Radiative corrections in $K \to \pi \ell^+ \ell^-$ and related decays

Bastian Kubis

Helmholtz-Institut für Strahlen- und Kernphysik (Theorie)
Bethe Center for Theoretical Physics
Universität Bonn, Germany

Working Group on Radiative Corrections

BINP Novosibirsk, 23/9/2011

BK, R. Schmidt, Eur. Phys. J. C70 (2010) 219

Contents

Let me try to get your attention...

Assume you couldn't care less about $K \to \pi\ell^+\ell^-$...

...why should you still listen?

Let me try to get your attention...

Assume you couldn't care less about $K \to \pi\ell^+\ell^-$...
...why should you still listen?

What are those "related decays" in the title?

Let me try to get your attention...

Assume you couldn't care less about $K \to \pi\ell^+\ell^-$...
...why should you still listen?

What are those "related decays" in the title?

- it's really mainly about the $\ell^+\ell^-$ part
- the radiative correction factors we calculated can directly be used also for $\omega \to \pi^0\ell^+\ell^-$, $\phi \to \eta\ell^+\ell^-$ ...
- the non-trivial part concerns e^+e^- final states—
  I believe the associated problems have not been appreciated in calculations of radiative corrections to
  e.g. $\pi^0 \to \gamma e^+e^-$ Kampf, Knecht, Novotný 2006
  or $\pi^0 \to e^+e^-$ Vaško, Novotný 2011

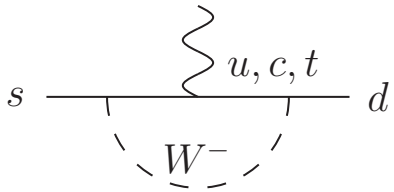
The physics case: $K \to \pi \ell^+ \ell^-$

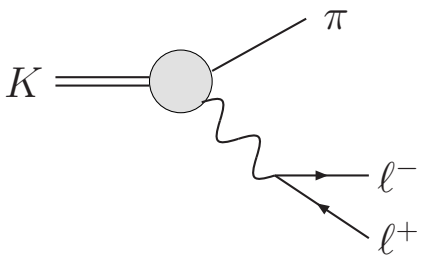
- flavour-changing neutral current process; rare:
  BR$(K^+ \to \pi^+ e^+ e^-) \approx 3 \times 10^{-7}$, ~ 10000 events at BNL + NA48/2
  BR$(K^+ \to \pi^+ \mu^+ \mu^-) \approx 8 \times 10^{-8}$, ~ 3000 events at NA48/2
  BR$(K_S \to \pi^0 \ell^+ \ell^-) \approx 3 \times 10^{-9}$, handful of events at NA48/1
  more to come from dedicated $K \to \pi \nu \bar{\nu}$ experiments (NA62 etc.)

The physics case: $K \to \pi\ell^+\ell^-$

- flavour-changing neutral current process; rare:
  BR$(K^+ \to \pi^+ e^+ e^-) \approx 3 \times 10^{-7}$, ~ 10000 events at BNL + NA48/2
  BR$(K^+ \to \pi^+ \mu^+ \mu^-) \approx 8 \times 10^{-8}$, ~ 3000 events at NA48/2
  BR$(K_S \to \pi^0 \ell^+ \ell^-) \approx 3 \times 10^{-9}$, handful of events at NA48/1
  more to come from dedicated $K \to \pi\nu\bar{\nu}$ experiments (NA62 etc.)

- Standard Model: one-loop, penguins $s \to d\gamma^*$:





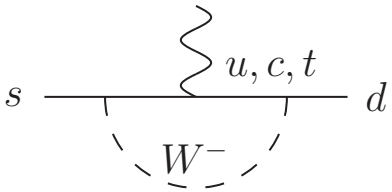
▷ $K \to \pi\gamma$ forbidden by gauge invariance

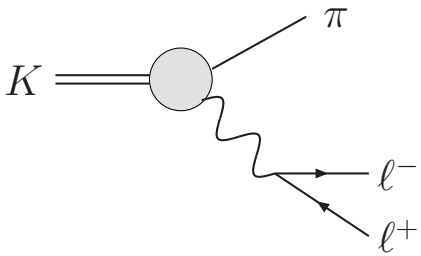
▷ $K \to \pi\gamma^*$ form factor $\bar{F}(s) \propto s$
$\Rightarrow$ essentially point-like $K\pi\ell^+\ell^-$ vertex

The physics case: $K \to \pi\ell^+\ell^-$

- flavour-changing neutral current process; rare:
  BR$(K^+ \to \pi^+ e^+ e^-) \approx 3 \times 10^{-7}$, ~ 10000 events at BNL + NA48/2
  BR$(K^+ \to \pi^+ \mu^+ \mu^-) \approx 8 \times 10^{-8}$, ~ 3000 events at NA48/2
  BR$(K_S \to \pi^0 \ell^+ \ell^-) \approx 3 \times 10^{-9}$, handful of events at NA48/1
  more to come from dedicated $K \to \pi\nu\bar{\nu}$ experiments (NA62 etc.)

- Standard Model: one-loop, penguins $s \to d\gamma^*$:

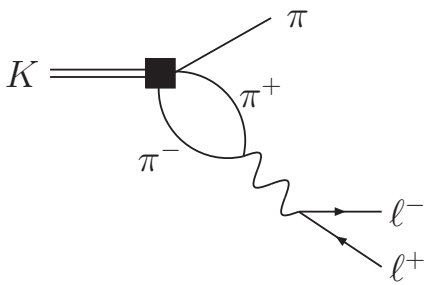




  - ▷ $K \to \pi\gamma$ forbidden by gauge invariance
  - ▷ $K \to \pi\gamma^*$ form factor $\bar{F}(s) \propto s$
    $\Rightarrow$ essentially point-like $K\pi\ell^+\ell^-$ vertex

- Chiral perturbation theory: leading contribution at one loop

  Ecker, Pich, de Rafael 1987



  - ▷ $\mathcal{L}_{\Delta S=1,\text{em}}$ low-energy constants
  - ▷ $\pi^+\pi^-$ P-wave only non-analytic piece
  - ▷ "effective" $\mathcal{O}(p^6)$ calculation

  D'Ambrosio, Ecker, Isidori, Portolés 1998

$K(k) \to \pi(p)\ell^+(p_+)\ell^-(p_-)$, dilepton spectrum

$$s = (k-p)^2 = (p_+ + p_-)^2 \ , \quad t = (p+p_-)^2 \ , \quad u = (p+p_-)^2$$

- essential form factor information resides in $d\Gamma/ds$:

$$\frac{d\Gamma}{ds} = \frac{\alpha^2 |F(s)|^2}{3(4\pi)^5 M_K^7} \lambda^{3/2}(M_K^2, s, M_\pi^2) \sqrt{1 - \frac{4m_\ell^2}{s}} \left(1 + \frac{2m_\ell^2}{s}\right)(1 + \Delta\Omega)$$

radiative corrections expressed in terms of $\Delta\Omega = \mathcal{O}(\alpha)$

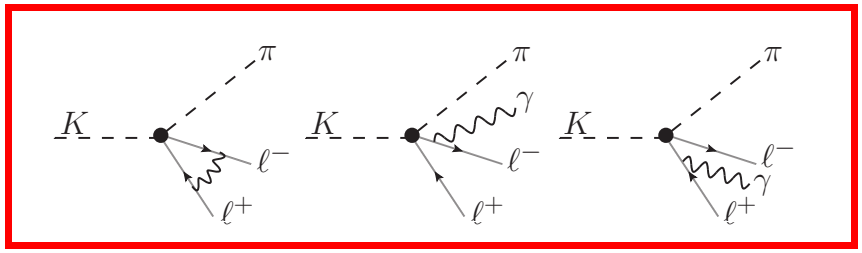
$K(k) \to \pi(p)\ell^+(p_+)\ell^-(p_-)$, dilepton spectrum

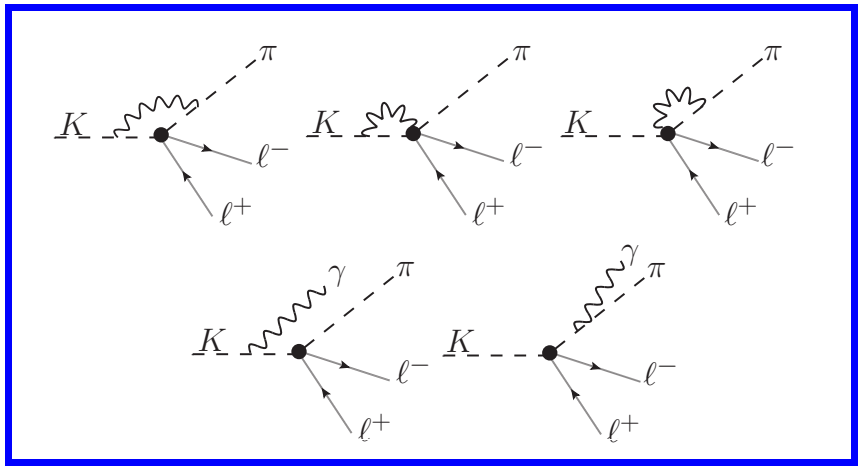
$$s = (k-p)^2 = (p_+ + p_-)^2 \,, \quad t = (p+p_-)^2 \,, \quad u = (p+p_-)^2$$

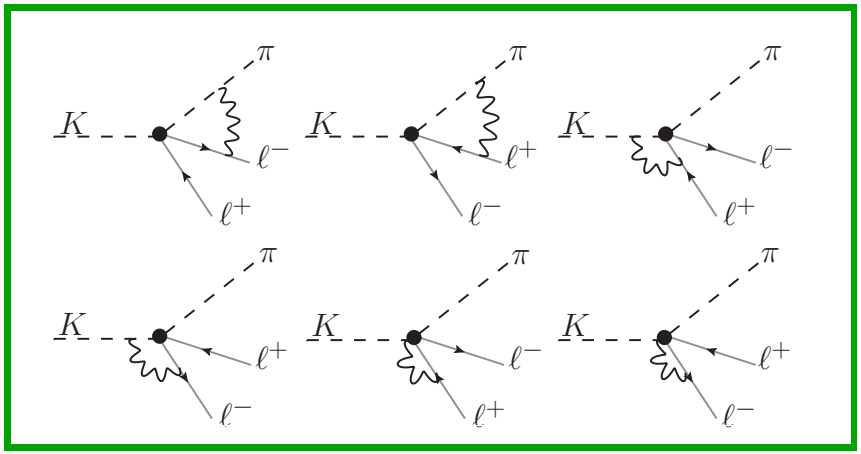
- essential form factor information resides in $d\Gamma/ds$:

$$\frac{d\Gamma}{ds} = \frac{\alpha^2 |F(s)|^2}{3(4\pi)^5 M_K^7} \lambda^{3/2}(M_K^2, s, M_\pi^2) \sqrt{1 - \frac{4m_\ell^2}{s}} \left(1 + \frac{2m_\ell^2}{s}\right)(1 + \Delta\Omega)$$

radiative corrections expressed in terms of $\Delta\Omega = \mathcal{O}(\alpha)$







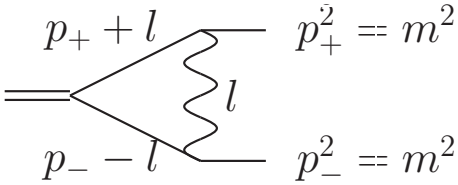
- $\Delta\Omega(K_S \to \pi^0 \ell^+\ell^-)$
- $\Delta\Omega(K^+ \to \pi^+\ell^+\ell^-)$

} in $d\Gamma/ds$ full form factor

- cancel in $d\Gamma/ds$ contribute to $\ell^+\ell^-$ asymmetry

Some general basics on radiative corrections

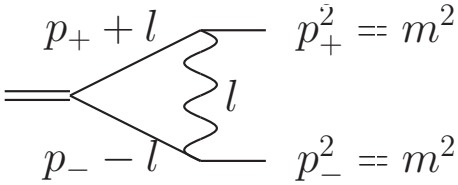
- (virtual) photon loops contain infrared divergences:



$$\propto \int \frac{d^4 l}{(2\pi)^4} \frac{1}{[(p_+ + l)^2 - m^2][(p_- - l)^2 - m^2] l^2}$$

Some general basics on radiative corrections

- (virtual) photon loops contain infrared divergences:

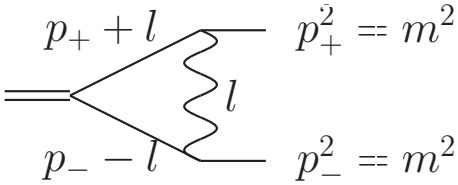


$$\propto \int \frac{d^4 l}{(2\pi)^4} \frac{1}{[2p_+ l + l^2][-2p_- l + l^2] l^2} \sim \int \frac{d^4 l}{l^4}$$

regulate with finite photon mass $m_\gamma \quad \Rightarrow \quad \log \frac{m_\gamma}{m}$ terms

Some general basics on radiative corrections

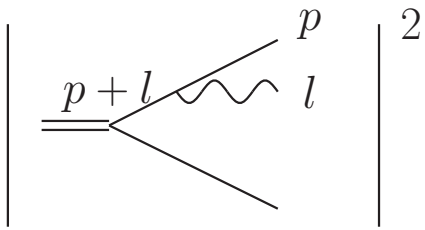
- (virtual) photon loops contain infrared divergences:



$$\propto \int \frac{d^4 l}{(2\pi)^4} \frac{1}{[2p_+ l + l^2][-2p_- l + l^2] l^2} \sim \int \frac{d^4 l}{l^4}$$

regulate with finite photon mass $m_\gamma \quad \Rightarrow \quad \log \frac{m_\gamma}{m}$ terms

- add bremsstrahlung (finite detector resolution!)

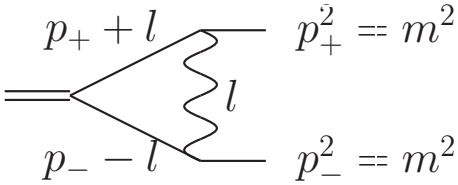


$$\propto \int_{m_\gamma}^{E_{\text{max}}} \frac{d^3 \mathbf{l}}{2 l^0} \frac{1}{(2p\, l)^2} \propto \int_{m_\gamma}^{E_{\text{max}}} \frac{d^3 l}{l^3} \propto \log \frac{E_{\text{max}}}{m_\gamma}$$

combine both contributions to $\log \frac{E_{\text{max}}}{m} \quad \Rightarrow \quad$ infrared finite!

Some general basics on radiative corrections

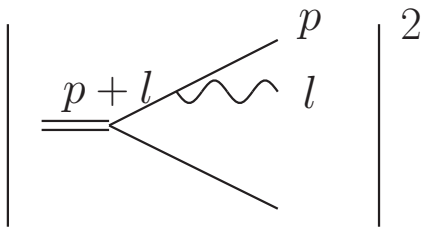
- (virtual) photon loops contain infrared divergences:



$$\propto \int \frac{d^4 l}{(2\pi)^4} \frac{1}{[2p_+ l + l^2][-2p_- l + l^2] l^2} \sim \int \frac{d^4 l}{l^4}$$

regulate with finite photon mass $m_\gamma \quad \Rightarrow \quad \log \frac{m_\gamma}{m}$ terms

- add bremsstrahlung (finite detector resolution!)



$$\propto \int_{m_\gamma}^{E_{\text{max}}} \frac{d^3 \mathbf{l}}{2 l^0} \frac{1}{(2p\, l)^2} \propto \int_{m_\gamma}^{E_{\text{max}}} \frac{d^3 l}{l^3} \propto \log \frac{E_{\text{max}}}{m_\gamma}$$

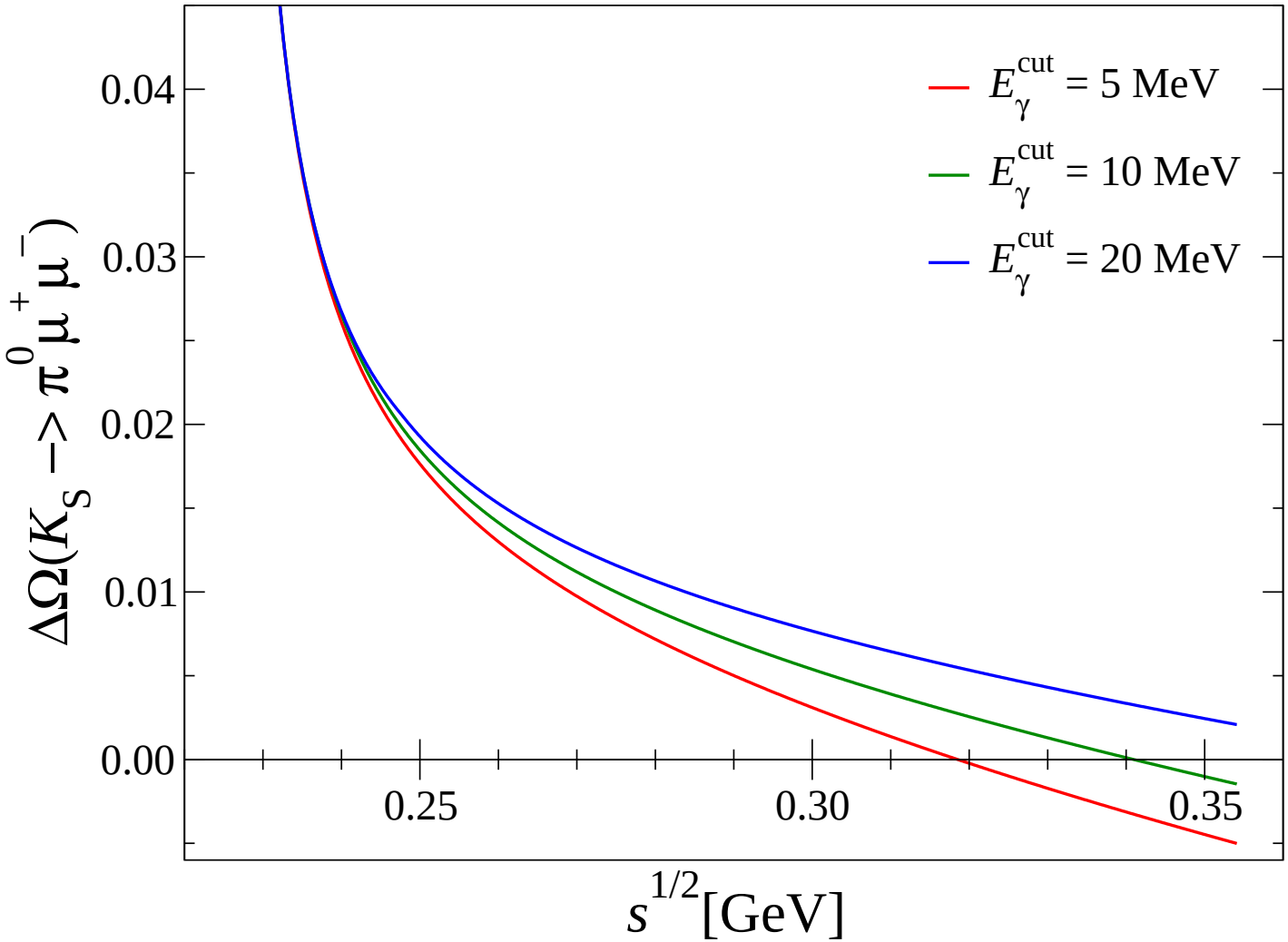
combine both contributions to $\log \frac{E_{\text{max}}}{m} \quad \Rightarrow \quad$ infrared finite!

- vast simplification: "soft-photon approximation"

neglect l except in the denominators: $\int_{m_\gamma}^{E_{\text{max}}} \frac{d^3 l}{l^3} l \propto E_{\text{max}} - m_\gamma$

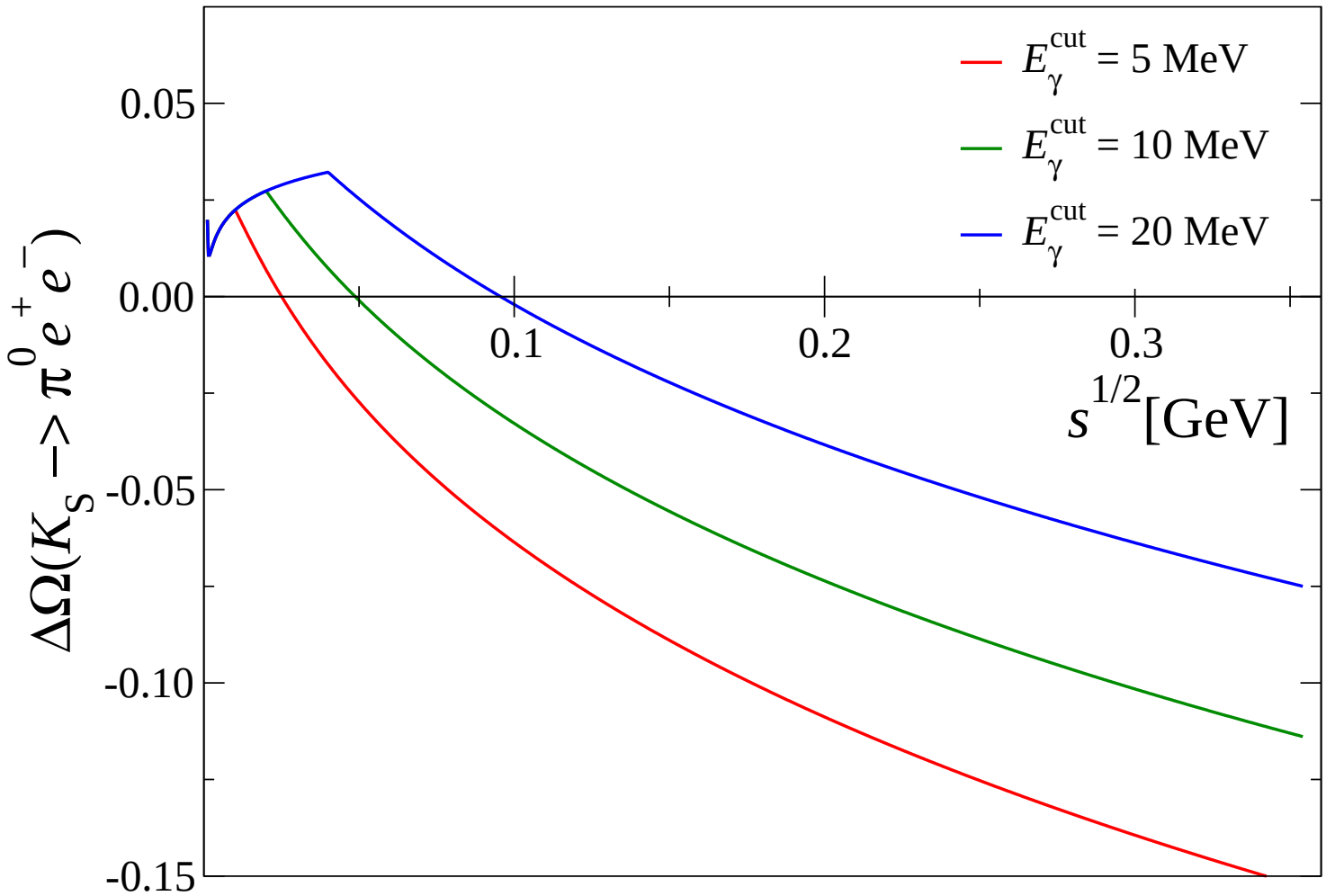
neglects terms of $\mathcal{O}(E_{\text{max}})$, correct including $\mathcal{O}(E_{\text{max}}^0)$

$K_S \to \pi^0 \mu^+ \mu^-$, soft-photon approximation



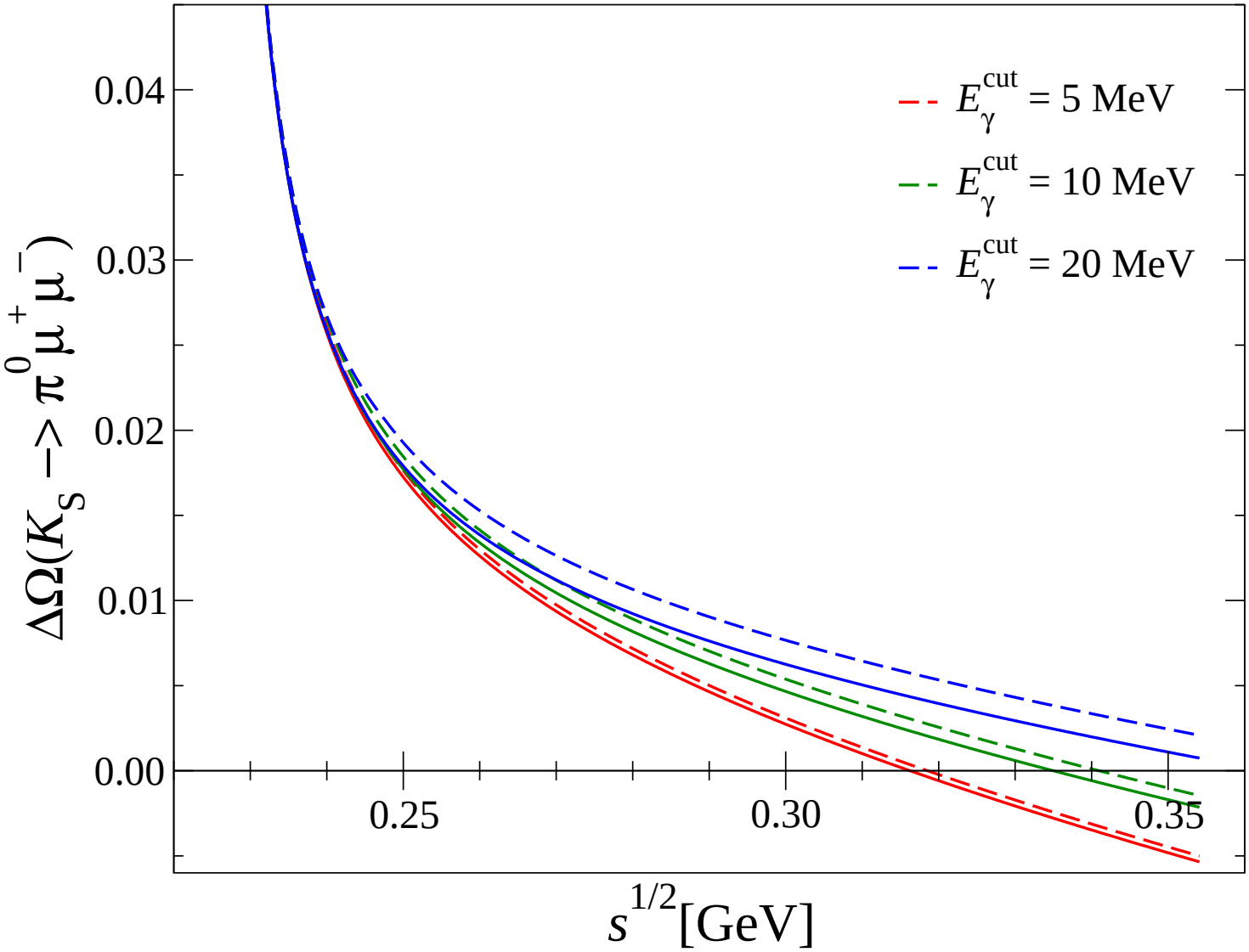
- dominated by Coulomb singularity $\propto \frac{\alpha}{\sqrt{s-4m_\mu^2}}$, otherwise small
- loop corrections ultraviolet-finite, no counterterm required

$K_S \to \pi^0 e^+ e^-$, soft-photon approximation



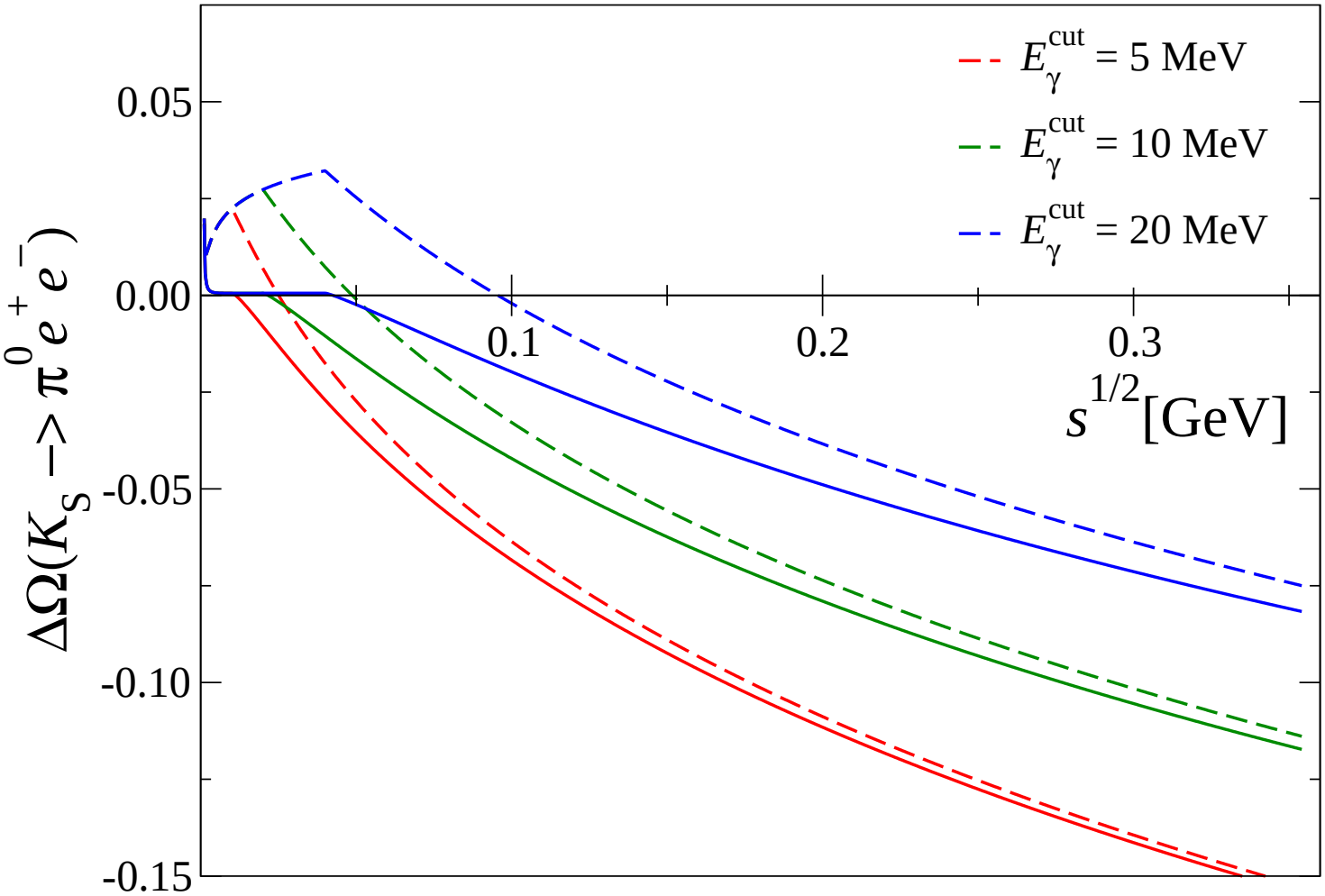
- huge radiative corrections – why??
- characteristic width of Coulomb pole $m_e^2 \Rightarrow$ hardly visible

$K_S \to \pi^0 \mu^+ \mu^-$, no soft-photon approximation



- corrections to soft-photon approximation (dashed) below 0.2%

$K_S \to \pi^0 e^+ e^-$, no soft-photon approximation



- corrections to soft-photon approximation (dashed) sizeable
- overall correction factor still very large

The KLN theorem and mass singularities

Kinoshita 1962; Lee, Nauenberg 1964

- enhancement of e^+e^- vs. $\mu^+\mu^-$: mass singularity??
- **Kinoshita–Lee–Nauenberg theorem:**
  no mass singularities in total/inclusive transition probabilities

The KLN theorem and mass singularities

Kinoshita 1962; Lee, Nauenberg 1964

- enhancement of e^+e^- vs. $\mu^+\mu^-$: mass singularity??
- **Kinoshita–Lee–Nauenberg theorem:**
  no mass singularities in total/inclusive transition probabilities

$$\Delta\Omega_{e^+e^-} = \frac{\alpha}{\pi}\left\{\frac{1}{4} - 2\left[\log\epsilon + \frac{(1-\epsilon)(3-\epsilon)}{4}\right]\log\delta - 2\log\epsilon + \frac{\pi^2}{3} - 2\mathrm{Li}\,(\epsilon)\right.$$
$$\left. + \frac{1-\epsilon}{2}\left[(3-\epsilon)\log(1-\epsilon) - \frac{11-3\epsilon}{2}\right]\right\} + \mathcal{O}(m_e)$$

$$\epsilon = \frac{2E_{\mathsf{max}}}{\sqrt{s}} \qquad \delta = \frac{m_e^2}{s}$$

The KLN theorem and mass singularities

Kinoshita 1962; Lee, Nauenberg 1964

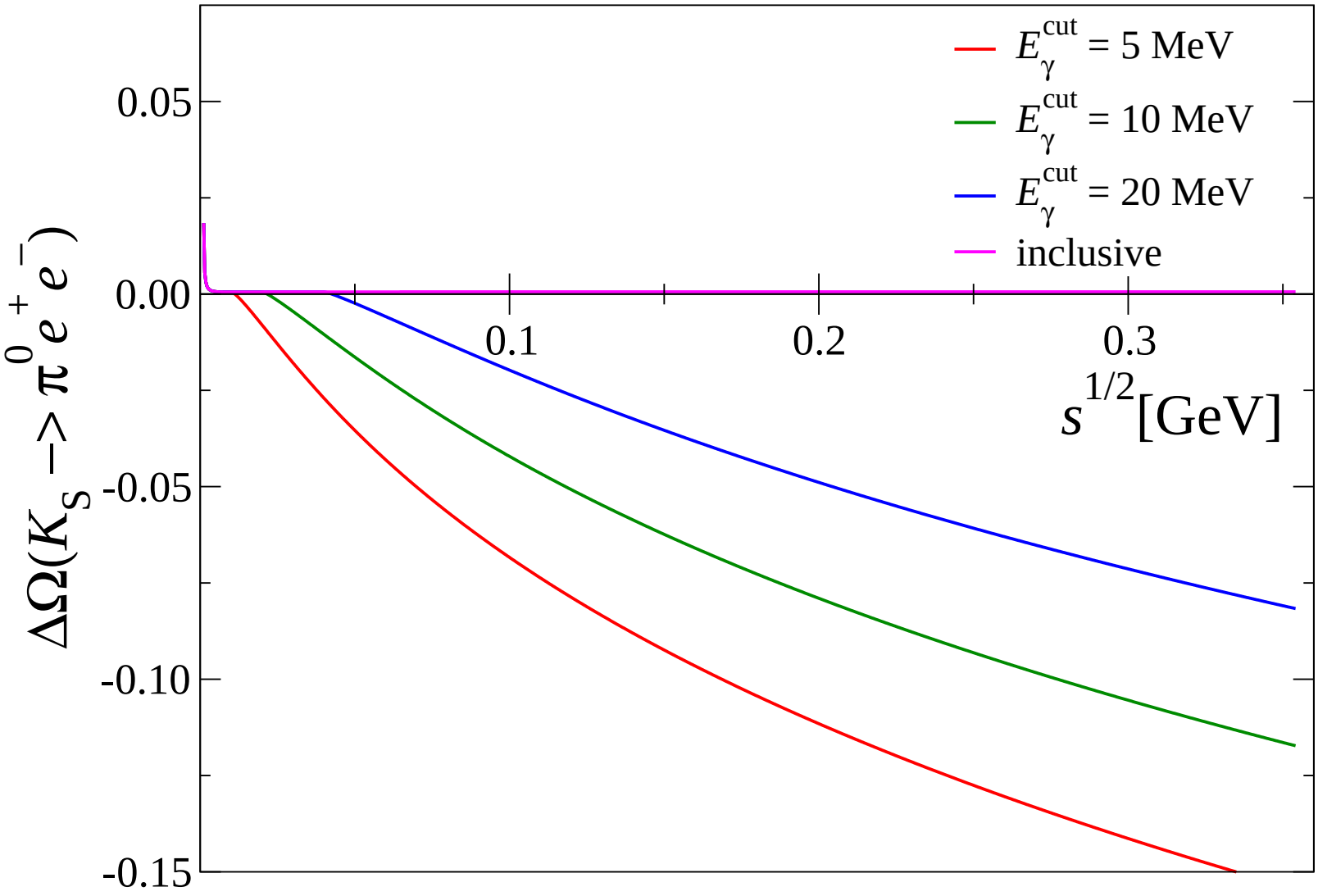
- enhancement of e^+e^- vs. $\mu^+\mu^-$: mass singularity??
- **Kinoshita–Lee–Nauenberg theorem:**
  no mass singularities in total/inclusive transition probabilities

$$\Delta\Omega_{e^+e^-} = \frac{\alpha}{\pi}\left\{\frac{1}{4} - 2\left[\log\epsilon + \frac{(1-\epsilon)(3-\epsilon)}{4}\right]\log\delta - 2\log\epsilon + \frac{\pi^2}{3} - 2\mathrm{Li}\,(\epsilon)\right.$$
$$\left. + \frac{1-\epsilon}{2}\left[(3-\epsilon)\log(1-\epsilon) - \frac{11-3\epsilon}{2}\right]\right\} + \mathcal{O}(m_e)$$

$$\epsilon = \frac{2E_{\mathsf{max}}}{\sqrt{s}} \qquad \delta = \frac{m_e^2}{s}$$

- inclusive limit $\epsilon \to 1 \quad \Rightarrow \quad$ all $\log\delta$ terms vanish, $\Delta\Omega_{\mathsf{incl}} = \dfrac{\alpha}{4\pi}$
- mass singularities $\propto \log\delta$ present for $\epsilon < 1$
- soft-photon approximation: neglect $\mathcal{O}(\epsilon)$ terms
  $\Rightarrow \quad$ no $\log\delta$ cancellation for $\epsilon \to 1$

$K_S \to \pi^0 e^+ e^- (\gamma)$ inclusive



- the more inclusive the bremsstrahlung, the smaller the correction

Collinear singularities and the restricted Dalitz plot

- origin of mass singularities in bremsstrahlung well understood:

$$\frac{1}{p\,l} = \frac{1}{(p^0 - |\mathbf{p}|z)|\mathbf{l}|} \overset{m_e \to 0}{=} \frac{1}{|\mathbf{p}||\mathbf{l}|(1-z)}$$

$p^0 = \sqrt{\mathbf{p}^2 + m_e^2}$, diverges for $z = \cos\theta_{e\gamma} \to 1$

- collinear singularity for radiation of photons off light particles

Collinear singularities and the restricted Dalitz plot

- origin of mass singularities in bremsstrahlung well understood:

$$\frac{1}{p\,l} = \frac{1}{(p^0 - |\mathbf{p}|z)|\mathbf{l}|} \overset{m_e \to 0}{=} \frac{1}{|\mathbf{p}||\mathbf{l}|(1-z)}$$

  $p^0 = \sqrt{\mathbf{p}^2 + m_e^2}$, diverges for $z = \cos\theta_{e\gamma} \to 1$

- collinear singularity for radiation of photons off light particles
- experimentally: cannot discriminate collinear photons
  (as one cannot discriminate soft photons) $\Rightarrow$ have to cut on $\theta_{e\gamma}$
- hard collinear photons lead to mass singularities $\propto \log m_e$
  unphysically enhanced radiative corrections shown so far

Collinear singularities and the restricted Dalitz plot

- origin of mass singularities in bremsstrahlung well understood:

$$\frac{1}{p\,l} = \frac{1}{(p^0 - |\mathbf{p}|z)|\mathbf{l}|} \overset{m_e \to 0}{=} \frac{1}{|\mathbf{p}||\mathbf{l}|(1-z)}$$

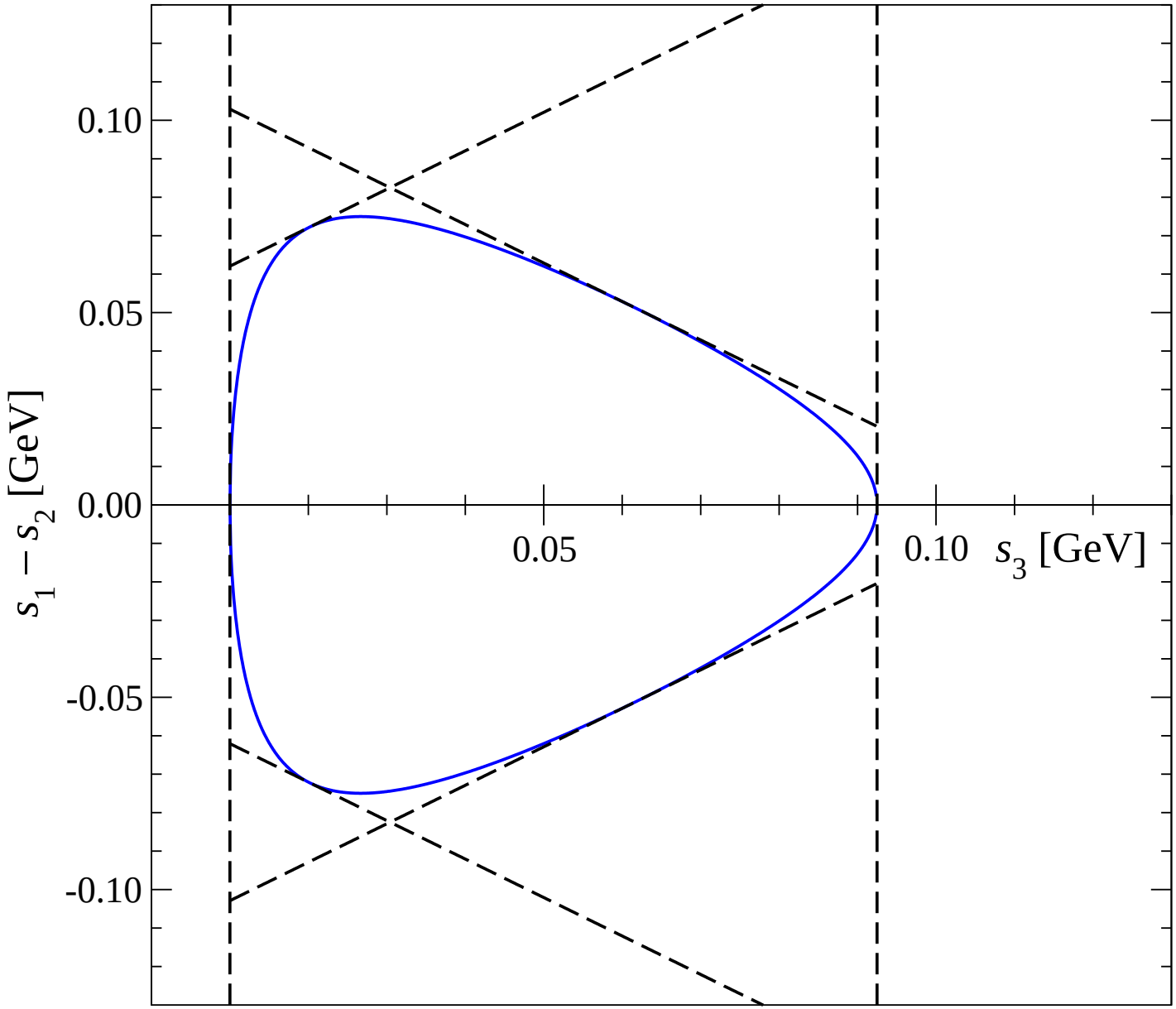
$p^0 = \sqrt{\mathbf{p}^2 + m_e^2}$, diverges for $z = \cos\theta_{e\gamma} \to 1$

- collinear singularity for radiation of photons off light particles
- experimentally: cannot discriminate collinear photons
  (as one cannot discriminate soft photons) $\Rightarrow$ have to cut on $\theta_{e\gamma}$
- hard collinear photons lead to mass singularities $\propto \log m_e$
  unphysically enhanced radiative corrections shown so far
- consider Dalitz plot for sub-decay

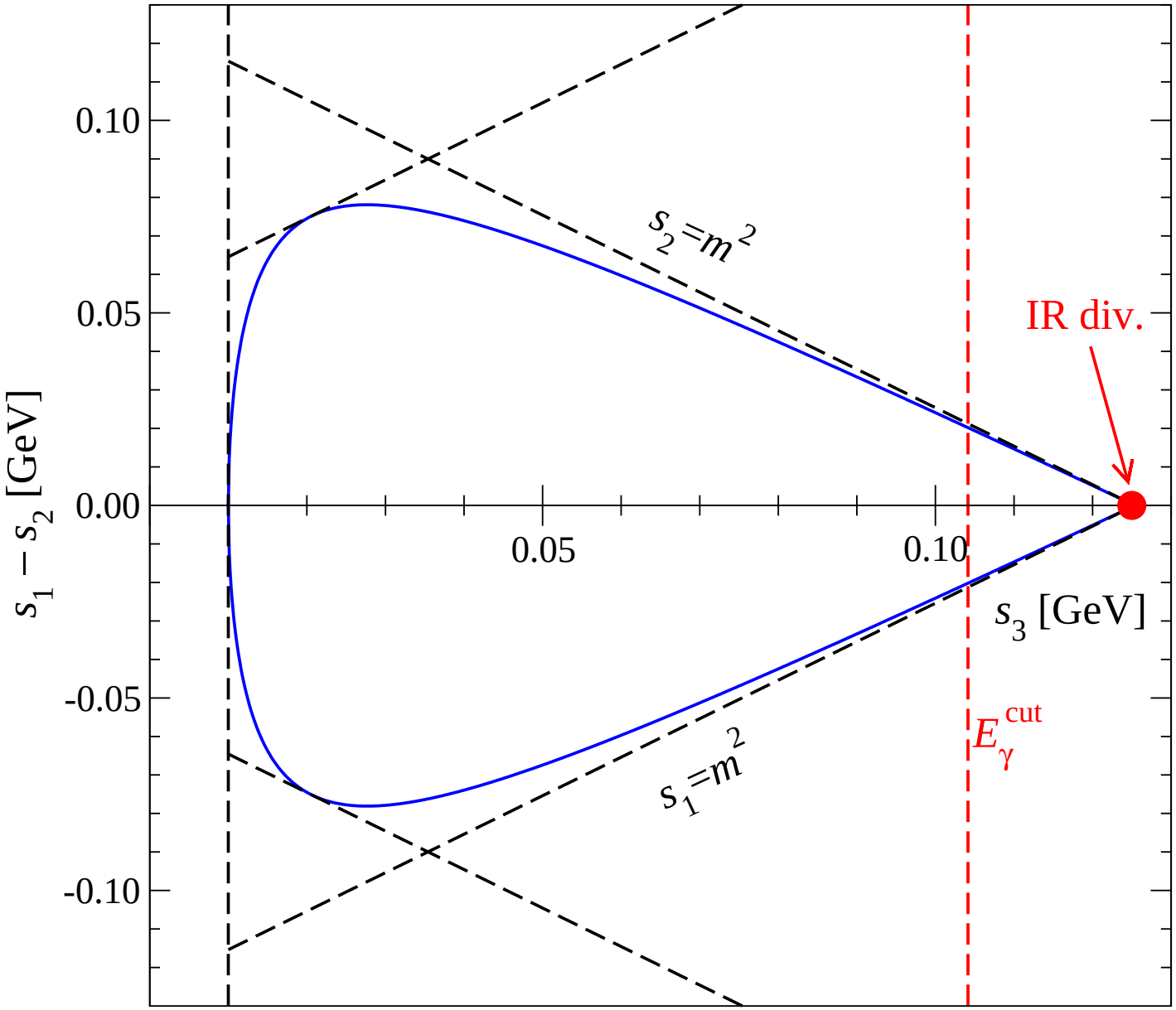
$$\gamma^*(\sqrt{s}) \to e^+(p_+)e^-(p_-)\gamma(l)\ ,\quad s_{1/2} = (p_\mp + l)^2\ ,\quad s_3 = (p_+ + p_-)^2$$

$$s_1 + s_2 + s_3 = s + 2m_e^2 \approx s$$

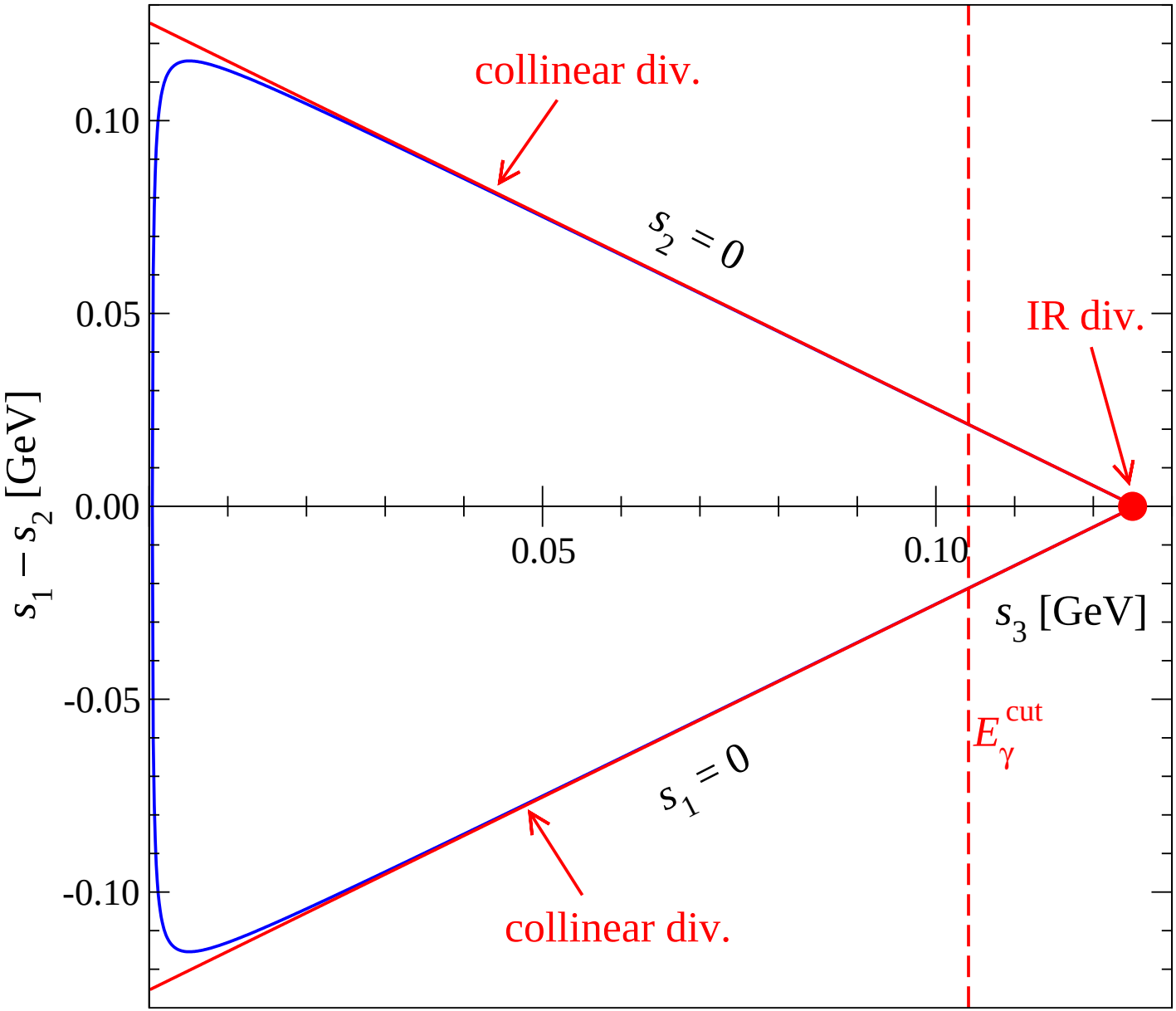
Dalitz plot:



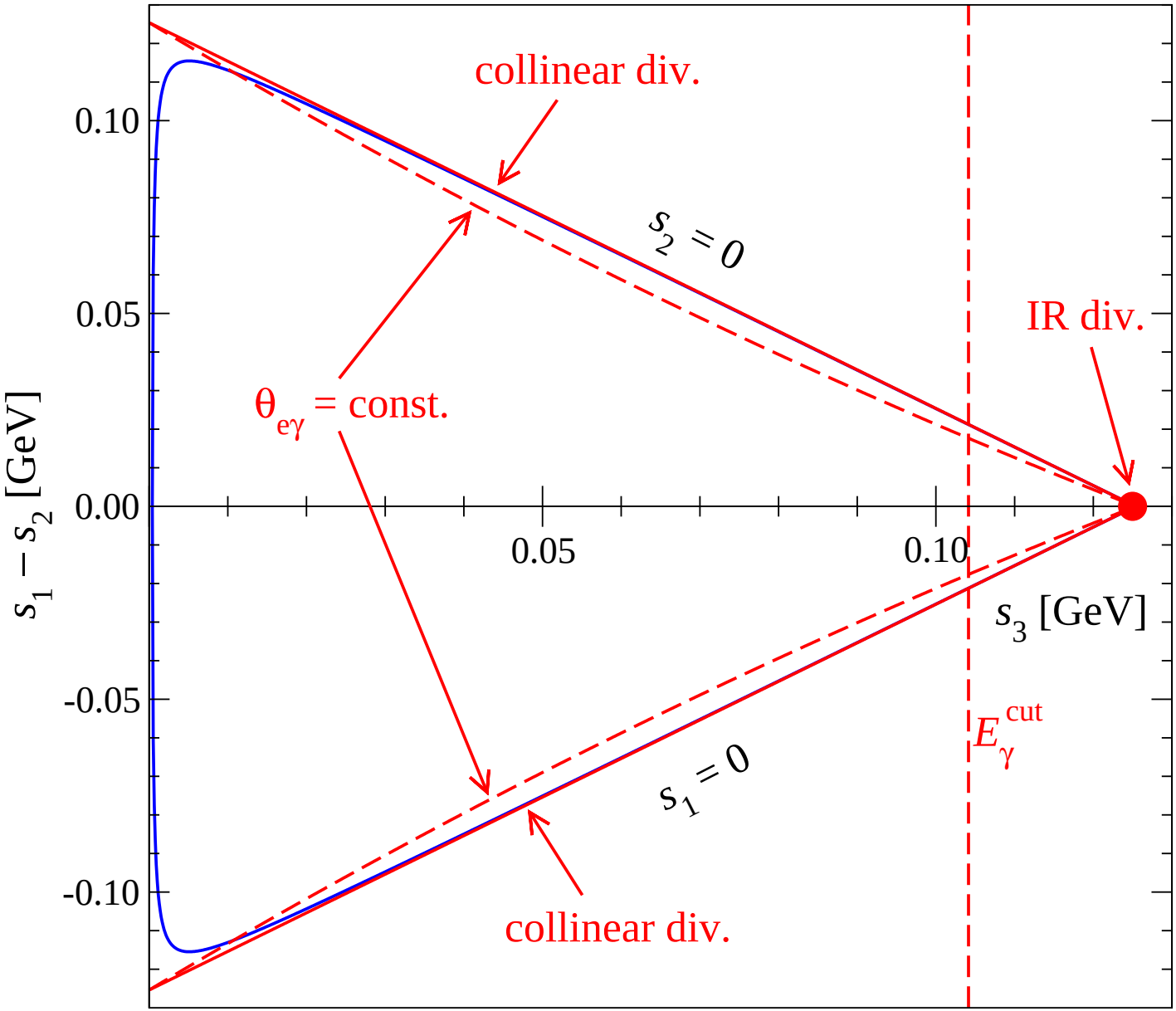
Dalitz plot: one massless particle



Dalitz plot: three (nearly) massless particles

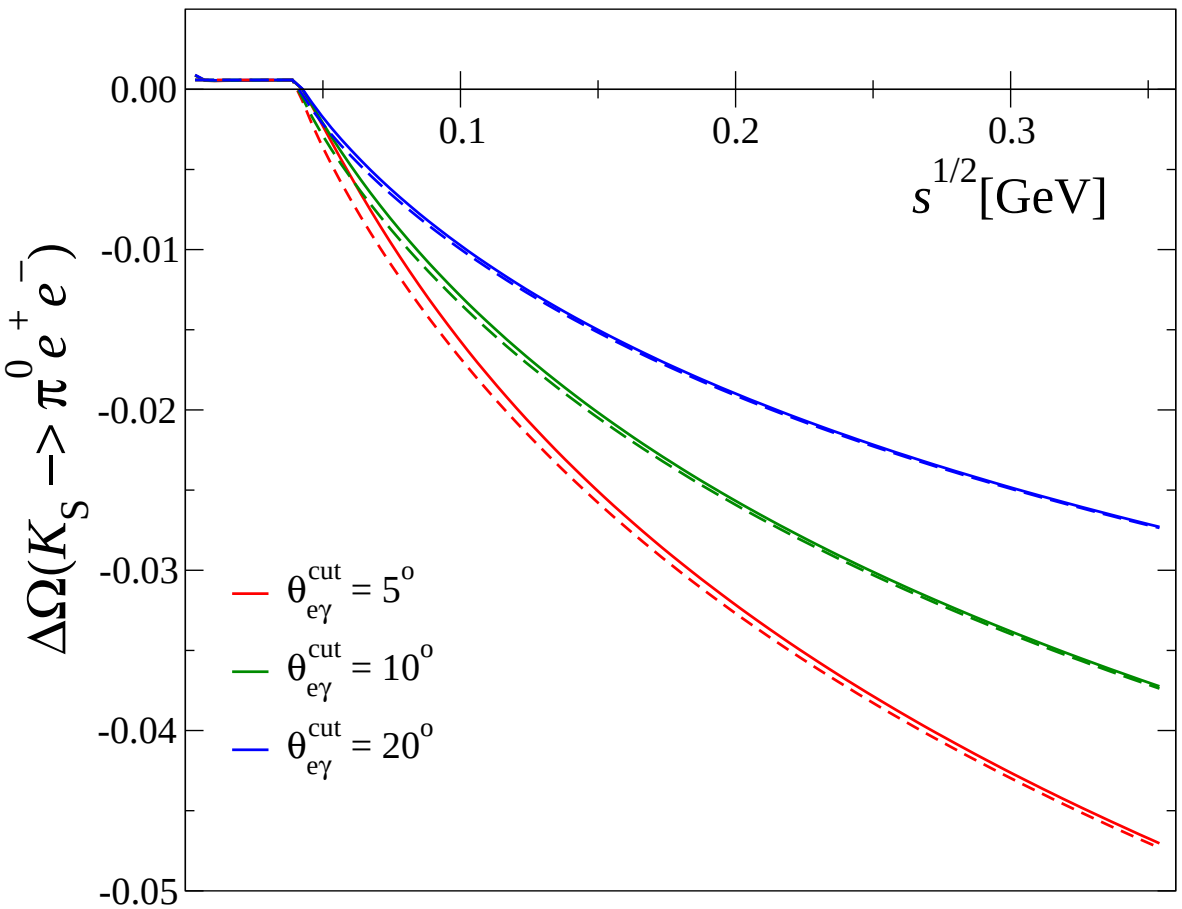


Dalitz plot: three (nearly) massless particles



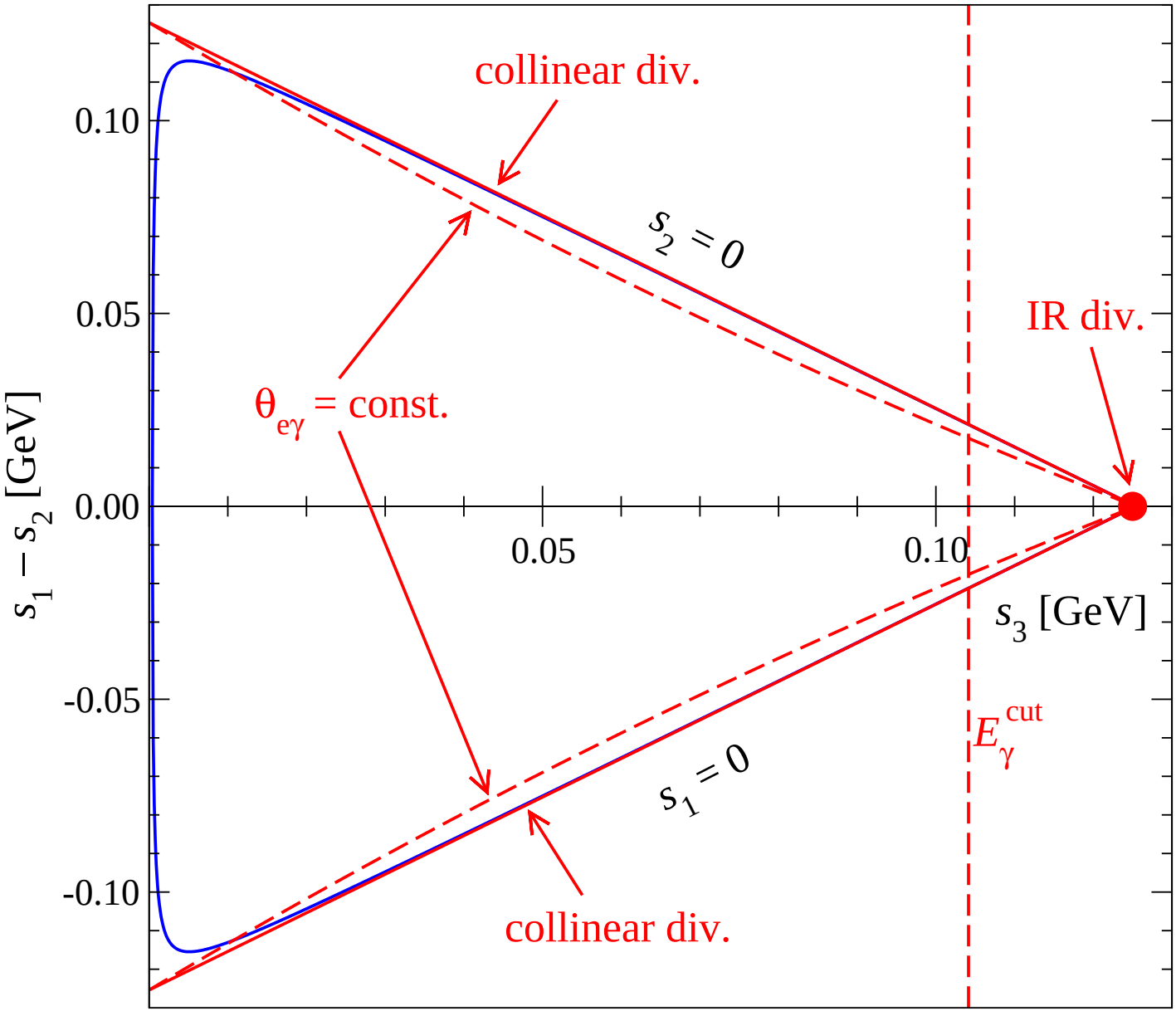
$K_S \to \pi^0 e^+ e^- (\gamma)$, angular cut

$$\Delta\Omega_{e^+e^-} = \frac{\alpha}{\pi}\left\{\frac{1}{4} - 2\left[\log\epsilon + \frac{(1-\epsilon)(3-\epsilon)}{4}\right]\log\mu + \ldots\right\}, \quad \mu = \frac{1-\cos\theta_{e\gamma}}{2}$$

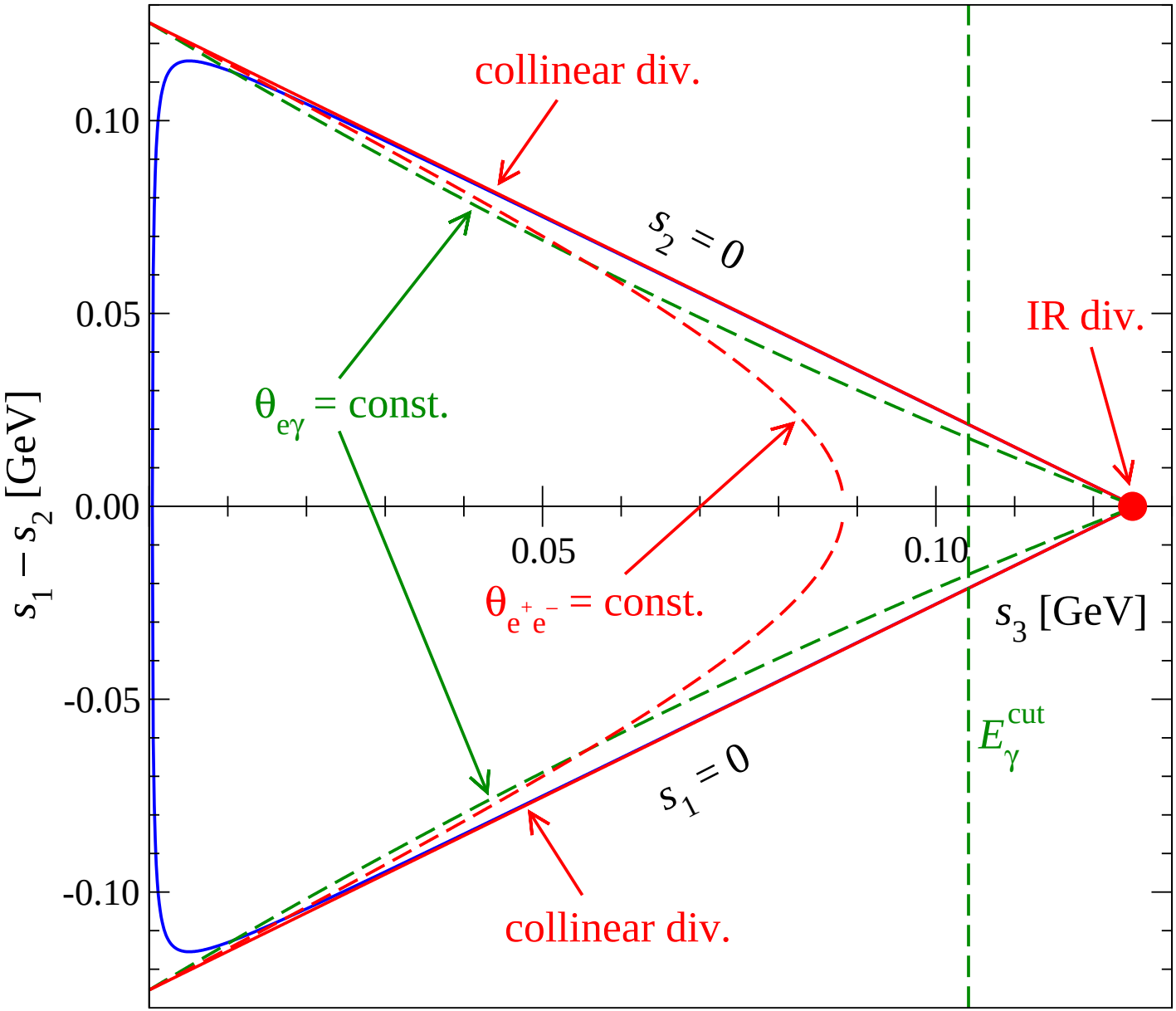


- $E_\gamma^{\text{cut}} = 20$ MeV fixed
- note: $\log\delta \to \log\mu$
- mass singularity translated into phase space singularity (RG resummation of phase space logs in SCET...)
- note: $\log\frac{m_e^2}{s} \approx -13$
  $\log\frac{1-\cos 20^\circ}{2} \approx -3.5$
- approximation $m_e = 0$ (dashed) very accurate

Dalitz plot: three (nearly) massless particles

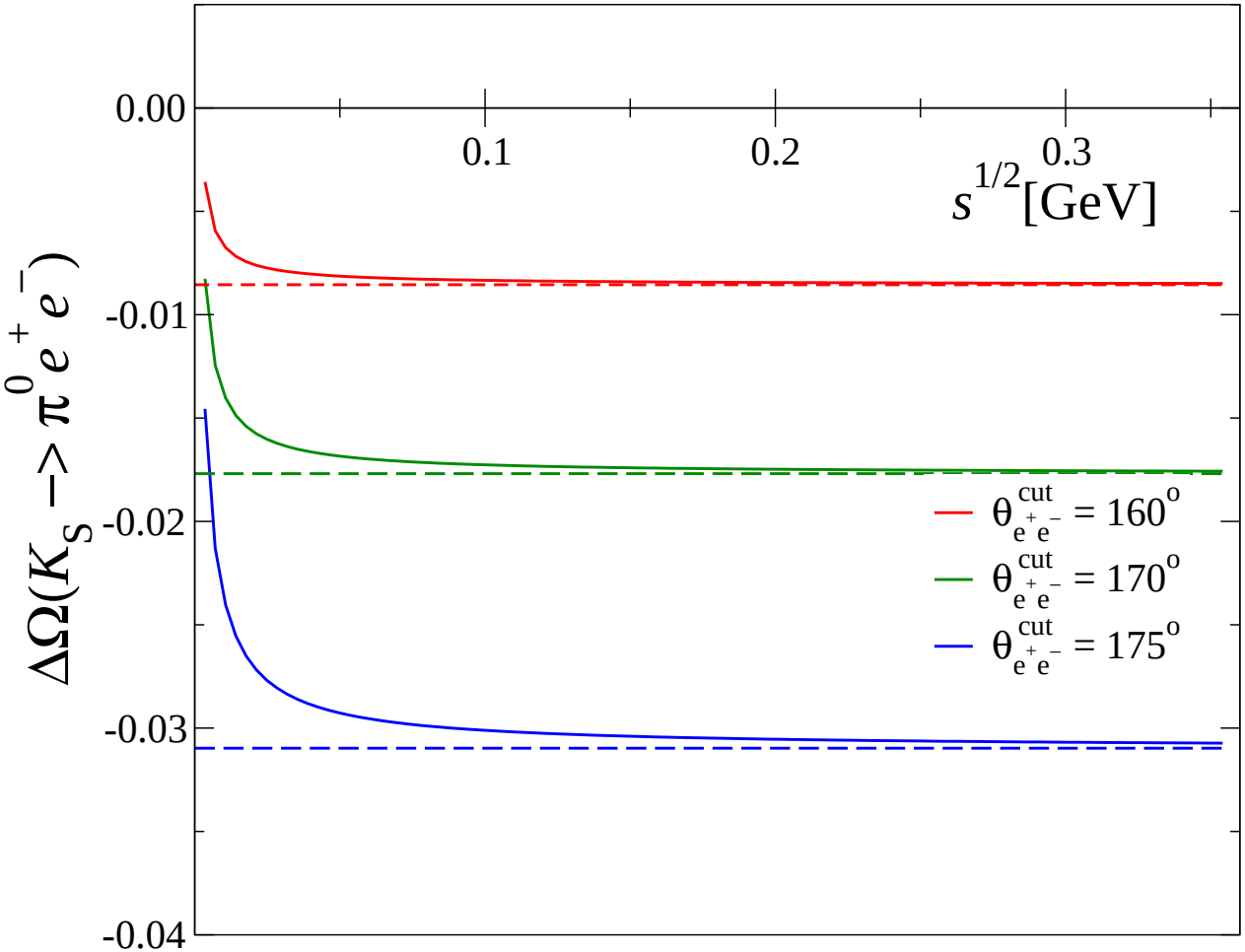


Dalitz plot: three (nearly) massless particles



$K_S \to \pi^0 e^+ e^- (\gamma), \theta_{e^+e^-}$ cut

- all singular parts of the bremsstrahlung (soft + collinear) occur for e^+e^- back to back $\Rightarrow$ single cut on $\theta_{e^+e^-}$ can cure both
- dimensionless cut $\Rightarrow \Delta\Omega_{e^+e^-}$ constant in massless limit (dashed)

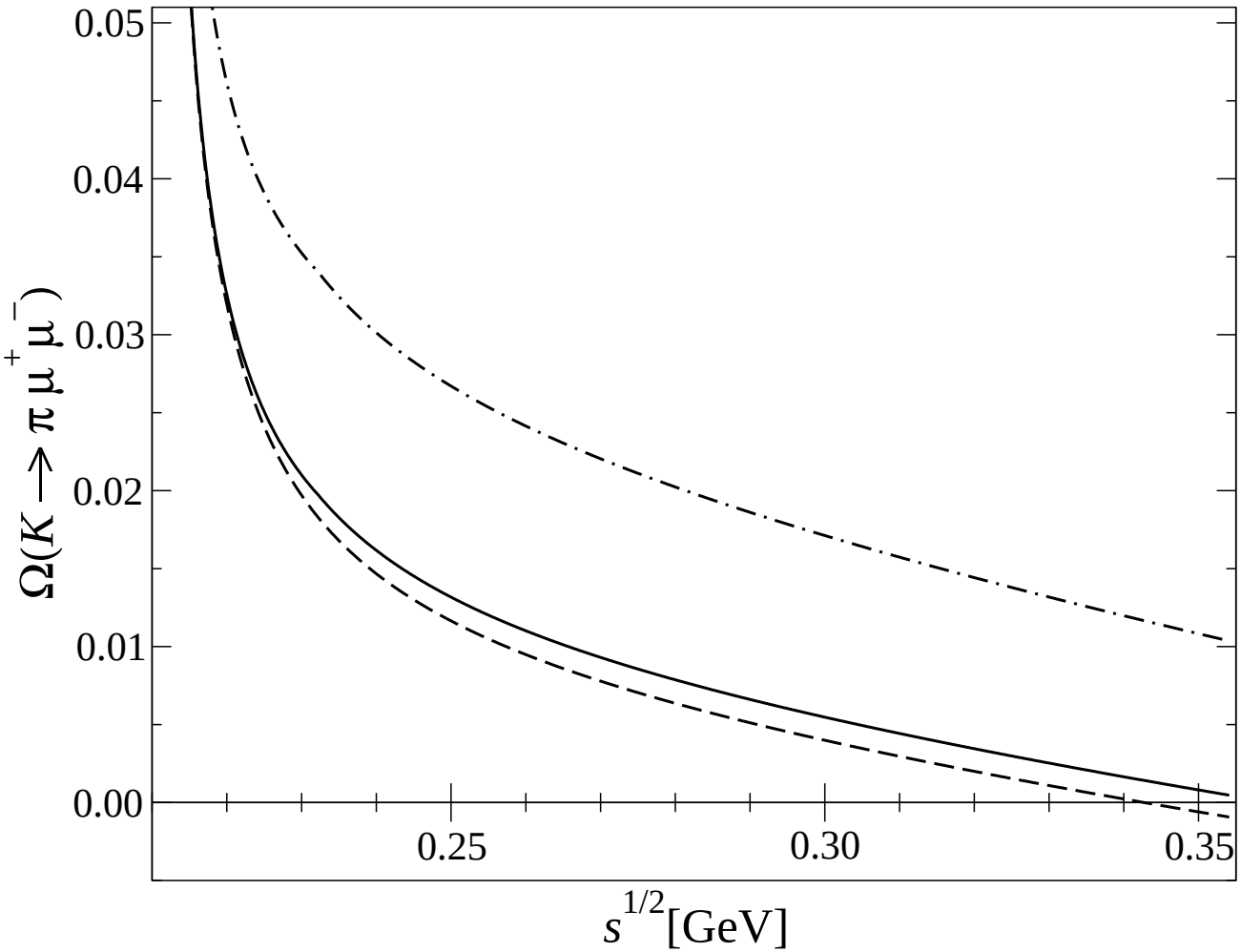


Summary / Conclusions

- Don't use the soft-photon approximation for e^+e^- final states!
  - ▷ hard collinear photons lead to enhanced logarithms
  - ▷ their coefficients not reproduced in the soft approximation
  - ▷ angular cuts (on $\theta_{e\gamma}$ or $\theta_{e^+e^-}$) required
  - ▷ compact analytic forms available in the massless limit
- $\mu^+\mu^-$ final states simpler, soft approximation justified
- radiative corrections for the dilepton spectrum $d\Gamma/ds$ are simple sums of corrections to hadronic and leptonic current
- no ultraviolet counterterms required
- universal correction factors $\Delta\Omega$ for dilepton spectra identical for vector meson conversion decays $\omega \to \pi^0\ell^+\ell^-$, $\phi \to \eta\ell^+\ell^-$ etc.
- consequences for existing studies of Dalitz decays $\pi^0, \eta \to \gamma e^+e^-$ should be investigated

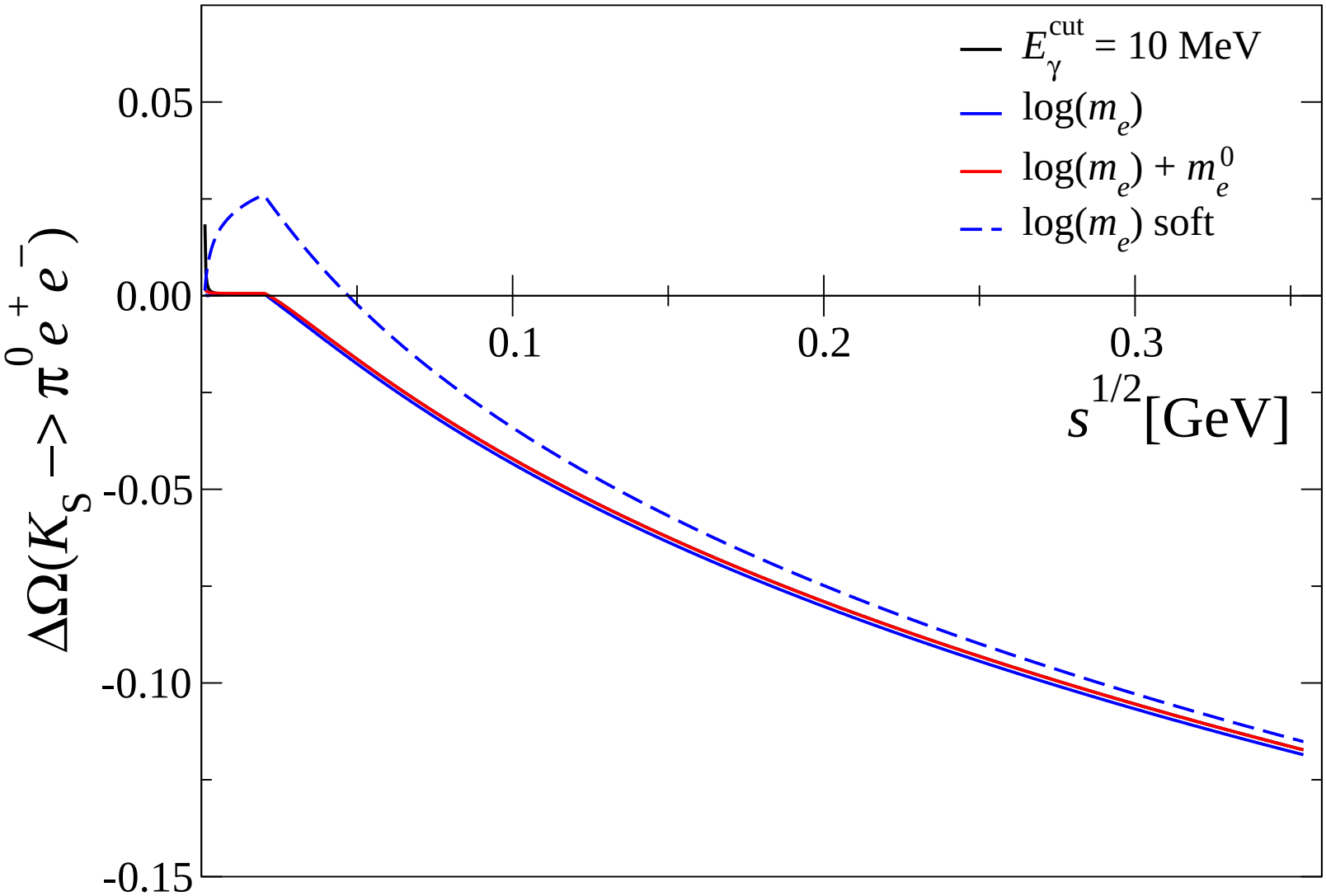
Spares

Hadronic-current corrections: $K^+ \to \pi^+\mu^+\mu^-$



- additional contribution to $K^+ \to \pi^+\mu^+\mu^-$ increases $\Delta\Omega(K_S \to \pi^0\mu^+\mu^-)$ by 1–1.5%

$K_S \to \pi^0 e^+ e^- (\gamma)$, small-mass approximation



- $\log m_e$ enhanced terms totally dominate

$\ell^+\ell^-$ or $t-u$ asymmetry

- radiative corrections linking leptonic and hadronic current are odd in $\nu \doteq t-u \quad \Rightarrow \quad$ cancel in $d\Gamma/ds$

$$A_\nu(s) = \left(\frac{d\Gamma}{ds}\right)^{-1} \int d\nu \, \mathrm{sgn}(\nu) \frac{d^2\Gamma}{ds\, d\nu}$$