Istituto Nazionale di Fisica Nucleare

Shear to entropy ratio of superfluids

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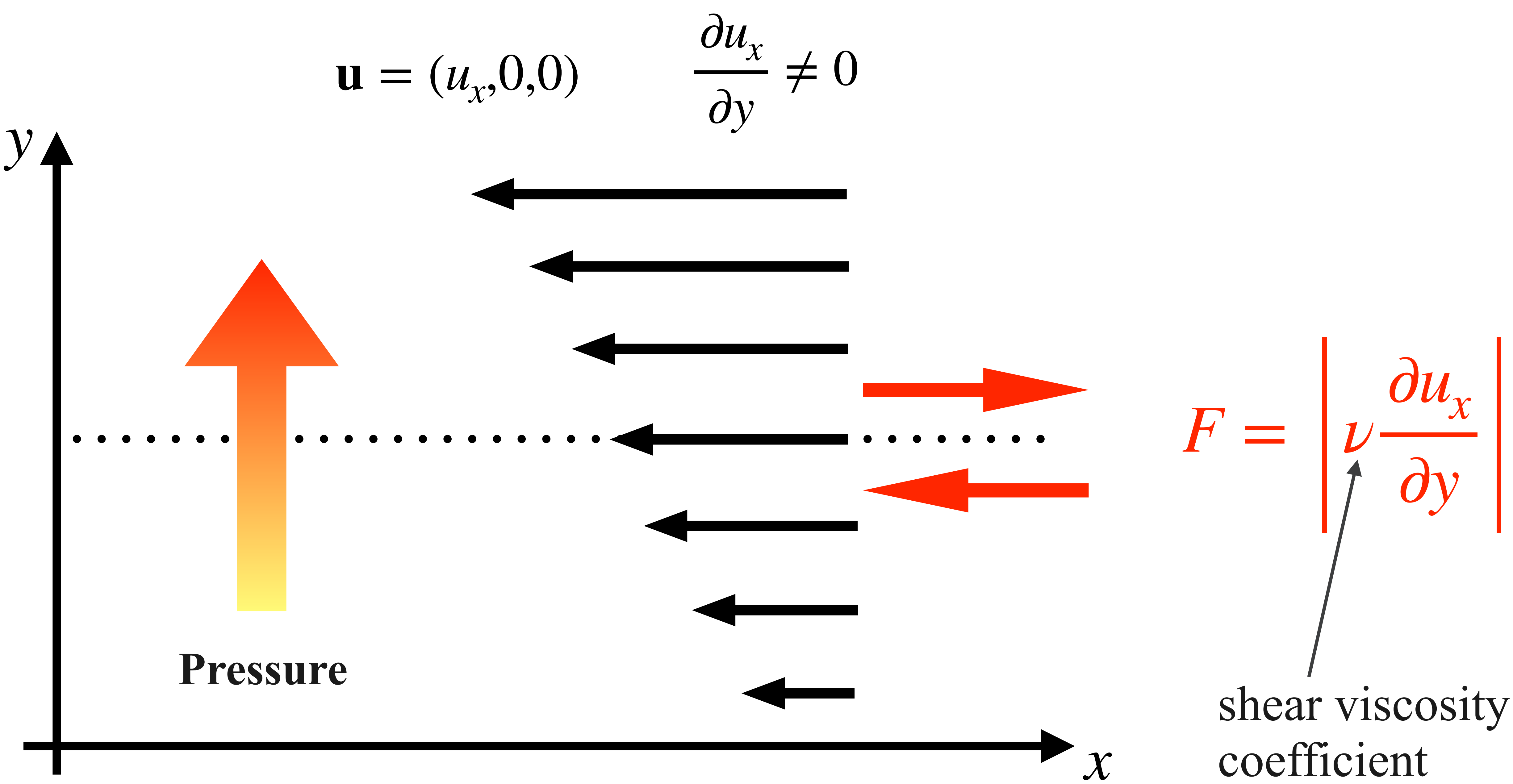
Outline

- Shear viscosity
- Acoustic analogs
- Dissipative processes
- Conclusions

Laminar flow
and
Shear viscosity

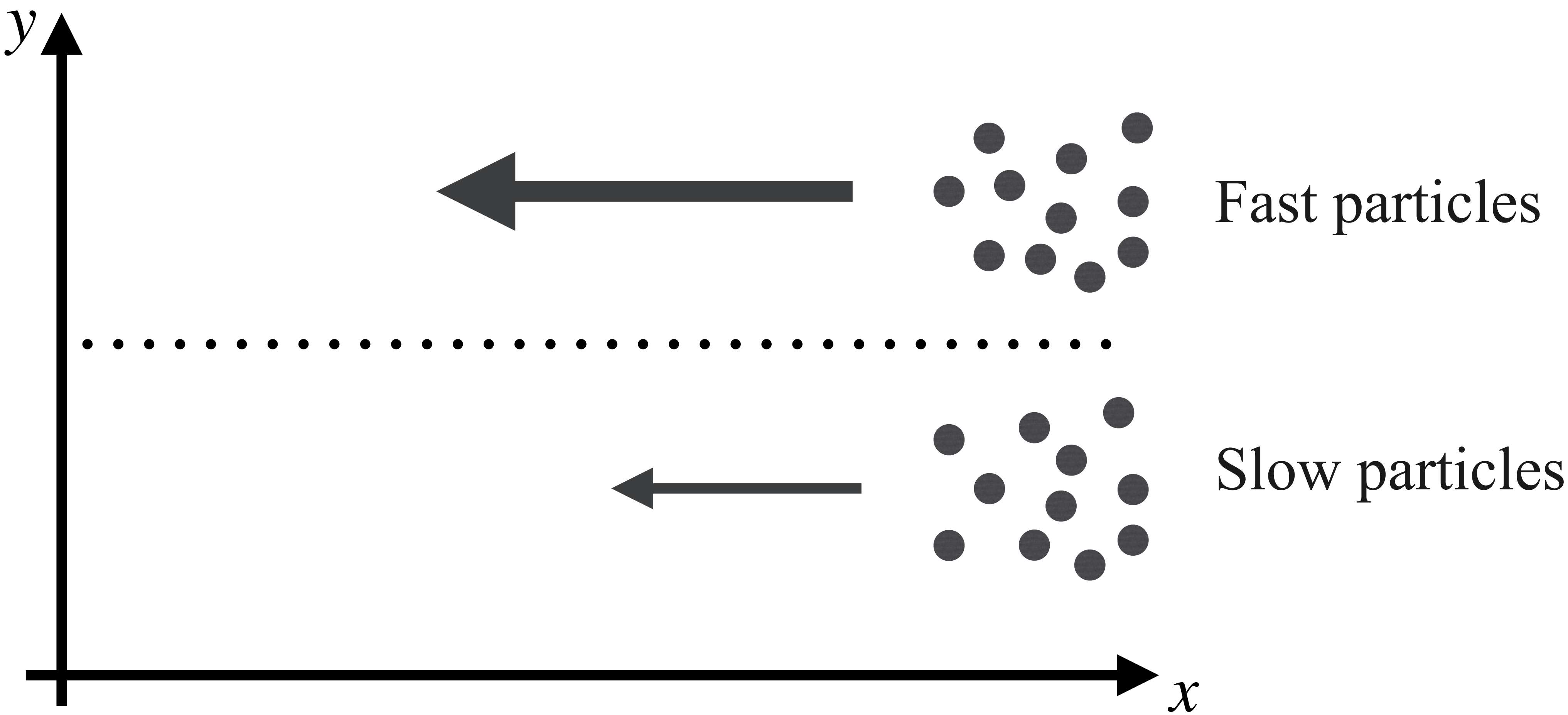
Laminar flow

Laminar flow



Unstable hydrodynamic configuration

Microscopic



Diffusion tends to isotropize the flow

Double role of interactions:

- **Interactions needed to scatter particles between the two layers (at large angles)**
- **Strong interactions reduce the mean free path: reduce the viscosity!**

Shear viscosity η

Diffusion between layers results in an effective **friction**

$$\eta \sim n\, p\, \lambda$$

λ	mean free path
p	average momentum
n	number density

From $p\lambda \geq \hbar$ it follows that $\dfrac{\eta}{n} \geq \hbar$

In relativistic systems **entropy** works better. Entropy density $s \propto k_B n$

$$\frac{\eta}{s} \sim p\lambda \geq \frac{\hbar}{k_B}$$

Multiple processes

Suppose there are N independent processes: $\eta_i \sim np\lambda_i \quad i = 1,\ldots,N$

$$\frac{1}{\eta_{eff}} \simeq \frac{1}{\eta_1} + \frac{1}{\eta_2} \ldots = \sum_{i=1}^{N} \frac{1}{\eta_i}$$

The smaller shear viscosity coefficient wins!

It is important to determine the minimum value of the shear viscosity

Shear viscosity η

Viscosity can be defined as the dissipative term linear in velocity gradients

$$T_{ij} = T^0_{ij} + \sigma'_{ij}$$

ideal fluid

dissipative

$$\sigma'_{ij} = \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{u}\right) + \zeta\delta_{ij}\nabla\cdot\mathbf{u}$$

Kinetic theory + hard sphere approximation $\quad \eta \approx \dfrac{\sqrt{mT}}{a^2}$

The KSS bound

- **Increasing** the temperature the **entropy increases**
- **Increasing** the interaction strength the **shear viscosity** should **decrease**

Does the η/s vanishes in some limit ?

It has been **conjectured** that in any **physical** system in 3+1 dimensions

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

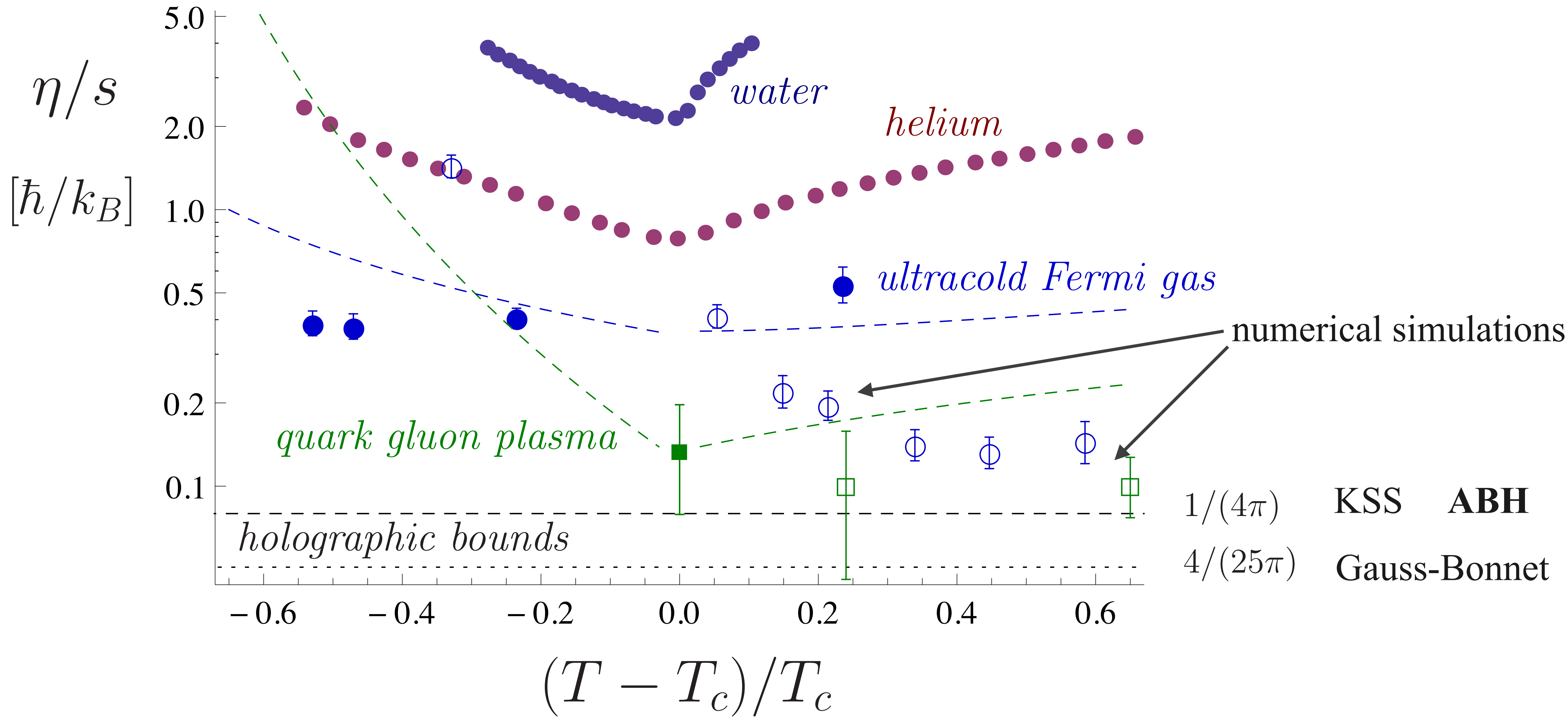
← no speed of light

KSS: P. Kovtun, D. T. Son, and A. O. Starinets, PRL 94, 111601 (2005)

Possible theoretical counterexamples, Cohen *Phys.Rev.Lett.* 99 (2007) 021602

Physical systems

Large η: weak interaction   Small η: strong interaction



Adams et al. New Journal of Physics 14 (2012)

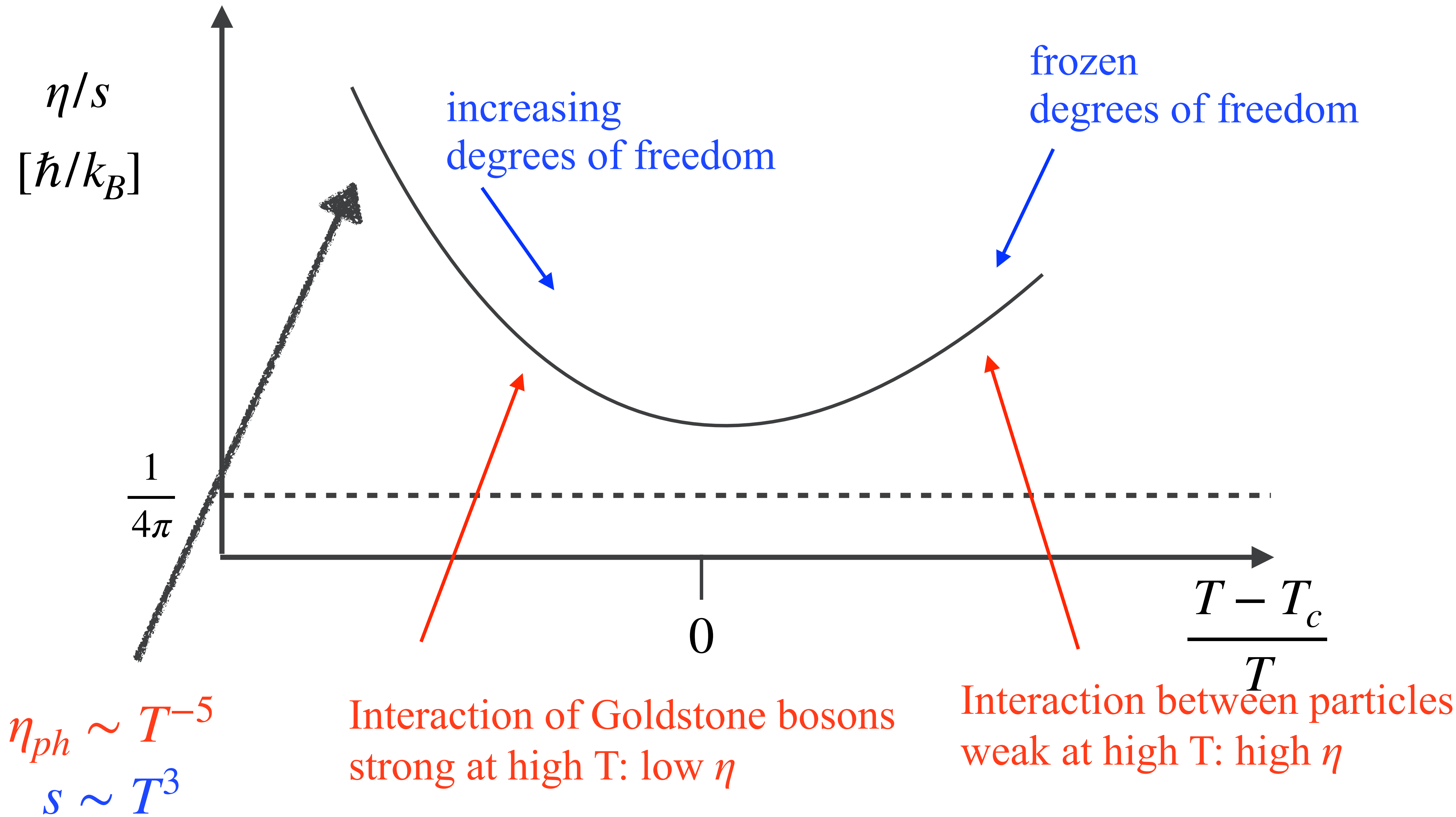
ABH: Acoustic Black Holes

L. Chiofalo, D. Grasso, MM and S. Trabucco, *New J.Phys.* 26 (2024) 5, 053021

Superfluids

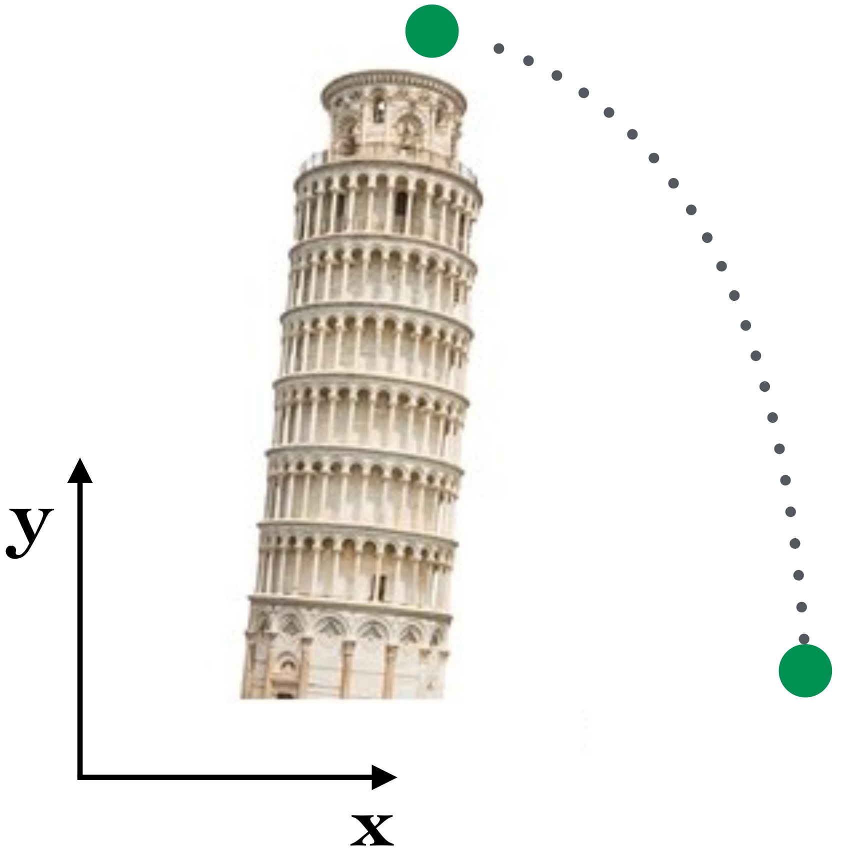
Large η: weak interaction | Small η: strong interaction

Large s: many degrees of freedom | Small s: few degrees of freedom



Gravity analogues

Fluid gradients to emulate gravity

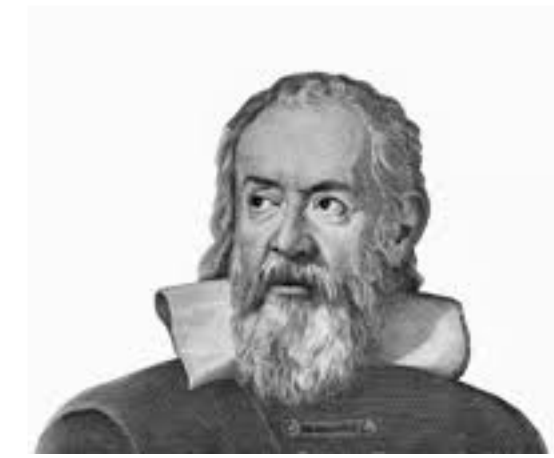


Gravity

$$u_x = u \qquad x = u\,t$$

$$u_y = gt \qquad y = \frac{1}{2} g t^2$$

trajectory $y = \dfrac{g}{2\,u^2} x^2$



Galileo

Fluid analog: u fluid velocity

Unrhu

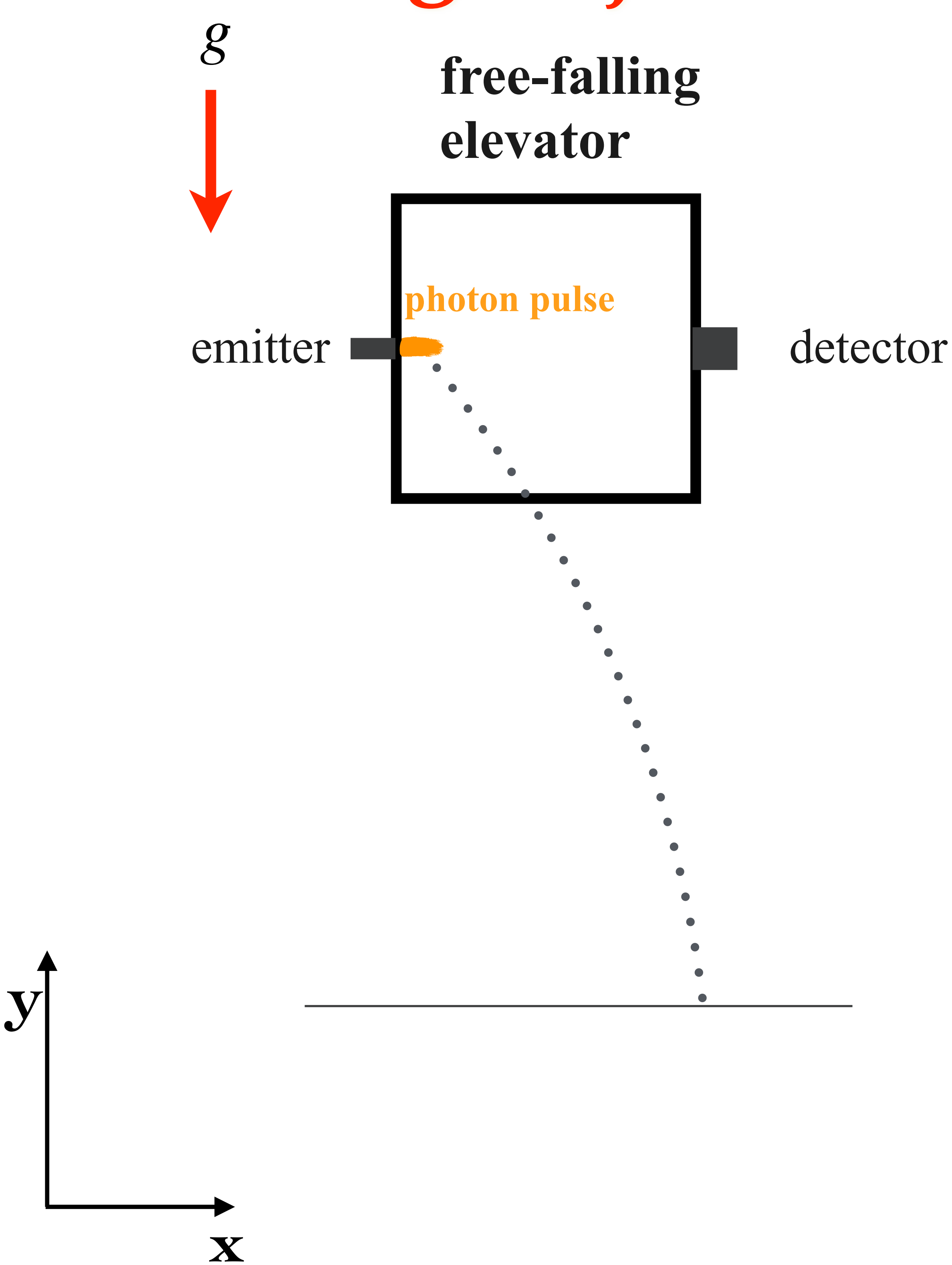
$$u_x = u \qquad x = u\,t$$

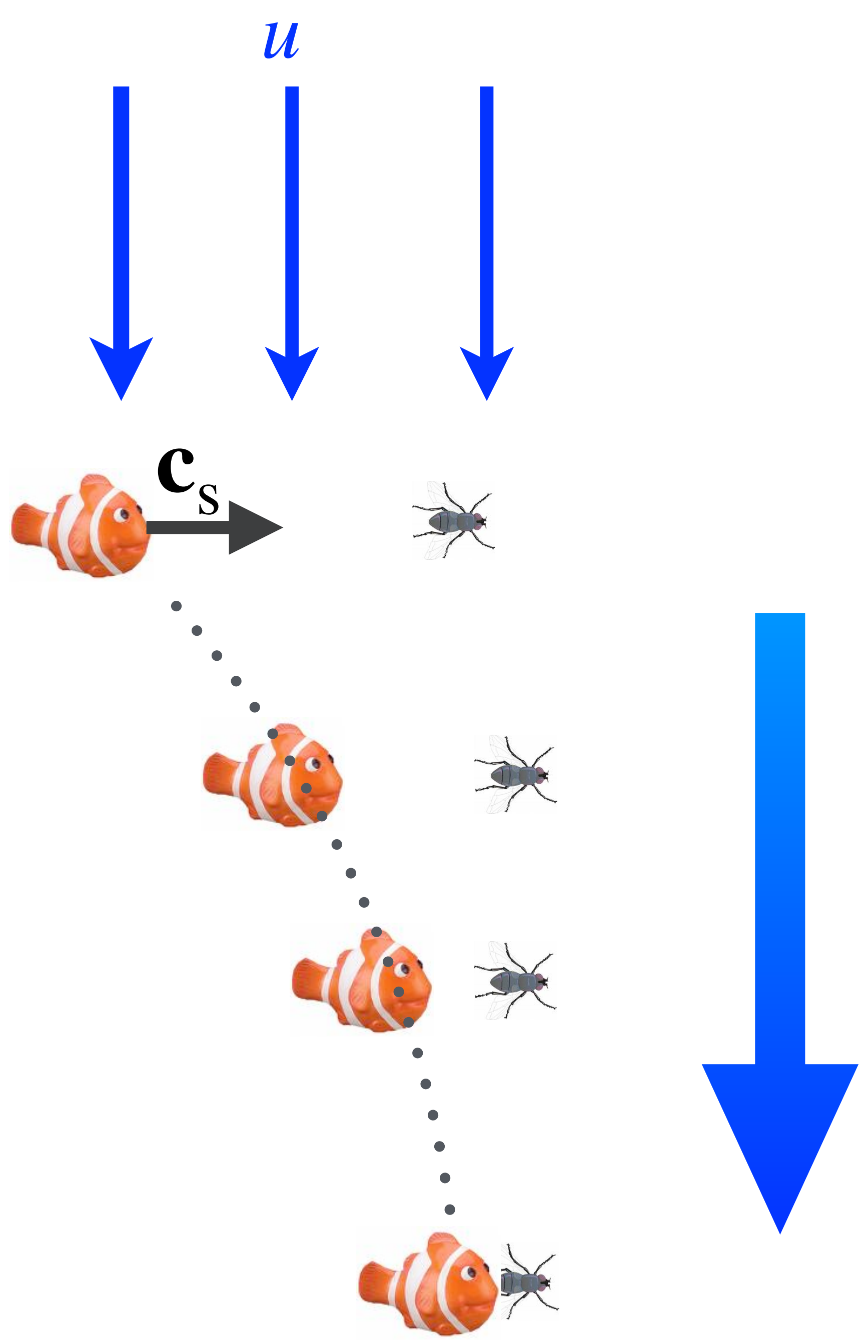
$$u_y = k\frac{x}{u} \qquad dy = k\frac{x}{u}dt \qquad dy = k\frac{x}{u^2}dx$$

trajectory $y = \dfrac{k}{2\,u^2} x^2$

k **is related to the "surface" acceleration**

Bending trajectories



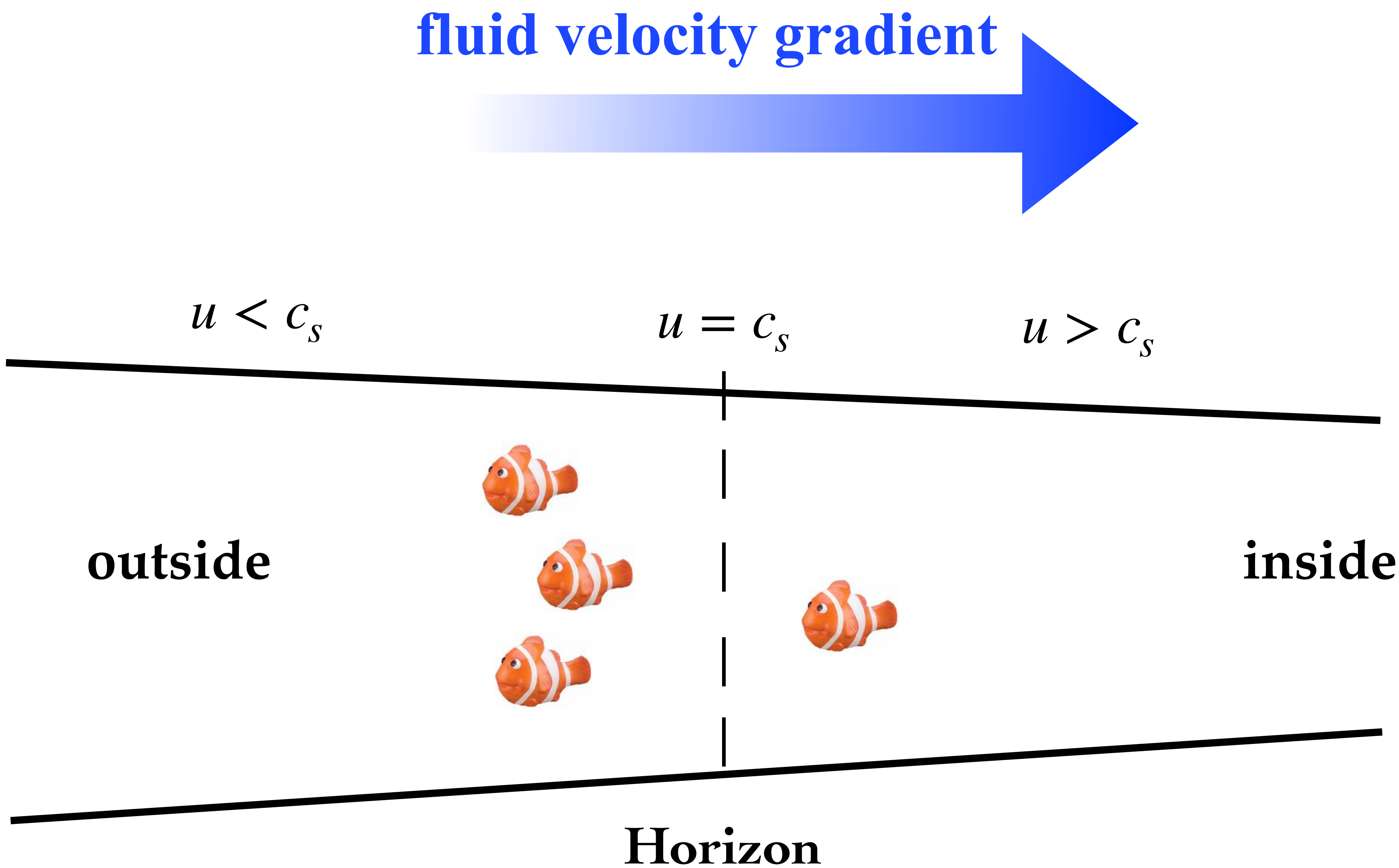


A bent trajectory

$$\frac{dy}{dx} = \frac{u}{c_s}$$

A **velocity space gradient** produces the analog of light bending

Sound trapping: Acoustic black hole



Fluid equations

Description of the fluid

Continuity equation $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Euler equation $$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = \mathbf{f}$$

Characteristics of the fluid

- barotropic $p \equiv p(\rho)$
- inviscid $\mathbf{f} = -\nabla p$
- irrotational $\mathbf{v} = \nabla \phi$

Small perturbations

Fluctuations around a background configuration

$$\begin{aligned}
\rho &= \rho_0 + \epsilon\rho_1 + \mathcal{O}(\epsilon^2) \\
p &= p_0 + \epsilon p_1 + \mathcal{O}(\epsilon^2) \\
\phi &= \phi_0 + \epsilon\phi_1 + \mathcal{O}(\epsilon^2)
\end{aligned}$$

$$\mathbf{v}_0 = \nabla\phi_0 \qquad \mathbf{v}_1 = \nabla\phi_1$$

Bulk → ϕ_0

Perturbation → $\epsilon\phi_1$

$$\frac{\partial\rho_0}{\partial t} + \nabla\cdot(\rho_0\mathbf{v_0}) = 0 \qquad \textbf{background}$$

$$\frac{\partial\rho_1}{\partial t} + \nabla\cdot(\rho_0\mathbf{v_1}) + \nabla\cdot(\rho_1\mathbf{v_0}) = 0 \qquad \textbf{perturbation}$$

Small perturbations

Combining linearized Euler and continuity equations:

$$\frac{\partial}{\partial t}\left(c_s^{-2}\rho_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)-\nabla\cdot\left(\rho_0\nabla\phi_1-c_s^{-2}\rho_0\mathbf{v}_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)=0$$

where $$c_s^2=\frac{\partial p}{\partial\rho}$$

check

$\mathbf{v_0}=0\,,\quad \rho_0=\text{const},\quad c_s=\text{const}$

$$\frac{\partial^2\phi_1}{\partial t^2}-c_s^2\nabla^2\phi_1=0$$

The non uniform medium changes the propagation

Gravity emerges

$$\frac{\partial}{\partial t}\left(c_s^{-2}\rho_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)-\nabla\cdot\left(\rho_0\nabla\phi_1-c_s^{-2}\rho_0\mathbf{v}_0\left(\frac{\partial\phi_1}{\partial t}+\mathbf{v}_0\cdot\nabla\phi_1\right)\right)=0$$

We can rewrite the above equation as

$$\boxed{\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi_1\right)=0}$$

where $\quad g_{\mu\nu}=\Omega\begin{pmatrix} c_s^2-v^2 & \mathbf{v}^t \\ \mathbf{v} & -I \end{pmatrix}$

Schwarzschild acoustic metric?

Acoustic metric

$$ds^2 = \frac{\rho}{c_s}\Big(-(c_s^2 - v^2)dt^2 + 2\mathbf{v}\cdot\mathbf{dx}\,dt + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big)$$

Painleve'–Gullstrand representation of Schwarzschid metric

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 \pm \sqrt{\frac{2GM}{r}}drdt + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$v \propto \frac{1}{\sqrt{r}}$ divergent flow at the origin

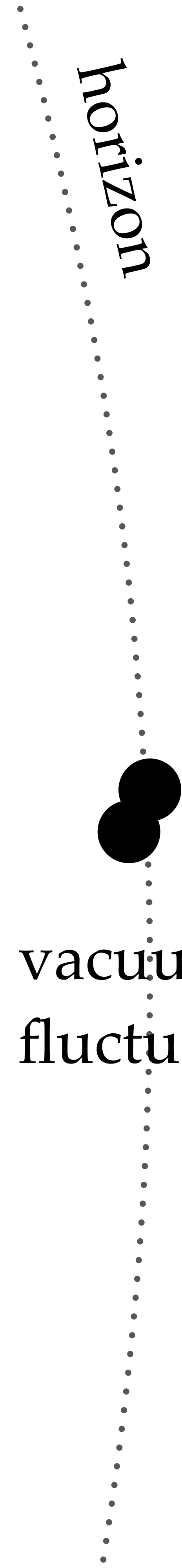
Abandon the 3D spherical geometry

Hawking radiation

S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys. 43, 199 (1975)*

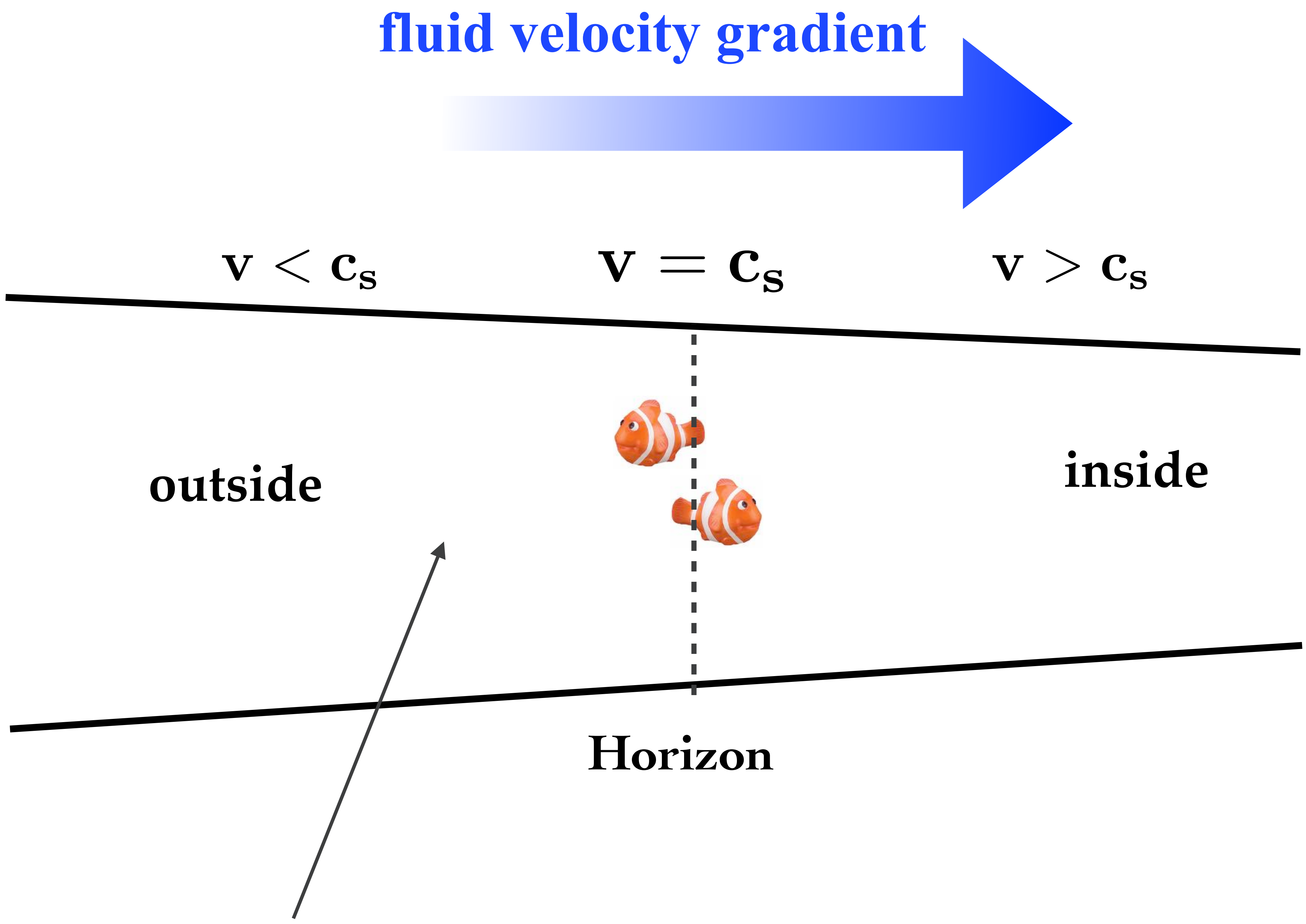
W. Unruh, Experimental black hole evaporation, *Phys.Rev.Lett. 46, 1351 (1981)*.

Hawking emission



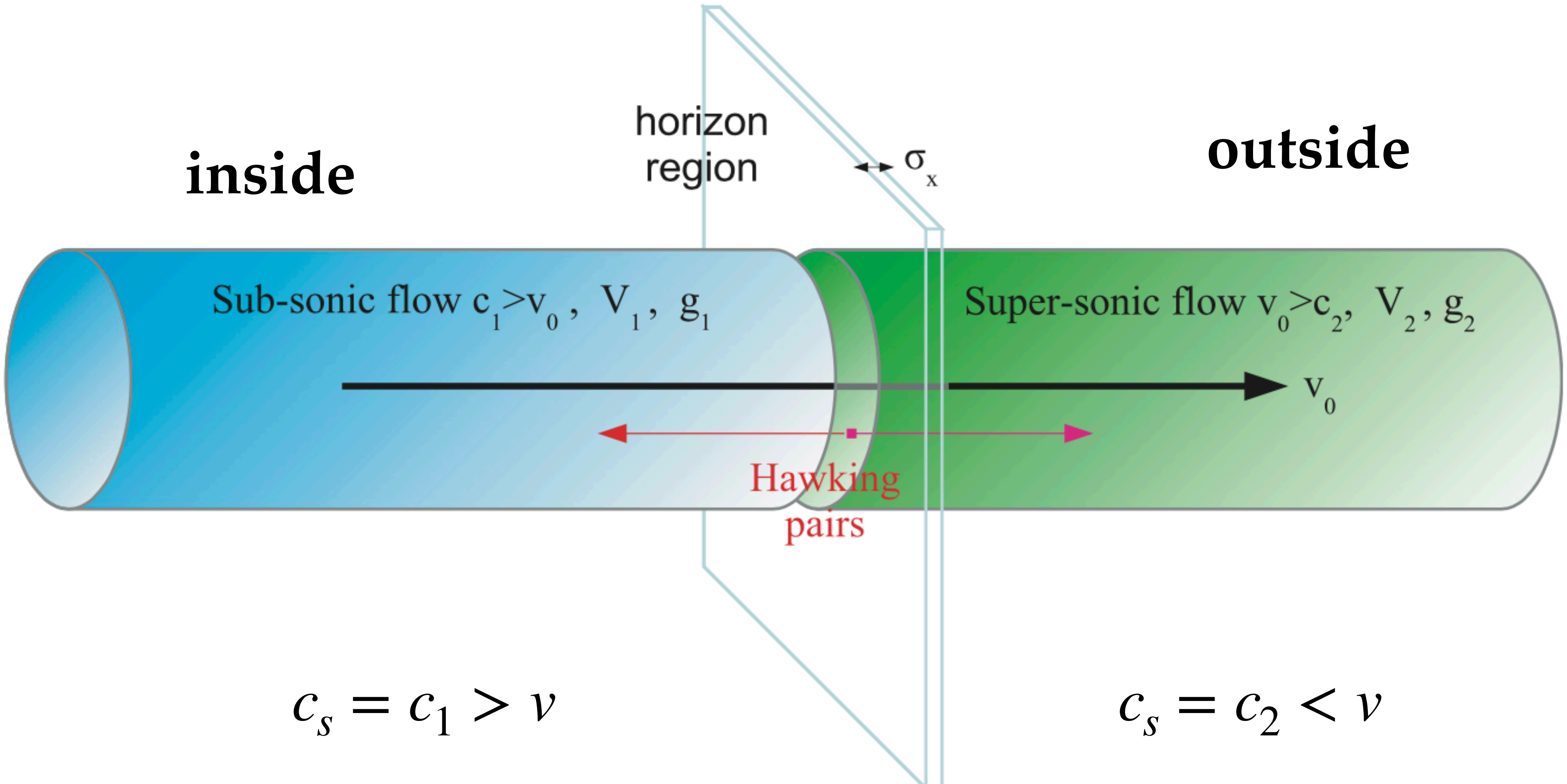
See for instance
Parikh, Wilczek *Phys.Rev.Lett.* 85 (2000) 5042

Analogue emission



Phonon escapes by quantum tunneling

Setup: trapped BEC condensate



$c_s = c_1 > v$

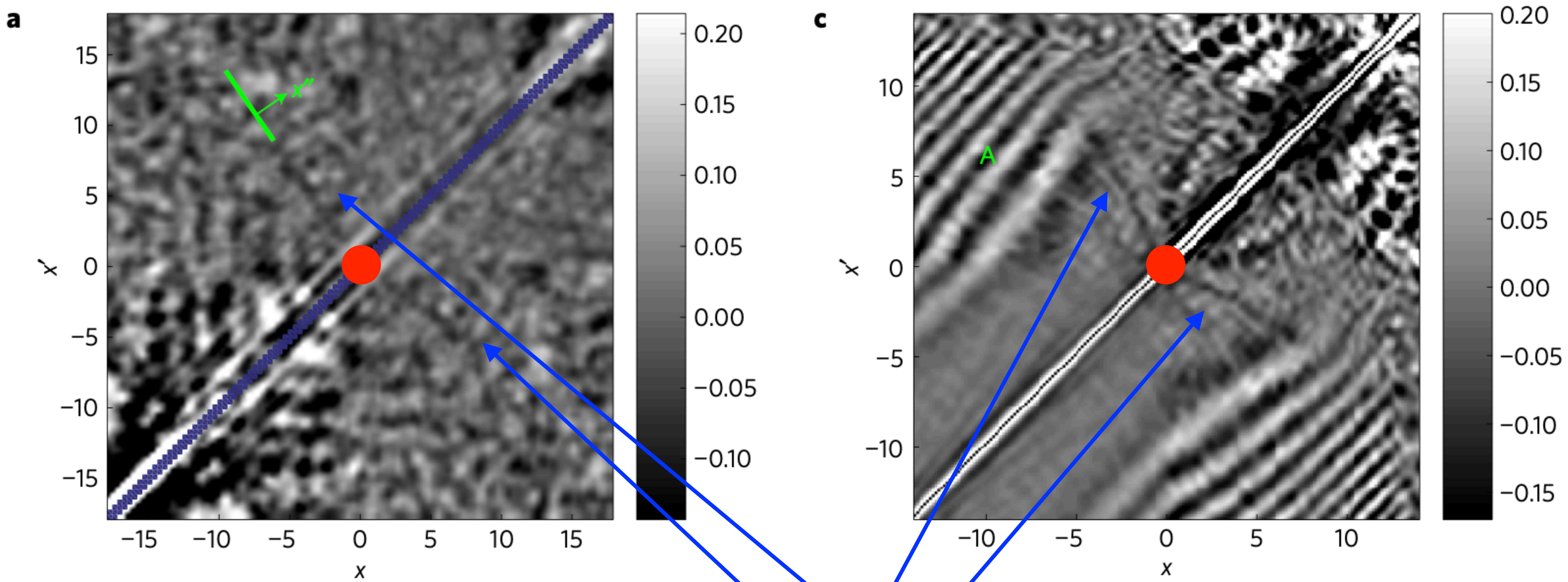
$c_s = c_2 < v$

Carusotto *et al New J. Phys.* 10 103001 (2008)

Instead of changing the velocity, change the speed of sound

Experimental observation

experiment numeric



acoustic hole position

effect of phonon emission on the density

Image obtained by 4600 repetitions of the experiment

Steinhauer, Nature Phys. 12 (2016) 95

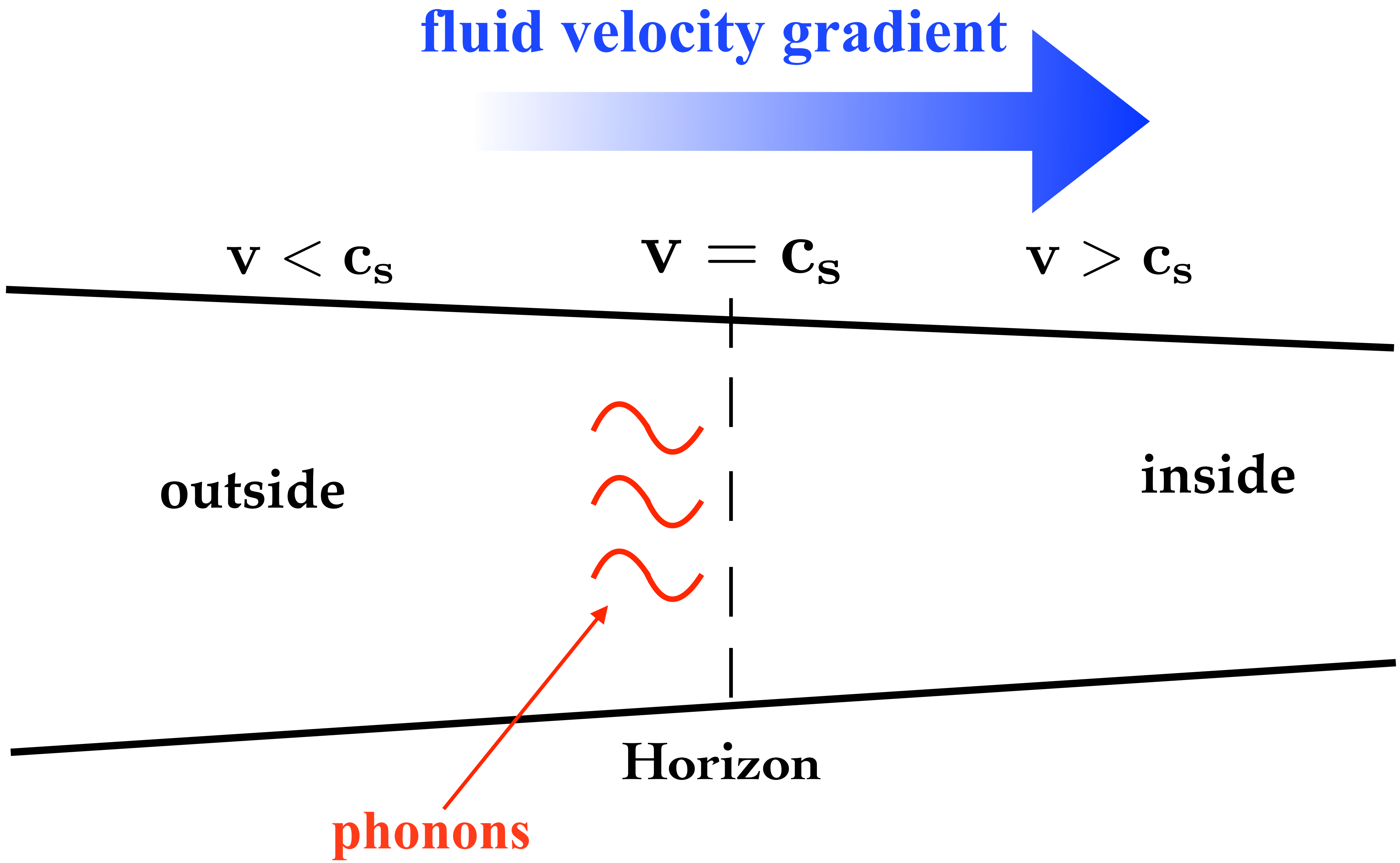
Fitted Hawking temperature $\sim 10^{-9} K$

See talk by S.Trabucco on friday.

Dissipative processes at the horizon

L. Chiofalo, D. Grasso, MM and S. Trabucco, *New J.Phys.* 26 (2024) 5, 053021

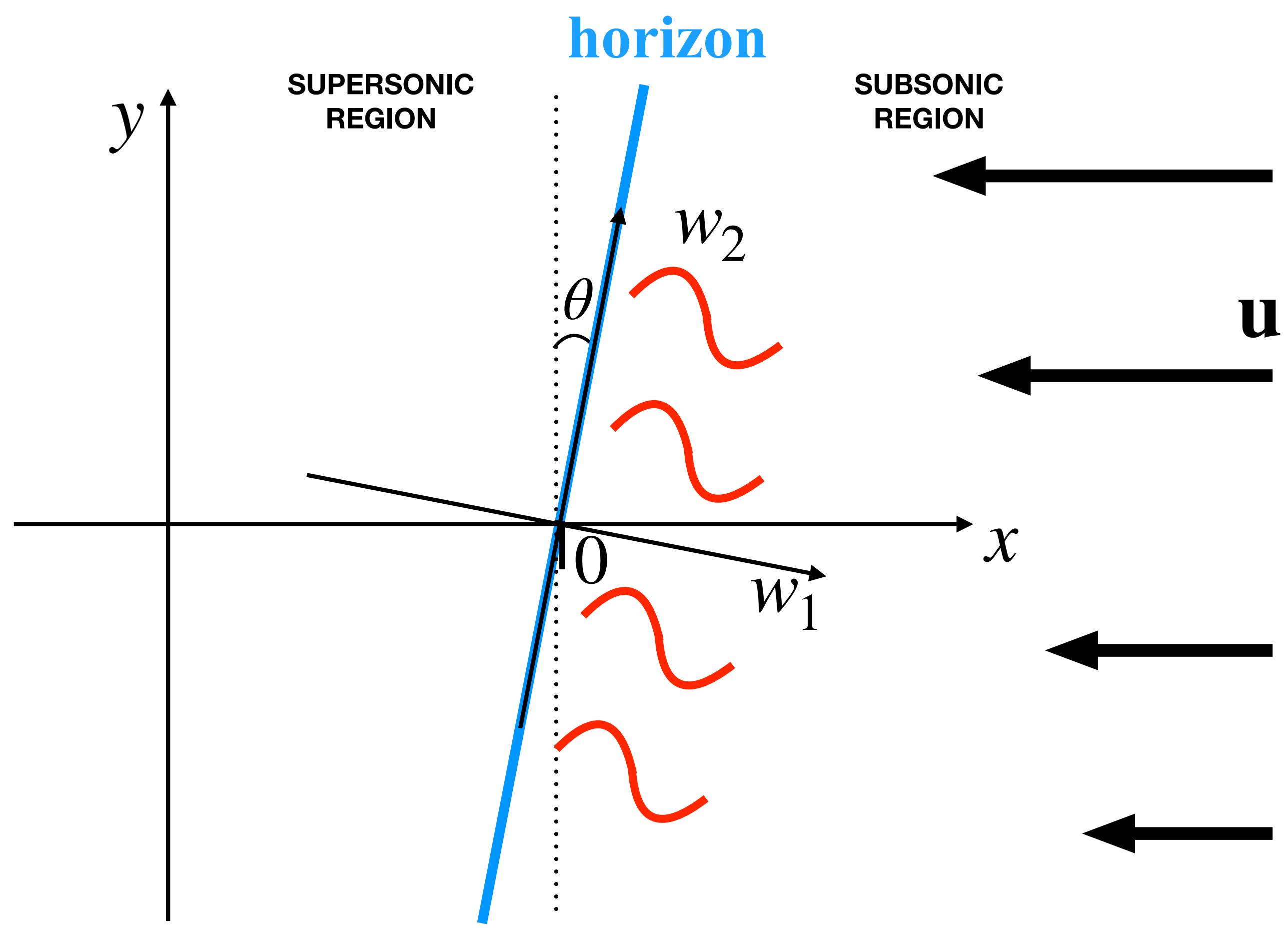
Viscosity of an acoustic hole



Energy conservation: phonon emission results in a decrease of the fluid velocity

More generally

Phonon momentum transfer



$$\mathbf{u} = (u_x, 0, 0)$$
$$u_x = c_s - Cx + ky$$

Viscous stress-tensor $\sigma'_{ik} = \eta\left(\partial_i u_k + \partial_k u_i\right) + \zeta \delta_{ix}\delta_{kx} \nabla \cdot \mathbf{u}$

Phonon stress-energy tensor $\tilde{T}^{\mu}_{\nu} = \int p^{\mu} p_{\nu} f(x,p) d\mathscr{P}$

MM, D. Grasso, S. Trabucco and L. Chiofalo, Phys.Rev.D 103 (2021) 7, 076001

Phonon in emergent gravity

Phonons distribution, *f*, solution of covariant Liouville equation

$$L[f] \equiv p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial f}{\partial p^\alpha} = C[f]$$

neglecting interactions $C[f] = 0$

Bose-Einstein like ansatz

$$f(x,p) = \frac{1}{\exp(p^\mu \beta_\mu) - 1}$$

$$\beta_{\lambda;\rho} + \beta_{\rho;\lambda} = 0$$ Killing's equation

MM, D. Grasso, S. Trabucco and L. Chiofalo, Phys.Rev.D 103 (2021) 7, 076001

Two expressions of the total stress tensor

$$\begin{cases} T_{ik} = T^0_{ik} + \sigma'_{ik} \\ T_{ik} = T^0_{ik} + \tilde{T}_{ik} \end{cases}$$

If dissipation is only due to phonon emission $\tilde{T}_{ik} = \sigma'_{ik}$

$$\frac{\zeta_{\rm eff}}{s_{\rm ph}} = \frac{\eta}{s_{\rm ph}} = \frac{1}{4\pi}$$

Saturation of the KSS bounds

Perturbation of the horizon: talk by Chiara Coviello on friday

Outlook: violating the KSS bound?

Massive phonons may arise as pseudo Nambu Goldstone bosons

If phonons are massive the phonon pressure is reduced

$$\frac{\zeta_{\rm eff}}{s_{\rm ph}} = \frac{\eta}{s_{\rm ph}} = \frac{1}{4\pi}(1-\delta)$$

where δ is proportional to the mass of the phonon

Conclusions

- The shear viscosity to density ratio measures strength of interactions and number of degrees of freedom

- We can emulate gravitational effects using fluids

- Dissipative processes are identified at the acoustic horizon

- They saturate the KSS bound

Thanks for your attention!

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Some references

MM and C. Manuel, Phys.Rev.D 77 (2008) 103014
MM, C. Manuel and B. A. Sa'd, Phys.Rev.Lett. 101 (2008) 241101
M. A. Escobedo, MM and C. Manuel, Phys.Rev.A 79 (2009) 063623
MM and C. Manuel, Phys.Rev.D 81 (2010) 043002
MM, C. Manuel and L. Tolos, Annals Phys. 336 (2013) 12-35
MM, D. Grasso, S. Trabucco and L. Chiofalo, Phys.Rev.D 103 (2021) 7, 076001
L. Chiofalo, D. Grasso, MM and S. Trabucco, *New J.Phys.* 26 (2024) 5, 053021
C.Coviello, L. Chiofalo, D. Grasso, S.Liberati, MM and S. Trabucco 2410.00264 [gr-qc]

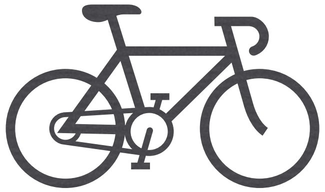
People

A. Biondi
M. Chiofalo
C. Coviello
D. Grasso
L. Lepori
S. Liberati
S. Trabucco

Kinetic theory for phonons

From GR

R. W. Lindquist, Annals of Physics 37, 487 (1966).
J. Stewart, Lecture Notes in Physics, Lecture Notes in Physics No. v. 10 (Springer-Verlag, 1969).



To analog models

MM and C. Manuel, Phys.Rev.D 77 (2008) 103014
MM, D. Grasso, S. Trabucco and L. Chiofalo   Phys.Rev.D 103 (2021) 7, 076001

Thermodynamics

Knowing the distribution function we can obtain the thermodynamics

distribution function

Phonon current: $n^{\mu}_{\rm ph} = \int p^{\mu} f(x,p)\, d\mathscr{P}$

integral measure

Energy momentum tensor: $T^{\mu\nu}_{\rm ph} = \int p^{\mu} p^{\nu} f(x,p)\, d\mathscr{P}$

Entropy current: $s^{\alpha}_{\rm ph} = -\int p^{\alpha} \left[f \ln f - (1+f)\ln(1+f) \right] \, d\mathscr{P}$

Transport of "phonon" number

Covariant conservation $\partial_\nu n^\nu_{\rm ph} + \Gamma^\mu_{\mu\nu} n^\nu_{\rm ph} = \int C[f] d\mathcal{P}$

collision integral

Where $\Gamma^\mu_{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\nu \sqrt{-g} = \frac{1}{c_s} \frac{\partial c_s}{\partial x^\nu}$

$g_{\mu\nu} = \eta_{\mu\nu} + \left(c_s^2 - 1\right) v_\mu v_\nu$

We keep $C[f] = 0$

Change in the flux of phonons due to the nonuniform background

Phonons emitted at the horizon

Phonons in collisionless regime

$$\textbf{Entropy}\ s^{\alpha}_{\rm ph} = -\int p^{\alpha}\left[f\ \ln f - (1+f)\ln(1+f)\right]\ d\mathscr{P}$$

The actual entropy flux

From the Fluid → To the Phonon gas

by means of the horizon $\Delta S_{\rm ph} = -\Delta S_{\rm H}$

We get that $T = T_H$

Entropy balance

Entropy variation of the fluid $$\Delta S_H = 2\pi \frac{r_H}{L_c^2} \Delta r_H$$

Entropy gain of the phonon gas $$\Delta S_{\rm ph} = 4\pi r_H^2 \, d_g \, s_{\rm ph} \Delta r_H$$

number of degrees of freedom

$$s_{\rm ph} = \frac{\pi T}{6 L_c^2 C x}$$

with $C = (v + c_s)'|_H$

$$\Delta S_{\rm ph} = -\Delta S_{\rm H} \quad \Longrightarrow \quad T = \frac{1}{2\pi} \left(\frac{c_s - |v|}{1 - c_s |v|} \right)' \Bigg|_H$$

Is gravity an emerging phenomenon?

Gravity as emerging theory has been proposed by many, including Sakharov

In Einstein's theory of gravitation one postulates that the action of space-time depends on the curvature (R is the invariant of the Ricci tensor):

$$S(R) = -\frac{1}{16\pi G}\int (dx)\sqrt{-g}R. \quad (1)$$

The presence of the action (1) leads to a "metrical elasticity" of space, i.e., to generalized forces which oppose the curving of space.

Here we consider the hypothesis which identifies the action (1) with the change in the action of quantum fluctuations of the vacuum if space is curved. Thus, we consider the metrical elasticity of space as a sort of level displacement effect (cf. also Ref. 1).[1]

An interesting one page reading

Vacuum quantum fluctuations in curved space and the theory of gravitation

A. D. Sakharov

Dokl. Akad. Nauk SSSR **177**, 70–71 (1967) [Sov. Phys. Dokl. **12**, 1040–1041 (1968). Also S14, pp. 167–169]

Usp. Fiz. Nauk **161**, 64–66 (May 1991)

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In present-day quantum field theory it is assumed that the energy-momentum tensor of the quantum fluctuations of the vacuum $T^i_k(0)$ and the corresponding action $S(0)$, formally proportional to a divergent integral of the fourth power over the momenta of the virtual particles of the form $\int k^3 dk$, are actually equal to zero.

Recently Ya. B. Zel'dovich[3] suggested that gravitational interactions could lead to a "small" disturbance of this equilibrium and thus to a finite value of Einstein's cosmological constant, in agreement with the recent interpretation of the astrophysical data. Here we are interested in the dependence of the action of the quantum fluctuations on the curvature of space. Expanding the density of the Lagrange function in a series in powers of the curvature, we have (A and $B \sim 1$)

$$(R) = \mathcal{L}(0) + A\int k dk \cdot R + B\int \frac{dk}{k} R^2 + \dots \qquad (2)$$

The first term corresponds to Einstein's cosmological constant.

The second term, according to our hypothesis, corresponds to the action (1), i.e.,

$$G = -\frac{1}{16\pi A\int k dk}, \quad A \sim 1. \qquad (3)$$

The third term in the expansion, written here in a provisional form, leads to corrections, nonlinear in R, to Einstein's equations.[2)]

The divergent integrals over the momenta of the virtual particles in (2) and (3) are constructed from dimensional considerations. Knowing the numerical value of the gravitational constant G, we find that the effective integration limit in (3) is

$$k_0 \sim 10^{28}\ \text{eV} \sim 10^{+33}\,\text{cm}^{-1}.$$

In a gravitational system of units, $G = \hbar = c = 1$. In this case $k_0 \sim 1$. According to the suggestion of M. A. Markov, the quantity k_0 determines the mass of the heaviest particles existing in nature, and which he calls "maximons." It is natural to suppose also that the quantity k_0 determines the limit of applicability of present-day notions of space and causality.

Consideration of the density of the vacuum Lagrange function in a simplified "model" of the theory for noninteracting free fields with particles $M \sim k_0$ shows that for fixed ratios of the masses of real particles and "ghost" particles (i.e., hypothetical particles which give an opposite contribution from that of the real particles to the R-dependent action), a finite change of action arises that is proportional to $M^2 R$ and which we identify with R/G. Thus, the magnitude of the gravitational interaction is determined by the masses and equations of motion of free particles, and also, probably, by the "momentum cutoff."

This approach to the theory of gravitation is analogous to the discussion of quantum electrodynamics in Refs. 4 to 6, where the possibility is mentioned of neglecting the Lagrangian of the free electromagnetic field for the calculation of the renormalization of the elementary electric charge. In the paper of L. D. Landau and I. Ya. Pomeranchuk the magnitude of the elementary charge is expressed in terms of the masses of the particles and the momentum cutoff. For a further development of these ideas see Ref. 7, in which the possibility is established of formulating the equations of quantum electrodynamics without the "bare" Lagrangian of the free electromagnetic field.

The author expresses his gratitude to Ya. B. Zel'dovich for the discussion which acted as a spur for the present paper, for acquainting him with Refs. 3 and 7 before their publication, and for helpful advice.

1) Here the molecular attraction of condensed bodies is calculated as the result of changes in the spectrum of electromagnetic fluctuations. As was pointed out by the author, the particular case of the attraction of metallic bodies was studied earlier by Casimir.[2]

2) A more accurate form of this term is $\int (dk/k)(BR^2 + CR^{ik}R_{ik} + DR^{iklm}R_{iklm} + ER^{iklm}R_{iklm})$ where $A, B, C, D, E \sim 1$. According to Refs. 4 to 7, $\int dk/k \sim 137$, so that the third term is important for $R \gtrsim 1/137$ (in gravitational units), i.e., in the neighborhood of the singular point in Friedman's model of the universe.

1 E. M. Lifshits, ZhETF 29:94 (1954); Sov. Phys. JETP 2:73 (1954), trans.
2 H. B. G. Casimir, Proc. Nederl. Akad. Wetensch. 51:793 (1948).
3 Ya. B. Zel'dovich, ZhETF Pis'ma 6:922 (1967); JETP Lett. 6:345 (1967), trans.
4 E. S. Fradkin, Dokl. Akad. Nauk SSSR 98:47 (1954).
5 E. S. Fradkin, Dokl. Akad. Nauk SSSR 100:897 (1955).
6 L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR 102:489 (1955), trans. in Landau's Collected Papers (D. ter Haar, ed.), Pergamon Press, 1965.
7 Ya. B. Zel'dovich, ZhETF Pis'ma 6:1233 (1967).

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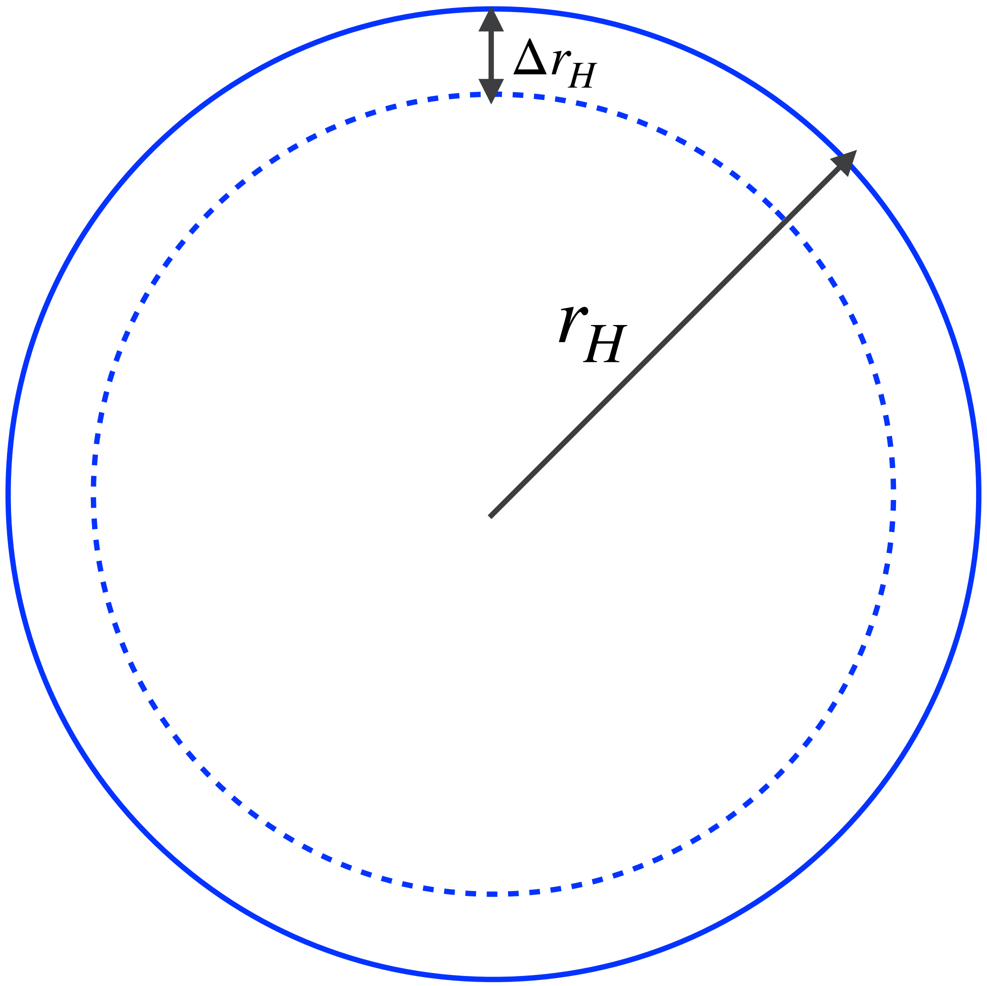
Backup slide

Hawking temperature



radius variation due to phonon emission

Associate an entropy to the sonic hole $S_H = \dfrac{A}{4L_c^2}$

Entropy variation due to horizon shrinking $\Delta S_H = 2\pi \dfrac{r_H}{L_c^2} \Delta r_H$

The phonon emission results in an entropy loss of the horizon

?

Particle in a moving medium ⟷ Particle in gravity

To which extent does it hold?

Quick recap

To have an horizon we need a transonic flow

$$v < c_s \qquad v = c_s \qquad v > c_s$$

It cannot be 3D

- We need to embed quantum effects

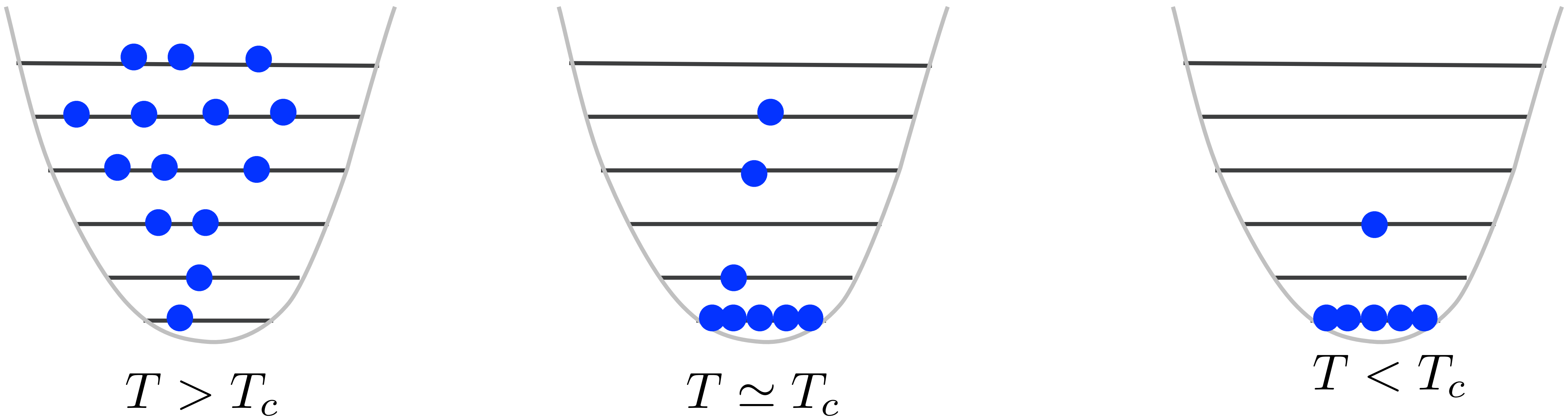
- Measure a dim phonon emission

- **How to avoid turbulence?** Use a Bose-Einstein condensate!

Bose-Einstein condensate (BEC)

It is a **coherent state of matter** with a “thermodynamically” large number of particles in the same quantum state

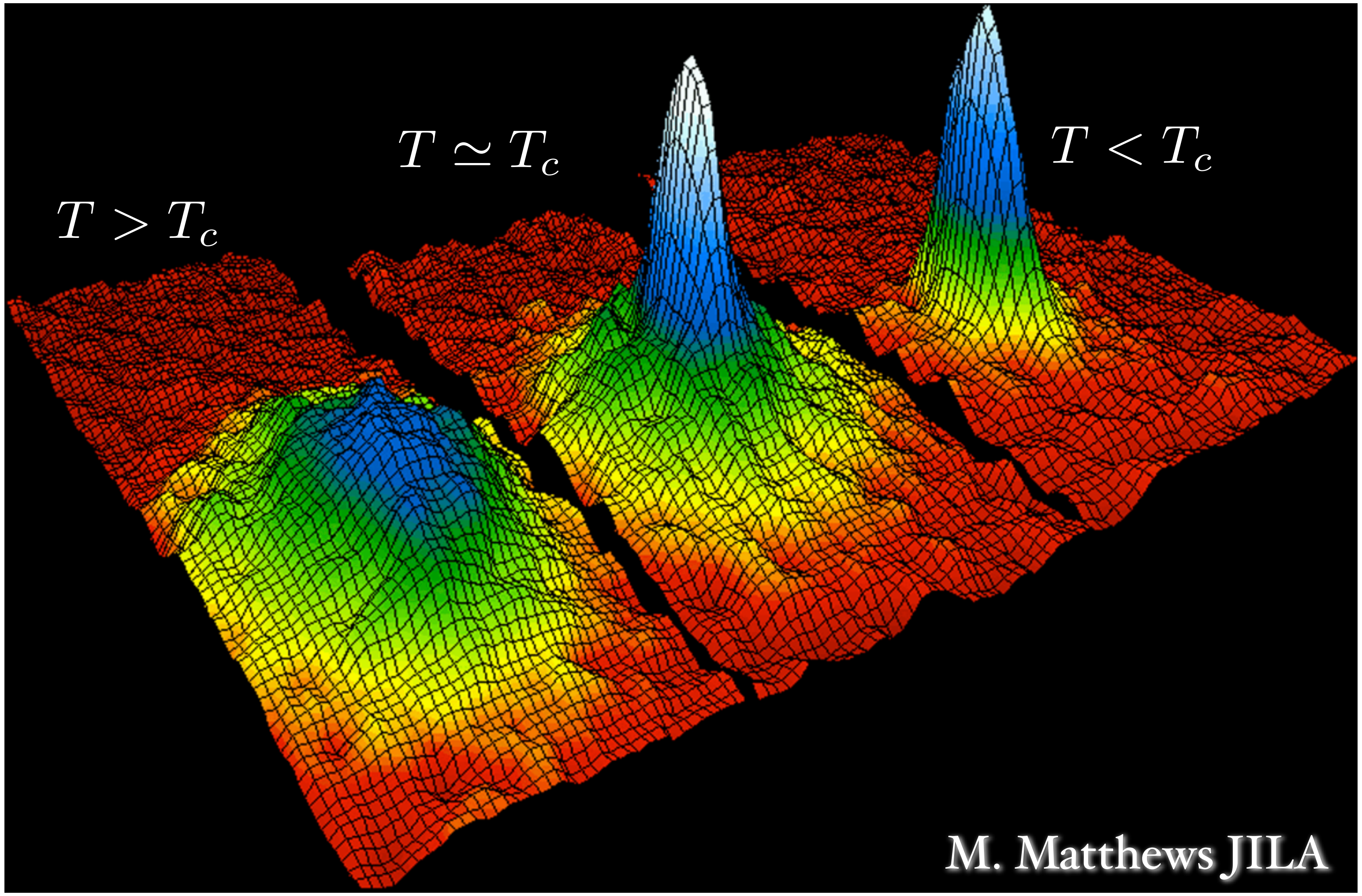
BOSONS@ low temperature in a potential well



Requirements:

1. Particles must be **bosons**
2. **Cold system**: A fight between thermal disorder and quantum coherence
3. Particles must be **stable**

Ultracold atoms in an optical trap



Velocity distribution of ^{87}Rb atoms $T_c \simeq 200$ nK

1. ^{87}Rb is **bosonic**
2. can be **cooled**
3. has a lifetime of about **10^{10} years** (the experiment lasts $\sim 10^3$s)

A simple geometrical picture

In medium phonon $\frac{d\mathbf{x}}{dt} = c_s \hat{\mathbf{n}} + \mathbf{v}$ as $c_s \hat{\mathbf{n}}\, dt = d\mathbf{x} - \mathbf{v}\, dt$

Square it $c_s^2 dt^2 - (d\mathbf{x} - \mathbf{v}dt)^2 = 0$

$\updownarrow$

Null geodesic $g_{\mu\nu} dx^\mu dx^\nu = 0$

acoustic metric $g_{\mu\nu} = \left(\begin{array}{c|c} c_s^2 - v^2 & \mathbf{v}^t \\ \hline \mathbf{v} & -I \end{array}\right)$

Note that $\sqrt{-g} = \sqrt{-\det g} = c_s$

Acoustic metric

Promoting to special relativity we have that

$$g_{\mu\nu} = \eta_{\mu\nu} + \left(c_s^2 - 1\right) v_\mu v_\nu \qquad \text{where} \qquad v_\mu = \gamma(1, -\mathbf{v})$$

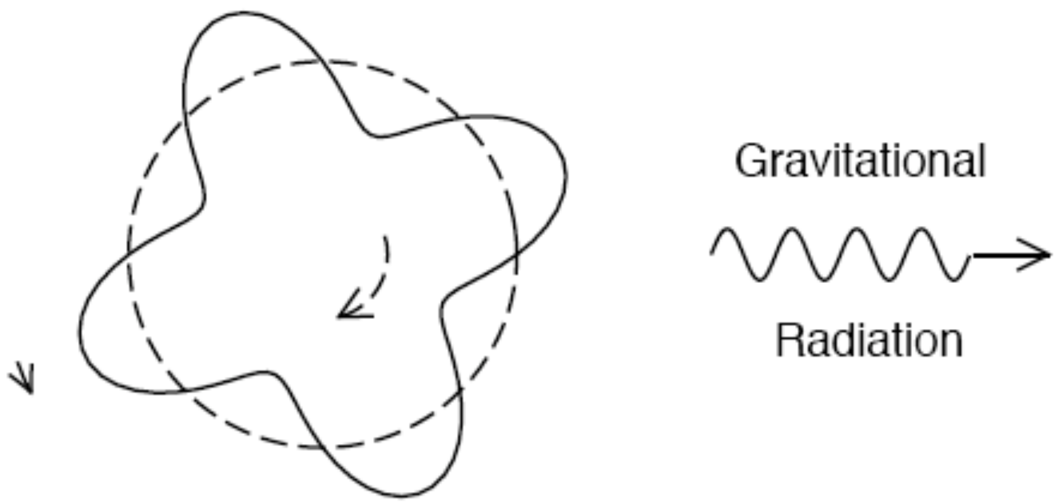
flat spacetime

in-medium effects

Description of the motion of point particles in a moving medium.

The gravity analog at work

R-mode instability of rotating stars

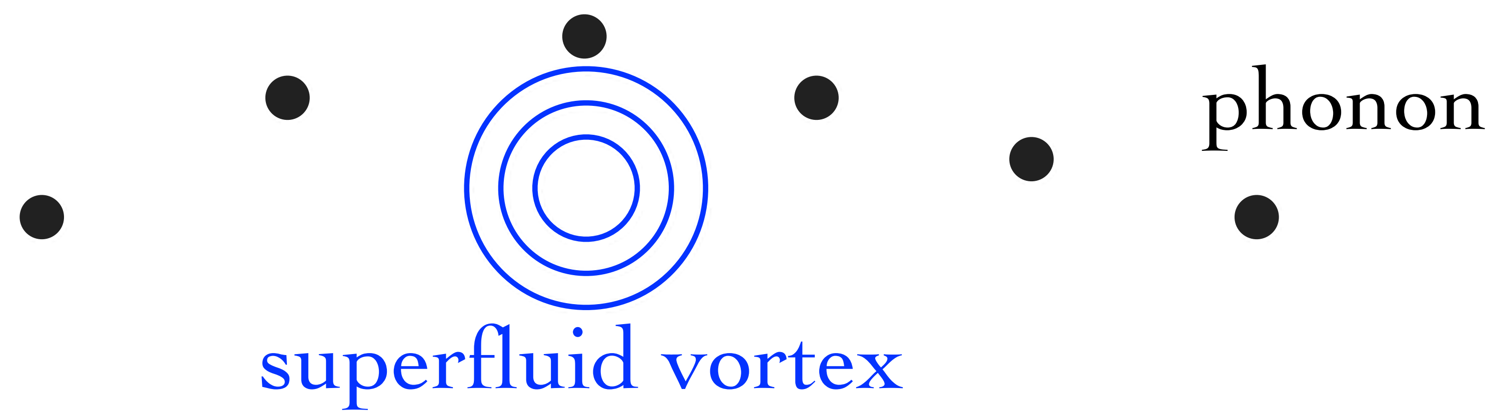


Quick spin down of pulsars

Lindblom, astro-ph/0101136
Andersson, Kokkotas
Int.J.Mod.Phys.D10:381-442,2001

Dissipative processes damp this mode

elastic phonon-vortex scattering



Analytic cross section

$$\frac{d\sigma}{d\theta} = \frac{c_s}{2\pi E} \frac{\cos^2\theta}{\tan^2\frac{\theta}{2}} \sin^2\frac{\pi E}{\Lambda}$$

MM, C. Manuel and B. A. Sa'd, Phys.Rev.Lett. 101 (2008) 241101

The realm of the analogy II

- ***Particle-wave duality***

Propagation of **massless bosons**

analogy

wave propagation in hydrodynamics

$$\frac{\partial^2 \psi}{\partial t^2} - c^2 \nabla^2 \psi = 0$$

This analogy is valid in the absence of interactions.

Including interactions the particle behavior is different: scattering, quantum corrections etc.

Gravity analogs

If we can rephrase a given problem as a geometrical problem we can look for a solution using the analogy with general relativity (GR)

Acoustic vs GR

- Sound wave propagation as a **scalar field** propagation in an **emerging GR background**

- **The background does not obey the Einstein equations, it obeys the Euler equations!**

One can certainly calculate the **Ricci and Einstein tensors** of the fluid using the acoustic metric.
However, they **do not satisfy the Hilbert-Einstein equation**.

A dim emission

$$T = \frac{\hbar c^3}{8\pi G k_B M} \simeq 6 \times 10^{-8} \left(\frac{M_\odot}{M}\right) K$$

$$T = \frac{g}{2\pi} \qquad g = \frac{M}{R_s^2} = \frac{M}{4M^2} = \frac{1}{4M} \qquad g = \left(1 - \frac{2M}{r}\right)'\Bigg|_H$$

If an acoustic hole is realizable and if it emits the Hawking radiation is it detectable?

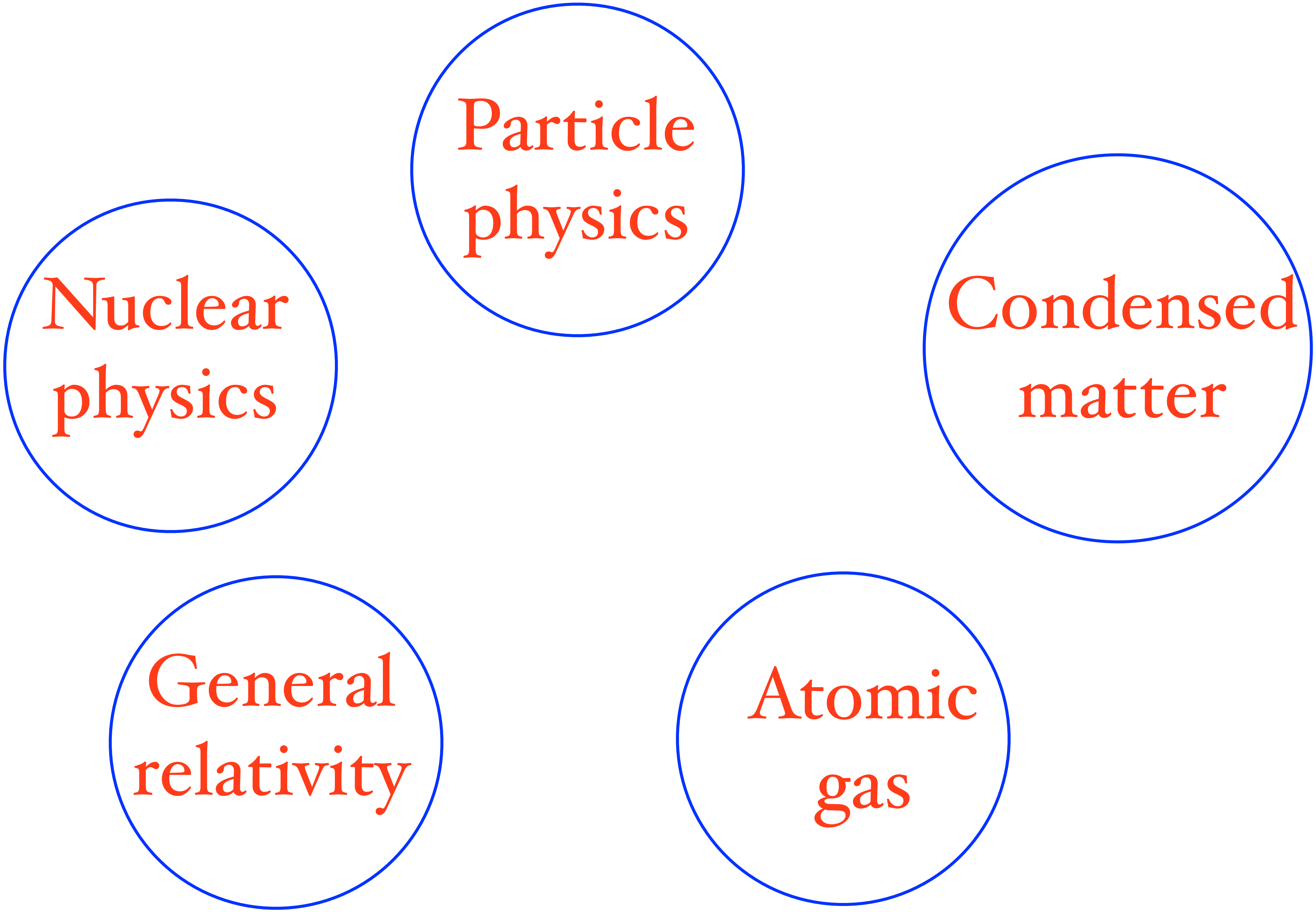
By analogy, the temperature of an acoustic hole $T = \frac{1}{2\pi} \frac{\partial |c_s - v|}{\partial n}\Bigg|_H$

$$T \simeq mc_s^2 \simeq 10^{-9} K$$

Boson isotope with a large mass: ^{87}Rb

The speed of sound is small $c_s \sim \mathrm{mm\,s^{-1}}$

The richness of physics



• Recap of the Higgs-Anderson mechanism

Physical process
Sponantenous breaking
of a local symmetry

→

Phenomenon
Gauge field acquires mass M

Range of the gauge field propagation $\sim 1/M$

Higgs mechanism
masses for $W^{\pm}$ and Z_0 bosons

analogy

Anderson effect
magnetic field screening in superconductors

The analogy is about **kinematics not dynamics**

The analogy works in ***restricted energy regions:*** at high energies one sees the microphysics.

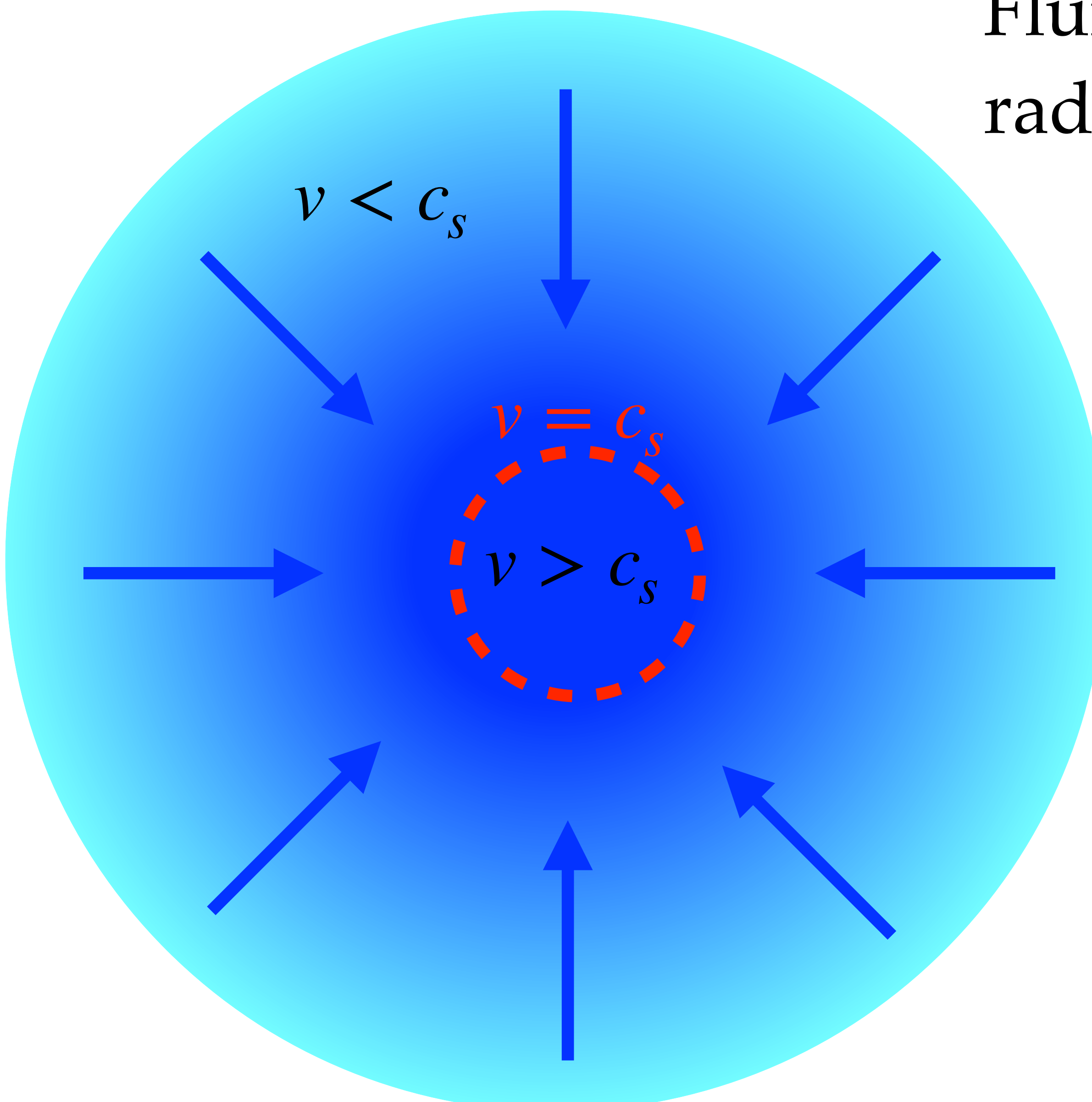
Schwarzschild acoustic metric?

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 \sin^2\theta d\phi^2)$$

Schwarzschild radius $R_s = 2M$

Fluid with radial flow

Does the fluid analog exist?



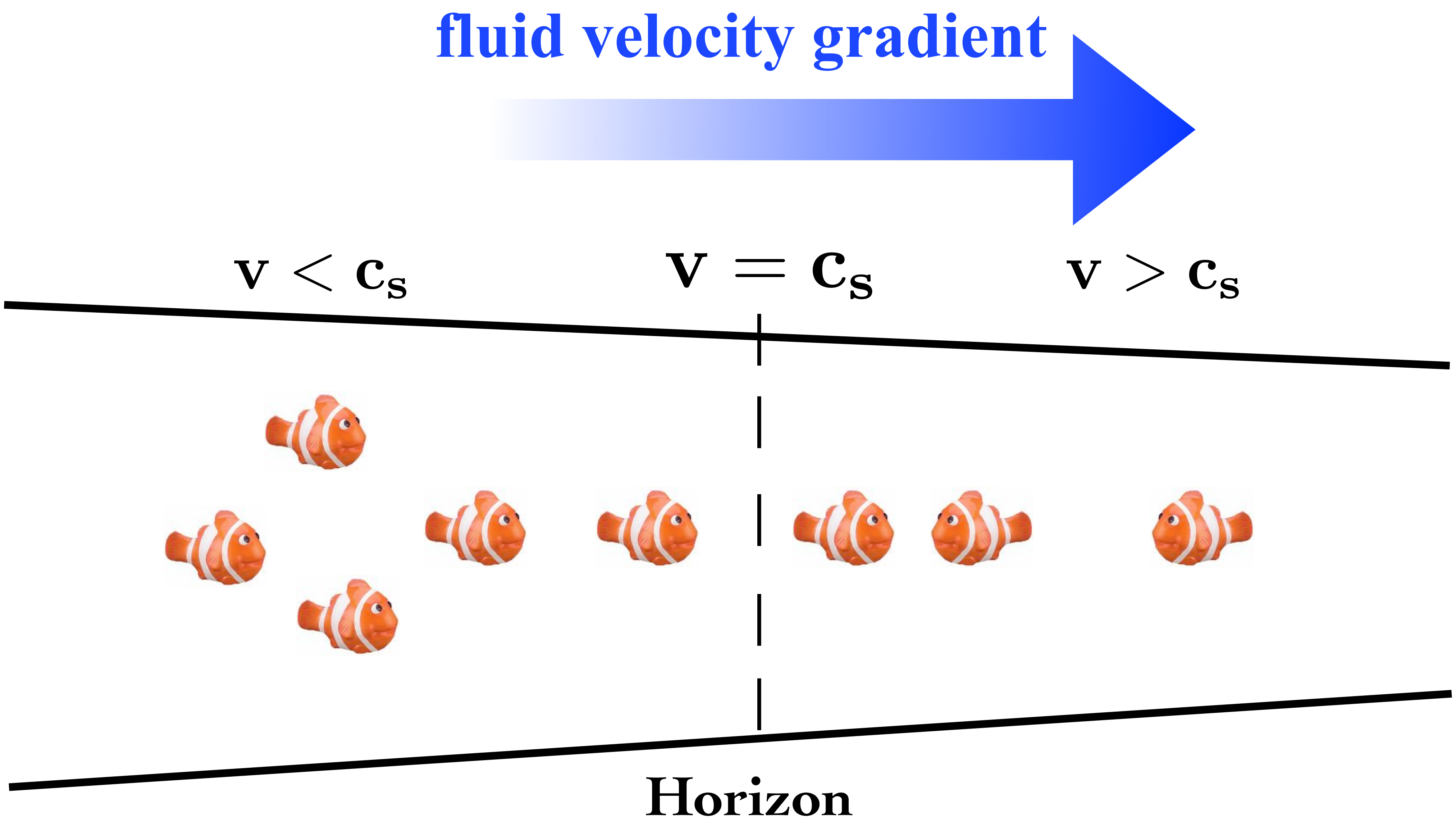
Gravity analogs

W. Unruh, Experimental black hole evaporation, Phys.Rev.Lett. 46 (1981) 1351–1353

M. Visser, Acoustic black holes: Horizons, ergospheres, and Hawking radiation, Class. Quant. Grav. 15 (1998) 1767–1791

C. Barcelo, S. Liberati, and M. Visser, Analogue gravity, Living Rev. Rel. 8 (2005) 12,

Horizon



fluid velocity gradient
$\mathbf{v} < \mathbf{c_s}$
$\mathbf{v} = \mathbf{c_s}$
$\mathbf{v} > \mathbf{c_s}$
Horizon

Lagrangian formulation

Consider the Lagrangian for a scalar field $\mathscr{L} \equiv \mathscr{L}(\phi, \partial_\mu \phi)$

Scale separation

background — "phonon"

$$\phi(x) = \phi_0(x) + \epsilon \phi_1(x)$$

long-wavelength — "short-wavelength"

Expand the action

$$S[\phi] = S[\phi_0] + \frac{\epsilon^2}{2} \int d^4x \left[\frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi_0)\partial(\partial_\nu \phi_0)} \partial_\mu \phi_1 \partial_\nu \phi_1 + \left(\frac{\partial^2 \mathcal{L}}{\partial \phi_0 \partial \phi_0} - \partial_\mu \frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi_0)\partial \phi_0} \right) \phi_1 \phi_1 \right]$$

Phonon's action $$S[\phi_1] = \frac{1}{2} \int d^4x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi_1 \partial_\nu \phi_1 - M^2_{\phi_0} \phi_1 \phi_1 \right)$$

BH thermodynamics

A particle/nuclear physics perspective

WKB tunneling amplitude $\Gamma \sim e^{-2\,\mathrm{Im}\,S}$

using the geodesic equation $\mathrm{Im}\,S = 4\pi\omega M$

$$\Gamma \sim e^{-8\pi M\omega} = e^{-\omega/T} \qquad T = \frac{1}{8\pi M} = \frac{g}{2\pi}$$

By analogy, the temperature of an acoustic hole $T = \frac{1}{2\pi} \frac{\partial |c_s - v|}{\partial n}\Big|_H$

$$T \simeq mc_s^2 \simeq 10^{-9}K$$