
VP reloaded



Thomas Teubner



- . Introduction
- .. HLMNT routine; status and comparison with other Vacuum Polarisation compilations
- ... Latest changes
- Narrow resonances: treatment and pitfalls

Thanks to my collaborators Kaoru Hagiwara, Ruofan Liao, Alan Martin and Daisuke Nomura.

Introduction

- Why Vacuum Polarisation / running α corrections ?

Precise knowledge of VP / $\alpha(q^2)$ needed for:

- Corrections for data used as input for $g - 2$: ‘undressed’ σ_{had}^0
$$a_{\mu}^{\text{had,LO}} = \frac{1}{4\pi^3} \int_{m_{\pi}^2}^{\infty} ds \, \sigma_{\text{had}}^0(s) K(s), \quad \text{with } K(s) = \frac{m_{\mu}^2}{3s} \cdot (0.63 \dots 1)$$
- Determination of α_s and quark masses from total hadronic cross section R_{had} at low energies and of resonance parameters.
- Part of higher order corrections in Bhabha scattering important for precise Luminosity determination.
- $\alpha(M_Z^2)$ a fundamental parameter at the Z scale (the least well known of $\{G_{\mu}, M_Z, \alpha(M_Z^2)\}$), needed to test the SM via precision fits/constrain new physics.
- Ingredient in MC generators for many processes.

- Dyson summation of Real part of one-particle irreducible blobs Π into the effective, real running coupling α_{QED} :

$$\Pi = \text{diagram: a wavy line with } \gamma^* \text{ and } q \text{ entering a shaded circular blob, with another wavy line exiting.}$$

$$\text{Full photon propagator} \sim 1 + \Pi + \Pi \cdot \Pi + \Pi \cdot \Pi \cdot \Pi + \dots$$

$$\rightsquigarrow \alpha(q^2) = \frac{\alpha}{1 - \text{Re}\Pi(q^2)} = \alpha / (1 - \Delta\alpha_{\text{lep}}(q^2) - \Delta\alpha_{\text{had}}(q^2))$$

- The Real part of the VP, $\text{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\text{Im}\Pi \sim \sigma(e^+e^- \rightarrow \text{hadrons})$:

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{q^2}{4\pi^2\alpha} \text{P} \int_{m_\pi^2}^{\infty} \frac{\sigma_{\text{had}}^0(s) ds}{s - q^2}, \quad \sigma_{\text{had}}(s) = \frac{\sigma_{\text{had}}^0(s)}{|1 - \Pi|^2}$$

$$[\rightarrow \sigma^0 \text{ requires 'undressing', e.g. via } \cdot(\alpha/\alpha(s))^2 \rightsquigarrow \text{iteration needed}]$$

- Observable cross sections σ_{had} contain the **|full photon propagator|²**, i.e. |infinite sum|².
 \rightarrow To include the subleading Imaginary part, use dressing factor $\frac{1}{|1 - \Pi|^2}$.

HLMNT routine; status and comparison

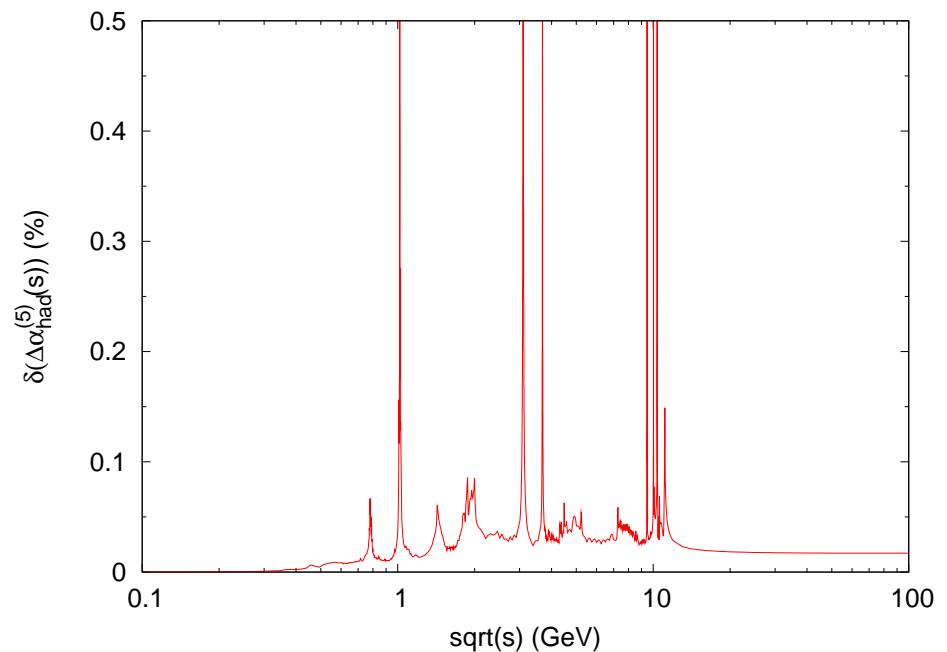
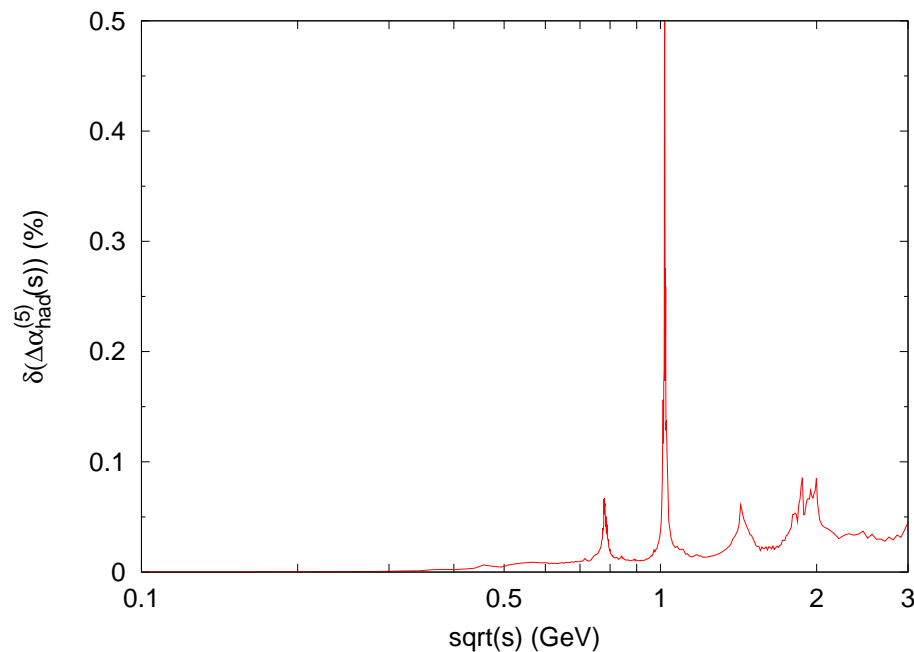
Features of the HLMNT VP code

new version based on HLMNT '11 imminent

- Latest version is VP_HLMNT_v2_0, version 2.0, 15 July 2010
- Simple set of (standard) Fortran routines; completely standalone, no libs needed; all explanations in comment-headers
- Gives separately real and imaginary part ($\Delta\alpha(s)$ and $R(s)$)
- Tabulation/interpolation of hadronic part, for both space- and time-like region, including errors; no input data files or rhad installation needed
- Leptonic part coded analytically; all special function included (partly with custom made expansions)
- top contribution in the same way
- Flag to **include or exclude** narrow resonances J/ψ , ψ' , $\Upsilon(1 - 6 S)$
[but ϕ always included via integral over final state data (3π , KK)]

- Typical accuracy $\delta \left(\Delta\alpha_{\text{had}}^{(5)}(s) \right)$

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):

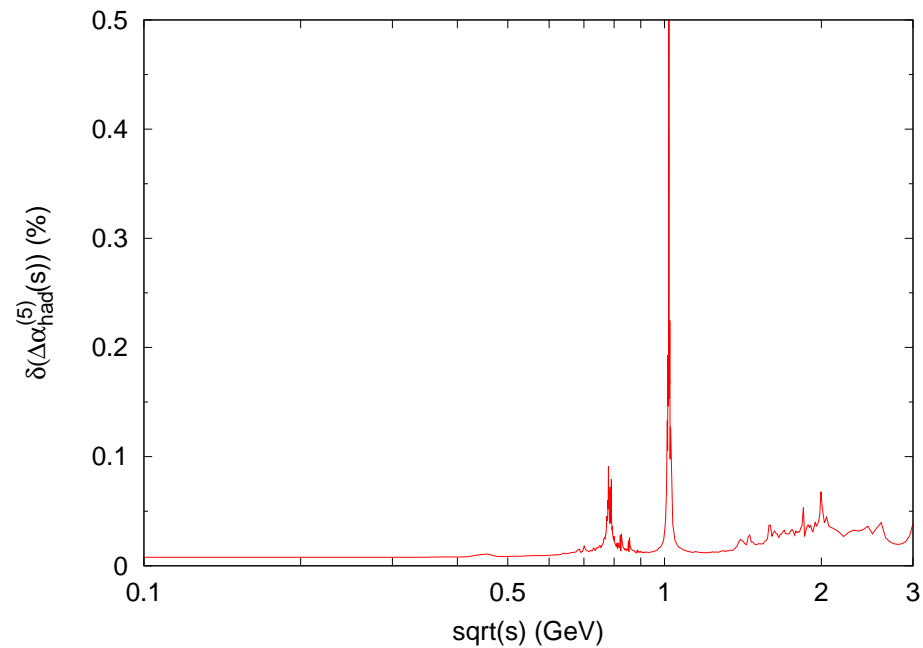
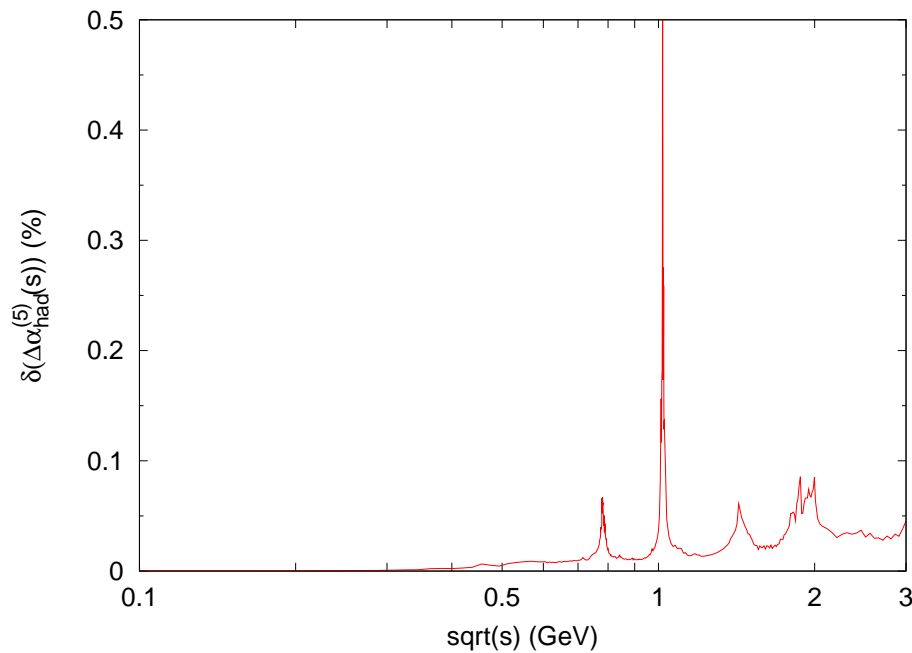


→ Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.

Enough in the long term? Need for more work in resonance regions.

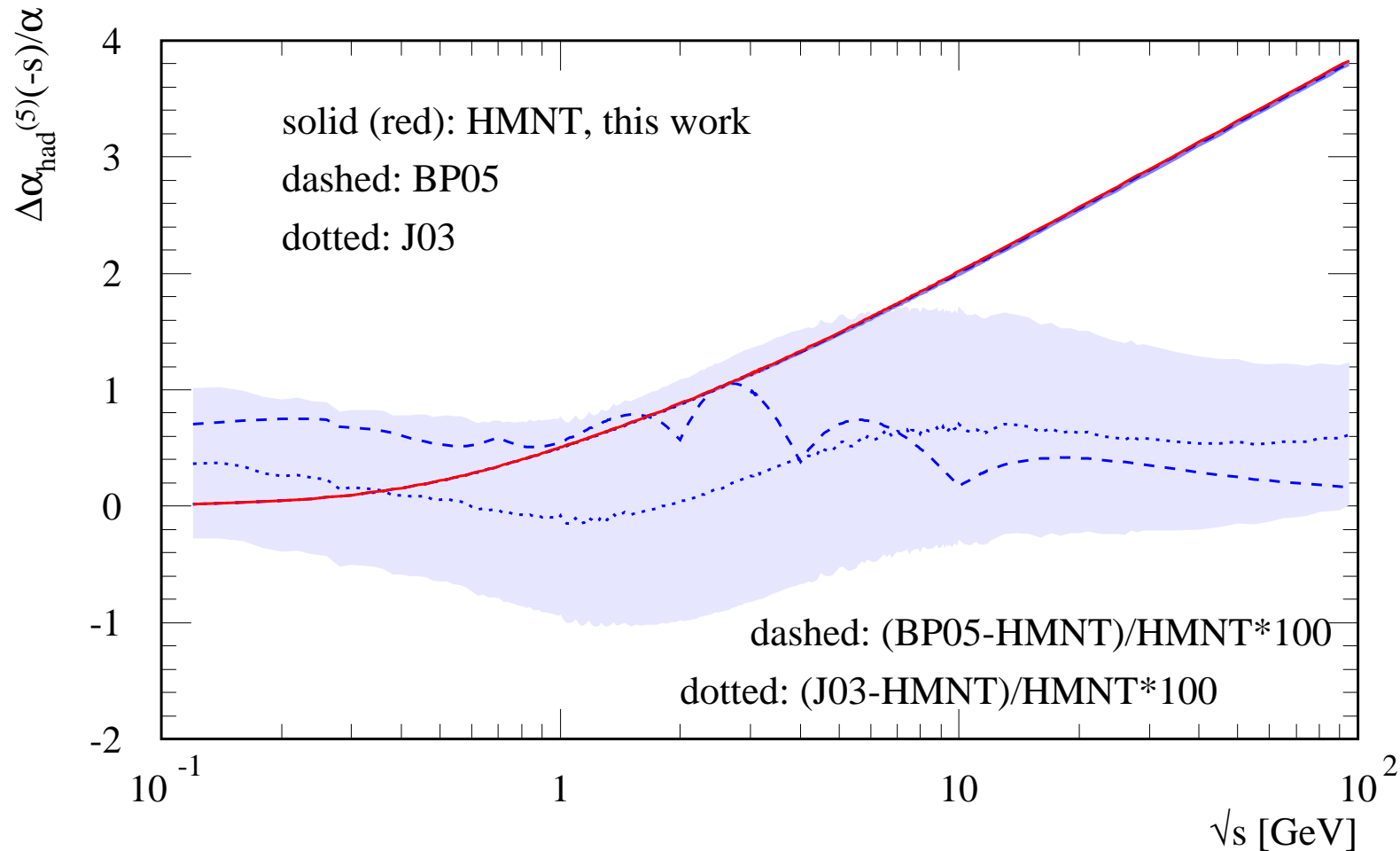
- Typical accuracy $\delta \left(\Delta\alpha_{\text{had}}^{(5)}(s) \right)$

Error of VP in the timelike regime: old vs. **new** HLMNT '11 compilation):



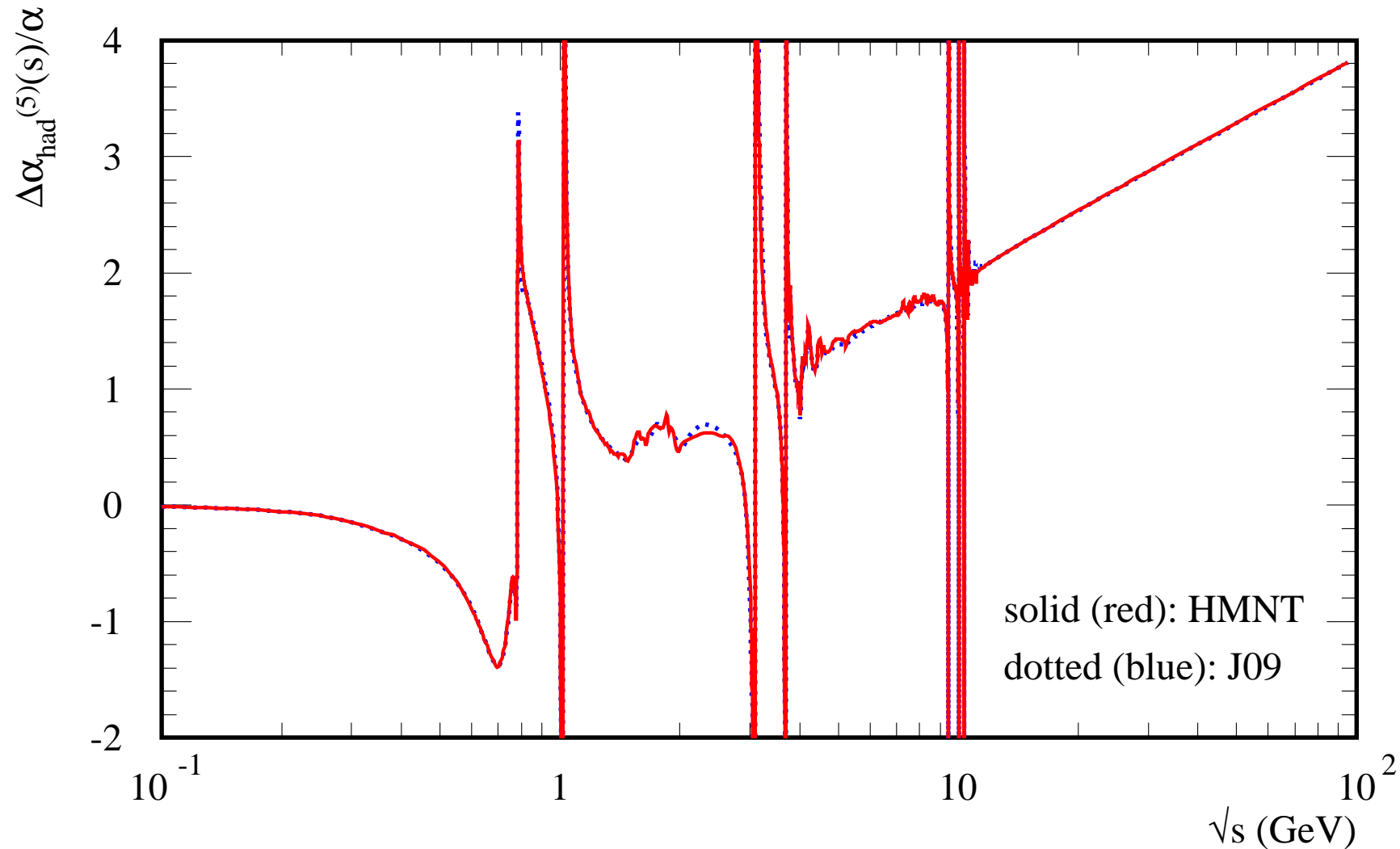
→ region $1 < \sqrt{s} < 2$ GeV (and higher) improved,
 ρ suffers from tension in 2π data (BaBar included).

- Comparison of Spacelike $\Delta\alpha_{\text{had}}^{(5)}(-s)/\alpha$ (smooth $\alpha(q^2 < 0)$)



- Differences between parametrisations clearly visible but within error band (of HLMNT)
- Few-parameter formula from Burkhardt+Pietrzyk slightly 'bumpy' but still o.k.
- Encourage use of more accurate recent tabulations; $\Delta\alpha(M_Z^2)$

- $\Delta\alpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner's new routines



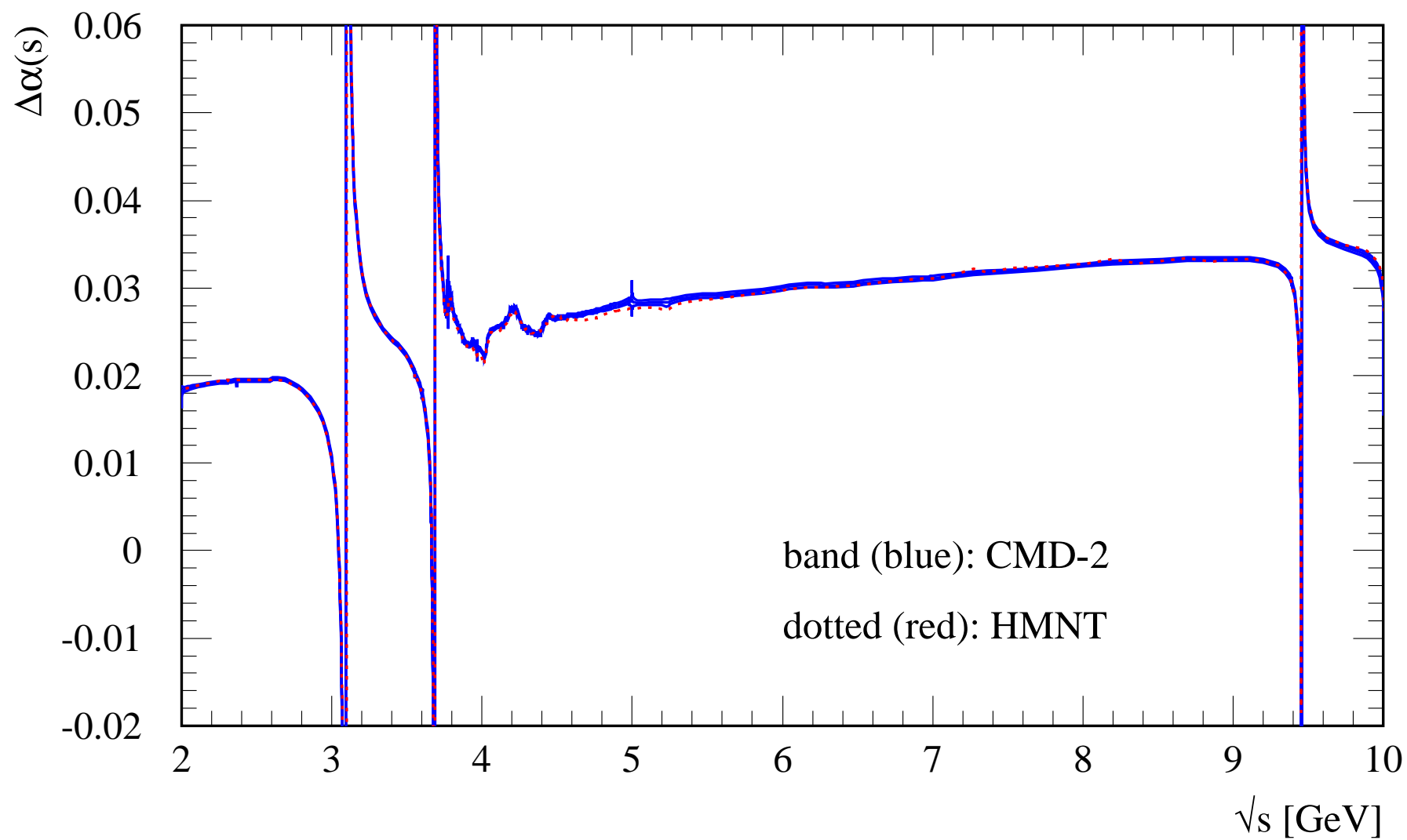
→ with new version big differences (with 2003 version) gone

— smaller differences remain and reflect different choices, smoothing etc.



HLMNT compared to Novosibirsk

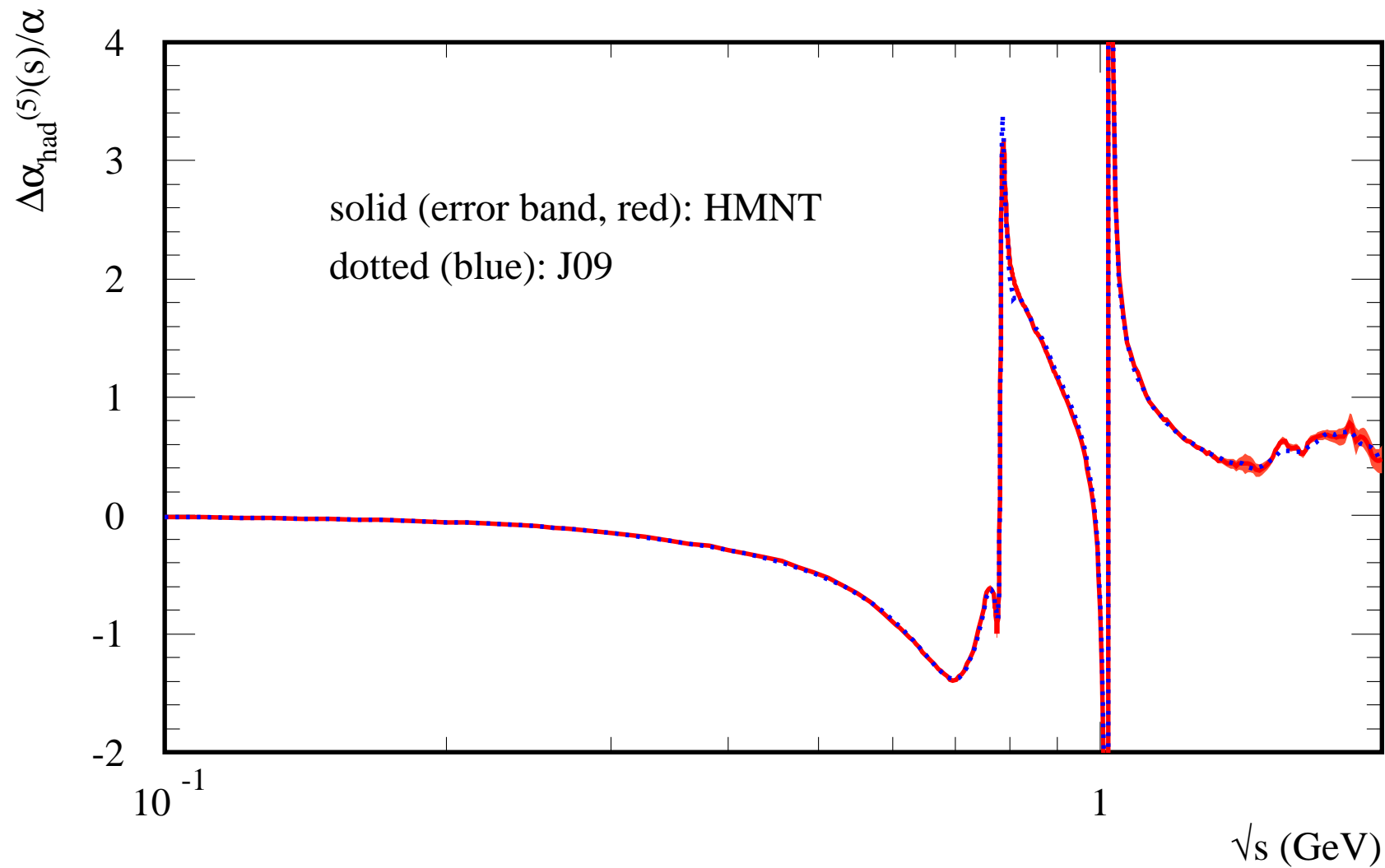
Timelike, $\Delta\alpha(q^2)$



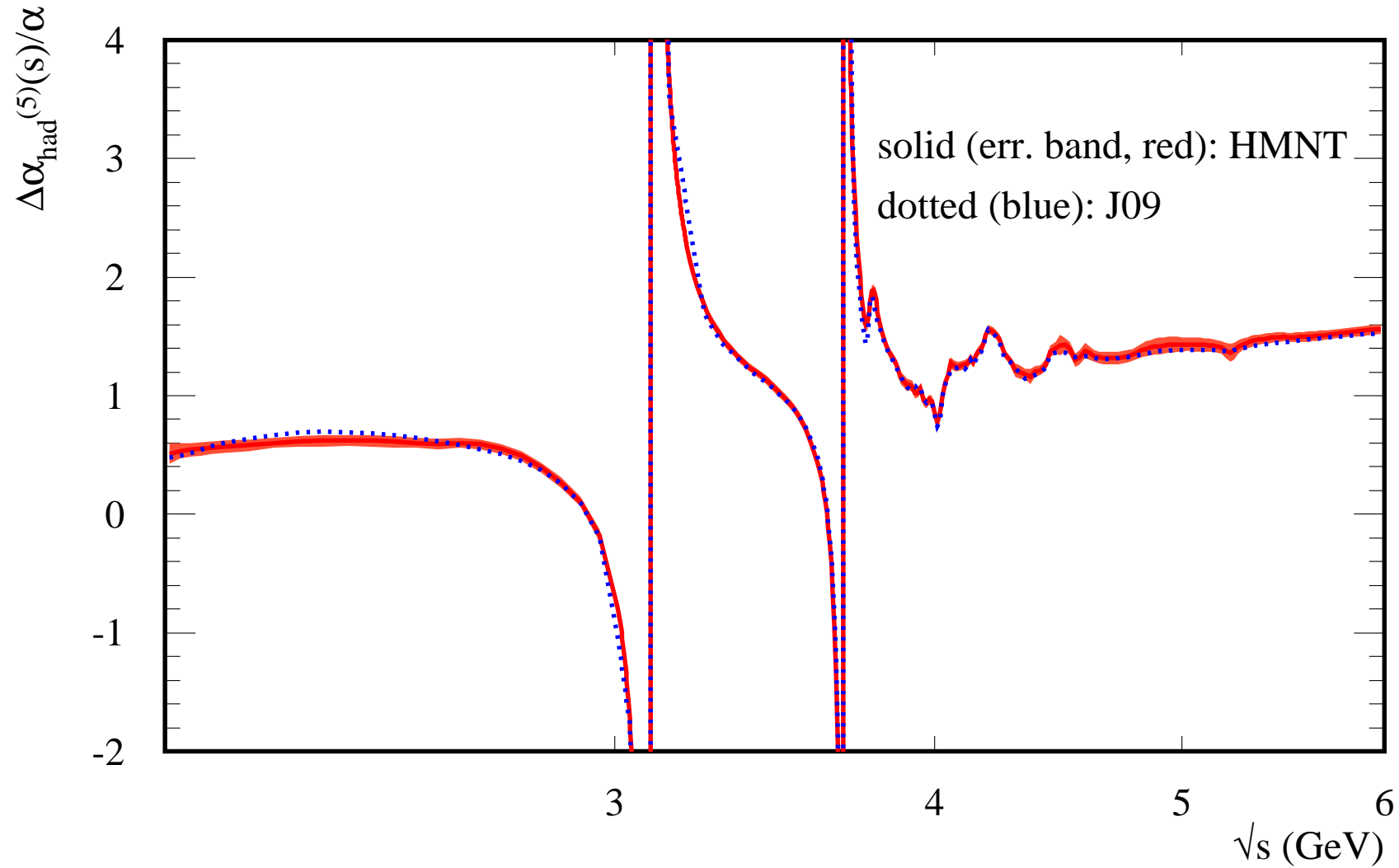
More comparison plots...

HLMNT compared to Fred Jegerlehner's new version: [Detailed look](#)

Low energies: ρ and ϕ

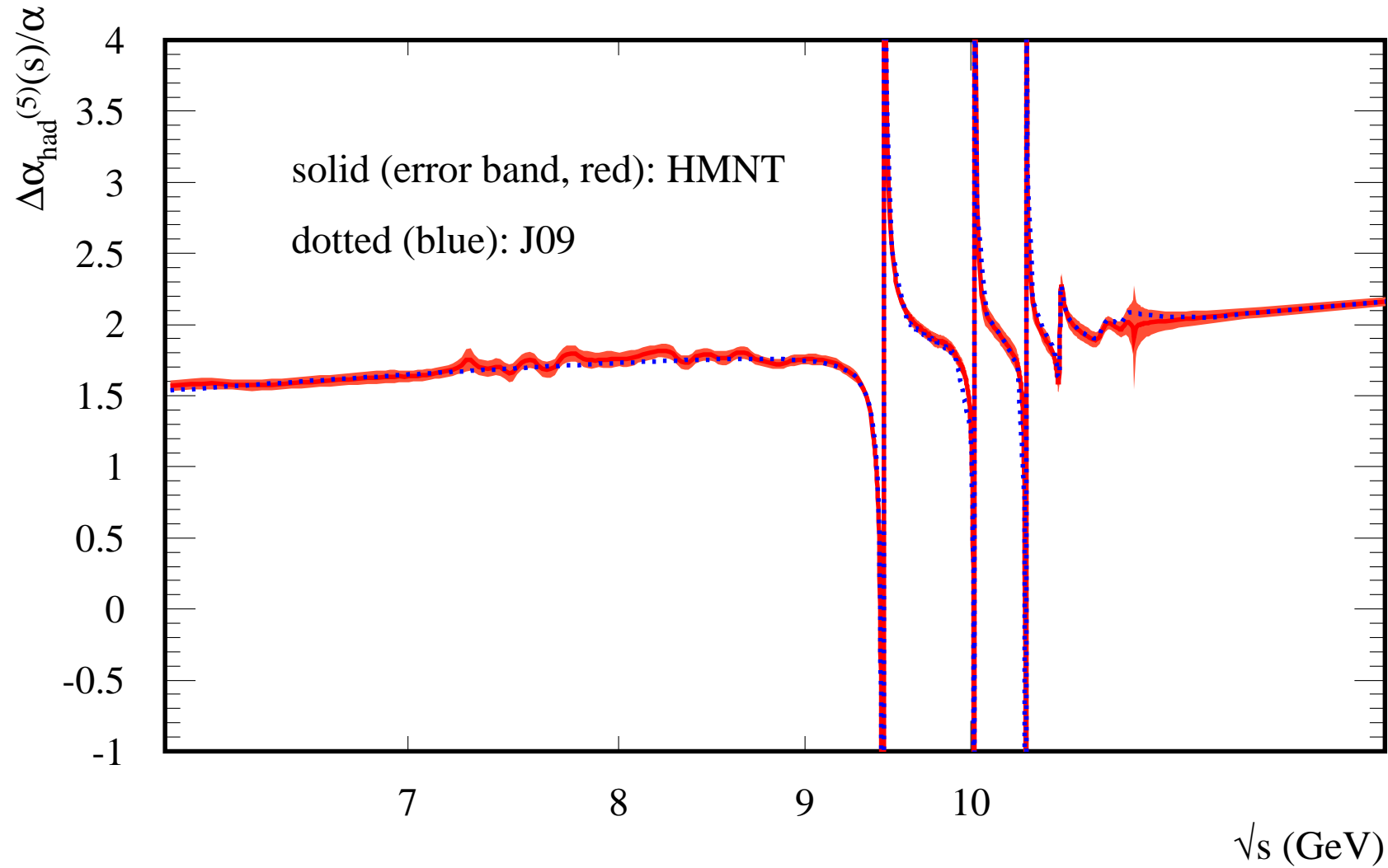


Medium energies: continuum and charm

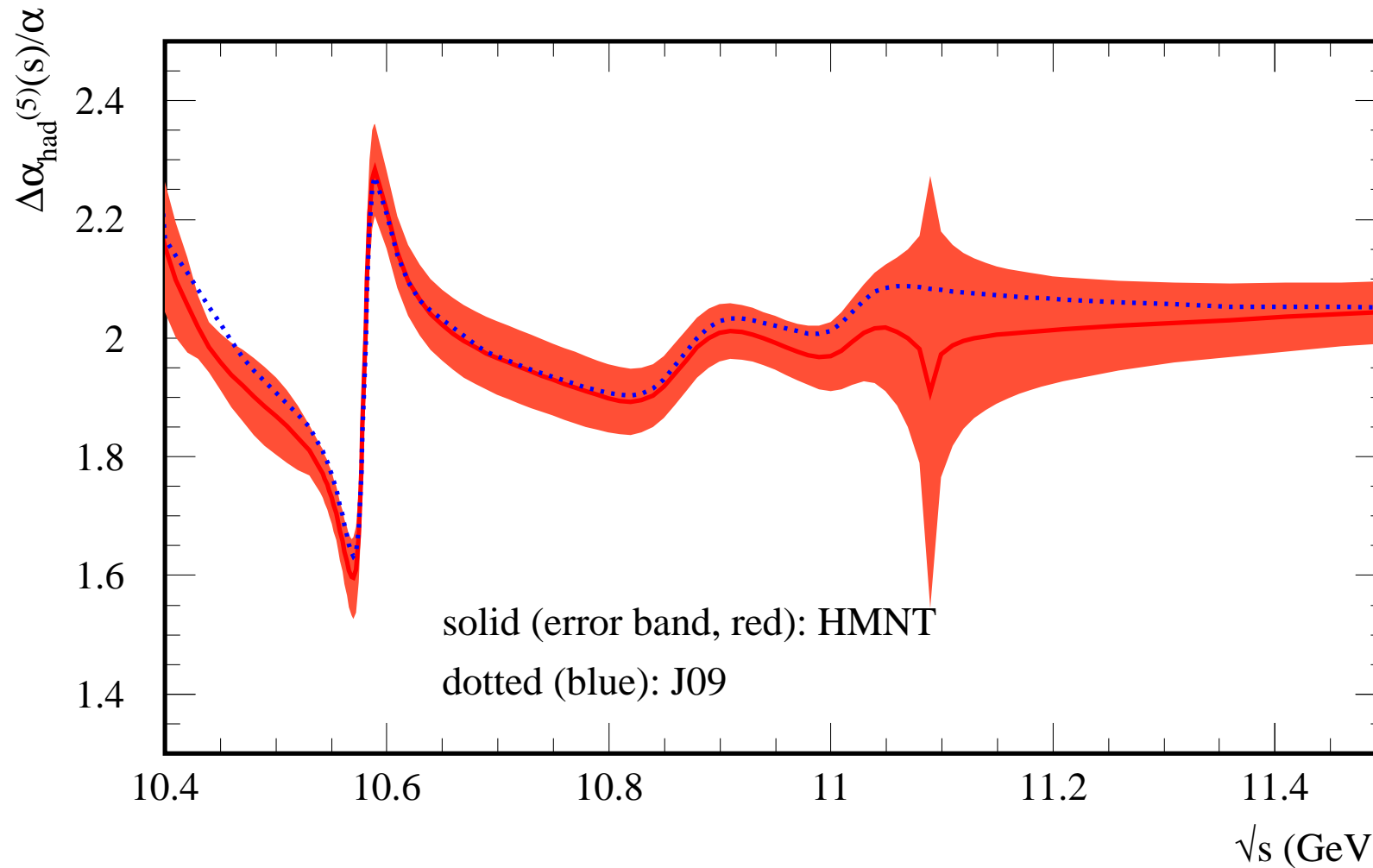


HLMNT compared to Fred Jegerlehner's new version: [Detailed look](#)

Higher energy continuum; bottom



Details of higher $\Upsilon(4, 5, 6 S)$ [10580, 10860, 11020] / open bottom region



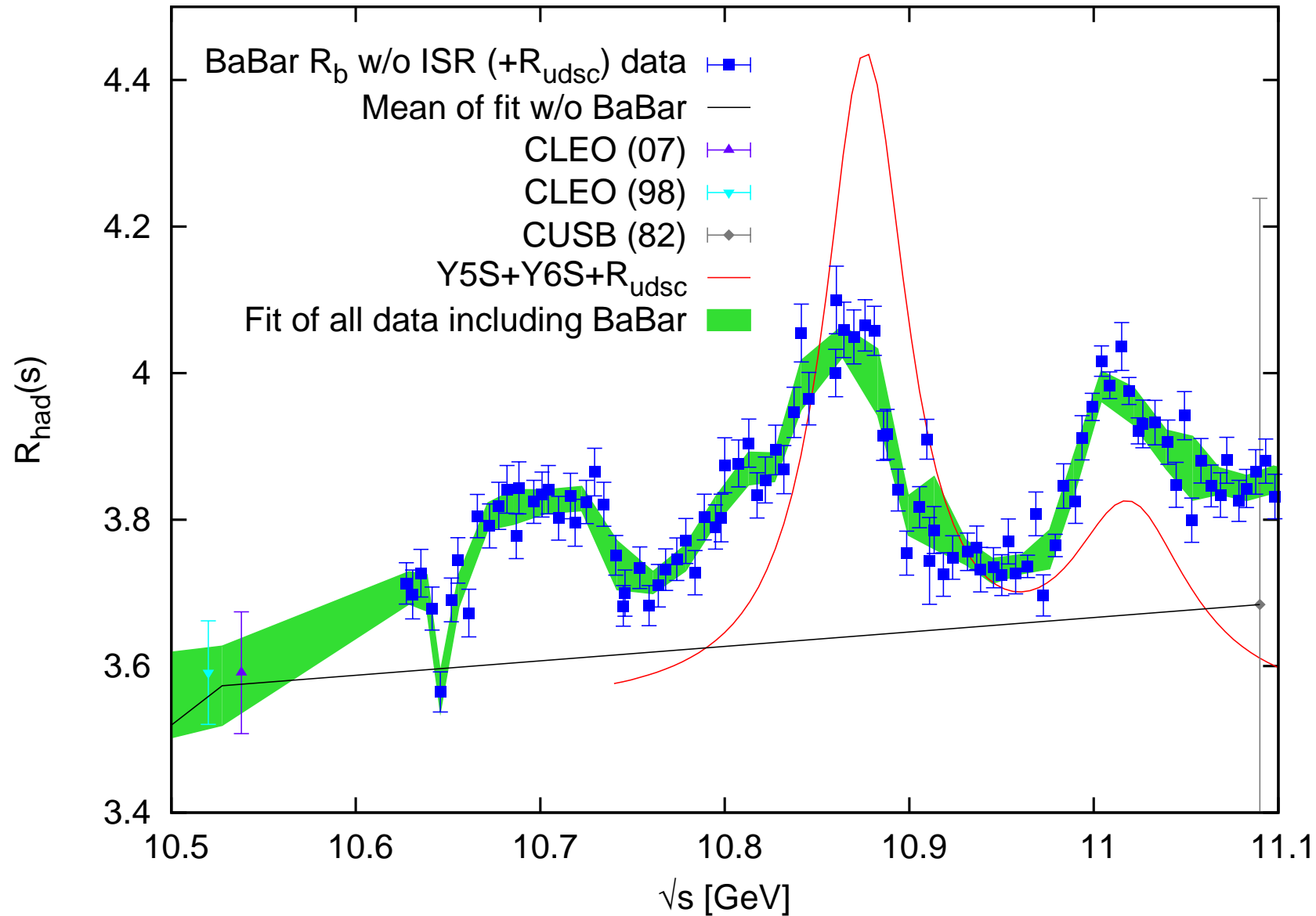
→ HLMNT still to include BaBar's $R_{b\bar{b}}$ data; ISR unfolding.. work in progress ✓

— expected to smooth and improve region above 11 GeV ✓

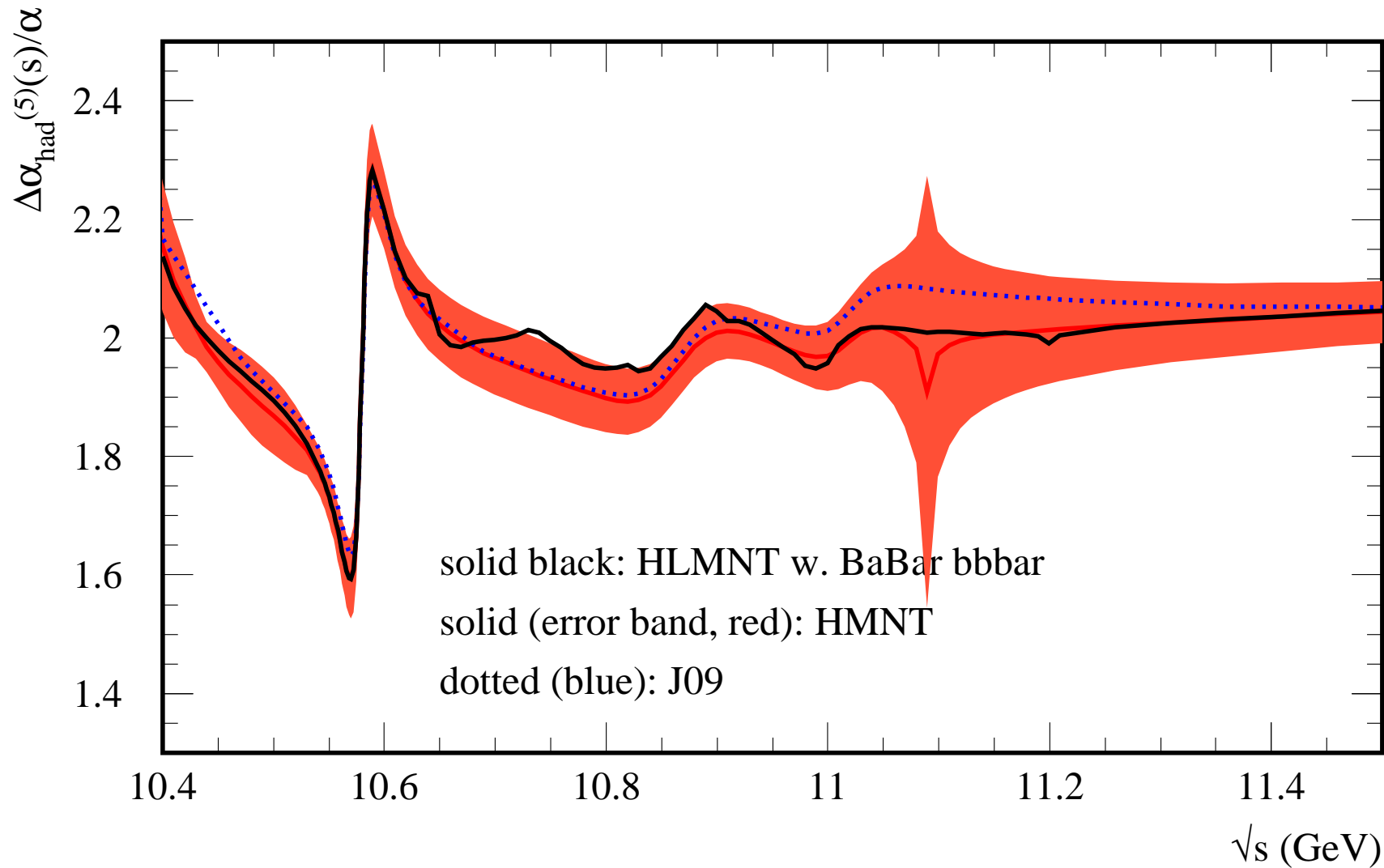
Latest changes

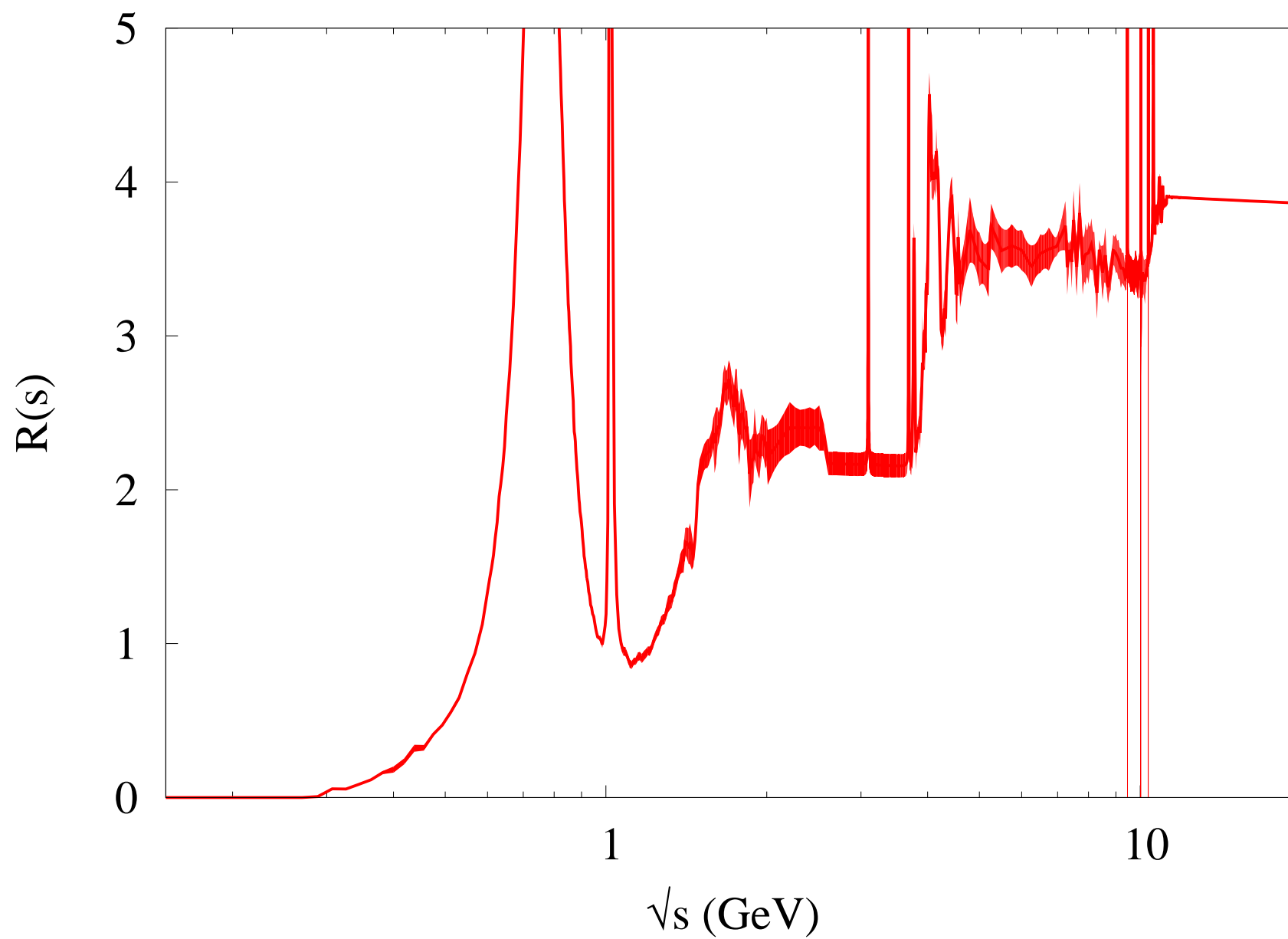
New HLMNT '11!

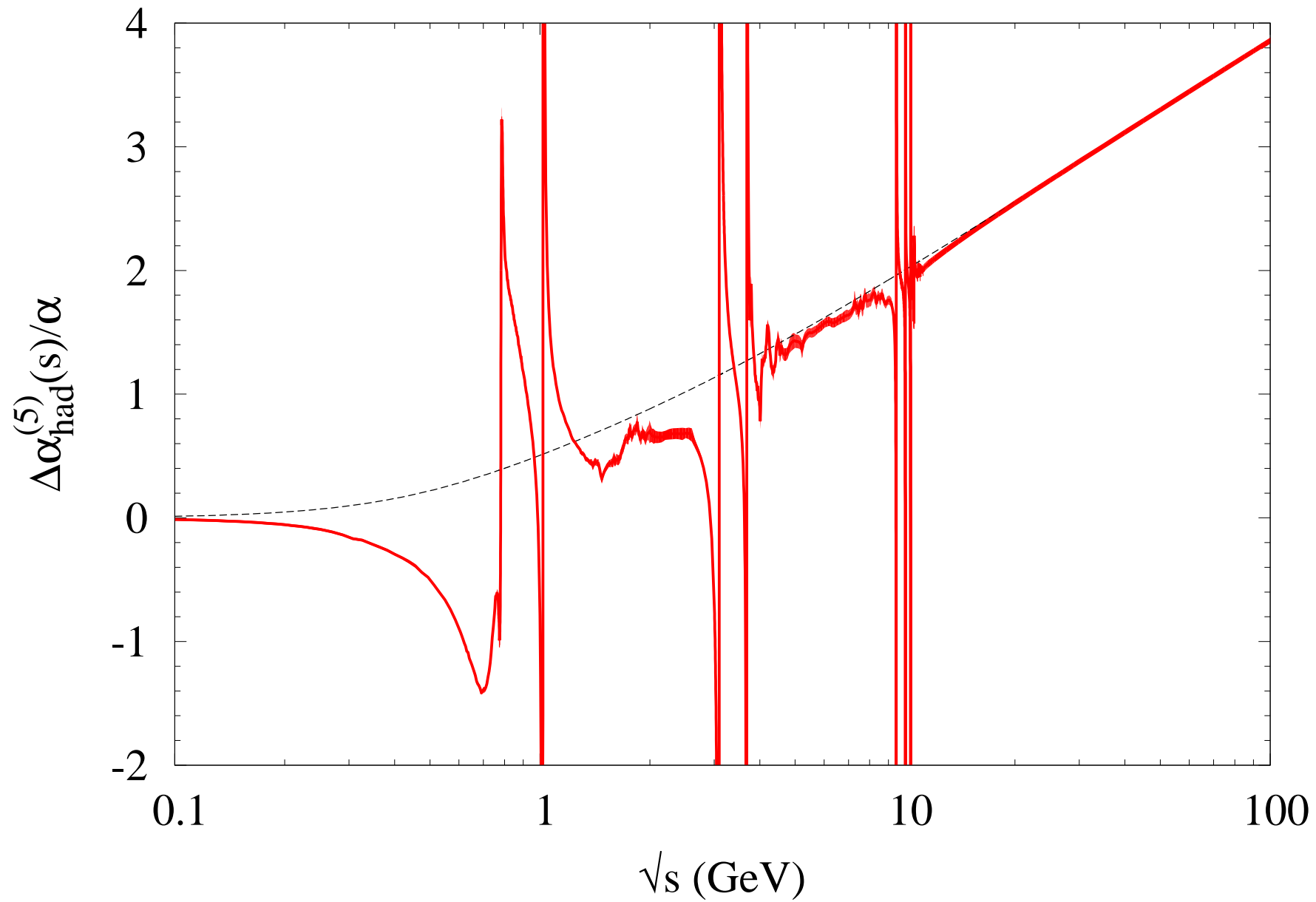
Inclusion of BaBar's $b\bar{b}$ after ISR deconvolution



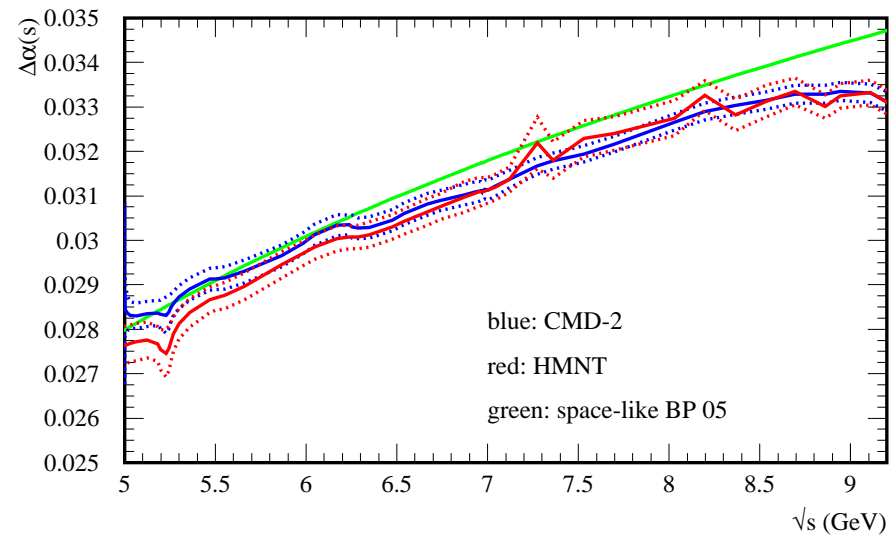
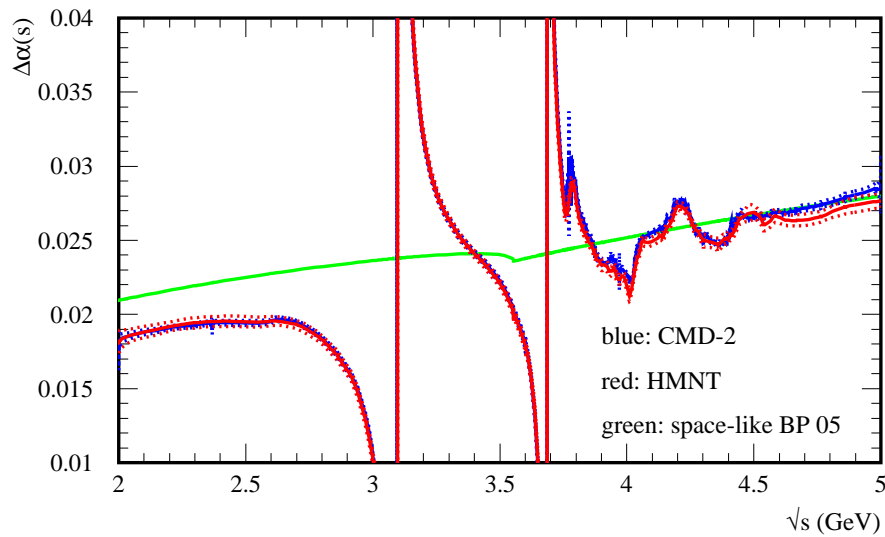
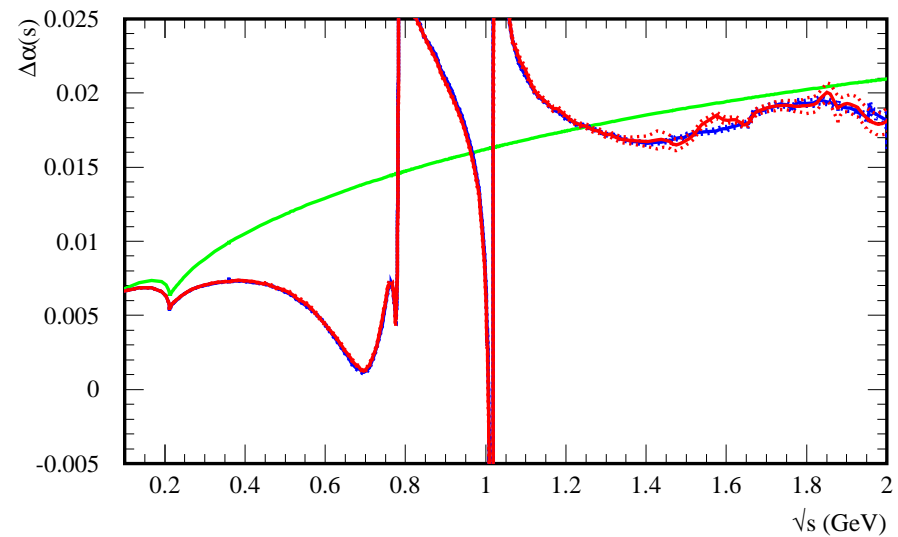
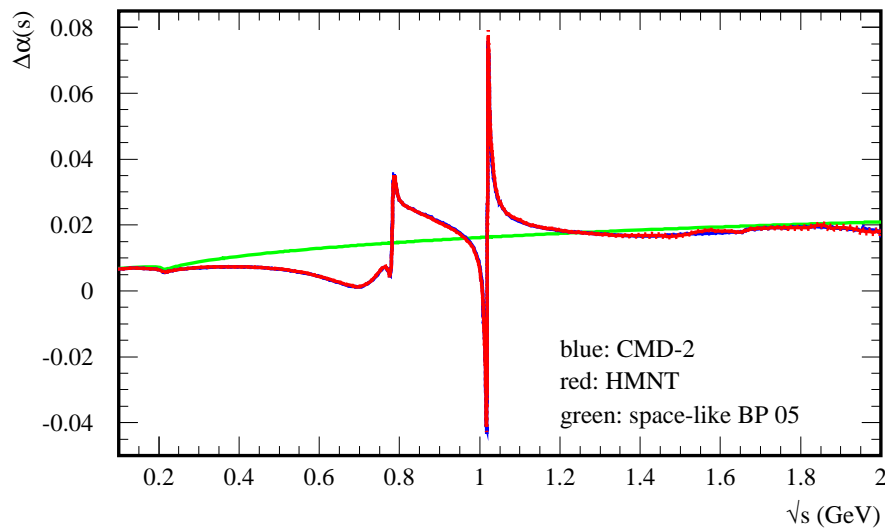
Higher energy continuum; bottom. No smoothing!



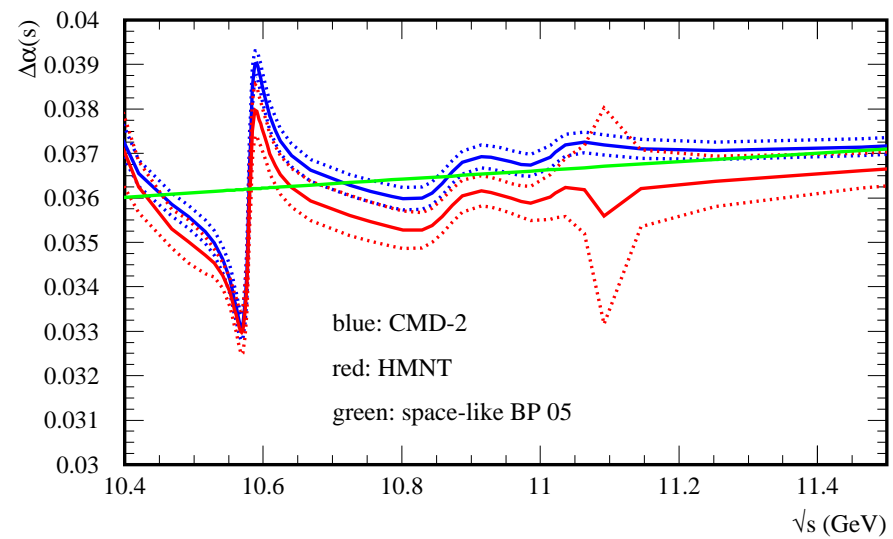
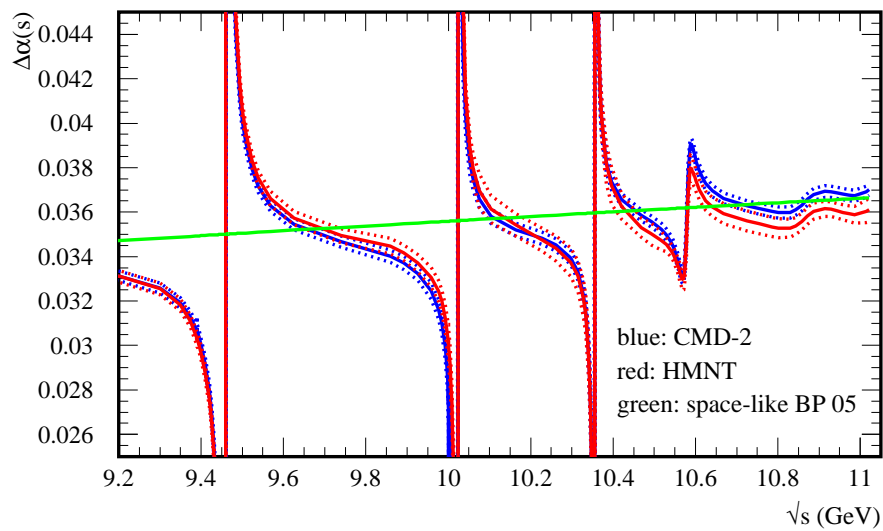
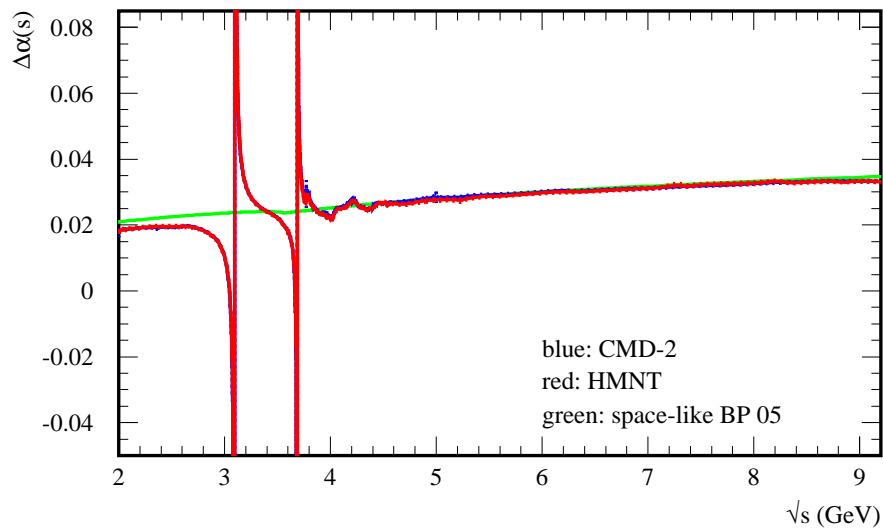




HLMNT compared to CMD-2's routine: Detailed looks



HLMNT compared to CMD-2's routine: three more zooms



Narrow Resonances: treatment and pitfalls

Note 1:

- For $\Delta\alpha$ or $g - 2$, using NR or BW formulae with the dressed width Γ_{ee} for a resonance V is inconsistent and introduces sizeable effects (a few percent).
- Undressing via the smooth spacelike running $\alpha(-M_V)$ comes closer numerically but is not fully correct.
- Use undressing formula

$$\Gamma_{ee}^0 = \frac{[\alpha/\alpha_{\text{no } V}(M_V^2)]^2}{1 + 3\alpha/(4\pi)} \Gamma_{ee},$$

where ‘no V ’ means that the resonance V is *excluded* from the running α .

Note 2:

- Close to narrow resonance energies $|\Pi| \sim 1$ and the summation breaks down
- ↪ Need other formulation, e.g. Breit-Wigner resonance propagator interfering with γ :

$$\left(\frac{\alpha(s)}{s}\right)^2 \rightarrow \frac{1}{s^2} \left| \alpha_{\text{no } V}(s) + \frac{3\Gamma_{ee}M_V}{s - M_V^2 + i\Gamma M_V} \right|^2.$$

Extras:

Comparison of different compilations

- **Timelike** $\alpha(s)$ from Fred Jegerlehner's (2003 routine as available from his web-page)

$$\alpha(s = E^2) = \alpha / \left(1 - \Delta\alpha_{\text{lep}}(s) - \Delta\alpha_{\text{had}}^{(5)}(s) - \Delta\alpha^{\text{top}}(s) \right)$$

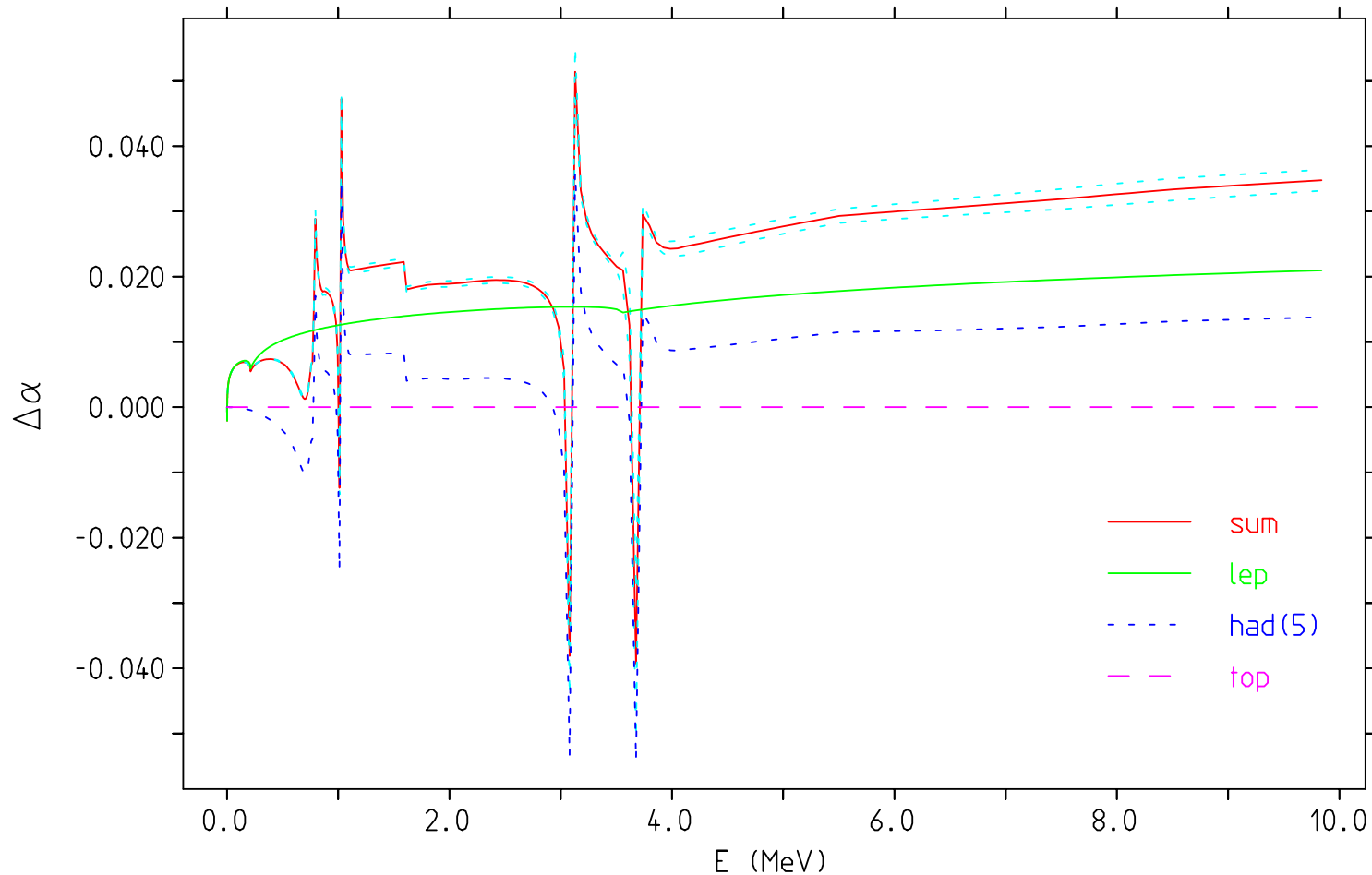
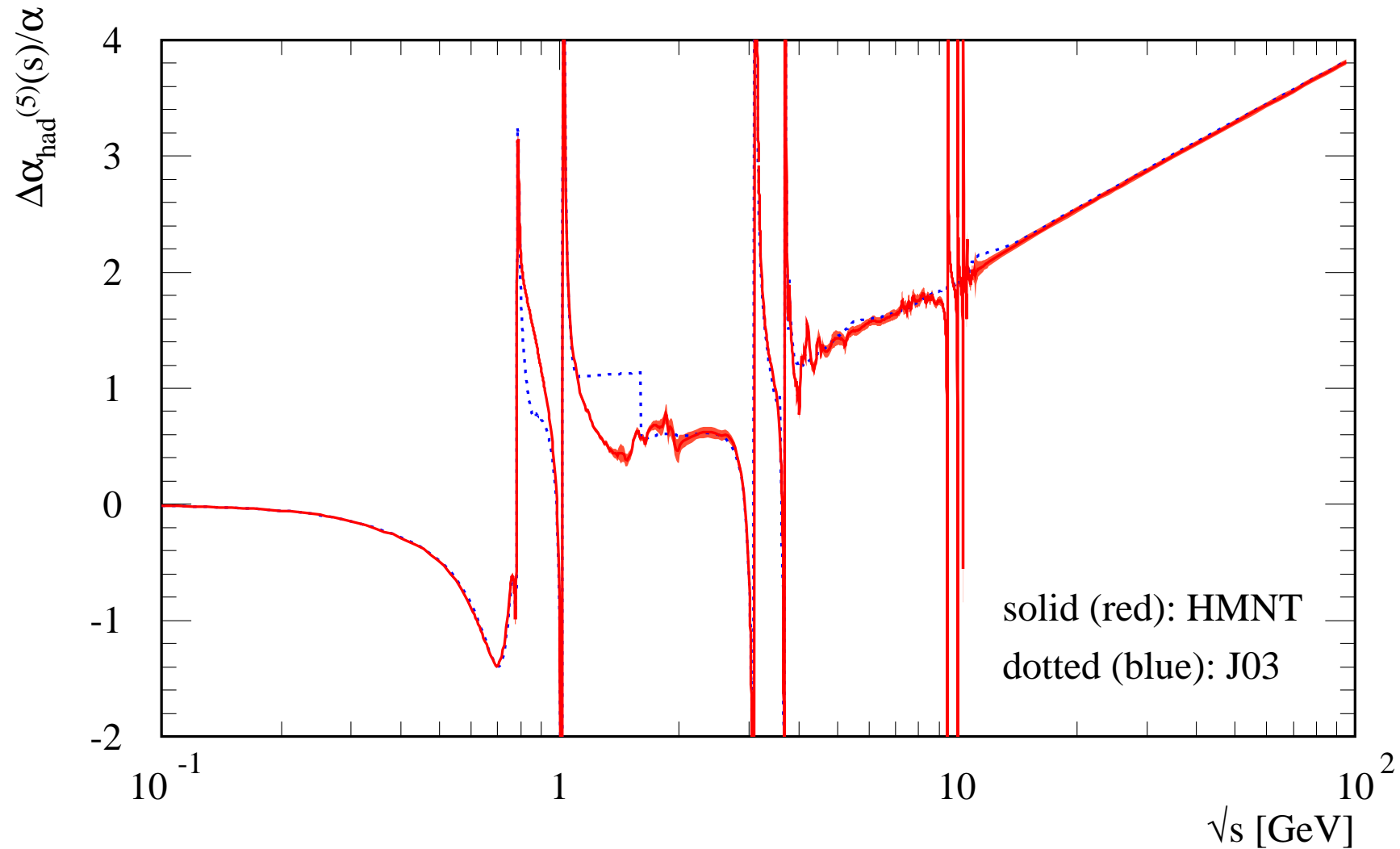


Figure from Fred Jegerlehner

Timelike $\alpha(s = q^2 > 0)$ follows resonance structure:

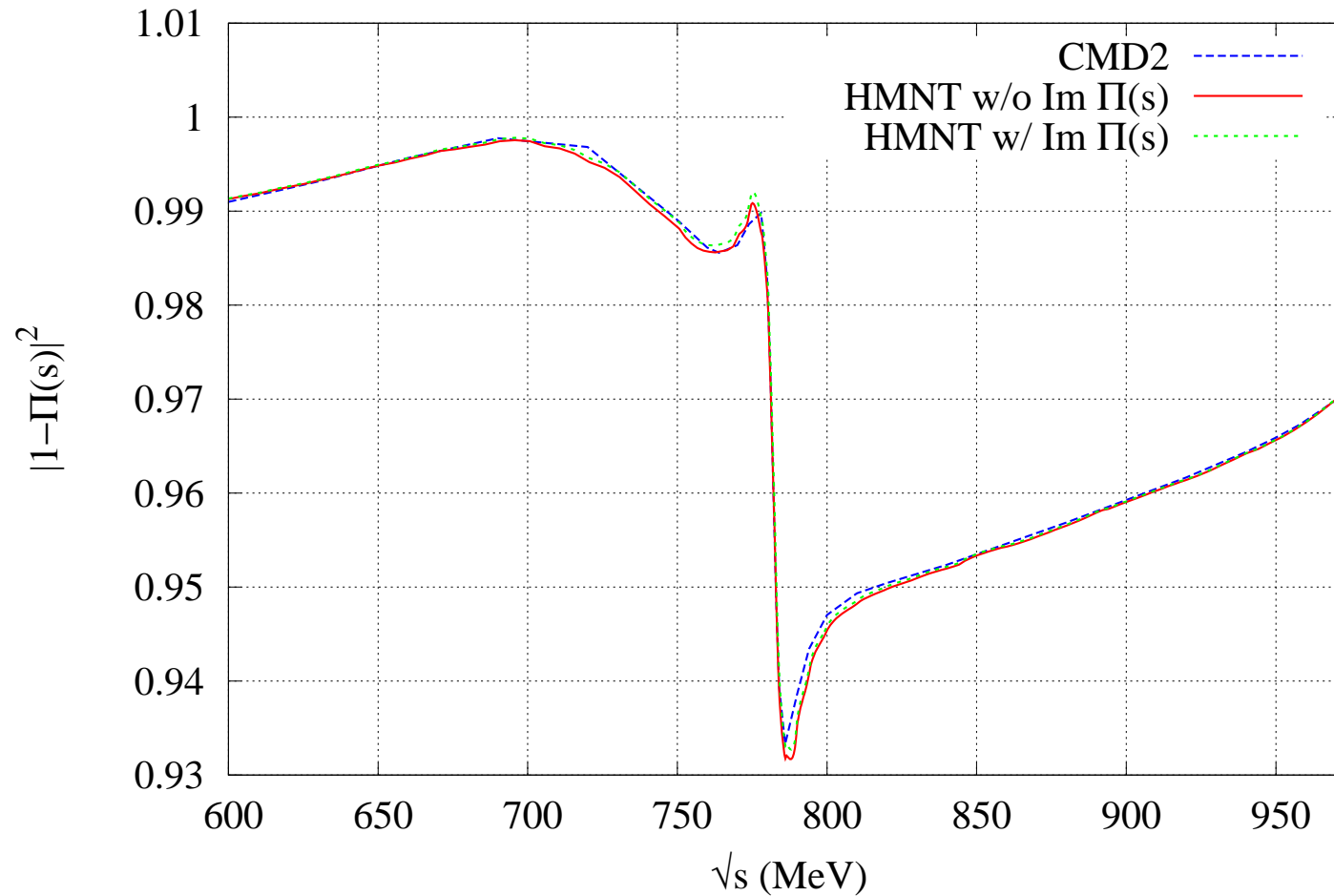


- Step below just a feature of unfortunate grid.
- Difference below 1 GeV not expected from data.

[Comparisons with other parametrisations confirm HMNT.]

- HMNT compared to Novosibirsk's parametrisation

Timelike $|1 - \Pi(s)|^2 \sim (\alpha(s)/\alpha)^2$ in ρ central energy region: A relevant correction!



→ Small but visible differences, as expected from independent compilations.

- What about $\Delta\alpha(M_Z^2)$? Obsolete/dated!!!

→ With the same data compilation of σ_{had}^0 as for $g - 2$ HLMNT find:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = 0.02760 \pm 0.00015 \quad (\text{HLMNT 09 prelim.})$$

$$\text{i.e. } \alpha(M_Z^2)^{-1} = 128.947 \pm 0.020 \quad [\text{HMNT '06: } \alpha(M_Z^2)^{-1} = 128.937 \pm 0.030]$$

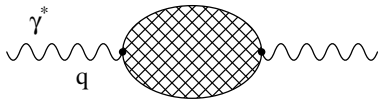
Earlier compilations:

Group	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	remarks
Burkhardt+Pietrzyk '05	0.02758 ± 0.00035	data driven
Troconiz+Yndurain '05	0.02749 ± 0.00012	pQCD
Kühn+Steinhauser '98	0.02775 ± 0.00017	pQCD
Jegerlehner '08	0.027594 ± 0.000219	data driven/pQCD
$(M_0 = 2.5 \text{ GeV})$	0.027515 ± 0.000149	Adler fct, pQCD
HMNT '06	0.02768 ± 0.00022	data driven

$$\text{Adler function: } D(-s) = \frac{3\pi}{\alpha} s \frac{d}{ds} \Delta\alpha(s) = -(12\pi^2) s \frac{d\Pi(s)}{ds}$$

allows use of pQCD and minimizes dependence on data.

The running QED coupling $\alpha(M_Z^2)$... and the Higgs mass



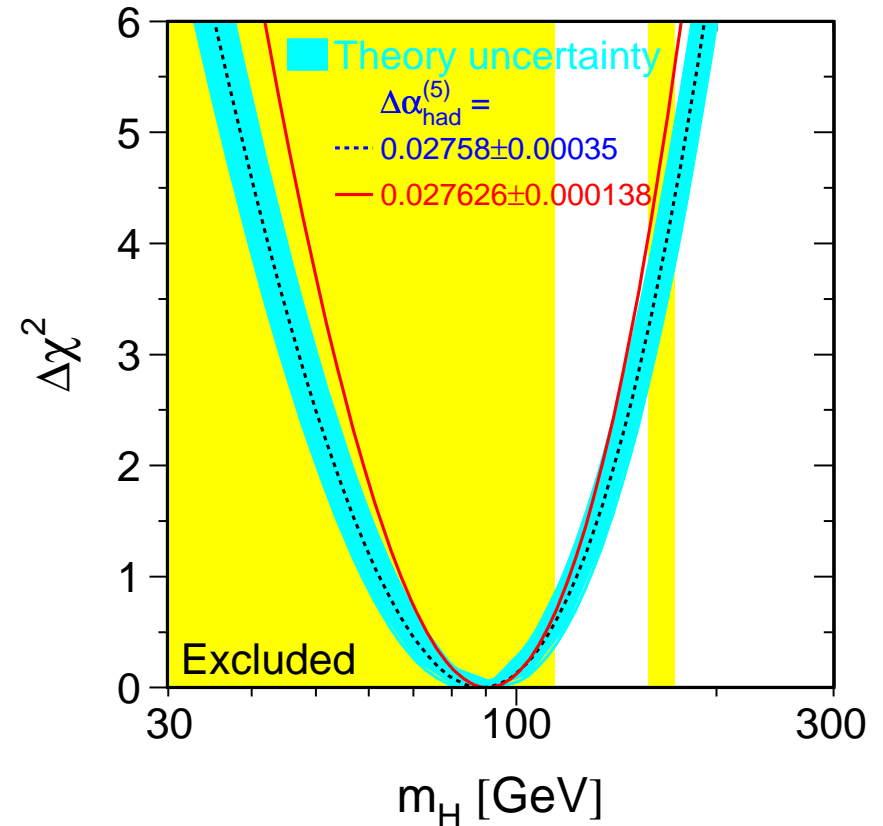
- Vacuum polarisation leads to the ‘running’ of α from $\alpha(q^2 = 0) = 1/137.035999084(51)$ to $\alpha(q^2 = M_Z^2) \sim 1/129$
- $\alpha(q^2) = \alpha / (1 - \Delta\alpha_{\text{lep}}(q^2) - \Delta\alpha_{\text{had}}(q^2))$
- Again use of a dispersion relation:

$$\Delta\alpha_{\text{had}}^{(5)}(q^2) = -\frac{\alpha q^2}{3\pi} P \int_{s_{\text{th}}}^{\infty} \frac{R_{\text{had}}(s) ds}{s(s-q^2)}$$
- **Hadronic uncertainties** \rightsquigarrow α the least well known EW param. of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$!
- We find (HLMNT '11):

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = 0.027626 \pm 0.000138$$

i.e. $\alpha(M_Z^2)^{-1} = 128.944 \pm 0.019$
- HLMNT-routine for $\alpha(q^2)$ & $R_{\text{had}}^{\text{data}}$ available

Fit of the SM Higgs mass: LEP EWWG



Fit and Fig. thanks to M. Grünewald

$$\hookrightarrow M_H = 91_{-23}^{+30} \text{ GeV}$$

$$[m_t = (173.3 \pm 1.1) \text{ GeV}]$$

Outlook

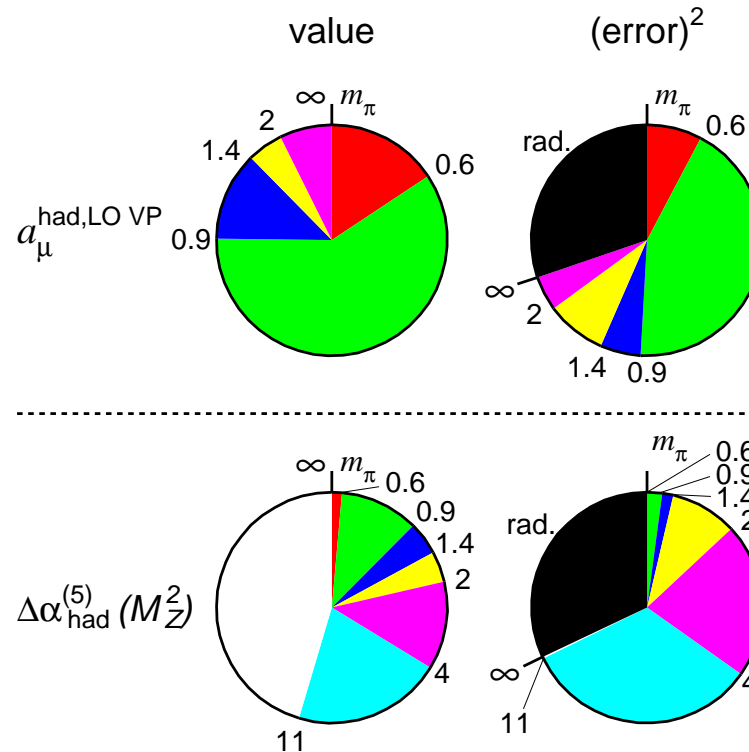
► Further improvements

Hadronic VP still the biggest error in a_μ^{SM} , soon L-by-L...

Pie diagrams for contr. to a_μ and $\alpha(M_Z)$ and their errors²

Prospects for further squeezing errors:

- More Rad. Ret. in progress at KLOE
- Great opportunity for KLOE-2, BELLE, **Super $\tau - c$** , in a few years **SUPER-Bs**, also strong case for DAFNE-HE
- Big improvement envisaged with CMD-3 and SND at VEPP2000
- Higher energies: BES-III at BEPCII in Beijing is on; KEDR at VEPP-4M



► New $g - 2$ experiments at Fermilab and J-PARC.

→ talks by G. Venanzoni, N. Saito

► Will a_μ^{SM} match the planned accuracy? \rightsquigarrow L-by-L may become the limiting factor!