analysis of their results have been recently performed in Ref. [8], where lattice data have been studied both individually and globally in order to have full control of the hadronic uncertainties. In this way, also the slight discrepancies existing among the different lattice results can be properly taken into account.

The first quantity that can be theoretically determined is then $R(D^*)$, which is by construction $|V_{cb}|$-independent. The final value obtained in Ref. [8] is $R(D^*)|_{\rm SM} = 0.262(9)$, to be compared with the experimental average $R(D^*)_{\rm exp} = 0.284(12)$ [4]. These two numbers are compatible at the 1.5σ level, thus reducing the tension reported by HFLAV [4]. However, these shapes of the FFs produce a 2.5σ discrepancy in the D^*-longitudinal polarization in case of production of light leptons, often referred to as F_L^ℓ, measured by the Belle and Belle-II experiments [9, 10]. This tension poses a new question, *i.e.* whether there can be NP coupled to light leptons (rather than to the τ), which is currently under investigation, see for instance Refs. [11, 12, 13].

For what concerns $|V_{cb}|$, as originally proposed in Refs. [14, 15], we may determine several values of this CKM matrix element by performing a bin-per-bin study of the experimental data [9, 10, 16], *i.e.* by computing

$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp}}{(d\Gamma/dx)_i^{th}}} \qquad (x = w,\, \cos\theta_\ell,\, \cos\theta_v,\, \chi) \tag{1}$$

for the i-th experimental bin. The comparison between the theoretical differential rates (integrated over specific bins) and the corresponding measurements gives, in fact, estimates of $|V_{cb}|$ in which the shape of the FFs cannot longer bias its extraction in a significant manner. At the same time, the bin-per-bin analysis can also provide direct detailed information about the agreement/disagreement of the shapes of the distributions between theory and experiments in the different phase space regions and for each of the different kinematical variables x. All the details of this analysis strategy can be found in Ref. [8]. The final result reads $|V_{cb}| = 39.92(64) \cdot 10^{-3}$.

Notice also that all the results in Ref. [8] (including in particular $R(D^*)$ and $|V_{cb}|$) are insensitive to the particular parametrization that we adopt for the hadronic FFs and are practically the same by using either the Dispersive Matrix method of Refs. [17, 18] (augmented with the importance sampling procedure of Ref. [19]) or the Boyd-Grinstein-Lebed parametrization [20]. Other analyses of (lattice and experimental) $B \to D^*$ data can be found in Refs. [21, 22] and lead to results very similar to the ones presented in this contribution.

2˙2. *Unitarity in the first row of the CKM matrix.* – As discussed in depth in [23], at present there exists an important tension concerning the determination of the CKM matrix elements $|V_{ud}|$ and $|V_{us}|$, which are fundamental ingredients to test the property of unitarity of the CKM matrix. In particular, by neglecting the contribution of $|V_{ub}|$, one has that [23]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \approx |V_{ud}|^2 + |V_{us}|^2 = 0.9985(6)_{|V_{ud}|}(4)_{|V_{us}|}, \tag{2}$$

pointing towards a 2σ violation of unitarity in the first row.

For what concerns $|V_{ud}|$, the three golden channels that allow its determination are the superallowed $0^+ - 0^+$ β-decays [24], the neutron β-decay [25] and, finally, the $\pi^+ \to \pi^0 e^+ \nu_e$ decay ($\pi_{\ell 3}$ decay) [25]. The results for $|V_{ud}|$ coming from these channels are perfectly consistent one with each other. An important improvement is expected in the future for $\pi_{\ell 3}$ decay. This transition is very clean from the theoretical point of view, but also very challenging to be measured. A significant reduction of the experimental uncertainty is expected at the future PIONEER experiment [26].

A direct access to $|V_{us}|$, instead, is offered by the semileptonic decay of K-mesons ($K_{\ell 3}$ decay) [3], by semileptonic decays of hyperons [25] and, finally, by the hadronic decays of τ-leptons. The latter transitions have been studied both exclusively [4] and inclusively, in particular in the second case some direct computations on the lattice have been performed [27, 28]. A further reduction of the uncertainties associated to these $|V_{us}|$ determinations will be very helpful in order to weaken or to strengthen the hypothetical violation of unitarity in the first row of the CKM matrix.

Last but not least, a complementary determination of the $|V_{us}|/|V_{ud}|$ ratio is possible through the ratio of the $K \to \ell\nu$ and $\pi \to \ell\nu$ decay rates ($K_{\ell 2}/\pi_{\ell 2}$). Many details on this issue can be found in Section 4 of Ref. [3].

3. – Loop-level processes

3˙1. *$b \to s$ quark transitions.* – $B \to K^{(*)}\ell\ell$ transitions are rare B-meson decays whose theoretical and experimental studies provide many challenges. Although past experimental indications of LFU violation have disappeared [29], many $b \to s\mu\mu$ data (including branching ratio measurements and angular ones) show tensions with the SM in the *low* di-muon invariant mass squared (q^2) region. A full control of the hadronic FFs is necessary in order to have reliable SM estimates of such quantities, which crucially depend on the following decay amplitudes (see for instance [30])

$$\mathcal{A}_\lambda^{L,R} \propto \left\{ (C_9 \mp C_{10})\mathcal{F}_\lambda(q^2) + \frac{2\, m_b\, M_B}{q^2} \left[C_7 \mathcal{F}_{T,\lambda}(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\} \quad (3)$$

at leading order in electromagnetic interactions. Here C_i are the Wilson Coefficients (WCs) at the weak scale, while $\mathcal{F}_\lambda(q^2)$, $\mathcal{F}_{T,\lambda}(q^2)$, $\mathcal{H}_\lambda(q^2)$ are FFs. Finally, λ represents the possible polarizations of the quark currents underlying the hadronic matrix elements.

At present, an important debate concerns the contribution of the non-local FFs $\mathcal{H}_\lambda(q^2)$ to the theoretical estimates of branching ratios and angular observables. In this sense, the authors of Ref. [31] (see also [32, 33] for previous analyses) have demonstrated that the results of global fits of all the available $b \to s\ell\ell$ data (including the new SM-like measurements of Ref. [29]) can give very different results for the NP contribution to the WC C_9 according to the way in which one parametrises the non-local FFs. While a data-driven approach (based on a naive q^2-expansion) finds a preferred value for C_9 perfectly compatible with the SM one, this is not the case if one puts to zero the contribution of the non-local FFs (see also Ref. [34] for another global analysis of $b \to s$ data in this sense). This last choice is, in reality, motivated by the findings of several studies scrutinizing the application of dispersion relations, analiticity and unitarity to the non-local FFs, as the ones in Refs. [30, 35]. While some efforts have also been done recently in order to estimate directly the magnitude of these contributions with phenomenological models [36, 37], further investigation is needed in order to find a conclusive statement on this issue.

In order to shed a new light on this problem, an important and parallel exercise consists in studying in detail other $b \to s$ channels. The first perfect candidate in this sense is the leptonic-and-radiative $B_s \to \mu\mu\gamma$ decay, whose branching ratio at *high* dimuon invariant mass squared is much less affected by the aforementioned long-distance effects. Hence, if NP contributions to C_9 are really there, they may be unambiguously identified in this channel [38, 39]. A lot of theoretical efforts have also been developed recently, including the first non-perturbative determination of the relevant FFs on the lattice [40]. Experimental searches for this transition have been performed by LHCb Collaboration in [41, 42], without having observed this transition yet. A second channel to be further investigated is $B \to K\nu\bar{\nu}$, which is free of long distance effects. A recent SM estimate of the total BR is $\mathcal{B}(B^\pm \to K^\pm\nu\bar{\nu})|_{\rm SM} = (4.44 \pm 0.30) \cdot 10^{-6}$ [43, 44], to be compared with the experimental value $\mathcal{B}(B^\pm \to K^\pm\nu\bar{\nu})|_{\rm exp} = (2.3 \pm 0.7) \cdot 10^{-5}$ [45], showing a 2.8σ discrepancy. It will be interesting in the future to see whether the same behaviour is seen also for $B \to K^*\nu\bar{\nu}$ decay, that has not been observed yet.

3·2. *Rare kaon decays.* – Rare kaon decays, and particularly the golden $K \to \pi\nu\bar{\nu}$ modes, offer a unique opportunity for indirect NP searches, since they may allow us to probe with high precision the short-distance effects present in the $s \to d$ quark transitions. The three major experiments involved in this field are NA62 for measurements of K^+ decays [46], KOTO for K_L ones [47] and finally LHCb for transitions involving both K_S-mesons and hyperons [48]. The analytic expressions of the total BRs of these decays have been computed many years ago (see for instance Ref. [49]) and, from the theory point of view, the major theoretical uncertainties on these quantities come at present from the UTA, although the experimental ones are still larger.

Thus, some efforts are still needed to increase the precision on both the theoretical and experimental sides and to have the possibility to scrutinize hypothetical NP effects with

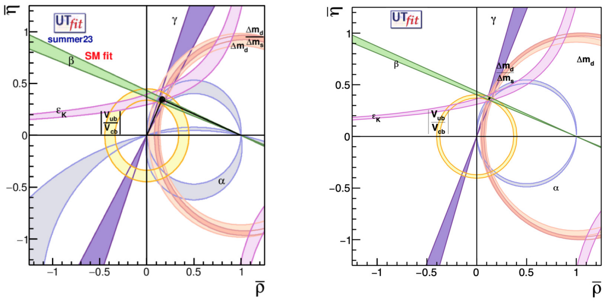

Fig. 1. – *Left panel:* present status (summer 2023) of the UTA as performed by the UTfit Collaboration [52, 53]. *Right panel:* future perspective for the UTA, assuming a scaling of the experimental uncertainties on some parameters (such as the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$) with the luminosity till 50 ab^{-1}. More details are contained in Section 18.2.2 of Ref. [54].

more constraining power. To give an explicit example, this issue is crucial at present in particular for the description of the QCD axion, whose interactions with ordinary matter are described by the Lagrangian terms

$$\mathcal{L}_{aff} = \frac{\partial_\mu a}{2 f_a} \bar{f}_i \gamma^\mu (c_{ij}^V + c_{ij}^A \gamma_5) f_j , \tag{4}$$

where $f_{i,j}$ are SM fermions. A very detailed study of the bounds that can be obtained on the effective flavor-violating couplings $c_{ij}^{V,\,A}$ (with $i \neq j$) from current flavour experiments are presented in [50], where the strongest limits concern the $s \to d$ sector. Let me also emphasize that an important interconnection with astroparticle physics here exists, since analogous bounds on flavor-violating $s \to d$ couplings can be derived from the study of compact astrophysical objects as supernovae [50, 51].

4. – Conclusions

In this proceeding I have discussed the state-of-the-art and the future (theoretical and experimental) challenges of some tree- and loop-level processes in flavour physics. Although the list can be evidently expanded, the key issue for precision flavour physics is that, in reality, all these transitions are interconnected, since they share many of the input ingredients of the UTA. Increasing the precision of the (both theoretical and experimental) description of each of these channels is thus fundamental in order to reduce the uncertainties on the SM parameters of the flavour sector and on the yet unmeasured observables. The two plots in Fig. 1 show exactly this point, by comparing the state-of-the-art of UTA [52, 53] (left panel) with a future projection based on the assumption of an important reduction of future experimental errors at Belle II [54]. It is thus clear that both the investigation of "new" channels and the increase in precision for the "old" ones will be two fundamental ingredients for the discovery / exclusion of many NP models in the near future.

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