

Event generators for two-photon hadrons production at KEDR

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Lowest-order cross section

In a process of the two-photon production $e^+e^- \rightarrow e^+e^- + f$ virtual photons emitted by colliding e^+ and e^- form C-even system f , consisting of n particles (see Fig. 1) with the four-momentum $k = k_1 + k_2$ and the invariant mass $W = \sqrt{k^2} = \sqrt{(k_1 + k_2)^2}$.

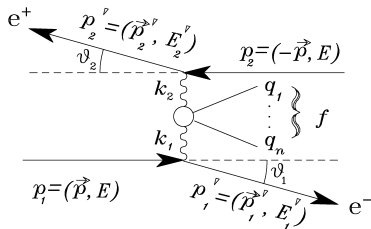


Fig. 1: The diagram of the process $e^+e^- \rightarrow e^+e^- + \text{hadrons}$.

The invariant variables of the reaction: $s = (p_1 + p_2)^2$,
 $t_1 = k_1^2 = (p_1 - p_1')^2$, $t_2 = k_2^2 = (p_2 - p_2')^2$, $s_1 = (p_1' + k)^2$, $s_2 = (p_2' + k)^2$.

For unpolarized beams the lowest order two-photon cross section is
(V.M.Budnev et al., Phys.Rep. C15(1975)181):

$$d\sigma = \frac{\alpha^2}{16\pi^4 t_1 t_2} \sqrt{\frac{(k_1 \cdot k_2)^2 - t_1 t_2}{(p_1 \cdot p_2)^2 - m_e^4}} \Sigma \frac{d^3 \vec{p}'_1}{E'_1} \frac{d^3 \vec{p}'_2}{E'_2}, \quad (1)$$

$$\begin{aligned} \Sigma = & 2\rho_1^{++} 2\rho_2^{++} \sigma_{TT} + 2\rho_1^{++} \rho_2^{00} \sigma_{TS} + \rho_1^{00} 2\rho_2^{++} \sigma_{ST} + \rho_1^{00} \rho_2^{00} \sigma_{SS} + \\ & + 2|\rho_1^{+-} \rho_2^{+-}| \tau_{TT} \cos 2\tilde{\phi} - 8|\rho_1^{+0} \rho_2^{+0}| \tau_{TS} \cos \tilde{\phi}. \end{aligned}$$

Here σ_{ab} and τ_{ab} include the effect of the strong interaction, while ρ_i^{++} , ρ_i^{+-} , ρ_i^{+0} , ρ_i^{00} ($i = 1, 2$) are the virtual photon density matrices in the $\gamma^* \gamma^*$ -helicity basis and are calculable with QED.

$\sigma_{TT} = (\sigma_{\parallel} + \sigma_{\perp})/2$ is cross section for unpolarized photons,

$\tau_{TT} = \sigma_{\parallel} - \sigma_{\perp}$.

σ_{\parallel} is cross section for transverse photons with parallel linear polarization,

σ_{\perp} is cross section for transverse photons with orthogonal linear polarization.

$\tilde{\phi}$ - angle between scattering planes of e^+ , e^- in the c.m. $\gamma^* \gamma^*$ frame.

For the KEDR tagger: $0 \leq \theta'_i \leq 0.01$ rad, $0.45 \leq E'_i/E_b \leq 0.98$.

So, $|k_i^2| \approx E'_i E_b (\theta'_i)^2 \leq 10^{-4} E_b^2 = 2.5 \cdot 10^{-3} \text{ GeV}^2$.

When $k_i^2 \rightarrow 0$ all cross sections except σ_{TT} and τ_{TT} disappears.

$\sigma_{TT} \rightarrow \sigma_{\gamma\gamma}$, where $\sigma_{\gamma\gamma}$ is the cross section for real photons. After the azimuthal integration, the contribution to the cross section (1) proportional to τ_{TT} disappears also. In this approximation we can write

$$d\sigma = \frac{\alpha^2}{8\pi^4 t_1 t_2} \frac{\sqrt{X}}{\sqrt{s(s-4m_e^2)}} K_{TT} \sigma_{TT} \frac{d^3 \vec{p}'_1}{E'_1} \frac{d^3 \vec{p}'_2}{E'_2}, \quad (2)$$

where

$$X = \frac{1}{4}(W^2 - t_1 - t_2)^2 - t_1 - t_2, \quad K_{TT} = 4\rho_1^{++} \rho_2^{++},$$

$$2\rho_1^{++} = \frac{(u_2 - \nu)^2}{K^2 W^2} + 1 + \frac{4m_e^2}{t_1}, \quad 2\rho_2^{++} = \frac{(u_1 - \nu)^2}{K^2 W^2} + 1 + \frac{4m_e^2}{t_2},$$

$$\nu = \frac{1}{2}(W^2 - t_1 - t_2), \quad K^2 = \frac{(\nu^2 - t_1 t_2)}{W^2},$$

$$u_1 = s_2 - m_e^2 - t_1, \quad u_2 = s_1 - m_e^2 - t_2.$$

Simulation of the process $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ can be split to two stages: (1) $e^+e^- \rightarrow e^+e^- + f$ ($2 \rightarrow 3$ reaction), (2) $f \rightarrow n$ particles.

The phase space of $n+2$ particles can be represented with the formula (E.Byckling, K.Kajante, Particle kinematics):

$$R_{n+2} = \int dW^2 R_3 R_n, \quad dR_n = \prod_i \frac{d^3 \vec{q}_i}{2E_i} \delta^4(k - \sum_{j=1}^n q_j) \quad (3)$$

where R_n – phase space of n final particles from the decay of system f .

The 3-particle phase space in the invariant variables is

$$R_3(s, W) = \frac{\pi}{16\sqrt{s(s-4m_e^2)}} \int \frac{dt_1 dt_2 ds_1 ds_2}{\sqrt{-\Delta_4(s, s_1, s_2, t_1, t_2, W^2, m_e^2)}}. \quad (4)$$

Here $\Delta_4(s, s_1, s_2, t_1, t_2, M_R^2, m_e^2)$ is the Gram determinant and the integration on azimuthal angle was performed. So, we have t_1, t_2, s_1, s_2 : 4 non-trivial variables for the $2 \rightarrow 3$ reaction ($3 \cdot n - 4 - 1$, where $n=3$).

From (2) and (3)-(4) we obtain the formula

$$d\sigma = \frac{\alpha^2 \sqrt{X} K_{TT}}{32\pi^3 s(s - 4m_e^2) t_1 t_2 \sqrt{-\Delta_4}} \cdot \sigma_{TT} dW^2 dt_1 dt_2 ds_1 dt_2, \quad (5)$$

where

$$\sigma_{TT} = \left| \frac{F(t_1, t_2)}{F(0, 0)} \right|^2 \sigma_{\gamma\gamma}.$$

Here in the σ_{TT} were included the transition form factors, $\sigma_{\gamma\gamma}$ is the cross section for real transverse photons.

Two options are used in the KEDR simulation for $|F(t_1, t_2)|^2$:

$|F(t_1, t_2)|^2 = |F(0, 0)|^2$ and the Vector meson Dominance Model (VDM):

$$|F|^2 = \frac{1}{(1 - t_1^2/\Lambda^2)^2 (1 - t_2/\Lambda^2)^2}, \quad (6)$$

where $\Lambda = m_\rho$.

The cross section $\sigma_{\gamma\gamma} = \text{const}$ corresponds to events with uniform distribution in the phase space. For low energy, $\sqrt{s} \lesssim 3 \text{ GeV}$, experimental data on $e^+e^- \rightarrow \text{hadrons}$ (Mark-I) do not contradict to this.

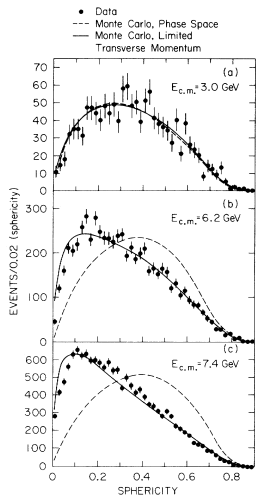


Fig. 2. The sphericity distribution of multihadron events.

Event generator $e^+e^- \rightarrow e^+e^- + \text{hadrons}$

A. Simulation of the process $e^+e^- \rightarrow e^+e^- + f$

For the MC integration of Eq. 5 in the GGHADRNS generator the value of variable W is simulated at first. Simulation of the invariants t_1, t_2, s_1, s_2 is performed using the method developed for the GALUGA two-photon event generator (G.A.Schuler, *Comp. Phys. Comm.* 108(1998)279.).

The values of the generated invariants t_1, t_2, s_1, s_2 , are then used together with a random azimuthal angle φ of the system of final particles to calculate the laboratory 4-momenta of the scattered e^-, e^+ and system f . For the simulation of the system f decay to n particles we use **K**-procedure from the book G.Kopylov. *Osnovy kinematiki rezonansov* (Basics of kinematics of resonances).

During the $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ events generation the laws of energy-momentum conservation are fulfilled exactly. Sum of weights of events is used to estimate for given $\sigma_{\gamma\gamma}$ the cross-section of the process $e^+e^- \rightarrow e^+e^- + \text{hadrons}$.

B. Simulation of multihadron system

The generator of multihadron ($n \geq 3$) system includes two regimes:

1. simulation of pion system with variable multiplicity, consisting of charged and neutral pions;
2. simulation of n particle system with fixed composition (particles can be of different types).

In both regimes at first the invariant mass W of the $\gamma\gamma$ system is generated.

For the 2-nd regime generator performs simulation of event as described above. Only the energy-momentum conservation should fulfil.

For the 1-st regime when the invariant mass W is generated, the average multiplicity of the event is calculated with the dependence, which describes data of the e^+e^- experiments (with $AMC=0.84$):

$$\langle n_c \rangle = 2.1 + AMC \cdot \ln(W^2), \text{ AMC - generator parameter.}$$

This dependence agrees with one, obtained in the MD-1/VEPP-4 for the reaction $\gamma\gamma \rightarrow \text{hadrons}$ studies:

$$\langle n_c \rangle = 1.62 \pm 0.37 + (1.83 \pm 0.45) \cdot \ln(W).$$

The Fig. 3 shows experimental data; the curve corresponds to the above formula with $AMC=0.84$ and $s = W^2$.

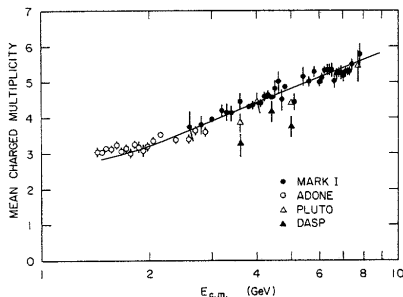


Fig. 3. Average charged multiplicity for the process $e^+e^- \rightarrow \text{hadrons}$ at low energy.

Then the average multiplicity is determined as

$\langle n \rangle = \langle n_c \rangle / (1 - AMN)$, where AMN - average fraction of π^0 .

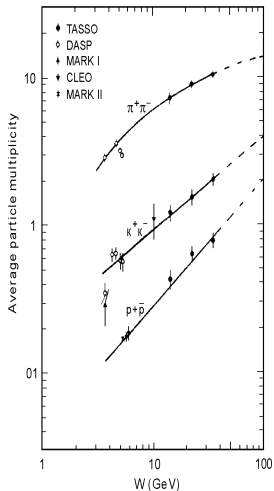
The pion system can approximately represent the multihadron system. It follows from the data (P.Mattig. Preprint DESY 83-38, Fig. 4). The composition of $e^+e^- \rightarrow \text{hadrons}$ event can be approximately written as

$$\langle n_\pi \rangle : \langle n_K \rangle : \langle n_p \rangle \approx 3 : 0.3 : 0.1.$$

Multiplicity n of pions in the event is simulated with the Poisson.

If $n < 3$ or $n > \min\{18, W/m_\pi\}$ the simulation should be repeated. Fraction of neutral pions n_n/n is generated using the binomial distribution with average AMN. This simulation should be repeated if number of charged pions $n_c = n - n_n$ is not even.

Fig. 4. Composition of event $e^+e^- \rightarrow \text{hadrons}$.



Density of $\gamma\gamma$ -luminosity

Rate of the 2γ -events with the invariant mass $W_{min} < W < W_{max}$ in e^+e^- annihilation can be written as

$$\dot{N} = L_{ee} \cdot \int_{W_{min}}^{W_{max}} dW \frac{d\sigma}{dW} = L_{ee} \cdot \int_{z_{min}}^{z_{max}} \frac{d\sigma}{dz} dz = L_{ee} \sigma_{\gamma\gamma} \cdot \int_{z_{min}}^{z_{max}} F_1(z) dz. \quad (7)$$

Here $z = W/\sqrt{s}$, L_{ee} is the e^+e^- luminosity, $F_1(z)$ is the density of $\gamma\gamma$ luminosity, $\sigma_{\gamma\gamma}$ is taken independent of z .

The formula for $F_1(z)$ can be obtained from (5) as:

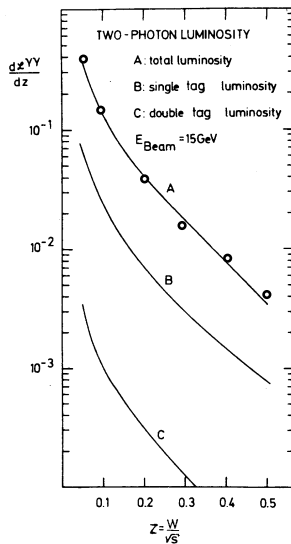
$$F_1(z) = \frac{z\alpha^2}{16\pi^3(s - 4m_e^2)} \cdot \int \frac{\sqrt{X} K_{TT}}{t_1 t_2 \sqrt{-\Delta_4}} \left| \frac{F(t_1, t_2)}{F(0, 0)} \right|^2 ds_1 ds_2 dt_1 dt_2. \quad (8)$$

Analytical formula for the density of $\gamma\gamma$ -luminosity F_1 was obtained in the work [J.H.Field, Nucl.Phys. B168\(1980\)477](#), where integration on phase volume of scattered e^\pm with account of exact kinematics was performed.

The comparison of this calculation with our formula (8) is shown in Fig 5. The figure was taken from work of the TASSO detector. Beam energy equals 15 GeV. Our calculation of the total luminosity shown with circles agrees well with the calculated curve A.

Fig. 5.

Density of $\gamma\gamma$ -luminosity at $\sqrt{s}=30$ GeV as a function of $z = W/\sqrt{s}$. Curves are from work of TASSO, circles - our $F_1(z)$ calculation with $F(t_1, t_2) = F(0, 0)$.



Some calculations of $e^+e^- \rightarrow e^+e^- + \text{hadrons}$

Calculations at $\sqrt{s} = 2 \times 1.5$ GeV are presented in Fig. 6,7. VDM factor is equal 1. Parameters of the generator: $W=0.5 - 1.5$ GeV; $AMC=0.84$ and $AMN=0.333$ for the variable pion multiplicity.

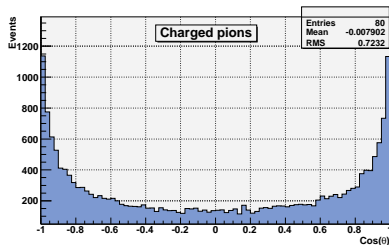
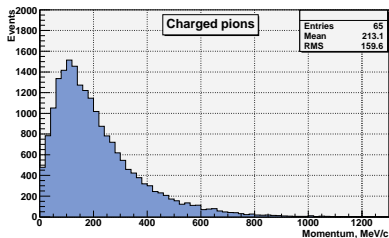


Fig. 6: The laboratory distributions for charged pions on momentum (left) and on polar angle.

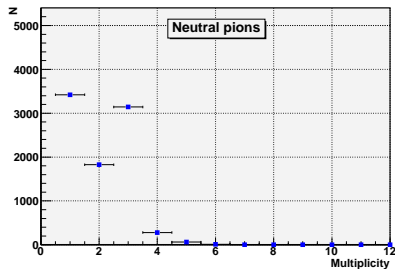
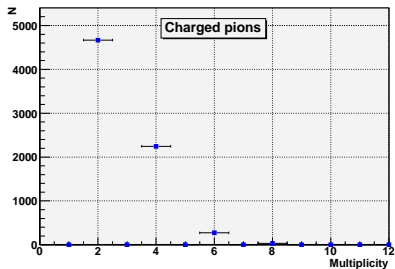


Fig. 7: The distributions on multiplicity for π^\pm (left) and π^0 .

Formulas, obtained in the equivalent photons approximation, are useful for estimates in the two-photon physics. Thus, for the differential cross section there is a formula (V.M.Budnev et al., Phys.Rep. C15(1975)181):

$$\frac{d\sigma}{dW} = \frac{4}{W} \left(\frac{\alpha}{\pi} \right)^2 \sigma_{\gamma\gamma}(W) \times \left[\ln^2 \left(\frac{\sqrt{s} \cdot m_\rho}{W \cdot m_e} \right) \cdot f \left(\frac{W}{\sqrt{s}} \right) - \frac{1}{3} \ln^3 \frac{s}{W^2} \right], \quad (9)$$

where $f(x) = (2 + x^2)^2 \ln(\frac{1}{x}) - (1 - x^2)(3 + x^2)$ and a restriction $|t_i| \leq m_\rho$ was used.

The measurement of the MD-1 detector at the VEPP-4 collider gave the following data for the total cross section $\gamma\gamma \rightarrow \text{hadrons}$ (based on 448 events).

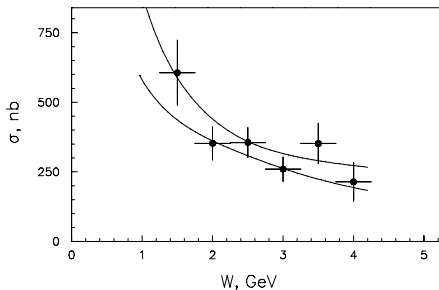


Table 1. Data of MD-1 on cross section $\gamma\gamma \rightarrow \text{hadrons}$.

W, GeV	$\sigma_{\gamma\gamma}$, nb	W, GeV	$\sigma_{\gamma\gamma}$, nb
1.25-1.75	606 ± 117	2.75-3.25	260 ± 43
1.75-2.25	352 ± 59	3.25-3.75	352 ± 72
2.25-2.75	356 ± 53	3.75-4.25	214 ± 70

Using the data from the Table 1 we calculated with the generator the total cross section $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ for some intervals ΔW . These calculations are presented in Table 2 in the comparison with integrals of (9).

Table 2. Total cross section $e^+e^- \rightarrow e^+e^- + \text{hadrons}$
at $\sqrt{s} = 2 \times 1.5$ GeV.

W, GeV	GGHADRNS, $ t_i \leq m_\rho^2$	Integral (9)
0.75 - 1.25	0.807 nb	1.05 nb
1.25 - 1.75	0.240 nb	0.313 nb
1.75 - 2.25	0.045 nb	0.060 nb
2.25 - 2.75	0.012 nb	0.018 nb

(For the interval $W = 0.75 - 1.25$ GeV used $\sigma_{\gamma\gamma} = 606$ nb.)

One can see, that the equivalent photons approximation overestimates cross sections on 20% or more.

Event generator $e^+e^- \rightarrow e^+e^- + R(J^{PC} = 0^{-+})$

In the case of pseudoscalar meson (PS) production (Fig. 8) in the Eq. 1 only σ_{TT} and τ_{TT} are non-zero and $\tau_{TT} = -2\sigma_{TT}$.

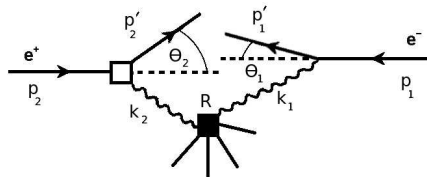


Fig. 8: The diagram of the process $e^+e^- \rightarrow e^+e^- + R$.

For a narrow PS meson with mass M_R and width $\Gamma_{\gamma\gamma}$ σ_{TT} can be written in terms of the transition formfactor (TFF) as:

$$\sigma_{TT}(W, Q_1^2, Q_2^2) = 8\pi \frac{\Gamma_{\gamma\gamma}}{M_R} \left| \frac{F(Q_1^2, Q_2^2)}{F(0, 0)} \right|^2, |F(0, 0)|^2 = \frac{4\Gamma_{\gamma\gamma}}{\pi\alpha^2 M_R^3}.$$

For a narrow PS resonance the equation (1) can be rewritten as

$$d\sigma = \frac{4\alpha^2\Gamma_{\gamma\gamma}}{\pi s^2 t_1^2 t_2^2 M_R^3} \left| \frac{F(t_1, t_2)}{F(0, 0)} \right|^2 B \frac{dt_2 dt_1 ds_1 ds_2}{\sqrt{-\Delta_4}}. \quad (10)$$

Here function B is a polynomial (S.J.Brodsky e.a., Phys.Rev. D4(1971)1532), depending on invariants of the problem. MC integration of the formula (10) with simulation invariants and the parameters of particles of the process $e^+e^- \rightarrow e^+e^- + R(J^{PC} = 0^{-+})$ is performed with the GGRESPS generator. Cross sections obtained with this generator are in agreement with calculations of other works.

At this workshop F.Nguyen reported the measured cross section $\sigma(e^+e^- \rightarrow e^+e^- + \eta)$ at $\sqrt{s}=1$ GeV. Comparison with our calculation:

$$\sigma_{exp} = 41.7 \pm 4 \text{ pb [KLOE]}$$

$$\sigma_{MC} = 35.5 \text{ pb [MC without VDM]}$$

$$\sigma_{MC} = 32.4 \text{ pb [MC with VDM]}$$

Thank you for attention