THE QCD AXION:
Some Like It Hot

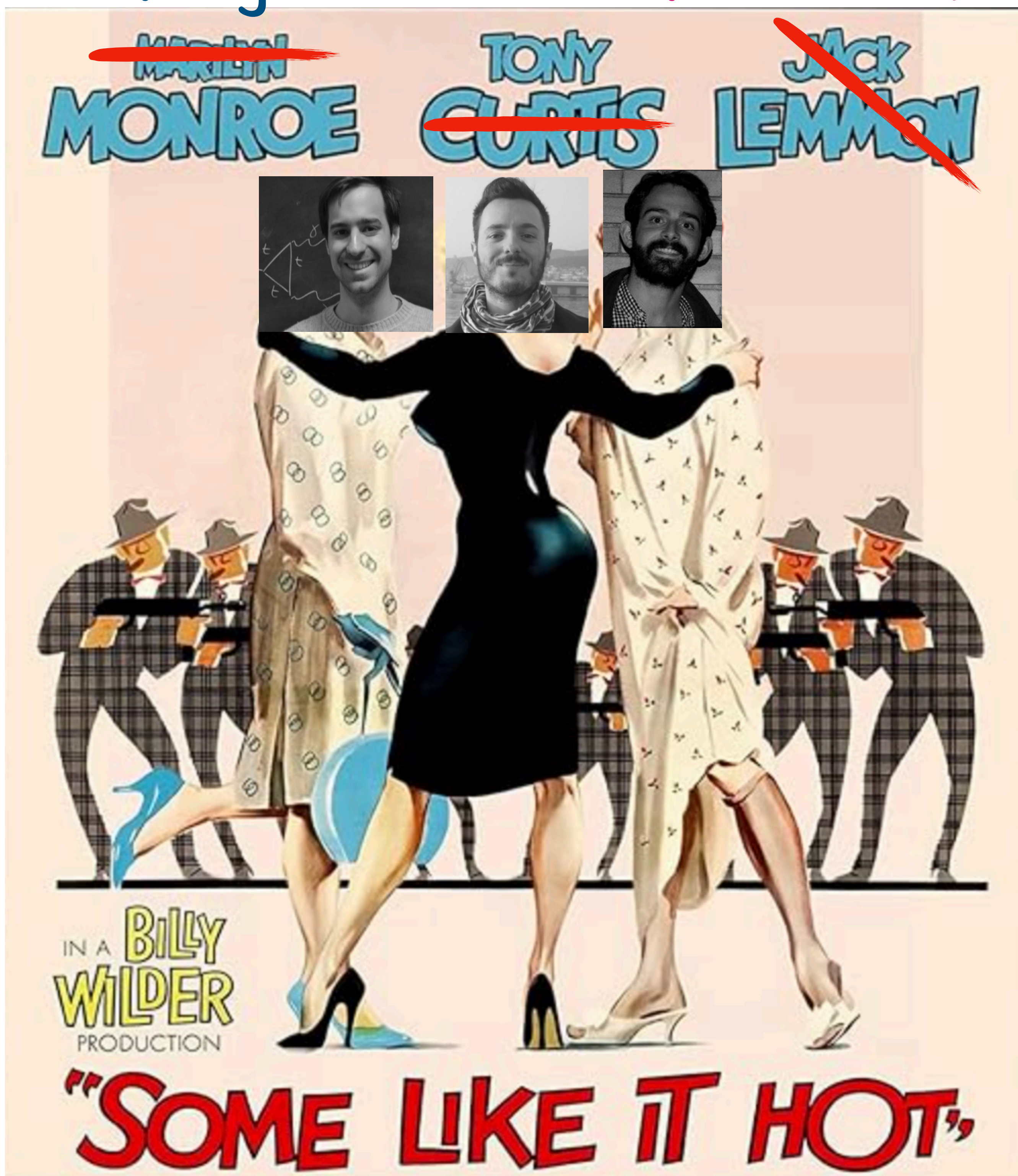
MAURO VALLI

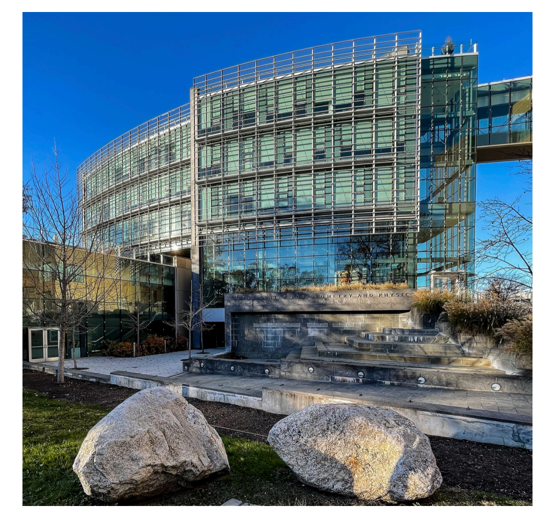
INFN Rome

A **SISSA** MOVIE

ASTROPARTICLE PHYSICS | THEORETICAL PARTICLE PHYSICS | ASTROPHYSICS

Maurilyn | di Cortona | Fred Whitey





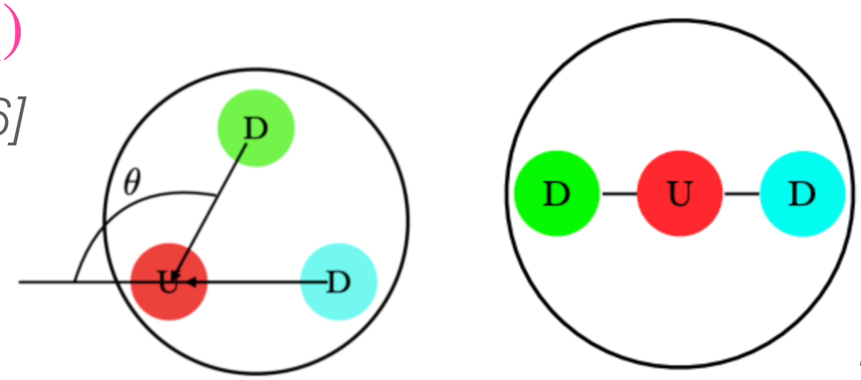
arXiv:**2310.08169**

Minimal QCD Axion

- It couples to Topological Charge Density: $\mathcal{Q} \equiv \frac{\alpha_s}{8\pi f_a} G\widetilde{G}$

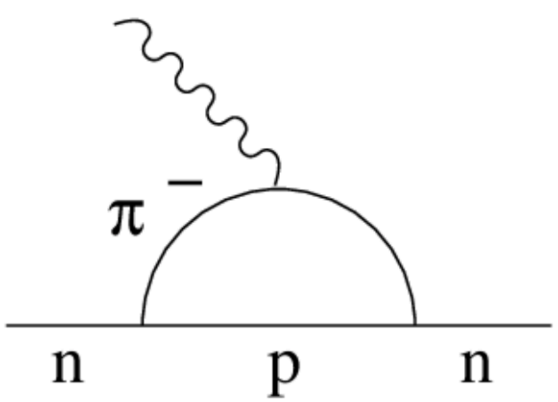
$f_a = 5.7 \times 10^6\,\text{GeV}\,(\text{eV}/m_a)$

[Grilli, Hardy, Pardo, Villadoro `16]



[Hook `21]

Coupling required to *"relax"* the Strong CP problem, namely:



$$\sim \theta \frac{m_q}{m_N^2} e \sim \frac{10^{-2}}{\text{GeV}} e \sim 10^{-16}\,\text{cm} \cdot e$$

$(\text{GeV}^{-1} \sim 10^{-14}\,\text{cm})$

VS d_n^{exp} $\lesssim 10^{-26}\,\text{cm} \cdot e$

$$\theta \lesssim 10^{-10}$$

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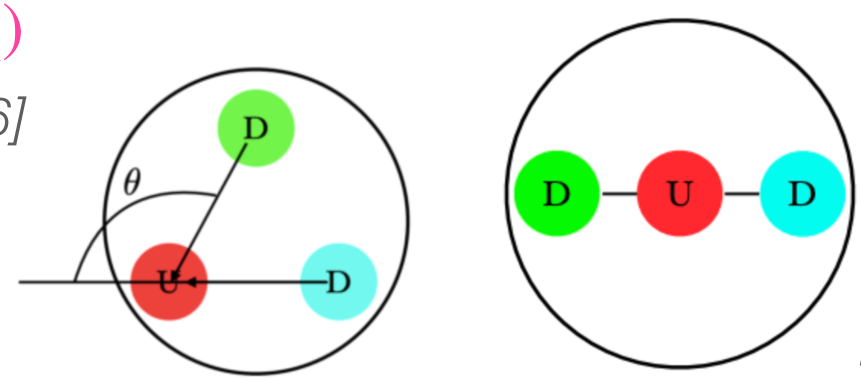
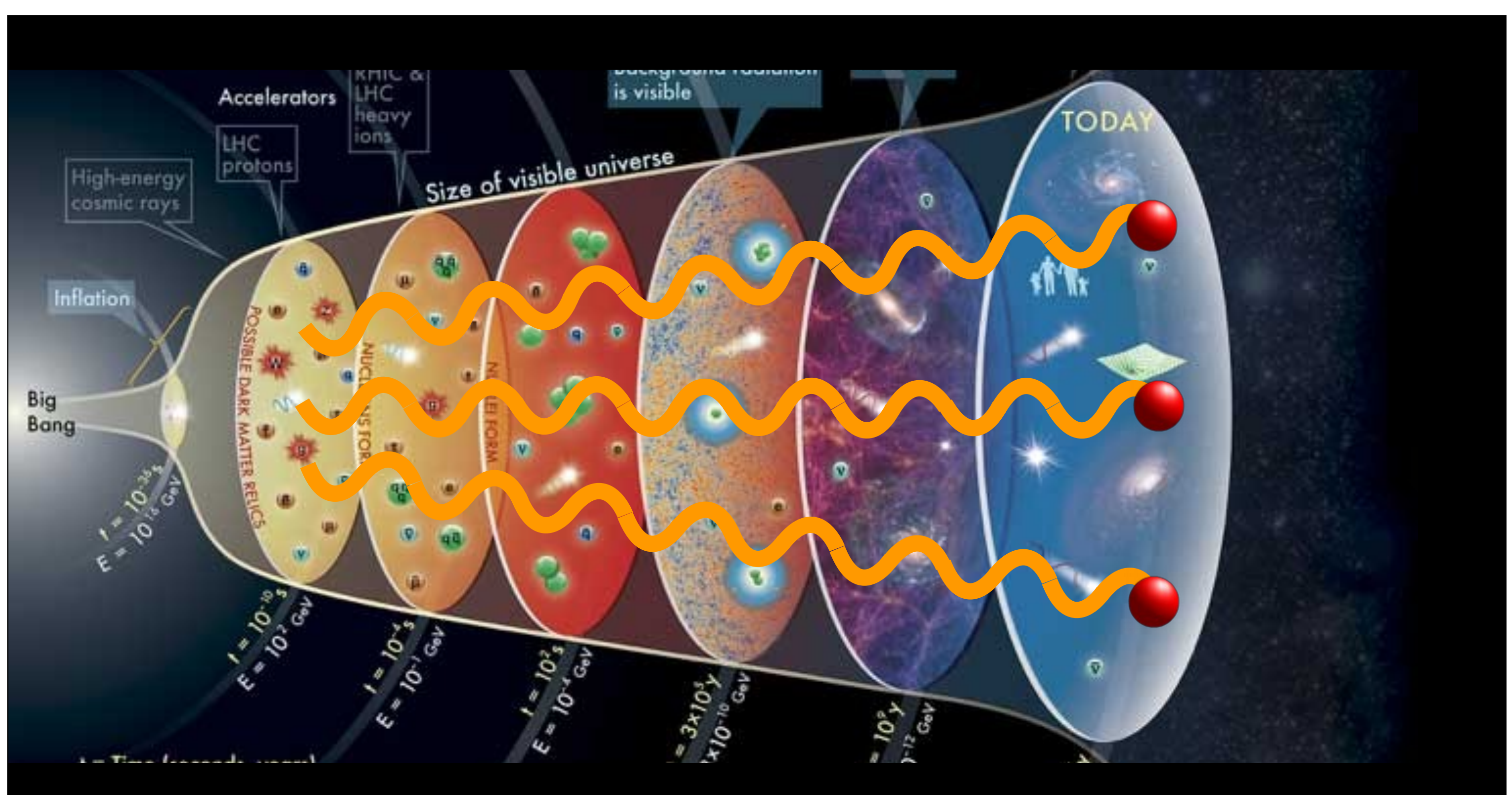


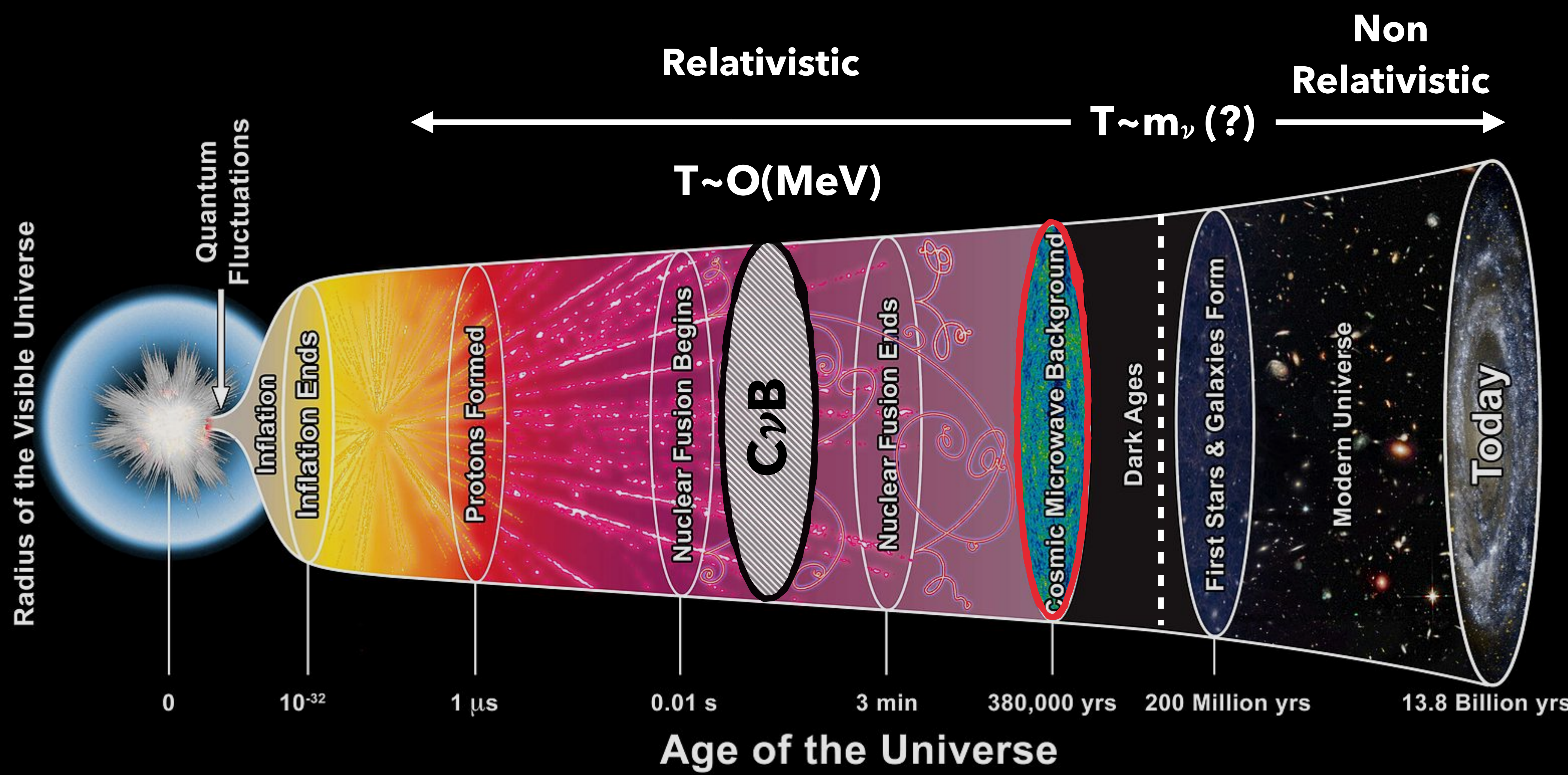
[Hook `21]

⇨ BSM contribution to Early Universe energy budget!



SPOILER: (Hot) Axions ~ Neutrinos

Neutrino cosmology 101



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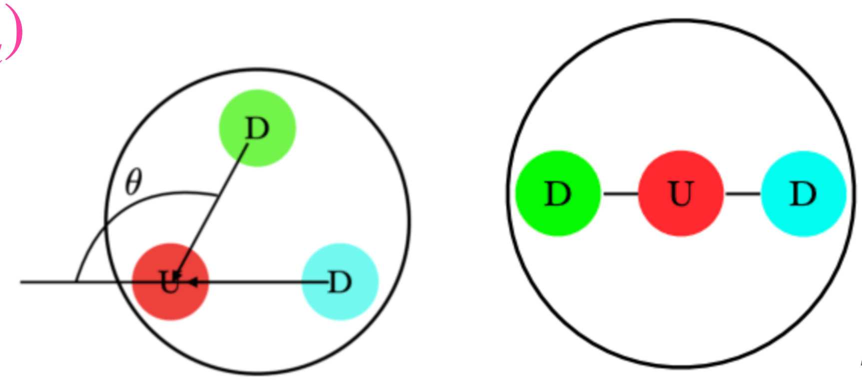
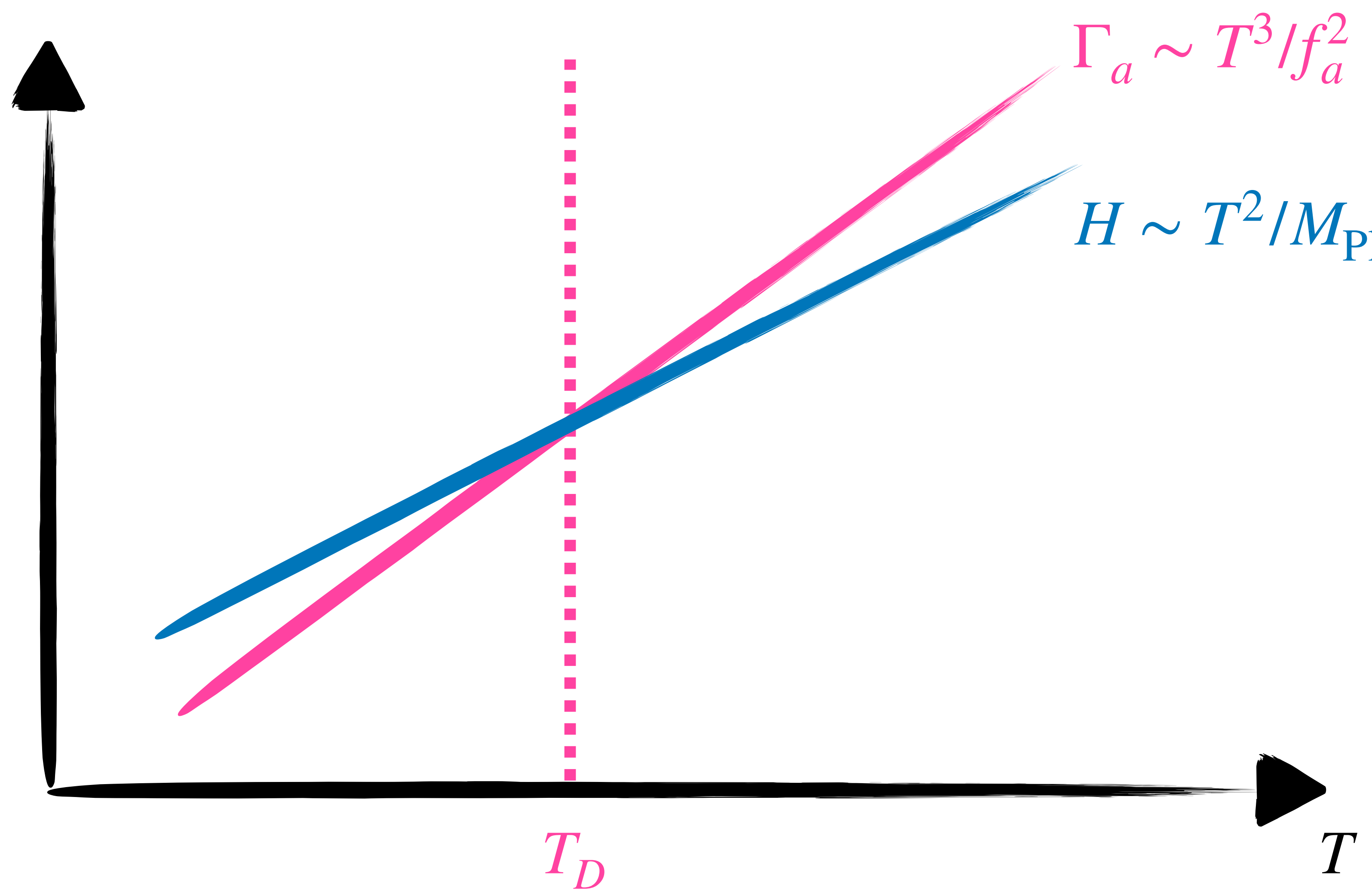


[Hook `21]

T_D

Decoupling roughly at the temperature:

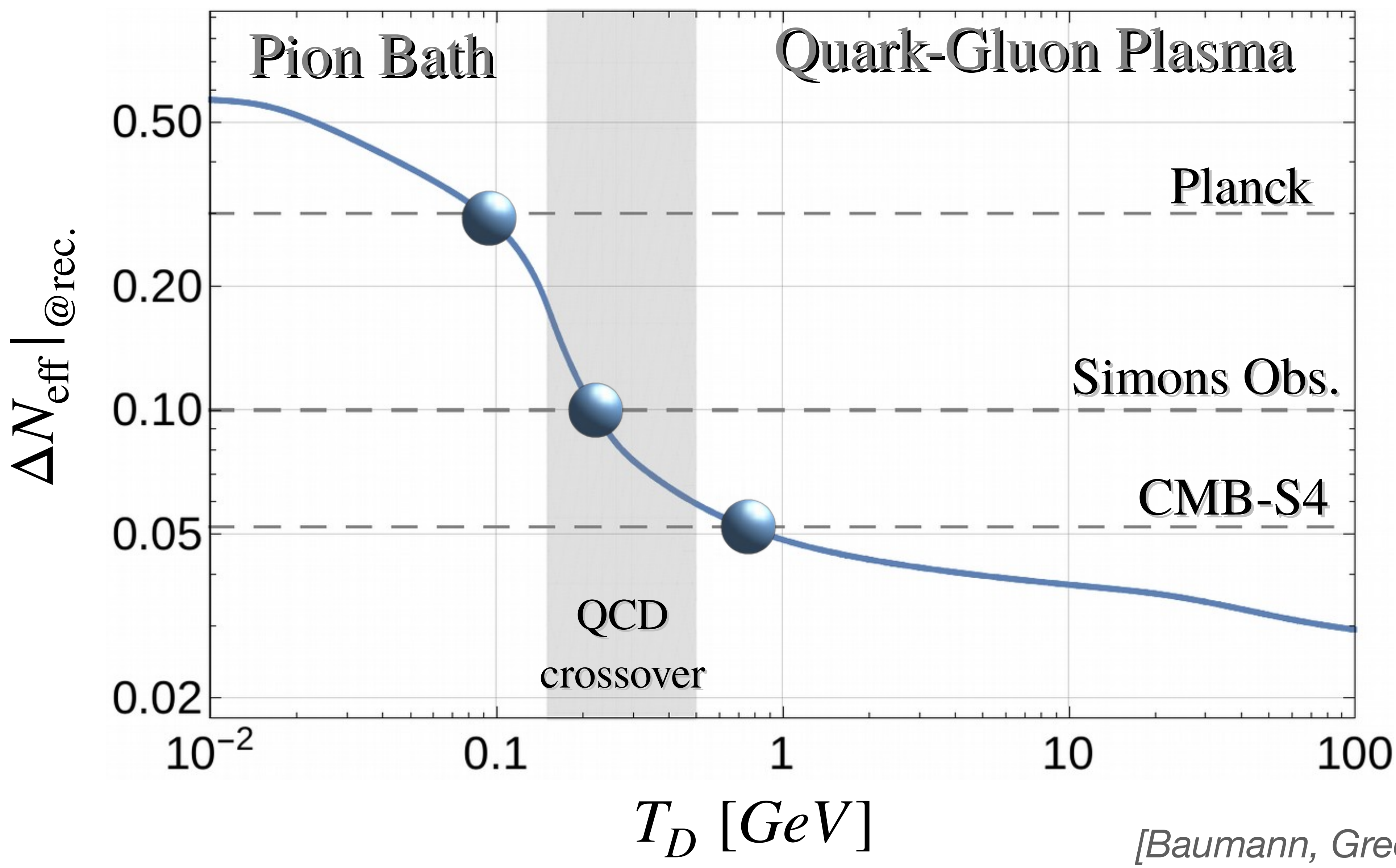
$\mathcal{O}(\text{MeV}) \times (\text{eV}/m_a)^2$



Minimal QCD Axion

$$\Delta N_{\rm eff}|_{\rm @rec.} \equiv \left.\frac{\rho_a(m_a)}{\rho_\nu}\right|_{T_{\rm CMB}} \simeq \frac{4}{7}\left(\frac{11}{4}\frac{g_\star(T_{\rm CMB})}{g_\star(T_D(m_a))}\right)^{4/3}$$

$\pi\pi \leftrightarrow a\pi$ $\quad gg \leftrightarrow ga$

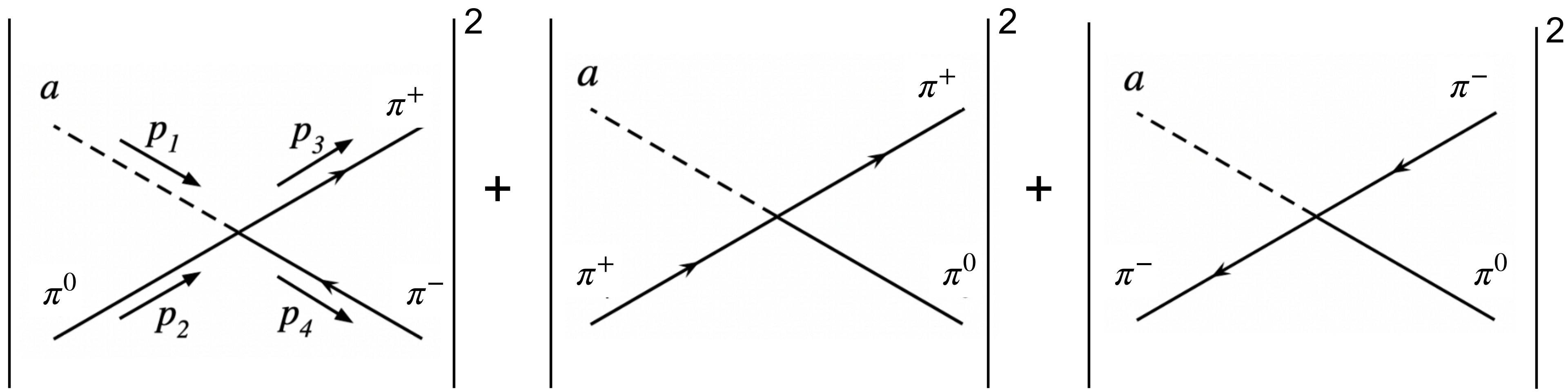


[Baumann, Green, Wallisch `16]

Hot Axions from pions

[Georgi, Kaplan, Randall `86]

$$\mathscr{L}_{a\pi}^{\rm LO} = \frac{C_{a\pi}}{f_a f_\pi}\partial^\mu a \left(2\partial_\mu \pi^0 \pi^+ \pi^- - \pi^0 \partial_\mu \pi^+ \pi^- - \pi^0 \pi^+ \partial_\mu \pi^-\right)$$



$$\sum |\mathcal{M}|^2_{\rm LO} = \left(\frac{C_{a\pi}}{f_a f_\pi}\right)^2 \frac{9}{4}\left[s^2 + t^2 + u^2 - 3m_\pi^4\right]$$

- $\langle E_\pi \rangle \simeq \rho_\pi / n_\pi \simeq 3T \;\Rightarrow \sqrt{s} \gtrsim 500\,\mathrm{MeV}$ @ $T \sim 100\,\mathrm{MeV}$
- $\Gamma_a^{\rm (NLO)} \sim \Gamma_a^{\rm (LO)}$ for $\mathrm{T} > 70\,\mathrm{MeV}$ [Di Luzio, Martinelli, Piazza `21]

Hot Axions from pions

G.Villadoro @ GGI `23

General form of low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right) q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}(1+c_3\sigma^3)}$$

$$\frac{\partial_\mu a}{2f_a} j_A^\mu \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\mathrm{phys}} + \sin(\theta_{a\pi})\,a_{\mathrm{phys}} \simeq \pi^0_{\mathrm{phys}} + \theta_{a\pi} a_{\mathrm{phys}}$$

@ all orders in χPT

$$\mathcal{M}_{a\pi^i\to\pi^j\pi^k} = \theta_{a\pi}\cdot\mathcal{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

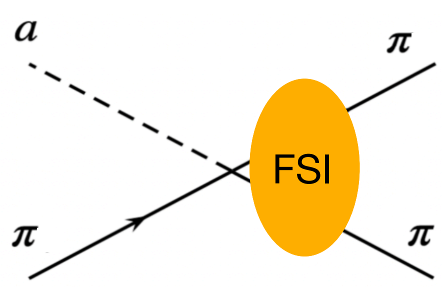
$\lesssim 10\%$

e.g. @ LO

$$|\mathcal{M}^{\mathrm{LO}}|^2 = \theta^2_{a\pi}\frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4} \qquad |\mathcal{M}^{\mathrm{LO}}_{\pi\pi}|^2 = \frac{s^2+t^2+u^2-4m_\pi^4}{f_\pi^4}$$

Hot Axions from pions

G.Piazza @ NP Signal `23



$\pi\pi$ final-state interactions (FSI) are resonant $\begin{cases} \sigma \text{ or } f_0(500) \text{ in } I = L = 0 \\ \rho(770) \text{ in } I = L = 1 \end{cases}$

<u>ChPT cannot produce resonances</u>

✤Inverse Amplitude Method (IAM): [Truong, PRL **61**, 2526]

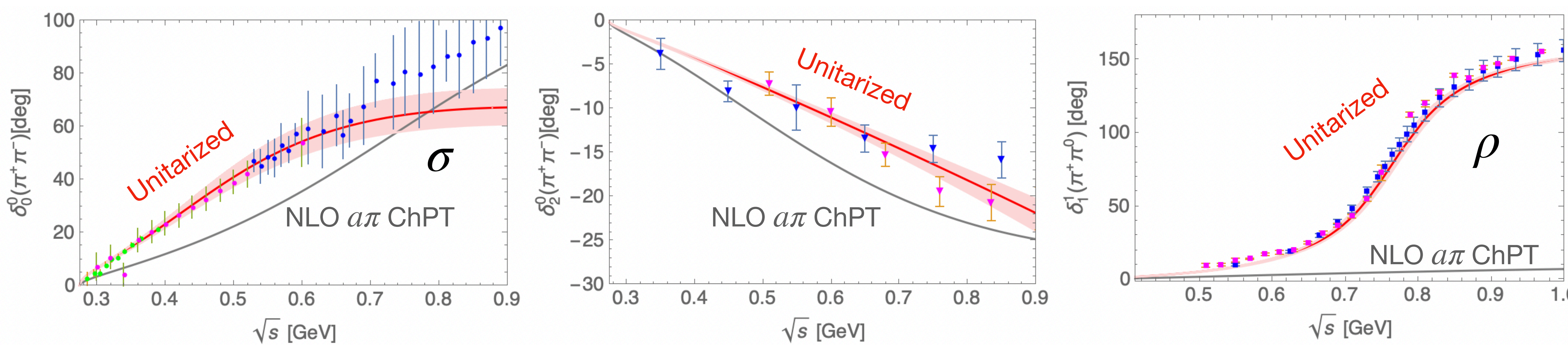
Definite I, J amplitudes

$$A_{IJ}(s) = \frac{A_{IJ}^{(2)}(s)}{1 - A_{IJ}^{(4)}(s)/A_{IJ}^{(2)}(s)}$$

The IAM amplitude satisfies unitarity and has the correct low-energy expansion of ChPT up to $\mathcal{O}(p^4)$

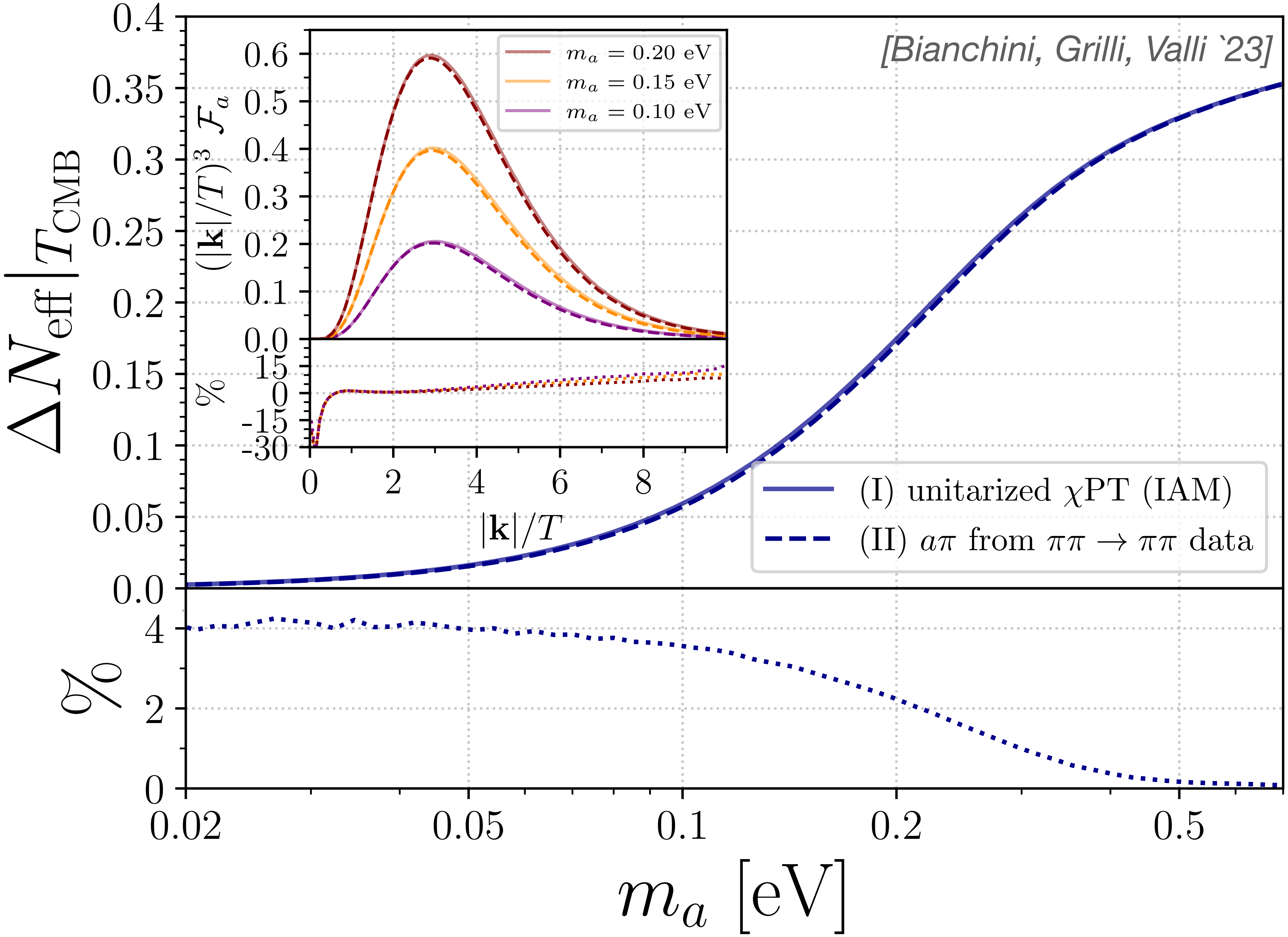
IAM LECs from fit to $\pi\pi$ scatt. [Dobado, Pelaez 1997]

✓ Phases obtained in IAM correspond to phases of $\pi\pi$ scattering: Watson th.!

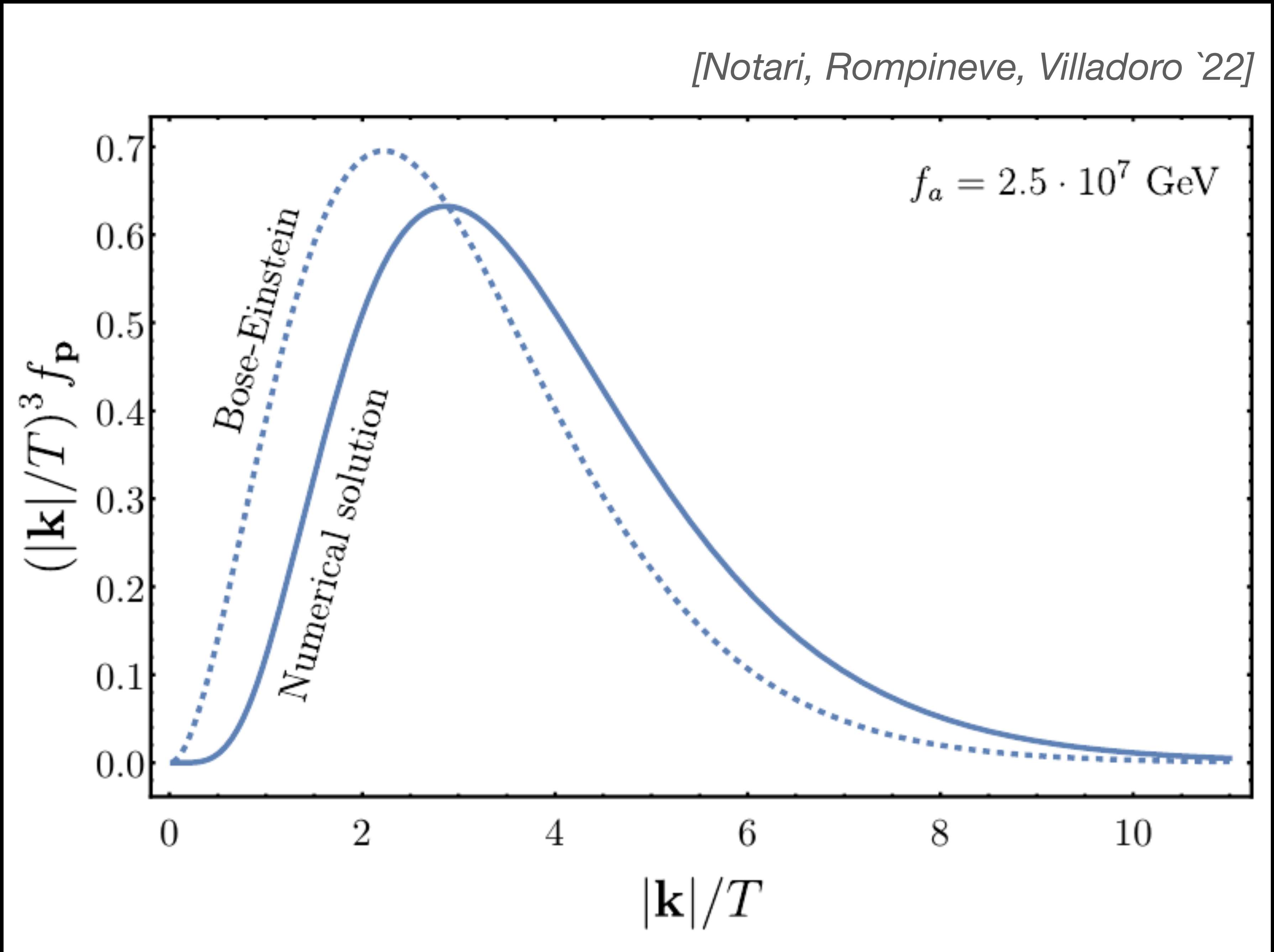


Hot Axions from pions

$$\frac{\partial \mathcal{F}_a}{\partial t} - H\,|\mathbf{k}|\,\frac{\partial \mathcal{F}_a}{\partial |\mathbf{k}|} = \Gamma_a\,(\mathcal{F}_a^{\rm eq} - \mathcal{F}_a)$$



Why momentum-dependent Boltzmann?

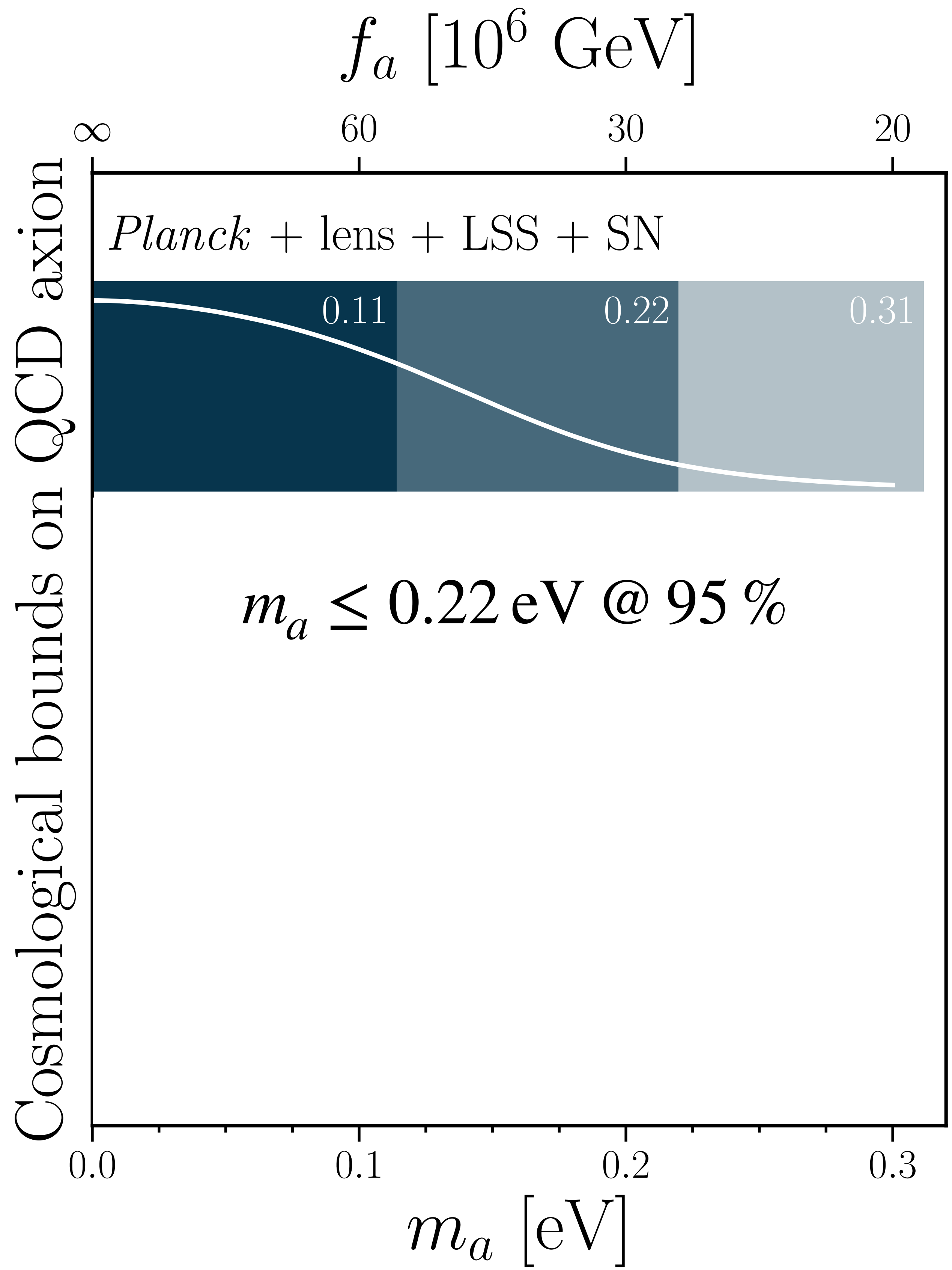


High momenta *k* decouple later than low *k*.

They see a lower g_* —> Greater $\Delta N_{\rm eff}$

Minimal QCD Axion ($T_D < T_C$)

[Bianchini, Grilli, Valli `23]



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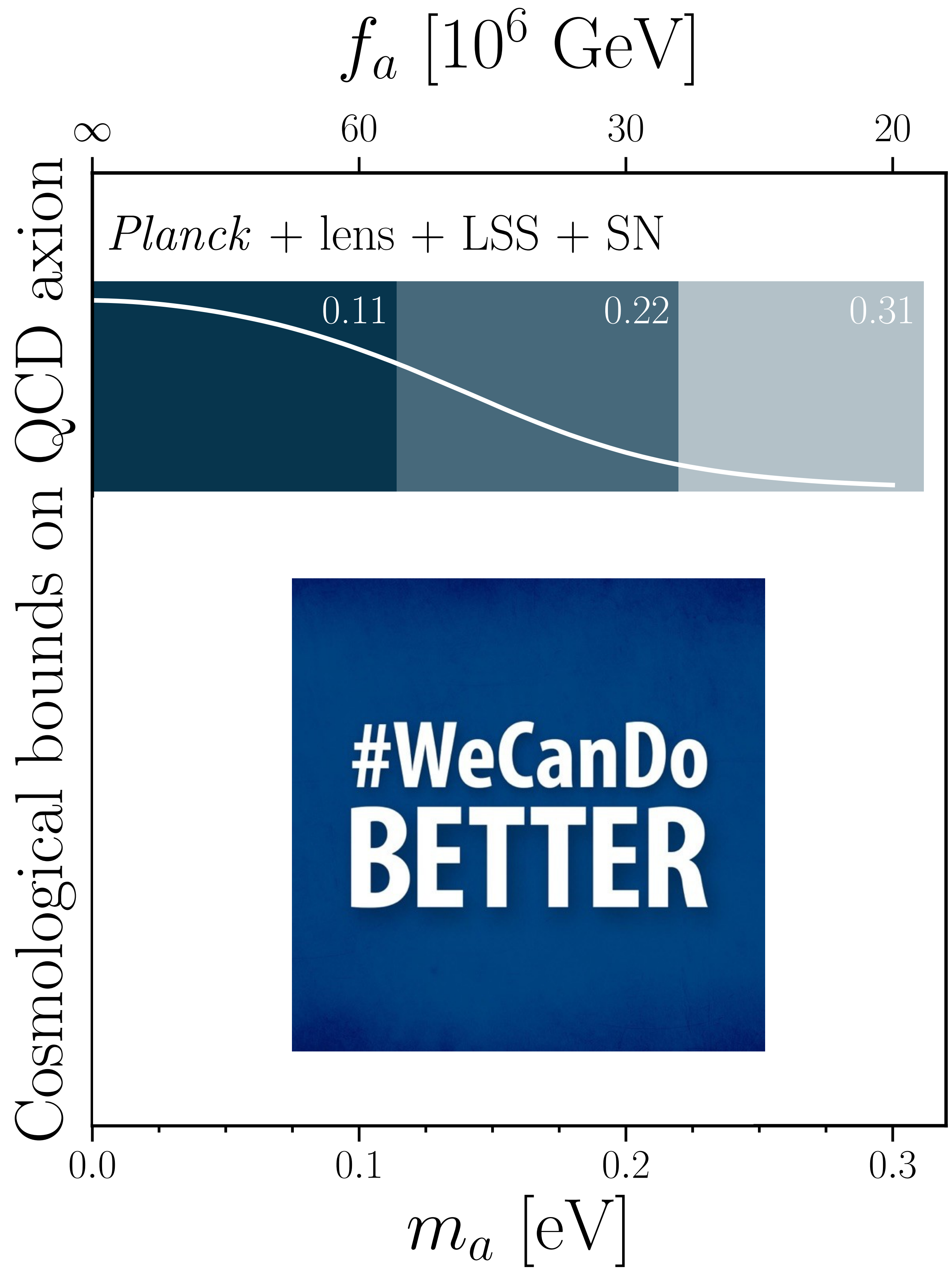
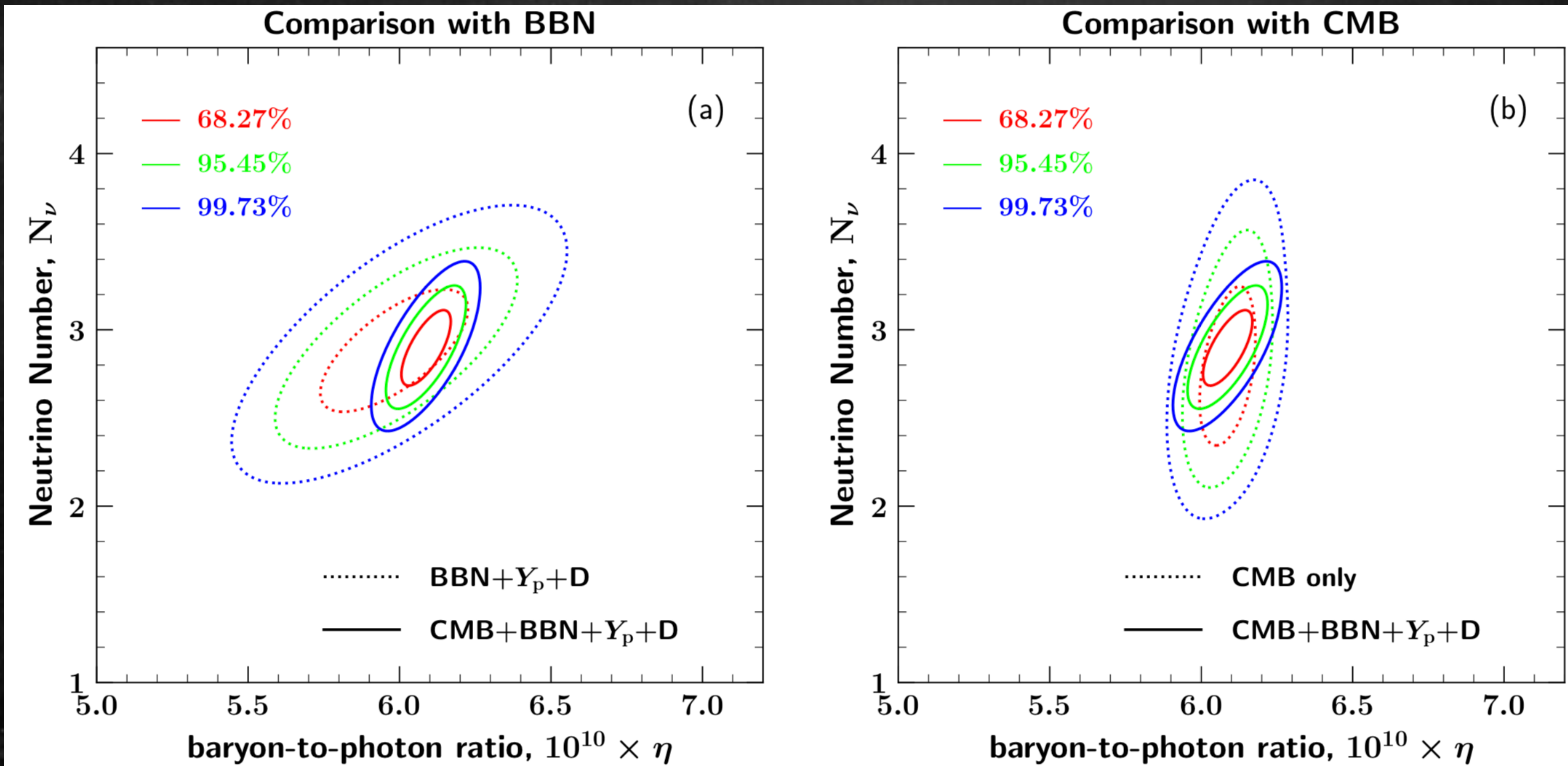


2nd SPOILER …

arXiv: **2207.13133**



BBN IS COMPETITIVE WITH CMB TO CONSTRAIN $\Delta N_{\rm eff}$

BBN era in ΛCDM

$$n + \nu_e \leftrightarrow p + e^-$$
$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$(n_n/n_p)|_{T \gtrsim MeV} \simeq \exp(-Q/T)$$

$m_n - m_p \simeq 1.3\, MeV$

$$(n_n/n_p)|_{T \simeq MeV} \simeq 1/6$$

Nucleosynthesis naively at $T_{nucl.} \sim B_D \simeq 2.2$ MeV … BUT:

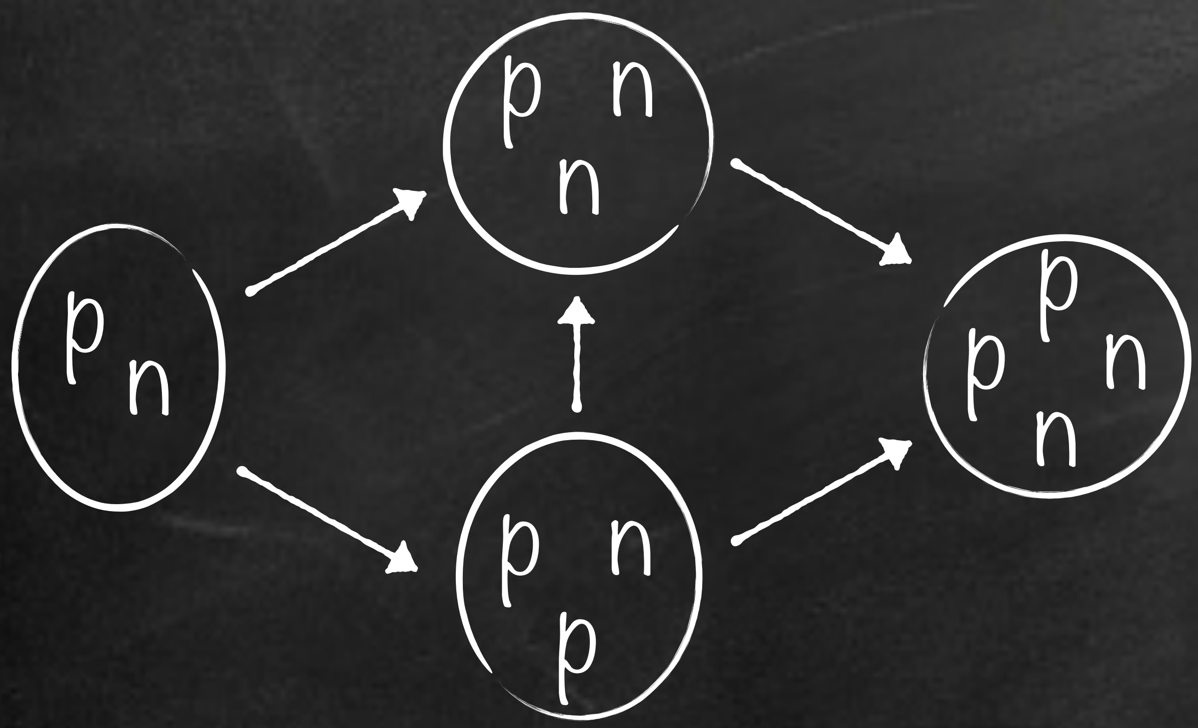
$$\Gamma(n + p \to D + \gamma) \sim n_B \langle \sigma v \rangle_{D\gamma}$$

$$\Gamma(n + p \leftarrow D + \gamma) \sim n_\gamma \exp(-B_D/T_\gamma) \langle \sigma v \rangle_{D\gamma}$$

i.e., it really starts at $T_{nucl.}$ such that: $\eta_B \simeq \exp(-B_D/T_{nucl.})$

BBN ERA IN ΛCDM

Deuterium "bottleneck" implies $T_{nucl.} \simeq 0.1$ MeV. After that :



~ all neutrons into helium-4

$$(n_n/n_p)\,|_{T\simeq 0.1 MeV} \simeq 1/7$$

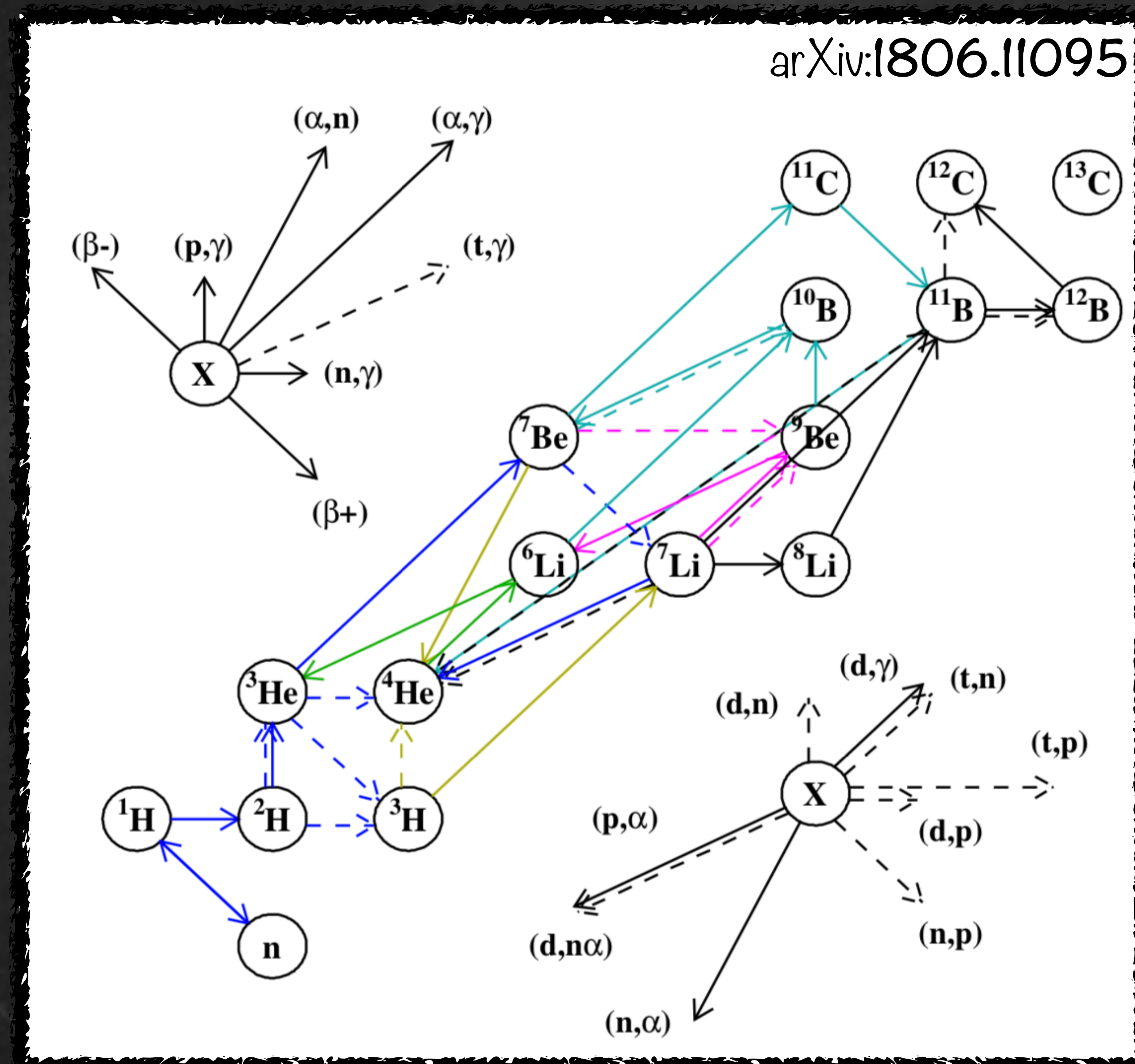
$$Y_P \equiv \frac{m_{^4He}}{m_B} \simeq \frac{4(n_n/2)}{n_n + n_p} \simeq 0.25$$

Baryon mass fraction in helium-4

$\mathcal{O}(10^{-5})$ residual amount of deuterium and helium-3 relative to p.

Lithium-7 "survives" in smaller relative abundance, $\mathcal{O}(10^{-10})$.

Of course … precise BBN predictions cannot be worked out by hand!



$$\dot{Y}_i \sim n_B \left(\langle \sigma v \rangle_{kl \to ij} Y_k Y_l - \langle \sigma v \rangle_{ij \to kl} Y_i Y_j \right)$$

THE ASTROPHYSICAL JOURNAL, Vol. 148, April 1967

ON THE SYNTHESIS OF ELEMENTS AT VERY HIGH TEMPERATURES*

ROBERT V. WAGONER, WILLIAM A. FOWLER, AND F. HOYLE
California Institute of Technology, Pasadena, California, and Cambridge University

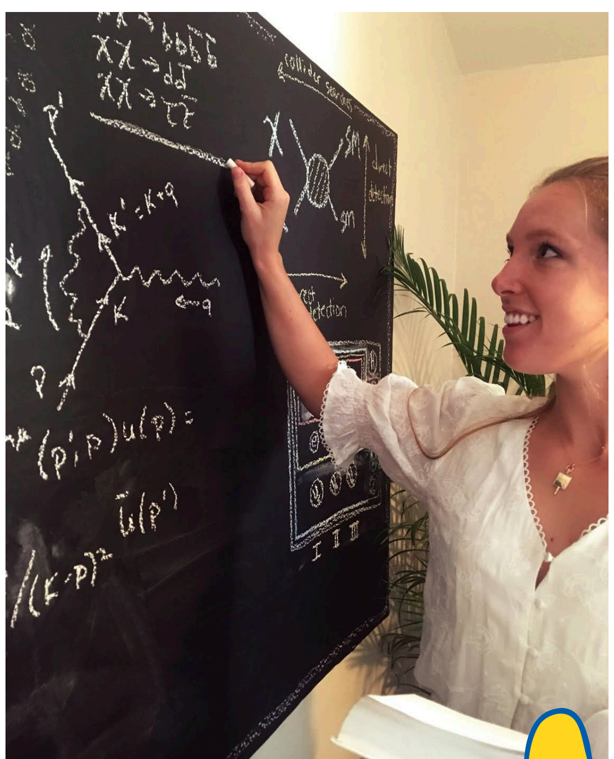
Received September 1, 1966

arXiv: 2307.07061

A new tool to investigate Big Bang Nucleosynthesis (BBN) within the Standard Model (SM) and Beyond (BSM)

PRyMordial The first 3 min in $\mathcal{O}(10)$ sec

Anne-Katherine Burns



Younger Tim

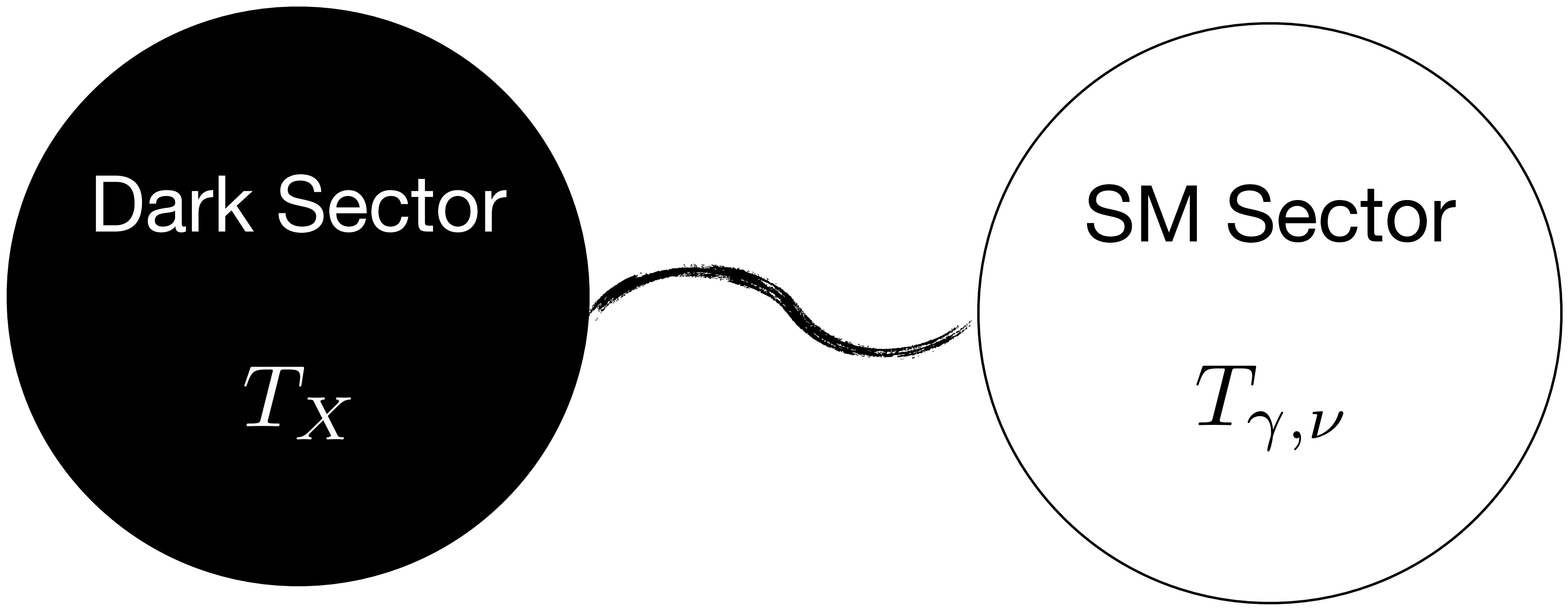
Californian Me

Reason # 1

ν Decoupling & N_{eff}

GO THERMAL: Approach extremely useful for BSM Physics!
2001.04466

E.g.:



$$T_\gamma(t), T_\nu(t), T_X(t)$$

→ $a(t)$, $a(T_\gamma)$

Primordial abundances affected in a 3-fold way:

H , $\Gamma_{n\leftrightarrow p}$, n_B

→ N_{eff}

Constraint from "Planck 2018"

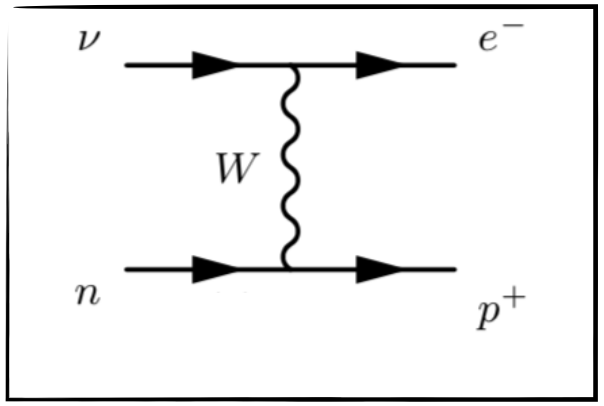
2.93 ± 0.29 @ 68 % CL

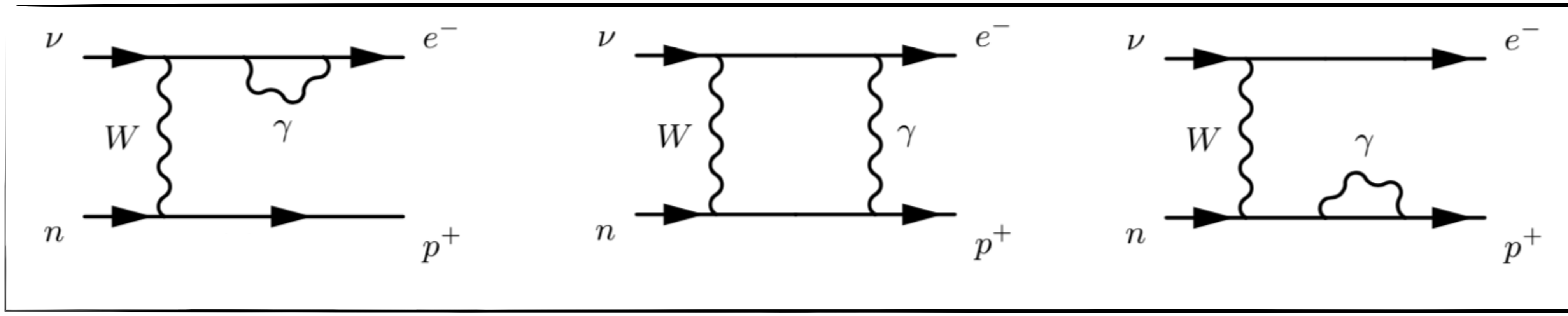
Reason # 2

Weak Rates $\Gamma_{n\leftrightarrow p}$

Both T_γ and T_ν relevant here !

Primordial ^{4}He & D/H precisely measured: **BBN precision tool**





LO: easy-peasy in Born … … but finite mass effects + $O(\alpha)$ corrections relevant for precision!

PRIMAT code offers a wonderful ab-initio computation, but takes time …

BLUE SWORD VS RED SWORD

SciPy

vegas 5.0

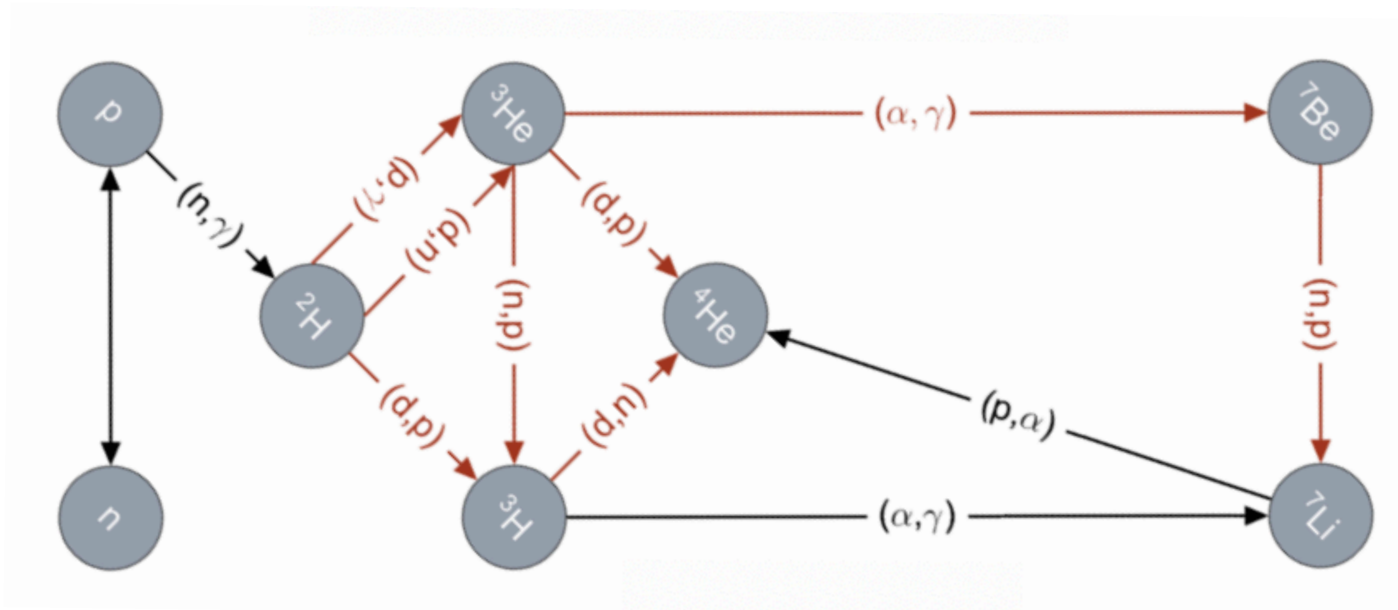
```
pip install vegas
```

WOLFRAM MATHEMATICA 12

Reason # 3

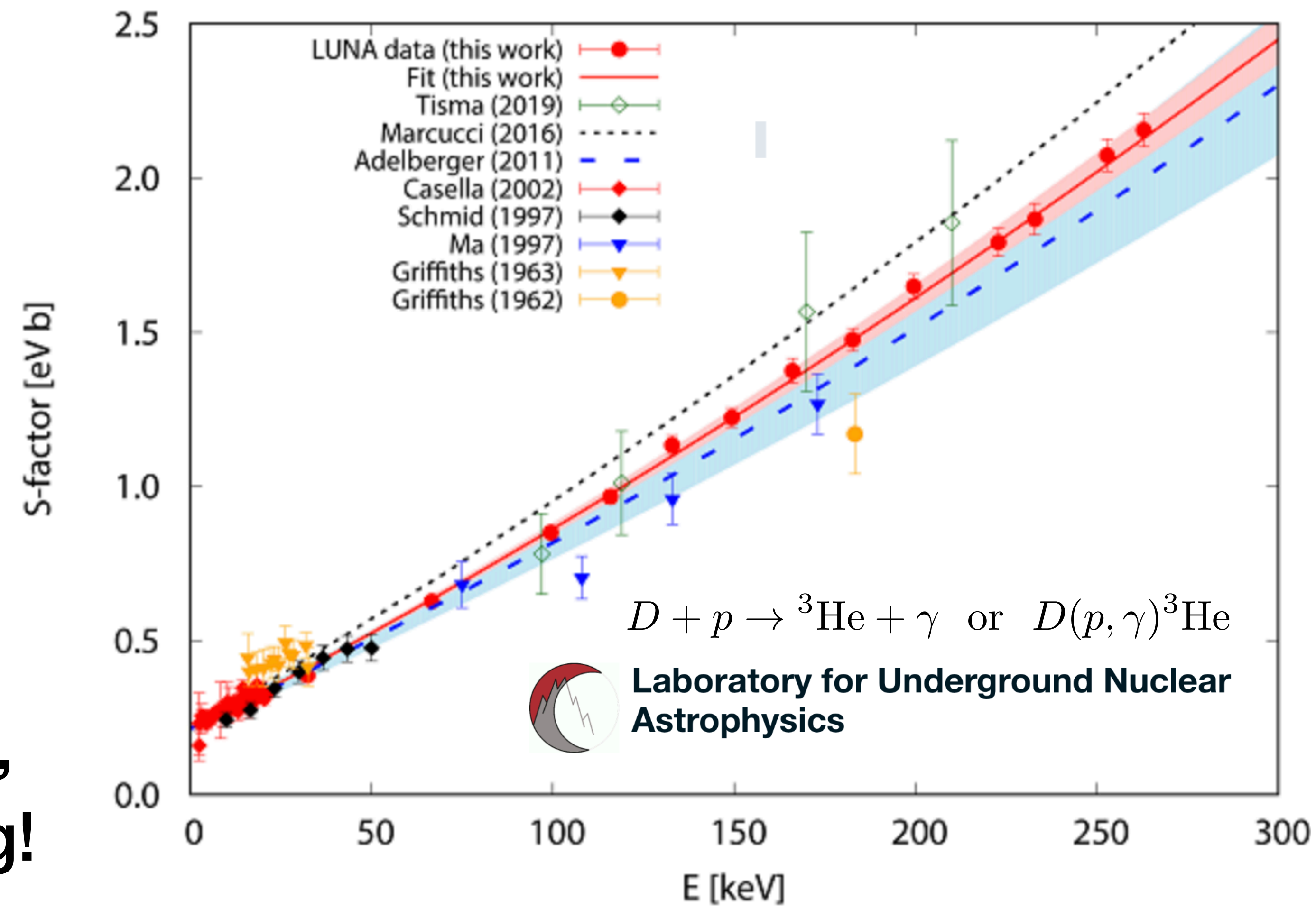
Nuclear Rates $\langle \sigma v \rangle_{\rm nucl}$

Nuclear net dependent also on $\eta(t)$



Existing codes implement own recipe for O(100) rates as function of T …

Nature **587**, 210–213 (2020)



Key reactions to study primordial light elements are only O(10).

Even those vary from group to group though, unless data driven (LUNA)

Exploring these systematics crucial, but a ready-to-use tool been missing!

PRyMordial: Overview

- **PRyMordial** **A new package for BBN phenomenology**
- Featuring:
  — simplified, but precise, method for ν decoupling
  — ab-initio efficient computation of n <—> p
  — a customizable up-to-date nuclear network
  — several built-in options for New Physics

Meets precision for state-of-the-art SM predictions.

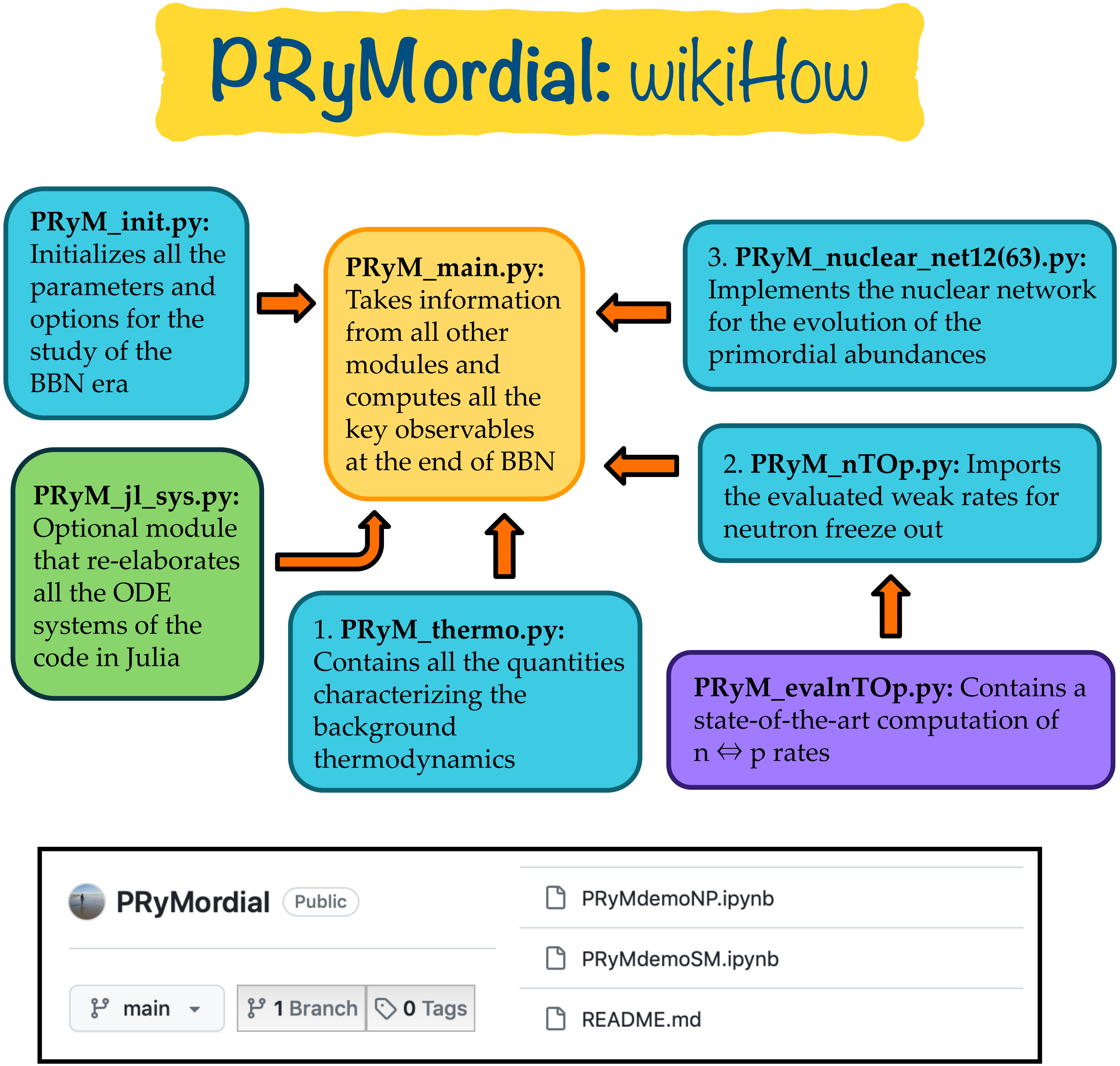
Opens up uncharted territory for BSM in BBN era.

- Fully Python-based, user-friendly & **numerically fast …**

vallima

github.com/vallima/PRyMordial

DiffEq.jl from Sci Machine Learning kit in julia



PRyMordial: wikiHow
PRyM_init.py: Initializes all the parameters and options for the study of the BBN era
PRyM_main.py: Takes information from all other modules and computes all the key observables at the end of BBN
3. PRyM_nuclear_net12(63).py: Implements the nuclear network for the evolution of the primordial abundances
PRyM_jl_sys.py: Optional module that re-elaborates all the ODE systems of the code in Julia
2. PRyM_nTOp.py: Imports the evaluated weak rates for neutron freeze out
1. PRyM_thermo.py: Contains all the quantities characterizing the background thermodynamics
PRyM_evalnTOp.py: Contains a state-of-the-art computation of n ⇔ p rates
PRyMordial
Public
main
1 Branch
0 Tags
PRyMdemoNP.ipynb
PRyMdemoSM.ipynb
README.md

BBN OBSERVATIONS

Primordial light elements predicted: $D,\ {}^3He,\ {}^4He,\ {}^7Li$

✅ Helium-4 observed in extragalactic HII regions

Emission spectra of gas clouds (detailed line modeling required)

✅ Deuterium observed in Quasar absorption systems

Damped Lyman-α spectra from intervening gas along l.o.s.

❌ Helium-3 observed in the Solar neighborhood

Solar winds, meteorites, ISM ... stellar nucleosynthesis uncertainties!

❓ Lithium-7 in the atmosphere of dwarf halo stars

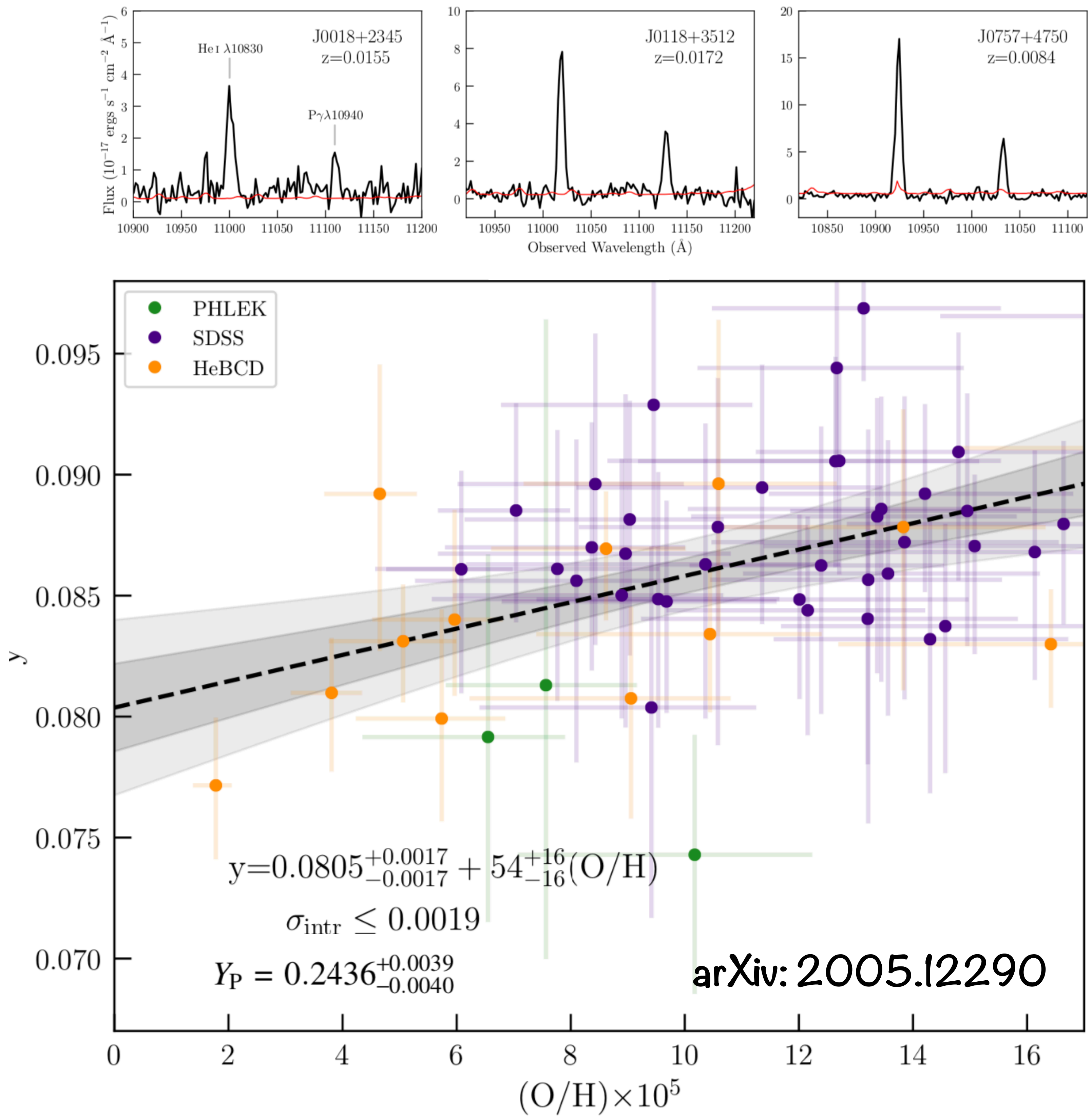
Physics of convection, depletion indicators ... needs support from data

arXiv:2204.03167

^4He

PDG 2021: $Y_P = 0.245 \pm 0.003$

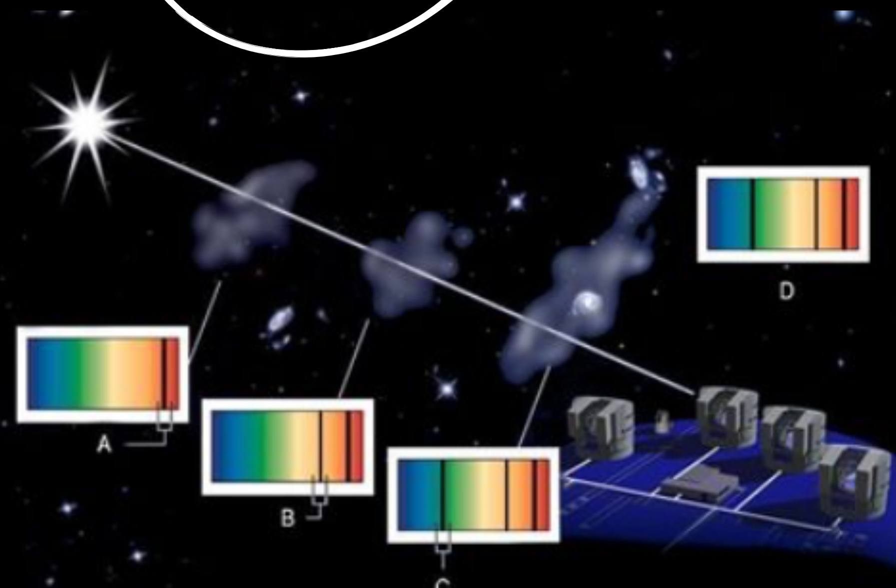
% level measurement

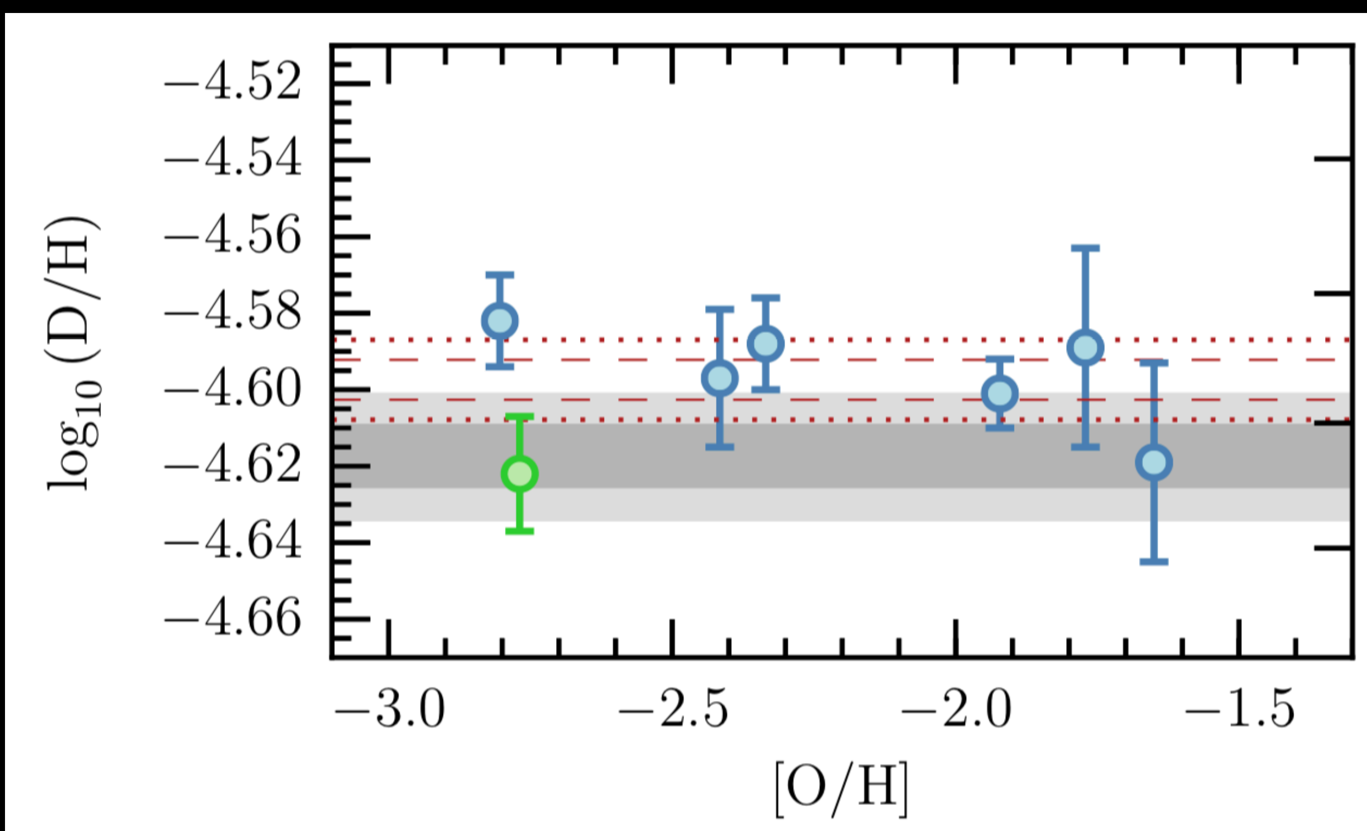


D

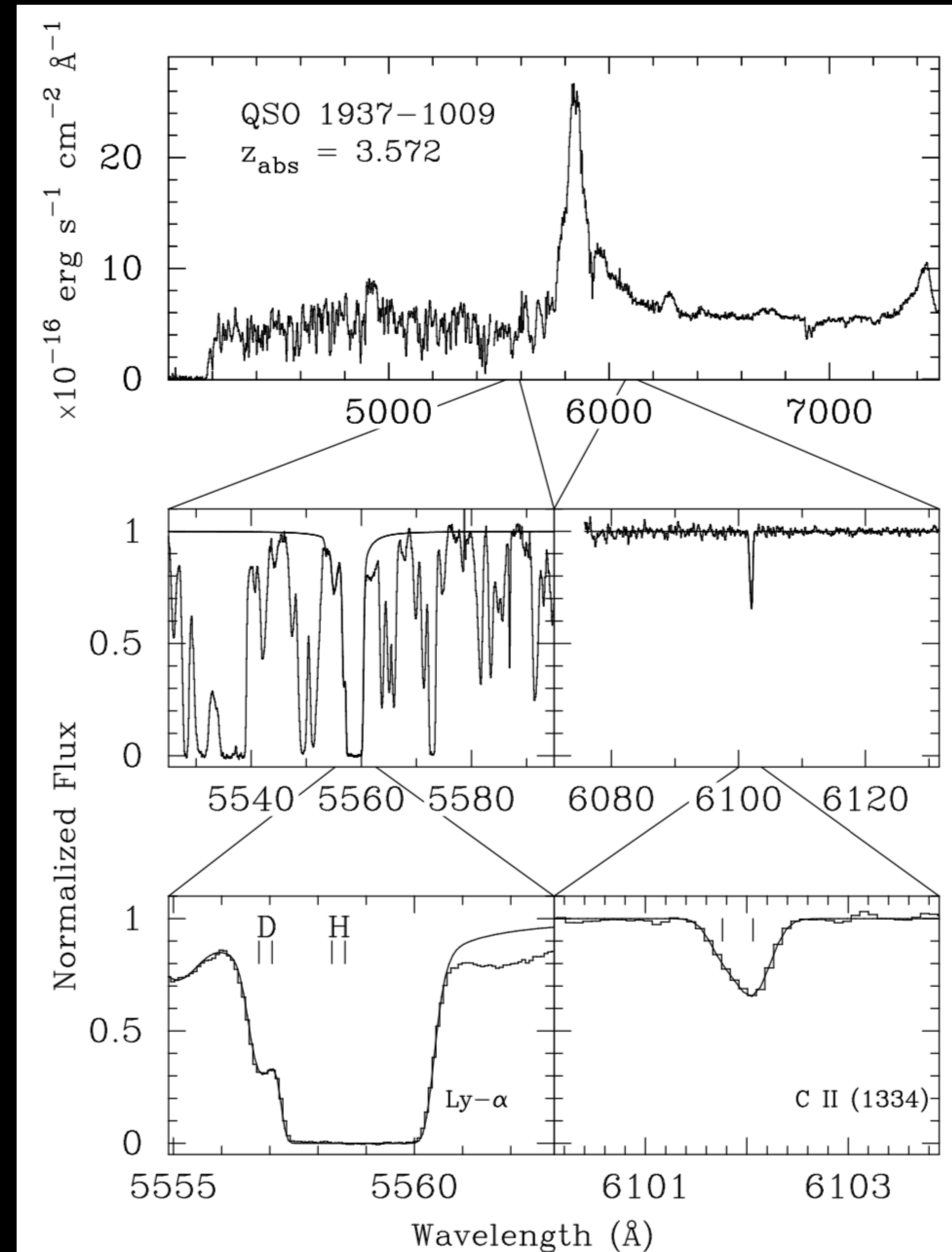
% level measurement

PDG 2021: $(D/H) \times 10^5 = 2.547 \pm 0.025$





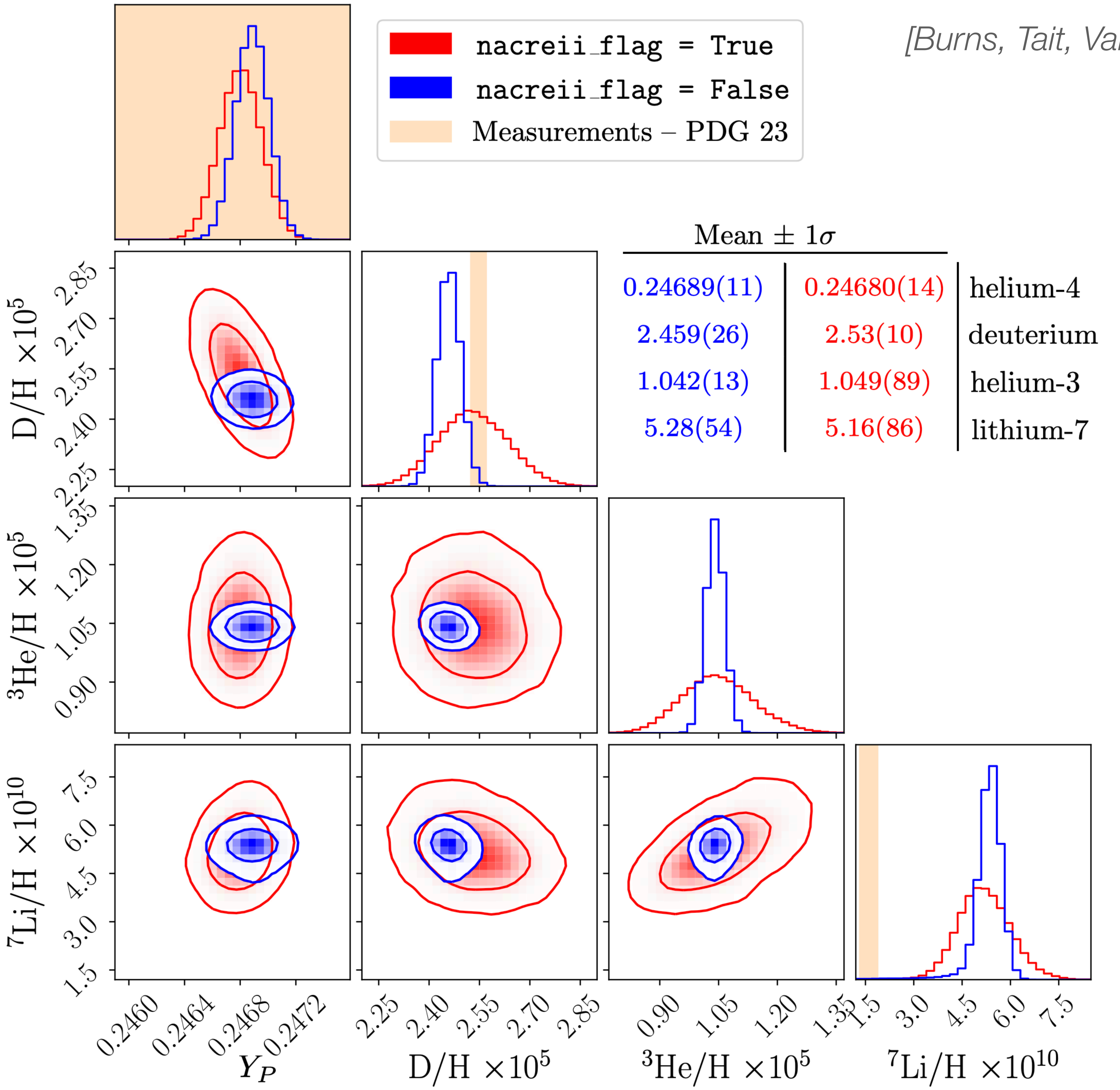
arXiv: 1710.11129



astro-ph/9803071

PRyMordial: SM state-of-the-art

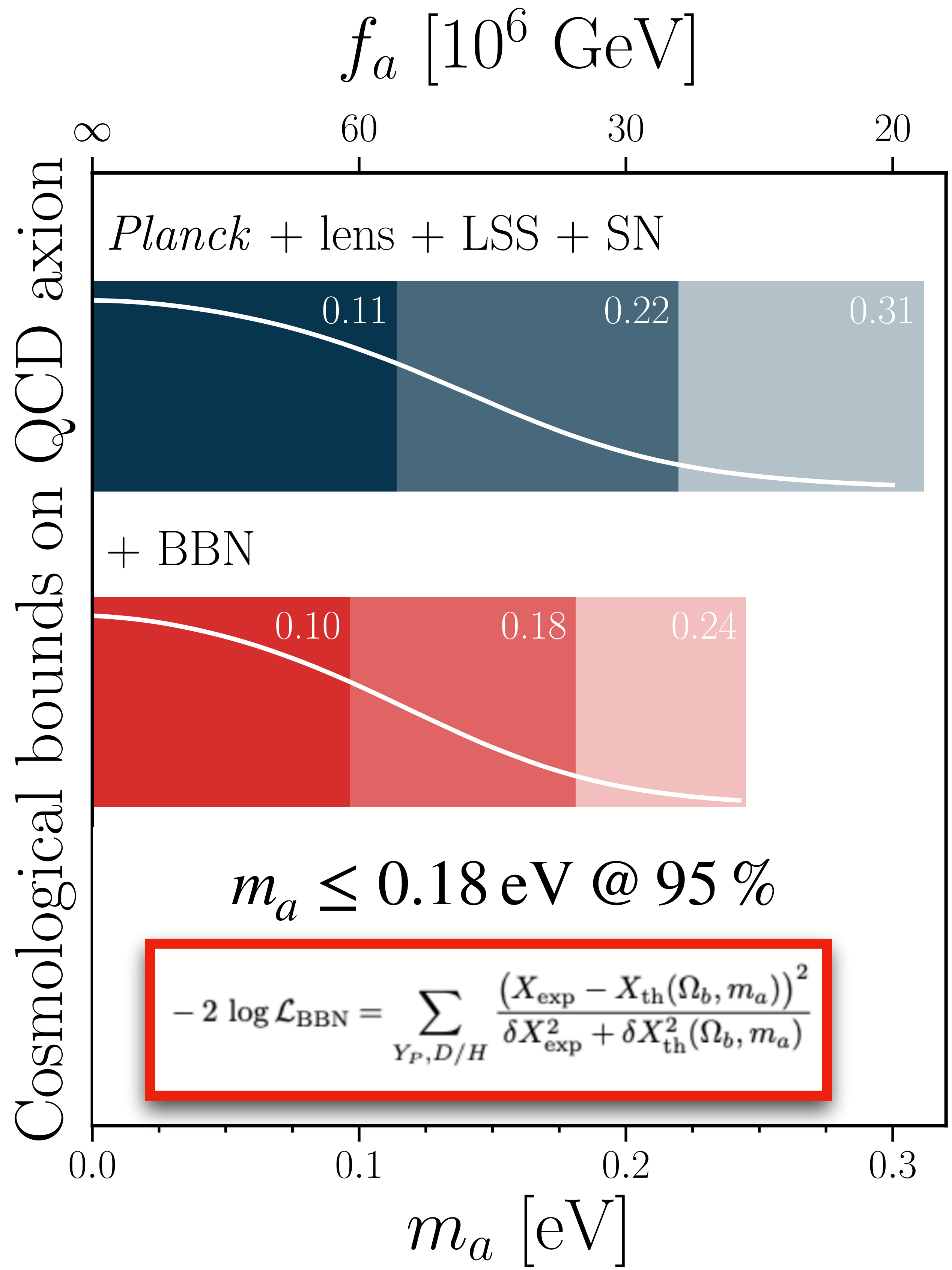
[Burns, Tait, Valli `23]



Mean ± 1σ		
0.24689(11)	0.24680(14)	helium-4
2.459(26)	2.53(10)	deuterium
1.042(13)	1.049(89)	helium-3
5.28(54)	5.16(86)	lithium-7

Minimal QCD Axion $(T_D < T_C)$

[Bianchini, Grilli, Valli `23]



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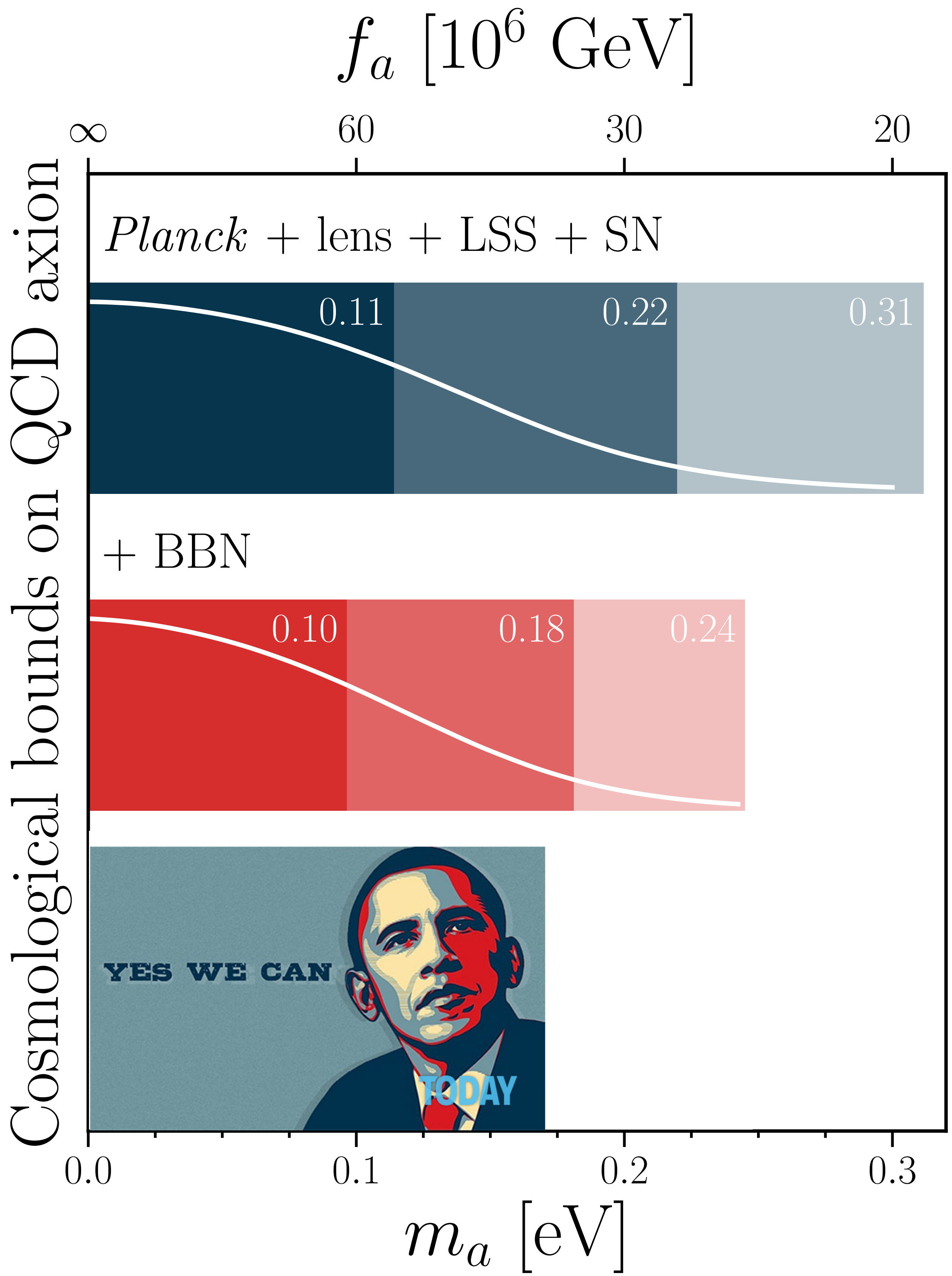
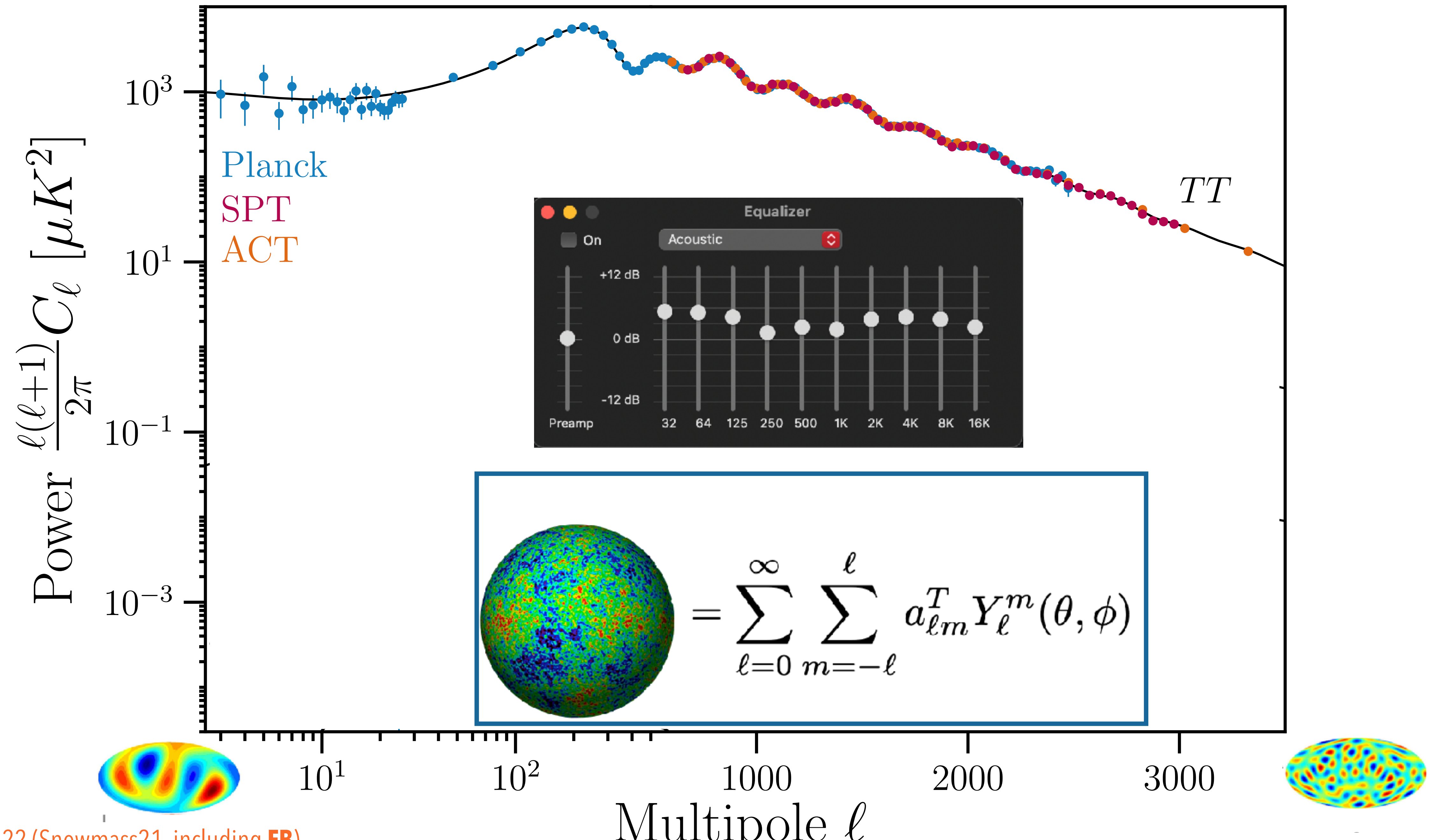
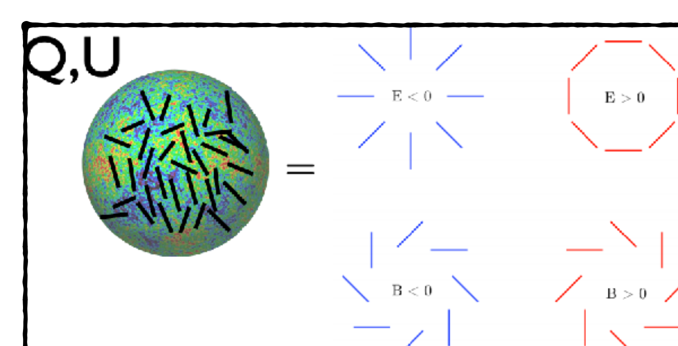


CMB temperature measurements



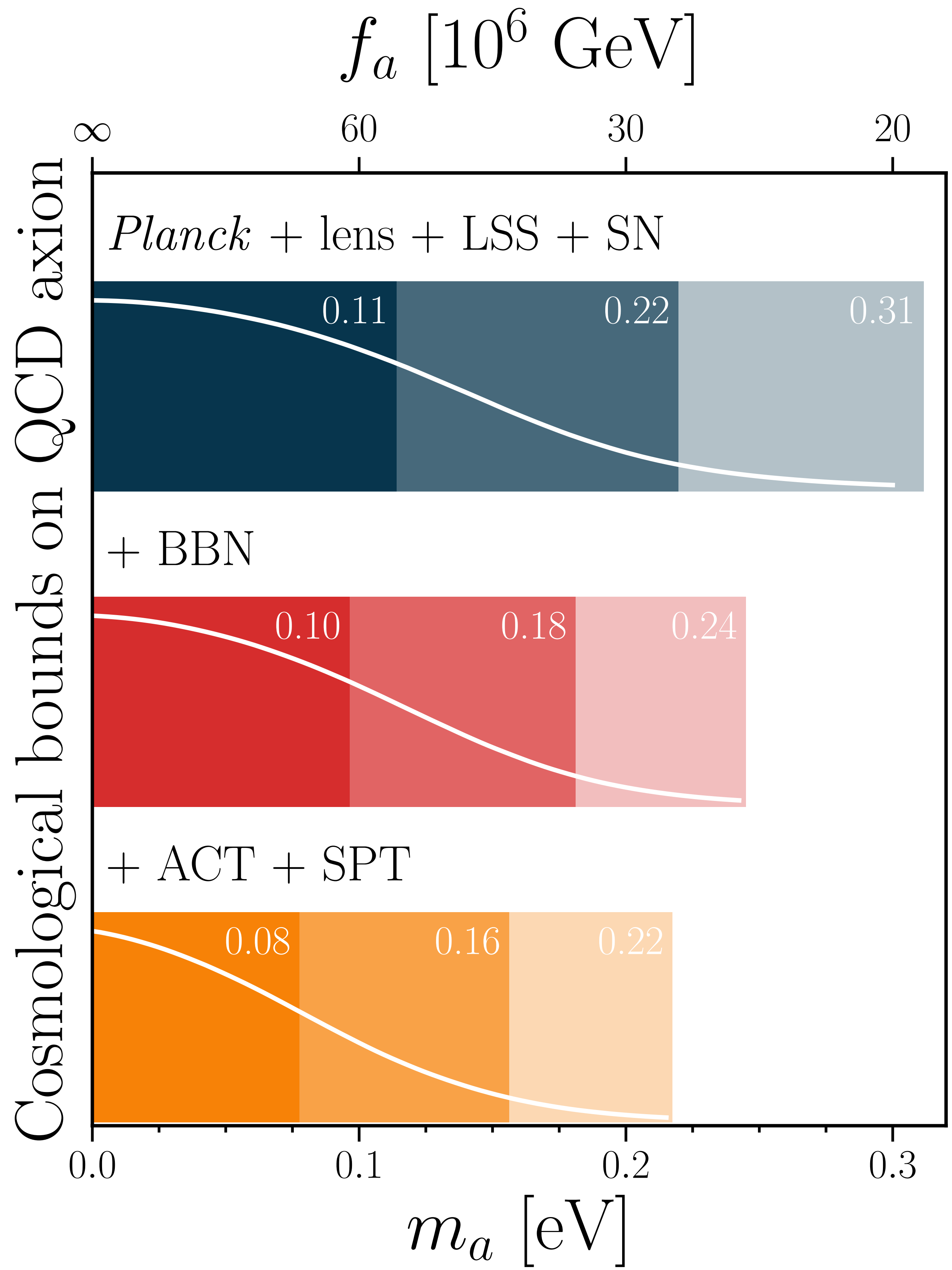
Chang+22 (Snowmass21, including **FB**)

CMB polarization



Minimal QCD Axion ($T_D < T_C$)

[Bianchini, Grilli, Valli `23]

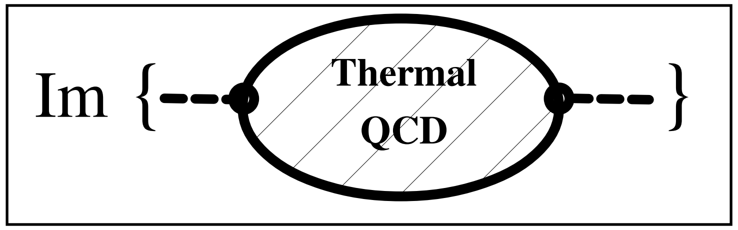


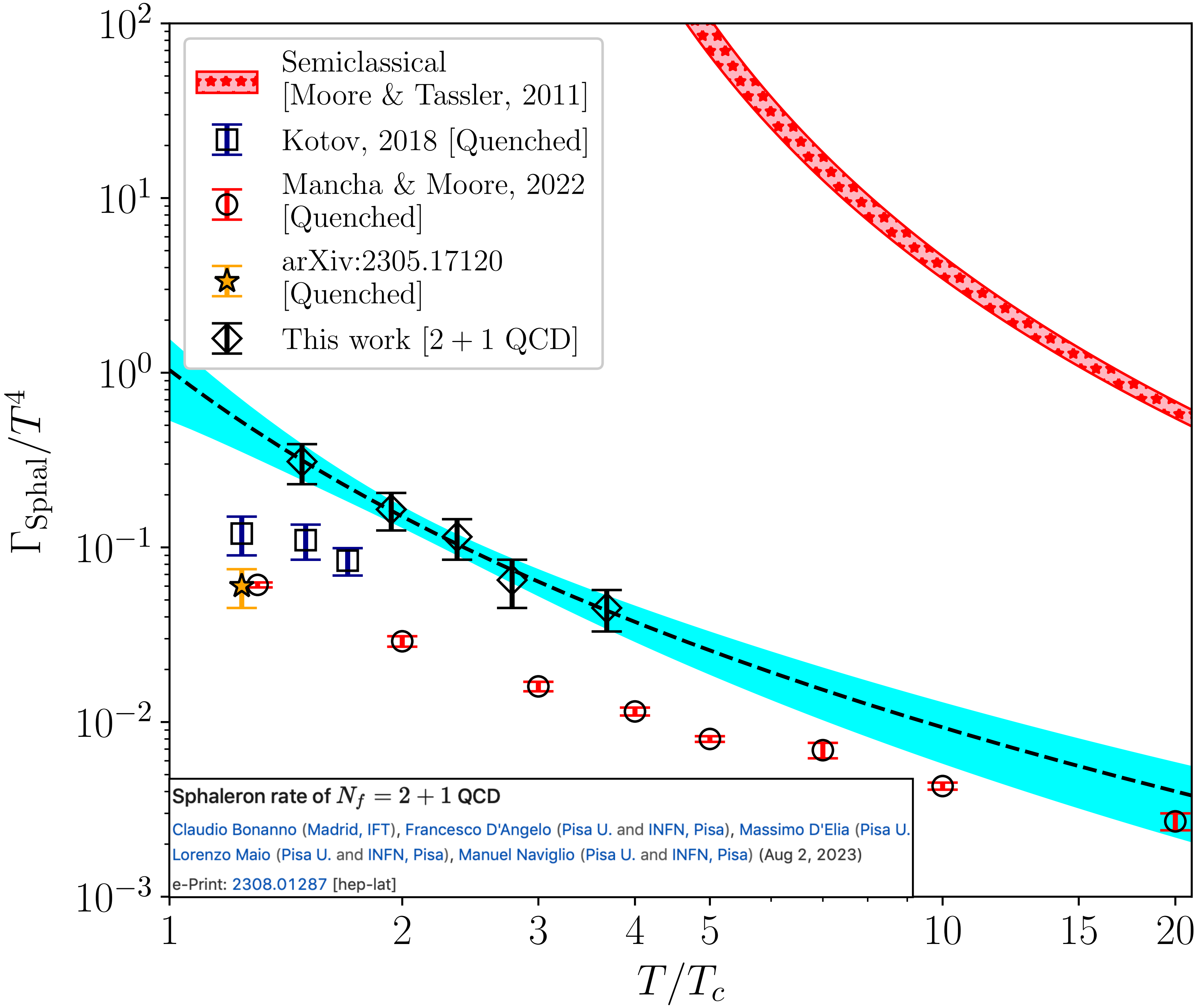
30% improvement with respect to [Notari, Rompineve, Villadoro`23]

$m_a \leq 0.16\,\mathrm{eV}$ @ 95 % HDI

Minimal QCD Axion ($T_D > T_C$)

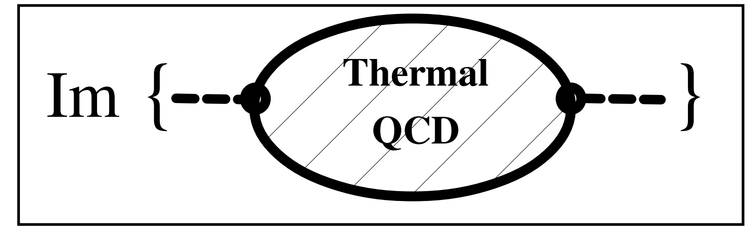
$$\Gamma_a = \int d^4x\, e^{ikx} \langle \mathcal{Q}(x)\mathcal{Q}(0)\rangle \quad \Longleftrightarrow$$





Minimal QCD Axion ($T_D > T_c$)

$$\Gamma_a = \int d^4x \, e^{ikx} \langle \mathcal{Q}(x) \mathcal{Q}(0) \rangle$$



150 MeV < T < 600 MeV

$$\Gamma_{\rm sph}(|\mathbf{k}| = 0) = \Lambda_0^4 \, (T/T_c)^\epsilon$$

$$\Lambda_0 \simeq 142.3 \text{ MeV}, \, \epsilon \simeq 1.81 \, , \, T_c = 155 \text{ MeV}$$

[Bonanno, D'Angelo, D'Elia, Maio, Naviglio `23]

- Recipe for a (reasonably) optimistic forecast:

(I) Axion initially in eq.: $Y_a(600\,\text{MeV}) \simeq Y_a^{\rm eq}(T_{\rm EW})$

(II) (Under)Estimate rate at non-zero momentum via:

$$n_a^{\rm eq} \, \overline{\Gamma}_a \gtrsim \frac{\Lambda_0^4}{4\pi^2 f_a^2} \left(\frac{T}{T_c} \right)^\epsilon \int_0^{3\alpha_s T} d|\mathbf{k}||\mathbf{k}| \exp(-|\mathbf{k}|/T)$$

(III) Set initial condition @ T_c from: $\dfrac{dY_a}{dt} = \dfrac{\overline{\Gamma}_a}{H} (Y_a^{\rm eq} - Y_a)$

arXiv: 2005.05290

The Art of **Forecasts**

cobaya

latest

Search docs

INSTALLATION AND QUICKSTART

- Installing cobaya
- Quickstart example
- Advanced example

GENERAL TOPICS AND COMPONENTS

- Input and invocation
- Output
- Parameters and priors
- Models: finer interaction with Cobaya's pipeline
- Likelihoods
- **one** likelihood

Read the Docs v: latest

Cobaya, a code for Bayesian analysis in Cosmology   Edit on GitHub

Cobaya, a code for Bayesian analysis in Cosmology

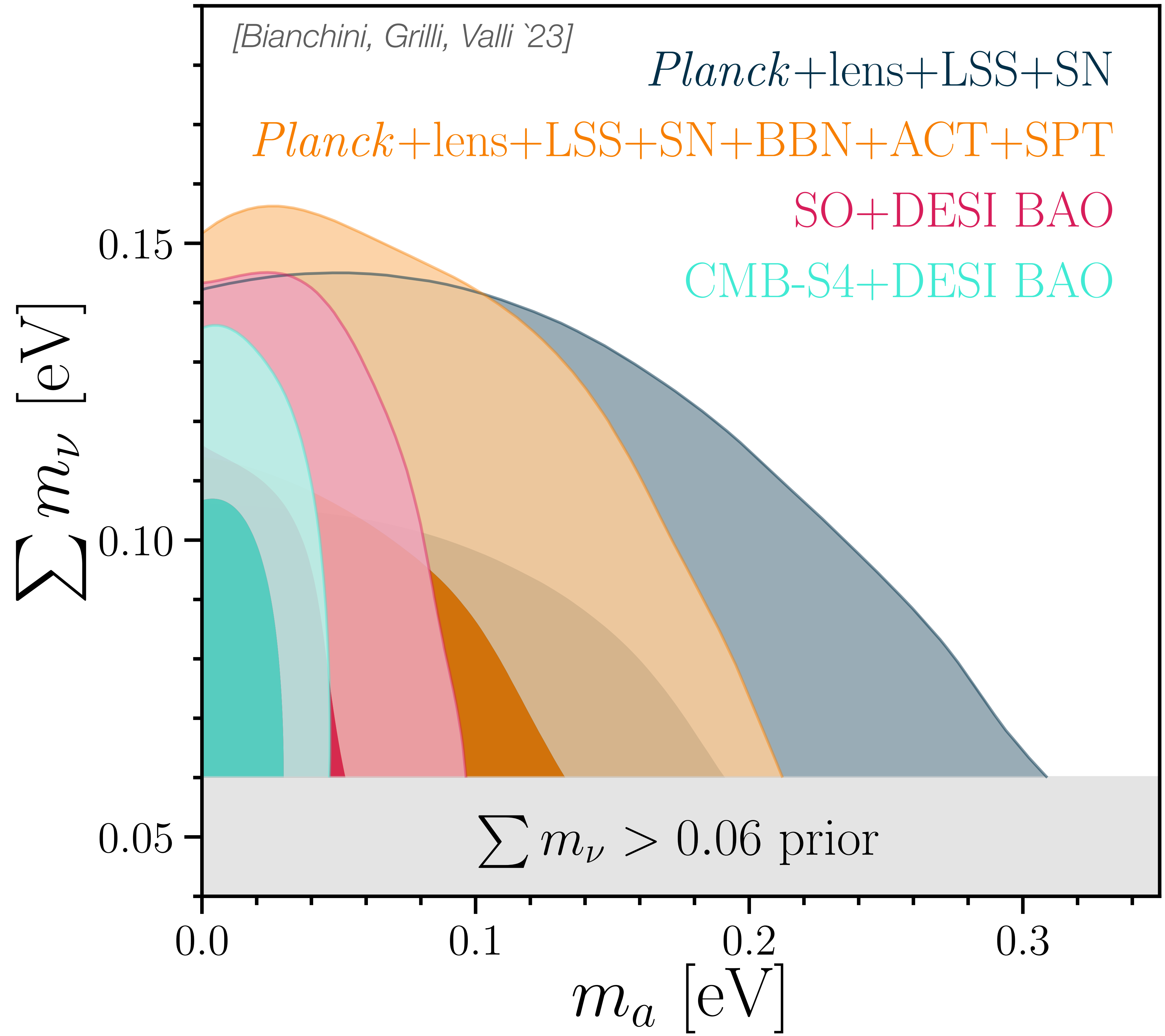
Author:	Jesus Torrado and Antony Lewis
Source:	Source code at GitHub
Documentation:	Documentation at Readthedocs
Licence:	LGPL + bug reporting asap + arXiv'ing of publications using it (see LICENCE.txt for details and exceptions). The documentation is licensed under the GFDL.
E-mail list:	https://cosmocoffee.info/cobaya/ – **sign up for important bugs and release announcements!**
Support:	For general support, CosmoCoffee; for bugs and issues, use the issue tracker.
Installation:	`pip install cobaya --upgrade` (see the installation instructions; in general do *not* clone)

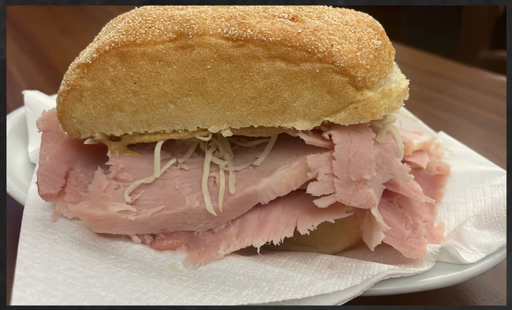
build canceled | docs passing | codecov 85% | pypi v3.4.1 | pypi downloads 110k

arXiv 2005.05290

Cobaya (**co**de for **bay**esian analysis, and Spanish for *Guinea Pig*) is a framework for sampling and statistical modelling: it allows you to explore an arbitrary prior or posterior using a range of Monte Carlo samplers (including the advanced MCMC sampler from CosmoMC, and the advanced nested sampler PolyChord). The results of the sampling can be analysed with GetDist. It supports MPI parallelization (and very soon HPC containerization with Docker/Shifter and Singularity).

Cosmo **Present & Future** of QCD Axion





Take Home

- Minimal QCD axion : "unavoidable" Hot Dark Matter

  W/ linear Cosmology + improved χPT, we found :

  $m_a \leq 0.16$ eV @ 95 % probability (CMB + LSS + BBN)

- Promising future: cosmo bound competitive w/ current astro ones

  Crucial aspects for "what's next" :

  * $\langle Q(x)\, Q(0) \rangle$ @ the crossover
  * Cosmology in the non-linear regime