Jet-Bundle Geometry of Scalar Field Theories

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based on 2308.00017, in collaboration
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COMETA

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The Standard Model Effective Field Theory – SMEFT

promoting the Standard Model to an EFT → add **higher-dimensional** terms made of SM **fields** and respecting the SM **symmetries**

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \dots \qquad \mathcal{L}_d = \sum_i C_i \mathcal{O}_i^{(d)}$$

C_i = Wilson coefficients

$\mathcal{O}_i^{(d)}$ = gauge-invariant operators forming a basis: a complete, non-redundant set

Buchmüller, Wyler 1986

- describes **any beyond-SM theory**, provided it lives at $\Lambda \gg v$
- a complete catalogue of all allowed beyond-SM effects, organized by expected size
- not experiment-specific! can be used as **a common framework** for LHC *and* other experiments
- a proper QFT! renormalizable order-by-order, systematically improvable in loops

SMEFT at $d = 6$: the Warsaw basis

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Grzadkowski,Iskrzynski,Misiak,Rosiek 1008.4884

free parameters

go down to O(100) imposing flavor symmetries, CP, B

Faroughy et al 2005.05366
Greljo et al 2203.09561
IB 2012.11343

they are ~never all relevant at the same time

SMEFT at $d = 6$: the Warsaw basis

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

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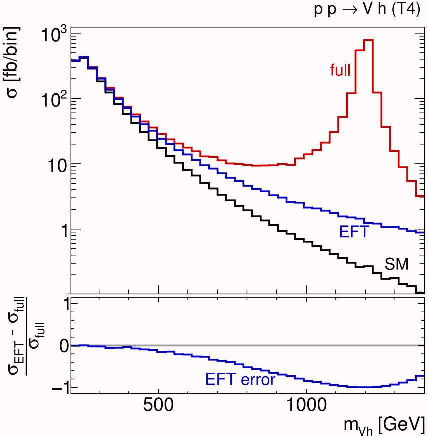
SMEFT for new physics searches at LHC

Top-Down

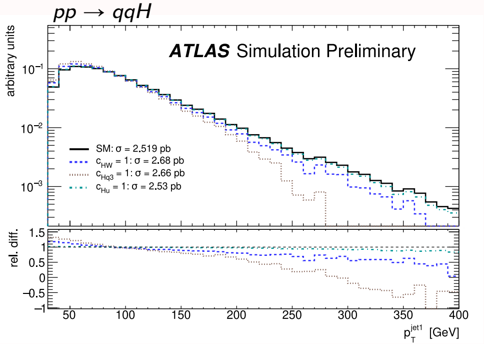
heavy BSM leaves residual footprints at visible energies



adapted from
Brehmer, Freitas, López-Val, Plehn 1510.03443

Bottom-Up

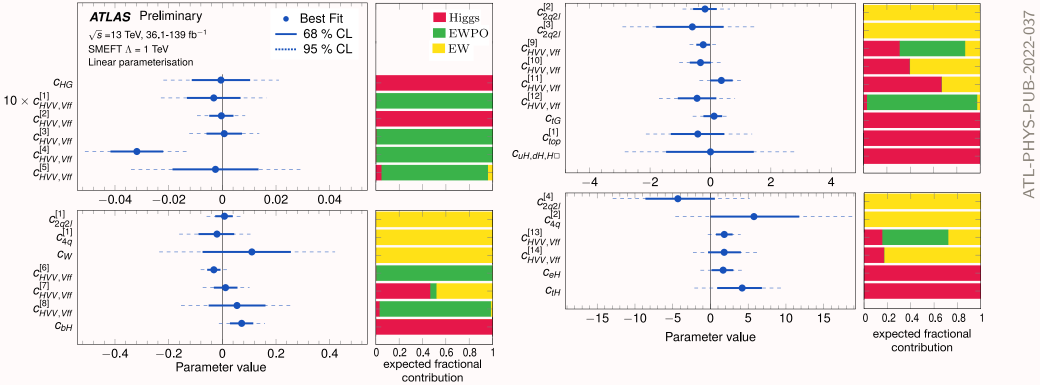
SMEFT operators cause deviations from SM predictions



ATLAS ATL-PHYS-PUB-2019-042

The SMEFT program for the LHC

- a vast campaign of measurements in Higgs, EW, top, Drell-Yan and other processes
- ultimate goal: large global analysis to **measure** as many Wilson coefficients as possible
- if deviations from SM are identified, SMEFT gives a powerful interpretation framework to **infer information** about the underlying BSM dynamics



A legitimate concern: EFT validity

- Λ is **unknown**
- LHC measurements often reach into **high energies** $(m, p_T, m_T \dots)$
- often **measurement** precision is not sufficient to guarantee that deviations from SM are small

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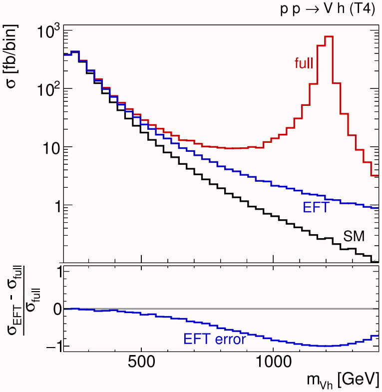
is $(E, v) \ll \Lambda$ a valid assumption?

are $d \geqslant 8$ terms always negligible?

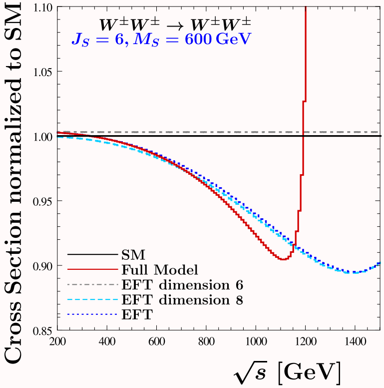
can there be UV scenarios for which SMEFT does not describe the low-E limit?

Possible issues with $d = 6$ SMEFT (1): poor convergence

EFT obtained from matching to full model



adapted from
Brehmer, Freitas, López-Val, Plehn 1510.03443

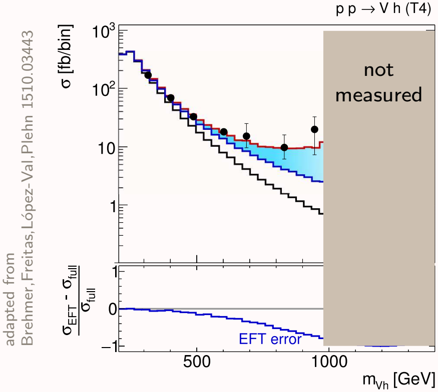


adapted from
Lang, Liebler, Schäfer-Siebert, Zeppenfeld 2103.16517

$d = 6$ contribution dominant at low m_{VH}

$d = 6$ contribution negligible

Possible issues with $d = 6$ SMEFT (1): poor convergence



- **top-down**: truncated EFT does not reproduce full model at high-E
- **bottom-up**: fit to data finds wrong values of C_i

→ several recent works on $d = 8$ impact

Hays,Martin,Sanz,Setford 1808.00442
Boughezal,Mereghetti,Petriello 2106.05337
Dawson et al 2110.06929, 2205.01561, 2212.03258, 2305.07689
Degrande, Li 2303.10493, Ellis,Mimasu,Zampedri 2304.06663
Corbett et al 2102.02819, 2107.07470, 2110.03694,2304.03305

op. bases: Murphy 2005.00059, Li,Ren,Shu,Xiao,Yu 2005.00008
RGE: Chala,Guedes et al 2106.05291, 2205.03301
matching: Chakrabortty et al 2306.09103, 2308.03849

⚠ $d = 8$ basis has 895 (36971) operators for 1 (3) flavors

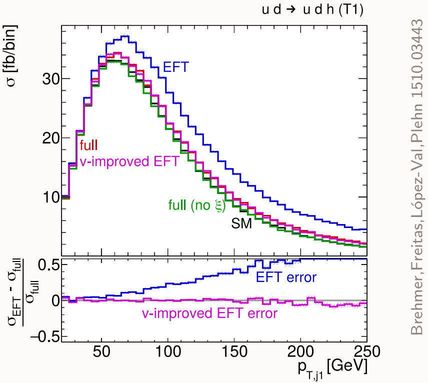
Powers of $H^\dagger H$ and geoSMEFT

a typical SMEFT feature: certain $d \geqslant 8$ operators only add $\mathbf{v^2/\Lambda^2}$ **corrections** to the same structures

$$[d=(6+2n)] \;=\; [d=6]\frac{(H^\dagger H)^n}{\Lambda^{2n}} \;\sim\; [d=6]\left(\frac{v^2}{\Lambda^2}\right)^n\left(1+2n\frac{h}{v}+n(2n-1)\frac{h^2}{v^2}+\cdots+\frac{h^{2n}}{v^{2n}}\right)$$

"v-improved" matching

reabsorbing powers of (v/Λ) in the matching *can* restore the agreement



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geometric SMEFT (**geoSMEFT**) idea: Helset,(Paraskevas,Martin),Trott 1803.08001, 2001.01453

for 2- and 3-point functions, there is only a **finite** number of kinematic structures.
they **saturate** at a certain $\bar{d}$. from there on, only powers of $(H^\dagger H)$ can be added

- resumming (v/Λ) series enables **all-orders** predictions for $2 \to 1$, $1 \to 2$ processes
- for $2 \to 2$ or higher multiplicities, resumming v/Λ is **not sufficient** in general!
  higher-d operators can introduce not only powers of (v/Λ), but also of s, t, u

The Higgs Effective Field Theory – HEFT

rather than H doublet:
singlet h + Goldstones $\mathbf{U}$

Feruglio 9301281, Grinstein,Trott 0704.1505, Buchalla,Catà 1203.6510, Alonso et al 1212.3305, IB et al 1311.1823,1604.06801, Buchalla et al 1307.5017,1511.00988. . .

$$H \mapsto \frac{v + h}{\sqrt{2}} \mathbf{U}, \qquad \mathbf{U} = \exp\left(\frac{i\vec{\sigma}\cdot\vec{\pi}}{v}\right)$$

HEFT **expands** directly **around the EW vacuum**! → removes completely the $(H^\dagger H)^n$ issue

HEFT ⊃ SMEFT ⊃ SM

- **more general** than SMEFT because implements weaker symmetry requirement
- **more complicated** power counting, mix of χPT and canonical dimensions
- **more operators** order-by-order in the expansions

The HEFT Lagrangian

Buchalla,Catà,Krause 1307.5017, IB,Gonzalez-Fraile,Gonzalez-Garcia,Merlo 1604.06801, Sun,Xiao,Yu 2206.07722

$$\mathbf{V}_\mu = (D_\mu \mathbf{U})\mathbf{U}^\dagger \sim W_\mu, Z_\mu \qquad \mathbf{T} = \mathbf{U}\sigma^3\mathbf{U}^\dagger \qquad Q_{L,R} = \begin{pmatrix} u_{L,R} \\ d_{L,R} \end{pmatrix} \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad L_R = \begin{pmatrix} 0 \\ e_R \end{pmatrix}$$

$$\mathcal{F}_i(h) = 1 + a_i \frac{h}{v} + b_i \frac{h^2}{v^2} + \dots \quad \langle \cdot \rangle = \underset{SU(2)}{\mathrm{Tr}}(\cdot)$$

$$\mathscr{L}_{HEFT} = \mathscr{L}_0 + \mathscr{L}_1 + \mathscr{L}_2 \dots$$

$$\begin{aligned} \mathscr{L}_0 = & -\frac{1}{4}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{2}\partial_\mu h \partial^\mu h \\ & - \frac{v^2}{4}\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \mathcal{F}_C(h) - \frac{v^2}{4}\langle \mathbf{T}\mathbf{V}_\mu\rangle^2 \mathcal{F}_T(h) \\ & + i\bar{Q}_L \not{D} Q_L + i\bar{Q}_R \not{D} Q_R + i\bar{L}_L \not{D} L_L + i\bar{L}_R \not{D} L_R \\ & - \frac{v}{\sqrt{2}}\left[\bar{Q}_L \mathbf{U}\mathcal{Y}_Q(h) Q_R + \text{h.c.}\right] - \frac{v}{\sqrt{2}}\left[\bar{L}_L \mathbf{U}\mathcal{Y}_L(h) L_R + \text{h.c.}\right] \end{aligned}$$

parameters for 3 flavors (L, B cons)

$\mathscr{L}_1$: 6573
$\mathscr{L}_2$: 10^6+

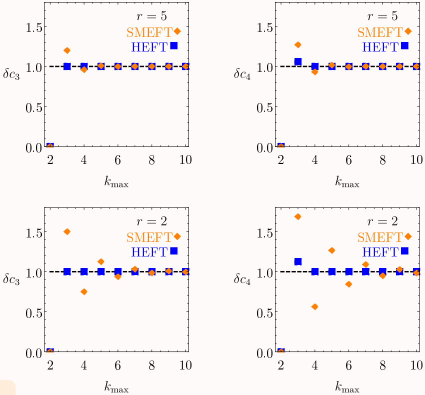
Sun,Xiao,Yu 2210.14939, Sun,Wang,Yu 2211.11598, Gráf,Henning,Lu,Melia,Murayama 2211.06725

39 operators (vs **15** in dim-6 Warsaw basis, **89** in dim-8 Murphy basis) Sun,Xiao,Yu 2206.07722

$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle^2 \mathcal{F}(h)$	$\langle \mathbf{V}_\mu \mathbf{V}_\nu \rangle^2 \mathcal{F}(h)$	$\partial_\mu \partial_\nu \mathcal{F}(h) \partial^\mu \partial^\nu \mathcal{F}(h)$
$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}_\nu \rangle \langle \mathbf{V}^\mu \mathbf{V}^\nu \rangle \mathcal{F}(h)$	$\langle \mathbf{T}\mathbf{V}_\mu \rangle^2 \langle \mathbf{V}_\nu \mathbf{V}^\nu \rangle \mathcal{F}(h)$	$\langle \mathbf{T}\mathbf{V}_\mu \rangle^4 \mathcal{F}(h)$
$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{V}_\nu \mathbf{V}^\nu \rangle \partial^\mu \mathcal{F}(h)$	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{V}^\mu \mathbf{V}^\nu \rangle \partial_\nu \mathcal{F}(h)$	$\langle \mathbf{T}\mathbf{V}_\mu \mathbf{V}_\nu \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \partial^\nu \mathcal{F}(h)$
$\langle \mathbf{T}\mathbf{V}_\mu \rangle^2 \langle \mathbf{T}\mathbf{V}_\nu \rangle \partial^\nu \mathcal{F}(h)$	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}_\nu \rangle \partial^\mu \partial^\nu \mathcal{F}(h)$	$\langle \mathbf{T}\mathbf{V}_\mu \rangle^2 \partial_\nu \mathcal{F}(h) \partial^\nu \mathcal{F}(h)$
$\langle \mathbf{V}_\mu \mathbf{V}_\nu \rangle \partial^\mu \partial^\nu \mathcal{F}(h)$	$\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \partial^\nu \mathcal{F}(h) \partial_\nu \mathcal{F}(h)$	$\langle \mathbf{T}\mathbf{V}_\mu \rangle \partial_\nu \mathcal{F}(h) \partial^\mu \partial^\nu \mathcal{F}(h)$
$\langle \tilde{W}_{\mu\nu} \mathbf{V}^\mu \rangle \langle \mathbf{T}\mathbf{V}^\nu \rangle \mathcal{F}(h)$	$\langle \mathbf{T}[\tilde{W}_{\mu\nu}, \mathbf{V}^\nu] \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \mathcal{F}(h)$	$\langle W_{\mu\nu} \mathbf{V}^\mu \rangle \langle \mathbf{T}\mathbf{V}^\nu \rangle \mathcal{F}(h)$
$\langle W_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h)$	$B_{\mu\nu} \langle \mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h)$	$\langle W_{\mu\nu} \mathbf{T} \rangle \langle \mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h)$
$\langle \tilde{W}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h)$	$\tilde{B}_{\mu\nu} \langle \mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h)$	$\langle W_{\mu\nu} \mathbf{T} \rangle \langle \tilde{W}^{\mu\nu} \mathbf{T} \rangle \mathcal{F}(h)$
$B_{\mu\nu} \langle W^{\mu\nu} \mathbf{T} \rangle \mathcal{F}(h)$	$\tilde{B}_{\mu\nu} \langle W^{\mu\nu} \mathbf{T} \rangle \mathcal{F}(h)$	$\langle W_{\mu\nu} \mathbf{T} \rangle^2 \mathcal{F}(h)$
$B_{\mu\nu} B^{\mu\nu} \mathcal{F}(h)$	$W_{\mu\nu} W^{\mu\nu} \mathcal{F}(h)$	$G_{\mu\nu} G^{\mu\nu} \mathcal{F}(h)$
$B_{\mu\nu} \tilde{B}^{\mu\nu} \mathcal{F}(h)$	$W_{\mu\nu} \tilde{W}^{\mu\nu} \mathcal{F}(h)$	$G_{\mu\nu} \tilde{G}^{\mu\nu} \mathcal{F}(h)$
$f_{abc} G^a_{\mu\nu} G^{b\nu\rho} G^{c\nu}_\rho \mathcal{F}(h)$	$\varepsilon_{ijk} W^i_{\mu\nu} W^{j\nu\rho} W^{k\nu}_\rho \mathcal{F}(h)$	$\varepsilon_{ijk} B_{\mu\nu} W^{i\nu\rho} W^{j\nu}_\rho \mathbf{T}^k \mathcal{F}(h)$
$f_{abc} \tilde{G}^a_{\mu\nu} G^{b\nu\rho} G^{c\nu}_\rho \mathcal{F}(h)$	$\varepsilon_{ijk} \tilde{W}^i_{\mu\nu} W^{j\nu\rho} W^{k\nu}_\rho \mathcal{F}(h)$	$\varepsilon_{ijk} \tilde{B}_{\mu\nu} W^{i\nu\rho} W^{j\nu}_\rho \mathbf{T}^k \mathcal{F}(h)$

SMEFT vs HEFT

- HEFT's $\mathscr{L}_1$ contains **more** parameters than dim-6 SMEFT, but **fewer** than dim-8
- there are scenarios where SMEFT does **not** apply, but HEFT **still works**
- there are cases in which SMEFT applies, but **converges** much slower than HEFT (composite H)



Cohen,Craig,Lu,Sutherland 2008.08597

👍 shall we move to HEFT for LHC searches?
👍 what are the phenomenological differences in the end?

❗ one main complication:

when SMEFT exists, the $H \to h, \mathbf{U}$ map must be an **unphysical** field redefinition

SMEFT/HEFT geometrical interpretation

let us consider only the 4 scalar fields : they can be seen as coordinates on 4D manifold

Alonso, Jenkins, Manohar 1511.00724, 1605.03602

	SMEFT $\sim$ **cartesian** coord.	HEFT $\sim$ **polar** coord.
$(\mathbb{R}^4)$	$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}$	$\vec{\phi} = (v+h)\exp\left[\frac{2\pi^i t_i}{v}\right]\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
$(SU(2))$	$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}$	$\mathbf{U} = \exp\left[\frac{\pi^i \sigma_i}{v}\right]$

- accidental $SU(2)_L \times SU(2)_R \sim O(4)$ symmetry: $H^\dagger H = \frac{|\vec{\phi}|^2}{2}$
- field redefinition $\leftrightarrow$ change of coordinates
- physics can be associated to geometry of the field space, independent of coordinates

Physics – Geometry connection

The kinetic term corresponds to a metric in field space

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi^i \partial^\mu \phi^j g_{ij}(\phi) + \dots$$

it captures **all operators with 2 derivatives**, up to arbitrary dimensions. e.g.

$$\partial_\mu H^\dagger \partial^\mu H\,(H^\dagger H)^n = \frac{1}{2}\partial_\mu \vec{\phi}\cdot\partial^\mu\vec{\phi}\left(\frac{\vec{\phi}\cdot\vec{\phi}}{2}\right)^n \rightarrow g_{ij} = \delta_{ij}\left(\frac{\vec{\phi}\cdot\vec{\phi}}{2}\right)^n$$

$$H^\dagger H \,\Box\, (H^\dagger H) = -(\vec{\phi}\cdot\partial_\mu\vec{\phi})^2 \rightarrow g_{ij} = -2\phi_i\phi_j$$

$$(iH^\dagger\partial_\mu H - i\partial_\mu H^\dagger H)^2 = 4(\partial_\mu\vec{\phi}\; t_{3R}\; \vec{\phi})^2 \rightarrow g_{ij} = 8(t_{3R}\phi)_i(t_{3R}\phi)_j$$

scattering amplitudes are proportional to the Riemann curvature invariants at the vacuum

$$\mathcal{A}(\phi_i\phi_j \to \phi_k\phi_l) = R_{ijkl}s_{ik} + R_{ikjl}s_{ij}$$

gauge sector and fermions can also be included in the formalism

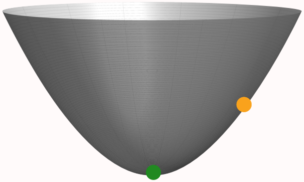
Cheung,Helset,Parra-Martinez 2111.03045,2202.06972
Helset,Jenkins,Manohar 2210.08000
Assi,Helset,Manohar,Pagès,Shen 2307.03187
Cohen,Lu,Sutherland 2312.06748

Possible issues with SMEFT (2): non existence

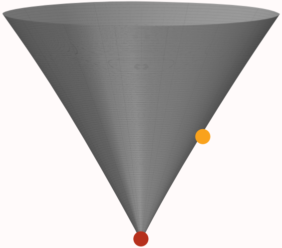
Cohen et al 2008.0597, 2108.03240, Banta et al 2110.02967
Gomez-Ambrosio et al 2204.01763, 2207.09848. figs by D. Sutherland

SMEFT expands around the **O(4) symmetric point**. HEFT expands around the **vacuum**.

there are cases where the SMEFT expansion **cannot be constructed**, or is not convergent at v



✔ SMEFT



✖ HEFT only

"loryons"

✖ HEFT only

BSM EWSB

Incompleteness of the geometric picture

1. only operators with **2 derivatives** are described

   no geometric interpretation of the scalar potential and of higher-derivative terms

   attempts to fix this in Cohen,Craig,Lu,Sutherland 2202.06965 Craig,Lee,Lu,Sutherland 2305.09722

2. in general, the Riemann curvature is **not** invariant under **derivative** field redefinitions

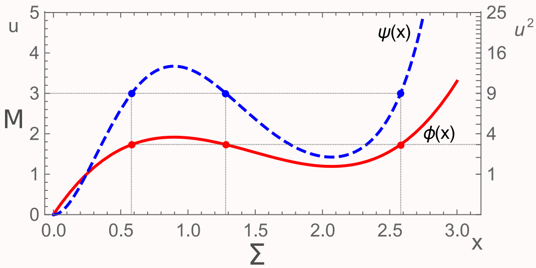
$$\mathscr{L} = \frac{1}{2}\partial_\mu r \partial^\mu r + \frac{r^2}{2}\partial_\mu \vec{n}\cdot\partial^\mu \vec{n} - \frac{m^2}{2} r^2 \qquad R = 0$$

$$\downarrow \quad r \mapsto r + A\,\Box\, r + B|\partial_\nu \vec{n}|^2 r$$

$$\mathscr{L}' = \frac{1}{2}(1-2m^2 A)\partial_\mu r \partial^\mu r + \frac{r^2}{2}(1-2m^2 B)\partial_\mu \vec{n}\cdot\partial^\mu \vec{n} - \frac{m^2}{2} r^2 + \mathcal{O}(A^2, B^2, 4\partial) \qquad R = \frac{12 m^2}{r^2}(B-A)$$

Fibre bundle picture

Alminawi,IB,Davighi 2308.00017



fibre bundle (E, Σ, π)
E = total space
Σ = base space = spacetime with coord x^μ
$\pi : E \to \Sigma$ projection map

locally: $E = \Sigma \times M$
M = fibre = field space with coord u^i

$\phi(x) : \Sigma \to E$ is a **(local) section** of the bundle

- section $\neq$ coordinates on M: $\phi \neq u$
- field redefinition = change of section. if non derivative: $\sim$ diffeomorphism $f : E \to E$

1. we define a **metric g** on E
2. we are more careful in the **mapping** from geometry $\to$ Lagrangians (function on E $\to$ function on Σ)

Scalar Lagrangian from Fibre bundle geometry

E metric: bundle has coordinates $y^I = (x^\mu, u^i)$. Poincaré invariance $\to g^{IJ}$ independent of x^μ

$$g = g_{IJ} dy^I \otimes dy^J = \begin{pmatrix} dx^\mu & du^i \end{pmatrix} \begin{pmatrix} g_{\mu\nu}(u) & g_{\mu j}(u) \\ g_{\nu i}(u) & g_{ij}(u) \end{pmatrix} \begin{pmatrix} dx^\nu \\ du^j \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu + 2 g_{\mu i} dx^\mu du^i + g_{ij} du^i du^j$$

pulling back to spacetime along the section $\phi \to$ **Lagrangian**

$$u^i \to \phi^i(x) = (u^i \circ \phi)(x)\,, \qquad du^i \to \partial_\rho \phi^i(x)$$

$$g \to \mathscr{L} = \frac{1}{2}\eta^{\rho\sigma}\langle \partial_\rho \otimes \partial_\sigma, \phi^*(g)\rangle = \eta^{\rho\sigma}\left[\frac{1}{2} g_{\rho\sigma}(\phi) + g_{\rho i}(\phi)\partial_\sigma\phi^i + \frac{1}{2} g_{ij}(\phi)\partial_\rho\phi^i\partial_\sigma\phi^j\right]$$

g_{ij} has the same interpretation as before. physics also requires

$$g_{\rho i}(\phi) \equiv 0 \qquad\qquad \eta^{\rho\sigma} g_{\rho\sigma}(\phi) = -2V(\phi)$$

$\to$ geometric description of the scalar potential!

SMEFT/HEFT in the Fibre bundle picture

SMEFT/HEFT = a theory of 4 scalar fields, with a $O(4)$ symmetry: $u^i, i = 1, 2, 3, 4$.

bundle metric entries

$$g_{\mu\nu}(u) = -\frac{\eta_{\mu\nu}\Lambda^4}{2} V\left[\frac{u \cdot u}{\Lambda^2}\right] \qquad g_{\mu i}(u) = 0$$

$$g_{ij}(u) = \delta_{ij} A\left[\frac{u \cdot u}{\Lambda^2}\right] + \delta_{ik}\delta_{jl}\frac{u^k u^l}{\Lambda^2} B\left[\frac{u \cdot u}{\Lambda^2}\right]$$

gives

$$\mathscr{L} = \frac{1}{2}\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi}\, A\left[\frac{\vec{\phi}\cdot\vec{\phi}}{\Lambda^2}\right] + \frac{1}{2\Lambda^2}(\vec{\phi}\cdot\partial_\mu\vec{\phi})(\vec{\phi}\cdot\partial^\mu\vec{\phi})\, B\left[\frac{\vec{\phi}\cdot\vec{\phi}}{\Lambda^2}\right] - \Lambda^4 V\left[\frac{\vec{\phi}\cdot\vec{\phi}}{\Lambda^2}\right]$$

→ most general effective Lagrangian with **up to 2** derivatives!

changes of coordinates do nothing! (metric is a tensor) → metric pulls back to the *same* Lagrangian

$$g' = g'_{ij}\, du'^i \otimes du'^j = \frac{\partial u^i}{\partial u'^k}\frac{\partial u^j}{\partial u'^l} g_{kl}\, \frac{\partial u'^i}{\partial u^m} du^m \otimes \frac{\partial u'^j}{\partial u^n} du^n = g_{ij}\, du^i \otimes du^j = g$$

instead: change of section, or equiv. diffeomorphism on E

SMEFT → HEFT. start from

$$g_{\mu\nu}(u) = -\frac{\eta_{\mu\nu} v^4}{2} V\left[\frac{u\cdot u}{v^2}\right], \qquad g_{\mu i}(u) = 0\,, \qquad g_{ij}(u) = \delta_{ij}$$

which gives

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - v^4 V\left(\frac{\vec{\phi}\cdot\vec{\phi}}{v^2}\right)$$

now we map

$$u^i \mapsto \left(1+\frac{u^4}{v}\right)u^i \quad i=1,2,3\,, \qquad u^4 \mapsto \left(1+\frac{u^4}{v}\right)\sqrt{v^2-\vec{u}^2}$$

Non-derivative field redefinitions in the Fibre bundle picture

changes of coordinates do nothing! (metric is a tensor) → metric pulls back to the *same* Lagrangian

$$g' = g'_{ij}\, du'^i \otimes du'^j = \frac{\partial u^i}{\partial u'^k}\frac{\partial u^j}{\partial u'^l} g_{kl}\, \frac{\partial u'^i}{\partial u^m} du^m \otimes \frac{\partial u'^j}{\partial u^n} du^n = g_{ij}\, du^i \otimes du^j = g$$

instead: change of section, or equiv. diffeomorphism on E

SMEFT → HEFT. we obtain

$$g_{\mu\nu}(u) = -\frac{\eta_{\mu\nu} v^4}{2} V \left(1 + \frac{u^4}{v}\right)^2 , \qquad g_{\mu i}(u) = 0 \,,$$

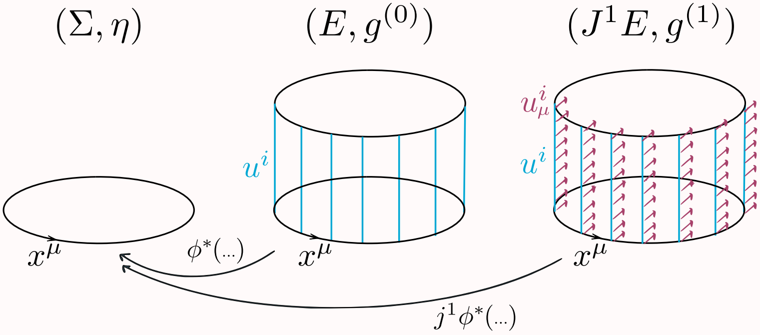
$$g_{ij}(u) = \left(1 + \frac{u^4}{v}\right)^2 \left[\delta_{ij} + \frac{u^i u^j}{v^2 - \vec{u}^2}\right] , \qquad g_{i4}(u) = 0 \,, \qquad g_{44}(u) = 1 \qquad (i, j = 1, 2, 3)$$

which gives

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi^4 \partial^\mu \phi^4 + \frac{1}{2}\left(1 + \frac{\phi^4}{v}\right)^2 \left[\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} + \frac{1}{v^2 - \vec{\phi}^2}(\vec{\phi} \cdot \vec{\phi})^2\right] - v^4 V \left(1 + \frac{\phi^4}{v}\right)^2$$

Jet bundles

Saunders 1989. see also Craig,Lee 2307.15742



$j^i_x\phi$ = r-jet of ϕ at x = equivalence class containing sections identical up to r-th derivative

J^rE = r-jet bundle = $\{j^r_x\phi | x \in \Sigma, \phi \in \Gamma_x(\pi)\}$ is a differentiable manifold. we use only J^1E

Scalar Lagrangian from 1-jet bundle geometry

J^1E metric: 1-jet bundle has coordinates $y^I = (x^\mu, u^i, u^i_\mu)$

$$g^{(1)} = g_{IJ} dy^I \otimes dy^J = \begin{pmatrix} dx^\mu & du^i & u^i_\mu \end{pmatrix} \begin{pmatrix} g_{\mu\nu} & g_{\mu j} & g^\nu_{\mu j} \\ g_{\nu i} & g_{ij} & g^\nu_{ij} \\ g^\mu_{\nu i} & g^\mu_{ij} & g^{\mu\nu}_{ij} \end{pmatrix} \begin{pmatrix} dx^\nu \\ du^j \\ du^i_\nu \end{pmatrix}$$

$$= g_{\mu\nu} dx^\mu dx^\nu + 2 g_{\mu i} dx^\mu du^i + 2 g^\nu_{\mu j} dx^\mu du^j_\nu + g_{ij} du^i du^j + 2 g^\nu_{ij} du^i du^j_\nu + g^{\mu\nu}_{ij} du^i_\mu du^j_\nu$$

pulling back to spacetime along the "prolongation" of the section $j^1\phi \to$ **Lagrangian**

$$u^i \to \phi^i(x)\,, \qquad u^i_\mu \to \partial_\mu \phi^i(x)\,, \qquad du^i \to \partial_\rho \phi^i(x)\,, \qquad du^i_\mu \to \partial_\rho \partial_\mu \phi^i(x)$$

$$g^{(1)} \to \mathscr{L} = \frac{1}{2}\eta^{\mu\nu} g_{\mu\nu} + g_{\mu i}\partial^\mu \phi^i + g^\nu_{\mu j}\partial^\mu \partial^\nu \phi^j + \frac{1}{2} g_{ij}\partial_\mu \phi^i \partial^\mu \phi^j + g^\nu_{ij}\partial_\rho \phi^i \partial^\rho \partial_\nu \phi^j + \frac{1}{2} g^{\mu\nu}_{ij}\partial_\rho \partial_\mu \phi^i \partial^\rho \partial_\nu \phi^j$$

- now all the metric entries are functions of $u_\mu, u^i_\mu \to \phi^i, \partial_\mu \phi^i$
- a 1-jet bundle metric maps to **a redundant basis of operators with up to 4 derivatives**

Scalar Lagrangian from 1-jet bundle metric: 1 scalar case

coordinates: (x^μ, u, u_μ).

→ we **expand** metric dependence on u_μ and leave dependence on u in analytic functions $A, B \dots$

→ retain only terms leading to operators with **up to 4 derivatives**

$$\frac{g_{\mu\nu}}{\Lambda^4} = -\frac{\eta_{\mu\nu}}{2}V(u) + \left[\frac{u_\mu u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho u^\rho}{\Lambda^4}\right]\frac{J(u)}{2} + \left[\frac{u_\mu u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho u^\rho}{\Lambda^4}\right]\frac{u_\sigma u^\sigma}{\Lambda^4}\frac{K(u)}{2}$$

$$\frac{g_{\mu u}}{\Lambda^2} = \frac{u_\mu}{\Lambda^2}G(u) + \frac{u_\mu u_\rho u^\rho}{\Lambda^6}H(u)$$

$$g_{\mu u}^{\nu} = \delta^\nu_\mu E(u) + \frac{u^\nu u_\mu}{\Lambda^4}F_1(u) + \delta^\nu_\mu \frac{u_\rho u^\rho}{\Lambda^4}F_2(u)$$

$$g_{uu} = C(u) + \frac{u_\rho u^\rho}{\Lambda^4}D(u) \qquad \Lambda g^\mu_{uu} = \frac{u^\mu}{\Lambda}B(u) \qquad \Lambda^2 g^{\mu\nu}_{uu} = \eta^{\mu\nu}A(u)$$

pulls back to

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\,(C + 2G + J) - \Lambda(\Box\phi)E - \Lambda^4 V$$
$$+ \frac{\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi}{\Lambda^2}\frac{A}{2} + \frac{\partial_\mu\partial_\nu\phi\partial^\mu\phi\partial^\nu\phi}{\Lambda^3}(B + F_1) + \frac{(\Box\phi)(\partial_\mu\phi\partial^\mu\phi)}{\Lambda^3}F_2 + \frac{(\partial_\mu\phi\partial^\mu\phi)^2}{\Lambda^4}\frac{D + 2H + K}{2}$$

Scalar Lagrangian from 1-jet bundle metric: 1 scalar case

coordinates: (x^μ, u, u_μ).

→ we **expand** metric dependence on u_μ and leave dependence on u in analytic functions $A, B \dots$

→ retain only terms leading to operators with **up to 4 derivatives**

$$\frac{g_{\mu\nu}}{\Lambda^4} = -\frac{\eta_{\mu\nu}}{2}V(u) + \left[\frac{u_\mu u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho u^\rho}{\Lambda^4}\right]\frac{J(u)}{2} + \left[\frac{u_\mu u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho u^\rho}{\Lambda^4}\right]\frac{u_\sigma u^\sigma}{\Lambda^4}\frac{K(u)}{2}$$

$$\frac{g_{\mu u}}{\Lambda^2} = \frac{u_\mu}{\Lambda^2}G(u) + \frac{u_\mu u_\rho u^\rho}{\Lambda^6}H(u)$$

$$g^{\nu}_{\mu u} = \delta^\nu_\mu E(u) + \frac{u^\nu u_\mu}{\Lambda^4}F_1(u) + \delta^\nu_\mu \frac{u_\rho u^\rho}{\Lambda^4}F_2(u)$$

$$g_{uu} = C(u) + \frac{u_\rho u^\rho}{\Lambda^4}D(u) \qquad \Lambda g^\mu_{uu} = \frac{u^\mu}{\Lambda}B(u) \qquad \Lambda^2 g^{\mu\nu}_{uu} = \eta^{\mu\nu}A(u)$$

pulls back to

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\left(C + 2G + J - 2E'\right) - \Lambda^4 V$$

blue = can be removed via EOM

$$+\frac{\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi}{\Lambda^2}\frac{A}{2} + \frac{\partial_\mu\partial_\nu\phi\partial^\mu\phi\partial^\nu\phi}{\Lambda^3}(B + F_1 - 2F_2) + \frac{(\partial_\mu\phi\partial^\mu\phi)^2}{\Lambda^4}\frac{D + 2H + K - 2F_2'}{2}$$

Scalar Lagrangian from 1-jet bundle metric: SMEFT/HEFT case

similar procedure as above, requiring also appropriate $O(4)$ transformations → more structures!

now $A, B, C \ldots$ are analytic functions of $(u \cdot u/\Lambda^2)$

$$\begin{aligned}
\frac{g_{\mu\nu}}{\Lambda^4} &= -\frac{\eta_{\mu\nu}}{2}V + \left[\frac{u_\mu \cdot u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho \cdot u^\rho}{\Lambda^4}\right]\frac{J_0}{2} + \left[\frac{u \cdot u_\mu \, u \cdot u_\nu}{\Lambda^6} + \frac{\eta_{\mu\nu}}{4}\frac{(u \cdot u_\rho)^2}{\Lambda^4}\right]\frac{J_1}{2} \\
&+ \left[\frac{u_\mu \cdot u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho \cdot u^\rho}{\Lambda^2}\right]\frac{u_\rho \cdot u^\rho}{\Lambda^4}\frac{K_0}{2} + \left[\frac{u_\rho \cdot u_\mu \, u^\rho \cdot u_\nu}{\Lambda^6} + \frac{\eta_{\mu\nu}}{4}\frac{(u_\rho \cdot u_\sigma)^2}{\Lambda^4}\right]\frac{K_1}{2} \\
\frac{g_{\mu j}}{\Lambda^2} &= \frac{u_{j\mu}}{\Lambda^2}G_0 + \frac{u_j \, u \cdot u_\mu}{\Lambda^4}G_1 + \frac{u_{j\mu}\, u_\rho \cdot u^\rho}{\Lambda^6}H_0 + \frac{u_{j\rho}\, u_\mu \cdot u^\rho}{\Lambda^6}H_1 + \left[\frac{u_j \, u \cdot u_\mu \, u_\rho \cdot u^\rho}{\Lambda^8} + \frac{u_{j\mu}(u \cdot u_\rho)^2}{\Lambda^8}\right]\frac{H_2}{2} \\
&+ \left[\frac{u_j \, u_\mu \cdot u_\rho \, u \cdot u^\rho}{\Lambda^8} + \frac{u_{j\rho}\, u \cdot u_\mu \, u \cdot u^\rho}{\Lambda^8}\right]\frac{H_3}{2} + \frac{u_j \, u \cdot u_\mu (u \cdot u_\rho)^2}{\Lambda^{10}}H_4 \\
g_{\mu j}^{\nu} &= \delta_\mu^\nu \frac{u_j}{\Lambda}E + \frac{u_j \, u^\nu \cdot u_\mu}{\Lambda^5}F_{10} + \frac{u_j^\nu \, u \cdot u_\mu + u \cdot u^\nu \, u_{j\mu}}{2\Lambda^5}F_{11} + \frac{u_j \, u \cdot u^\nu \, u \cdot u_\mu}{\Lambda^7}F_{12} + \delta_\mu^\nu \frac{u_j \, u_\rho \cdot u^\rho}{\Lambda^5}F_{20} + \delta_\mu^\nu \frac{u_{j\rho}\, u \cdot u^\rho}{\Lambda^5}F_{21} + \delta_\mu^\nu \frac{u_j (u \cdot u^\rho)^2}{\Lambda^7}F_{22} \\
g_{ij} &= \delta_{ij}C_0 + \frac{u_i u_j}{\Lambda^2}C_1 + \delta_{ij}\frac{u_\rho \cdot u^\rho}{\Lambda^4}D_0 + \frac{u_{i\rho}u_j^\rho}{\Lambda^4}D_1 + \left[\delta_{ij}\frac{(u \cdot u_\rho)^2}{\Lambda^6} + \frac{u_i u_j \, u_\rho \cdot u^\rho}{\Lambda^6}\right]\frac{D_2}{2} + \frac{(u_i u_{j\rho} + u_j u_{i\rho})\, u \cdot u^\rho}{\Lambda^6}\frac{D_3}{2} + \frac{u_i u_j (u \cdot u_\rho)^2}{\Lambda^8}D_4 \\
\Lambda g_{ij}^\mu &= \frac{u_i^\mu u_j}{\Lambda^3}B_0 + \frac{u_i u_j^\mu}{\Lambda^3}B_1 + \delta_{ij}\frac{u \cdot u^\mu}{\Lambda^3}B_2 + \frac{u_i u_j \, u \cdot u^\mu}{\Lambda^5}B_3 \\
\Lambda^2 g_{ij}^{\mu\nu} &= \eta^{\mu\nu}\delta_{ij}A_0 + \eta^{\mu\nu}\frac{u_i u_j}{\Lambda^2}A_1
\end{aligned}$$

Scalar Lagrangian from 1-jet bundle metric: SMEFT/HEFT case

similar procedure as above, requiring also appropriate $O(4)$ transformations → more structures!

leading to

$$
\begin{aligned}
\mathscr{L} = {} & \frac{1}{2}(\partial_\mu\phi\cdot\partial^\mu\phi)\,(C_0+2G_0+J_0) + \frac{(\partial_\mu\phi\cdot\phi)^2}{\Lambda^2}\,\frac{C_1+2G_1+J_1}{2} + (\Box\phi\cdot\phi)E - \Lambda^4 V \\
& + \frac{(\partial_\mu\partial_\nu\phi\cdot\partial^\mu\partial^\nu\phi)}{\Lambda^2}\,\frac{A_0}{2} + \frac{(\partial_\mu\partial_\nu\phi\cdot\phi)(\partial^\mu\partial^\nu\phi\cdot\phi)}{\Lambda^4}\,\frac{A_1}{2} \\
& + \frac{(\partial_\mu\partial_\nu\phi\cdot\partial^\mu\phi)(\partial^\nu\phi\cdot\phi)}{\Lambda^4}\,\frac{B_0+B_1+2B_2+2F_{11}}{2} + \frac{(\partial_\mu\partial_\nu\phi\cdot\phi)(\partial^\mu\phi\cdot\partial^\nu\phi)}{\Lambda^4}\,\frac{B_0+B_1+2F_{10}}{2} \\
& + \frac{(\Box\phi\cdot\phi)(\partial_\mu\phi\cdot\partial^\mu\phi)}{\Lambda^4}F_{20} + \frac{(\Box\phi\cdot\partial_\mu\phi)(\partial^\mu\phi\cdot\phi)}{\Lambda^4}F_{21} + \frac{(\partial_\mu\phi\cdot\partial^\mu\phi)^2}{\Lambda^4}\,\frac{D_0+2H_0+K_0}{2} + \frac{(\partial_\mu\phi\cdot\partial_\nu\phi)^2}{\Lambda^4}\,\frac{D_1+2H_1+K_1}{2} \\
& + \frac{(\partial_\mu\partial_\nu\phi\cdot\phi)(\partial^\mu\phi\cdot\phi)(\partial^\nu\phi\cdot\phi)}{\Lambda^6}(B_3+F_{12}) + \frac{(\Box\phi\cdot\phi)(\partial_\mu\phi\cdot\phi)^2}{\Lambda^6}F_{22} \\
& + \frac{(\partial_\mu\phi\cdot\partial^\mu\phi)(\partial_\nu\phi\cdot\phi)^2}{\Lambda^6}\,\frac{D_2+2H_2+K_2}{2} + \frac{(\partial_\mu\phi\cdot\partial_\nu\phi)(\partial^\mu\phi\cdot\phi)(\partial^\nu\phi\cdot\phi)}{\Lambda^6}\,\frac{D_3+2H_3+K_3}{2} \\
& + \frac{(\partial_\mu\phi\cdot\phi)^4}{\Lambda^8}\,\frac{D_4+2H_4+K_4}{2}
\end{aligned}
$$

Scalar Lagrangian from 1-jet bundle metric: SMEFT/HEFT case

similar procedure as above, requiring also appropriate $O(4)$ transformations → more structures!

leading to

blue = can be removed via EOM

$$
\begin{aligned}
\mathscr{L} = &\frac{1}{2}(\partial_\mu\phi\cdot\partial^\mu\phi)(C_0+2G_0+J_0-2E) + \frac{(\partial_\mu\phi\cdot\phi)^2}{\Lambda^2}\frac{C_1+2G_1+J_1-4E'}{2} - \Lambda^4 V \\
&+\frac{(\partial_\mu\partial_\nu\phi\cdot\partial^\mu\partial^\nu\phi)}{\Lambda^2}\frac{A_0}{2} + \frac{(\partial_\mu\partial_\nu\phi\cdot\phi)(\partial^\mu\partial^\nu\phi\cdot\phi)}{\Lambda^4}\frac{A_1}{2} \\
&+\frac{(\partial_\mu\partial_\nu\phi\cdot\partial^\mu\phi)(\partial^\nu\phi\cdot\phi)}{\Lambda^4}\frac{B_0+B_1+2B_2+2F_{11}-4F_{20}-2F_{21}}{2} \\
&+\frac{(\partial_\mu\partial_\nu\phi\cdot\phi)(\partial^\mu\phi\cdot\partial^\nu\phi)}{\Lambda^4}\frac{B_0+B_1+2F_{10}-2F_{21}}{2} + \frac{(\partial_\mu\phi\cdot\partial^\mu\phi)^2}{\Lambda^4}\frac{D_0+2H_0+K_{10}-2F_{20}}{2} \\
&+\frac{(\partial_\mu\phi\cdot\partial_\nu\phi)^2}{\Lambda^4}\frac{D_1+2H_1+K_1-2F_{21}}{2} + \frac{(\partial_\mu\partial_\nu\phi\cdot\phi)(\partial^\mu\phi\cdot\phi)(\partial^\nu\phi\cdot\phi)}{\Lambda^6}(B_3+F_{12}-2F_{22}) \\
&+\frac{(\partial_\mu\phi\cdot\partial^\mu\phi)(\partial_\nu\phi\cdot\phi)^2}{\Lambda^6}\frac{D_2+2H_2+K_2-4F'_{20}-2F_{22}}{2} \\
&+\frac{(\partial_\mu\phi\cdot\partial_\nu\phi)(\partial^\mu\phi\cdot\phi)(\partial^\nu\phi\cdot\phi)}{\Lambda^6}\frac{D_3+2H_3+K_3-4F'_{21}-4F_{22}}{2} + \frac{(\partial_\mu\phi\cdot\phi)^4}{\Lambda^8}\frac{D_4+2H_4+K_4-4F'_{22}}{2},
\end{aligned}
$$

Extension to higher derivatives

proved that: metric $g^{(r)}$ of a r-jet bundle $\to$ **redundant** basis of operators with up to $2(r+1)$ deriv.

r-jet bundle has coordinates $y^I = (x^\mu, u^i, u^i_{\mu_1}, u^i_{\mu_1\mu_2}, \ldots, u^i_{\mu_1\ldots\mu_r})$

$$g^{(r)} = \begin{pmatrix} dx^\mu & du^i & du^i_{\mu_1} & \cdots & du^i_{\mu_1\ldots\mu_r} \end{pmatrix} \begin{pmatrix} g_{\mu\nu} & g_{\mu j} & g^{\nu_1}_{\mu j} & \cdots & g^{\nu_1\ldots\nu_r}_{\mu j} \\ g_{\nu i} & g_{ij} & g^{\nu_1}_{ij} & \cdots & g^{\nu_1\ldots\nu_r}_{ij} \\ g^{\mu_1}_{\nu i} & g^{\mu_1}_{ij} & g^{\mu_1\nu_1}_{ij} & \cdots & g^{\mu_1\nu_1\ldots\nu_r}_{ij} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g^{\mu_1\ldots\mu_r}_{\nu i} & g^{\mu_1\ldots\mu_r}_{ij} & g^{\mu_1\ldots\mu_r\nu_1}_{ij} & \cdots & g^{\mu_1\ldots\mu_r\nu_1\ldots\nu_r}_{ij} \end{pmatrix} \begin{pmatrix} dx^\nu \\ du^j \\ du^j_{\nu_1} \\ \cdots \\ du^j_{\nu_1\ldots\nu_r} \end{pmatrix}$$

- arbitrary internal symmetries (or absence thereof) can always be implemented
- many redundancies! different metric entries mapping to same operators, IBP, EOM, diffeos...

Wrapping up

- SMEFT is a very popular theory for indirect new physics searches.
  an ambitious program underway for **LHC**, interest in combining with **other experiments**
- validity of SMEFT assumptions so far only postulated (but not disproved either)
- recently, several groups considered possible **alternatives** to SMEFT truncated to dim-6:
  geoSMEFT, direct study of $d = 8$, "primaries"/on-shell amplitudes, HEFT
- HEFT is an alternative to SMEFT, that adopts a different description of the scalar sector
- HEFT is more general then SMEFT, but their characterisation is obscured by field redefinitions.
  differential geometry used in past years to work around this issue
- we proposed a new formulation based on field space bundles and their higher **jet bundles**
  - 👍 gives a geometric interpretation to **scalar potential and higher-∂ terms**
  - significant degeneracy. relation to amplitudes less clear than in field space picture
  - gauge and fermion fields not incorporated yet