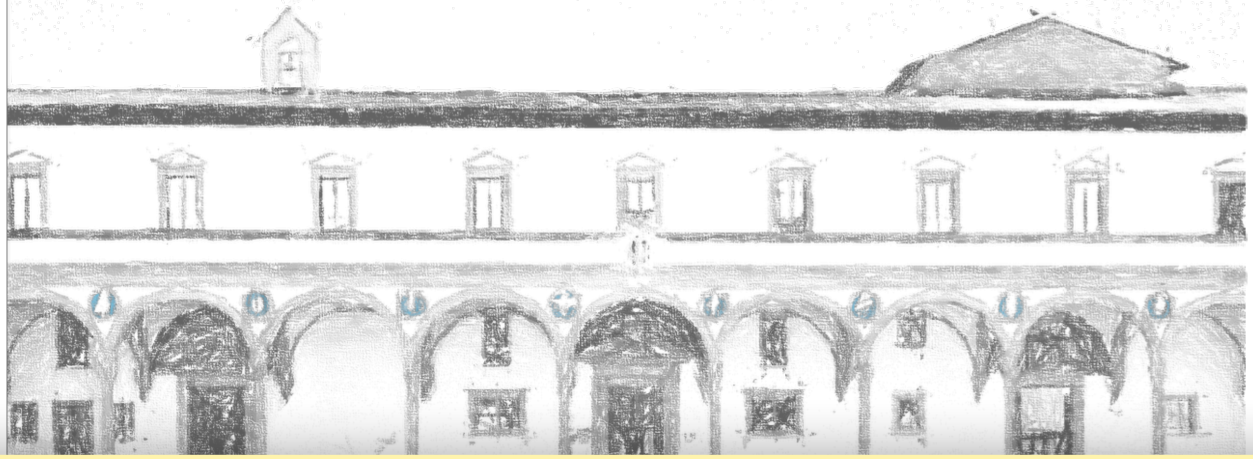
(2) INFN Istituto Nazionale di Fisica Nucleare

(1) ROMA TRE UNIVERSITÀ DEGLI STUDI



Un più semplice modello di sapore per i neutrini: il revival del gruppo modulare S_3

Matteo Parriciatu (1)(2)

Basato su: JHEP09(2023)043, in collaborazione con D. Meloni (1)(2)

IFAE, Incontri di Fisica delle Alte Energie, Firenze 2024

3-5 Aprile 2024

Paradigma $3\,\nu$

$$\underbrace{|\nu_\alpha\rangle}_{\text{Base di interazione}} = \sum_{i=1}^{3} U^*_{\alpha i} \underbrace{|\nu_i\rangle}_{\text{Base di massa}}$$

$\theta_{12}\ \theta_{13}\ \theta_{23}$

δ_{CP} $\alpha_1\,,\alpha_2$

Majorana

$$U_{\mathrm{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \times \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)$$

$c_{ij} \equiv \cos\theta_{ij}\ ,\ s_{ij} \equiv \sin\theta_{ij}$

Pontecorvo - Maki - Nakagawa - Sakata

$$P(\nu_\alpha \to \nu_\beta) = \sum_{i=1}^{3}\sum_{j=1}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \exp\left(-i\frac{\Delta m^2_{ij} L}{2E}\right)$$

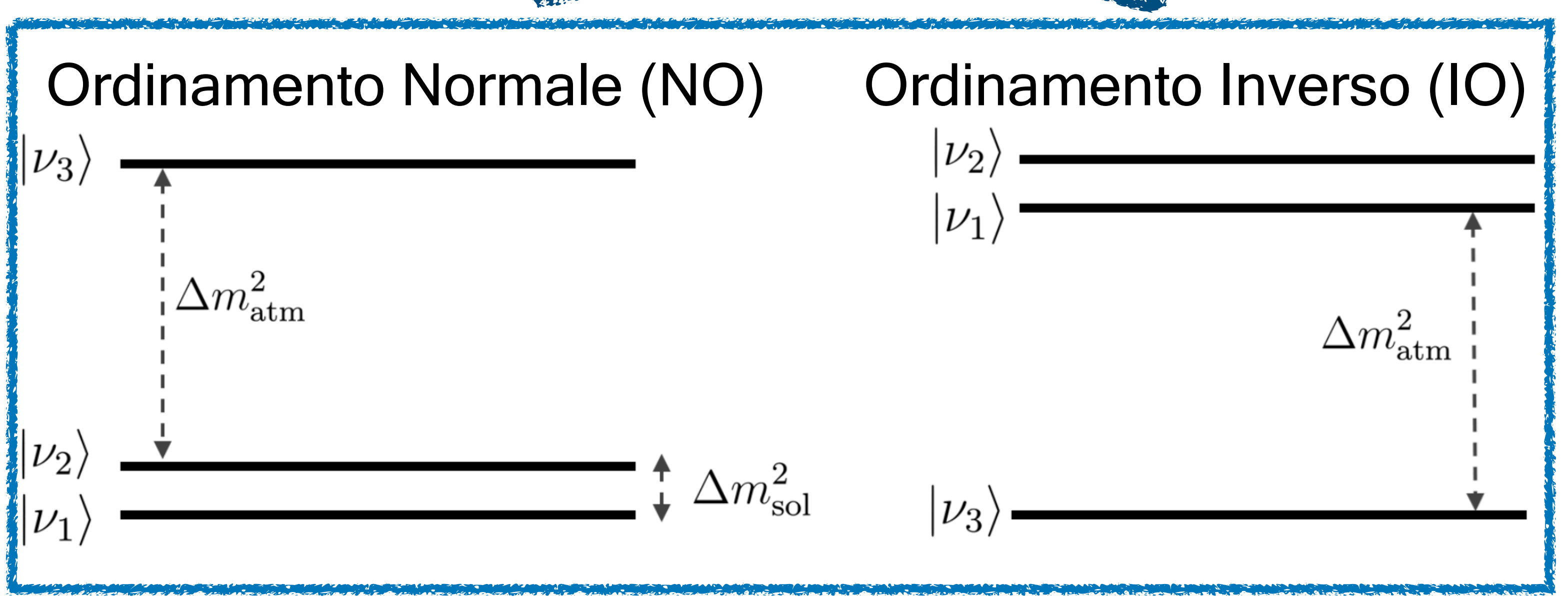
Segno determinato da effetto MSW

$|\Delta m^2_{\text{sol}}| \equiv |m_2^2 - m_1^2|$ $\qquad$ $|\Delta m^2_{\text{atm}}| \equiv |m_3^2 - m_{1,2}^2|$

$$r \equiv \frac{\Delta m^2_{\text{sol}}}{|\Delta m^2_{\text{atm}}|} \approx 1/30$$

Segno indeterminato

?



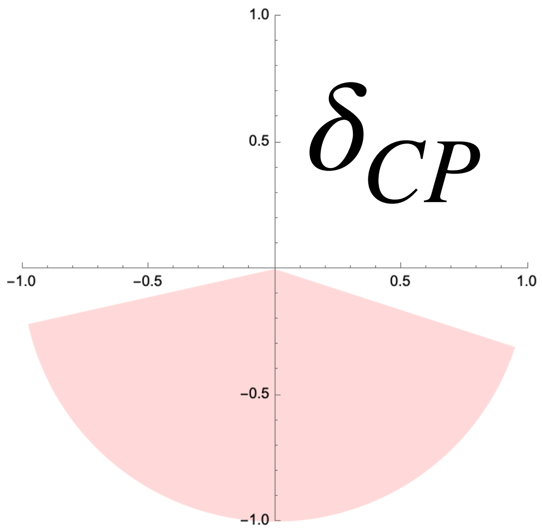
Il puzzle del sapore e il mixing dei neutrini

Valori di best fit

$$\theta_{12} \approx 33.4^\circ \quad , \quad \theta_{13} \approx 8.5^\circ \quad , \quad \theta_{23} \approx 42.4^\circ \qquad (\mathbf{NO})$$
$$\theta_{12} \approx 33.4^\circ \quad , \quad \theta_{13} \approx 8.5^\circ \quad , \quad \theta_{23} \approx 48.9^\circ \qquad (\mathbf{IO})$$

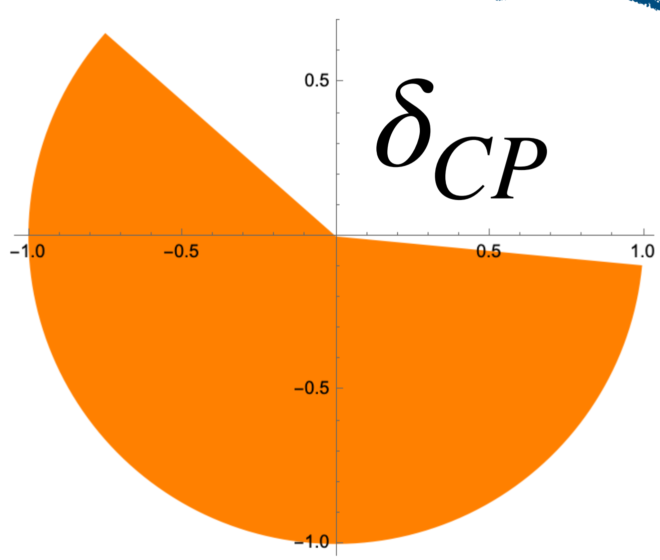
F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

Due grossi mixing uno piccolo



NO

IO

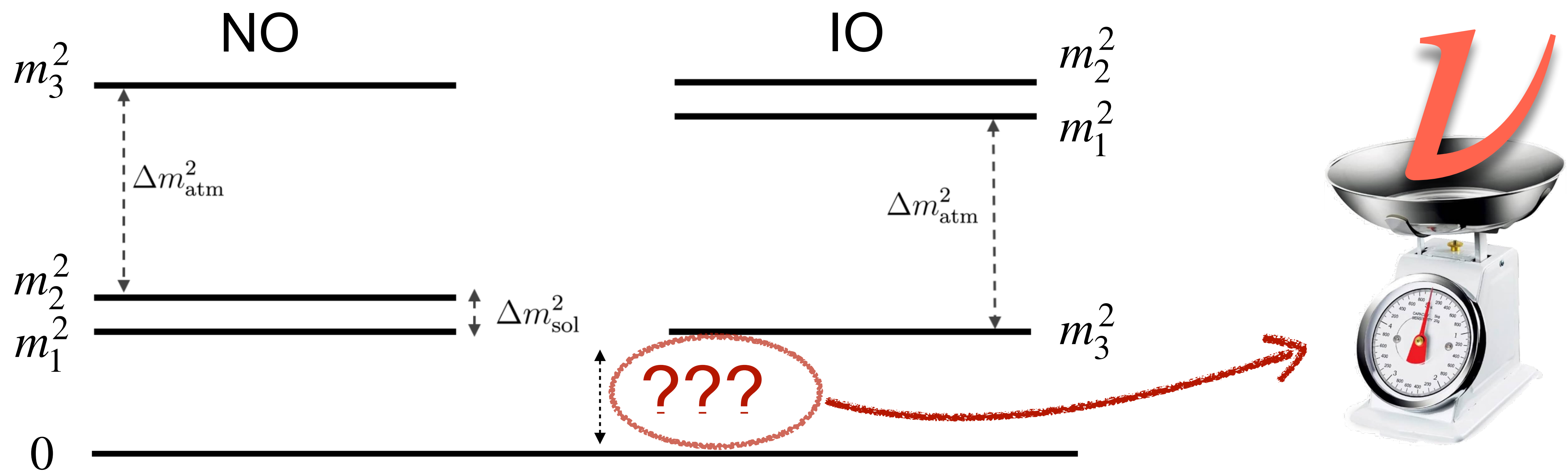


Violazione per

$$\frac{\delta_{CP}}{\pi} \neq \{0,1\}$$

Conservazione CP ancora ammessa a 2σ (NO)

3σ ranges da
F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)



$$\sum_i^3 m_i < 0.115 \text{ eV} \qquad (95\% \text{ C.L.})$$

A. Shadab et al.

Phys. Rev. D **103** (Apr, 2021)

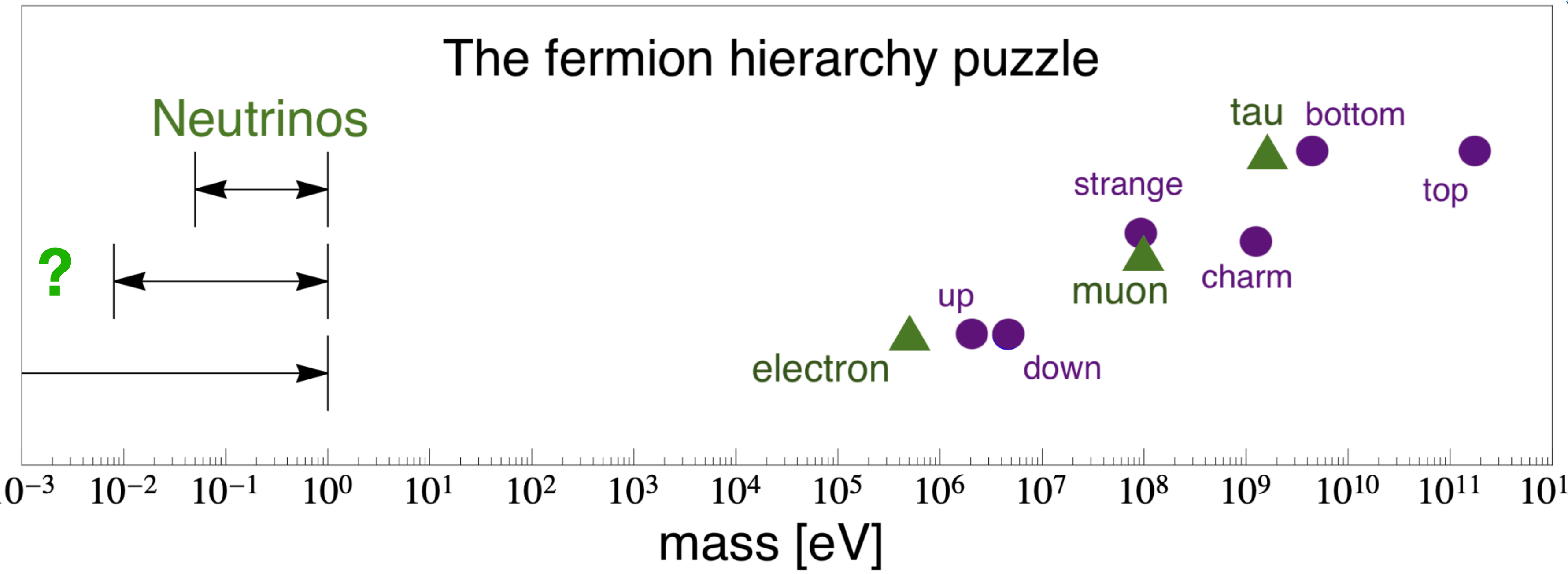
Cosmologia

$$\sqrt{\sum_i m_i^2 |U_{ei}|^2} < 0.8 \text{ eV}$$

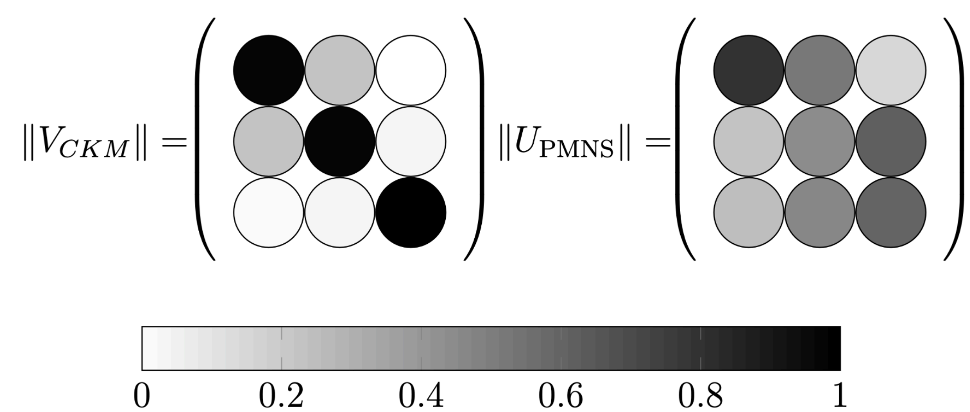
Nat. Phys. **18** (Feb, 2022)

KATRIN

▲ leptons
● quarks



$$m_e/m_\mu \simeq \frac{1}{200} \qquad m_\mu/m_\tau \simeq \frac{1}{17}$$



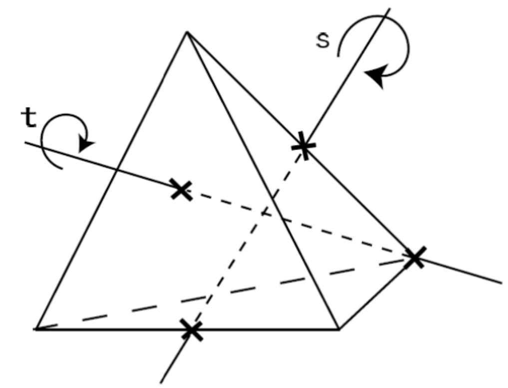
D. Meloni [1709.02662v2]

Tre “copie” con masse diverse!

ν_e	ν_μ	ν_τ
e	μ	τ

$S_3 \quad A_4 \quad S_4 \quad A_5 \ldots\ldots$

Simmetrie discrete non-abeliane?



Isidor I. Rabi

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-bimassimale compatibile con i dati fino al 2012

$$\theta_{13} = 0 \quad \theta_{12} \simeq 35^\circ \quad \theta_{23} = 45^\circ$$

↓

Non nullo

F. P. An *et al.*, “Observation of electron-antineutrino disappearance at daya bay,” *Phys. Rev. Lett.* **108** (Apr, 2012)

Problematiche dell’approccio tradizionale

EFT con scalari “flavoni” ϕ_i

$$\mathscr{W}_{Yukawa} \supset \frac{\alpha}{\Lambda} E^c (L\phi_i)_{\mathbf{1}} H_d$$

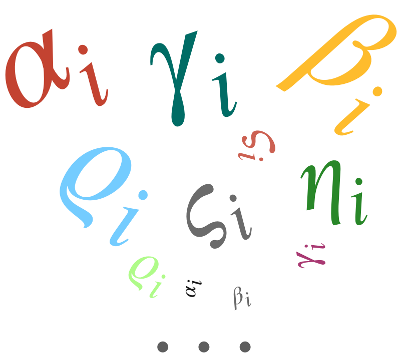
❌ $V(\phi_i) \to$ Complicato!

❌ $U_{\text{PMNS}} = U^0_{\text{TBM}} + \ldots \text{correzioni}$

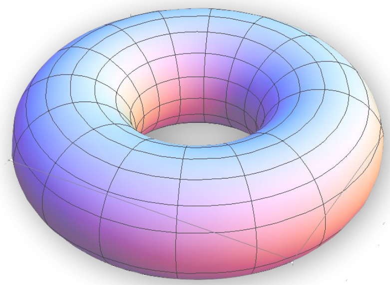
$\theta_{13} = 0$ — Automatico

$\theta_{13} \approx 8.5^\circ$?

$\to \delta\theta_{23}\ ,\ \delta\theta_{12} \approx 8.5^\circ$

❌ 

F. Feruglio
[1706.08749]



$\tau \equiv$ modulus

$$(y_{jk}\bar{L}_j H \ell_{Rk} + \text{h.c.})$$

Nel MS sono parametri liberi senza struttura

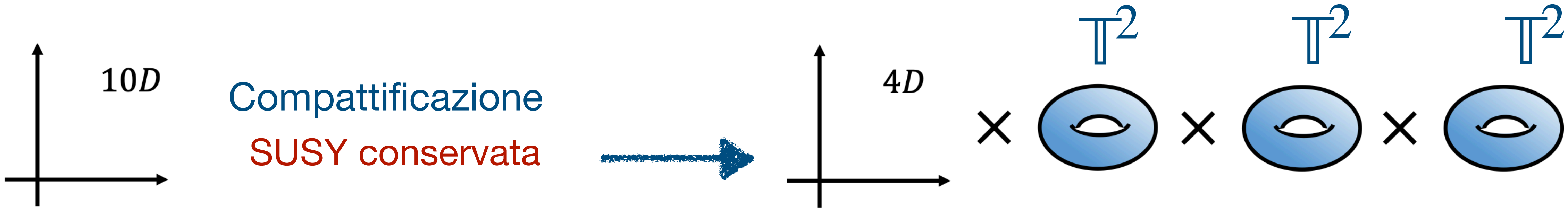
Simmetria modulare

$$Y(\tau)$$

Forme modulari: meno parametri liberi

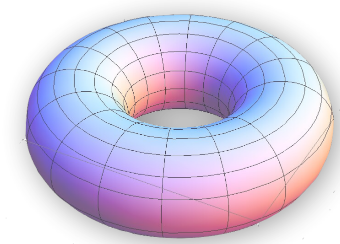
Predittività ✓

User-friendly ✗



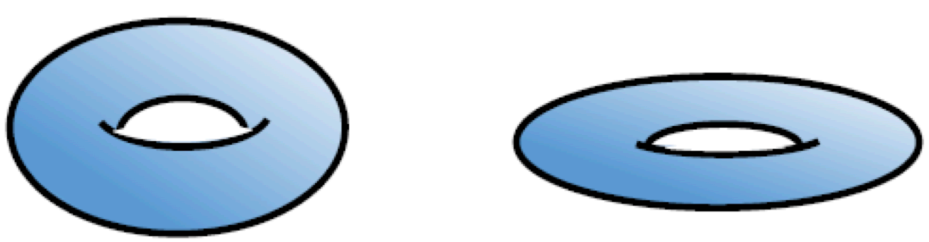
Reticolo discreto Λ

$\omega_1, \omega_2 \in \mathbb{C}$

$\mathbb{C}/\Lambda \equiv$ 

$$\tau \equiv \frac{\omega_2}{\omega_1}$$

Modulus



$\tau = \tau_1$ $\tau = \tau_2$

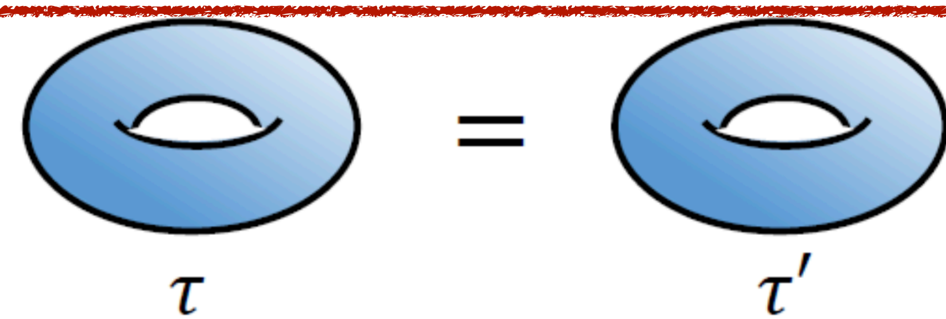
E se cambiamo base? $a, b, c, d \in \mathbb{Z}$

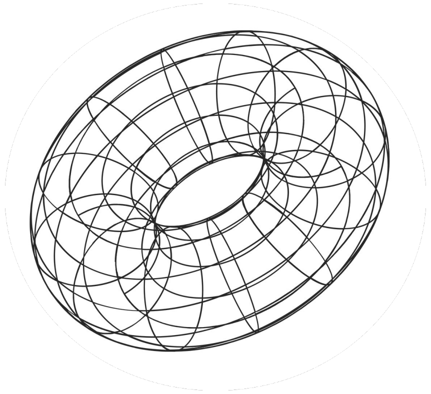
$$\begin{pmatrix} \omega_2' \\ \omega_1' \end{pmatrix} = \gamma \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix}$$

$\gamma \in$ SL$(2,\mathbb{Z}) \equiv$ Gruppo modulare $\equiv \Gamma$

$$\tau \equiv \frac{\omega_2}{\omega_1} \xrightarrow{\text{SL}(2,\mathbb{Z})} \tau' = \frac{a\,\omega_2 + b\,\omega_1}{c\,\omega_2 + d\,\omega_1} = \frac{a\tau + b}{c\tau + d}$$

$\tau \to \tau'$





$$\int d^4x\, d^6y\, \mathscr{L}_{10D} \Longrightarrow \int d^4x\, \mathscr{L}_{EFT}$$

Cosa è una forma modulare? $Y(\tau)$

- $Y(\gamma(\tau)) = (c\tau + d)^k\, Y(\tau)$

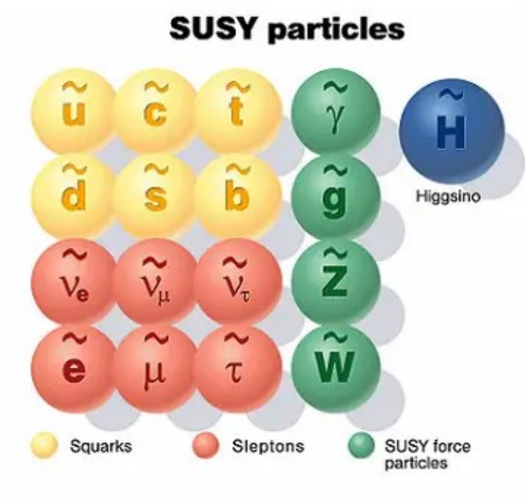
$c, d \in \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \gamma$

$a, b, c, d \in \mathbb{Z} \quad , \quad ad - bc = 1$

Olomorfa in: $\{\tau \in \mathbb{C} \,|\, \mathrm{Im}(\tau) > 0\}$

- “Peso” k>0

Condizione molto vincolante!



Come trasformano i supercampi

$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} & \gamma \in \Gamma \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}$$

$\varphi^{(I)}$: Campi di materia usuali

$\rho^{(I)}(\gamma)$: Rapp. unitaria dif $\Gamma_N \subset \Gamma$

$N = 1, 2, 3, \ldots$ "Livello"

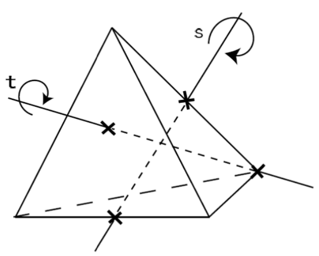
Gruppo modulare finito

$$\Gamma_N$$

per $N \leq 5$ isomorfo a

$N = 2$	$N = 3$	$N = 4$	$N = 5$
S_3	A_4	S_4	A_5

gruppi discreti non-abeliani



- $\mathcal{W}(\Phi) = \sum (Y_{I_1 \ldots I_n}(\tau)\, \varphi^{(I_1)} \ldots \varphi^{(I_n)})_{\mathbf{1}}$

$\mathscr{W}(\Phi)$ è invariante modulare se:

$$\begin{cases} \rho \otimes \rho_{I_1} \otimes \rho_{I_2} \ldots \otimes \rho_{I_n} \supset \mathbf{1} \\ k_Y = k_{I_1} + k_{I_2} + \ldots + k_{I_n} \end{cases}$$

$\rho \otimes \rho_{I_1} \otimes \ldots \supset \mathbf{1}$ ⟶ Usuale

$k_Y = k_{I_1} + \ldots + k_{I_n}$ ⟶ Novità

$$Y_{I_1 \ldots I_n}(\tau) \to (c\tau + d)^{k_Y} \rho(\gamma) Y_{I_1 \ldots I_n}(\tau)$$

Yukawa: forme modulari di peso k_Y

$$\varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)}$$

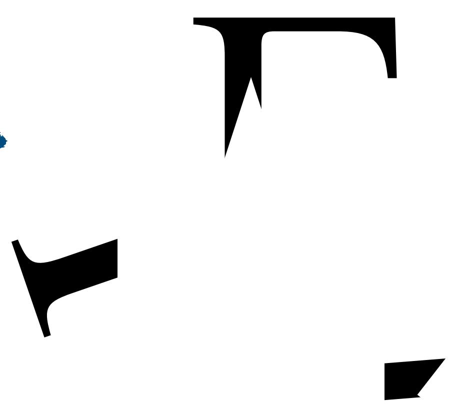
Supercampi di cariche modulari $-k_I$

$$(c\tau + d)^{k_Y} (c\tau + d)^{-\sum k_{I_n}} = 1$$

Rottura della simmetria

$$\Gamma$$

$$< \tau > \equiv \text{VEV}$$



$$Y(\tau)$$

determinate univocamente da τ

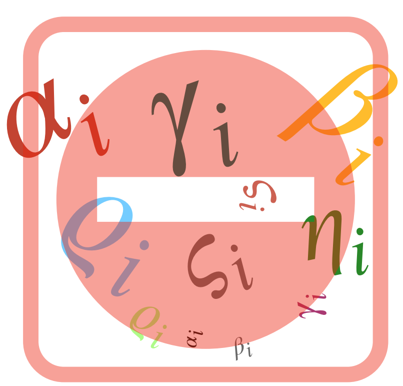
$$\text{Fourier} \sim \sum_{n}^{\infty} a_n q^n$$

$$q \equiv e^{2\pi i\, \tau}$$

Matrici di massa dei leptoni

$$M_e \sim \sum_i \alpha_i \begin{pmatrix} f_{11}(\tau) & f_{12}(\tau) & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & f_{33}(\tau) \end{pmatrix}$$

$f_{ij} \equiv$ funzioni pre-determinate di τ



$\alpha_i \equiv$ numero limitato di parametri liberi ✓

Un numero considerevole di modelli funzionanti dal 2018

$N = 2$

S_3	A_4	S_4	A_5	…
✗	✓	✓	✓	

T. Kobayashi, K. Tanaka, T.H. Tatsuishi
Phys. Rev. D **98** (Jul, 2018)

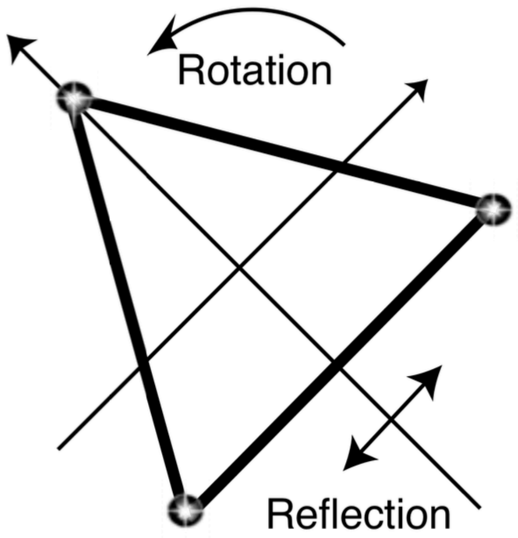
- 

- Flavoni extra
- Gerarchia dei leptoni carichi “fine-tuned”

$N = 2$

S_3

Gruppo di permutazione di 3 oggetti



- Più piccolo gruppo discreto non-abeliano ✓

- Tre rappresentazioni irriducibili

$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$
singoletto	pseudo-singoletto	doppietto

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \to \notin \mathbf{2}\ (S_3)$$

Una base per le forme modulari di peso k

- $\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)$ ⟶ $Y_1(\tau), Y_2(\tau) \sim \mathbf{2}$

Funzione eta di Dedekind $(q \equiv e^{2\pi i \tau})$

Peso più basso: k=2

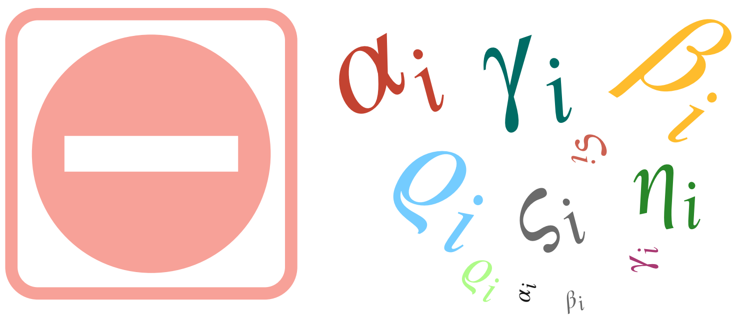
$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}} \to (c\tau + d)^2 \rho(\gamma)_{\mathbf{2}} \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$$

Un numero molto limitato! ✔

-

N	$d_k(\Gamma(N))$	Γ_N
2	$k/2+1$	S_3

$$\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1} \quad , \quad \mathbf{1}' \otimes \mathbf{2} = \mathbf{2} \quad , \quad \mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$$



Imporre simmetria CP nel modello

- gCP $\Rightarrow$ $\alpha_i \in \mathbb{R}$

P. Novichkov, J. Penedo, S. Petcov, A. Titov

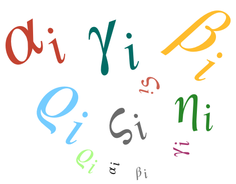
Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

$$\tau = \text{Re}\,\tau + i\,\text{Im}\,\tau$$

Unica sorgente di CPV del modello è il VEV del modulus τ

I princìpi guida

Flavoni oltre τ (modulus)



$$\frac{y_e}{y_\mu} \sim \mathcal{O}(10^{-3})$$

$$Y_1(\tau) = \frac{7}{100} + \frac{42}{25}q + \frac{42}{25}q^2 + \frac{168}{25}q^3 + \ldots$$

$$Y_2(\tau) = \frac{14\sqrt{3}}{25}q^{1/2}(1 + 4q + 6q^2 + \ldots)$$

Sviluppo di Fourier $(q \equiv e^{2\pi i \tau})$

$\longrightarrow$ $|Y_2(\tau)| \lesssim |Y_1(\tau)|$

$\mathrm{Im}\,\tau \gtrsim 1$

Seguendo i princìpi guida...

$D_\ell \equiv \begin{pmatrix}\text{elettrone}\\ \text{muone}\end{pmatrix} \sim \mathbf{2}$ $\qquad$ $\ell_3 \equiv \text{tau} \sim \mathbf{1}'$

		E_1^c	E_2^c	E_3^c	D_ℓ	ℓ_3	$H_{d,u}$
	$SU(2)_L \times U(1)_Y$	$(\mathbf{1}, +1)$	$(\mathbf{1}, +1)$	$(\mathbf{1}, +1)$	$(\mathbf{2}, -1/2)$	$(\mathbf{2}, -1/2)$	$(\mathbf{2}, \mp 1/2)$
Irreps	$\Gamma_2 \cong S_3$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{1}'$	$\mathbf{1}$
Pesi	k_I	4	0	-2	2	2	0

Leptoni carichi

$$\mathcal{W}_e^H = \alpha E_1^c H_d (D_\ell Y_{\mathbf{2}}^{(3)})_{\mathbf{1}} + \beta E_2^c H_d (D_\ell Y_{\mathbf{2}})_{\mathbf{1}'} + \gamma E_3^c H_d \ell_3 + \alpha_D E_1^c H_d \ell_3 Y_{\mathbf{1}'}^{(3)}$$

$(m_\tau, m_\mu, m_e) \sim m_\tau (1, |Y_1|, |Y_1^3|)$ $\qquad$ $|Y_1| \sim \mathcal{O}(10^{-2})$

Neutrini di Majorana da operatori di Weinberg

$$\mathcal{W}_\nu^{k_\ell=2} \supset \frac{g}{\Lambda} H_u H_u (D_\ell D_\ell)_{\mathbf{2}} Y_{\mathbf{2}}^{(2)} + \frac{g'}{\Lambda} H_u H_u D_\ell \ell_3 (Y_{\mathbf{2}}^{(2)}) + \\ + \frac{g''}{\Lambda} H_u H_u (D_\ell D_\ell)_{\mathbf{1}} Y_{\mathbf{1}}^{(2)} + \frac{g_p}{\Lambda} H_u H_u \ell_3 \ell_3 (Y_{\mathbf{1}}^{(2)}) \,.$$

- $\Lambda \to$ Scala di nuova Fisica
- $\{g'/g\,, g''/g\,, g_p/g\} \in \mathbb{R}$ parametri liberi adimensionali
- Riproduce mixing e Δm^2 dei neutrini ✓

JHEP 09 (2023) 043
D. Meloni, M.Parriciatu

Il “revival” di S_3: risultati numerici

- Fit VS 6 osservabili adimensionali (no CP)

$$q_j = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, m_e/m_\mu, m_\mu/m_\tau, r\}$$

$$\langle\tau\rangle = \boxed{\mathrm{Re}\,\tau} + i\,\mathrm{Im}\tau$$

Unica sorgente CPV

$$\boxed{q \equiv e^{2\pi i\tau}}$$

Parameter	Best-fit value and 1σ range	
$\Delta m^2_{\rm sol}/(10^{-5}\ {\rm eV}^2)$	$7.36^{+0.16}_{-0.15}$	
	NO	**IO**
$\|\Delta m^2_{\rm atm}\|/(10^{-3}\ {\rm eV}^2)$	$2.485^{+0.023}_{-0.031}$	$2.455^{+0.030}_{-0.025}$
$r \equiv \Delta m^2_{\rm sol}/\|\Delta m^2_{\rm atm}\|$	0.0296 ± 0.0008	0.0299 ± 0.0008
$\sin^2\theta_{12}$	$0.303^{+0.013}_{-0.013}$	$0.303^{+0.013}_{-0.013}$
$\sin^2\theta_{13}$	$0.0223^{+0.0007}_{-0.0006}$	$0.0223^{+0.0006}_{-0.0006}$
$\sin^2\theta_{23}$	$0.455^{+0.018}_{-0.015}$	$0.569^{+0.013}_{-0.021}$
$\delta_{\rm CP}/\pi$	$1.24^{+0.18}_{-0.13}$	$1.52^{+0.14}_{-0.15}$
m_e/m_μ	0.0048 ± 0.0002	
m_μ/m_τ	0.0565 ± 0.0045	

F. Capozzi et al.
Phys. Rev. D **104** (Oct, 2021)

$\sin^2\theta_{12}$ ✓

$\sin^2\theta_{13}$ ✓

$\sin^2\theta_{23}$ ✓

$r = \dfrac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}$ ✓

Tutti riprodotti entro 1σ sperimentale

10 (6) parametri

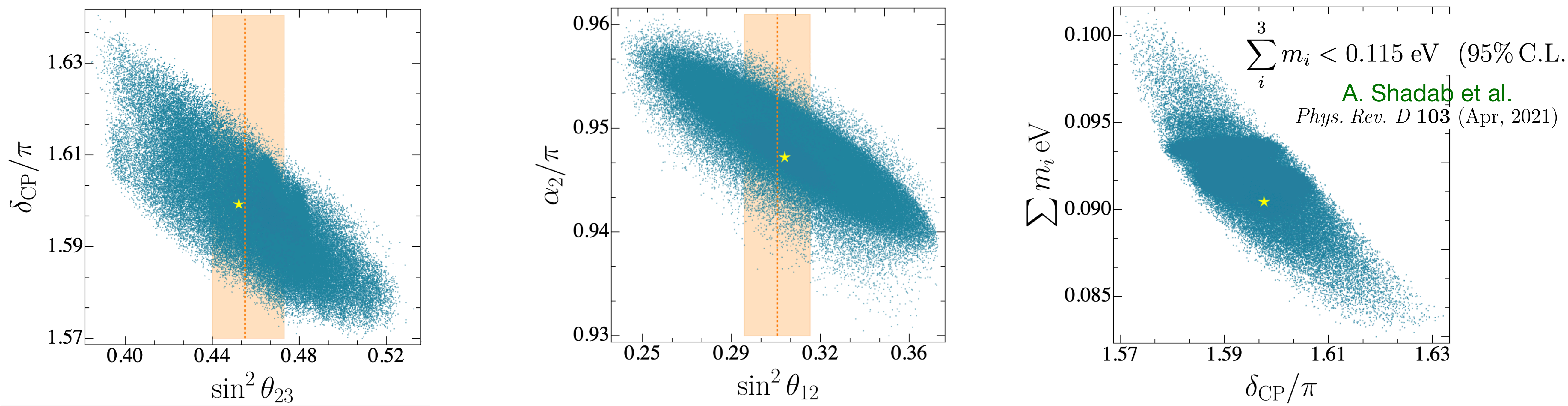
VS

12 (9) osservabili

Predizioni del modello

- Predizione: Ordinamento Normale per i neutrini
- Predizione: fase CP vicino a massima violazione $\delta_{CP} \sim 1.6\pi$
- Correlazioni e scale di massa dei neutrini

VEV del modulus
$\tau \sim \pm 0.09 + 1.7i$



★ Punto di minimo χ^2

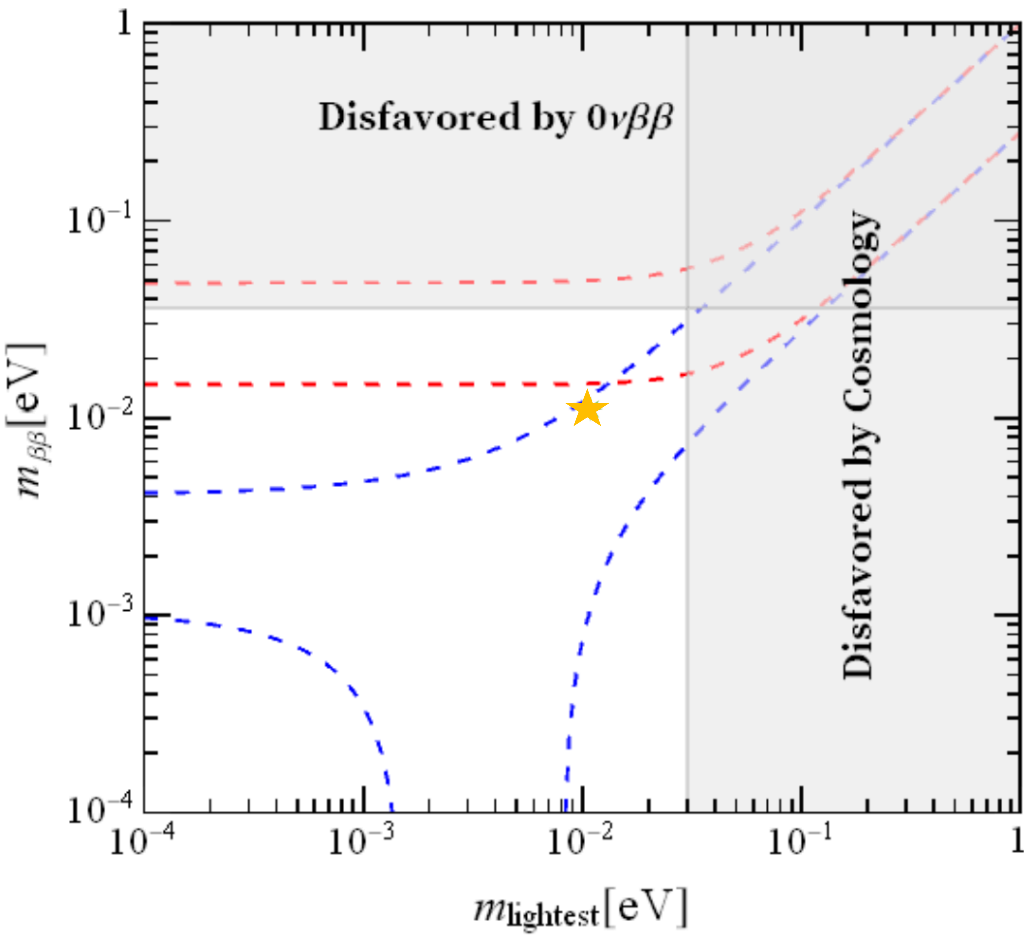
Predizioni del modello

- Precise predizioni su masse dei neutrini e altre osservabili di interesse sperimentale

	Best fit $\pm 1\sigma$
m_1 [eV]	$0.0174^{+0.0011}_{-0.0014}$
m_2 [eV]	$0.0194^{+0.0010}_{-0.0012}$
m_3 [eV]	$0.0535^{+0.0004}_{-0.0004}$
$\sum_i m_i$ [eV]	$0.090^{+0.002}_{-0.003}$
$\|m_{\beta\beta}\|$ [meV]	$18.14^{+1.17}_{-1.48}$
m_β^{eff} [meV]	$19.60^{+1.02}_{-1.25}$
α_1/π	$\pm 1.129^{+0.019}_{-0.013}$
α_2/π	$\pm 0.946^{+0.004}_{-0.004}$

Doppio decadimento beta senza neutrini

Decadimento beta (e.g. KATRIN)

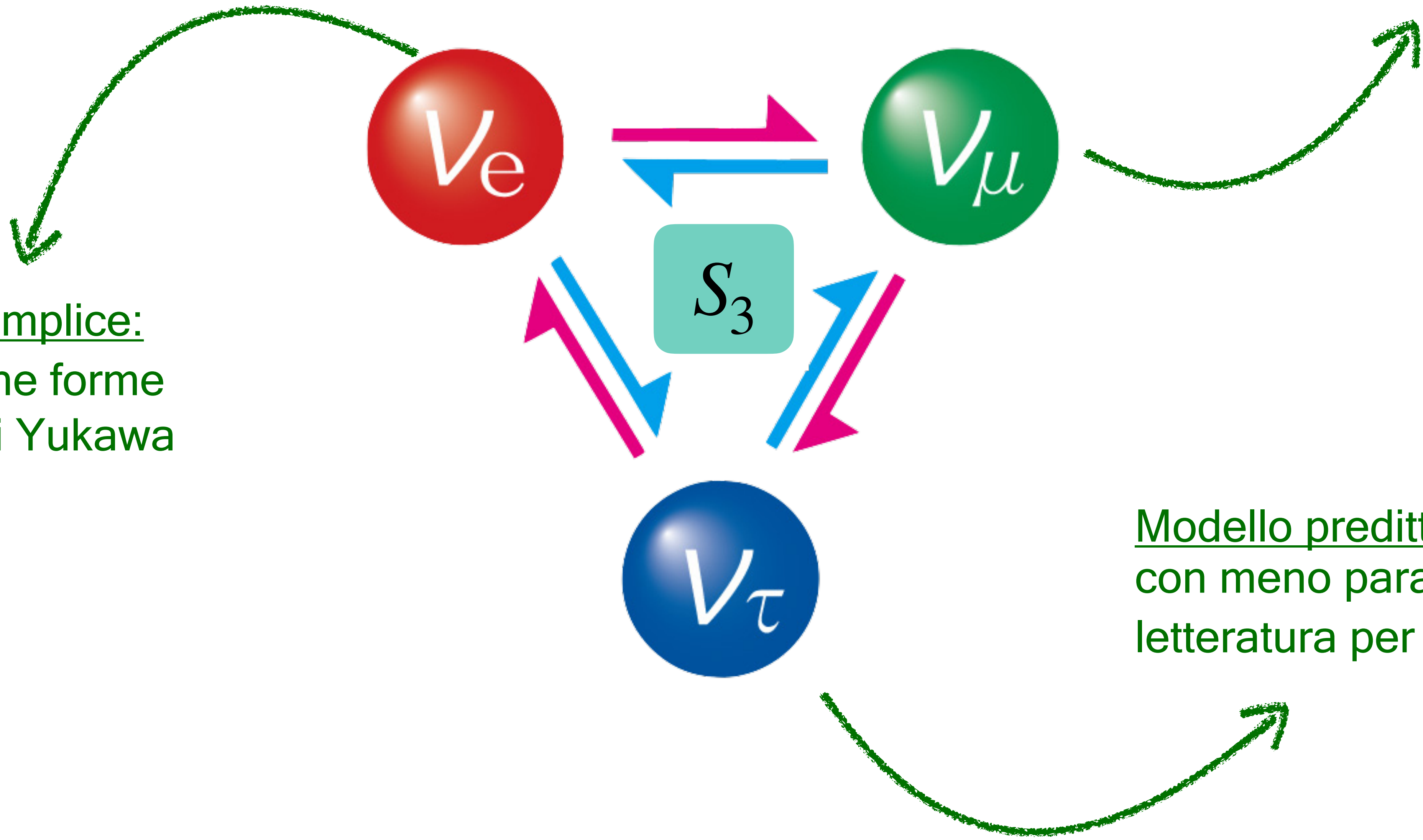


$\langle m_{\beta\beta}\rangle < (36 - 156)\,\text{meV}$ KamLAND-Zen
Phys. Rev. Lett. **130** (Jan, 2023)

★ Punto di minimo χ^2

Il “revival” di S_3: conclusioni

Quarks con S_3?



<u>Modello semplice:</u>
$\Gamma_2 \to$ poche forme modulari di Yukawa

<u>Modello predittivo:</u>
con meno parametri in letteratura per $\Gamma_2 \cong S_3$

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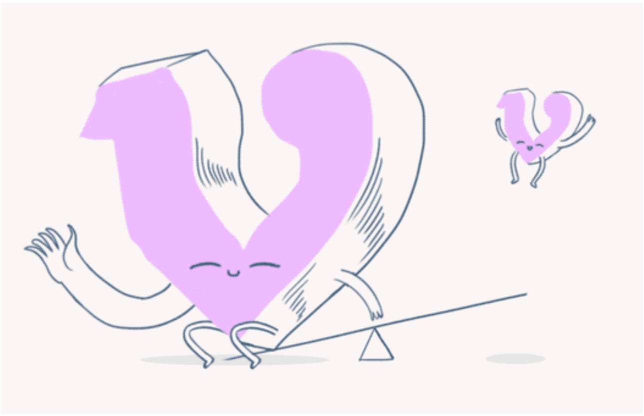
Grazie per l'attenzione

BACKUP

But what about a $\Gamma_2 \simeq S_3$ seesaw version?

Recent work with S.Marciano, D.Meloni: arxiv:2402.18547



- Reproduces low-energy CP-violation and matter-antimatter asymmetry of the Universe through Leptogenesis

Introduce Minimal seesaw scenario with only 2 RHN transforming as $\sim \mathbf{2}$ under S_3 with weight 2

$$\mathcal{W}_\nu = gH_uN^cD_\ell Y_{\mathbf{2}}^{(2)} + g'H_u(N^cY_{\mathbf{2}}^{(2)})_{\mathbf{1}'}\ell_3 + g''H_u(N^cD_\ell)_{\mathbf{1}}Y_{\mathbf{1}}^{(2)} + \\ +\Lambda[(N^cN^c)_{\mathbf{2}}Y_{\mathbf{2}}^{(2)} + \lambda(N^cN^c)_{\mathbf{1}}Y_{\mathbf{1}}^{(2)}]\,,$$

$$M_D = gv_u\begin{pmatrix} -(Y_2^2-Y_1^2)+\frac{g''}{g}(Y_1^2+Y_2^2) & 2Y_1Y_2 & \frac{g'}{g}(2Y_1Y_2) \\ 2Y_1Y_2 & (Y_2^2-Y_1^2)+\frac{g''}{g}(Y_1^2+Y_2^2) & -\frac{g'}{g}(Y_2^2-Y_1^2)\end{pmatrix}_{\mathrm{RL}}$$

$$m_\nu = -M_D^T\mathcal{M}_R^{-1}M_D\,.$$

$$\mathcal{M}_R = \Lambda\begin{pmatrix} -(Y_2^2-Y_1^2)+\lambda(Y_1^2+Y_2^2) & 2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2-Y_1^2)+\lambda(Y_1^2+Y_2^2)\end{pmatrix}_{\mathrm{RR}}\,.$$

- Massless neutrino in the spectrum, Normal Ordering
- Excellent fit: $\chi^2 \sim 0.98$ (but now δ_{CP} is fitted)

Backup slides: Clebsch-Gordan

- Clebsch-Gordan coefficients for S_3

$$\mathbf{2}\otimes\mathbf{2}=\mathbf{1}\otimes\mathbf{1}'\otimes\mathbf{2}\begin{cases}\mathbf{1} & \sim \quad \psi_1\varphi_1+\psi_2\varphi_2\\ \mathbf{1}' & \sim \quad \psi_1\varphi_2-\psi_2\varphi_1\\ \mathbf{2} & \sim \quad \begin{pmatrix}\psi_2\varphi_2-\psi_1\varphi_1\\ \psi_1\varphi_2+\psi_2\varphi_1\end{pmatrix}\end{cases}$$

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}_{\mathbf{2}} \qquad \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}_{\mathbf{2}}$$

S_3 doublets

$$\mathbf{1}'\otimes\mathbf{1}'=\mathbf{1} \quad \sim \quad y_1y_2$$

$$\mathbf{1}'\otimes\mathbf{2}=\mathbf{2} \quad \sim \quad \begin{pmatrix}-y_1\psi_2\\ y_1\psi_1\end{pmatrix}$$

$y_1, y_2 \equiv$ pseudo-singlets $(\mathbf{1}')$

e.g. $(D_\ell Y_{\mathbf{2}}(\tau))_{\mathbf{1}'}=(D_\ell)_1(Y_{\mathbf{2}}(\tau))_2-(D_\ell)_2(Y_{\mathbf{2}}(\tau))_1\sim\mathbf{1}'$

Modular symmetry vs traditional: an example

Non-Abelian discrete flavour symmetry

$$\mathscr{W}_L \supset \frac{\alpha}{\Lambda} E_1^c (L\varphi)_{\mathbf{1}} H_d$$

$$\varphi \sim \mathbf{2} = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

Free parameters

Structure dependent on the symmetry breaking sector

Modular flavour symmetry

$$\mathscr{W}_L \supset \alpha E_1^c [L Y_{\mathbf{2}}(\tau)]_{\mathbf{1}} H_d$$

$$Y_{\mathbf{2}}(\tau) \sim \mathbf{2} = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}$$

Completely fixed by modular group

The only unknown is the complex VEV of τ

The Modular symmetry approach

Modular-invariant SUSY action

$$\mathcal{S} = \int d^4x \int d^2\theta d^2\bar{\theta}\, K(\Phi, \bar{\Phi}) + \left[\int d^4x \int d^2\theta\, \mathcal{W}(\Phi) + \text{h.c.}\right]$$

Kähler potential

- Gives the kinetic terms after the modulus acquires a VEV
- A minimalistic form is chosen

Superpotential

- Holomorphic function of superfields
- Encodes the Higgs Yukawa interactions

$\sigma \equiv \Lambda_\tau \tau$

The superfields transform as:

$$\begin{cases} \tau \to \gamma(\tau) = \dfrac{a\tau + b}{c\tau + d} \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}, \quad \text{with} \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_N$$

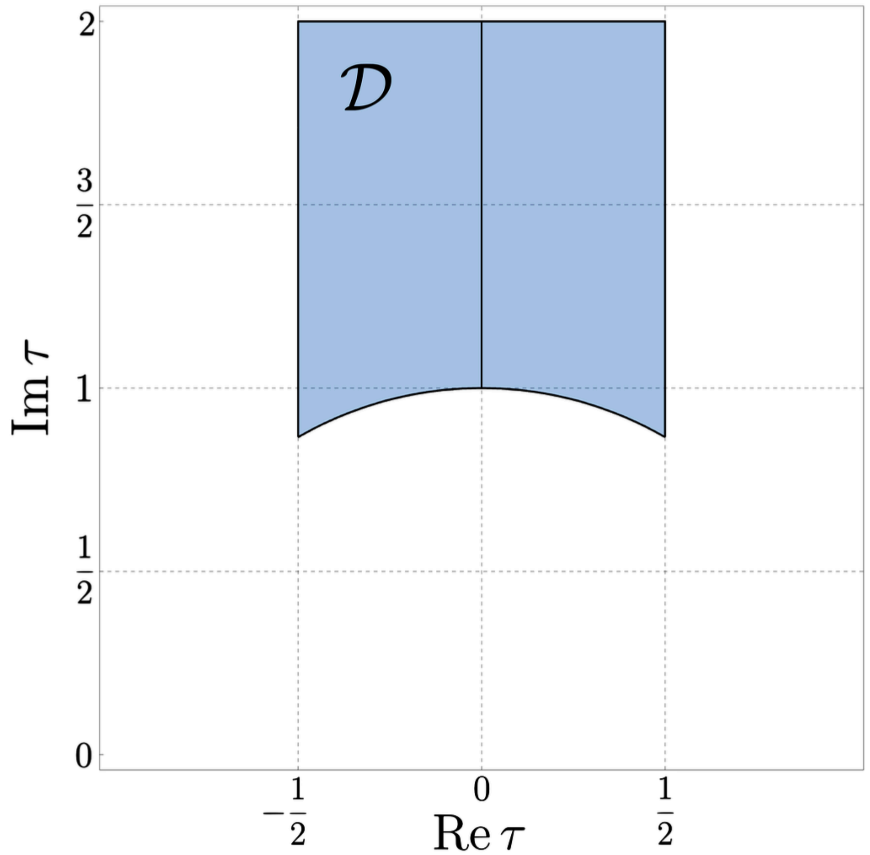
Action is invariant if, under Γ_N:

$$\begin{cases} \mathcal{W}(\Phi) \to \mathcal{W}(\Phi) \\ K(\Phi, \bar{\Phi}) \to \underbrace{K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi})}_{\text{Kähler transformation}} \end{cases}$$

$\theta, \bar{\theta}$ Grassmann spinor coordinates
$\Phi = (\tau, \varphi)$ Chiral superfields
φ Usual matter supermultiplets
$\rho(\gamma)$ Unitary representation of Γ_N

Backup slides

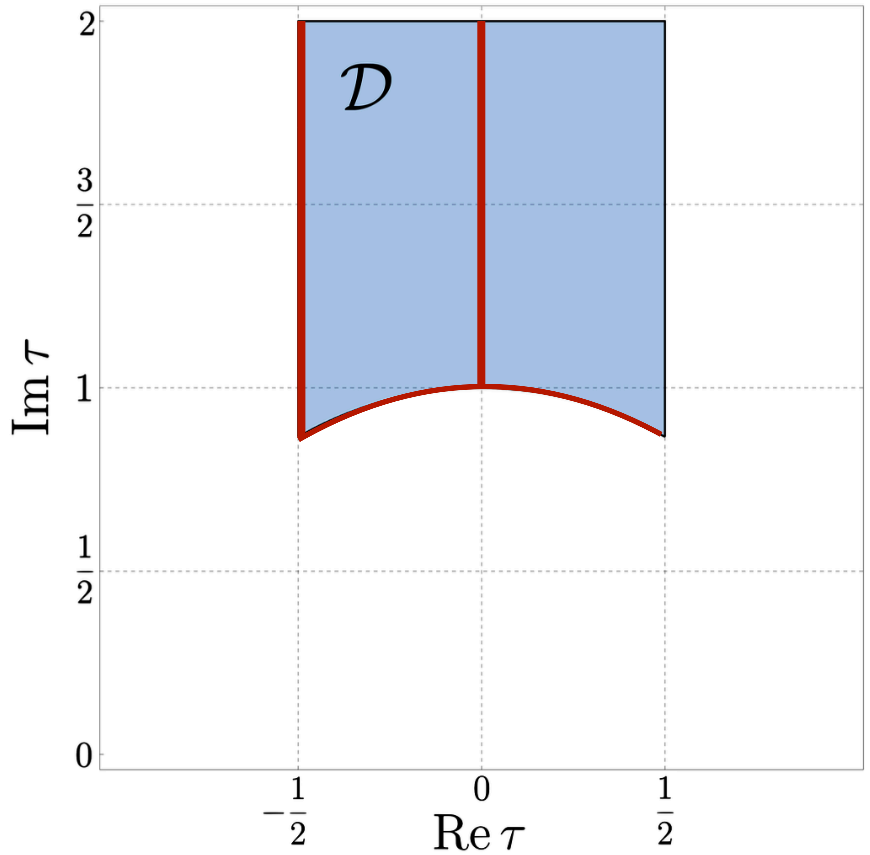
- The fundamental domain $$\mathcal{D} = \left\{ \tau \in \mathbb{C} \,:\, \mathrm{Im}\,\tau > 0\,,\;\; |\mathrm{Re}\,\tau| \leq \frac{1}{2}\,,\;\; |\tau| \geq 1 \right\}$$



- Every VEV outside this domain can be mapped here through modular symmetry
- In our case, even a small departure from imaginary axis results in sizeable CP-violating phase

Backup slides

- The fundamental domain $$\mathcal{D} = \left\{ \tau \in \mathbb{C} \,:\, \mathrm{Im}\,\tau > 0\,,\;\; |\mathrm{Re}\,\tau| \leq \frac{1}{2}\,,\;\; |\tau| \geq 1 \right\}$$



- Every VEV outside this domain can be mapped here through modular symmetry

- CP conserving values

The Kähler potential...

- Minimalist choice for the Kähler

$h \equiv$ positive constant $\Lambda_\tau \equiv$ dimensions of mass

$$K(\Phi, \bar{\Phi}) = -h\Lambda_\tau^2 \log(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\varphi^{(I)}|^2$$

Satisfies $K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + f(\Phi) + f(\bar{\Phi})$ under Γ_N

- In a bottom-up approach, this is unjustified

- Corrections of the Kähler potential can spoil the predictivity of the model

M.-C. Chen, S. Ramos-Sánchez, and M. Ratz, "A note on the predictions of models with modular flavor symmetries," *Physics Letters B* **801** (Feb, 2020) 135153.

- This question is an open one

The Modular symmetry approach

The group generators

- Finite modular group can be defined: $\Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$

$$\overline{\Gamma} \equiv \Gamma/\{\pm\mathbb{I}\}$$
$$\overline{\Gamma}(N) \equiv \Gamma(N)/\{\pm\mathbb{I}\}$$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z}) \middle| \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

subgroups of Γ N=1,2,3...called “level”

- Generators S and T of the modular group Γ_N

$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau + 1 \qquad S^2 = T^N = (ST)^3 = \mathbb{I}$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

- S_3 Generators S and T satisfy:

$$S^2 = T^2 = (ST)^3 = \mathbb{I}$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I},$$

The Modular S_3 model: lowest weights

Level 2 modular forms of lowest weight (2) constructed from Dedekind's Eta "seed function"

- $\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \quad , \quad q \equiv e^{2\pi i \tau}$ → $\left\{\eta(\tau/2), \eta\left(\frac{\tau+1}{2}\right), \eta(2\tau)\right\}$

Closed set under the modular group

$$\alpha + \beta + \gamma = 0$$

$$Y(\alpha,\beta,\gamma|\tau) = \frac{d}{d\tau}\left[\alpha \log \eta(\tau/2) + \beta \log \eta((\tau+1)/2) + \gamma \log \eta(2\tau)\right]$$

Impose transformation properties under S_3 generators

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$(\rho(S))^2 = \mathbb{I}, \qquad (\rho(S)\rho(T))^3 = \mathbb{I}, \qquad (\rho(T))^2 = \mathbb{I},$$

This fixes the constants

$$\begin{cases} Y_1(\tau) = \dfrac{C}{2}\left[\dfrac{\eta'(\tau/2)}{\eta(\tau/2)} + \dfrac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)} - \dfrac{8\eta'(2\tau)}{\eta(2\tau)}\right] \\[2ex] Y_2(\tau) = \dfrac{C}{2}\sqrt{3}\left[\dfrac{\eta'(\tau/2)}{\eta(\tau/2)} - \dfrac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)}\right] \end{cases}$$

"C" arbitrary

- $\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}} \rightarrow (c\tau + d)^2 \rho(\gamma)_{\mathbf{2}} \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}}$

The Modular S_3 model: the normalisation

Level 2 modular forms of lowest weight (2) constructed from Dedekind's Eta "seed function"

$$\eta(\tau) \equiv q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \quad , \quad q \equiv e^{2\pi i \tau} \longrightarrow \left\{\eta(\tau/2), \eta\left(\frac{\tau+1}{2}\right), \eta(2\tau)\right\}$$

Closed set under the modular group

"C" arbitrary

$$\begin{cases} Y_1(\tau) = \dfrac{C}{2}\left[\dfrac{\eta'(\tau/2)}{\eta(\tau/2)} + \dfrac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)} - \dfrac{8\eta'(2\tau)}{\eta(2\tau)}\right] \\ Y_2(\tau) = \dfrac{C}{2}\sqrt{3}\left[\dfrac{\eta'(\tau/2)}{\eta(\tau/2)} - \dfrac{\eta'\left(\frac{\tau+1}{2}\right)}{\eta\left(\frac{\tau+1}{2}\right)}\right] \end{cases}$$

Impose CP symmetry on the model

- Superpotential parameters must be real: less free parameters

P. Novichkov, J. Penedo, S. Petcov, A. Titov

Journal of High Energy Physics **2019** no. 7, (Jul, 2019)

$$Y(\tau) \xrightarrow{\text{CP}} Y(-\tau^*) = Y^*(\tau)$$

- In our case, this is true if C is purely imaginary

$$C = \frac{7i}{25\pi}$$

The choice made in this work

Only source of CPV is the VEV of modulus τ

The Modular S_3 model: charged-leptons sector

Found two viable choices for modular charges and weights

$$k_D = k_\ell$$

$$\begin{cases} k_{E_1} = 6 - k_\ell \\ k_{E_2} = 2 - k_\ell \\ k_{E_3} = -k_\ell \end{cases}$$

"Hierarchical"

$$M_\ell^\dagger = \begin{pmatrix} \alpha(Y_{\mathbf{2}}^{(3)})_1 & \alpha(Y_{\mathbf{2}}^{(3)})_2 & \alpha_D Y_{\mathbf{1}'}^{(3)} \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix} v_d$$

Charged-leptons masses reproduced with:

$$\frac{\beta}{\alpha} \sim \frac{\gamma}{\alpha} \sim \frac{\alpha_D}{\alpha} \approx \mathcal{O}(1)$$

$$\begin{cases} k_{E_1} = 4 - k_\ell \\ k_{E_2} = 2 - k_\ell \\ k_{E_3} = -k_\ell \end{cases}$$

"Minimal"

$$M_\ell^\dagger = \begin{pmatrix} \alpha(Y_{\mathbf{2}}^{(2)})_1 & \alpha(Y_{\mathbf{2}}^{(2)})_2 & 0 \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix} v_d$$

Charged-leptons masses reproduced with:

$$\frac{\beta}{\alpha} \sim \frac{\gamma}{\alpha} \approx \mathcal{O}(10)$$

$$\begin{pmatrix} (Y_{\mathbf{2}}^{(3)})_1 \\ (Y_{\mathbf{2}}^{(3)})_2 \end{pmatrix}_{\mathbf{2}} = \begin{pmatrix} Y_1(Y_1^2 + Y_2^2) \\ Y_2(Y_1^2 + Y_2^2) \end{pmatrix}_{\mathbf{2}} \quad , \quad Y_{\mathbf{1}'}^{(3)} = (Y_2^3 - 3Y_1^2 Y_2)_{\mathbf{1}'}$$

$v_d \equiv$ VEV of H_d

Backup slides

Numerical procedure

- Define a "figure of merit", i.e. chi-square for every set of parameters $l(p_i) \equiv \sqrt{\chi^2(p_i)}$

$$\chi^2(p_i) = \sum_{j=1}^{6} \left(\frac{q_j(p_i) - q_j^{\text{b-f}}}{\sigma_j} \right)^2$$

$$p_i = \{\tau, \beta/\alpha, \gamma/\alpha, ..., g'/g, g_p/g, ...\}$$

$$q_j = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, m_e/m_\mu, m_\mu/m_\tau, r\}$$

- Define a "potential" with a given temperature T and a threshold

$$V(p_i) = \begin{cases} l(p_i) & , \quad l(p_i) \leq l_{\max} \\ +\infty & , \quad \text{otherwise} \end{cases}$$

P. P. Novichkov, J. T. Penedo, S. T. Petcov, and A. V. Titov, "Modular S_4 models of lepton masses and mixing," (2019)

- At iteration "t", generate a new point from a Gaussian centred on the previous one →
- Accept the new point with a probability given by:

$$P_\alpha = \min[1, \exp(V(p_i^{(t)}) - V(p_i'))/T]$$

Backup slides

A measure of fine-tuning:
Altarelli-Blankenburg

$$\text{Fine-tuning} = \frac{\sum_i \left| \frac{\text{par}_i}{\delta\text{par}_i} \right|}{\sum_i \left| \frac{\text{obs}_i}{\sigma_i} \right|}$$

- δpar_i The shift of the parameter from the point of minimum, which increases the chi-square by one unit, while keeping all other parameters fixed.
- σ_i Experimental errors for each observable as extracted from F. Capozzi et al. *Phys. Rev. D* **104** (Oct, 2021)
- The model with less fine-tuning, in our case, seems to be the one with the least number of free parameters (model I)

Backup slides

Model I [7]

	Best-fit and 1σ range
$\mathrm{Re}\,\tau$	$\pm 0.0895^{+0.0032}_{-0.0055}$
$\mathrm{Im}\,\tau$	$1.697^{+0.023}_{-0.016}$
β/α	$14.33^{+0.58}_{-0.38}$
γ/α	$17.39^{+1.38}_{-0.87}$
g'/g	$31.57^{+27.59}_{-10.29}$
g''/g	$7.17^{+6.36}_{-2.36}$
g_p/g	$8.51^{+7.99}_{-3.03}$
$v_d\,\alpha$ [MeV]	102.14
$v_u^2\,g/\Lambda$ [eV]	0.47
$\sin^2\theta_{12}$	$0.300^{+0.013}_{-0.006}$
$\sin^2\theta_{13}$	$0.0223^{+0.0004}_{-0.0006}$
$\sin^2\theta_{23}$	$0.452^{+0.015}_{-0.009}$
r	$0.0295^{+0.0007}_{-0.0006}$
m_e/m_μ	$0.0048^{+0.0001}_{-0.0002}$
m_μ/m_τ	$0.0578^{+0.0023}_{-0.0040}$
Ordering	**NO**
δ/π	$\pm 1.594^{+0.007}_{-0.010}$
m_1 [eV]	$0.0182^{+0.0018}_{-0.0014}$
m_2 [eV]	$0.0201^{+0.0017}_{-0.0013}$
m_3 [eV]	$0.0537^{+0.0006}_{-0.0005}$
$\sum_i m_i$ [eV]	$0.092^{+0.004}_{-0.003}$
$\langle m_{\beta\beta}\rangle$ [meV]	$18.89^{+1.90}_{-1.47}$
m_β^{eff} [meV]	$20.26^{+1.69}_{-1.30}$
α_1/π	$\pm 1.124^{+0.014}_{-0.017}$
α_2/π	$\pm 0.949^{+0.005}_{-0.005}$
Fine-tuning	12.2
χ^2_{min}	0.16

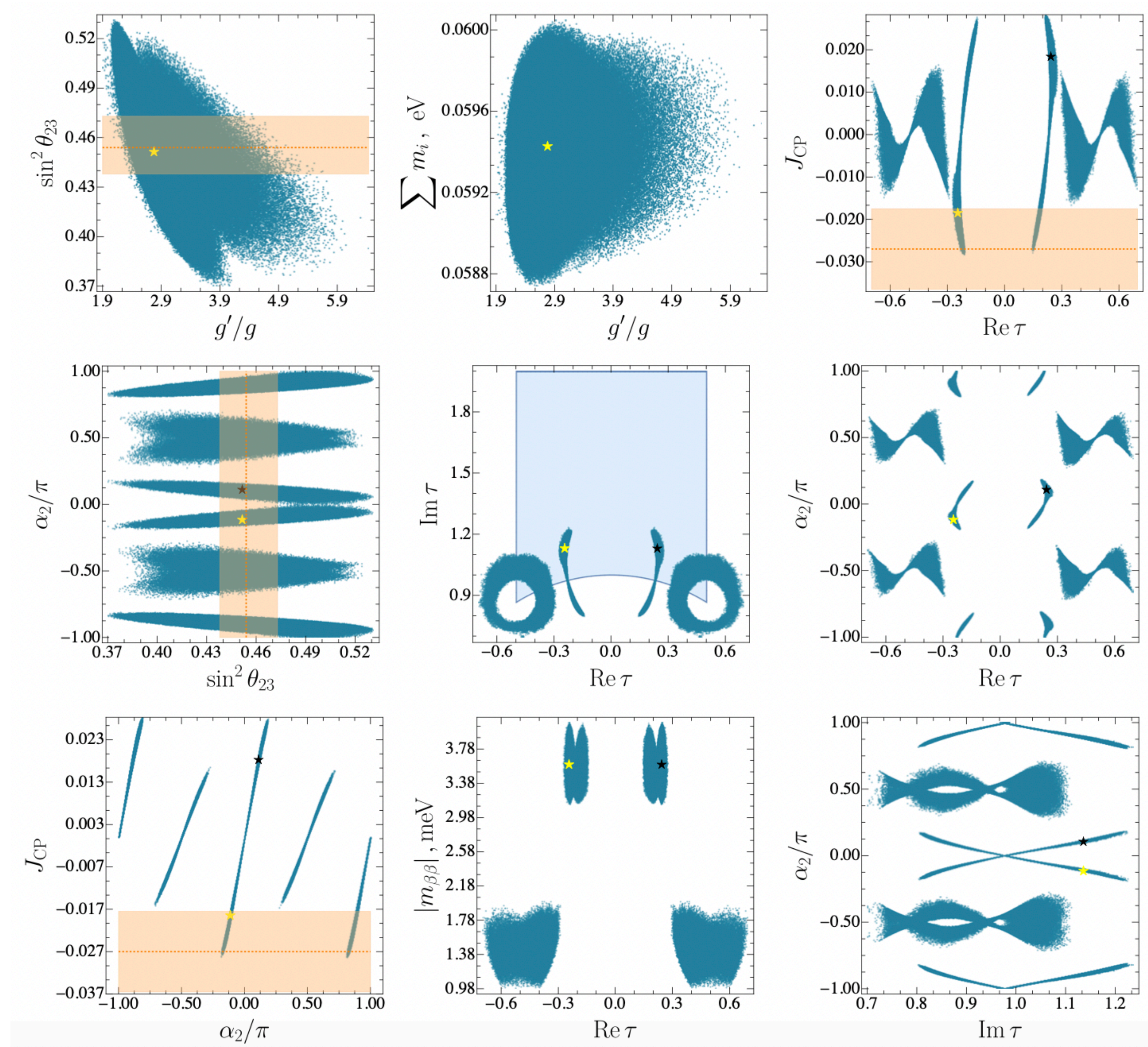
Model II [8]

	Best-fit and 1σ range
$\mathrm{Re}\,\tau$	$\pm 0.090^{+0.004}_{-0.004}$
$\mathrm{Im}\,\tau$	$1.688^{+0.015}_{-0.018}$
β/α	$1.03^{+0.04}_{-0.04}$
γ/α	$1.26^{+0.12}_{-0.08}$
α_D/α	$1.33^{+1.51}_{-1.05}$
g'/g	$41.9^{+73.7}_{-12.8}$
g''/g	$9.55^{+16.81}_{-2.91}$
g_p/g	$11.5^{+21.2}_{-3.8}$
$v_d\,\alpha$ [MeV]	1404.6
$v_u^2\,g/\Lambda$ [eV]	0.35
$\sin^2\theta_{12}$	$0.305^{+0.009}_{-0.015}$
$\sin^2\theta_{13}$	$0.0222^{+0.0007}_{-0.0006}$
$\sin^2\theta_{23}$	$0.454^{+0.007}_{-0.008}$
r	$0.0295^{+0.0007}_{-0.0007}$
m_e/m_μ	$0.0048^{+0.0002}_{-0.0002}$
m_μ/m_τ	$0.0570^{+0.0034}_{-0.0048}$
Ordering	**NO**
δ/π	$\pm 1.597^{+0.009}_{-0.006}$
m_1 [eV]	$0.0174^{+0.0011}_{-0.0014}$
m_2 [eV]	$0.0194^{+0.0010}_{-0.0012}$
m_3 [eV]	$0.0535^{+0.0004}_{-0.0004}$
$\sum_i m_i$ [eV]	$0.090^{+0.002}_{-0.003}$
$\langle m_{\beta\beta}\rangle$ [meV]	$18.14^{+1.17}_{-1.48}$
m_β^{eff} [meV]	$19.60^{+1.02}_{-1.25}$
α_1/π	$\pm 1.129^{+0.019}_{-0.013}$
α_2/π	$\pm 0.946^{+0.004}_{-0.004}$
Fine-tuning	11.2
χ^2_{min}	0.074

Backup slides

	Best-fit and 1σ range
$\mathrm{Re}\,\tau$	$\pm 0.244^{+0.012}_{-0.067}$
$\mathrm{Im}\,\tau$	$1.132^{+0.027}_{-0.297}$
β/α	$0.92^{+0.85}_{-0.03}$
γ/α	$-1.20^{+0.06}_{-2.14}$
$\log_{10}(\alpha_D/\alpha)$	$-13.4^{+13.2}_{-76.3}$
g'/g	$2.76^{+0.21}_{-0.23}$
g''/g	$-2.53^{+0.13}_{-0.03}$
$\log_{10}(\lvert\lambda\rvert)$	$-12.2^{+10.9}_{-59.2}$
$v_d\,\alpha$, [GeV]	$1.08^{+0.06}_{-0.69}$
$v_u^2\,g^2/\Lambda$ [eV]	$3.46^{+0.55}_{-1.65}$
$\sin^2\theta_{12}$	$0.305^{+0.011}_{-0.011}$
$\sin^2\theta_{13}$	$0.0221^{+0.0006}_{-0.0005}$
$\sin^2\theta_{23}$	$0.448^{+0.014}_{-0.016}$
r	$0.0296^{+0.0006}_{-0.0008}$
m_e/m_μ	$0.0048^{+0.0001}_{-0.0002}$
m_μ/m_τ	$0.0574^{+0.0032}_{-0.0050}$
Ordering	**NO**
J_{CP}	$-0.018^{+0.002}_{-0.002}$
α_1/π	0
α_2/π	$\pm 0.112^{+0.792}_{-0.014}$
m_1 [meV]	0
m_2 [meV]	$8.620^{+0.095}_{-0.123}$
m_3 [meV]	$50.806^{+0.016}_{-0.021}$
$\sum_i m_i$ [eV]	$0.0594^{+0.0001}_{-0.0001}$
$\lvert m_{\beta\beta}\rvert$ [meV]	$3.61^{+0.09}_{-0.09}$
m_β^{eff} [meV]	$8.90^{+0.10}_{-0.09}$
d_{FT}	3.03
χ^2_{min}	0.98

Minimal seesaw model: arxiv:2402.18547
with S.Marciano, D.Meloni



Ideas to explore: sterile neutrino (3+1) scheme

Minimal construction: Zhang MES mechanism

He Zhang
[1110.6838]

$S \sim$ fermionic gauge singlet

$\nu_R \sim$ Majorana heavy singlets

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2} \overline{\nu_R^c} M_R \nu_R + \text{h.c.},$$

Arbitrary assumption: no Majorana mass term for S: can it be “modularly imposed”?

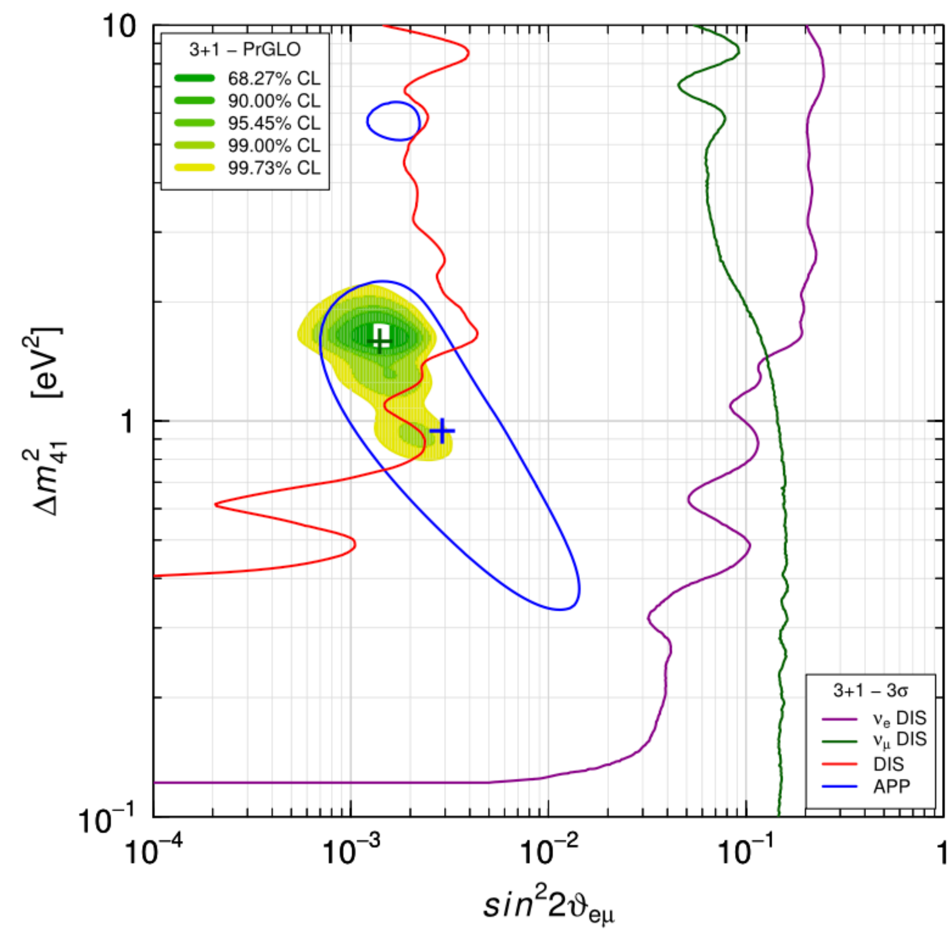
$$M_\nu^{7\times7} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix}$$

Sterile-active mixing

$$R \sim \frac{\mathcal{O}(M_D)}{\mathcal{O}(M_S)} = \text{Modular suppression?}$$

Only existing study: A_4 model

M.Singh et al.
[2303.10922]



Non-standard interactions

tests of modulus couplings

G-J. Ding, FF, 2003.13448

non standard neutrino interactions

$$\mathcal{L} = i \sum_{f=e,e^c,\nu} \overline{f}\, \overline{\sigma}^\mu \partial_\mu f + \frac{1}{2}\partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha - \frac{1}{2} M_\alpha^2 \varphi_\alpha^2$$
$$- (m_e + \mathcal{Z}_\alpha^e \varphi_\alpha) e^c e - \frac{1}{2}\nu (m_\nu + \mathcal{Z}_\alpha^\nu \varphi_\alpha)\nu + h.c. + \dots .$$

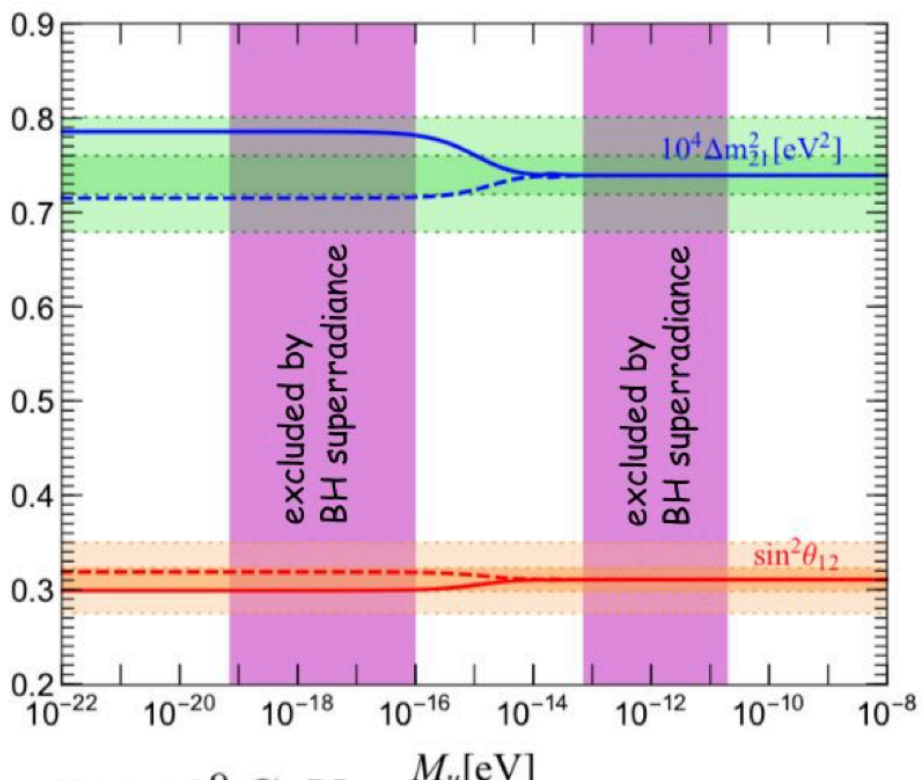
$$\tau = \langle \tau \rangle + \frac{\varphi_u + i\,\varphi_v}{\sqrt{2}}$$

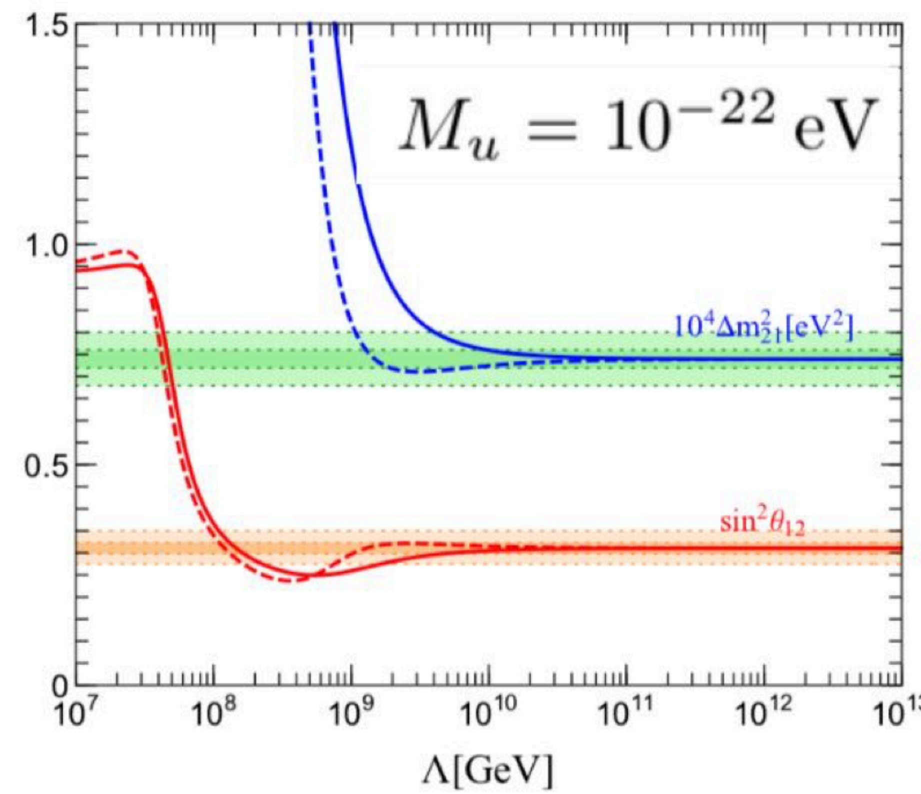
in medium with non-zero electron number density

small, unless the modulus is very light

$$\delta m_\nu(0) = -n_e^0 \frac{\mathrm{Re}(\mathcal{Z}^e)\mathcal{Z}^\nu}{M^2(R)} ,$$

in the sun:





$\Lambda = 5 \times 10^9$ GeV

[modulus VEV]

from Feruglio's slides at Mod. Symmetry Bethe Workshop