Gravitational Waves and Black Hole perturbations in Acoustic Analogues

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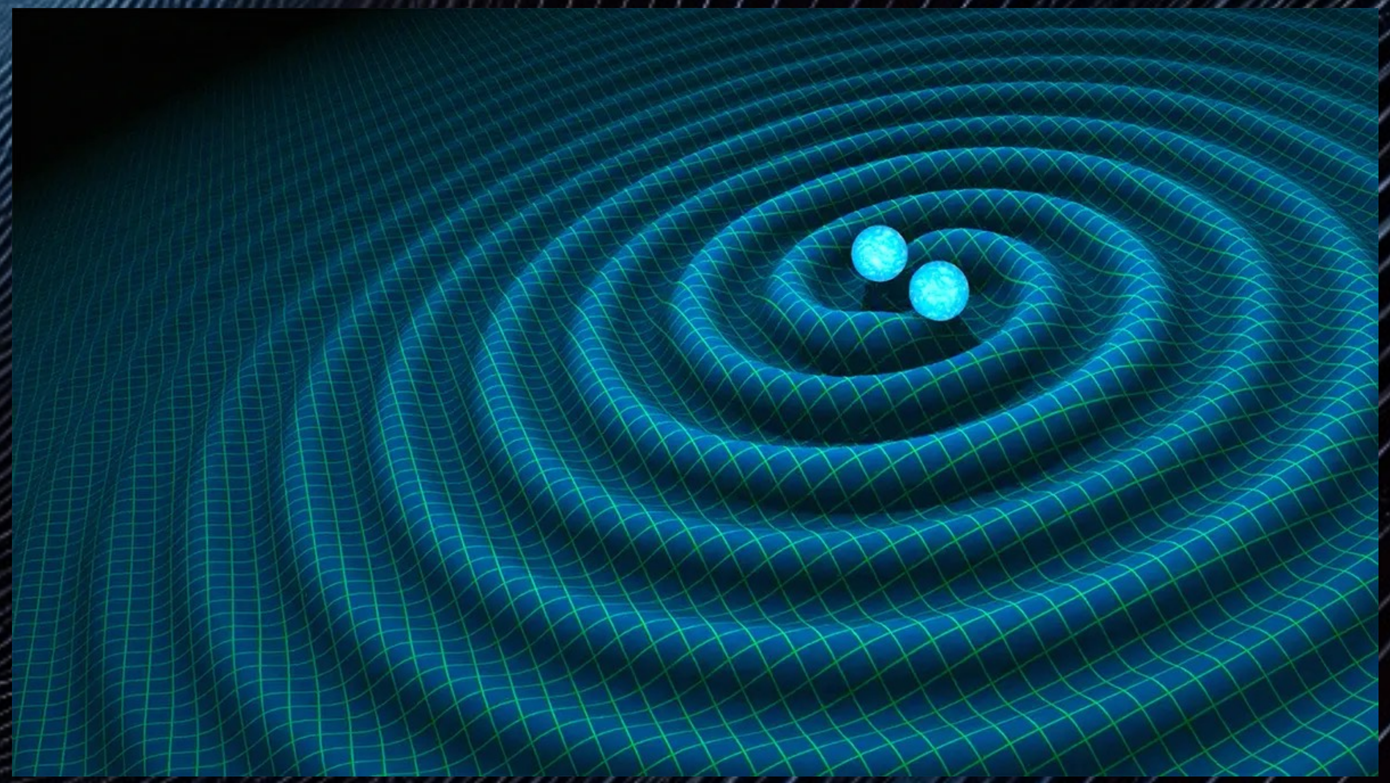
Based on *C. Coviello et al., arXiv:2410.00264 (2024)*

PRESENTATION OUTLINE

- Analogue Gravity

- Results → analogue gravitational wave; acoustic black hole perturbation

- Conclusions and future perspectives



From R. Hurt/Caltech-JPL

ANALOGUE GRAVITY

ANALOGUE GRAVITY

- **reproduce gravitational phenomena:** light waves in a curved spacetime ↕ sound waves in a moving fluid

- **experimentally accessible** ⟶ Hawking radiation

  S. W. Hawking, Nat. 1974

- **universal properties of black holes?**

C. Barceló, S. Liberati, M. Visser, Living Rev. Relativ. 2011

EMERGENT ACOUSTIC METRIC

Continuity

$$\partial_t mn + \nabla \cdot (mn\mathbf{v}) = 0$$

Euler

$$mn[\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v}] = \mathbf{f}$$

linearize around some background

$$n = n_0 + \epsilon n_1$$

$$p = p_0 + \epsilon p_1$$

$$\theta = \theta_0 + \epsilon \theta_1$$

inviscid $\mathbf{f} = -\nabla p$

irrotational $\mathbf{v} = \nabla \theta / m$

$$-\partial_t \left(\frac{n_0}{c_s^2 m} \left(\partial_t \theta_1 + \mathbf{v}_0 \cdot \nabla \theta_1 \right) \right) + \nabla \cdot \left(\frac{n_0}{m} \nabla \theta_1 - \frac{n_0}{c_s^2 m} \mathbf{v}_0 \left(\partial_t \theta_1 + \mathbf{v}_0 \cdot \nabla \theta_1 \right) \right) = 0$$

W. Unruh, Phys. Rev. Lett. 1981

EMERGENT ACOUSTIC METRIC

$$-\partial_t\left(\frac{n_0}{c_s^2 m}\left(\partial_t\theta_1+\mathbf{v}_0\cdot\nabla\theta_1\right)\right)+\nabla\cdot\left(\frac{n_0}{m}\nabla\theta_1-\frac{n_0}{c_s^2 m}\mathbf{v}_0\left(\partial_t\theta_1+\mathbf{v}_0\cdot\nabla\theta_1\right)\right)=0$$

rewriting

$$\square\theta_1=\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\theta_1\right)=0$$

with

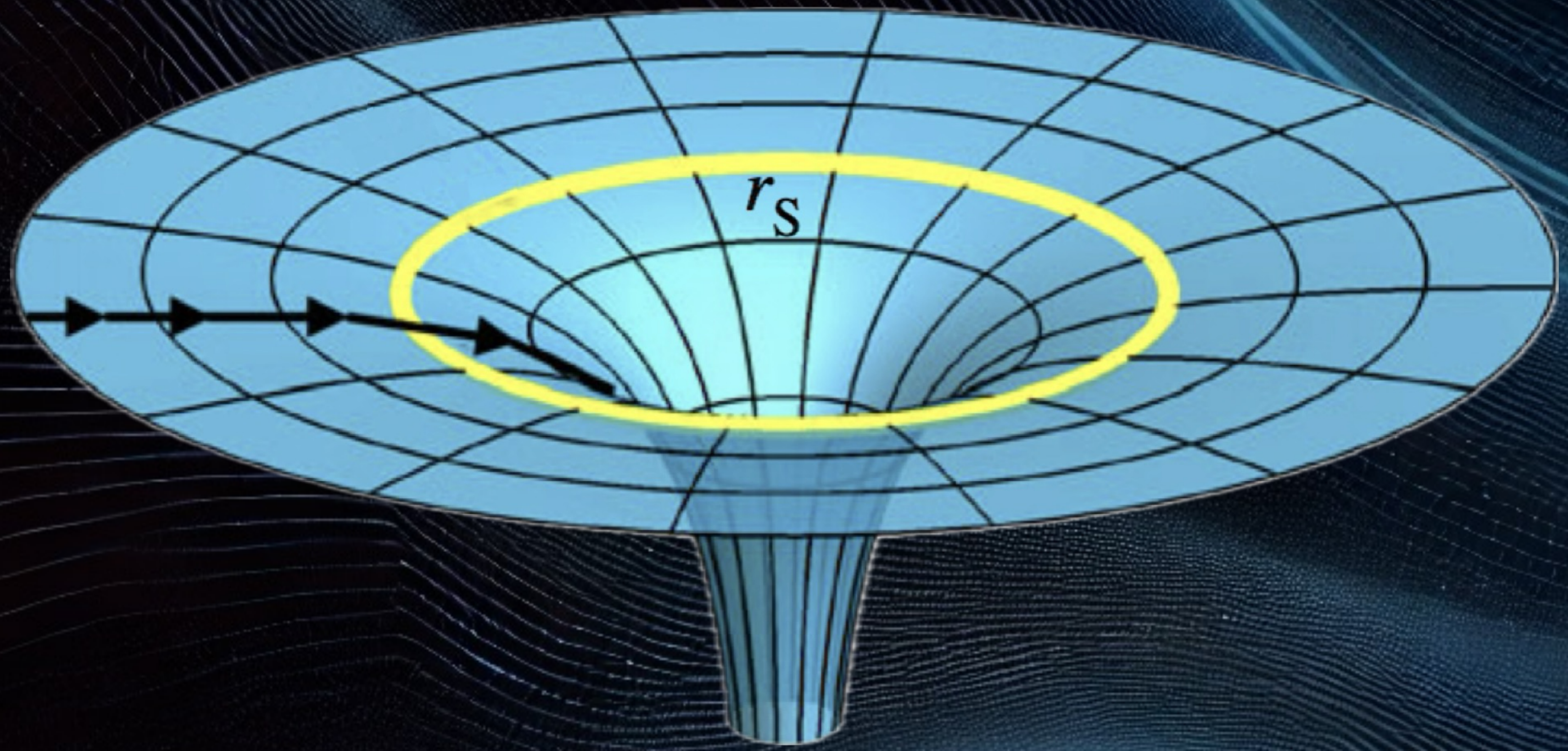
acoustic metric

$$g_{\mu\nu}(t,\mathbf{x})=\frac{n_0}{mc_s}\begin{pmatrix}-(c_s^2-v_0^2) & -(v_0)_j\\ -(v_0)_i & \delta_{ij}\end{pmatrix}$$

W. Unruh, Phys. Rev. Lett. 1981

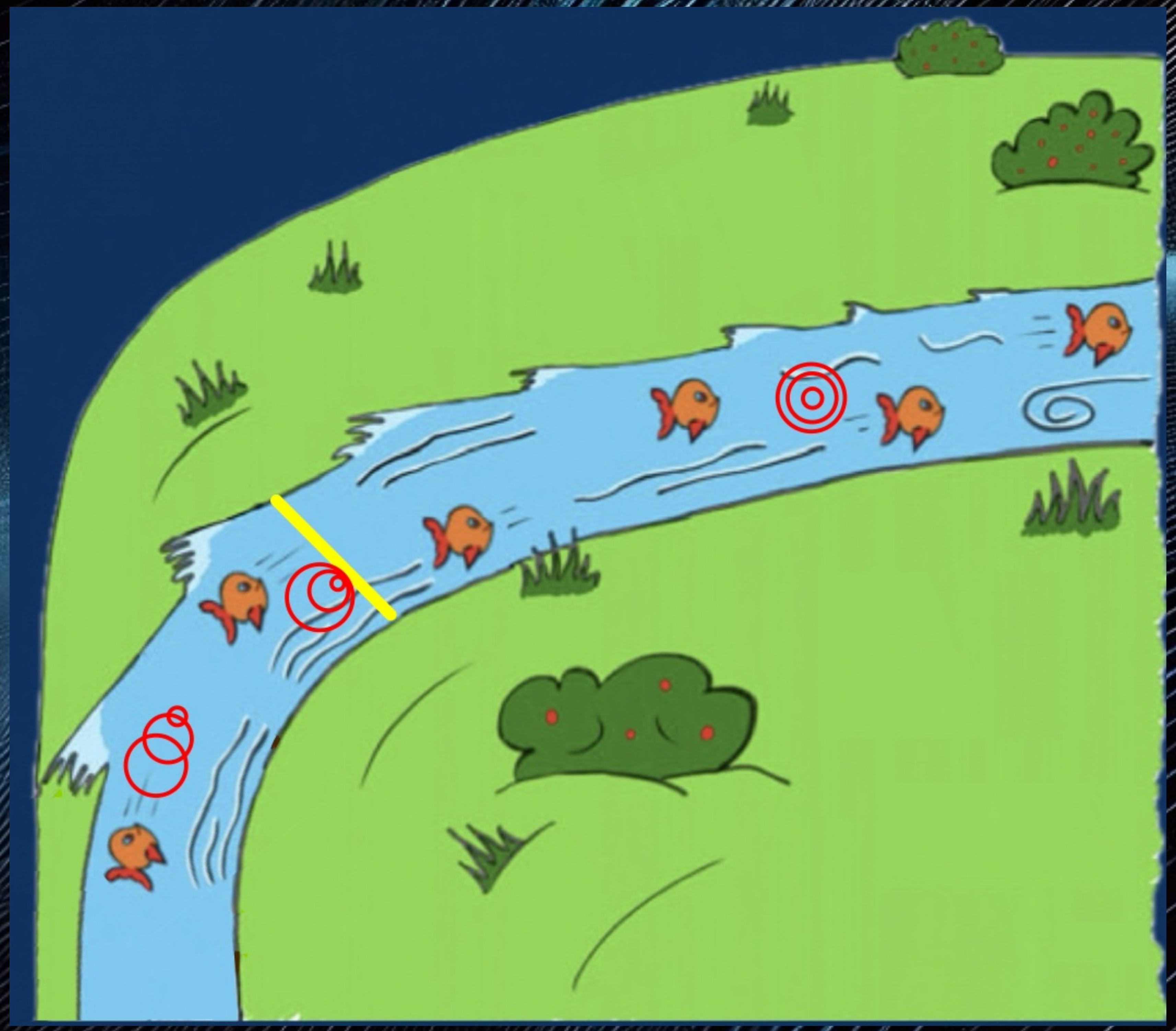
ANALOGUE BLACK HOLE

black hole



analogue black hole

W. Unruh, Phys. Rev. Lett. 1981



photons crossing the event horizon ⟷ phonons transitioning subsonic→supersonic

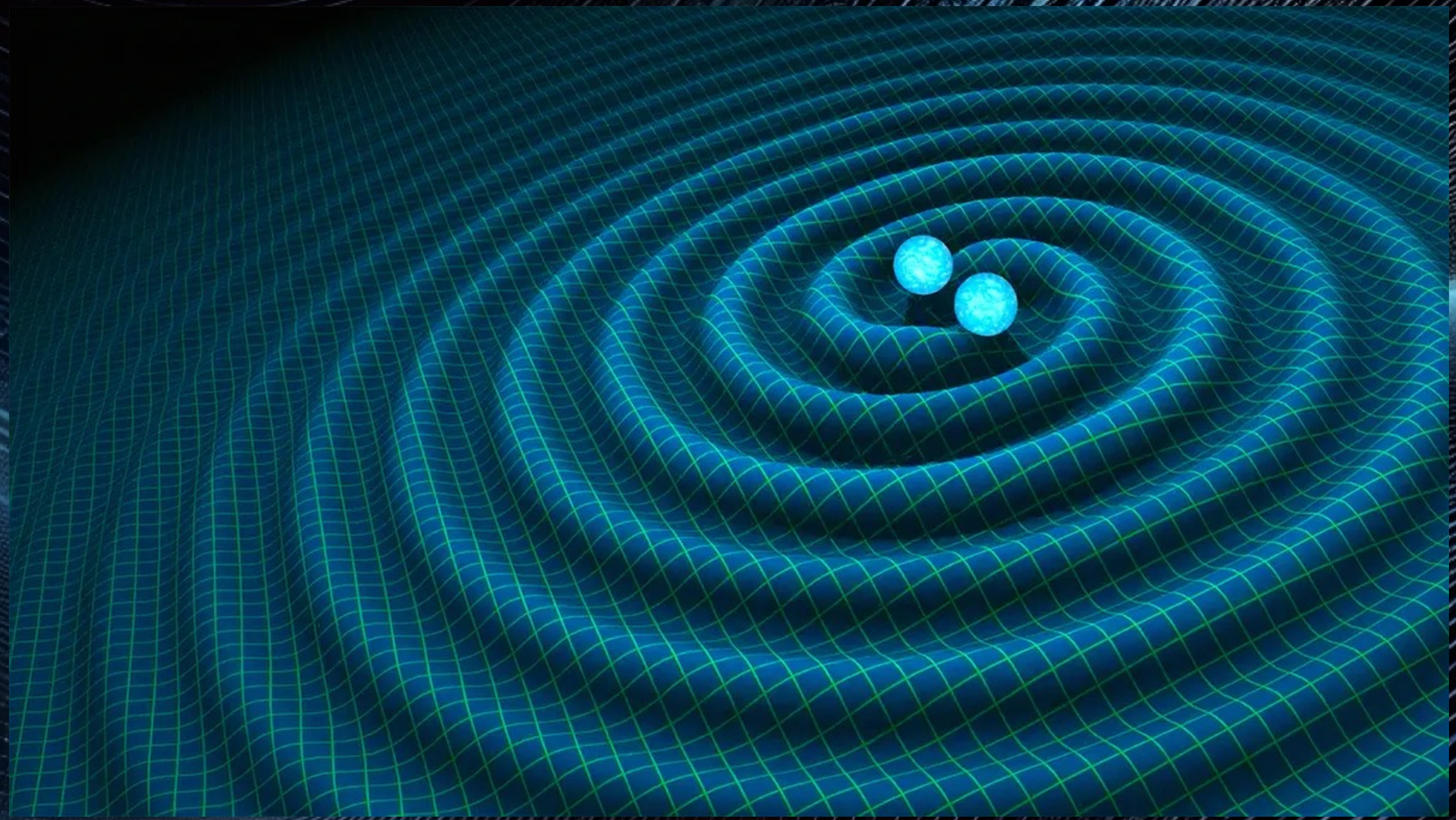
Adapted from R. Aguero-Santacruz, D. Bermudez, Phyl. Trans. R. Soc. A 2020

RESULTS

PURPOSE

Acoustic horizon excited by a gravitational wave-like perturbation

dynamical situation



From R. Hurt/Caltech-JPL

Steps:

1. Analogue of a gravitational wave
2. Introduce the acoustic horizon

STEP 1:

ANALOGUE GRAVITATIONAL WAVE

ACOUSTIC METRIC IN A BOSE-EINSTEIN CONDENSATE

Gross-Pitaevskii

$$i\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext} + \frac{4\pi a\hbar^2}{m}\psi^\dagger\psi\right)\psi$$

$$\psi = \sqrt{n_c}e^{-i\theta/\hbar}$$

hydrodynamic regime

linearization

phonon's equation of motion

$$\Box\,\delta\theta = 0$$

$$g^{(an)}_{\mu\nu} = \frac{n_c}{mc_s}\begin{pmatrix} -(c_s^2 - v^2) & -v_j \\ -v_i & \delta_{ij} \end{pmatrix}$$

emergent acoustic metric

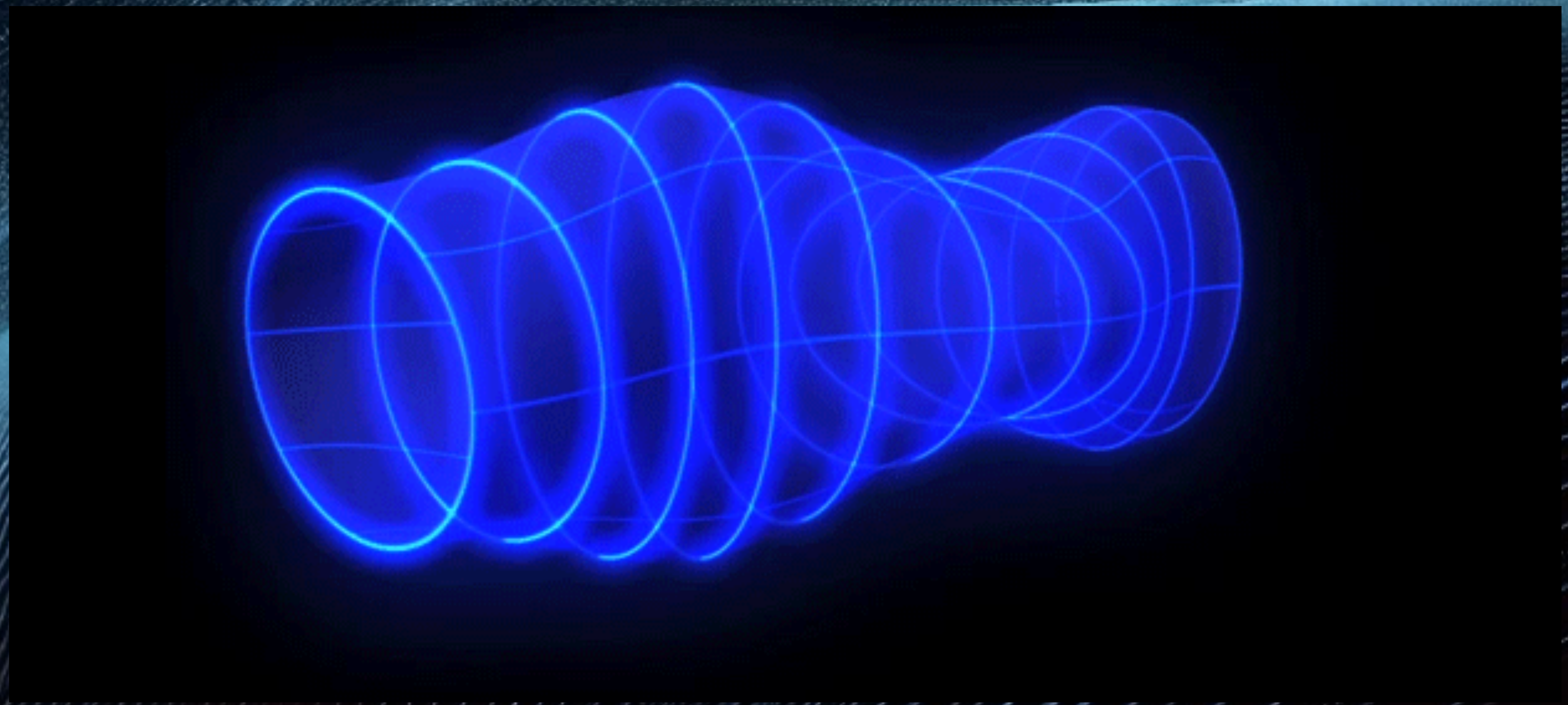
C. Barceló, S. Liberati, M. Visser, Living Rev. Relativ. 2011

GRAVITATIONAL WAVES

Einstein's equations in vacuum $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$ — $g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}$ linearized theory

Gravitational wave moving along z in the TT gauge

$$h^{TT}_{\mu\nu}(t,z) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cos\left(\omega(t - z/c)\right)$$



gauge symmetry $x^\mu \rightarrow x'^\mu = x^\mu + \epsilon \zeta^\mu$, implying $h_{\mu\nu}(x^\rho) \rightarrow \boxed{h'_{\mu\nu}(x'^\rho)} = h_{\mu\nu} - \epsilon(\partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu)$

GOAL: ANALOGUE GRAVITATIONAL WAVE

goal: $g_{\mu\nu}^{(an)} = \eta_{\mu\nu}^{(an)} + \epsilon \boxed{h_{\mu\nu}^{(an)}}$

$\eta_{\mu\nu}^{(an)}$ ↓ Minkowski

$h_{\mu\nu}^{(an)}$ ↓ gravitational wave

scale separation

↓

two types of fluctuations:

- background's ⟶ metric perturbations
- phonons ⟶ propagate on top of acoustic metric

METHOD

- Perturb background quantities
$$\begin{aligned} n_c &\to n_c + \epsilon \delta n_c \\ v_i &\to v_i + \epsilon \delta v_i \\ c_s &\to c_s + \epsilon \delta c_s \end{aligned} \longrightarrow g_{\mu\nu}^{(an)} = \eta_{\mu\nu}^{(an)} + \epsilon h_{\mu\nu}^{(an)}$$

- Ad hoc coordinate transformations have been invented to match $h_{\mu\nu}$ with $h_{\mu\nu}^{(an)}$

- Align condensate characteristics for shared math expressions between $h_{\mu\nu}^{(an)}$ and $h_{\mu\nu}$

PHYSICAL REQUIREMENTS

Check physical requirements:

- *irrotational condition* $\nabla \times \mathbf{v} = 0$
- *continuity equation* $\partial_t n_c + \nabla \cdot (n_c \mathbf{v}) = 0$
- Euler's *equation* $m\partial_t \mathbf{v} + \nabla \left(m\frac{v^2}{2} + V_{ext} + \frac{4\pi a \hbar^2}{m} n_c - \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n_c}}{\sqrt{n_c}} \right) = 0$

RESULT

constant and uniform

background metric $\eta_{\mu\nu}^{(an)} = \frac{n_c}{mc_s}\text{diag}(-1,+1,+1,+1)$ if $\mathbf{v}=\mathbf{0}$

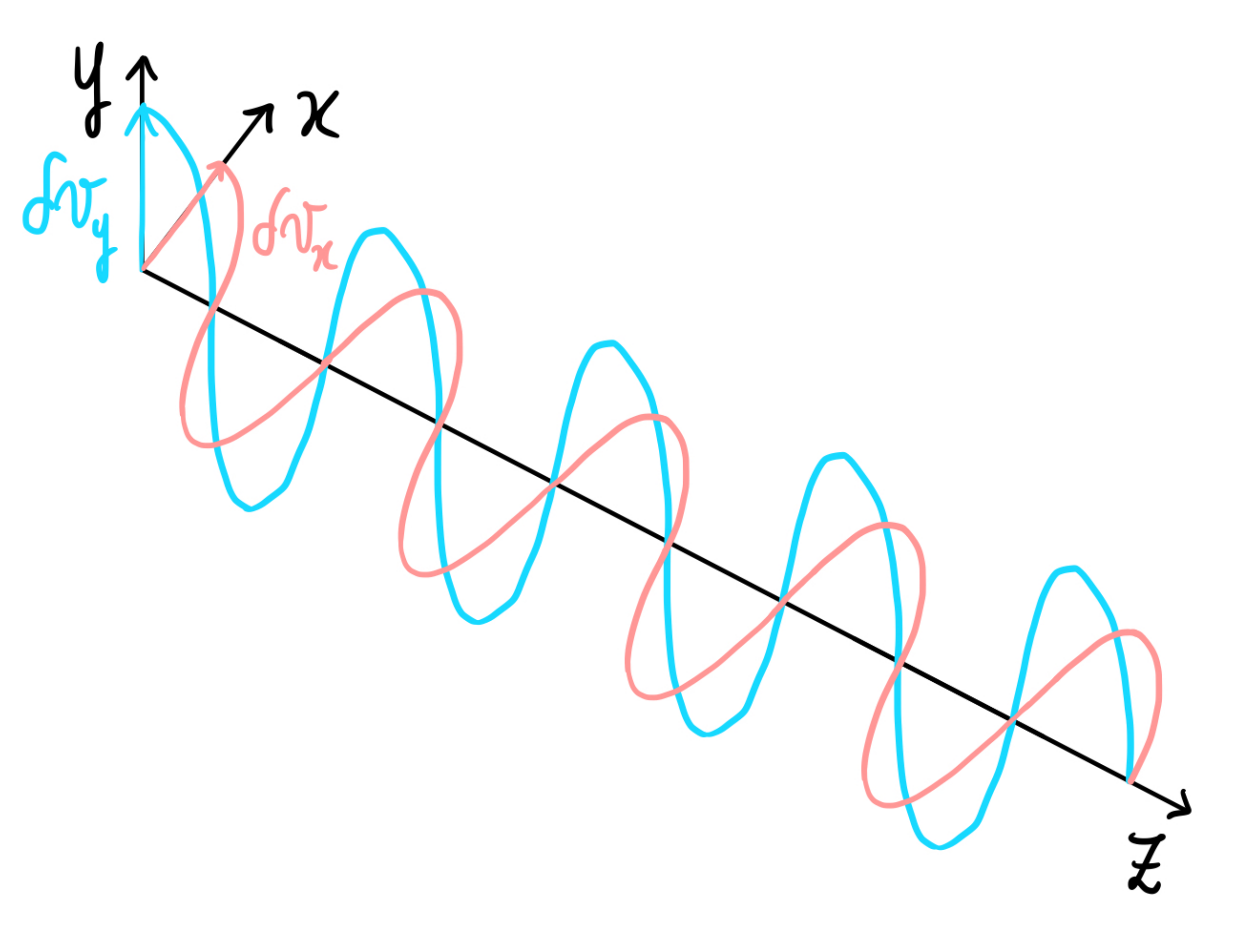
perturbation metric

$$h_{\mu\nu}^{(an)} = \frac{n_c}{mc_s}\begin{pmatrix} 0 & -\frac{\delta v_x}{c_s} & -\frac{\delta v_y}{c_s} & 0 \\ -\frac{\delta v_x}{c_s} & 0 & 0 & 0 \\ -\frac{\delta v_y}{c_s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\frac{\delta v_x}{c_s} = l(fh_+ + gh_\times)\cos(\omega(t - z/c_s))$$

$$\frac{\delta v_y}{c_s} = b(fh_+ + gh_\times)\cos(\omega(t - z/c_s))$$

$l, b, f, g \in \mathbb{R}$



the same of $h_{\mu\nu}$ of a gravitational wave written in a given gauge

with $x\omega/c_s \ll 1$ *and* $y\omega/c_s \ll 1$

STEP 2:

ACOUSTIC BLACK HOLE PERTURBATION

METHOD

- Choose a geometry for the acoustic black hole ⟶ *cylindrical*

- Extend the gravitational wave-like perturbation in the new geometry

- Check physical requirements: ⟶
  - *irrotational condition*
  - *continuity equation*
  - *Euler's equation*

- Study the deformation of the horizon

No relation with astrophysical objects

ACOUSTIC BLACK HOLE

cylindrical geometry → more general calculation
experimentally accessible with Bose-Einstein condensates

M. Visser, Class. Quantum Grav. 1998

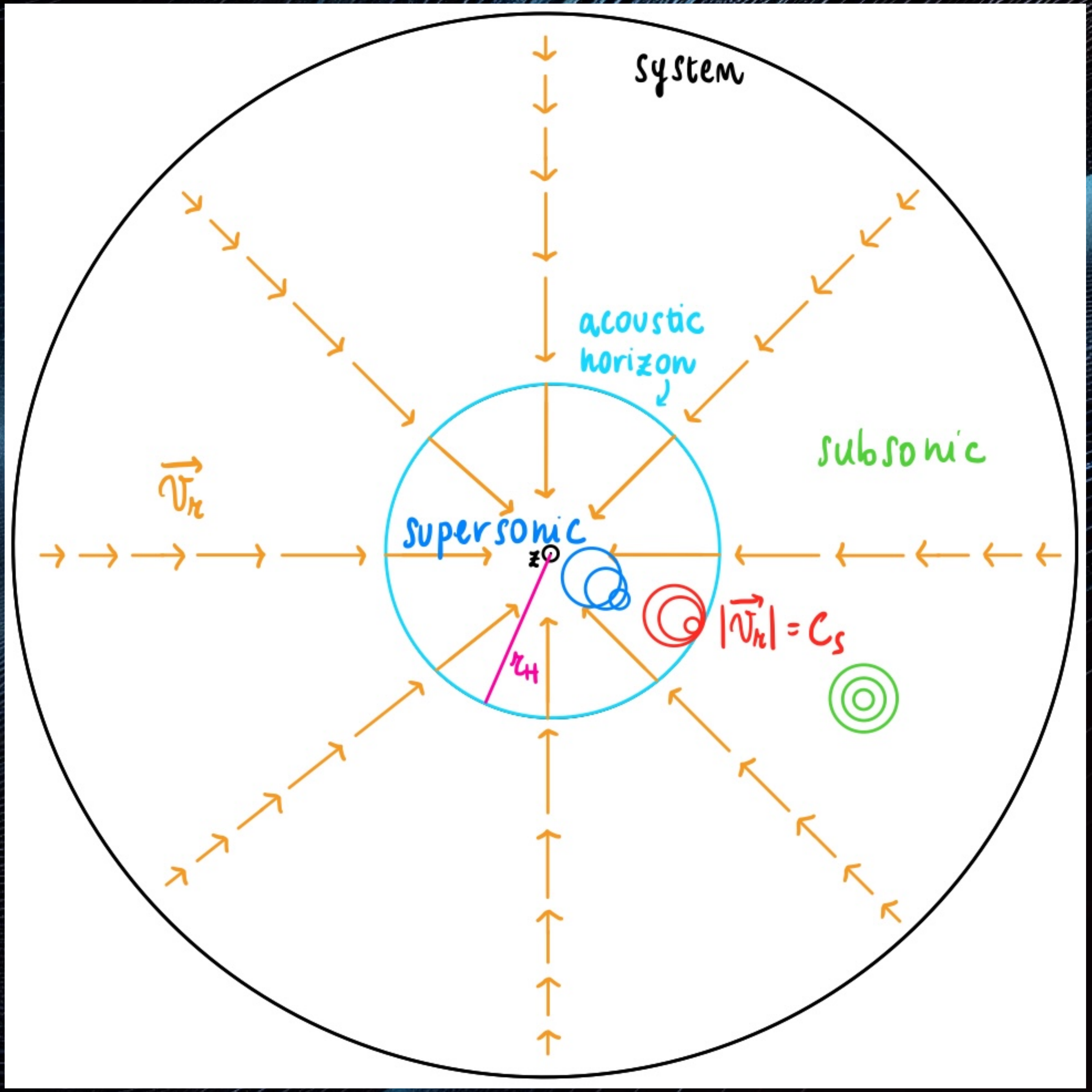
$$ds^2 \propto -c_s^2 dt^2 + \left(dr - \frac{A}{r}dt\right)^2 + r^2 d\theta^2 + dz^2$$

with $\mathbf{v} = \frac{A}{r}\hat{r}$ and n_c, c_s constant and uniform

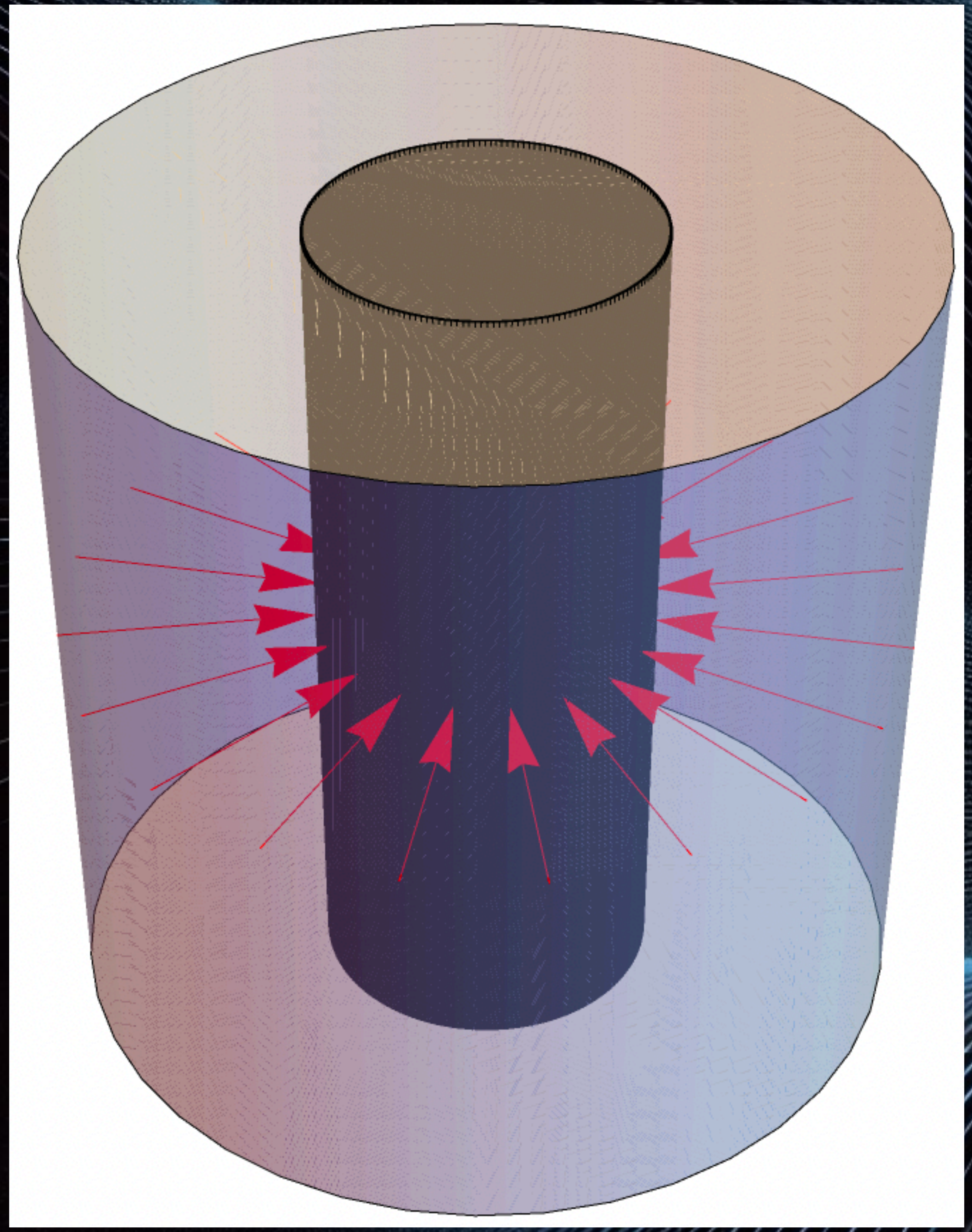
the acoustic event horizon forms where $|v_r| = c_s$

↓

$$r_H = |A|/c_s$$



GRAVITATIONAL WAVE-LIKE PERTURBATION



$$\frac{\delta v_\phi}{c_s} = \frac{r_0}{r} b(fh_+ + gh_\times)\cos(\omega(t - r/c_s))$$

$f, g > 0$

$$\frac{\delta v_z}{c_s} = l(fh_+ + gh_\times)\cos(\omega(t - r/c_s))$$

PHYSICAL REQUIREMENTS

+ dimensionless coordinates $(\tilde{r}, \tilde{z} \in [0,1])$, $\omega_r = 2\pi c_s / L_r$

analogue "GW" $$\frac{\delta v_z}{c_s} = \frac{1}{2\pi}(fh_+ + gh_\times)\cos(\omega(t - 2\pi\tilde{r}/\omega_r)), \quad \delta v_\phi = 0$$

induced perturbation $$\frac{\delta v_r}{c_s} = \tilde{z}\frac{\omega}{\omega_z}(fh_+ + gh_\times)\sin(\omega(t - 2\pi\tilde{r}/\omega_r))$$

→ deforms the horizon

PERTURBED ACOUSTIC HORIZON

the perturbed acoustic horizon forms where:

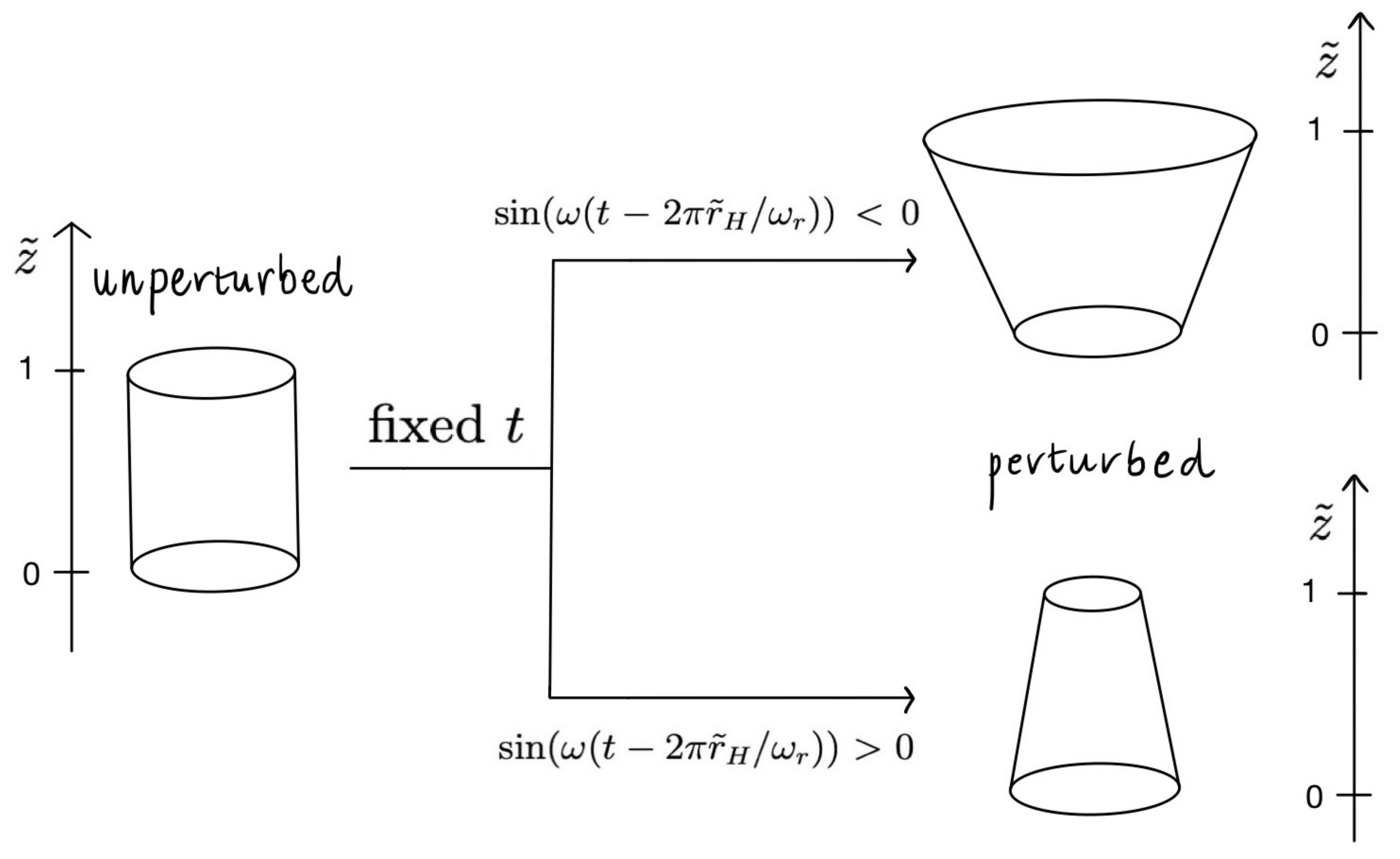
$$|v_r + \epsilon \delta v_r|_{\tilde{r}_H^{new}} = c_s$$

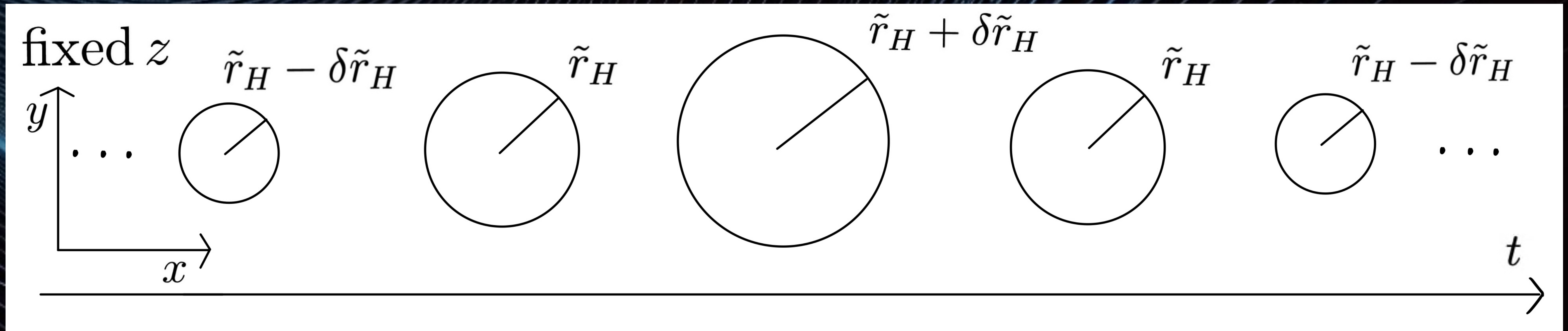
with $\tilde{r}_H^{new} = \tilde{r}_H + \epsilon\delta\tilde{r}_H , \quad \tilde{r}_H = \dfrac{|A|}{c_s L_r} ,$

$$\frac{\delta\tilde{r}_H}{\tilde{r}_H} = -\,\tilde{z}\frac{\omega}{\omega_z}(fh_+ + gh_\times)\sin(\omega(t - 2\pi\tilde{r}_H/\omega_r))$$

↓

tilted oscillating horizon



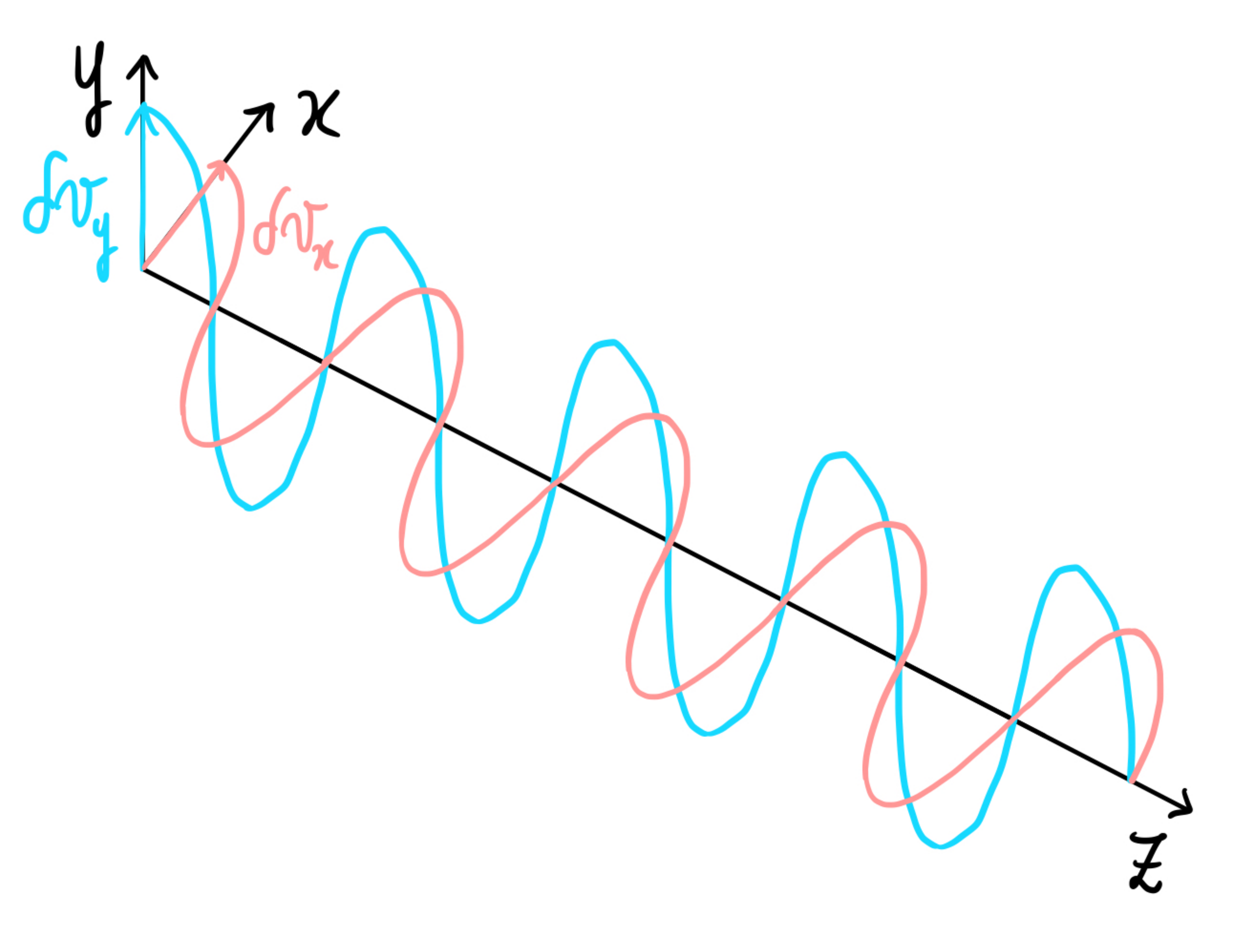


CONCLUSIONS AND FUTURE PERSPECTIVES

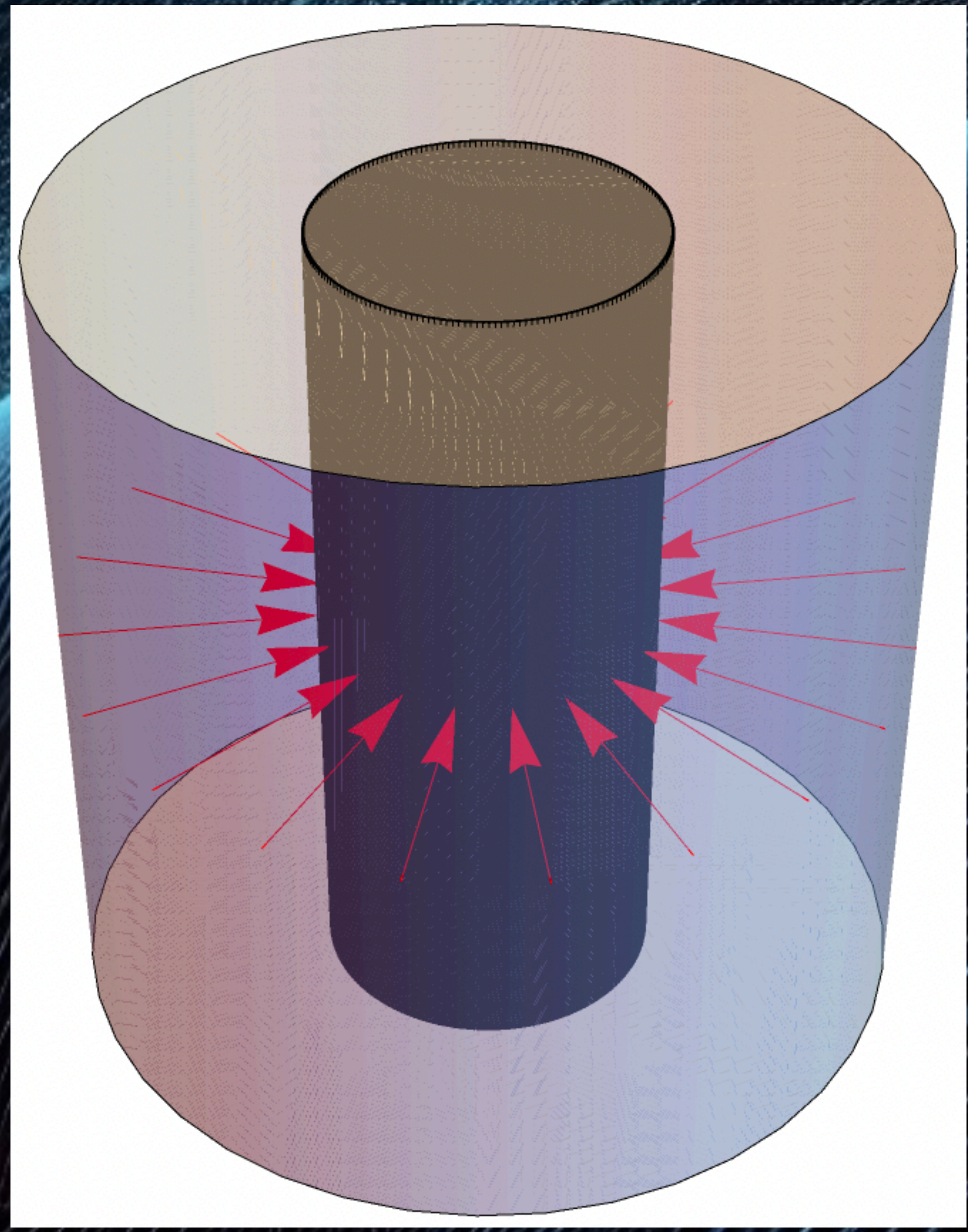
CONCLUSIONS

We have obtained the following results:

1. analogue gravitational wave



2. perturbed acoustic black hole



Laboratory-reproducible system where to study how an acoustic horizon responds to perturbations closely modeled after GWs

FUTURE PERSPECTIVES

- **Dissipative properties of acoustic horizons**

$\frac{\eta}{s} \geq \frac{1}{4\pi}$ Kovtun-Son-Starinets bound $\longrightarrow$ saturated? universal?

P. Kovtun, D. T. Son, A. O. Stearinets, JHEP 2003

- **Reflectivity properties of acoustic horizons**

for astrophysical black holes: N. Oshita et al., JCAP 2020

- **Quantization of our spin-2 perturbation of the metric?**

- **Quasi-normal modes of the analogue black hole**

for astrophysical black holes: B. Toshmatov et al., Phys. Rev. D, 2015

- **Gravitational memory?**

M. Favata, Class. Quantum Gravity., 2010

- **Experimental implementation**

A. L. Gaunt et al., Phys. Rev. Lett. 2013

THANK YOU FOR YOUR ATTENTION !

BACKUP SLIDES

RESULT I: ANALOGUE GW

$$\frac{\delta v_x}{c_s} = l(fh_+ + gh_\times)\cos(\omega(t - z/c_s))$$

$$\frac{\delta v_y}{c_s} = b(fh_+ + gh_\times)\cos(\omega(t - z/c_s))$$

continuity: homogeneous n_c

Euler:

$$\delta V_{ext} = m\omega c_s(lx + by)(fh_+ + gh_\times)\sin(\omega(t - z/c_s))$$

irrotationality:

$$\frac{\delta v_z}{c_s} = \frac{\omega}{c_s}(lx + by)(fh_+ + gh_\times)\sin(\omega(t - z/c_s))$$

negligible

$h^{(an)}_{\mu\nu}$ the same of $h_{\mu\nu}$ of a gravitational wave written in the gauge obtained from the TT using:

$$\zeta^\mu = \left(-(lx + by)(fh^{TT}_+ + gh^{TT}_\times), \frac{1}{2}(xh^{TT}_+ + yh^{TT}_\times), \frac{1}{2}(xh^{TT}_\times - yh^{TT}_+), 0\right)$$

with $x\omega/c_s \ll 1$ and $y\omega/c_s \ll 1$

RESULT II: ANALOGUE BH+GW

$$\delta S = \frac{2\pi}{3} n_c \frac{\bar{z}\bar{r}^3}{\bar{r}_H^2} \frac{\omega^3}{\omega_r \omega_z} (f h_+ + g h_\times) \times \left(-1 - \frac{\bar{r}_H}{\bar{r}} \right) \sin(\omega(t - 2\pi\bar{r}/\omega_r))$$

$$\frac{\delta n_c}{n_c} = \frac{\bar{z}\bar{r}}{\bar{r}_H} \frac{\omega}{\omega_z} (f h_+ + g h_\times) \sin(\omega(t - 2\pi\bar{r}/\omega_r)) + \frac{2\pi}{3} \frac{\bar{z}\bar{r}^3}{\bar{r}_H^2} \frac{\omega^2}{\omega_r \omega_z} (f h_+ + g h_\times) \cos(\omega(t - 2\pi\bar{r}/\omega_r))$$

$$\frac{\delta a}{a} = - \frac{\delta n_c}{n_c} \longrightarrow \delta c_s = 0$$

$$\delta V_{\text{ext}} = \left(\frac{3}{8\pi} \frac{\bar{z}(\bar{r} - \bar{r}_H)}{\bar{r}_H^2} \frac{\omega^2 \omega_r}{\omega_z c_s^2} - \frac{\pi}{6} \frac{\bar{z}\bar{r}^3}{\bar{r}_H^2} \frac{\omega^4}{\omega_z \omega_r c_s^2} \right) \frac{\hbar^2}{m} (f h_+ + g h_\times) \cos(\omega(t - 2\pi\bar{r}/\omega_r)) - \left(\bar{z} \frac{c_s^2 \omega}{\omega_z} m + \frac{1}{16\pi^2} \frac{\bar{z}}{\bar{r}\bar{r}_H} \frac{\omega \omega_r^2}{\omega_z c_s^2} \frac{\hbar^2}{m} + \frac{\bar{z}\bar{r}}{\bar{r}_H^2} \left(\frac{7}{12}\bar{r} - \frac{1}{4}\bar{r}_H \right) \frac{\omega^3}{\omega_z c_s^2} \frac{\hbar^2}{m} \right) (f h_+ + g h_\times) \sin(\omega(t - 2\pi\bar{r}/\omega_r))$$

PERTURBED HORIZON'S GENERATORS

$l^\mu \partial_\mu$ tangent to the null geodesic congruence that generates the horizon

$$l^0 = c_s + \epsilon(fh_+ + gh_\times)\left[3c_s\frac{\omega}{\omega_z}\tilde{z}\sin(\omega(t - 2\pi\tilde{r}_H/\omega_r)) - \frac{8\pi}{3}c_s\frac{\omega^2}{\omega_r\omega_z}\tilde{r}_H\tilde{z}\cos(\omega(t - 2\pi\tilde{r}_H/\omega_r))\right]$$

$$l^1 = \epsilon\tilde{r}_H\tilde{z}\frac{\omega^2}{\omega_z}(fh_+ + gh_\times)\cos(\omega(t - 2\pi\tilde{r}_H/\omega_r))$$

$$l^2 = 0$$

$$l^3 = \epsilon(fh_+ + gh_\times)\left[\frac{1}{2\pi}\frac{\omega_z\omega}{\omega_r}\tilde{r}_H\sin(\omega(t - 2\pi\tilde{r}_H/\omega_r)) + \frac{1}{4\pi^2}\omega_z\cos(\omega(t - 2\pi\tilde{r}_H/\omega_r))\right]$$

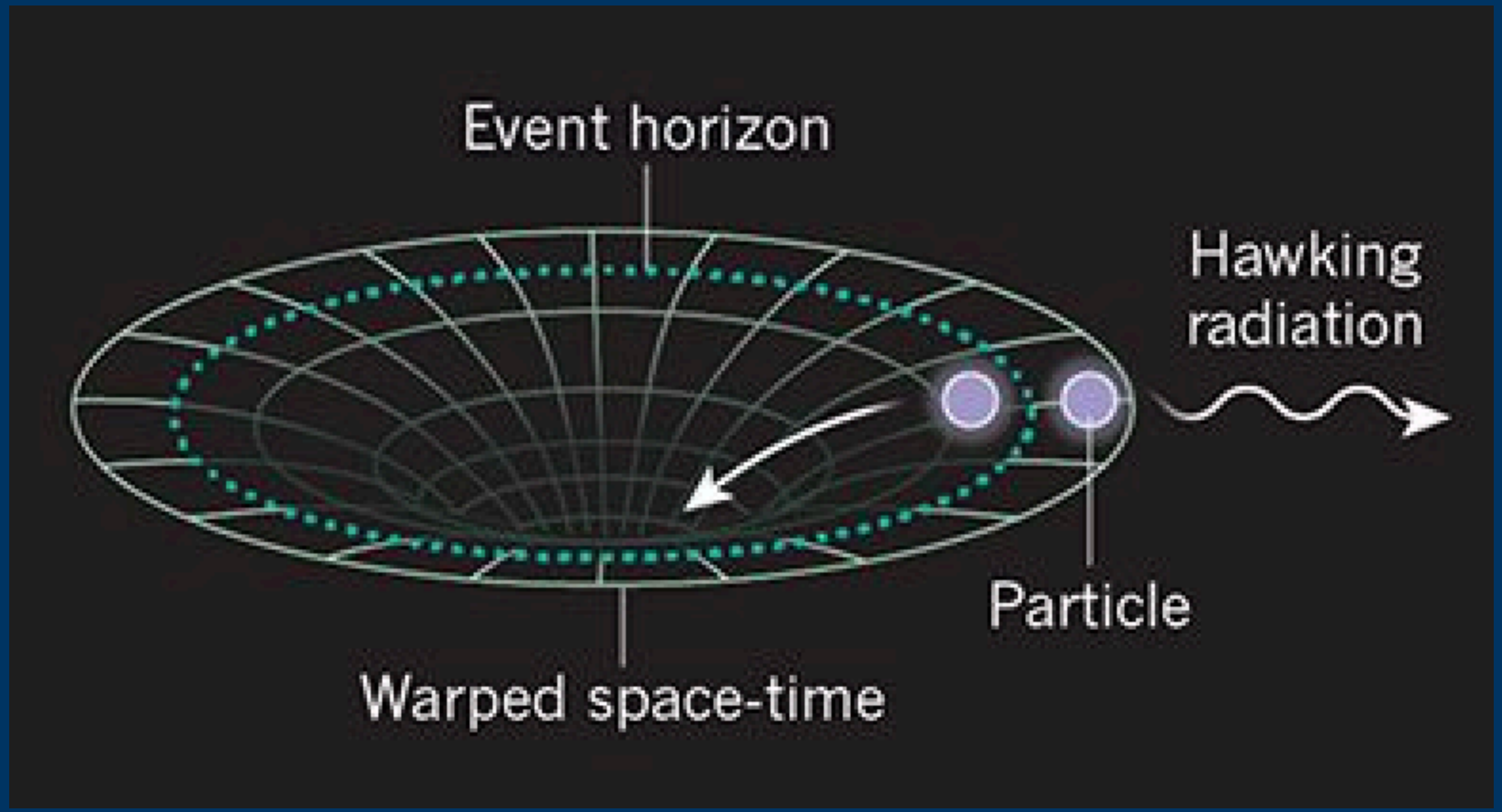
↓

useful to study the horizon's evolution

HAWKING RADIATION

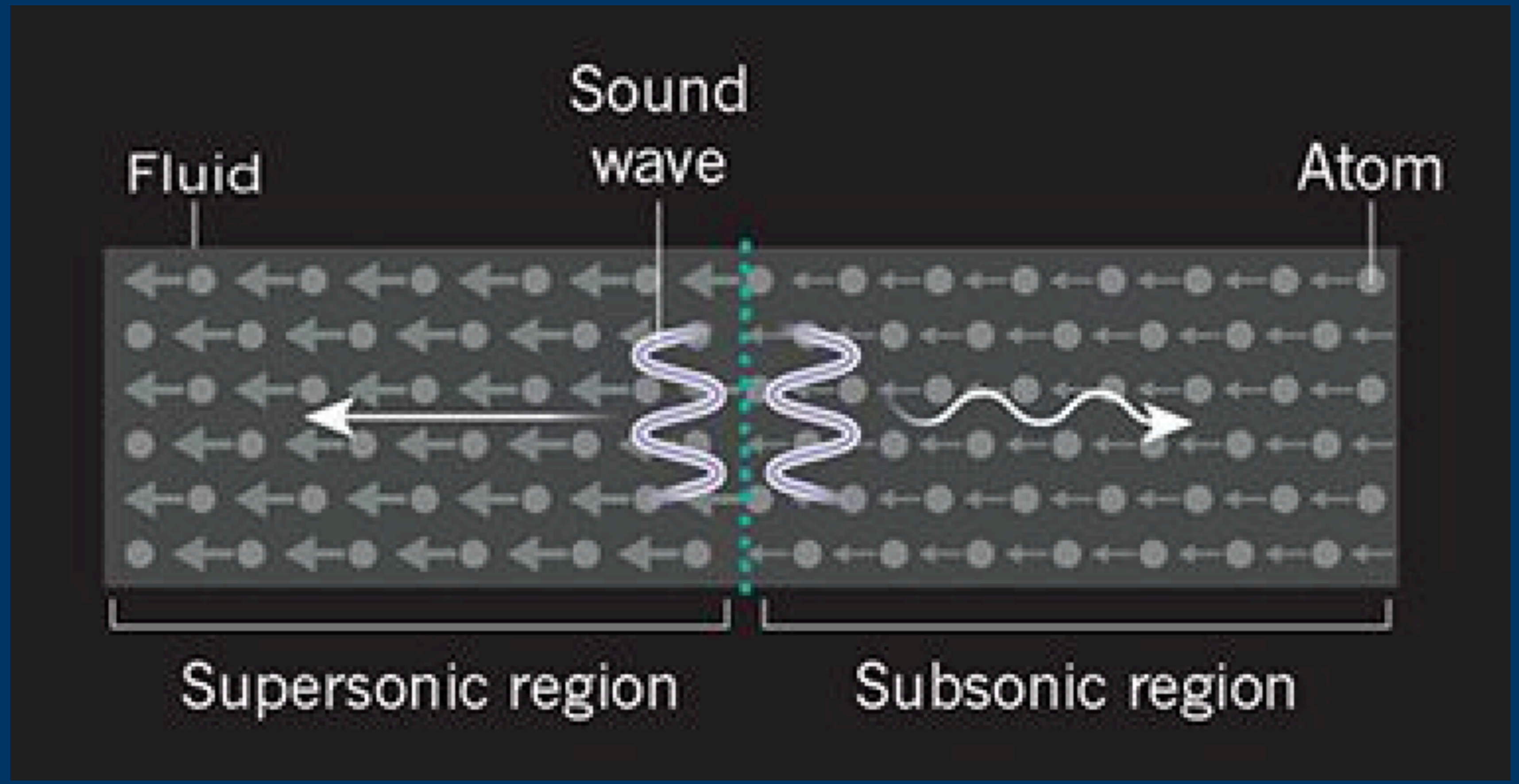
A distant observer will detect a thermal radiation flux emitted from the black hole at $T_h = \dfrac{\hbar\kappa}{2\pi c k_B}$

astrophysical black hole



$T_h \sim 10^{-8} K$
$T_{cmb} \sim 3K$ ⟶ no direct measurement

analogue black hole



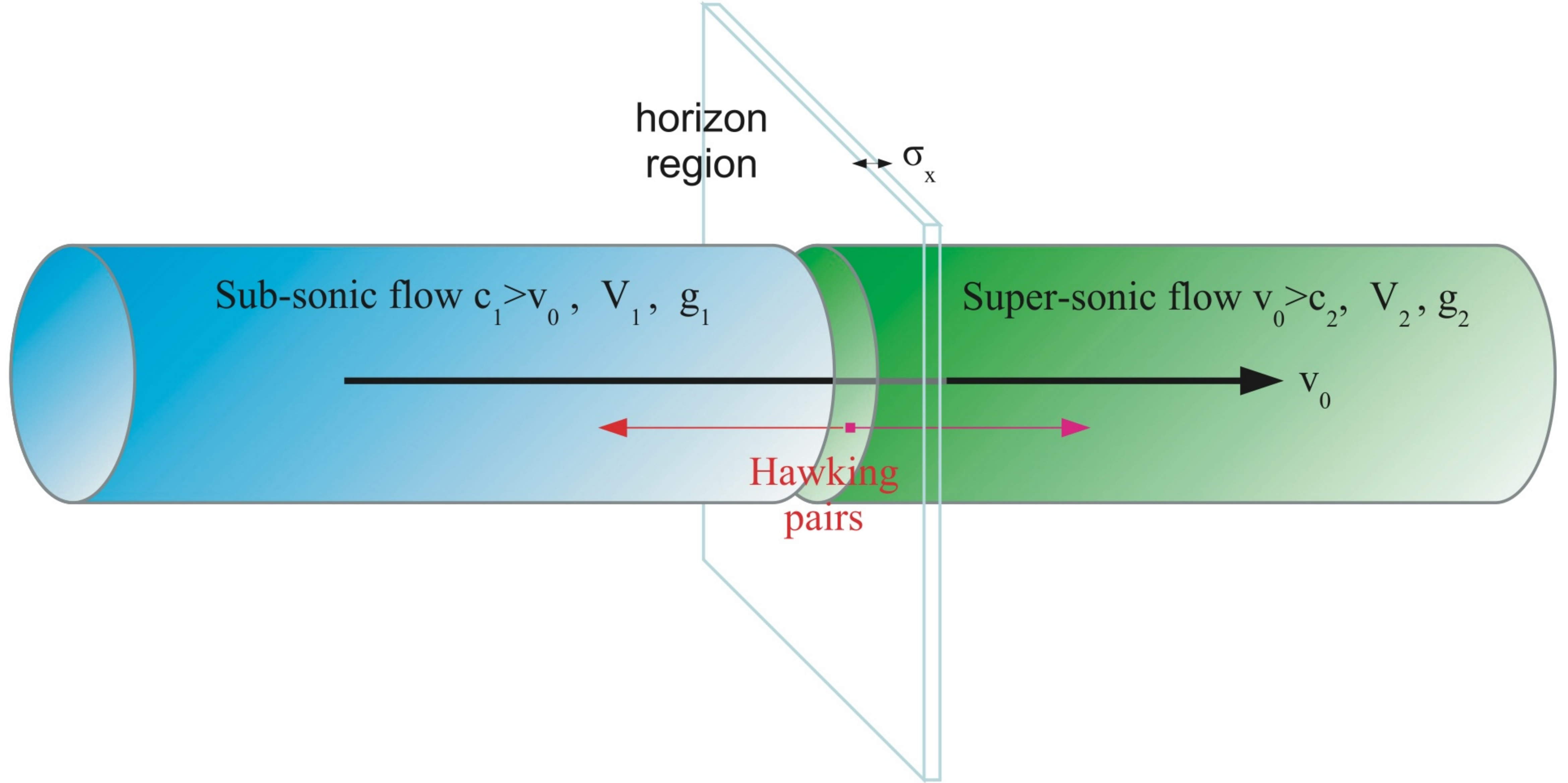
From S. Weinfurtner, Nat.

evidence in the laboratory

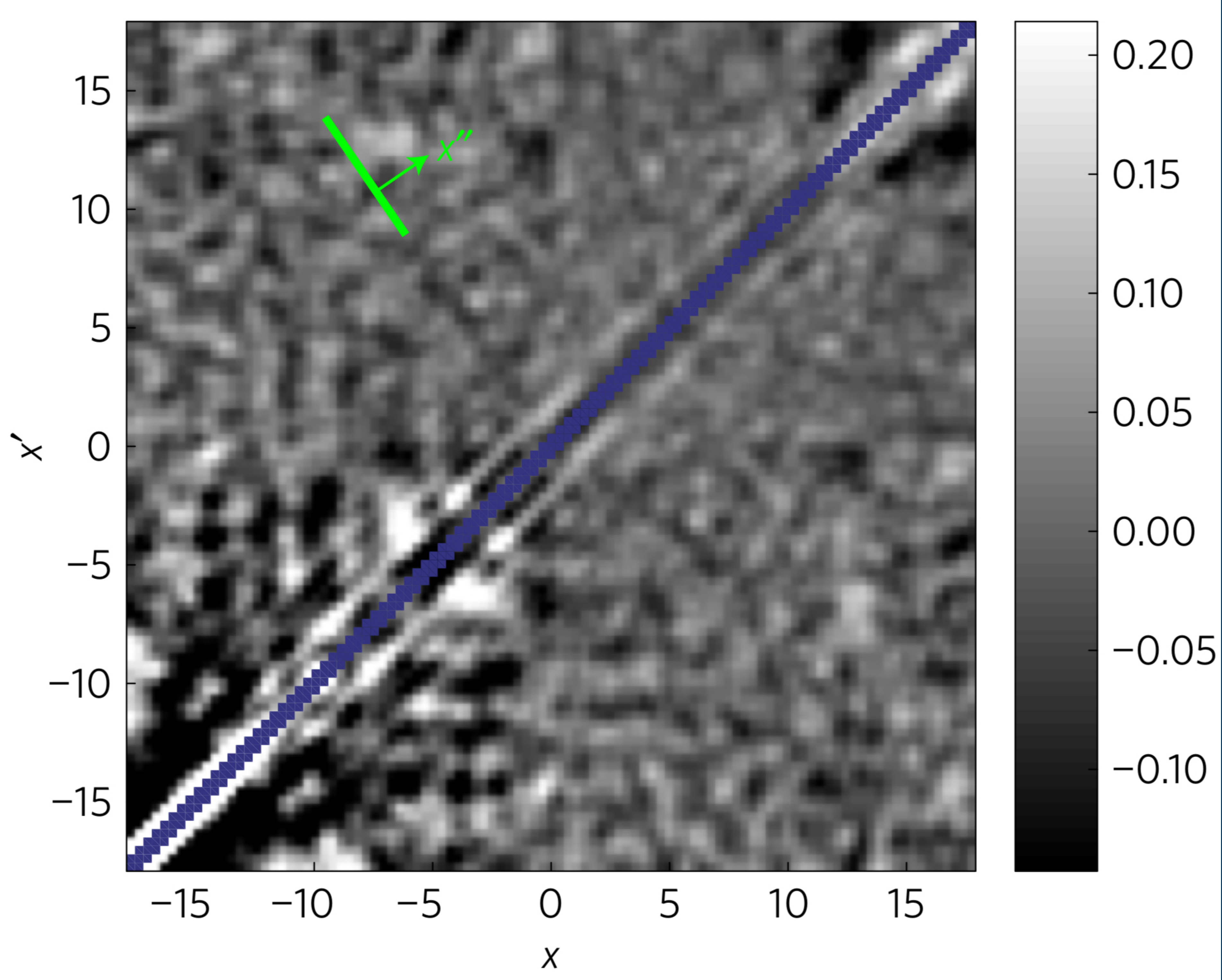
J. Steinhauer, Nat. 2016

J. R. de Nova, K. Golubkov, V.Kolobov, J. Steinhauer, Nat. 2019

OBSERVATION OF THE ANALOGUE HAWKING RADIATION



R. Balbinot, A. Fabbri, S. Fagnocchi, A. Recati, I. Carusotto, Phys. Rev. A 2008



J. Steinhauer, Nat. 2016

$$T_h^{meas} = 0.348(7)\, nK$$

$$T_h^{expec} = 0.351(4)\, nK$$

I. Carusotto, S. Fagnocchi, A. Recati, R. Balbinot, A. Fabbri, New J. Phys. 2008