Betatron radiation

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Outline

- **X ray sources**
- **Plasma accelerator**

Context

- **Xrays from plasma (Betatron Radiation)**
- **Electron motion (wiggler/undulator analogy)**

Theory

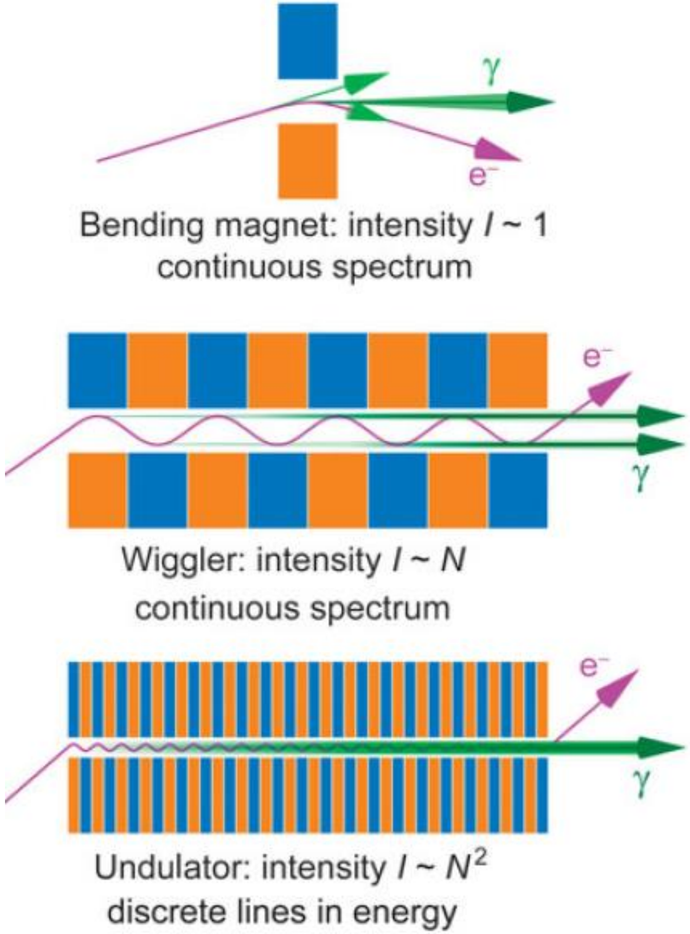
- **Radiation properties**
- **Single electron**
- **Electron distribution**
- **3d theoretical model**
- **Conference and summer school**
- **Perspectives**

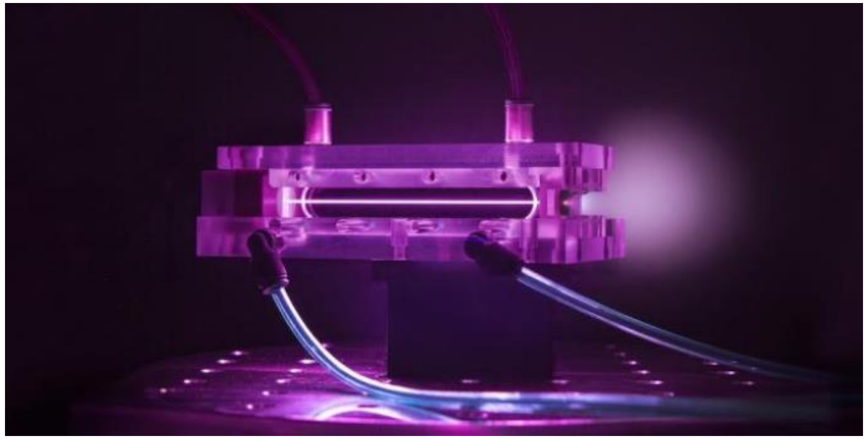
My work

X ray sources

Emission stimulation

Synchrotron



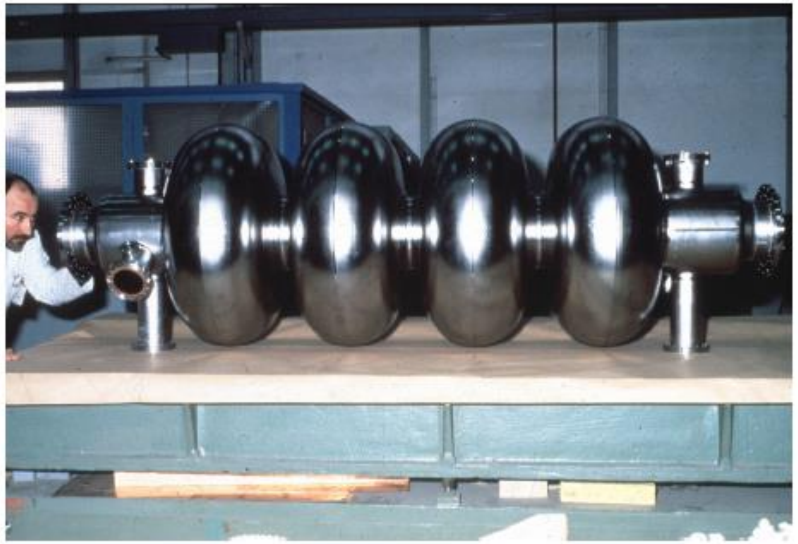


Plasma

Free Electron Laser

Plasma accelerators

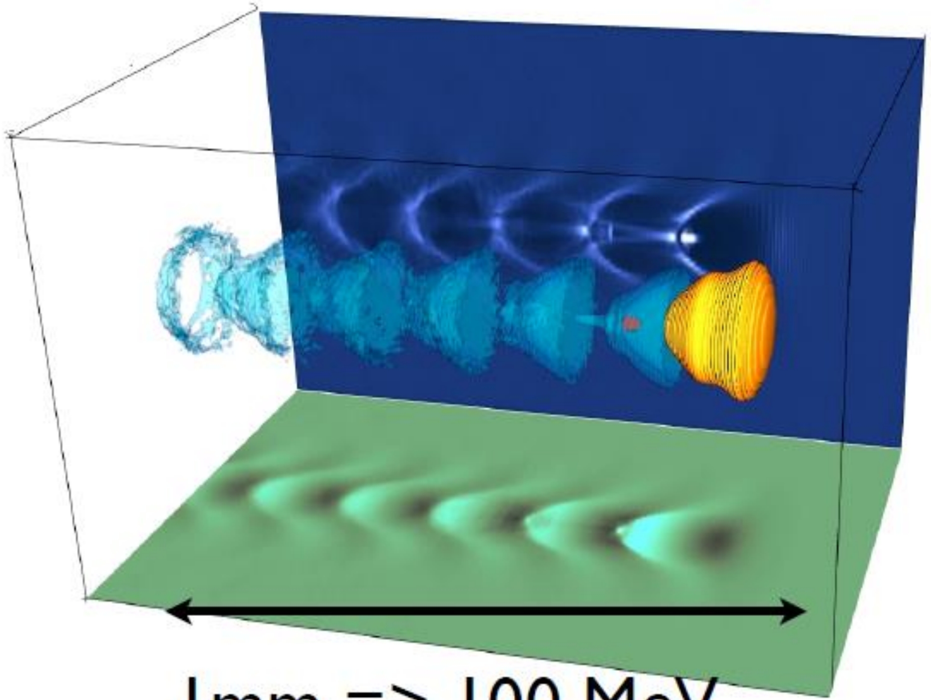
RF Cavity



1 m => 100 MeV Gain

Electric field < 100 MV/m

Plasma Cavity

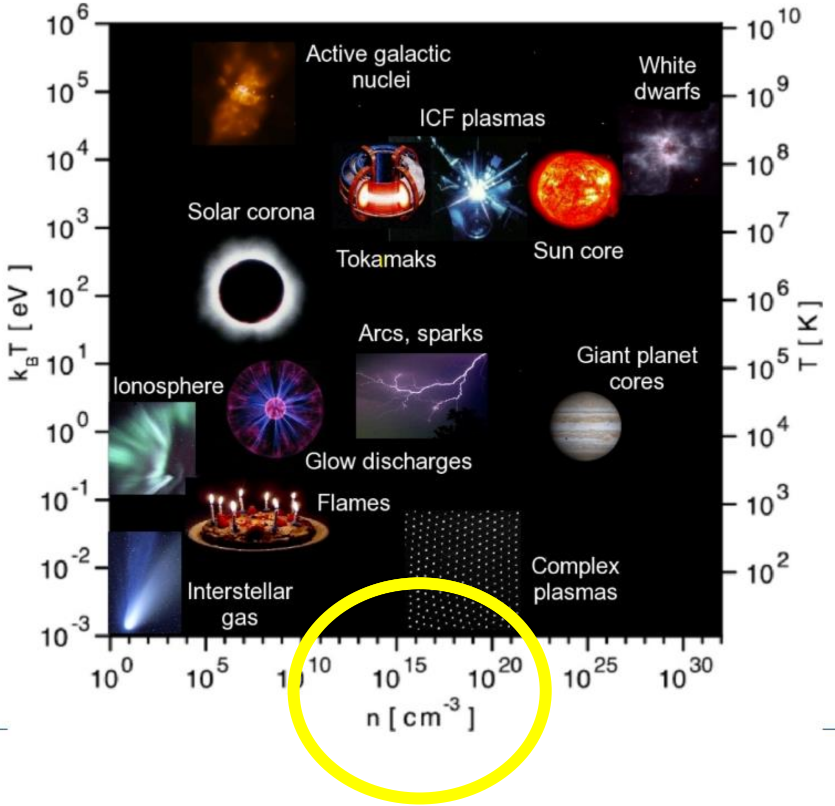


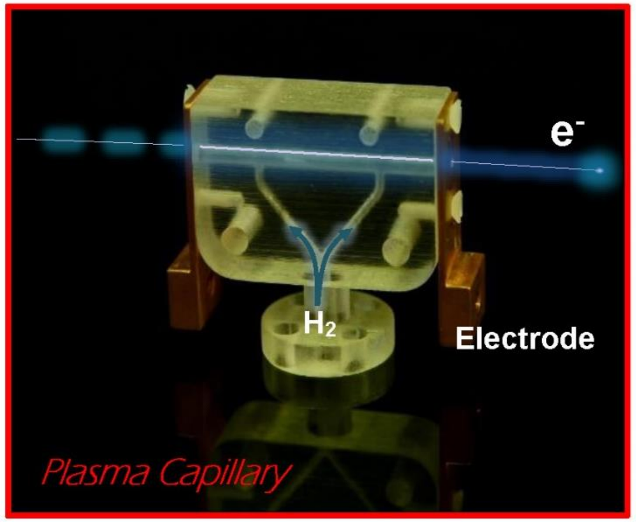
1mm => 100 MeV

Electric field > 100 GV/m

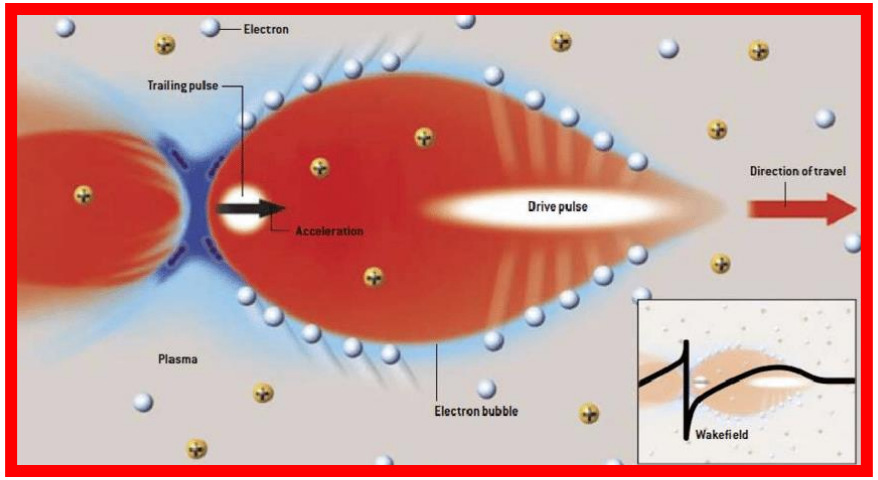
V. Malka *et al.*, Science **298**, 1596 (2002)

Plasma accelerator parameters





LWFA/PWFA



$$n_p = 10^{17}\ cm^{-3}$$

$$\omega_p = 2\times 10^{13}\ Hz$$

$$E_0 = 30\ \frac{GV}{m}$$

$$\lambda_p = 100\ \mu m\ (300\ fs)$$

$$\sigma_{zD} \approx 25\ \mu m\ (75\ fs)$$

$$\sigma_{xD} \approx 2\ \ \mu m$$

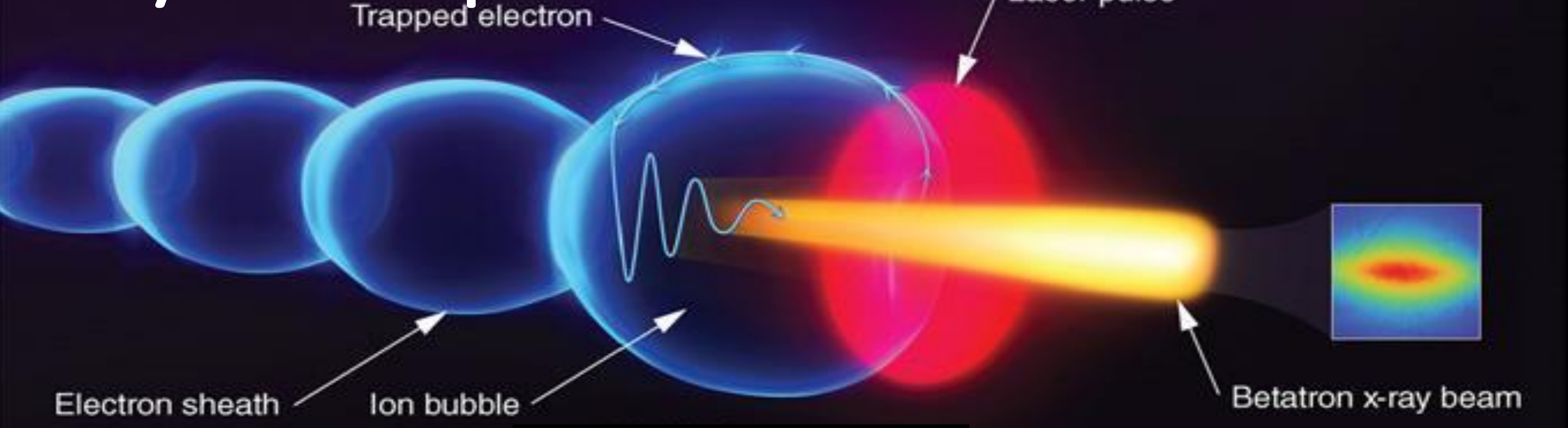
Plasma frequency

$$E_{max}\left[\frac{V}{m}\right] = \frac{m_e c \omega_p}{e}$$

Plasma frequency

$$\omega_p = \sqrt{e^2 n_e / m_e \epsilon_0}$$

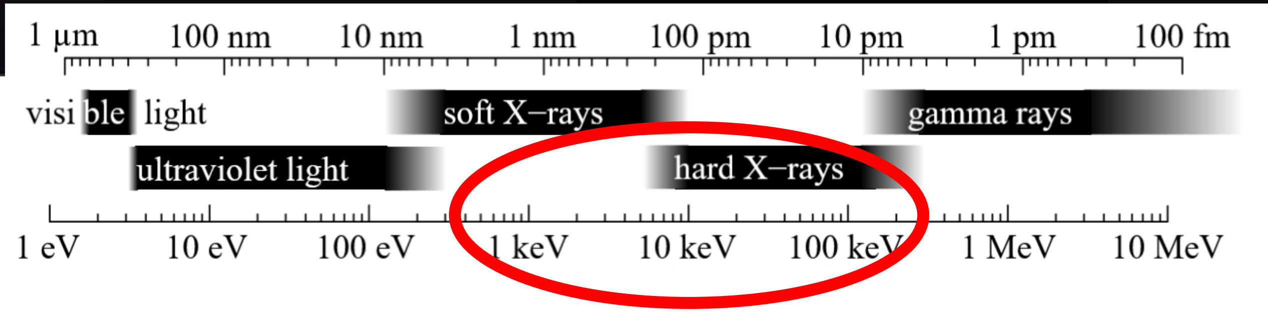
Xrays from plasma



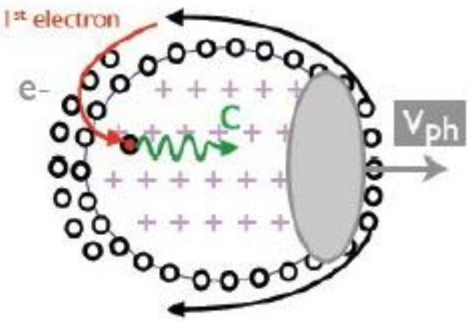
[redacted]

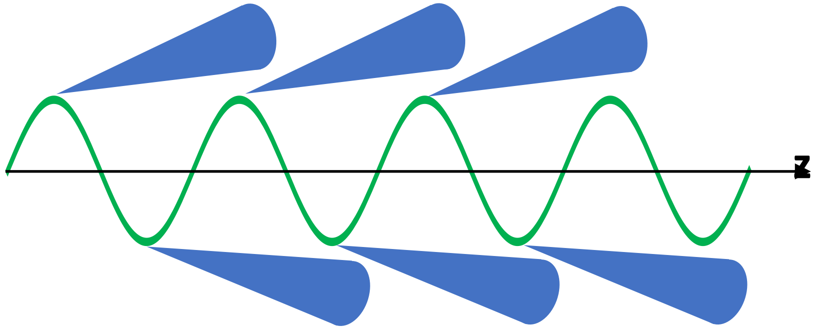
$$\omega_p = \sqrt{e^2 n_e / m_e \epsilon_0}$$

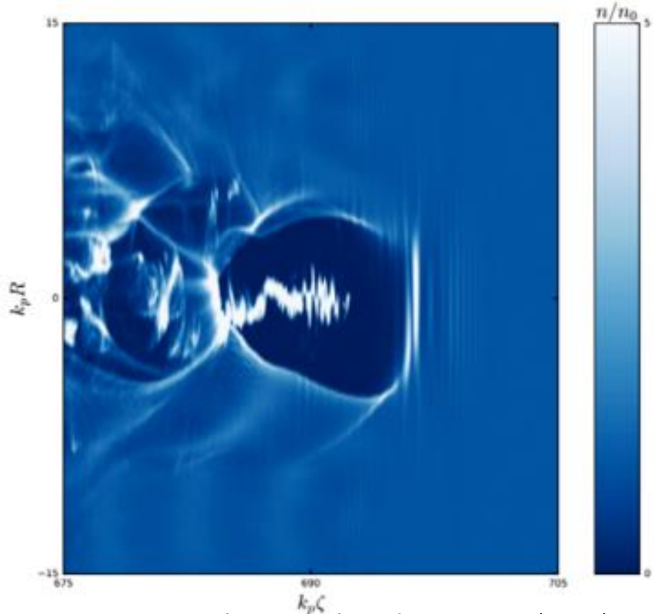
$$\omega_\beta = \frac{\omega_p}{\sqrt{2\gamma}}$$



Refs.
F.Albert et al. “Laser wakefield accelerator based light sources: Potential applications and Requirements “, Plasma Phys. Control. Fusion







Refs.
(Left Fig.) Curcio, A., et al. “First measurements of betatron radiation at FLAME laser facility." *Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms* 402 (2017): 388-392

Wiggler / Undulator

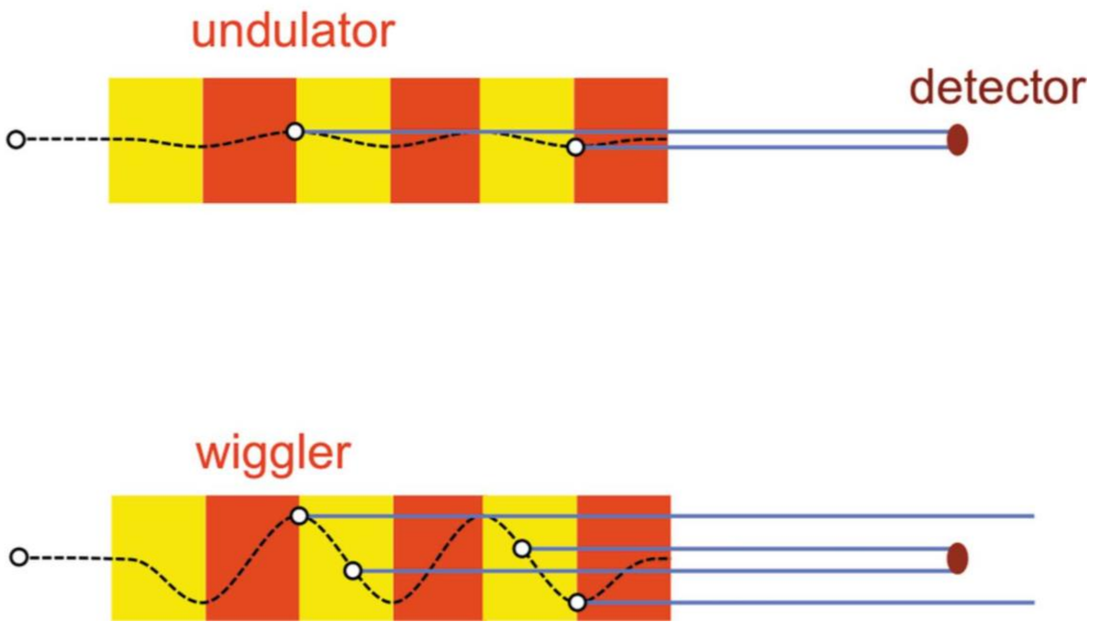
Equation of motion (1D)

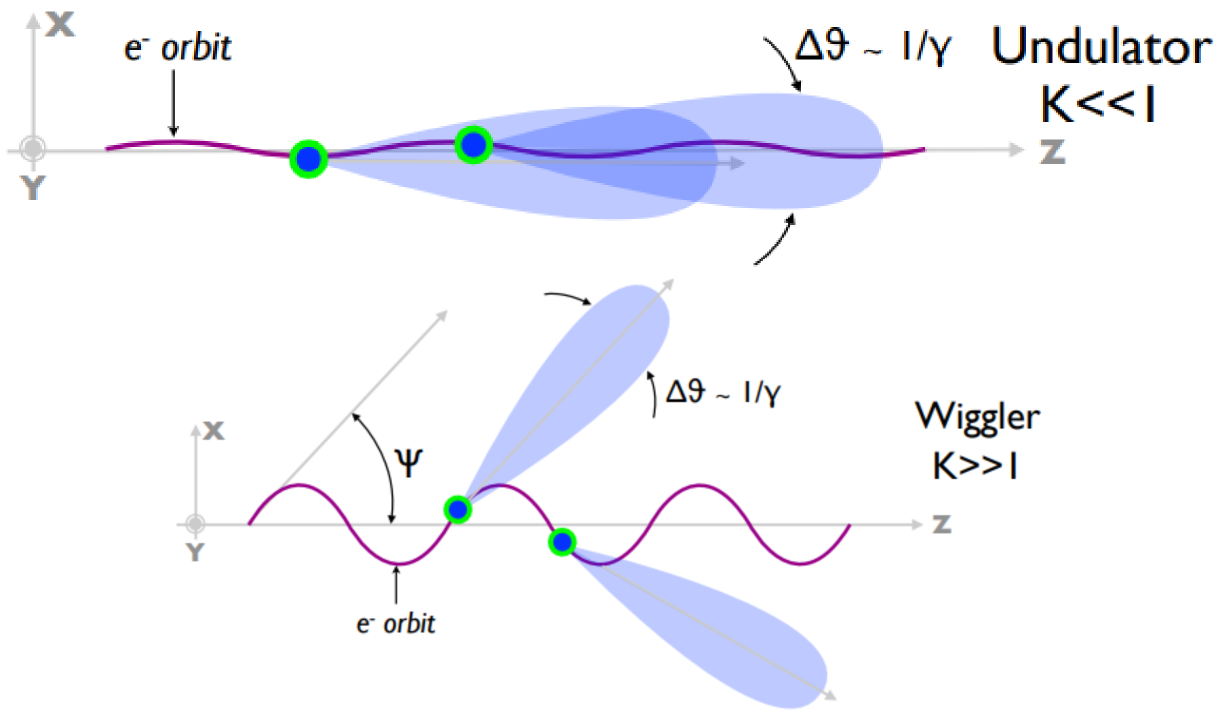
$$x = r\sin(k_\beta ct)$$

$$z = z_0 + \beta_z\left(1 - \frac{r^2 k_\beta^2}{4\beta_z^2}\right)ct - \frac{r^2 k_\beta^2}{8\beta_z^2}\cos(2k_\beta ct)$$

Strength parameter

$$K = \frac{eB\lambda_u}{2\pi mc}$$





Refs.
Corde, Sébastien, et al. "Femtosecond x rays from laser-plasma accelerators." *Reviews of Modern Physics* 85.1 (2013): 1.

Betatron radiation

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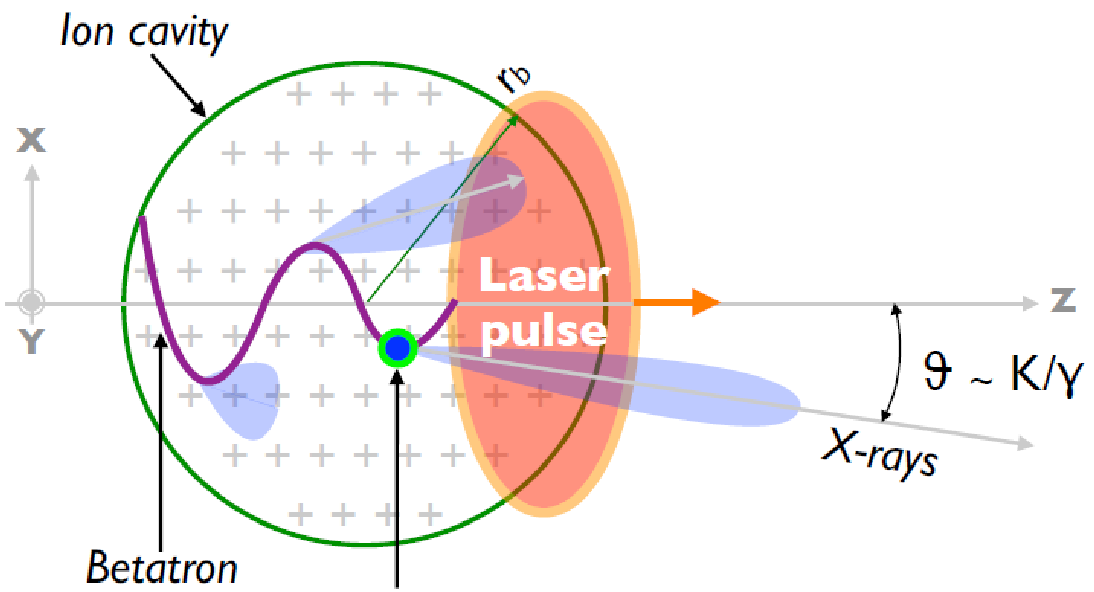
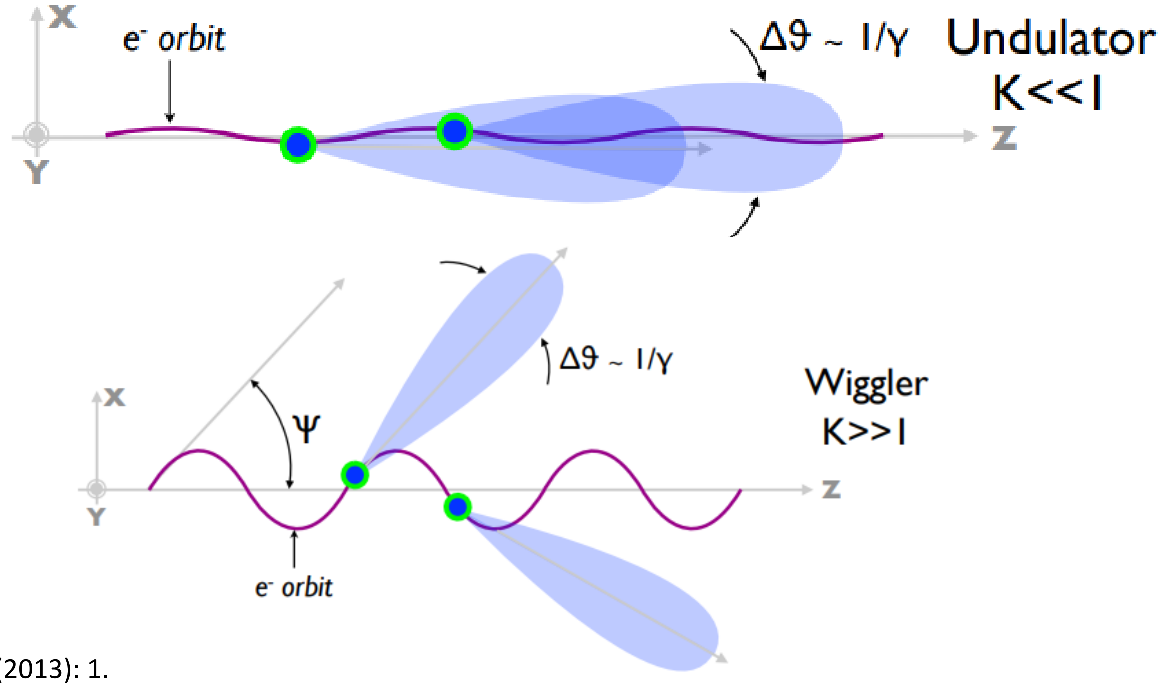
Betatron strength parameter

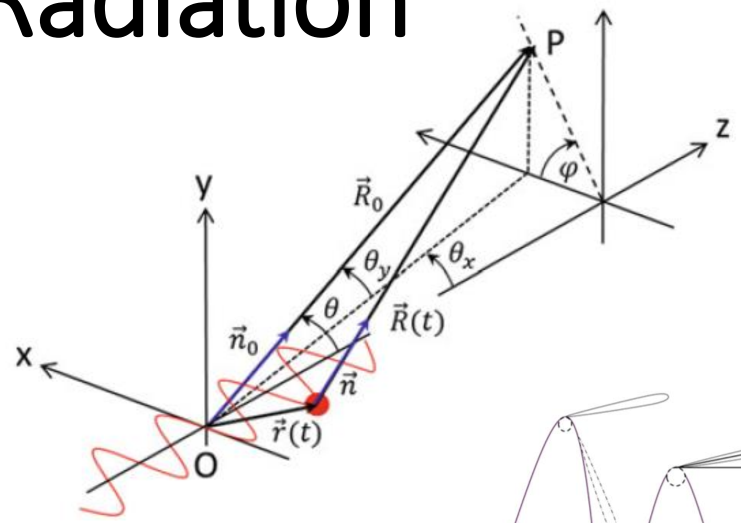
$$K_\beta = \gamma r k_\beta$$



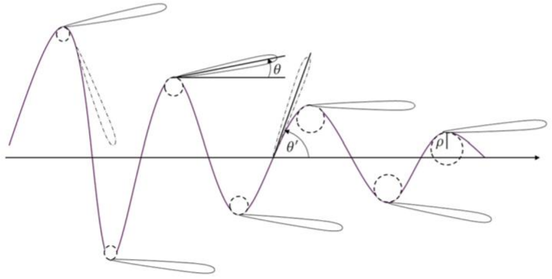


Refs.
Corde, Sébastien, et al. "Femtosecond x rays from laser-plasma accelerators." *Reviews of Modern Physics* 85.1 (2013): 1.

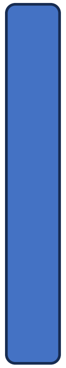
Radiation



$$\frac{\mathrm{d}^2 I}{d\omega\,\mathrm{d}\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-T/2}^{T/2} \boldsymbol{n}\times(\boldsymbol{n}\times\boldsymbol{\beta})\, e^{i\omega\left(t-\frac{\boldsymbol{n}\cdot\boldsymbol{r}}{c}\right)}\,\mathrm{d}t\right|^2$$



Detector



$$\frac{d^2 I}{d\omega\Omega}(\theta=0) = \sum_{n=1}^{\infty}\frac{e^2}{\pi\epsilon_0 c}\frac{\omega}{\omega_n}\frac{\gamma^2 N_\beta^2 F_n R_n}{1+K_\beta^2/2}$$

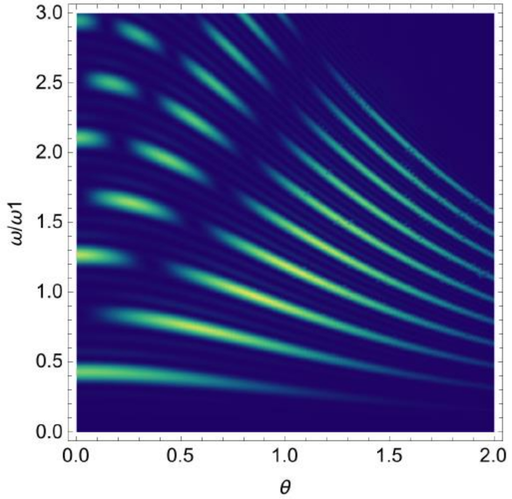
Resonant function

$$R_n = \frac{sin^2[n\pi N_\beta\,(\omega/\omega_n - 1)]}{[n\pi N_\beta\,(\omega/\omega_n - 1)]^2}$$

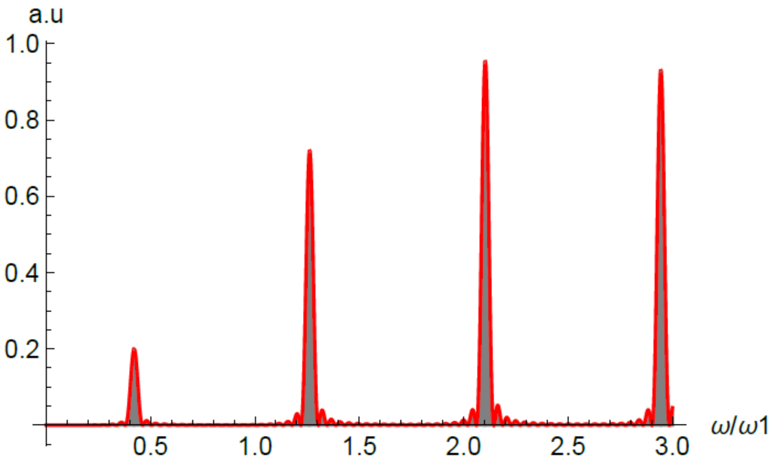
n-frequency

$$\omega_n = 2n\omega_\beta\frac{\gamma^2}{1+K_\beta^2}$$

$$\frac{\mathrm{d}^2 I}{d\omega\,\mathrm{d}\Omega}$$



$$\frac{\mathrm{d}^2 I}{d\omega\,\mathrm{d}\Omega}$$

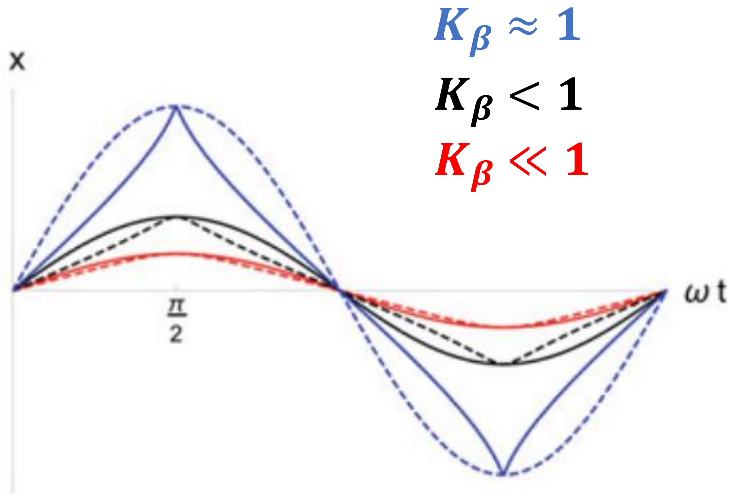


harmonic amplitude function

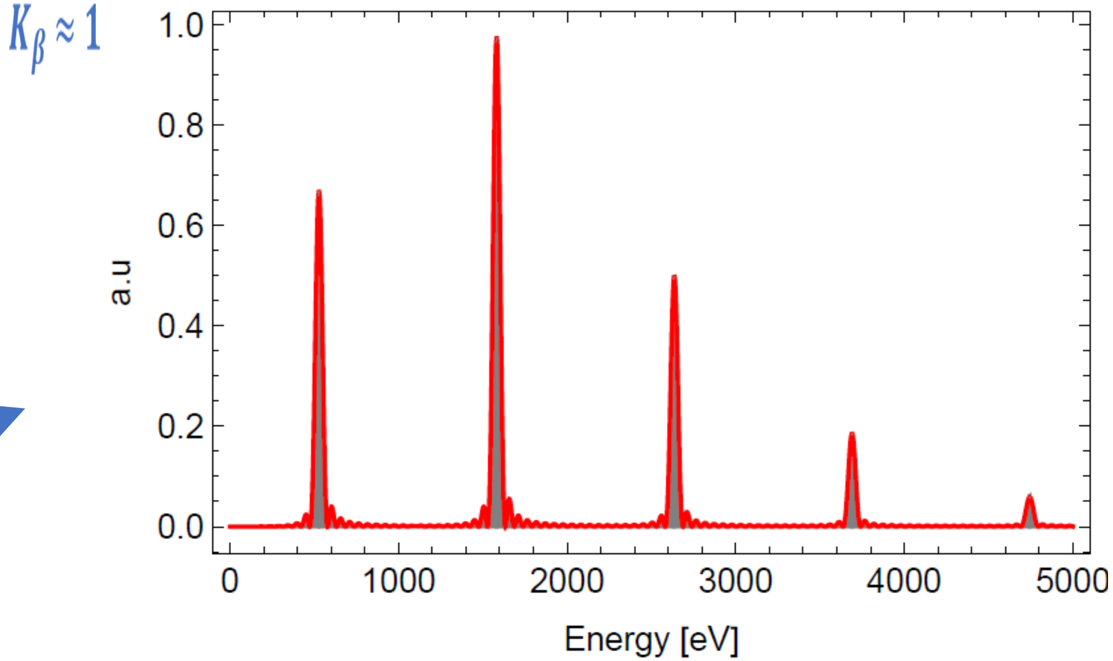
$$F_n = n\alpha_n[J_{(n-1)/2}(\alpha_n) - J_{(n+1)/2}(\alpha_n)]^2$$

$$\alpha_n = \frac{n\left(\frac{\omega}{\omega_n}\right)K_\beta^2}{4\left(1+K_\beta^2/2\right)}$$

Electron motion

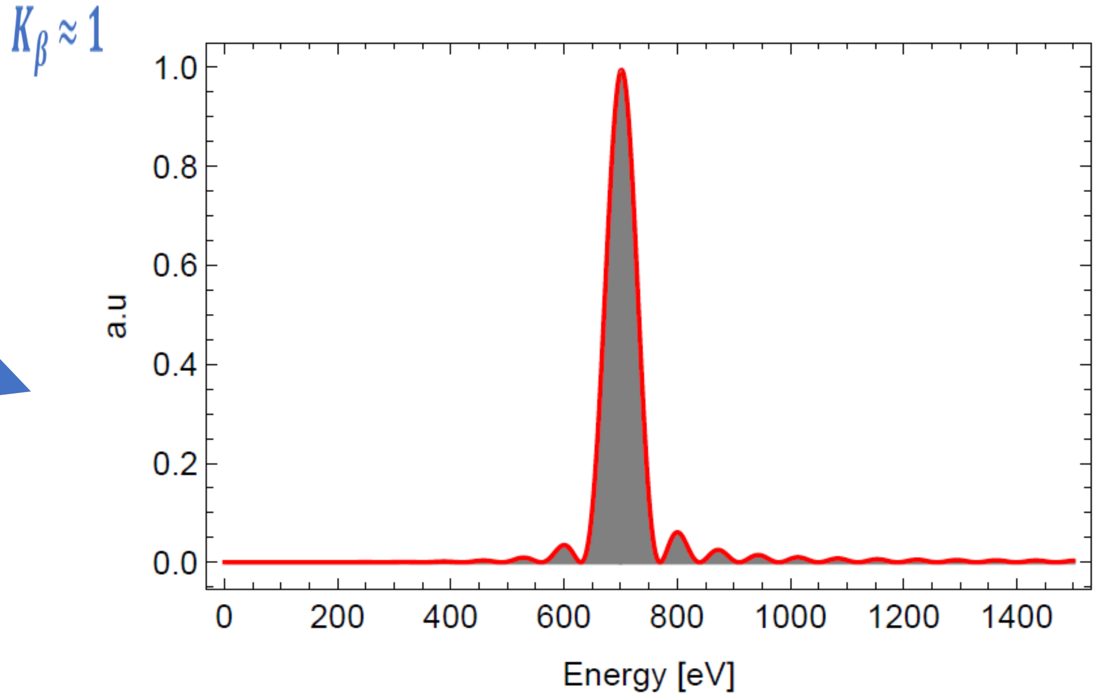


Wiggler
$K_\beta \approx 1$



$K_\beta \ll 1$

Undulator



Radiation

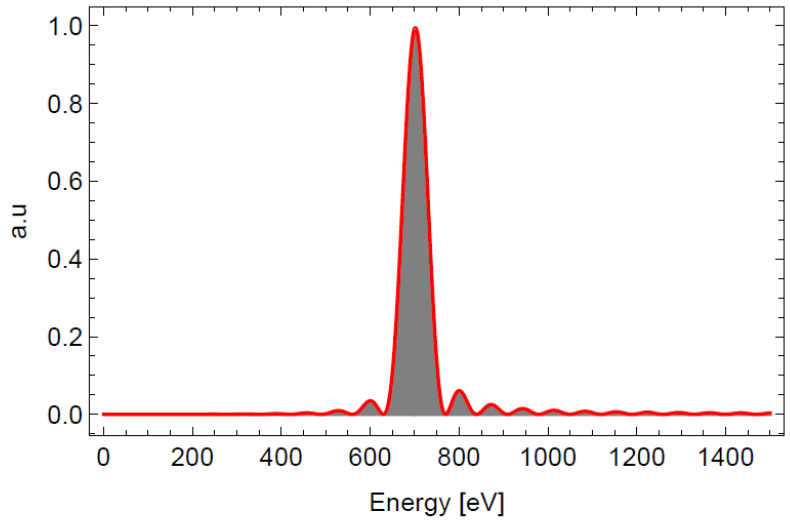
K<<1

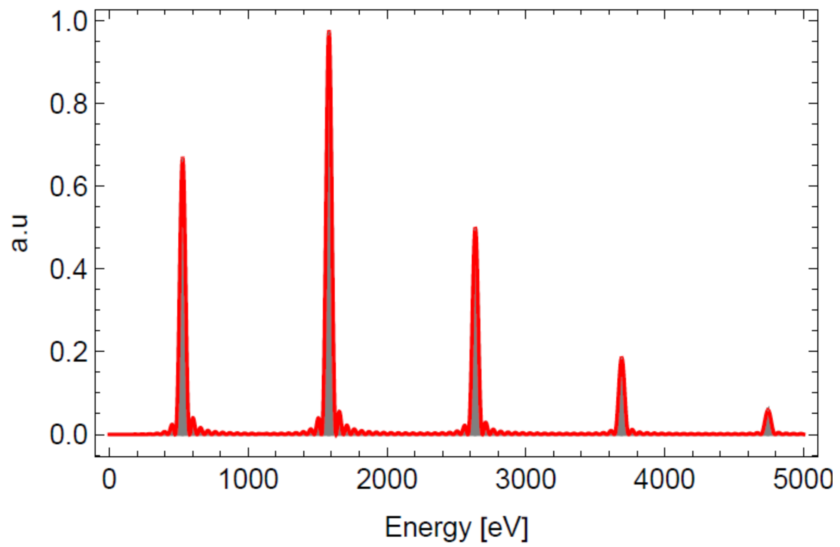
$\frac{d^2 I}{d\omega\, d\Omega}$

K=1

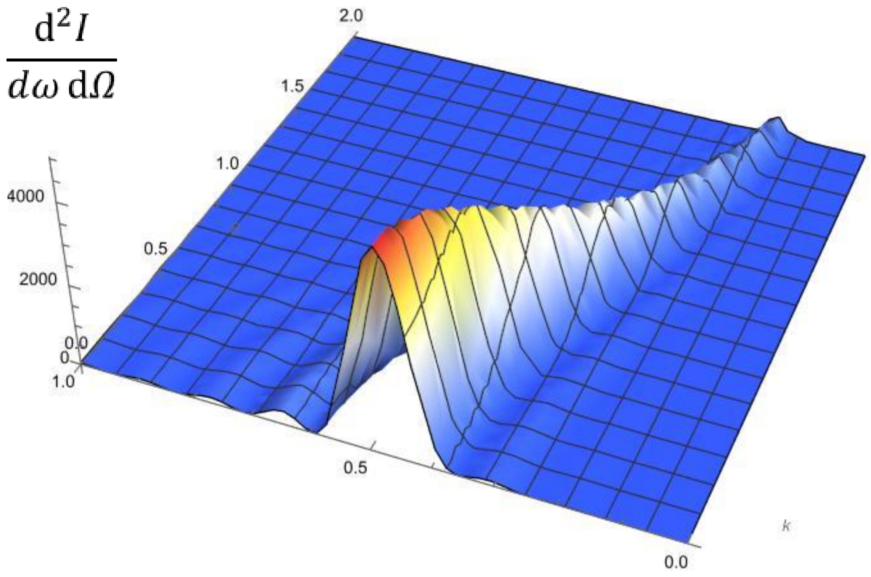
$\frac{d^2 I}{d\omega\, d\Omega}$

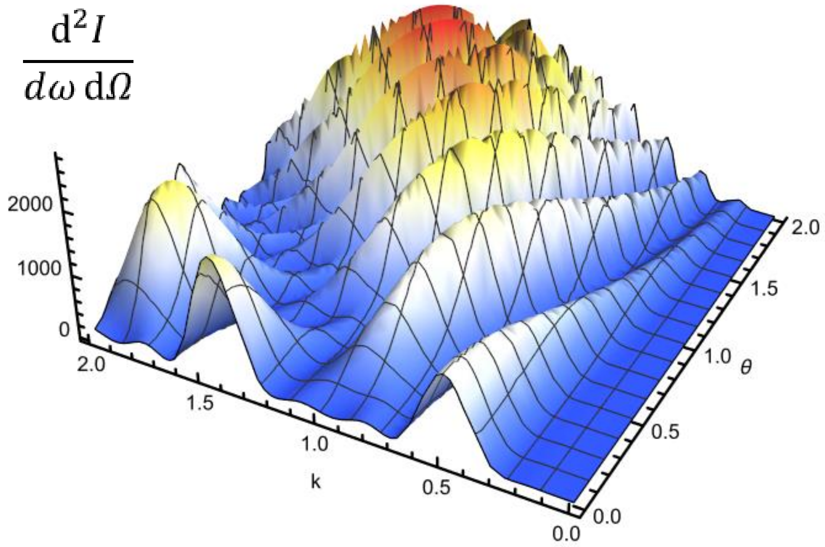
On axis





Off axis





Energy radiated

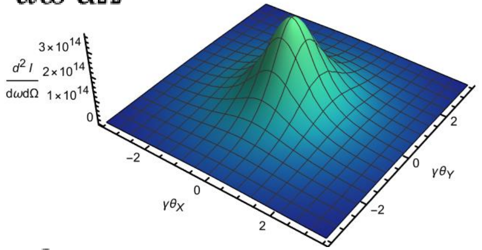
$$\frac{\mathrm{d}^2 I}{d\omega\,\mathrm{d}\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-T/2}^{T/2} \boldsymbol{n}\times(\boldsymbol{n}\times\boldsymbol{\beta})\, e^{i\omega\left(t-\frac{\boldsymbol{n}\cdot\boldsymbol{r}}{c}\right)}\,\mathrm{d}t\right|^2$$

$$K_\beta = \gamma r k_\beta$$

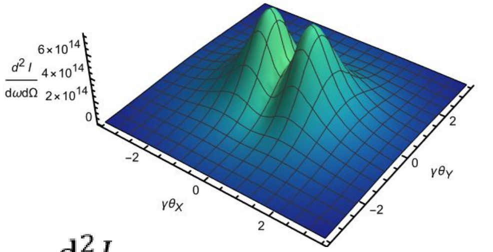
$r = const.$

$k_\beta = const.$

$\frac{\mathrm{d}^2 I}{d\omega\,\mathrm{d}\Omega}$



$\frac{\mathrm{d}^2 I}{d\omega\,\mathrm{d}\Omega}$



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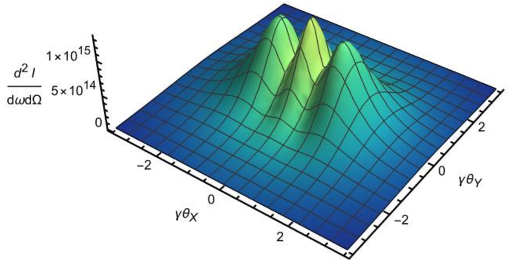

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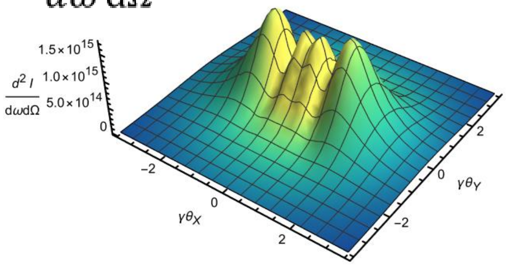

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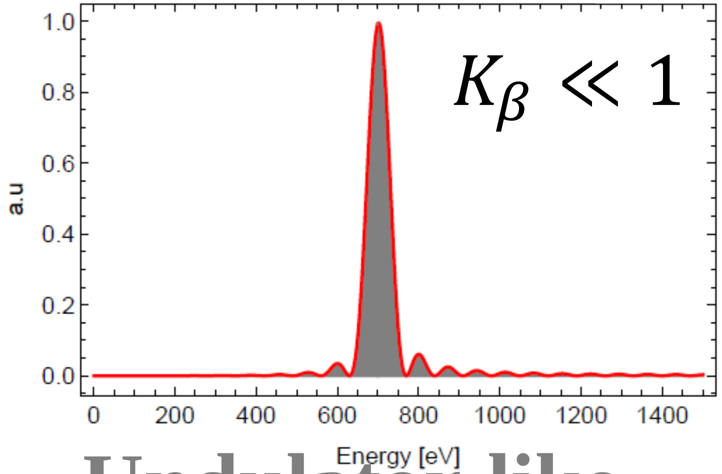

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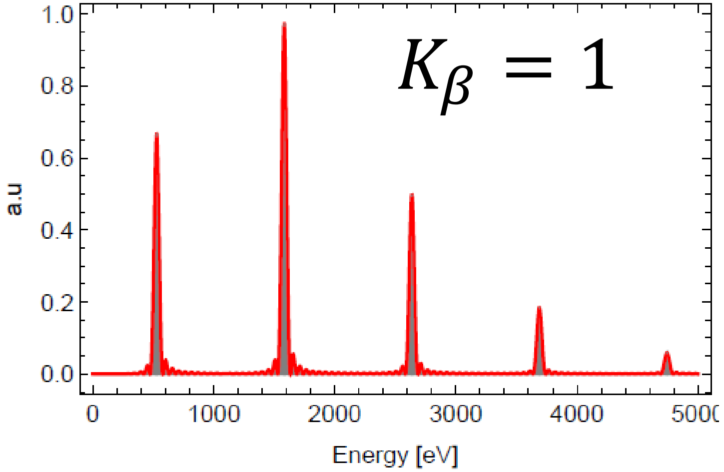
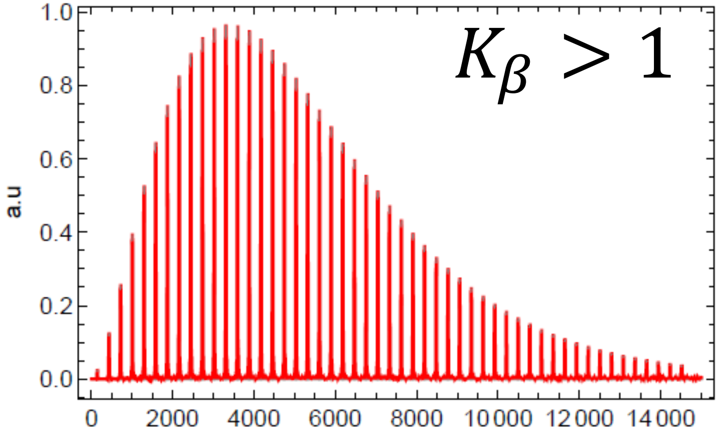


$\frac{\mathrm{d}^2 I}{d\omega\,\mathrm{d}\Omega}$



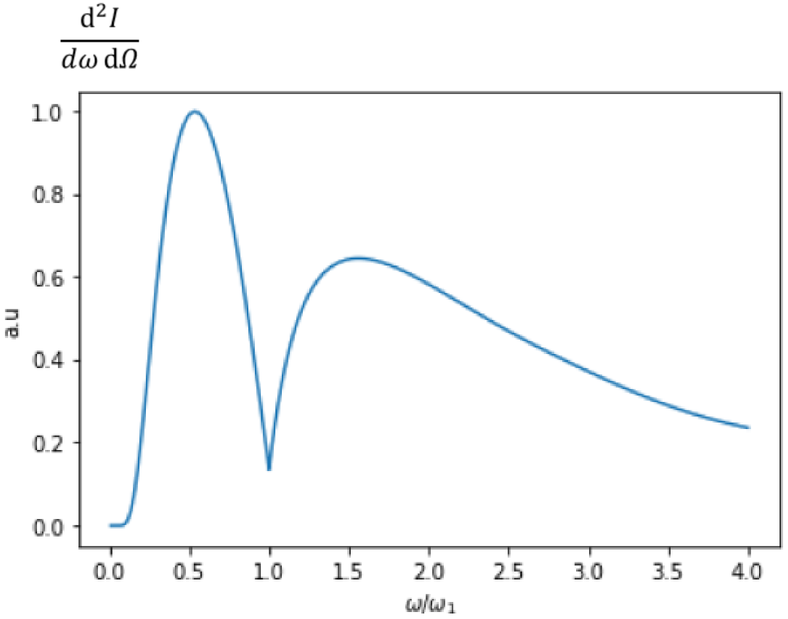
Undulator-like ⟷ **Wiggler like**

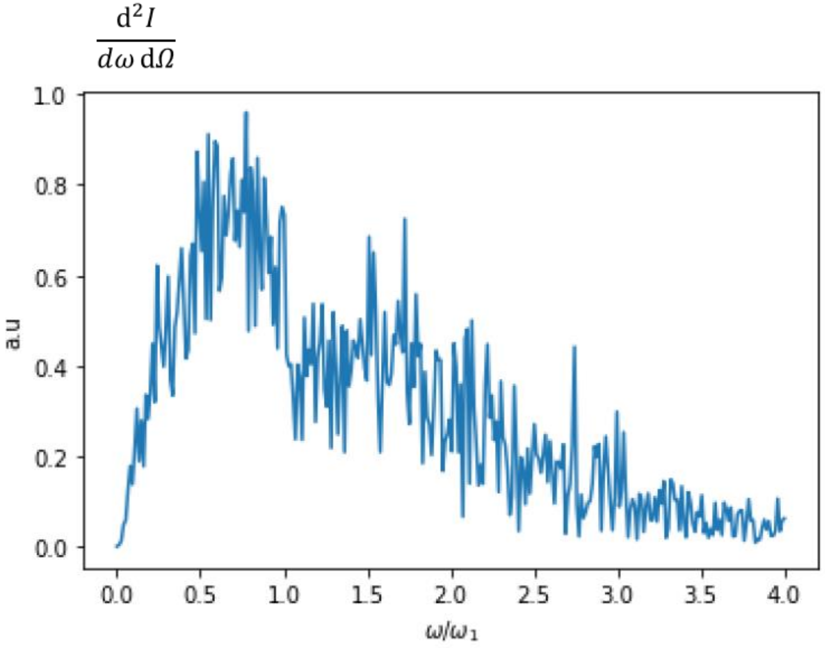
Gaussian distribution of radius

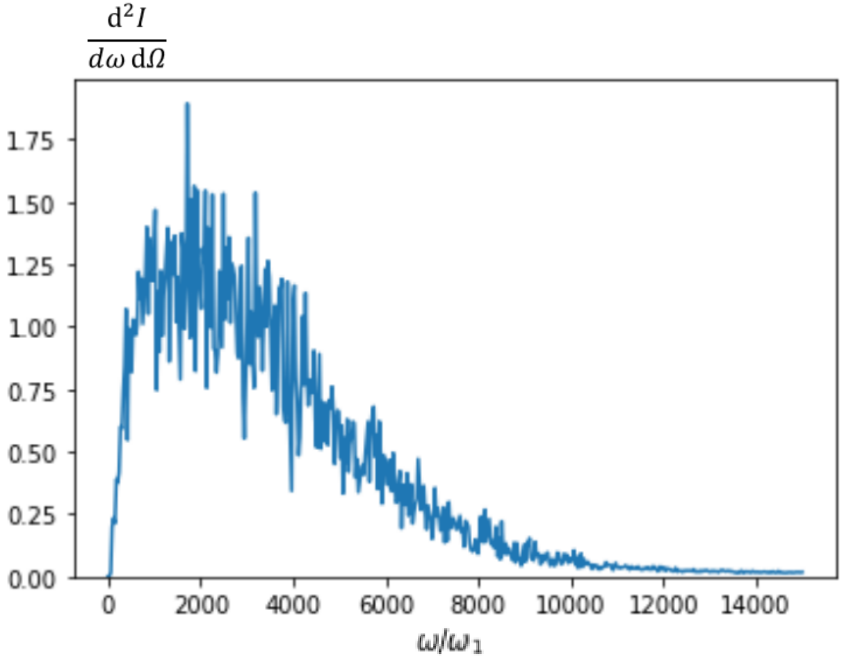
$$K_\beta(t) = \gamma(t)\mathbf{r}(\mathbf{t})k_\beta(t)$$

$$K_\beta = \gamma r k_\beta$$

Single particles #harmonics=5	200 particles #harmonics =5	200 particles #harmonics =60

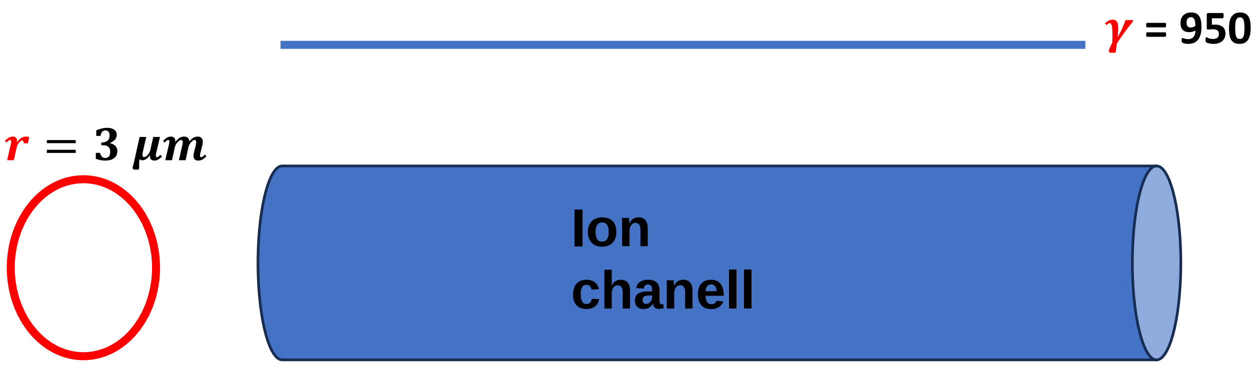






$$\frac{d^2 I_B(\omega,\theta)}{d\omega d\Omega} = \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^{\infty} da_\beta a_\beta f_e(a_\beta) \frac{d^2 I(\omega, a_\beta, \theta, \phi)}{d\omega d\Omega}$$

Ideal spectrum



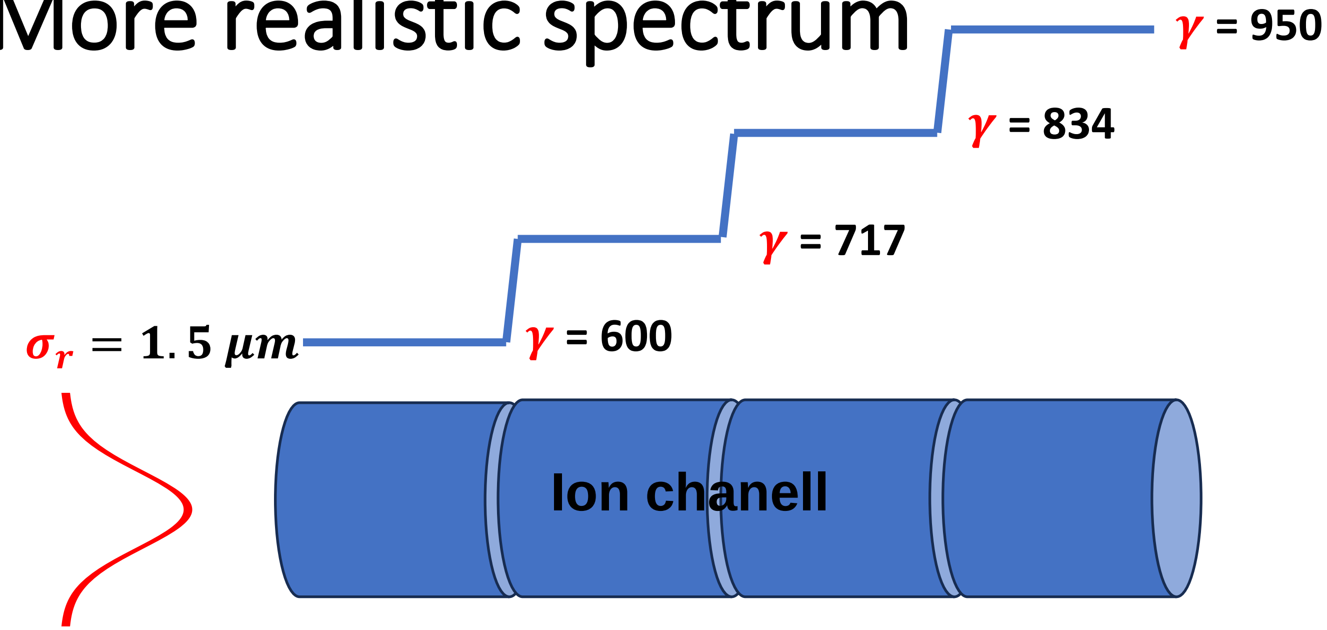
Constant

$$K_\beta(t) = \gamma(t) r(t) k_\beta(t)$$

Constant

Constant

More realistic spectrum



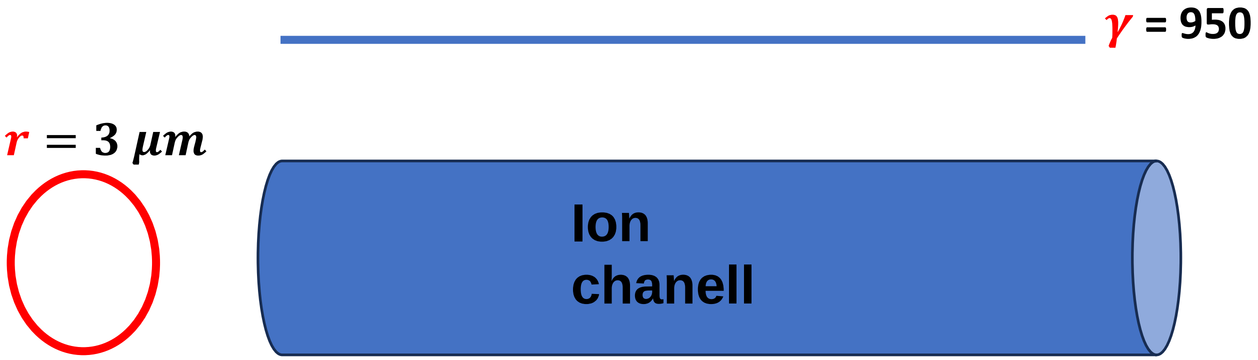
Each particle has a different oscillation amplitude

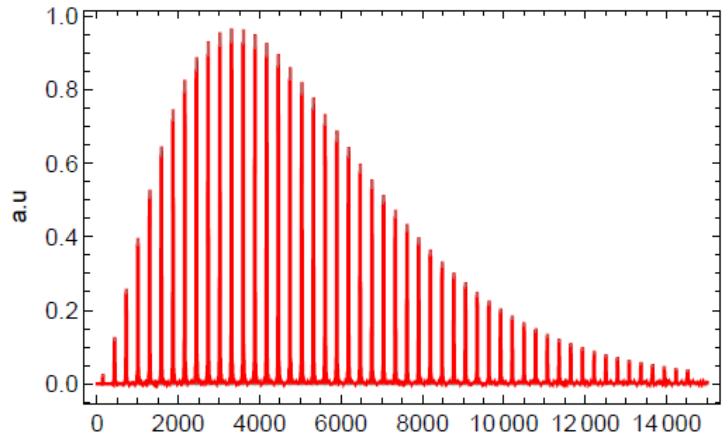
$$K_\beta(t) = \boldsymbol{\gamma(t)} \boldsymbol{r(t)} \boldsymbol{k_\beta(t)}$$

Particle are accelerated

$k_\beta(t) = k_p/\sqrt{2\,\boldsymbol{\gamma(t)}}$

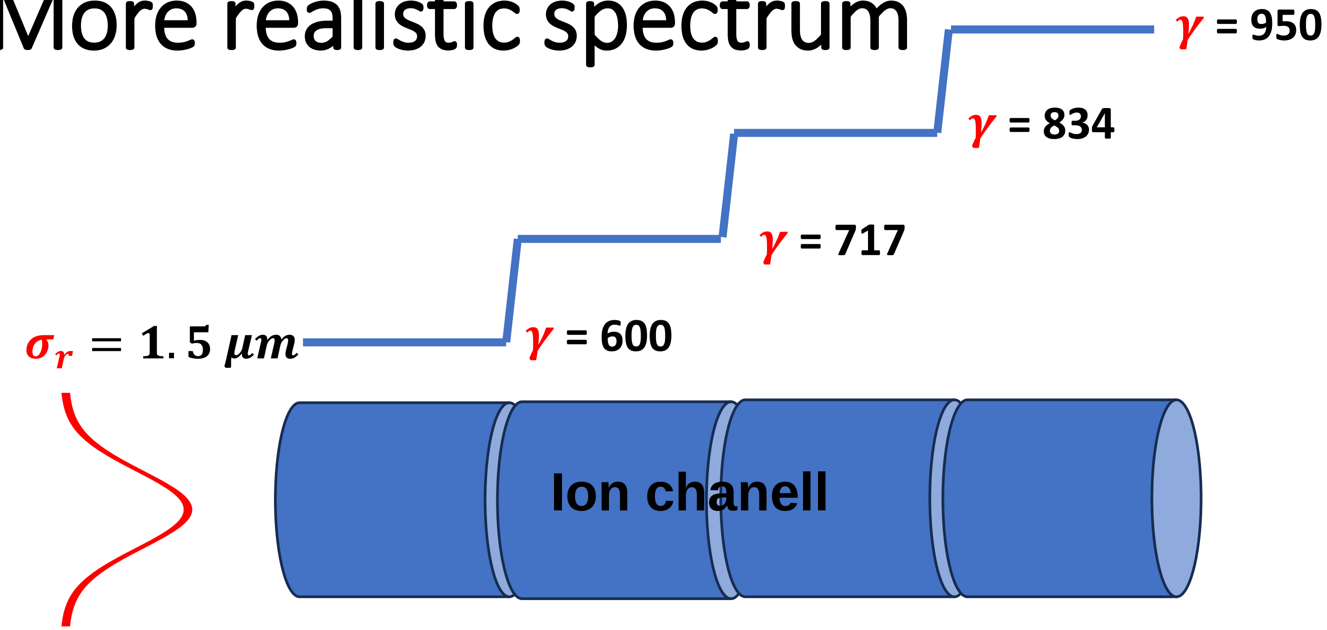
Ideal spectrum

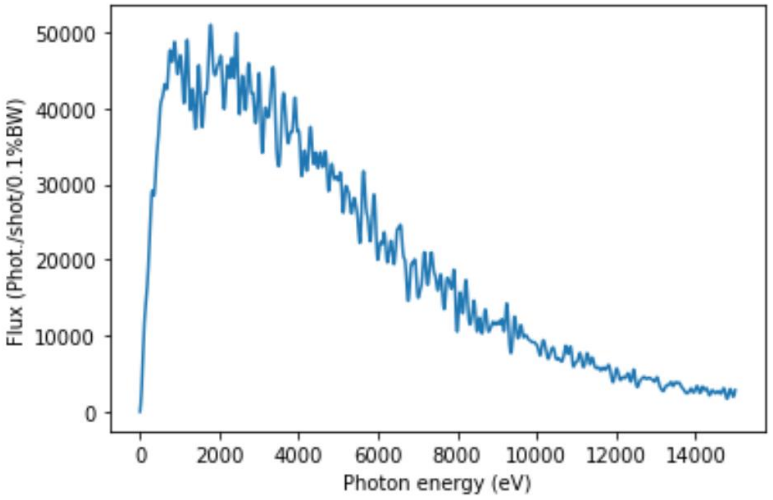




$$K_\beta(t) = \gamma(t) r(t) k_\beta(t)$$

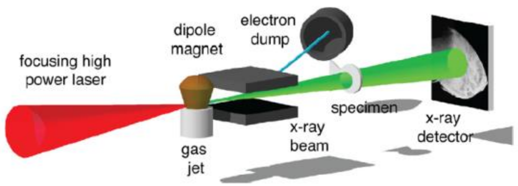
More realistic spectrum





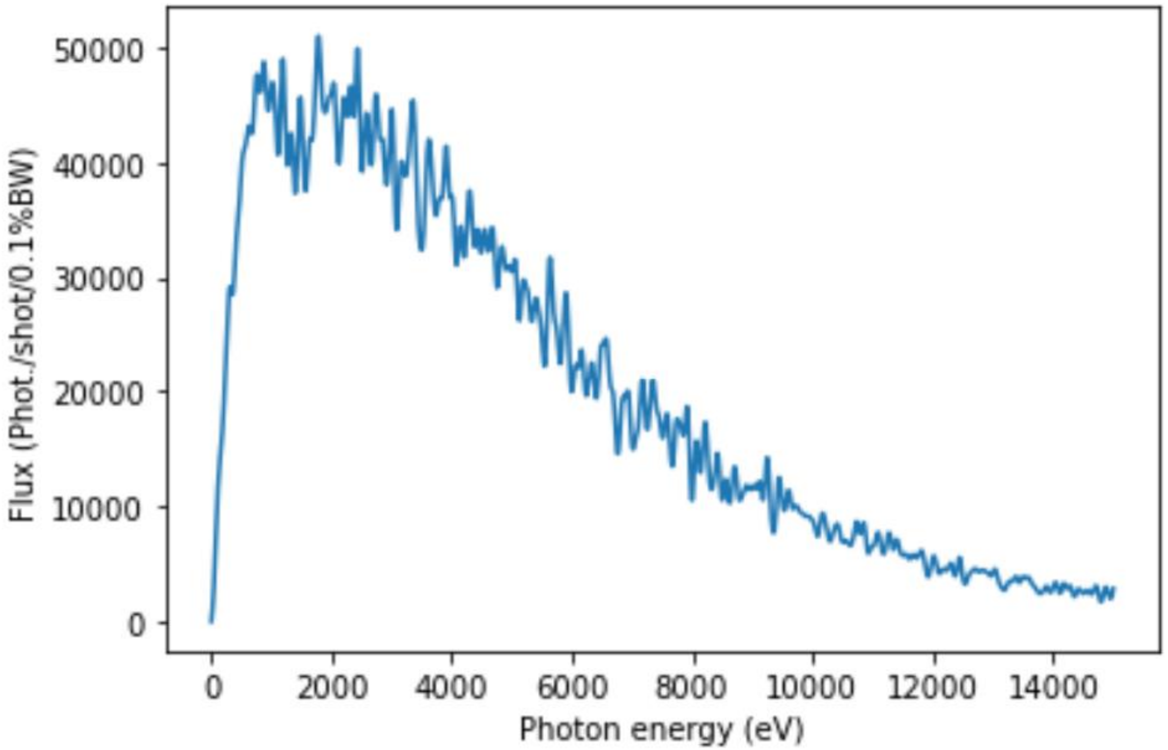
$$K_\beta(t) = \boldsymbol{\gamma(t)}\boldsymbol{r(t)}\boldsymbol{k_\beta(t)}$$

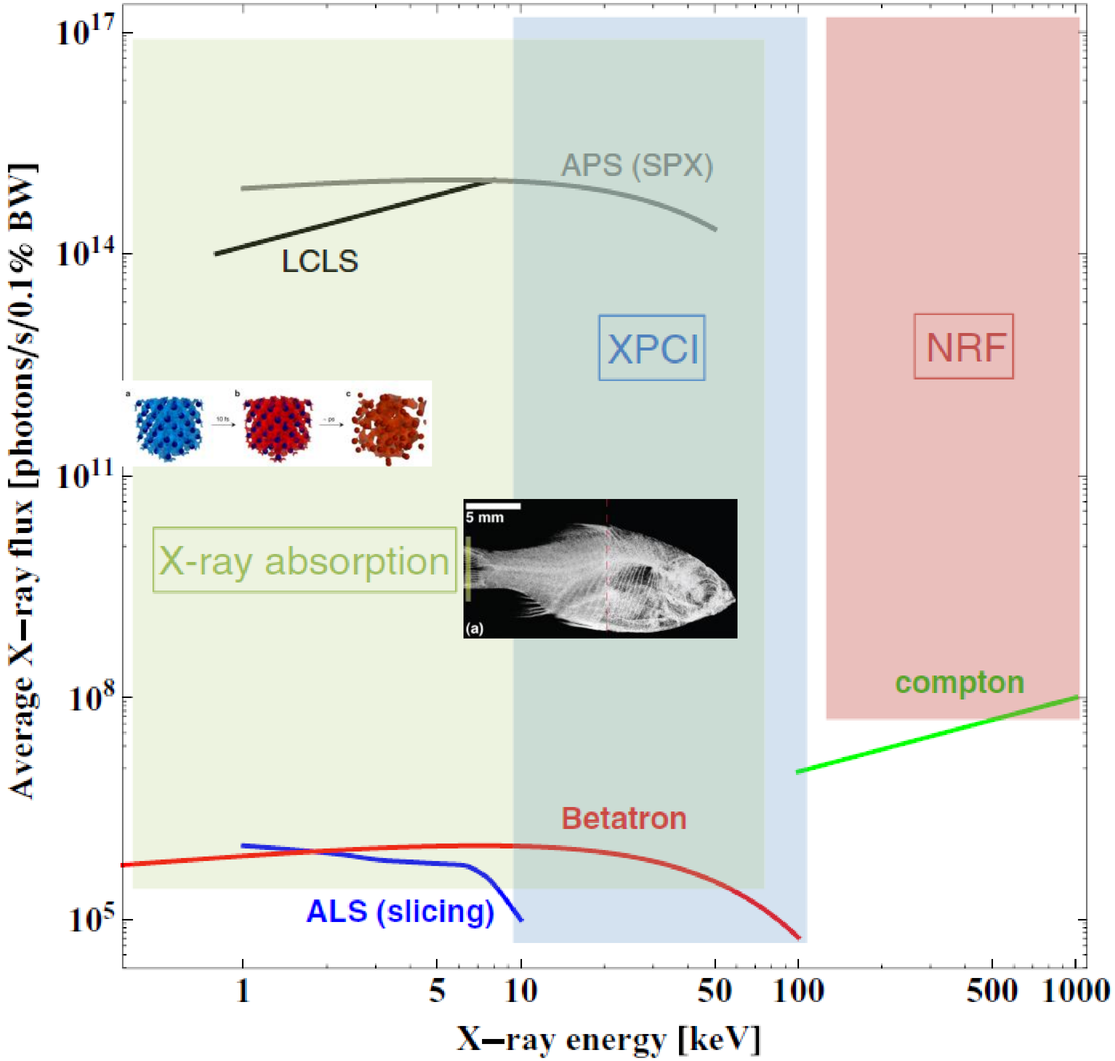
Calculation of theFlux



EuPRAXIA@SPARC_LAB

Parameter [Units]	Value
Beam energy [MeV]	600–950
Plasma density [cm^{-3}]	3.0×10^{16}
RMS transverse beam size [μm]	3.0
Charge [pC]	45
Capillary (plasma) length [m]	0.6
Pulse duration [fs]	10





Ref.
Antoine Rousse, Production of a keV X-Ray Beam from Synchrotron Radiation in Relativistic Laser-Plasma Interaction, Nature communication
Ref.
S. Kneip et al. "X-ray phase contrast imaging of biological specimens with femtosecond pulses of betatron radiation from a compact laser plasma wakefield accelerator", APPLIED PHYSICS LETTERS
Refs.
F. Stellato et al. "Plasma-Generated X-ray Pulses: Betatron Radiation Opportunities at EuPRAXIA@SPARC_LAB" Condensed matter

Theoretical development of 3D model of electron motion

More accurate trajectory

$$x_e(t) = x_\beta(t)cos(\int_0^t \Omega_\beta(t')dt' + \phi_x)$$

$$y_e(t) = y_\beta(t)cos(\int_0^t \Omega_\beta(t')dt' + \phi_y)$$

← Different initial phase

$$z_e(t) = \int_0^t v_z(t')dt' + z_0$$

$$x_\beta(t) = x_\beta(0)(\frac{1}{\gamma(t)})^{\frac{1}{4}}$$

$$y_\beta(t) = y_\beta(0)(\frac{1}{\gamma(t)})^{\frac{1}{4}}$$

← Amplitude damping

$$\Omega_\beta(t) = \frac{\omega_\beta(0)}{\sqrt{2\gamma(t)}}$$

← Frequency shift

$$\gamma(t)$$

← Acceleration

Conference and school

- September 2022- **European Network for Novel Accelerators (EuroNNac) conference Elba island** (poster session)
- September 2023 **- European Advanced Accelerator Concepts Workshop** (poster session)
- May 2023 -14 **International Particle Accelerator Conference (IPAC)**

(poster session)

- February 2023 – **Winter school at Bad Honnef Physics** Schools (Germany) on Plasma Acceleration

- July 2023 – **Summer school at Erice (Italy): International School of Particle Accelerators**

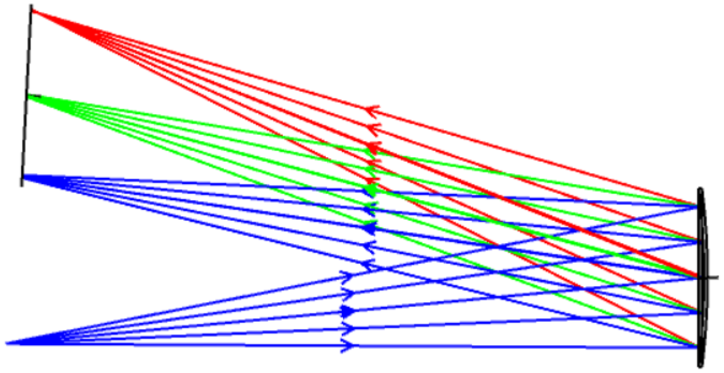
Accelerator School at E. Majorana CENTER - Erice (Sicily)

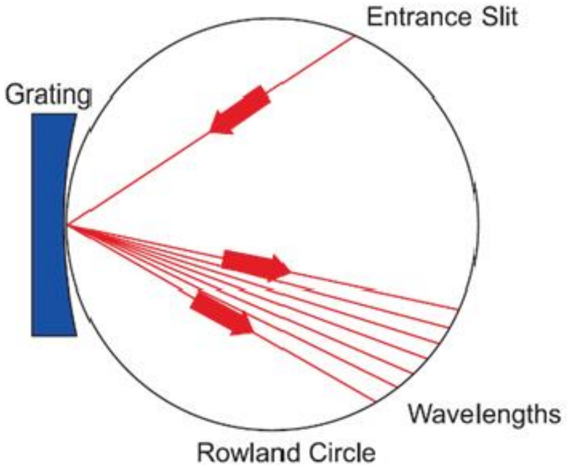
Conclusion

- I have studied the theory behind betatron radiation, starting from the relativistic motion of a single electron within what is called an ion channel
- I have explored the connection between the motion of a single electron and its radiation.
- I tried to develop a more realistic computational model that takes into account the distribution of electrons and the fact that electrons are accelerated discretizing the ion chanell
- I started a theoretical model of more accurate trajectory.

Perspectives

Measurement of the bunch length starting from the analysis of the incoherent radiation fluctuations





Spectrometer simulation

Spectrometer measurment

Data analysis

Thanks for the attention

Parameter	APS	ALS	LCLS	Betatron	Compton
Pulse duration	20–100 ps	<1 ps	10–80 fs	30–60 fs	30–60 fs
Repetition rate	6.5 MHz	kHz	120 Hz	1 Hz	1 Hz
Energy range	0.2–40 keV	0.25–9 keV	0.5–24 keV	1–80 keV	0.1–2 MeV
Bandwidth	2–100%	100%	0.1%	100%	50%
Tunability	Variable undulator gap	Limited	e-beam energy	e-beam energy	e-beam energy
Photons/pulse	10^8	10^7	10^{13}	10^8	10^7
Reproducibility	Excellent	Excellent	Limited (SASE)	Poor	Poor

Parameter	Betatron	Compton
Repetition rate	>30 Hz	>30 Hz
Energy range	1–150 keV	1–10 MeV
Bandwidth	100%	<1%
Photons/second	10^8	10^{13}
Jitter	1% rms	1% rms