Searching for Binary Supermassive Black Holes via Optical Spectroscopy

by Mike Eracleous

Including an introduction that sets the stage for later talks.

Gravi-γ-ν
11 October 2024

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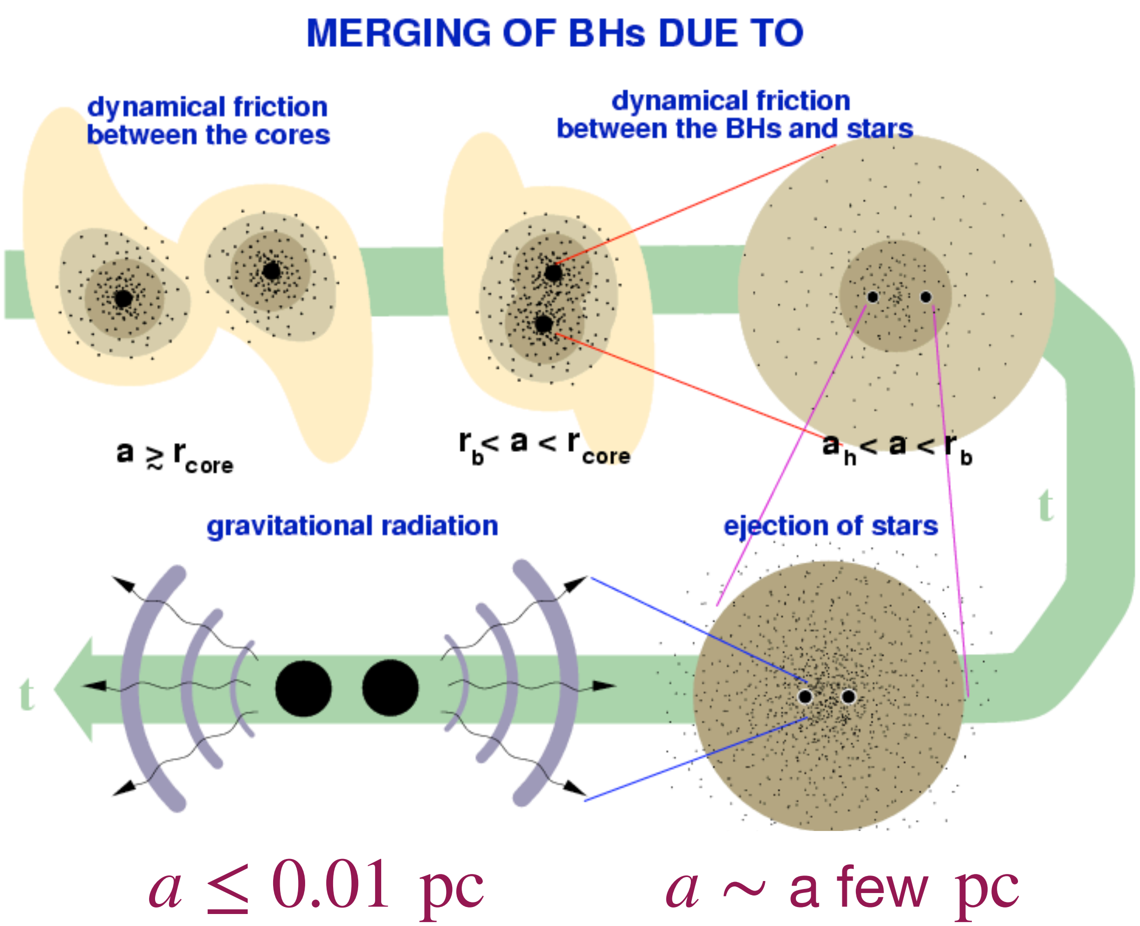
Exercise in patience and persistence

- Jessie Runnoe (Vanderbilt), .P.I.
  Tamara Bogdanovic, (Georgia Tech)
- **Current Penn State Students**: Niana Mohammed, Mary Ogborn, Kaitlyn Szekerczes
- **Graduate Students at Other Universities**: Carolyn Drake, (Vanderbilt), Katie Futrowsky (Georgia Tech)
- **Past Undergraduates**: Gavin Mathes, Alison Pennell, Stephanie Brown, Mary Kaldor
- **Other Collaborators**: Khai Nguyen, Peter Breiding, Sarah Burke-Spolaor, Jia Liu, Dan Doan, Jules Halpern, Todd Boroson, Steinn Sigurðsson, Joe Lazio, Helene Flohic

The stages of SBHB evolution



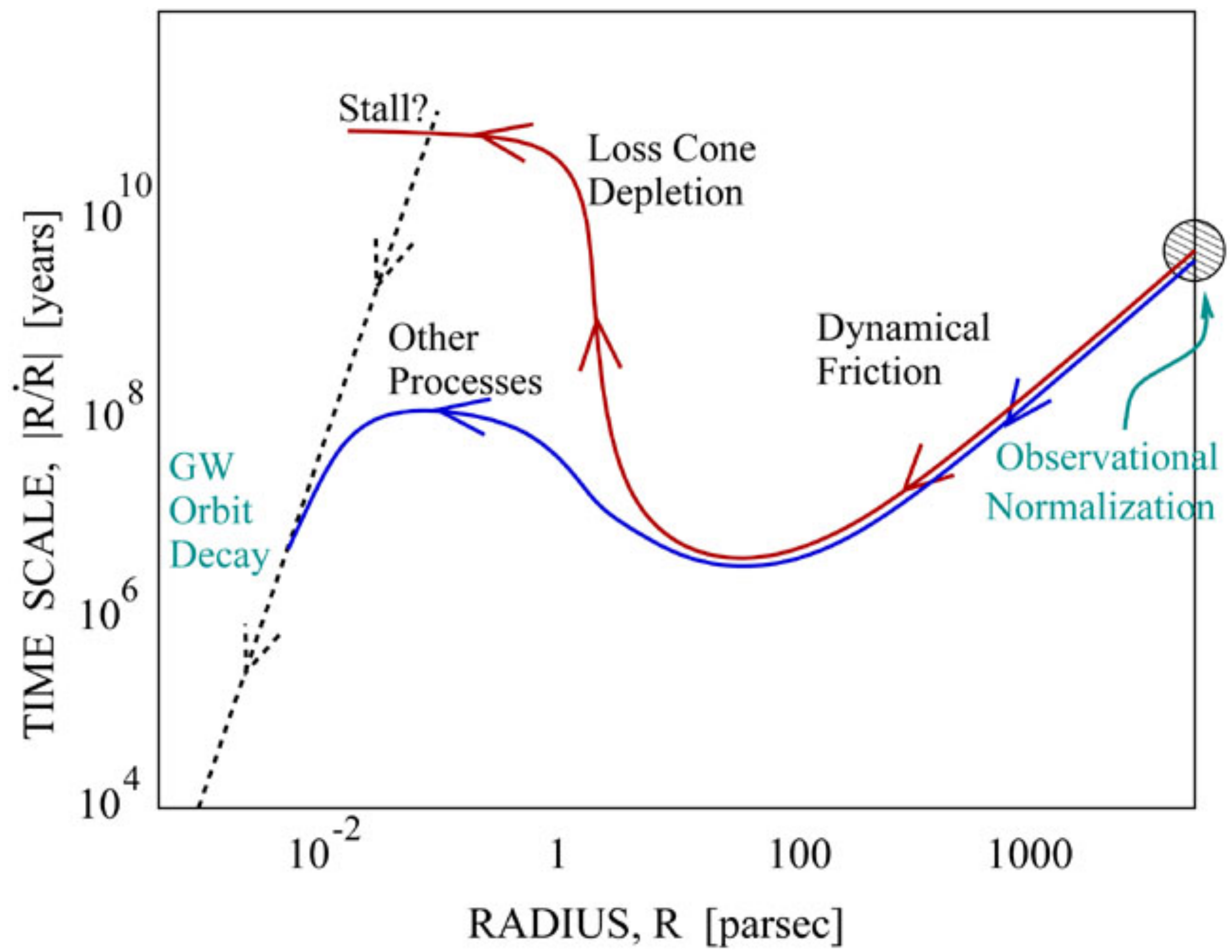


figure from Christian Zier's web site
Max-Planck-Institut für Radioastronomie

from Backer+04, Carnegie Observatories Astrophysics Series 1, 238
based on the work of Begelman+80, Nature 287, 307

SBHB evolution according to modern stellar dynamics

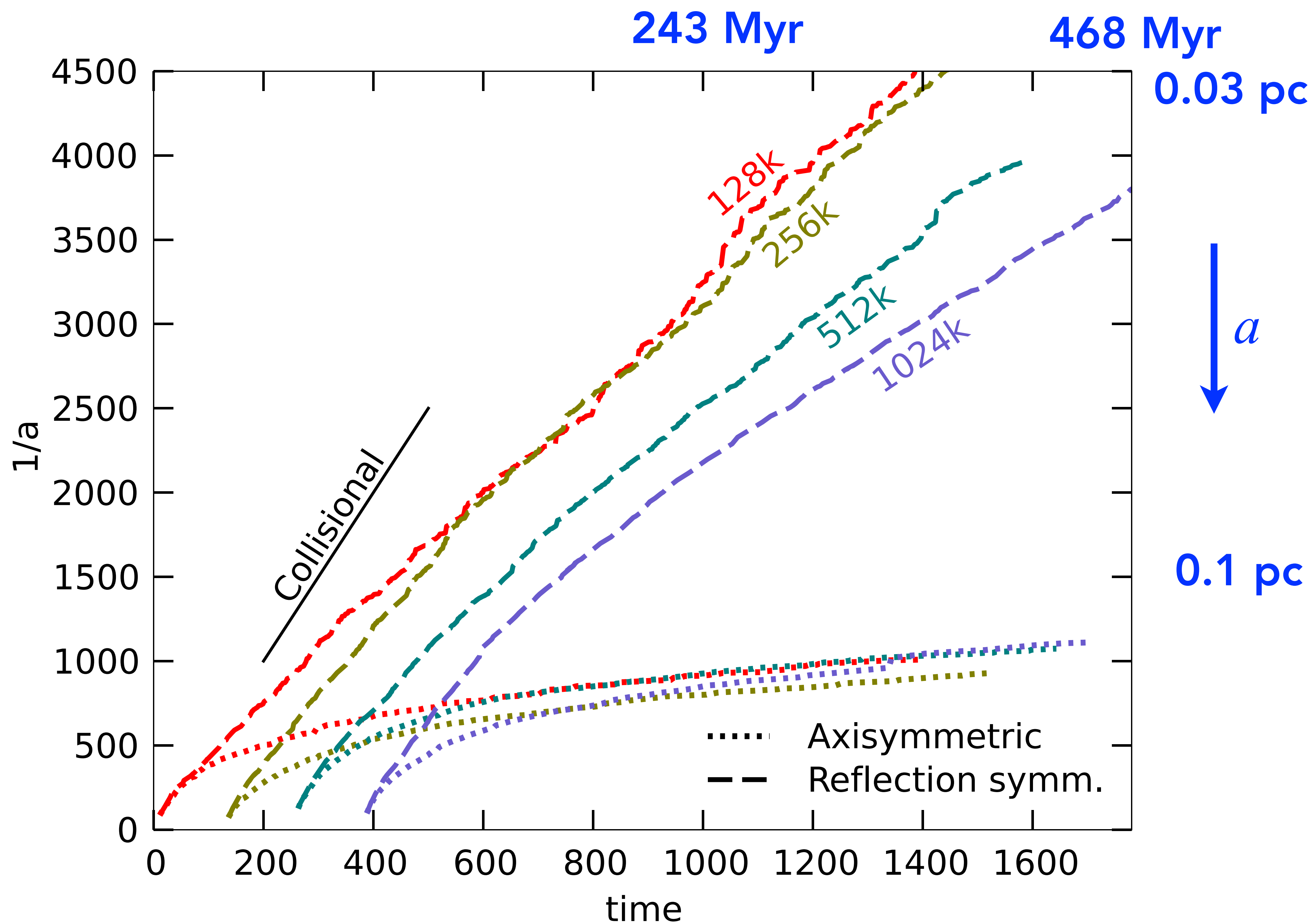
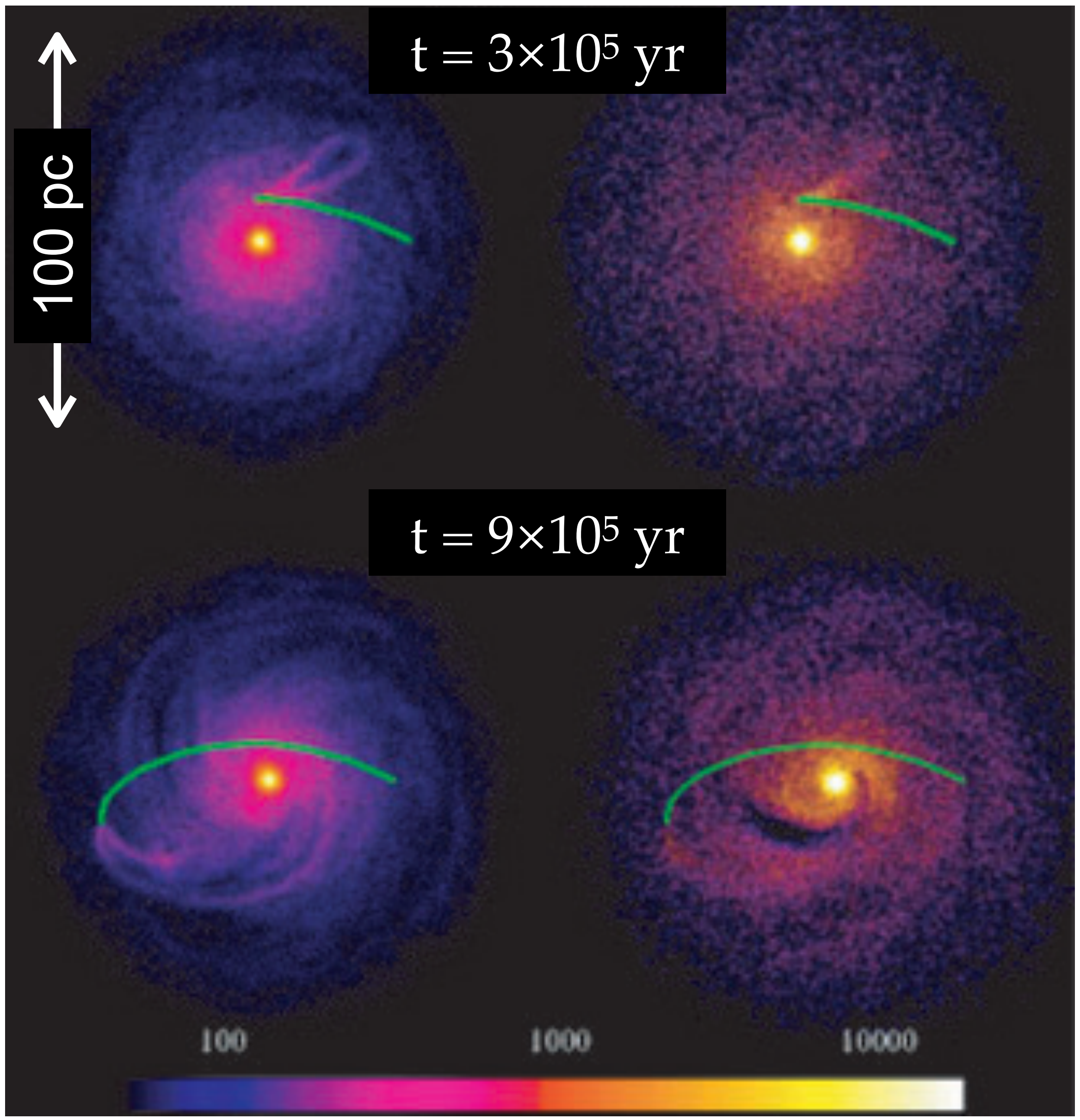
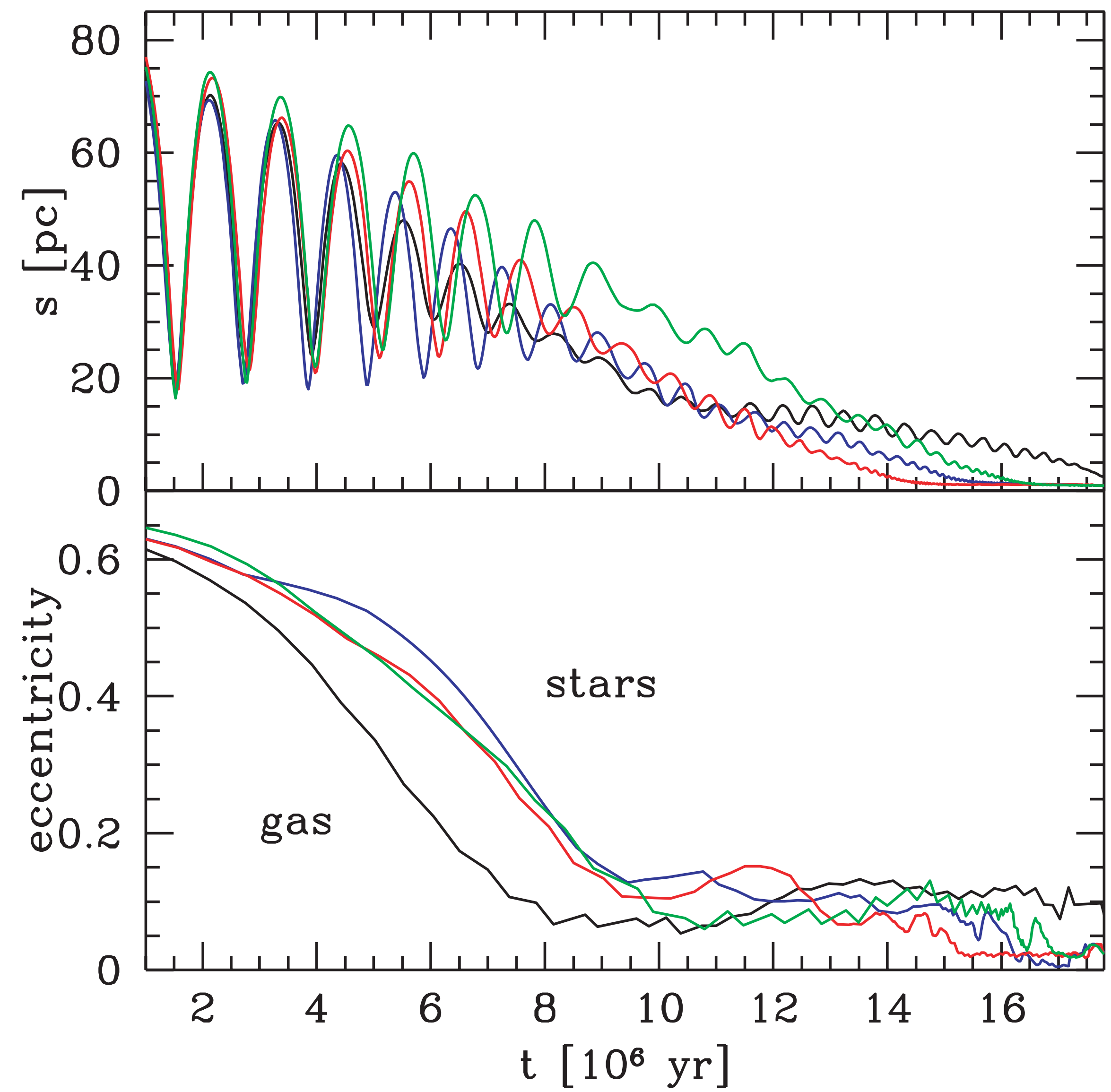


figure from Vasiliev+15, ApJ, 810, 49

SBHB evolution in a gas-rich environment





Gas density Star density

from Dotti+07, MNRAS, 379, 156

Last phase of evolution: torques from gaseous, circumbinary disk and accretion

SPH simulations
(scale free)

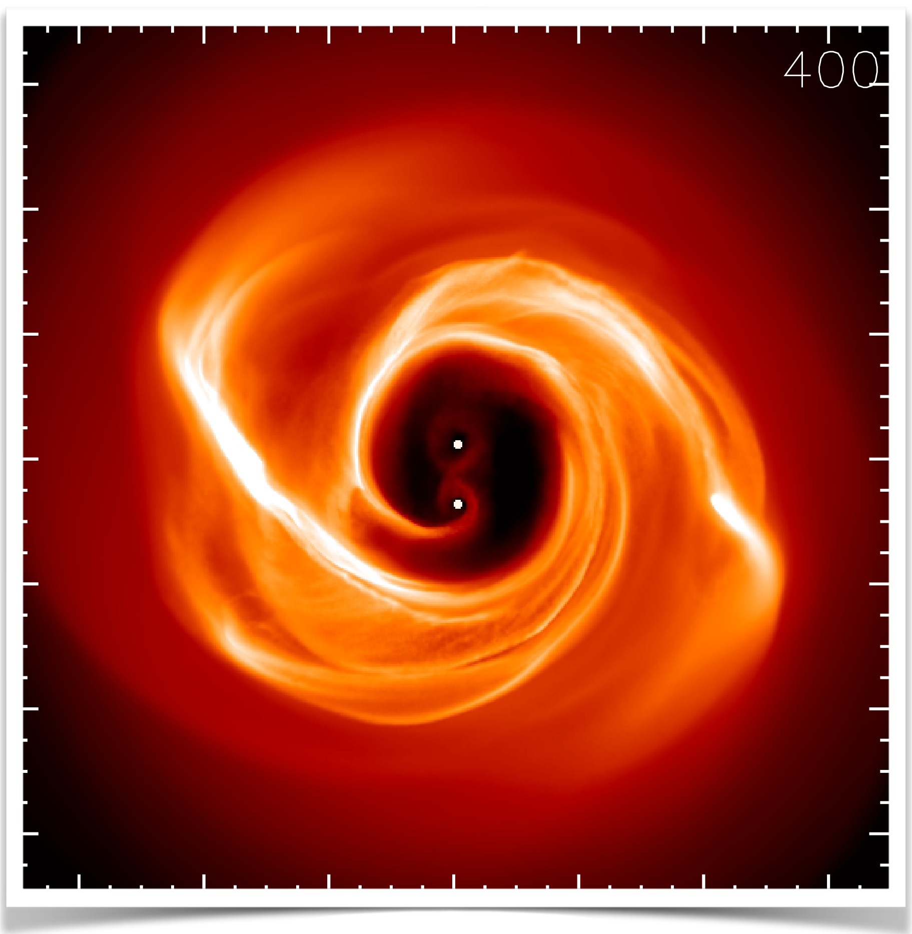
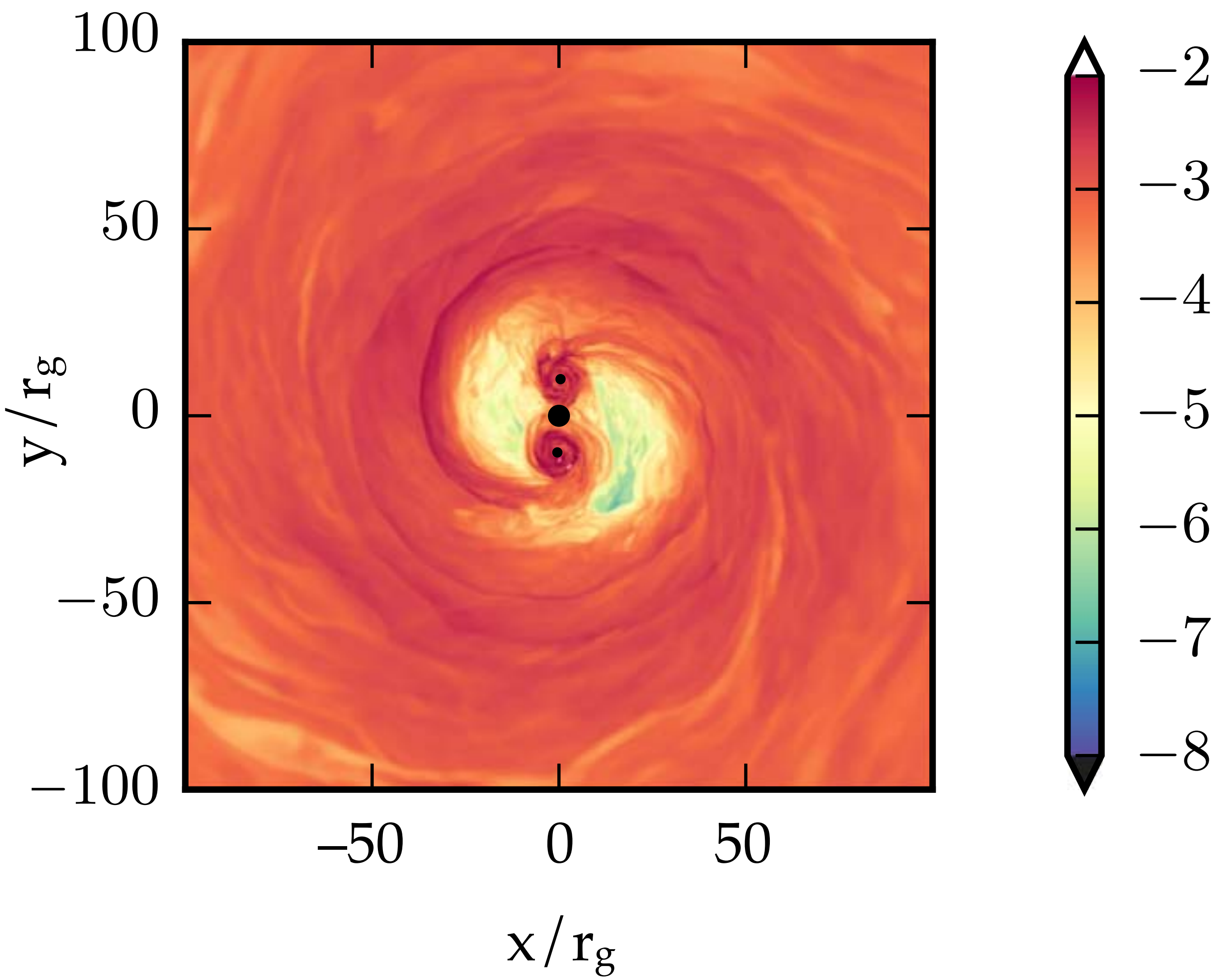


figure from Cuadra+09, MNRAS, 393, 1423

GRMHD simulations
(short separations)



from d'Ascoli+18, ApJ, 865, 140

Some things we could learn from finding and studying binary supermassive black holes at $a \sim 0.1 - 10$ pc.

Do they exist? How common are they?

Validate the physical scenarios that have invoked them.

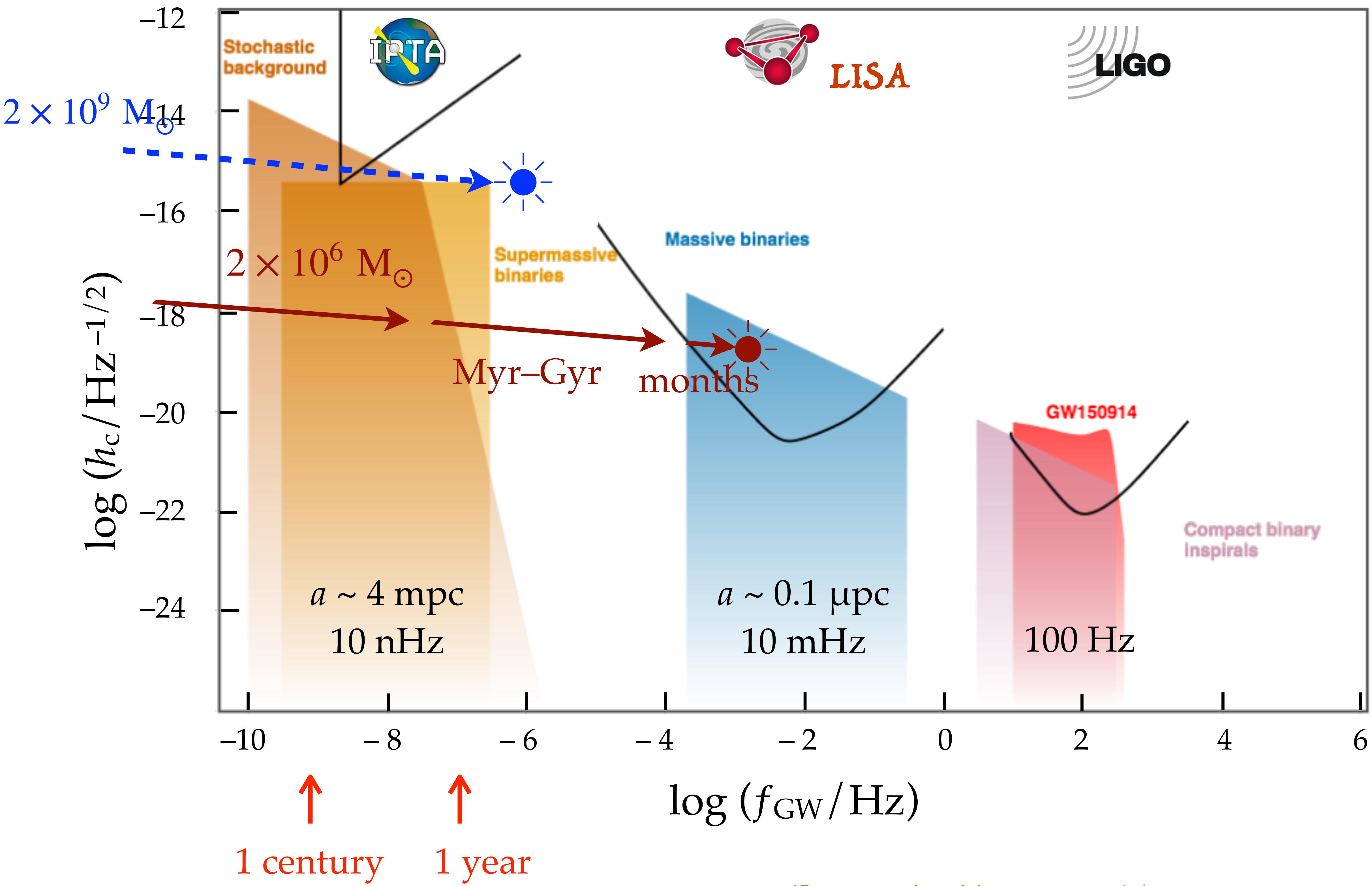
What is the distribution of orbital separations and other system properties?

Track the earlier stages of the evolution of the binary and test models for angular momentum loss from the binary.

Host galaxies, their redshift distribution, and evolutionary state.

Identify progenitors of a big family of low-frequency (mHz) gravitational wave sources.

Journey of a 2×10⁶ $M_\odot$ supermassive binary (@ $z = 3$) through the GW frequency spectrum starting at 1 pc



(figure made with `http://gwplotter.com/`)

The early evolutionary phases of a (super)massive binary can be studied by electromagnetic observations.

Observing the different evolutionary phases

Slide modified from N. Mohammed

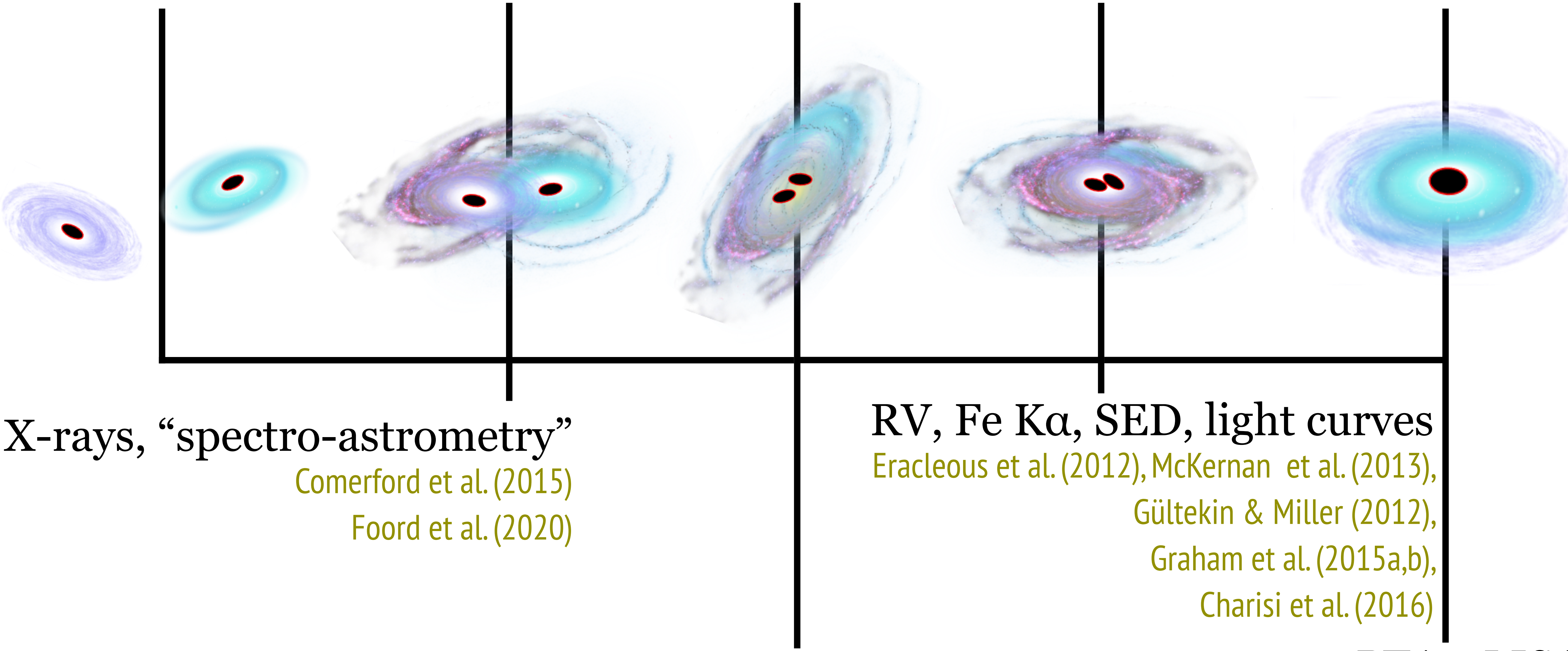
Galaxies approach each other

Dual AGN 0.1-10 kpc

Bound BH <10 pc

Close binary <1 pc

Coalescence



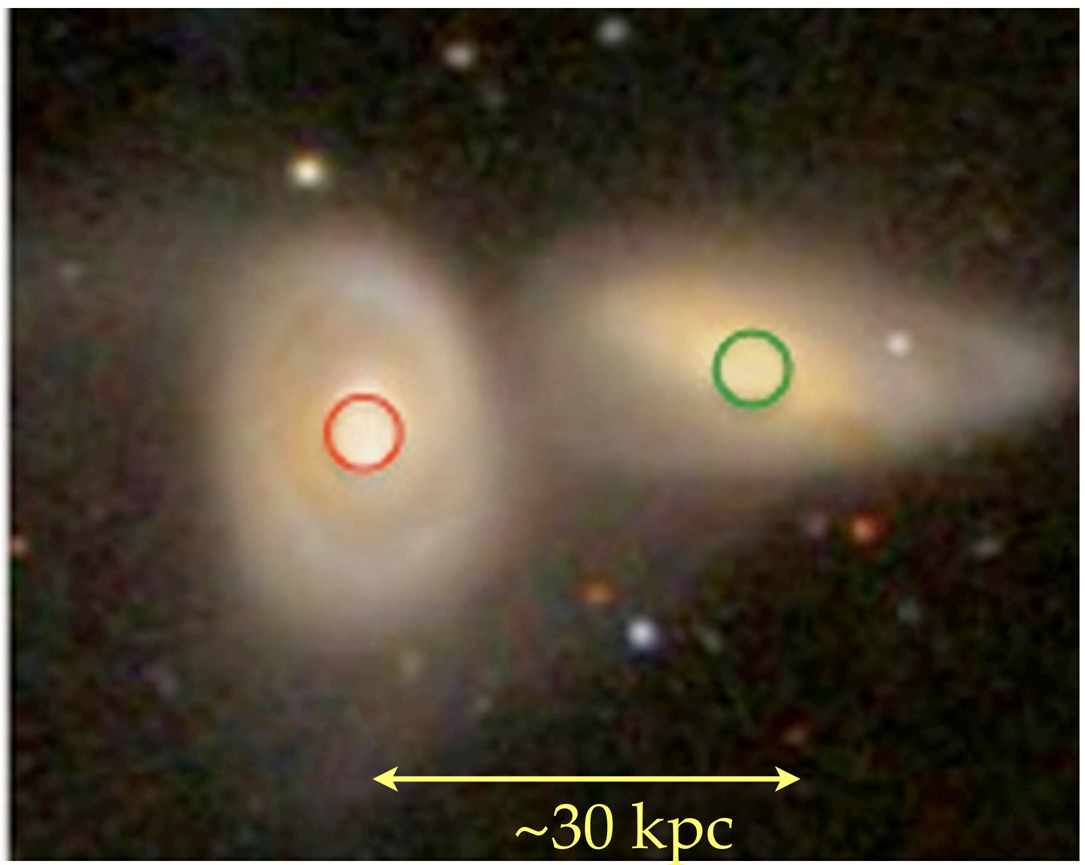
X-rays, "spectro-astrometry"
Comerford et al. (2015)
Foord et al. (2020)

RV, Fe Kα, SED, light curves
Eracleous et al. (2012), McKernan et al. (2013), Gültekin & Miller (2012), Graham et al. (2015a,b), Charisi et al. (2016)

radio interferometry
Rodriguez et al. (2006), Burke-Spolaor et al.(2011), Bansal et al. (2017), Breiding et al. (2021)

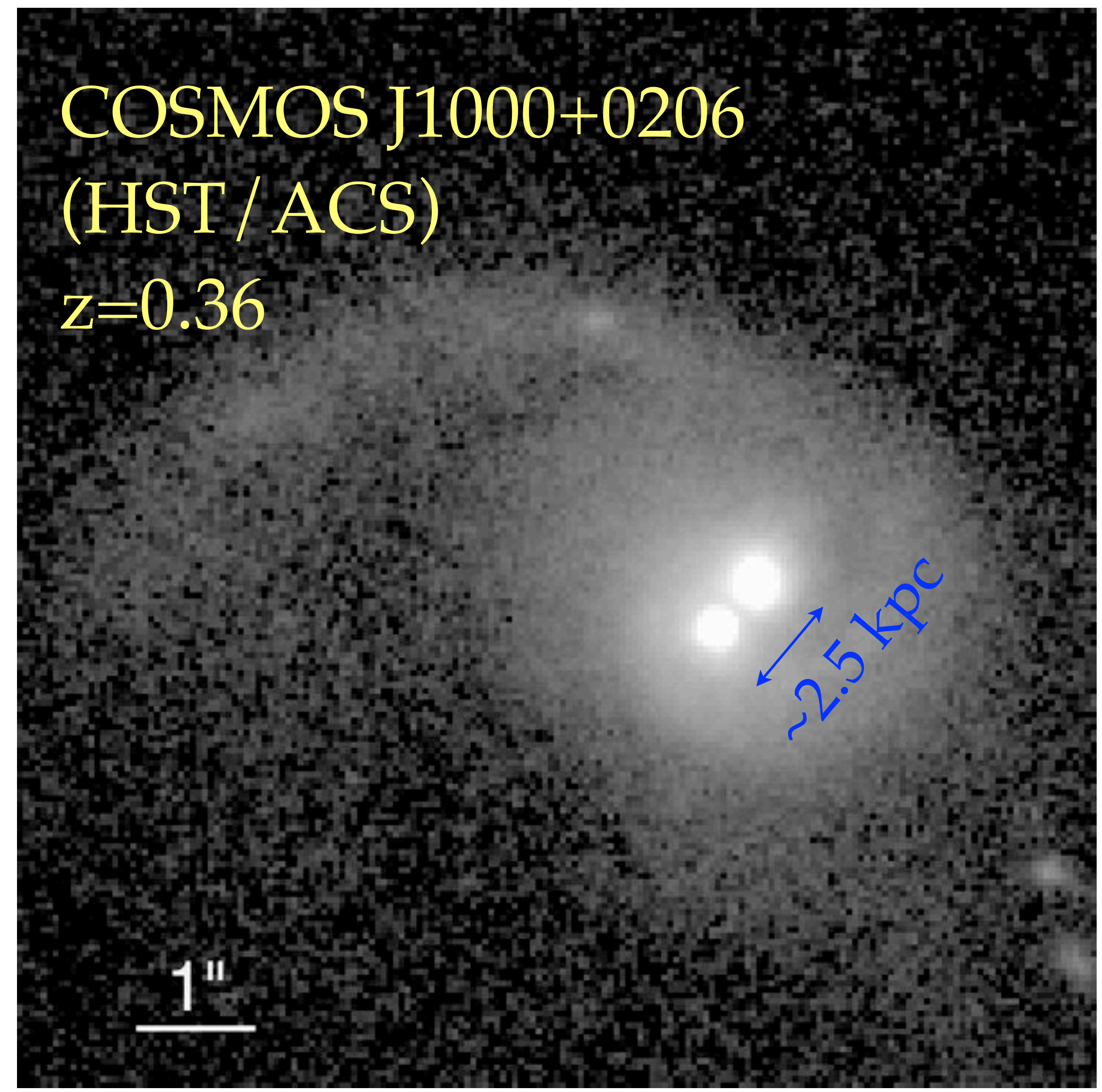
PTAs, LISA
Arzoumanian et al. (2020)
Sesana (2015)

Widely separated, dual active nuclei



SDSS image of dual AGN from BAT survey

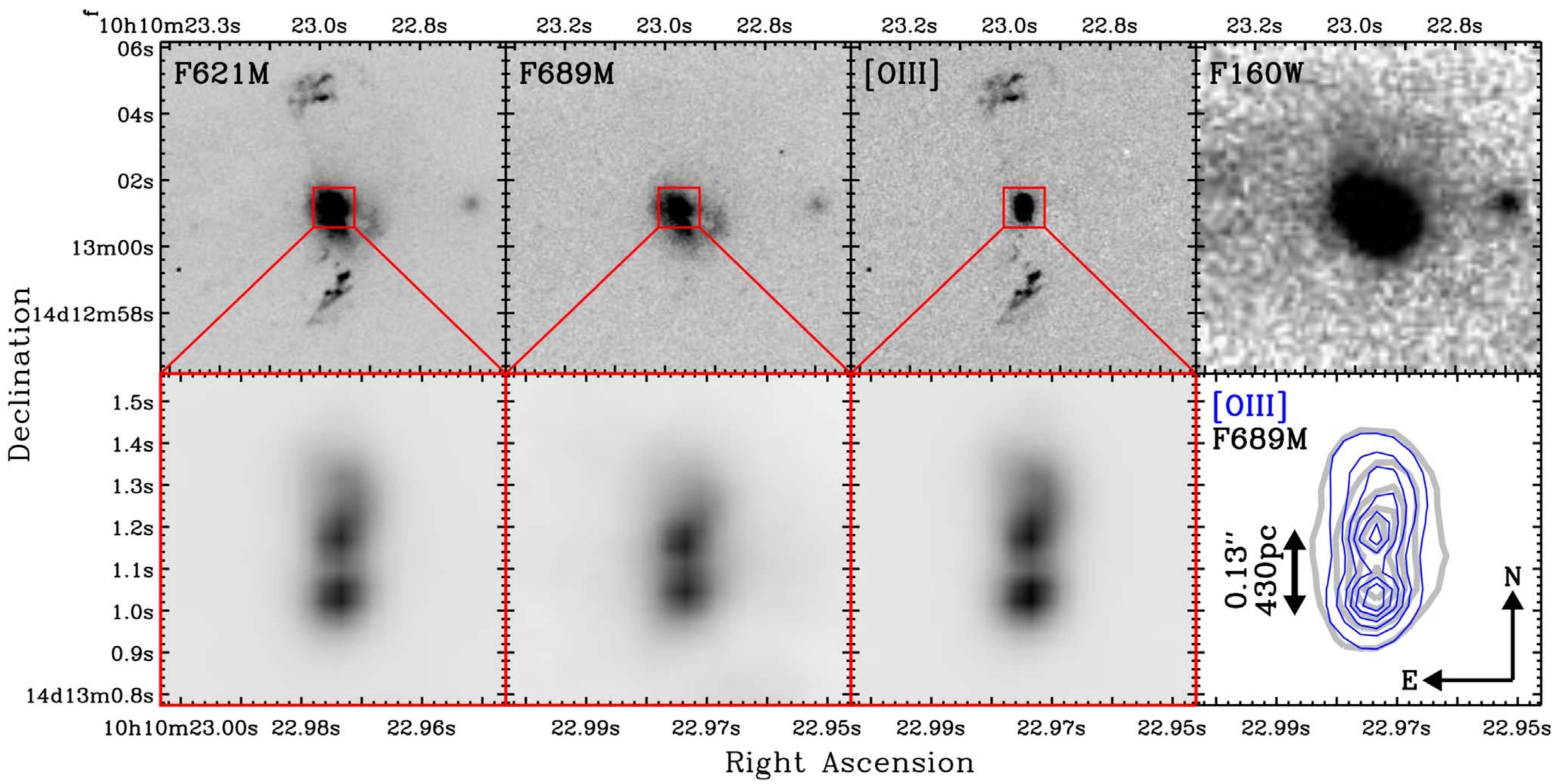
(from Koss+12, ApJL, 746, L22)



(from Comerford+09, ApJ, 702, L82

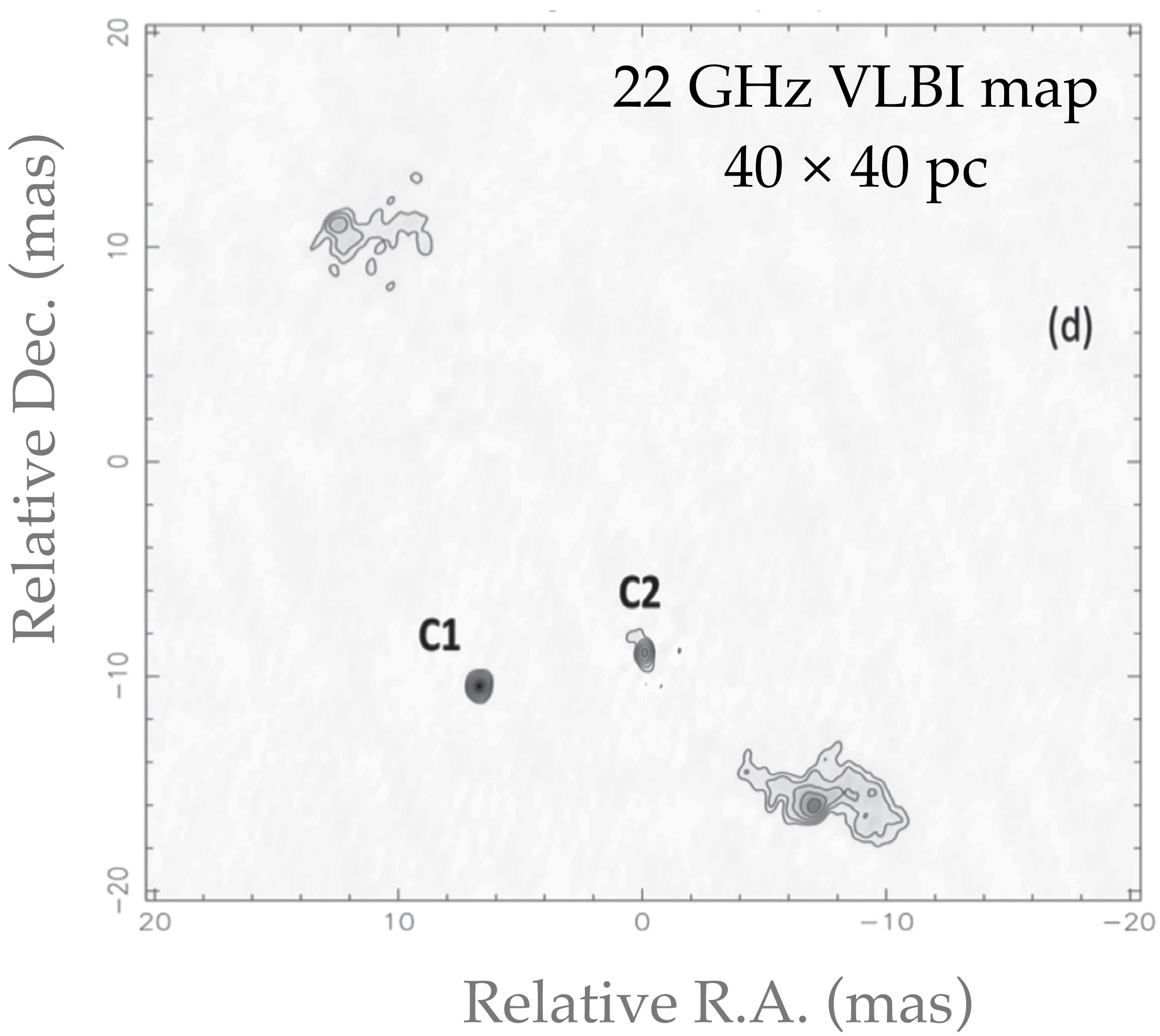
Sub-kpc binary in SDSS J1010+1413

HST / WFC3 imaging



(from Gouldring+19, ApJL, 879, L21)

4C+37.11, a.k.a. CSO 0402+379



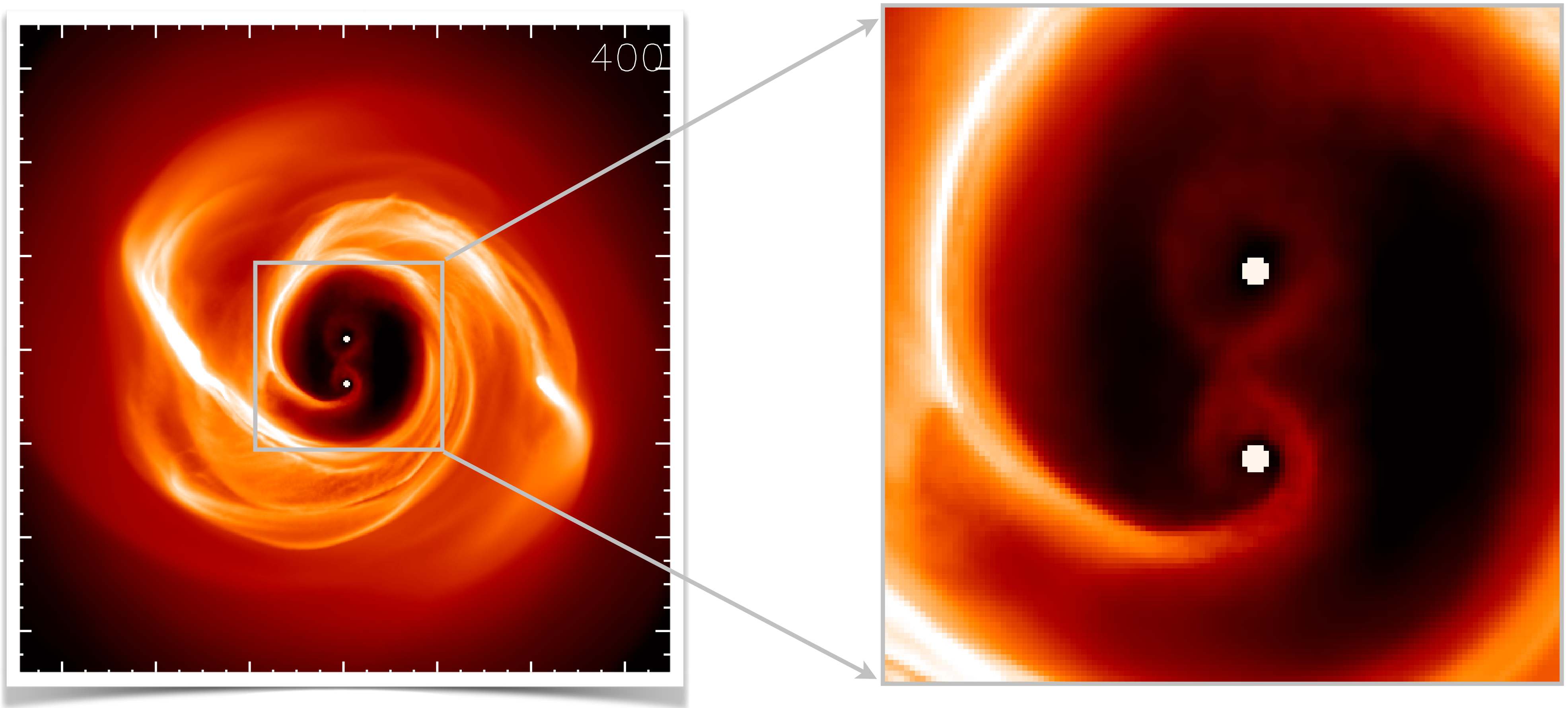
(from Bansal+17, ApJ, 843, 14)

- Separation 7.3 pc @ z = 0.055
- Small proper motion detected in 12 years
- Infer (assuming circular orbit):
  - $P = 3\times10^4$ yr
  - $M = 1.5\times10^{10}\ M_\odot$

Limited by angular resolution since 1 pc → 0.3 mas @ *z* = 0.2

Possible observational signatures of the (uncertain) final phases

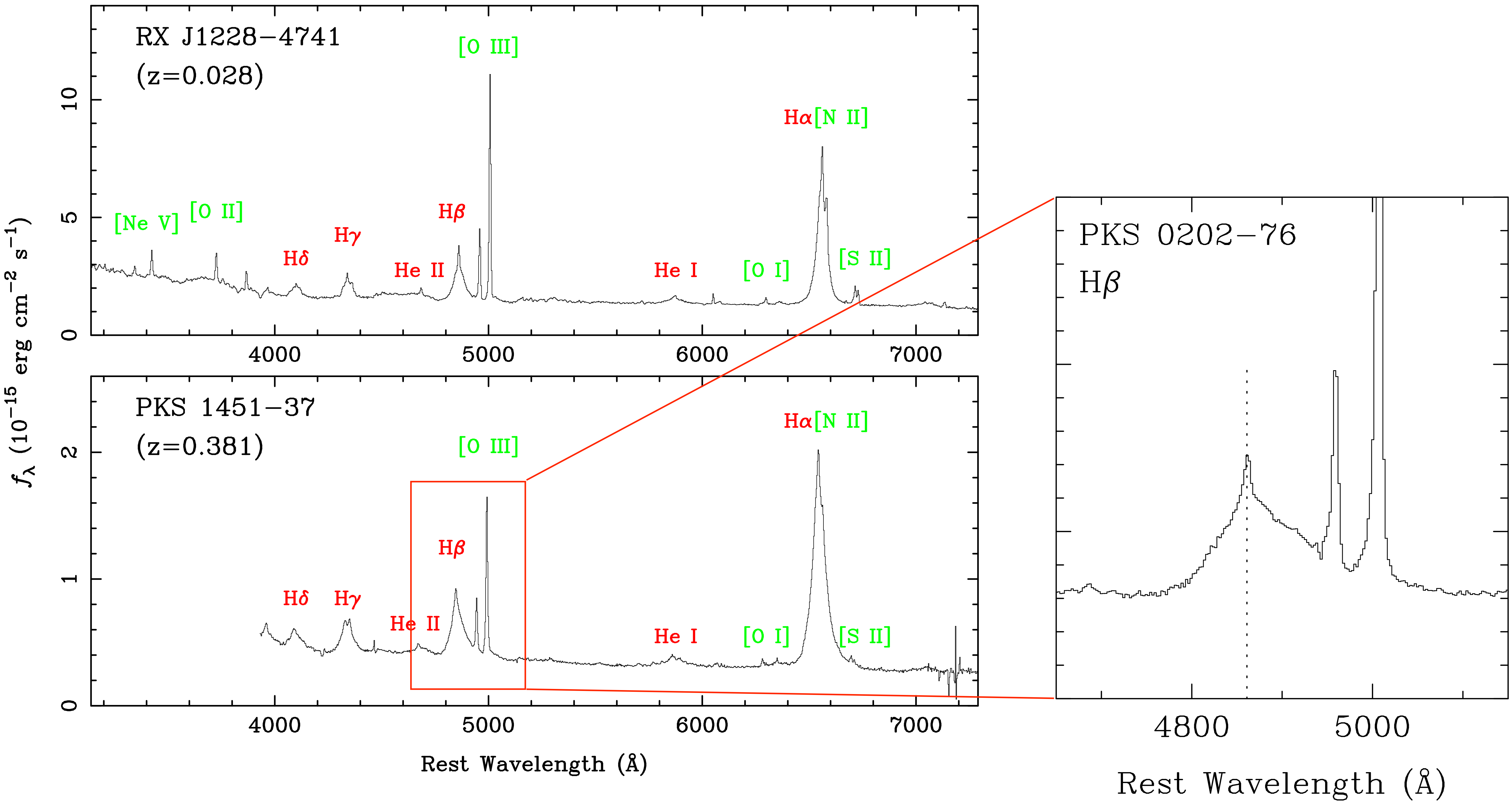
Geometry of accretion flow and possible signatures



- Binary period introduces a characteristic time scale ➡ photometric variations
- Gas bound to individual black holes follows them in their orbits ➡ spectroscopic variations (line strengths and profiles)

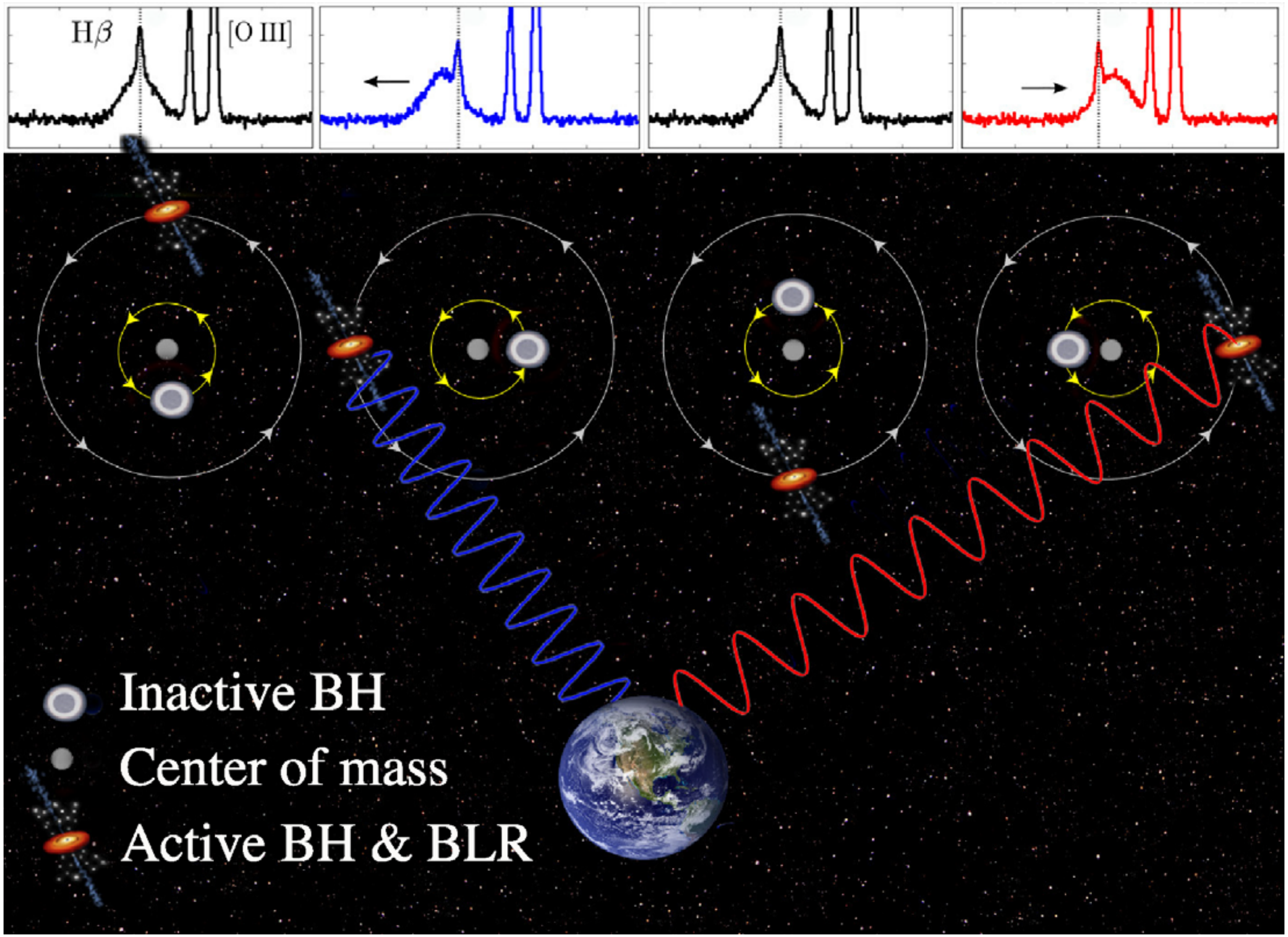
(figure from Cuadra+09, MNRAS, 393, 1423)

Examples of optical spectra of quasars

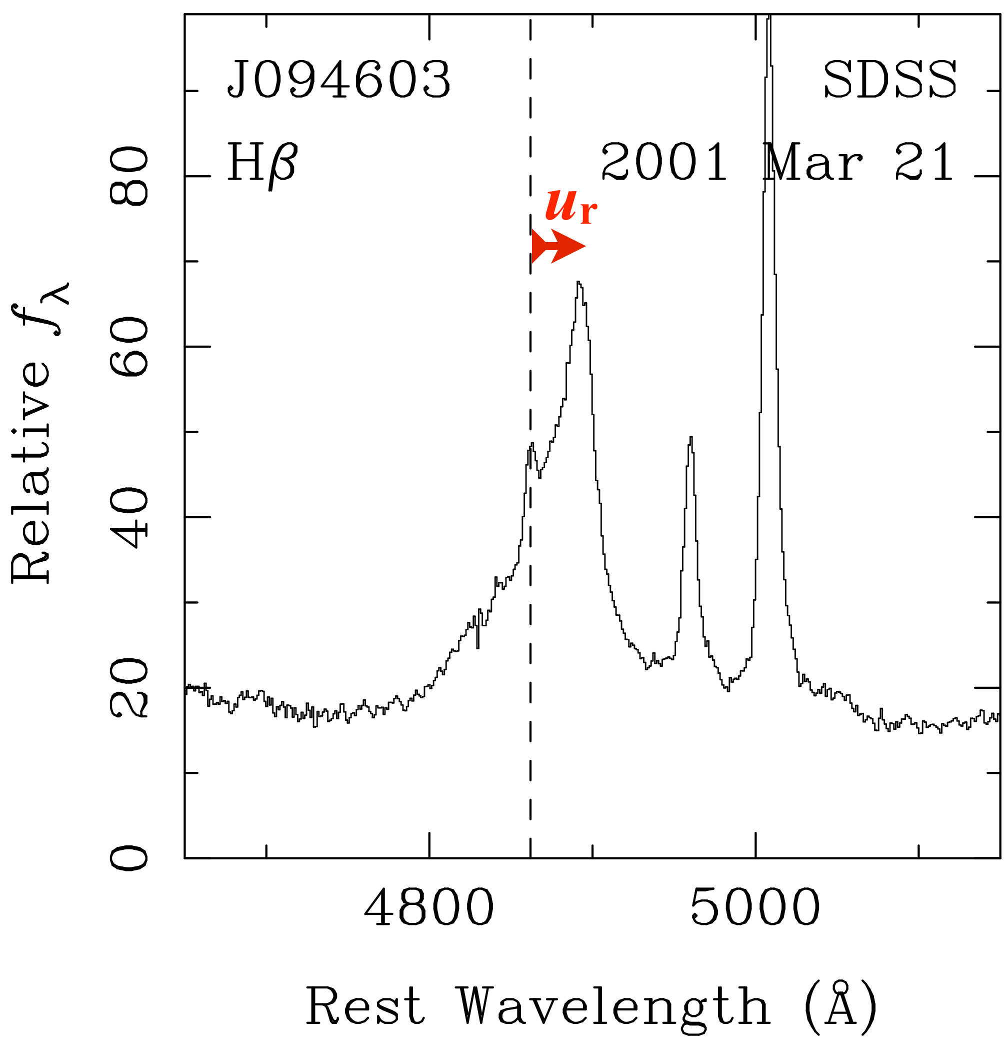


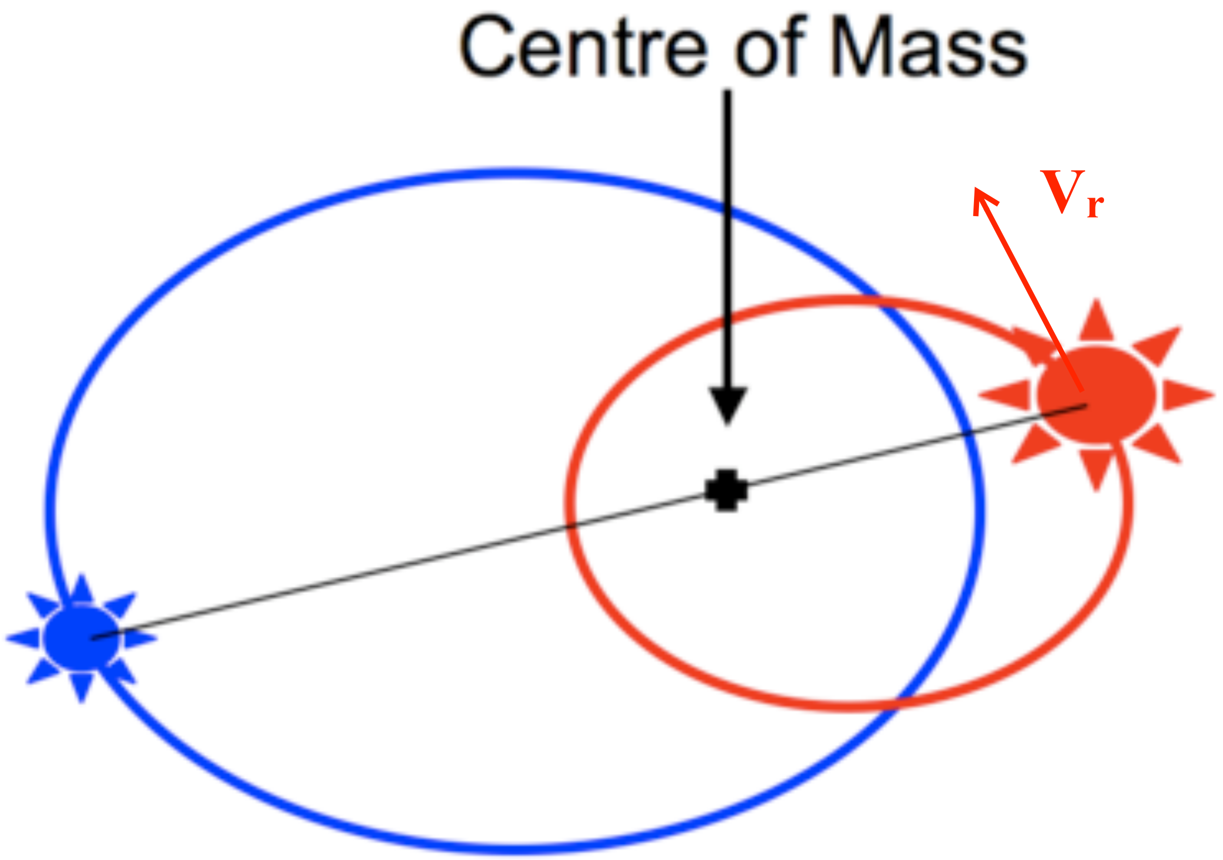
Principles of the radial velocity test

Figure from Guo+19, MNRAS, 482, 3288

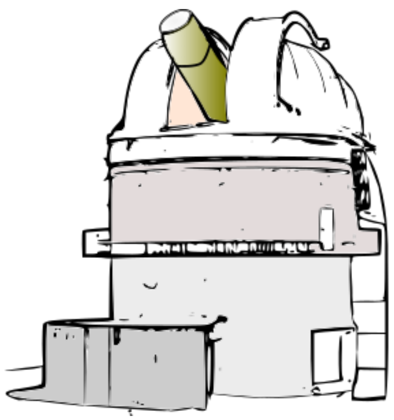


Examples of shifted line profiles

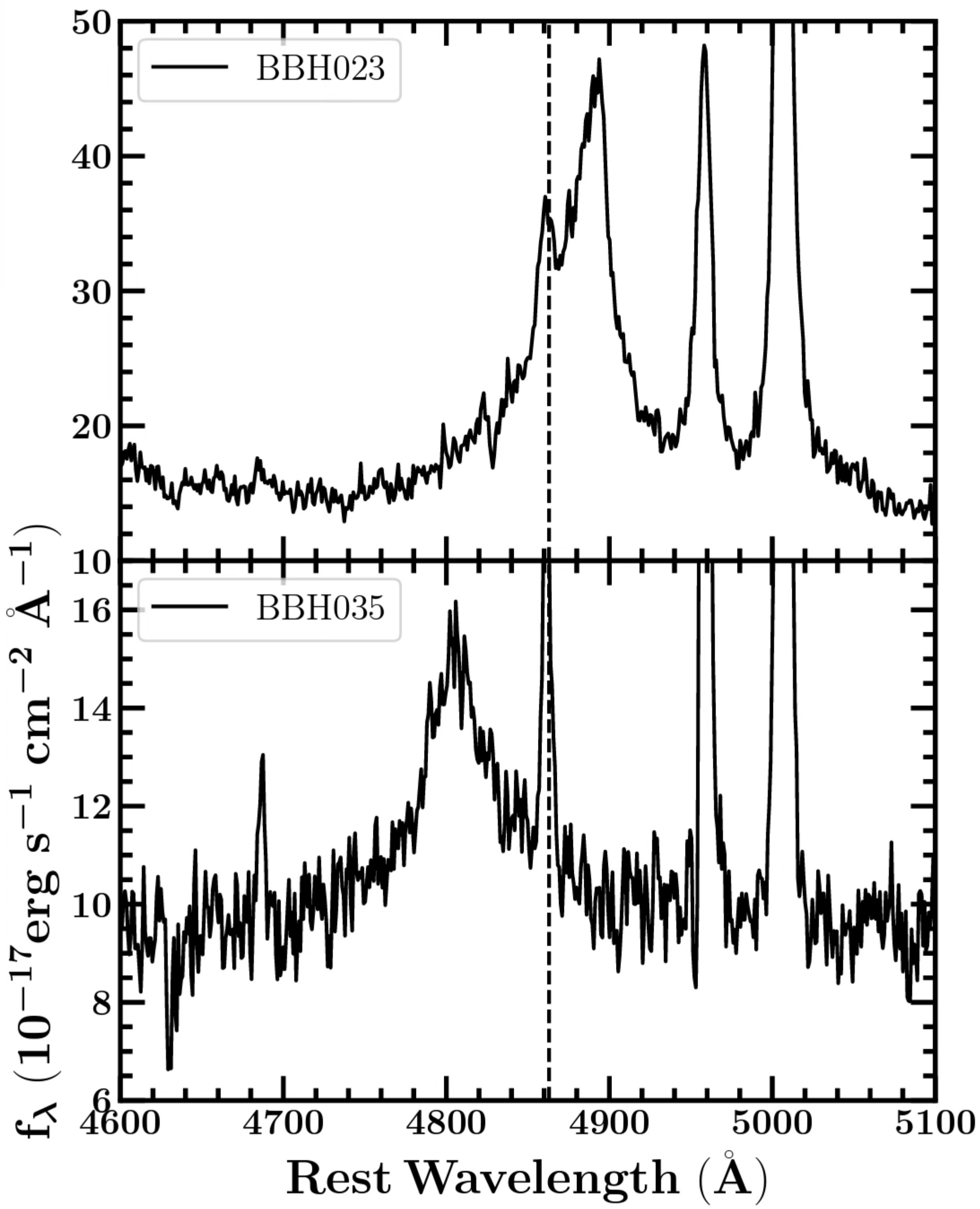




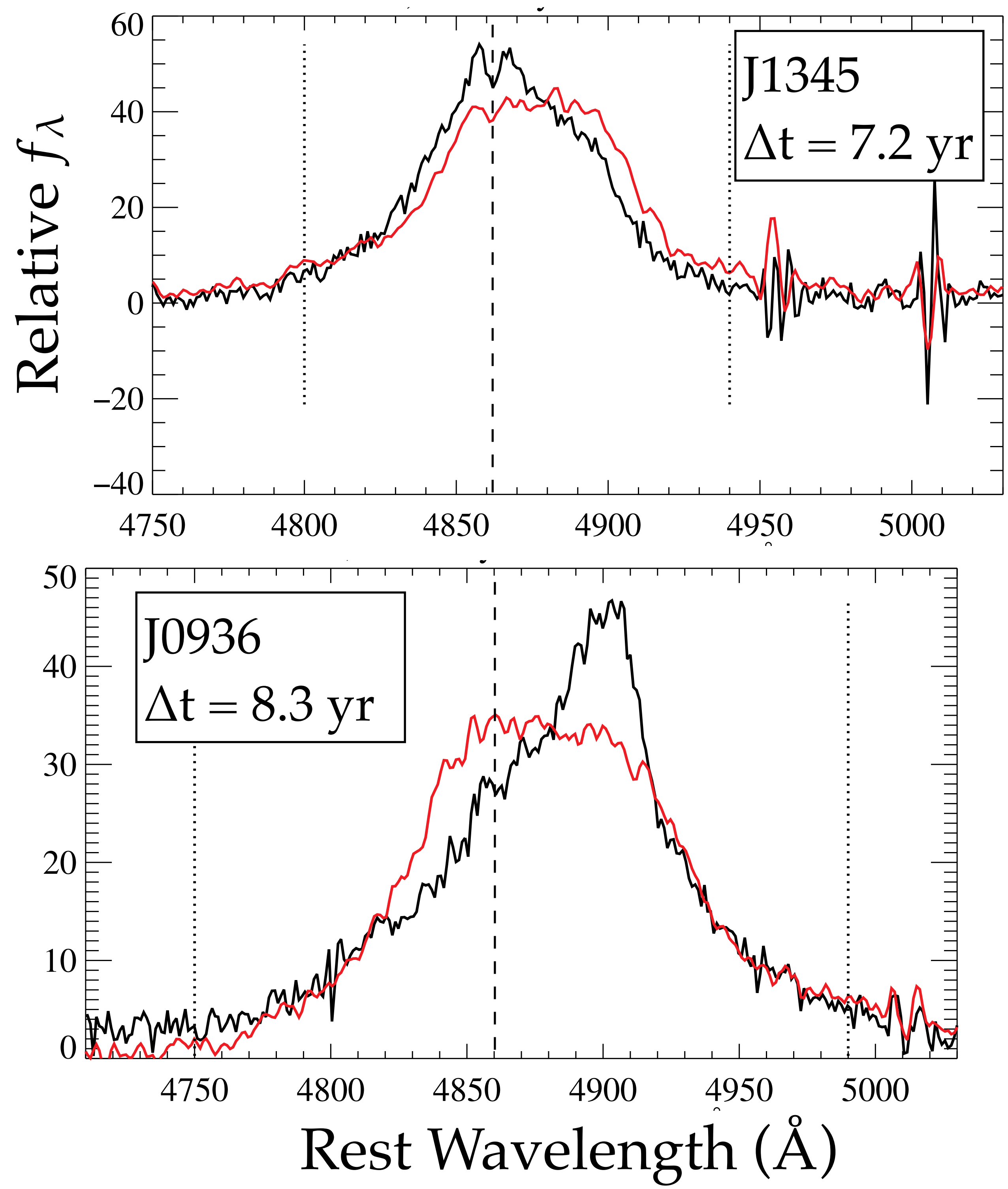
(spectrum from Eracleous+12, ApJS, 201, 23)



Examples of variations of shifted line profiles

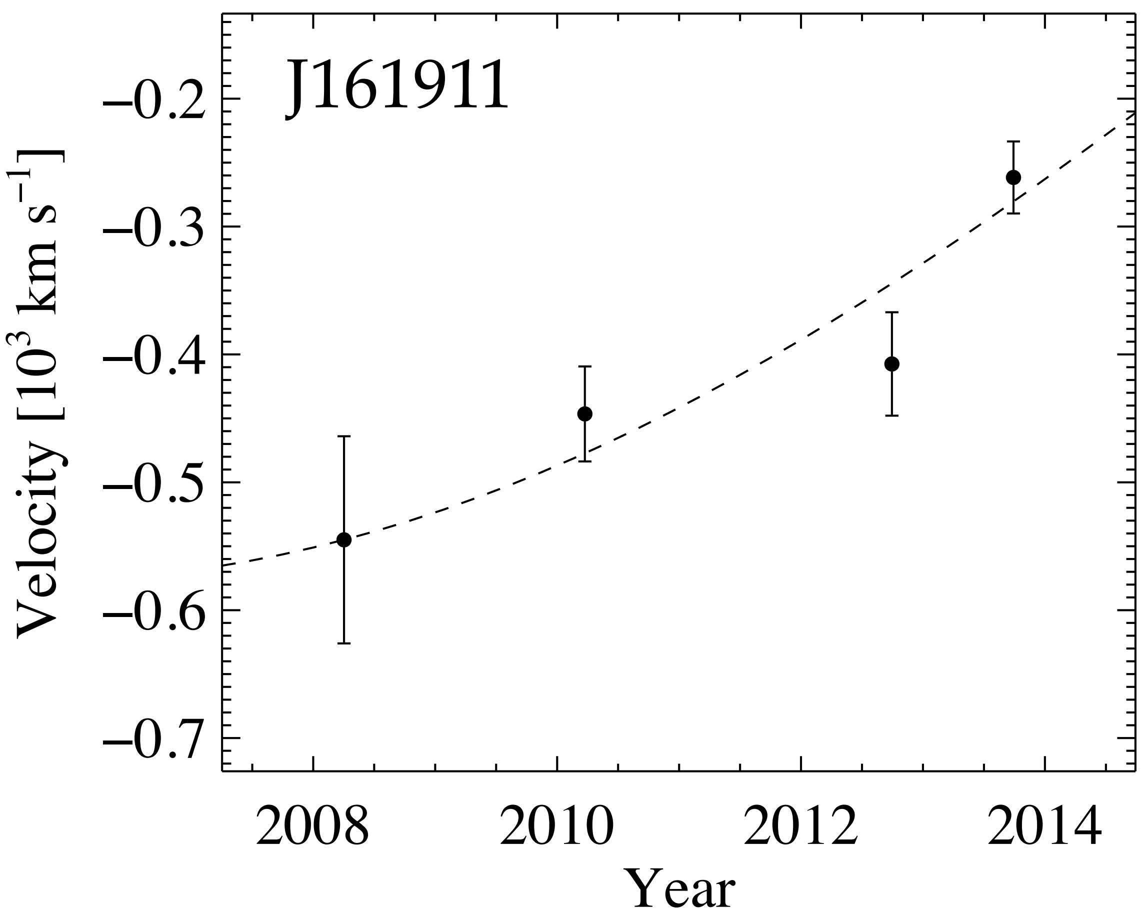


(from Eracleous+12, ApJS, 201, 23)

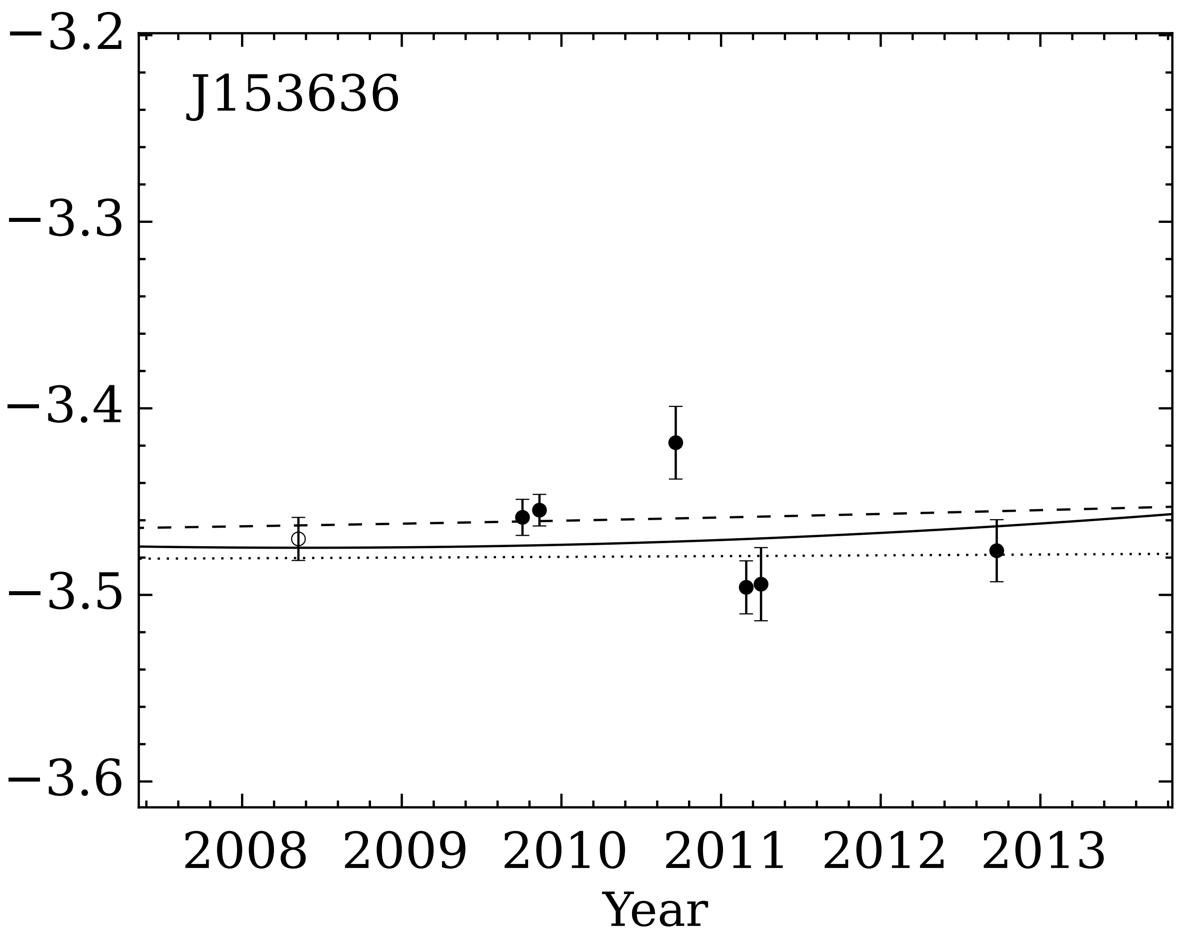


(from Liu+12, ApJ, 789, 140)

45 promising candidates from Runnoe+17 and Guo+19



Example of significant radial velocity variations



Example of radial velocity variations consistent with zero

(Figures from Runnoe+17, ApJS, 201, 23)

The main limitation in most methods is ambiguity.

Ambiguity comes from incomplete understanding of the underlying physical processes that shape the observational appearance of “typical” quasars.

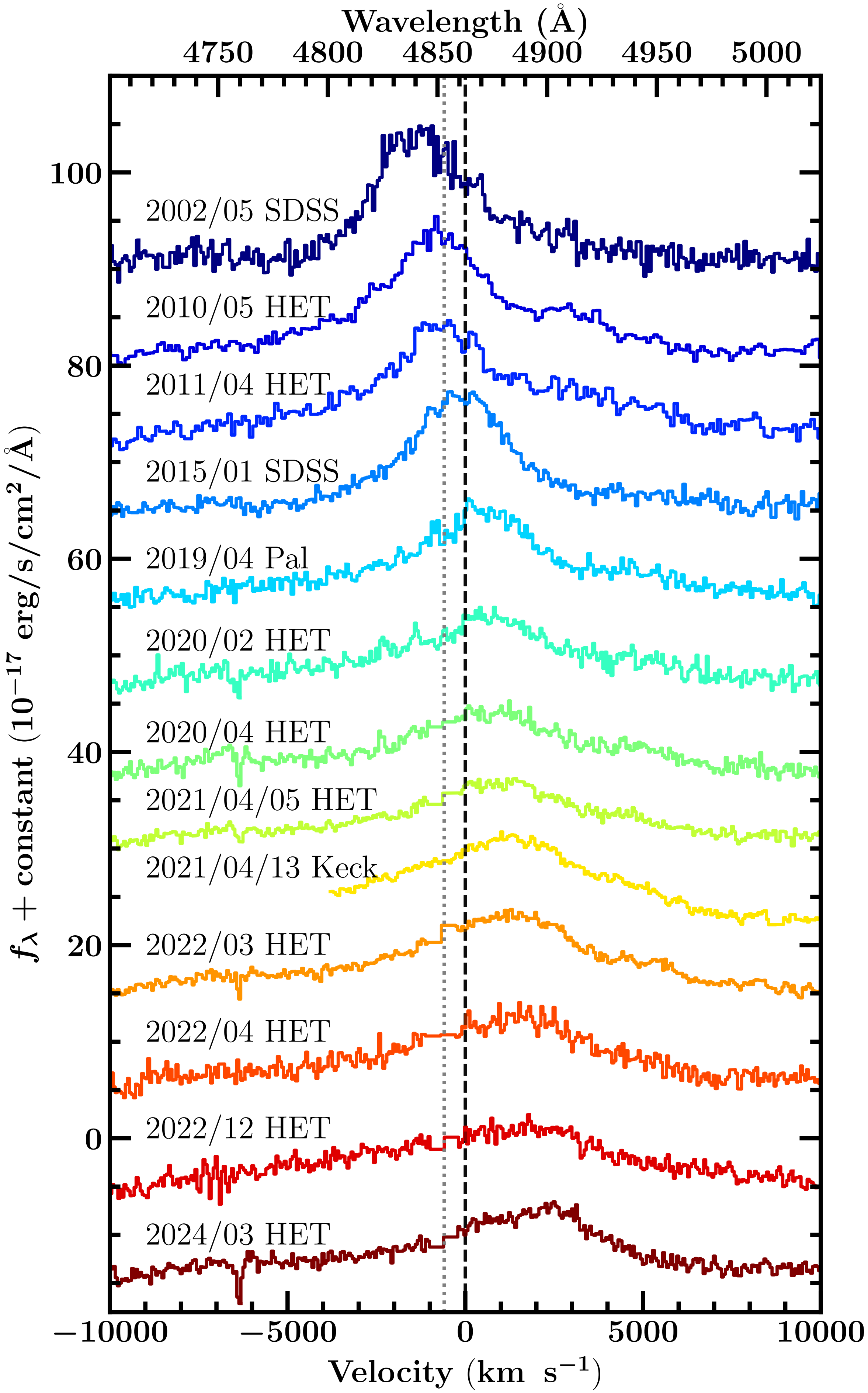
For every theoretical prediction of what a **binary** black hole will do, we have to be sure that a **single** black hole in a typical quasar cannot do the same thing…

Spectroscopy (work by our group)

- Displaced peaks of broad emission lines
  - Observe $\Delta v \sim 1000$ km s^{-1} and assume $M{\sim}10^8\,M_\odot$
    $\rightarrow$ $a \sim 0.1$ pc, P ~ 300 yr, $dv/dt \sim 20$ km s^{-1} yr^{-1}
  - cf., P ~ few yr from photometric methods
- 2–3 dozen candidates selected after long-term spectroscopic monitoring. One of them promising!
- but…
  - Orbital periods expected to be very long, unlikely to observe complete cycle in a human lifetime
  - Not sure if displaced lines are the signature to look for
  - Profiles may vary on much shorter time scales than orbital period

Status of spectroscopic searches in summary

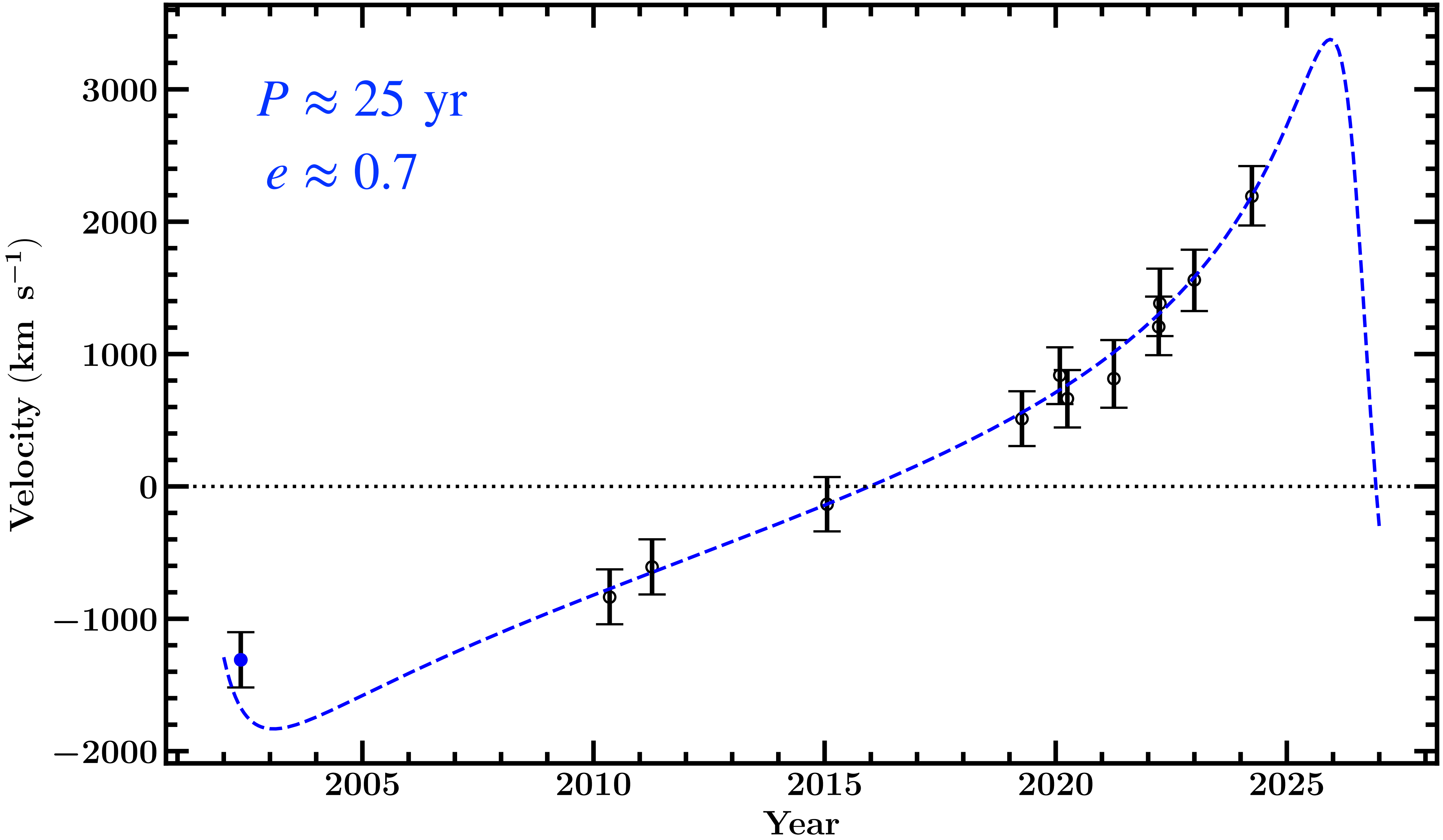
- Double-peaked emission lines not consistent with orbital motion
- Single-peaked offset lines from Runnoe+17 and Guo+19
  - Monitoring of 2–3 dozen candidates
  - Consistent with: a~0.1 pc and P~100 yr, $M_\bullet$ > few×10^7 $M_\odot$
  - One candidate appears particularly promising at the moment
- Looking for corroborating evidence via radio observations (Breiding+21, ApJ, 914, 37)
  - VLBA imaging shows unresolved sources, a < 20 pc
  - Higher-frequency observations coming next to increase spacial resolution
- Comparison with gravitational wave background
  - Calibrating on the candidates selected so far we do not violate the constraints from the GW background.



The best spectroscopic candidate so far

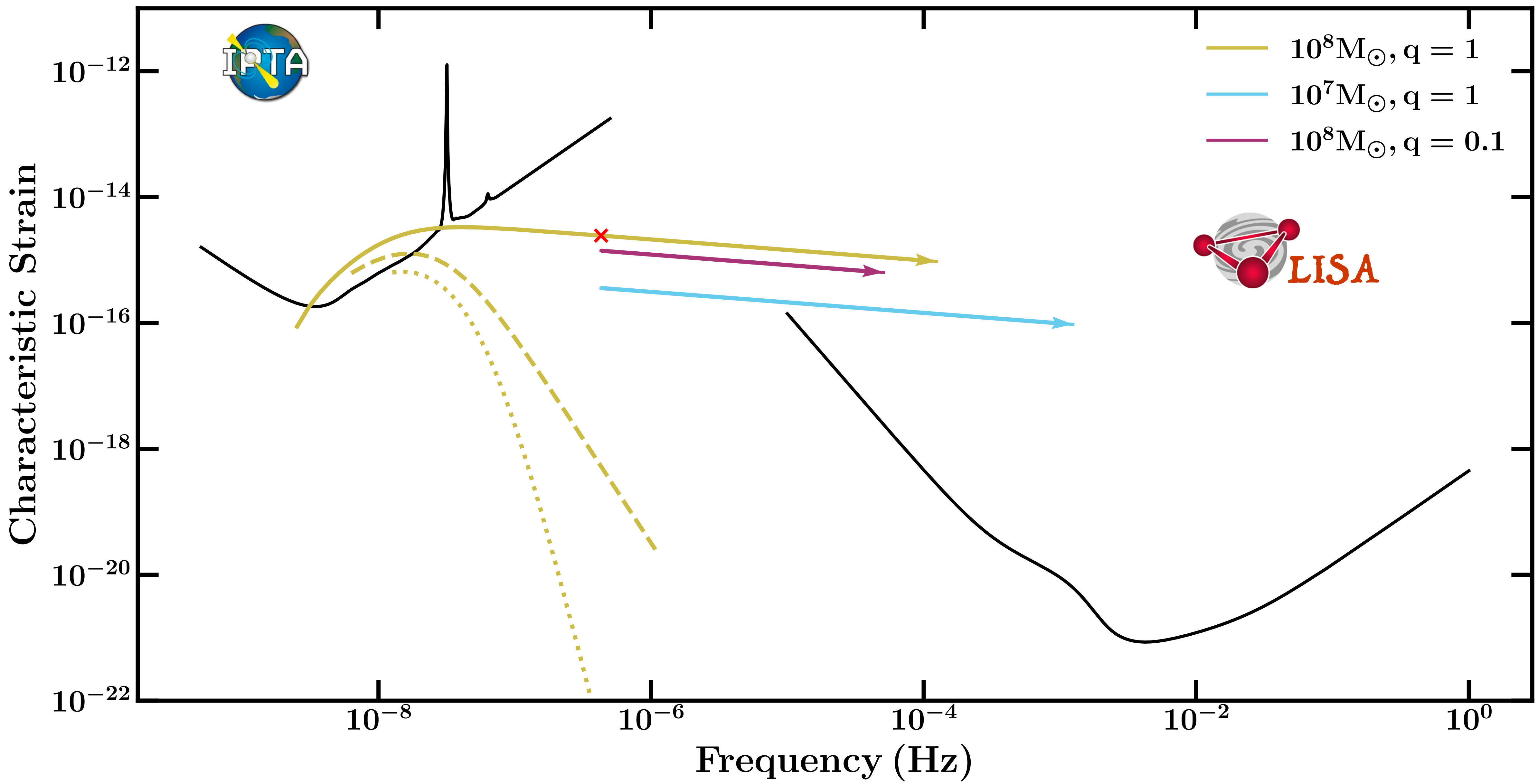
Work by Niana Mohammed
paper in preparation

Radial velocity curve and Keplerian model fit



Work by Niana Mohammed
paper in preparation

Projected evolution of J0950, assuming e=0, q=1, and $M = 10^8$ $M_\odot$

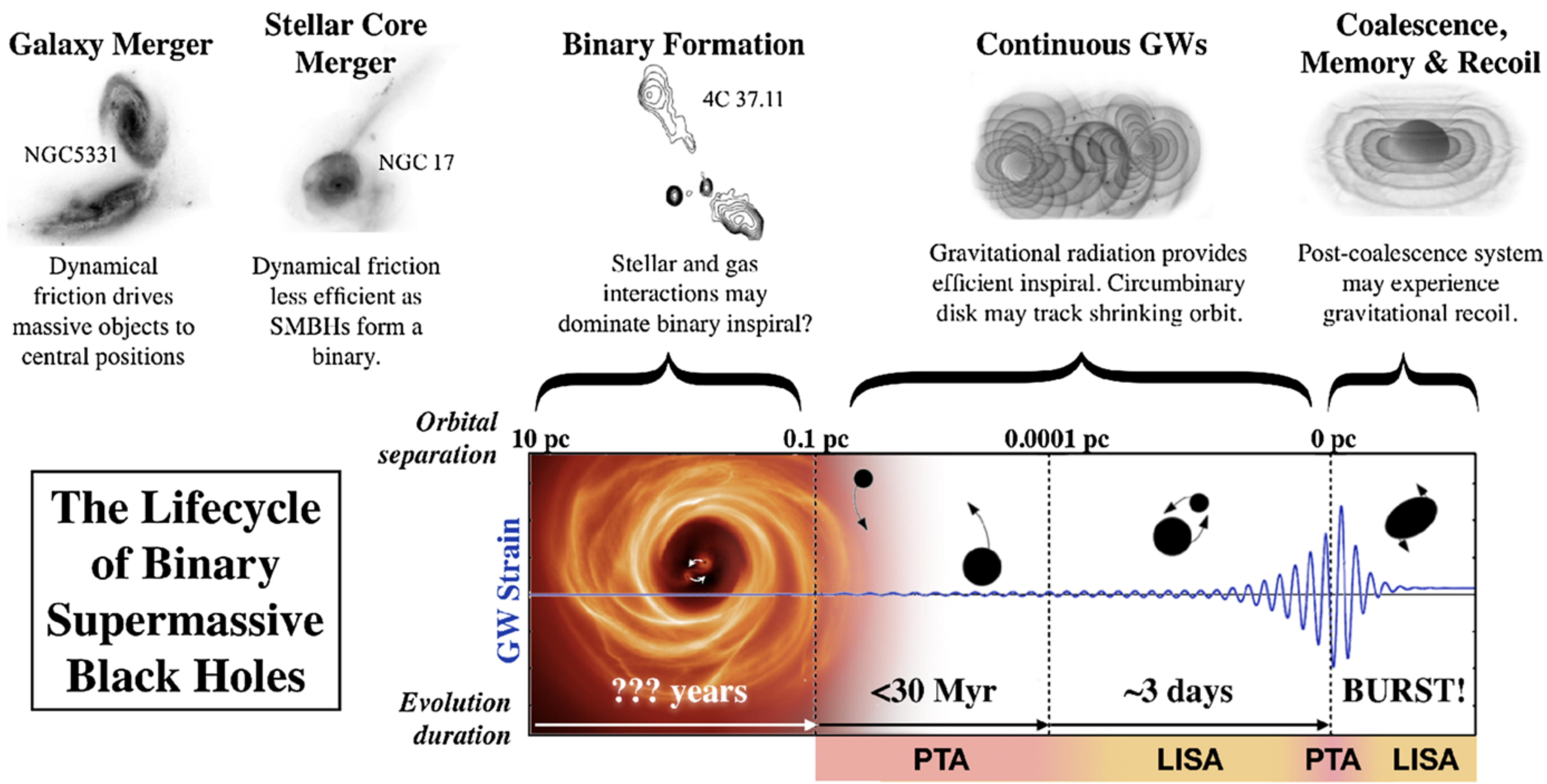


Work by Niana Mohammed
paper in preparation

The End (of this talk)

The real action begins in 10–15 years…

BACKUP SLIDES


Galaxy Merger
NGC5331
Dynamical friction drives massive objects to central positions
Stellar Core Merger
NGC 17
Dynamical friction less efficient as SMBHs form a binary.
Binary Formation
4C 37.11
Stellar and gas interactions may dominate binary inspiral?
Continuous GWs
Gravitational radiation provides efficient inspiral. Circumbinary disk may track shrinking orbit.
Coalescence, Memory & Recoil
Post-coalescence system may experience gravitational recoil.
The Lifecycle of Binary Supermassive Black Holes
Orbital separation
10 pc
0.1 pc
0.0001 pc
0 pc
GW Strain
??? years
<30 Myr
~3 days
BURST!
Evolution duration
PTA
LISA
PTA
LISA

E.M. probes of five stages of the journey of a $2\times(10^6\ M_\odot)$ supermassive binary from ~10 pc to coalescence.

separation

evolution

a	P_{orb}	f_{GW}	E.M. Observability
10 pc	2 Myr	30 fHz	resolved by radio interferometry (proper motion may be observable)
0.1 pc	2 kyr	30 pHz	spectroscopy (emission line shifts) infra-red interferometry
2 mpc	6 yr	11 nHz	(f_{GW} within PTA band) modulation of optical light curves attendant spectroscopic variations
2 μpc	1.6 hr	0.3 mHz	(entering LISA band) fast modulation of X-ray light curves
0.1 μpc	1 min	33 mHz	(chirp and merger in LISA band) polychromatic E.M. flare

Potential E.M. signatures of supermassive binaries at separations: a ~ pc – μpc

- Direct imaging via radio interferometry

  Burke-Spolaor+11, MNRAS, 410, 2113; Bansal+17, ApJ, 843, 14; Kharb 217, Nat. Ast., 1, 727, Breiding+21, ApJ, 914, 37

  observational searches

- Radial velocity variations of broad emission lines

  Runnoe+17, ApJS, 201, 23; Guo+19, MNRAS, 482, 3288; Decarli+13, MNRAS, 433, 1492; Wang+17, ApJ, 834, 129

  observational searches

- Infrared interferometry (shift of photocenter)

  Dexter+20, ApJ, 905, 33

  predictions only

- Modulation of optical light curves

  Graham+15, MNRAS, 453, 1562; Charisi+16, MNRAS, 463, 2145; Liu+2019, ApJ, 884, 36; Vaughan+16, MNRAS, 461, 3145

  observational searches

- Reverberation (various forms)

  Wang+18, ApJ, 862, 171; Ji+21, ApJ, 910, 101

  predictions only

Potential E.M. signatures of supermassive binaries at separations: a ~ pc – μpc (*continued*)

- Combination of photometric and radial velocity modulations

  Bon+12, ApJ, 759, 118; Li+16, ApJ 822, 1

  observational searches

- Unusual relative intensities and profiles of broad lines

  Montuori+11, MNRAS, 412, 26 and 2012, MNRAS, 425, 1633

  predictions only

- Unusual spectral energy distribution because of mini- and circumbinary disks.

  Gükltekin & Miller 12, ApJ, 761, 90; Roedig+14, ApJ, 785, 115; Tang+18, MNRAS, 476, 2249

  predictions only

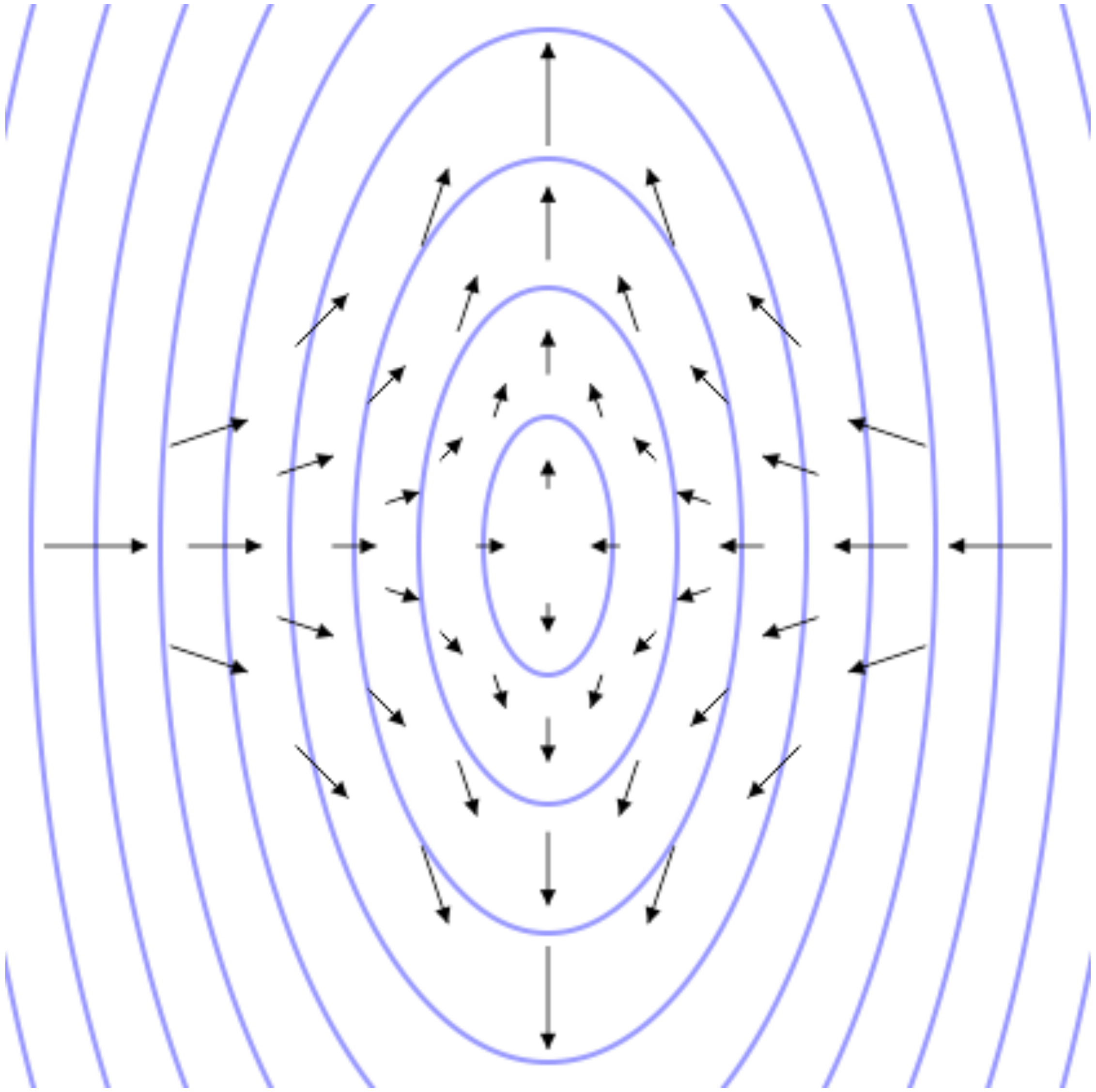
- Modulated extreme-UV / X-ray emission during late stages of inspiral and periodic shifts of X-ray emission lines.

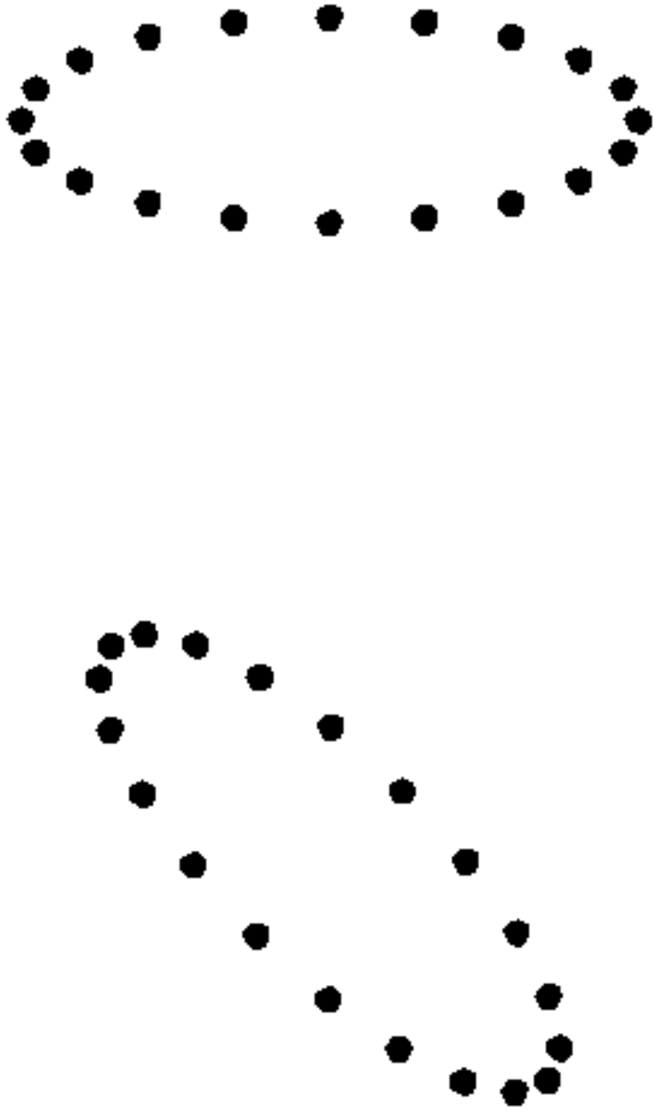
  Bode+10, ApJ, 715, 1117 and 2012, ApJ, 744, 45; McKernan+13, MNRAS, 432, 1468; d'Ascoli+18, ApJ, 865, 140

  predictions only

Speaking of gravitational waves…

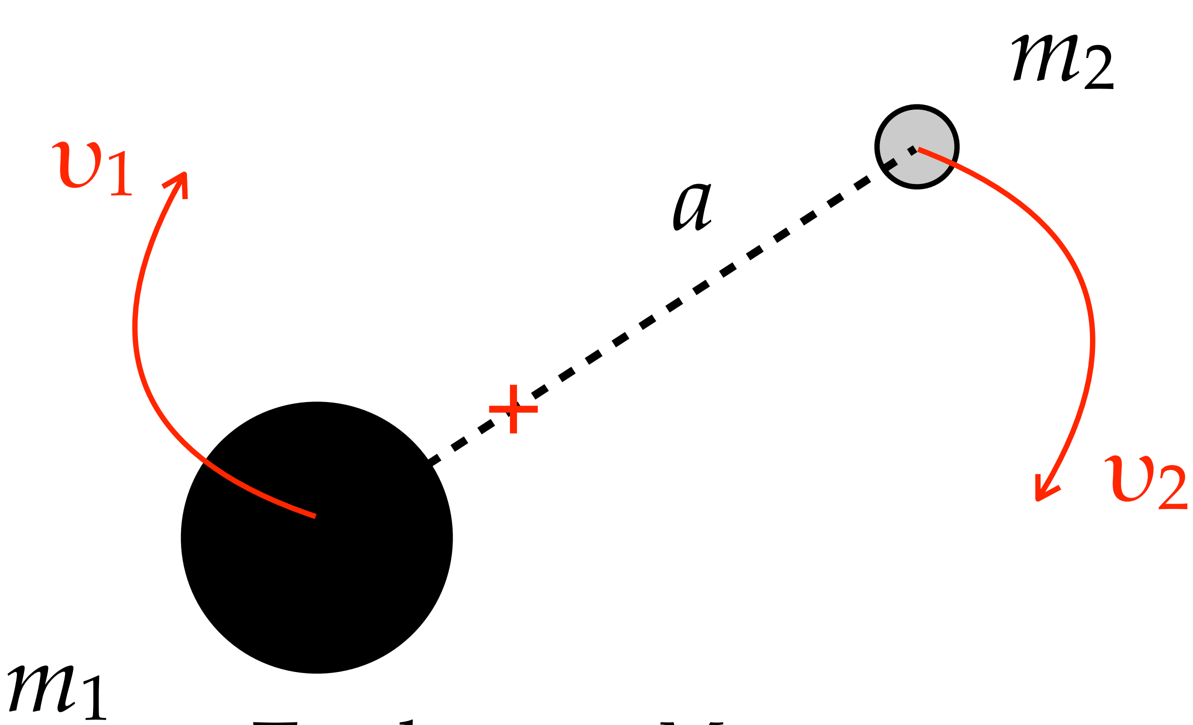
Illustrations of Deformation (from Wikipedia)





Linearly polarized wave (face on)

The Fiducial Binary System (in a **circular** orbit)



Total mass: $M = m_1 + m_2$

Mass ratio: $q = m_2/m_1 < 1$

$$t_{\rm decay} \sim \frac{c^5}{G^3}\frac{a^4}{M^3}$$

Gravitational Wave Frequency

$$f_{\rm GW} = \frac{2}{P} = \frac{1}{\pi}\left(\frac{GM}{a^3}\right)^{1/2}$$

At the end of inspiral $a \to$ ISCO

$$f_{\rm GW} = 4.38\left(\frac{M}{M_\odot}\right)^{-1} \text{ kHz}$$

- **LIGO**, $M \sim 60\ M_\odot \to 73$ Hz
- **LISA**, $M \sim 10^6\ M_\odot \to 4$ mHz
- **PTA**, $M \sim 10^9\ M_\odot \to 4\ \mu$Hz

LISA mission design

- 2.5 Million km arm length (0.0167 AU)
- Earth-trailing heliocentric orbit
- Passively maintained constellation of 3 spacecraft. Stable for 5 years. Nominal mission life time 4 years
- Adopted by the ESA in 2024. Nominal launch in mid 2030s
- Positional accuracy: > 10 arcmin²
- Approximately uniform time-averaged sky sensitivity

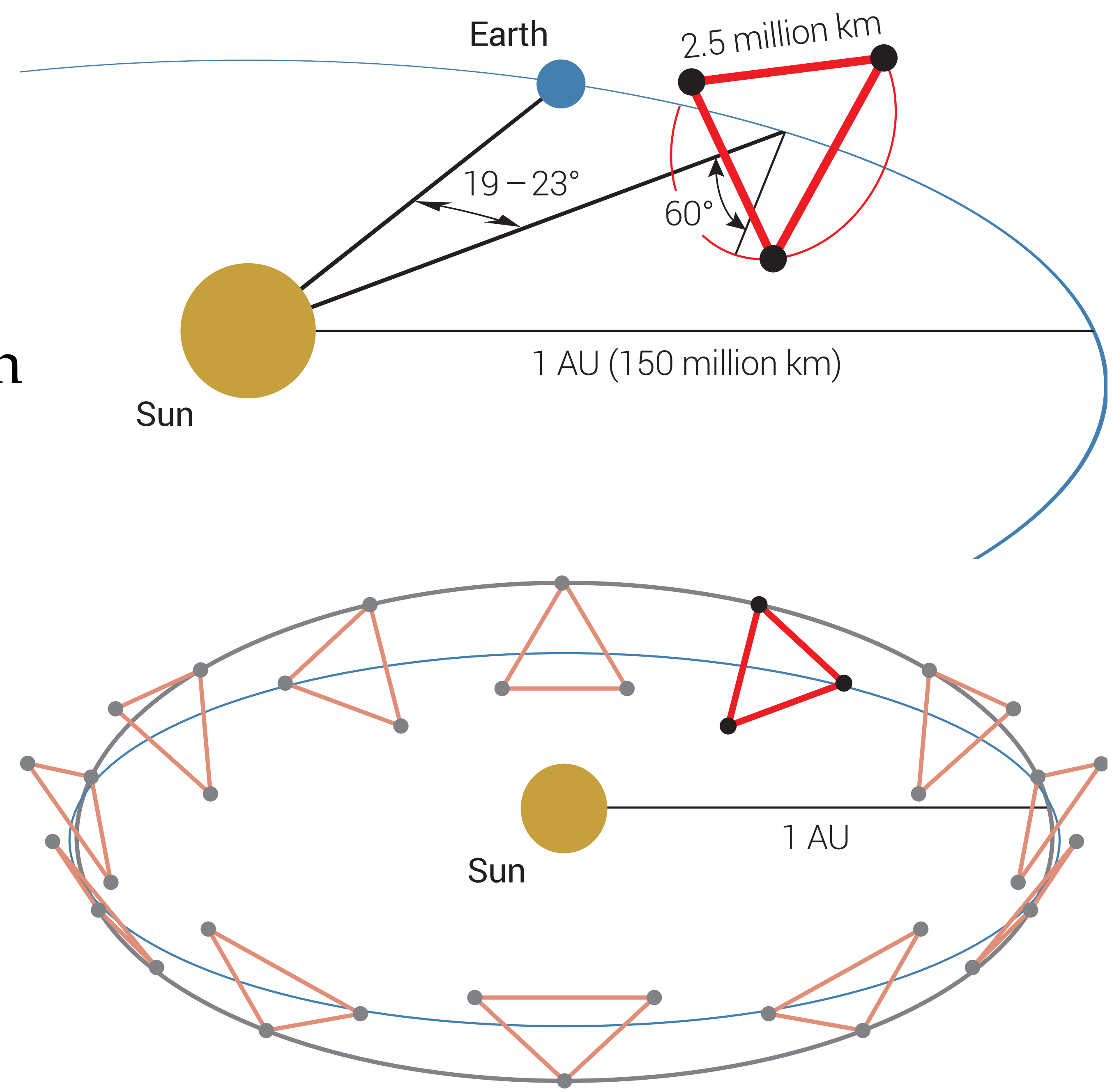


Illustration from Amaro-Seoane+17, arXiv:1702.00786

What will LISA be able to measure and how well?

- Signal detection and parameter estimation via matched filtering
  - can infer main orbital parameters (including eccentricity)
  - can estimate BH spin if a merger is observed
- Parameter accuracy scales with signal-to-noise ratio
- Can exploit many cycles of a periodic/regular signal and incorporate constraints from electromagnetic observations to reduce the uncertainties
- Expected uncertainties:
  - $\delta \mathcal{M} / \mathcal{M} \sim 0.01\text{–}1\ \%$ (chirp mass)
  - $\delta D_L / D_L \sim 3\text{–}10\ \%$ (luminosity distance)
  - $\delta\Omega \sim 10\ \text{arcmin}^2 - 10\ \text{deg}^2$ (position)

LISA sources and science themes illustrated

$$h_c \propto \frac{dh}{df}\sqrt{N_{cyc}}$$

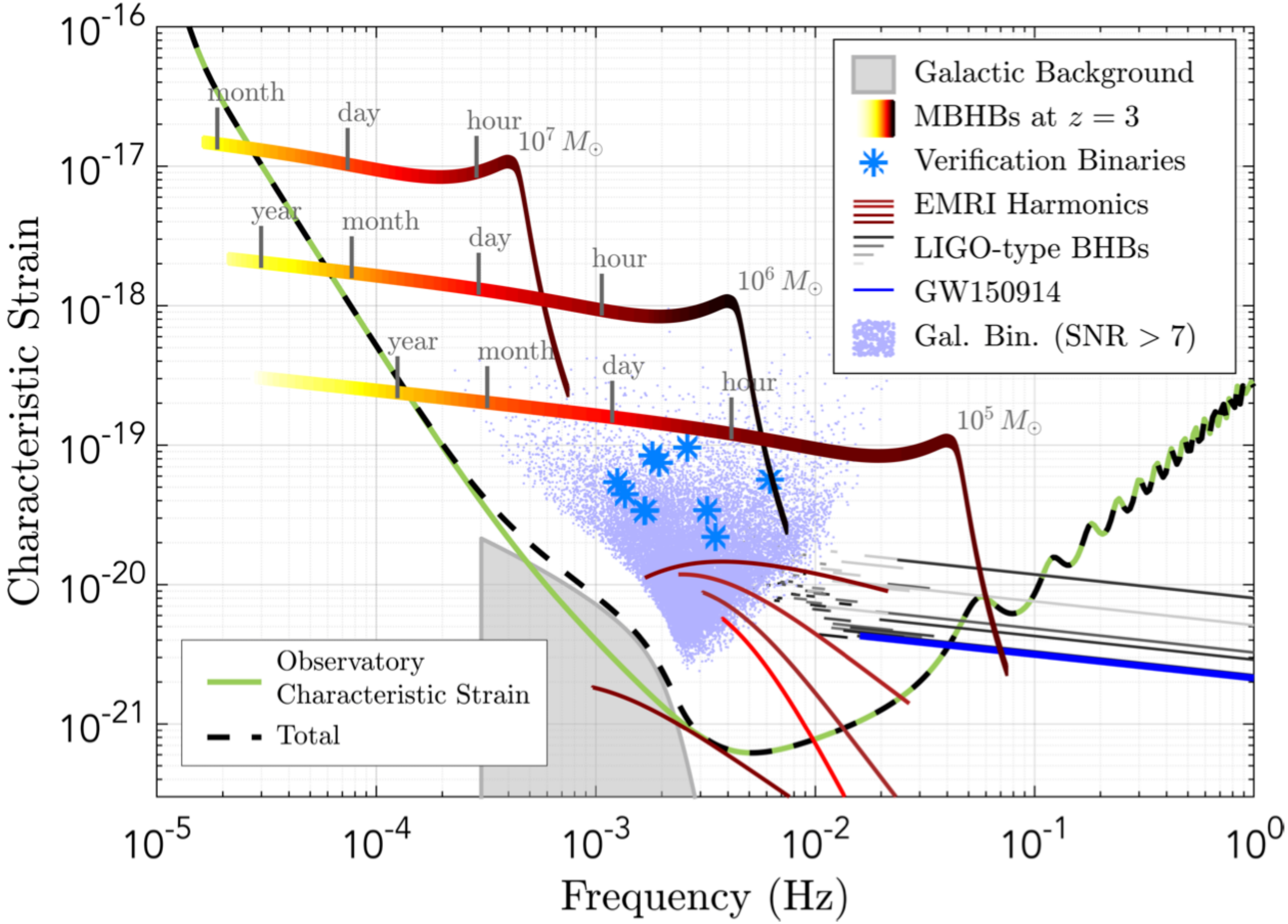
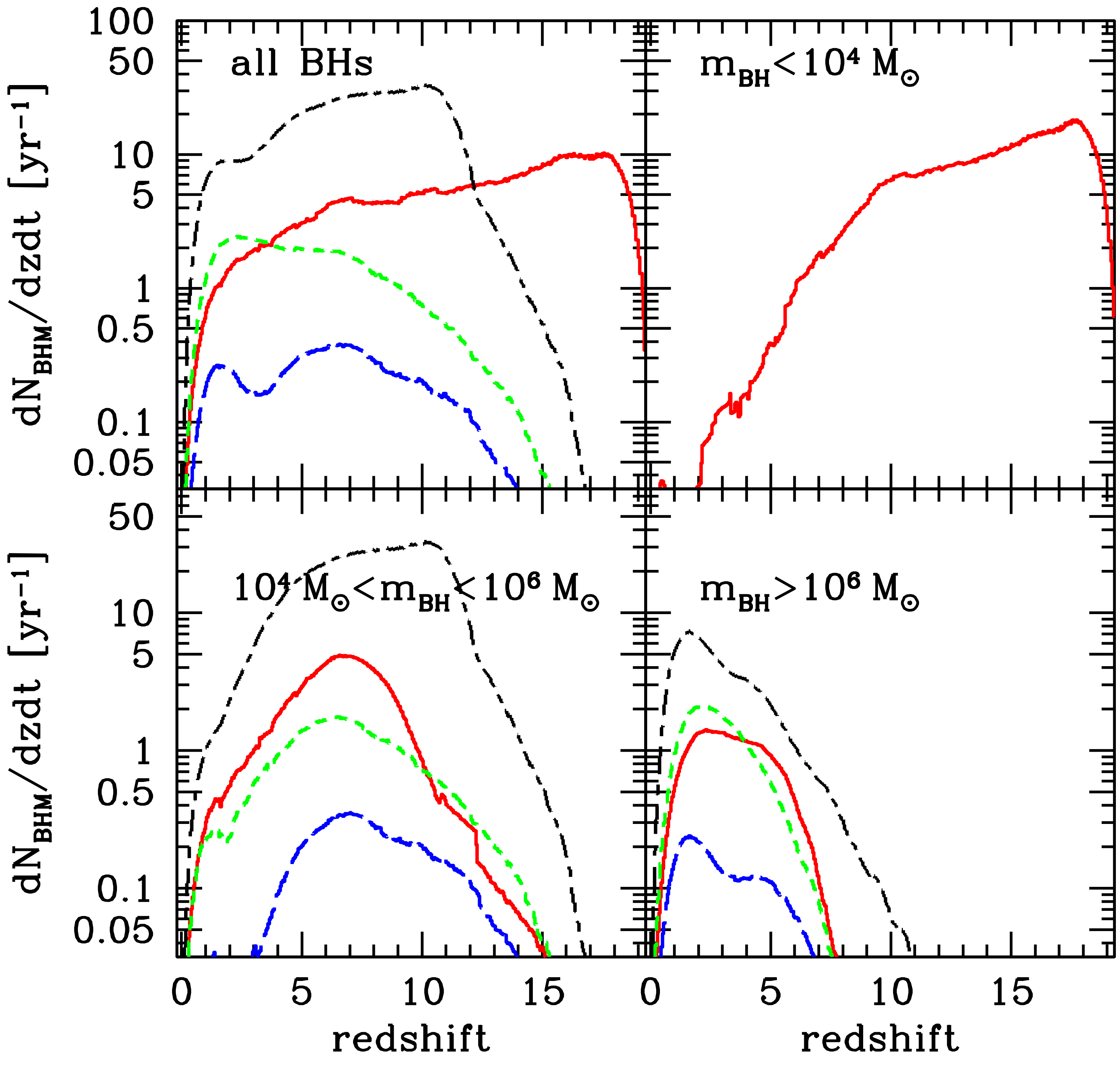


figure from LISA Proposal (K. Danzmann PI, arXiv:1702.00786)

Predicted SMBH Merger Rates (with large uncertainties)



(figure from Sesana+07, MNRAS, 377, 1711)

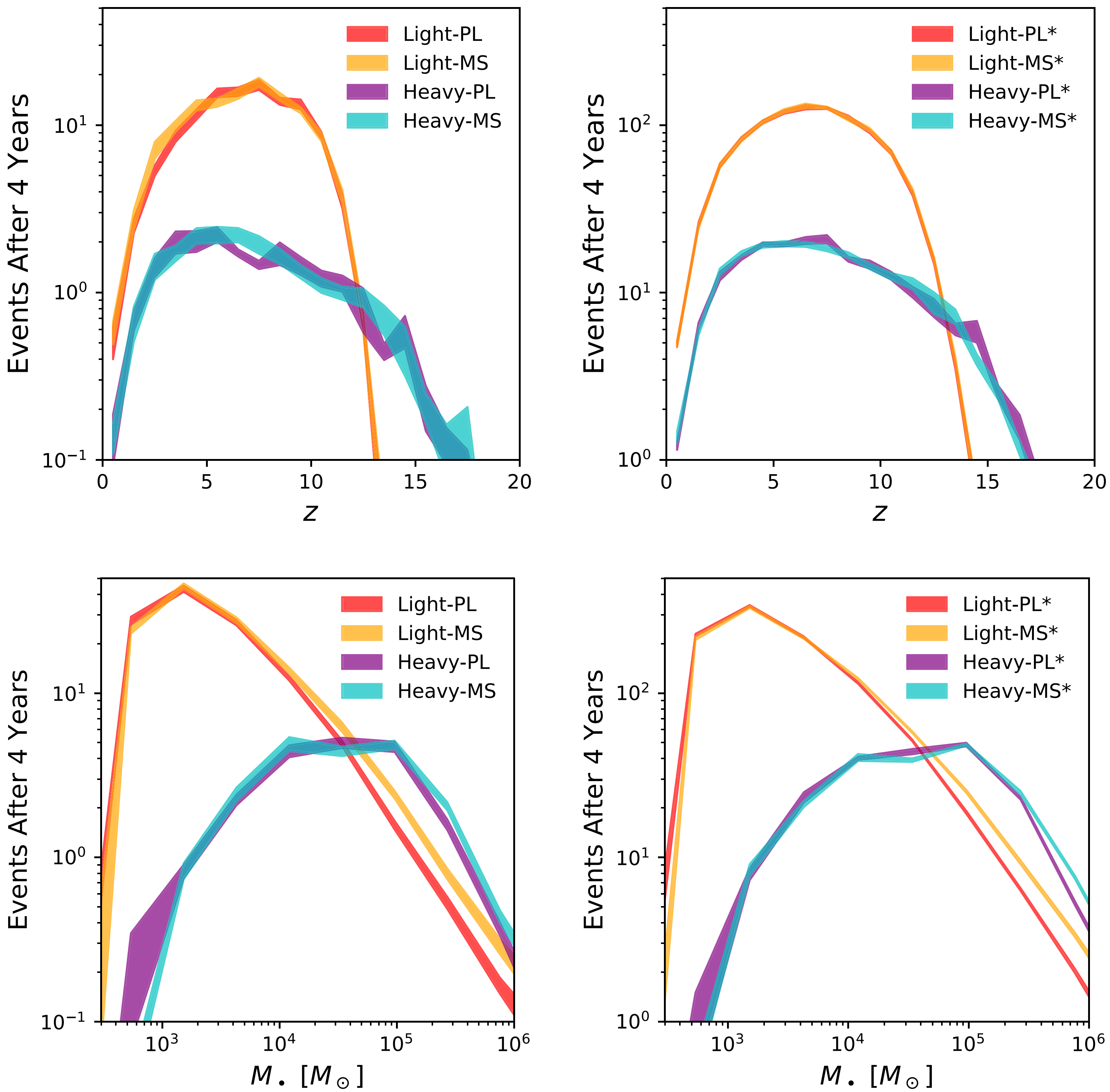
Red: ~100 $M_\odot$ seeds from first generation of stars

Black: ~10^4 $M_\odot$ seeds from direct collapse

Green: ~10^5 $M_\odot$ seeds from gravitational instabilities with slow cooling.

Blue: ~10^5 $M_\odot$ seeds from gravitational instabilities with rapid cooling.

Predicted SMBH Merger Rates (with large uncertainties)



Red: ~

Orange: ~

Purple: ~

Blue: ~

Ricarte & Natarajan 2018, MNRAS, 481, 3278

Merger histories of today's massive black holes

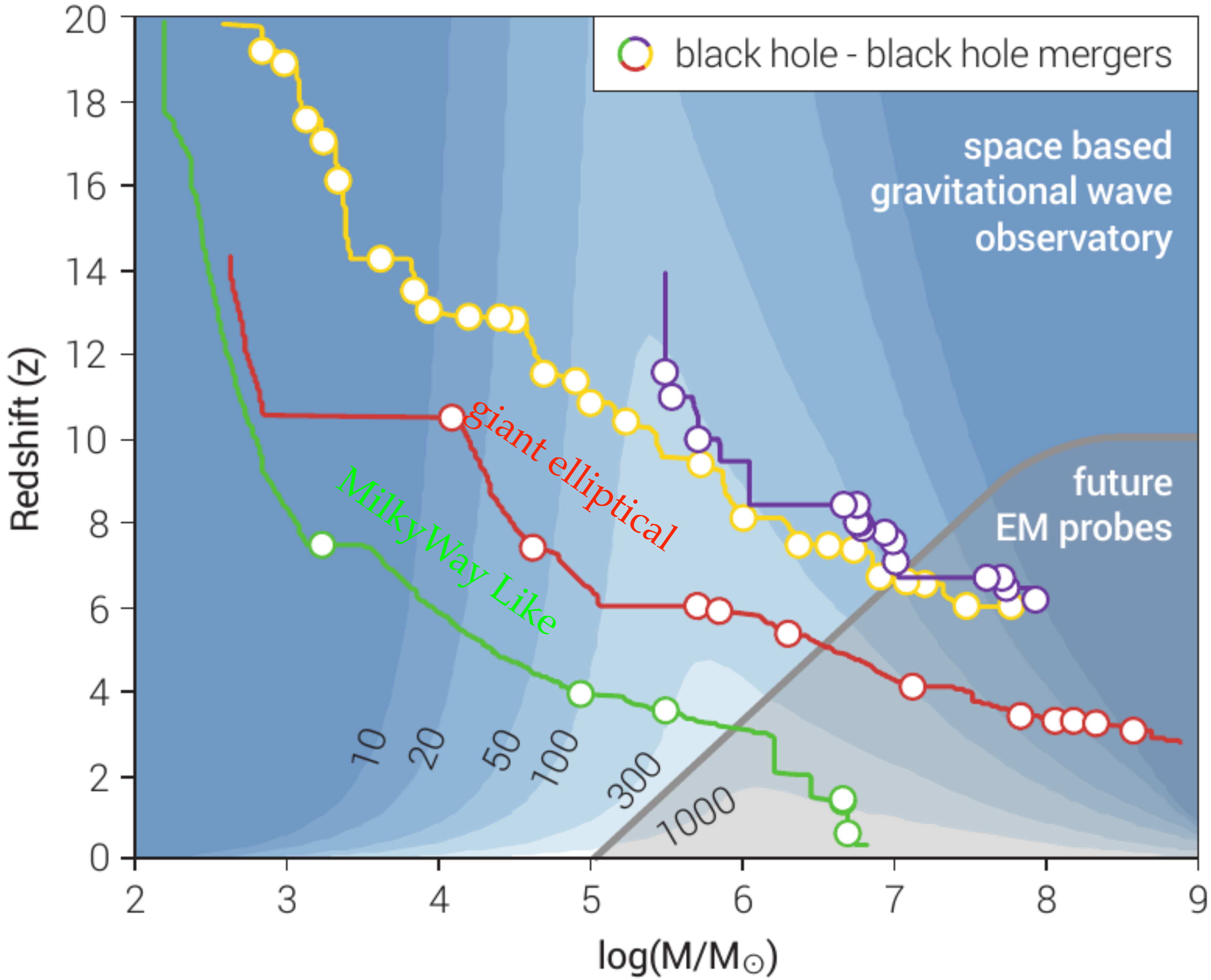
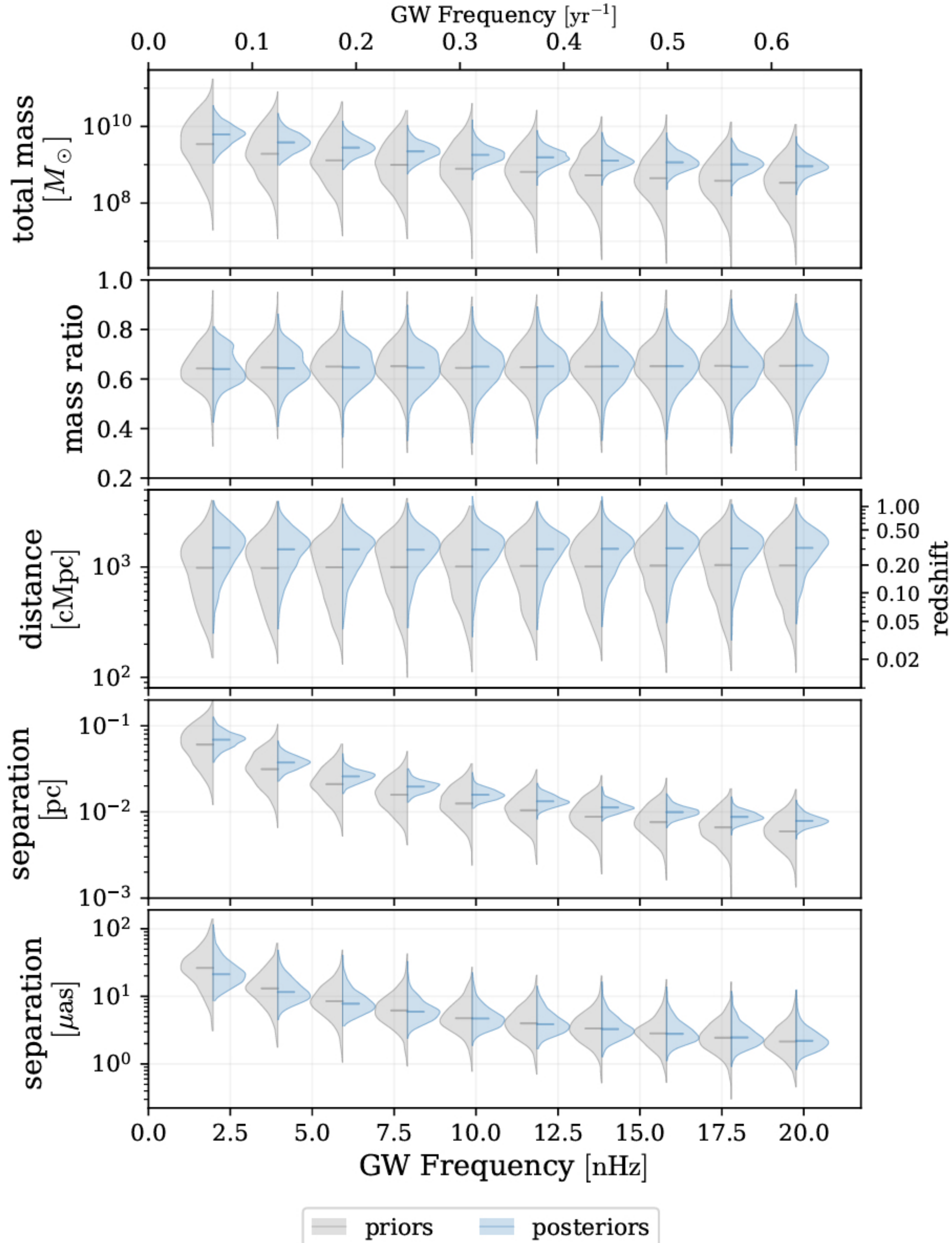


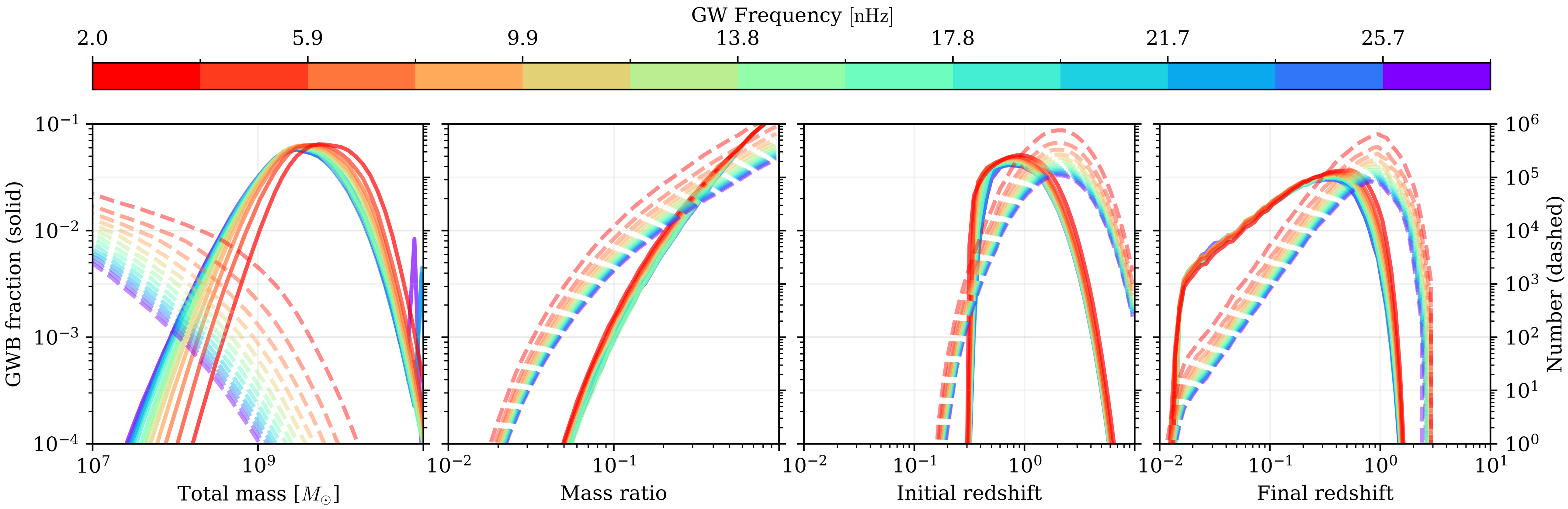
figure from "The Gravitational Universe" (eLISA Consortium, arXiv:1305.5720)

Supermassive binary population inferred from NanoGrav background

- Redshift: 0.1 – 0.5
- Mass: $10^9 - 10^{10}$ $M_\odot$
- Separation: 0.1 – 0.01 pc



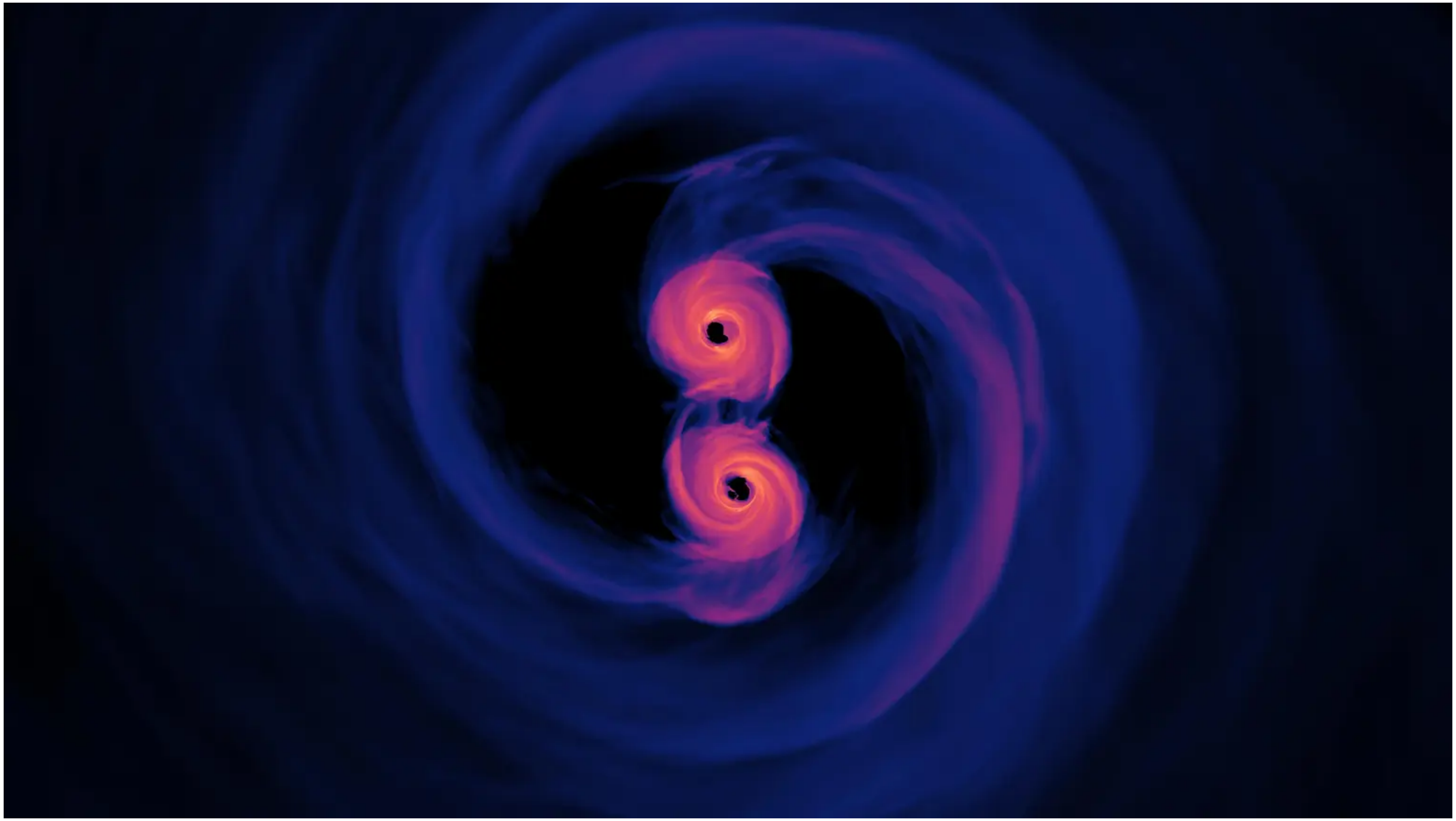
Binary properties inferred from the NanoGrav background



Agazie+13, ApJL, 952, L37

On the theoretical side of things…

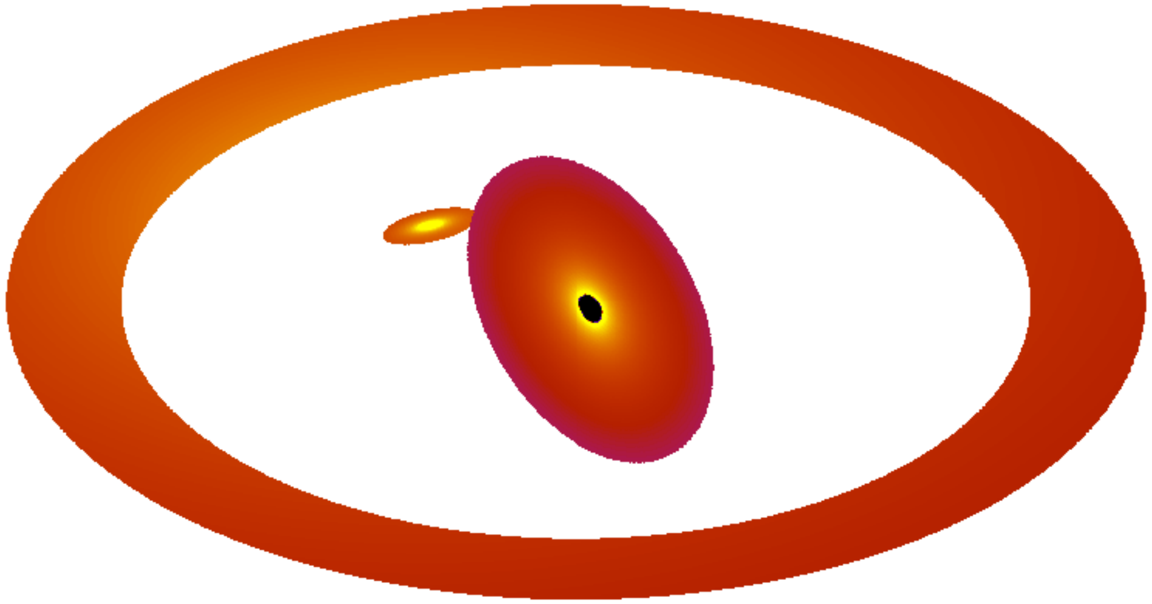
Credit: NASA's Goddard Space Flight Center/Scott Noble
simulation data by d'Ascoli et al. 2018, ApJ, 865, 140

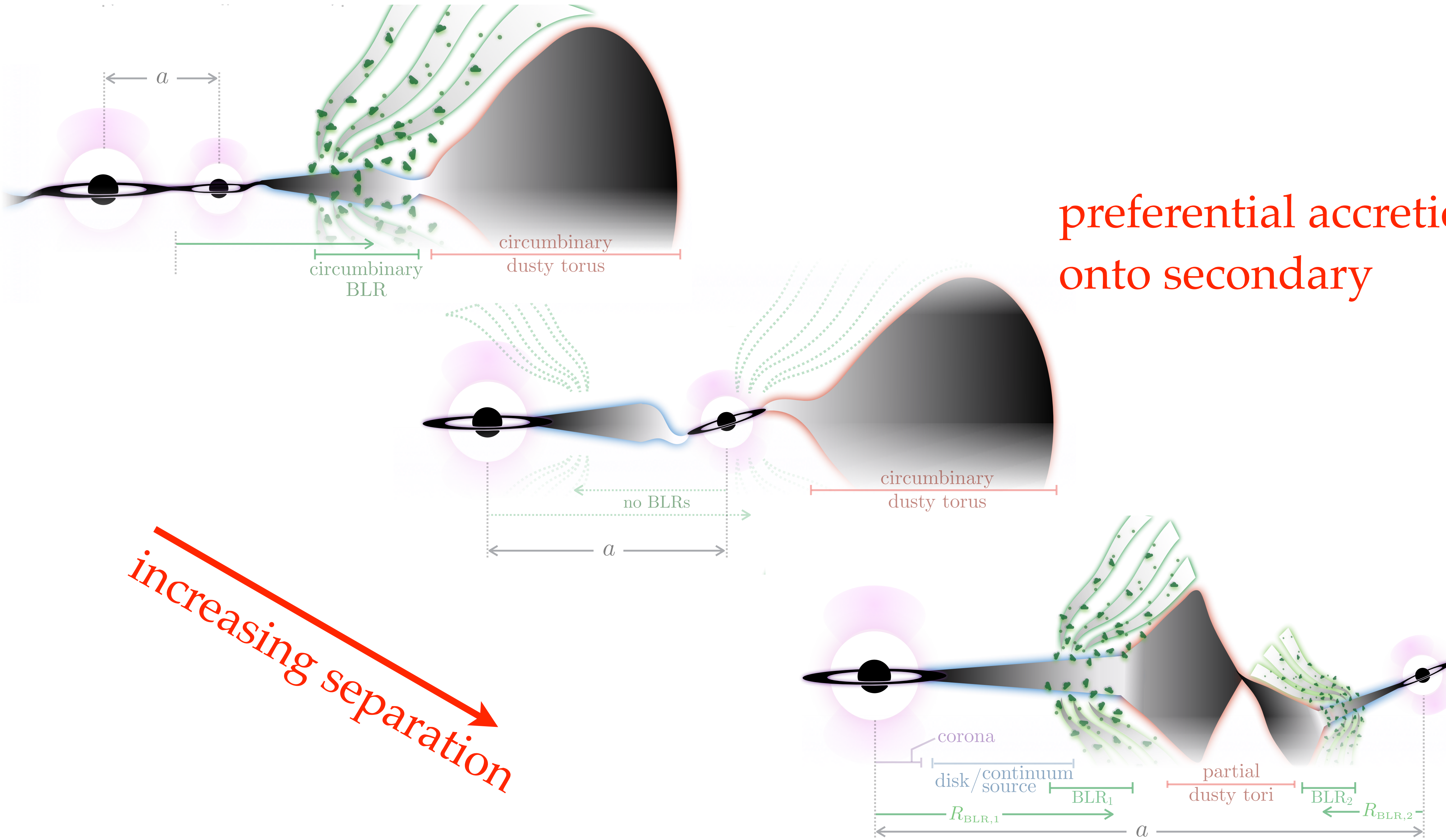


See animation at https://svs.gsfc.nasa.gov/13043
or https://www.youtube.com/watch?v=i2u-7LMhwvE

A very general geometry:
Two misaligned disks orbiting each other.

(movie made by Khai Nguyen)





preferential accretion onto secondary

increasing separation

figures from Kelley 21, MNRAS, 500, 4065

Inferences from simple models of line profiles and populations

- **Line profiles and population properties**
  Nguyen & Bogdanovic 2016, ApJ, 828, 68; Pflugger+18, ApJ, 861, 69; Nguyen+19 ApJ, 870, 16
  - Calculation of large library of synthetic line profiles.
  - Development of method to find location of an observed spectrum in the parameter space of the library

    Can infer basic properties of system but *cannot prove* that system is a SBHB

- **Detectability**
  Kelley+21, MNRAS, 500, 406
  - Secondary black hole most likely detectable in less than 1 in 10^4 quasars
  - 1 in 200 binaries have detectable velocity offsets and ~1 in 3000 detectable accelerations

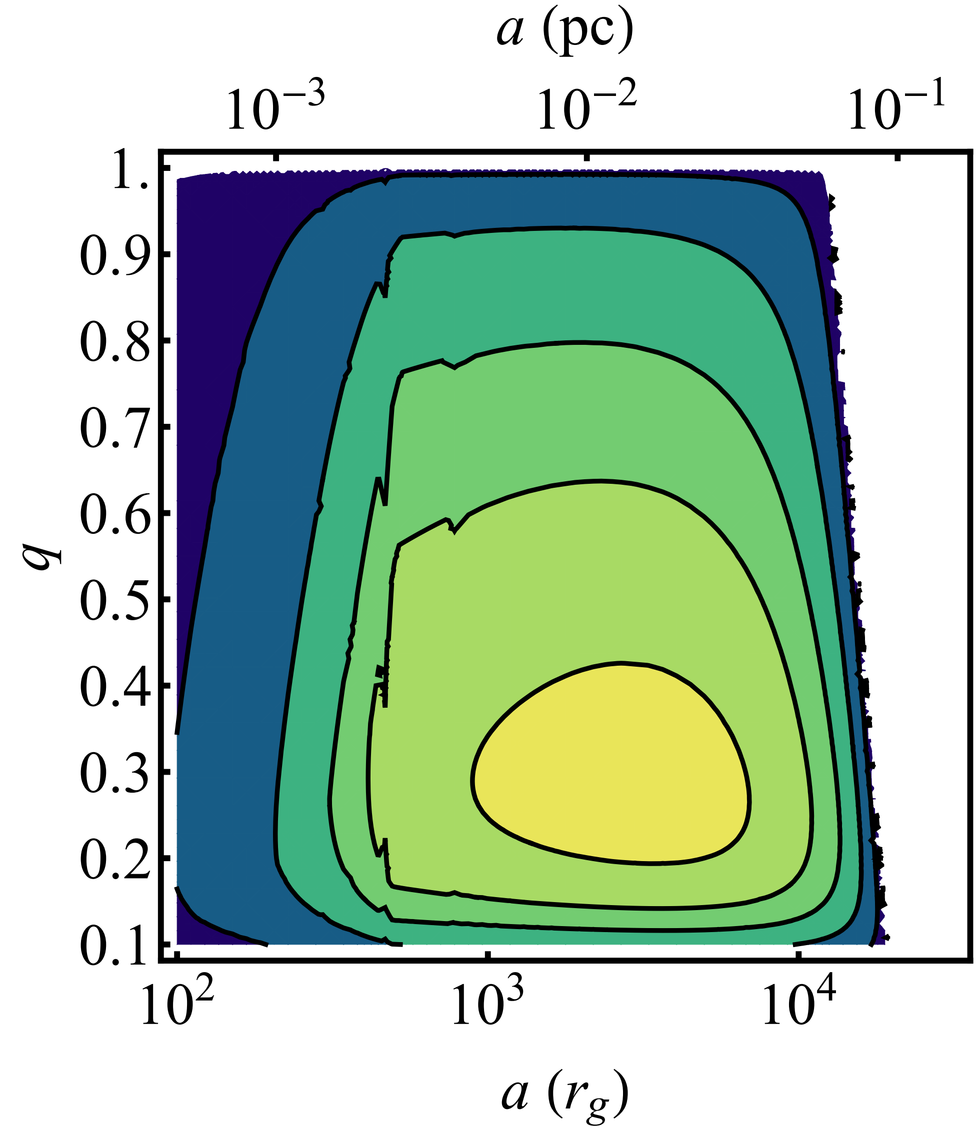
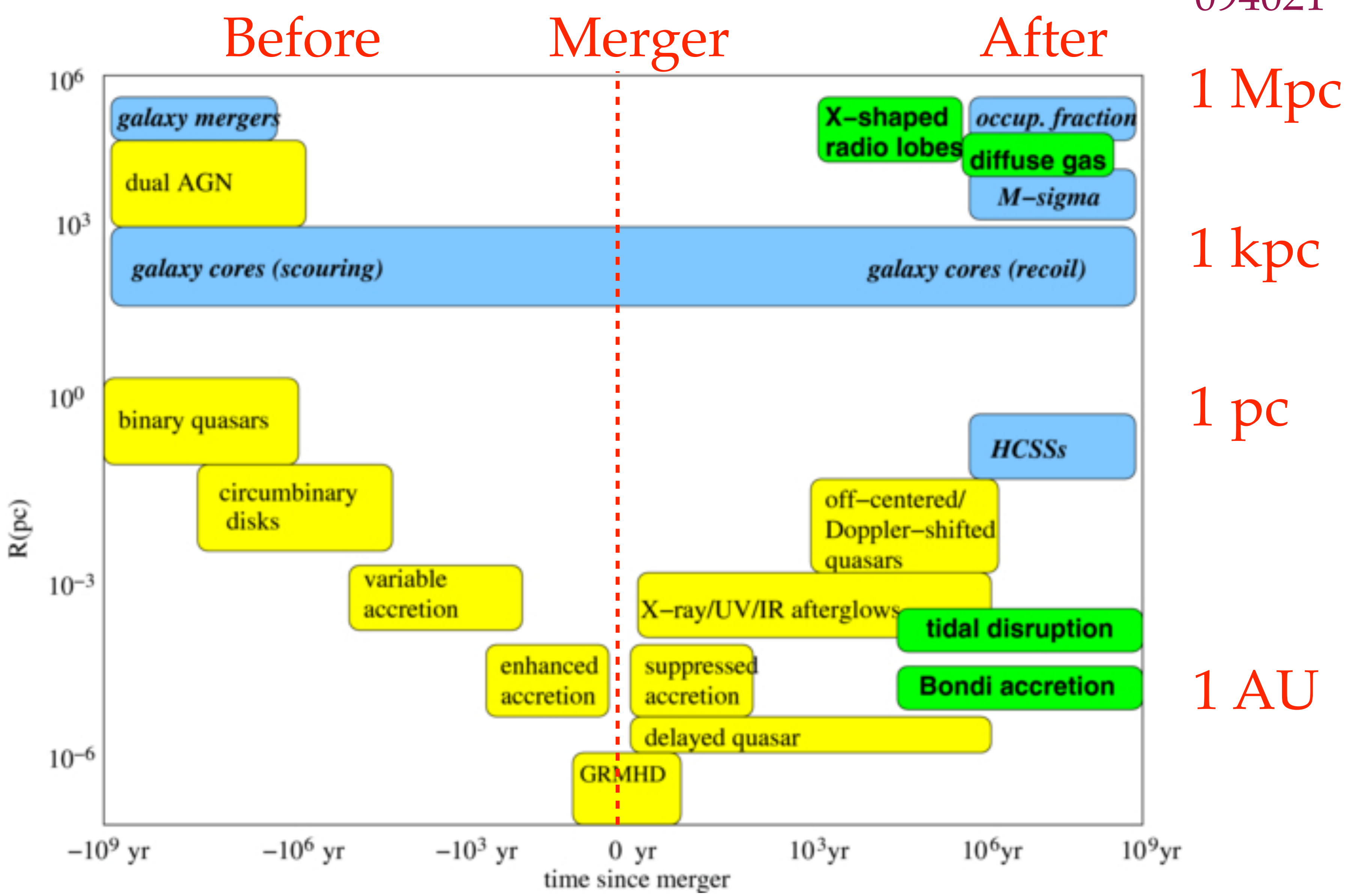


Figure from Nguyen+19 ApJ, 870, 16

More on observational tests

EM signatures of SMBHs on a variety of length and time scales

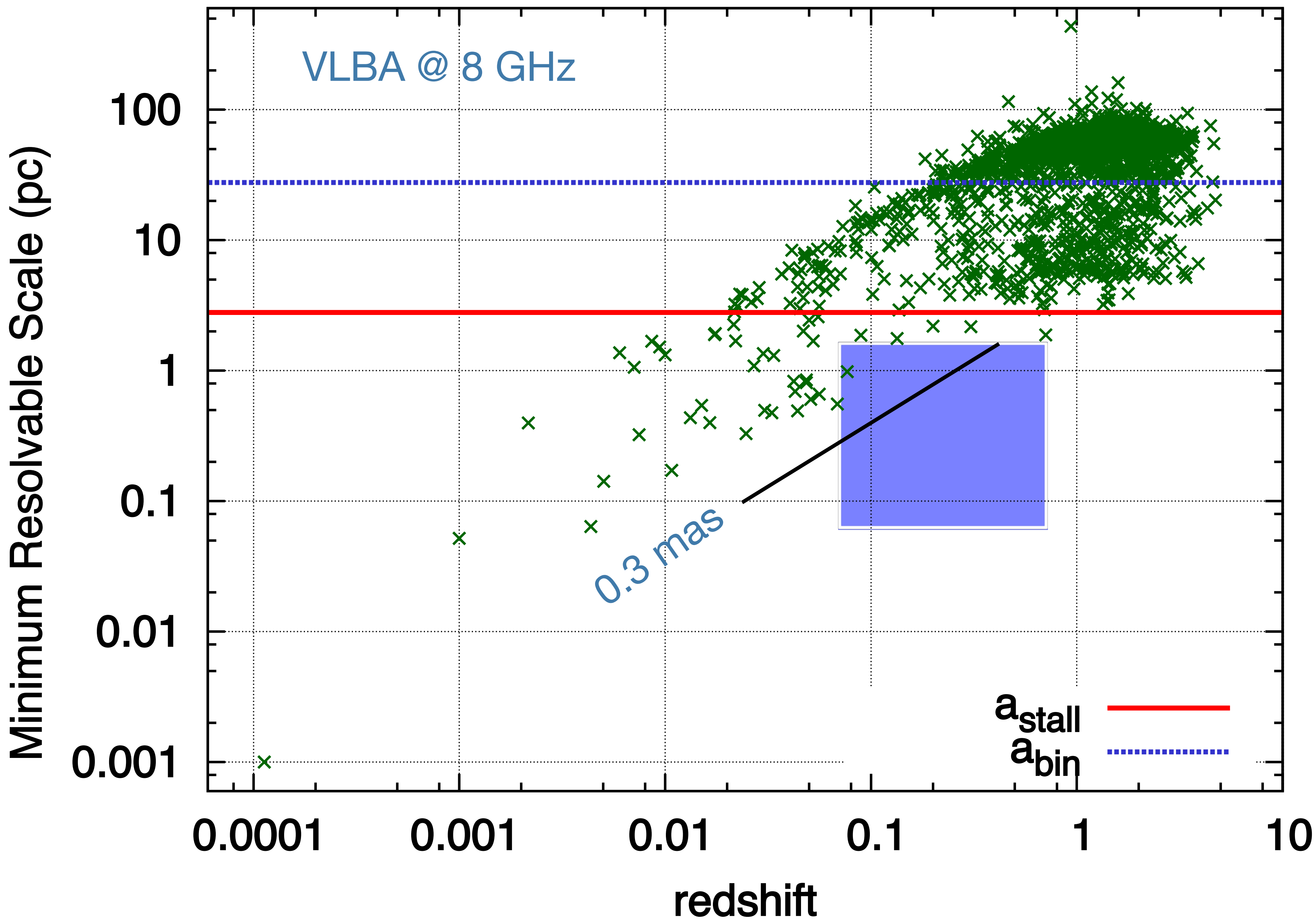
Figure from Schnittman 2011, *Class. Quant. Grav.*, **28**, 094021



Direct Imaging: requires our best instruments and optimism

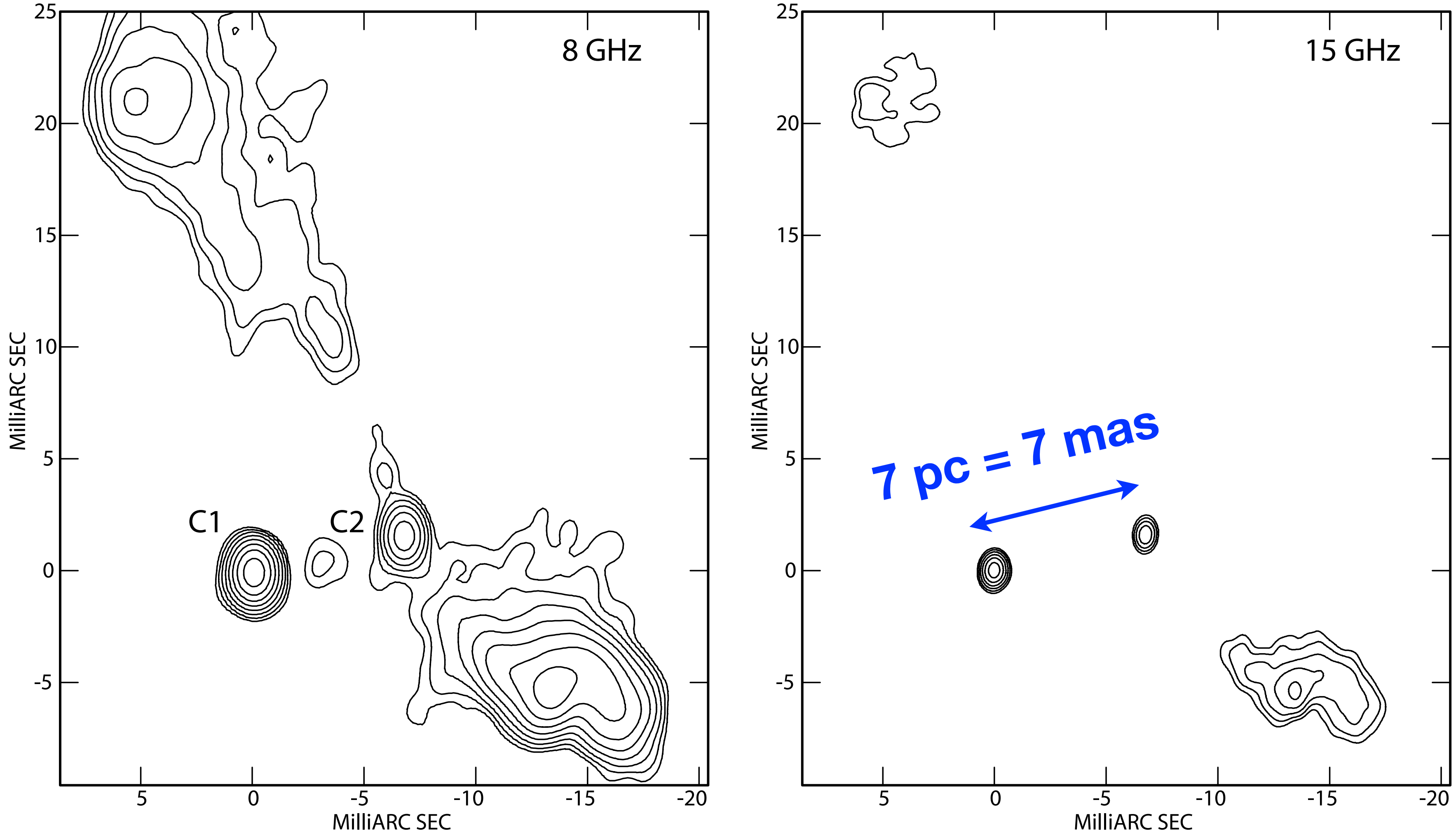
- Angular size corresponding to 1 pc
  - 10 mas @ 20 Mpc (~Virgo cluster)
  - 1 mas @ z=0.05 (Seyfert galaxies)
  - 0.23 mas @ z=0.3 (nearby quasars)
  - 0.13 mas @ z=3 ("cosmic noon")
- Resolution attainable by our best instruments
  - Hubble Space Telescope (UV-O-IR) → 100 mas
  - VLBA @ 3 mm → 0.12 mas
  - GMVA @ 3 mm → 40 μas
  - EHT @ 1.3 mm → 25 μas

VLBI imaging



(from Burke-Spolaor 11, MNRAS, 410, 2113)

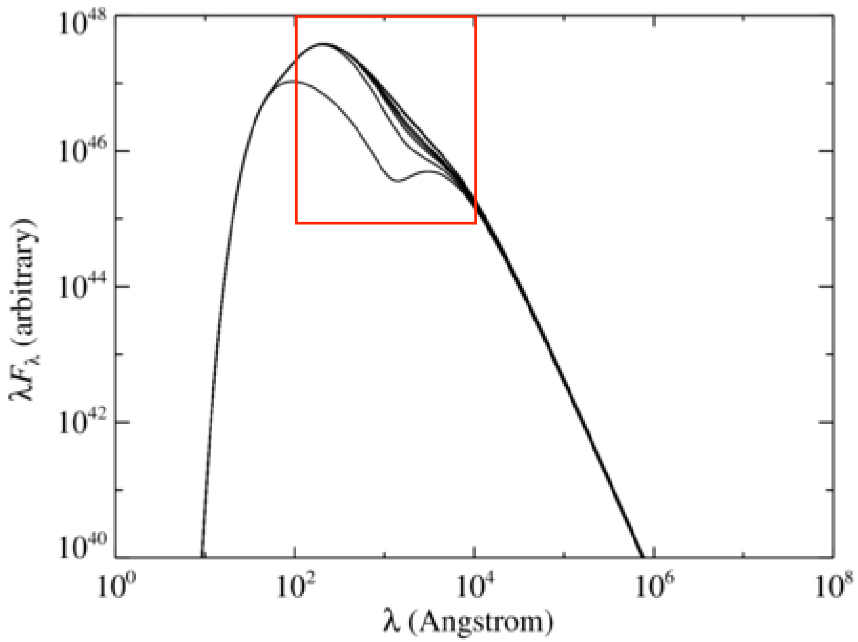
4C+37.11: Smallest known SMBH Pair

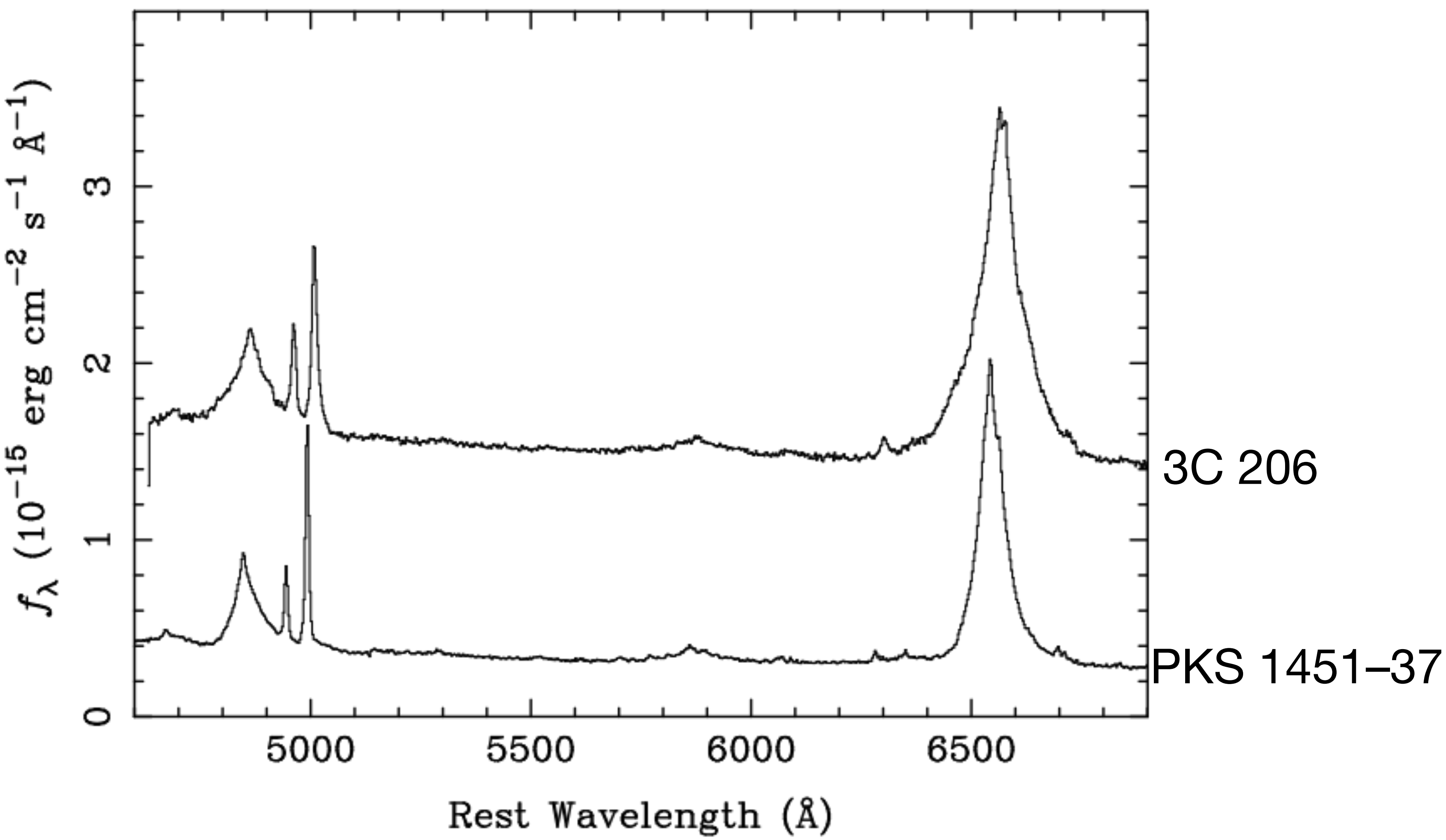


Rodriguez et al. 2006, ApJ, 2006, ApJ, 646, 49

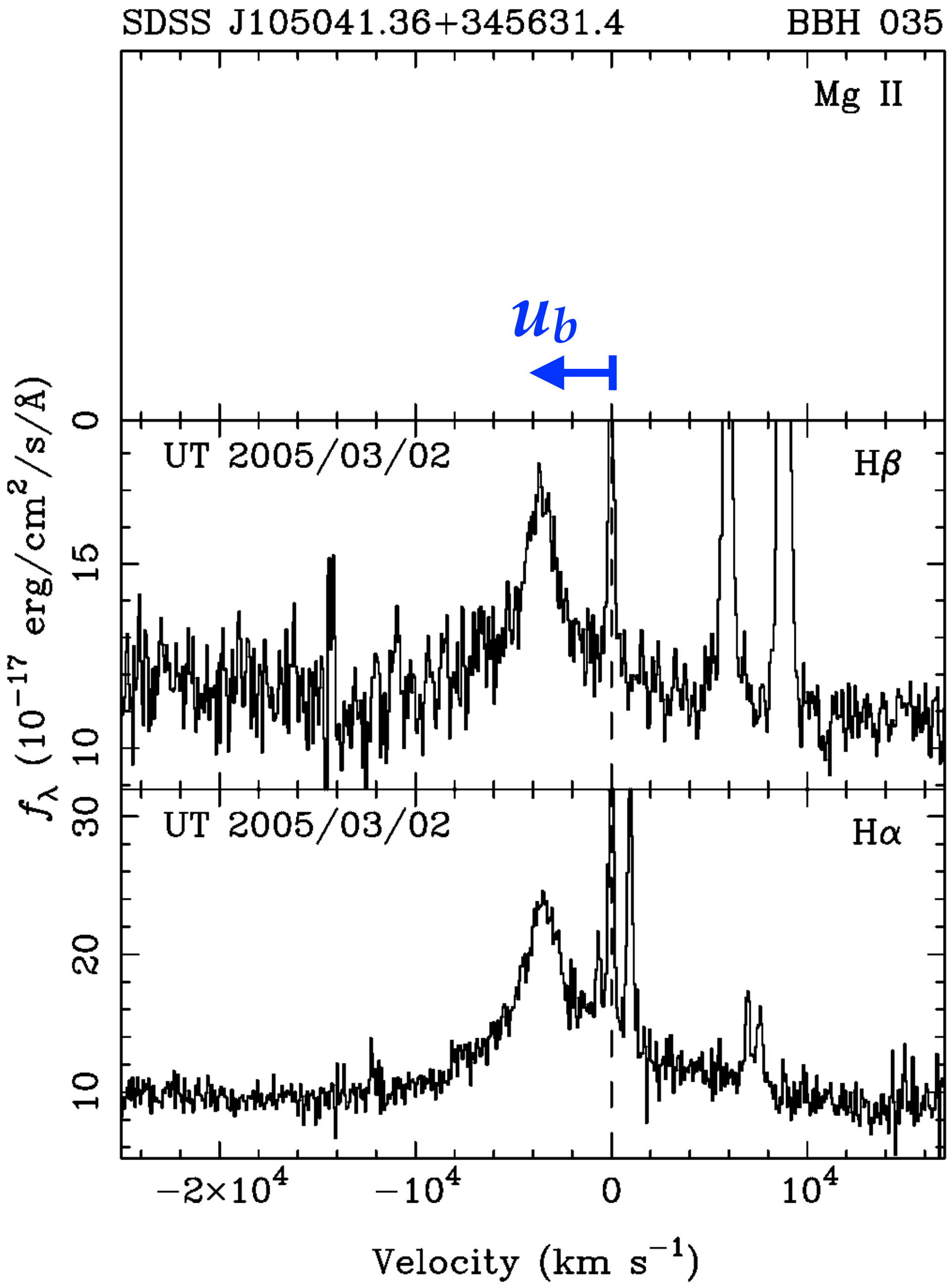
Spectral Energy Distributions of Quasars

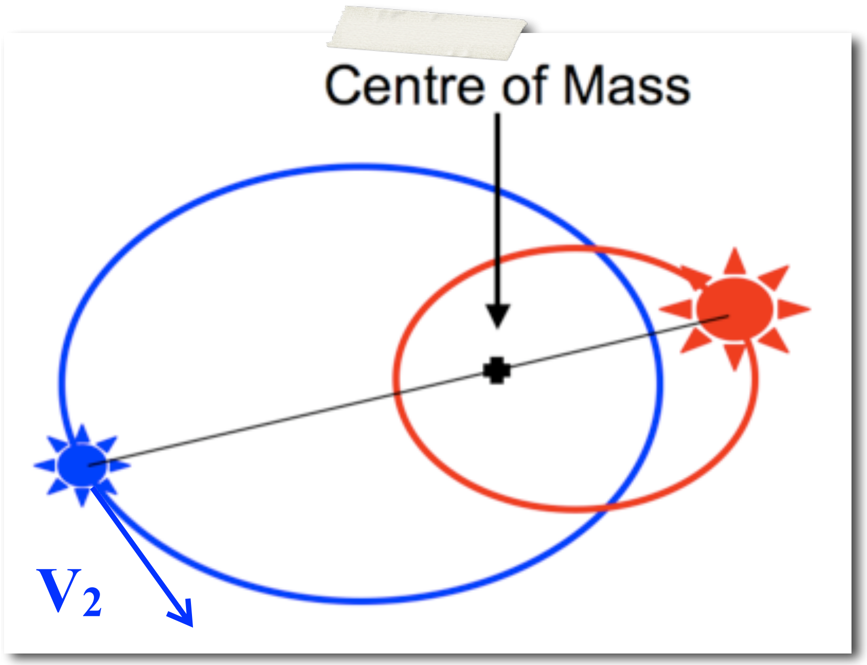
(Gükltekin & Miller 2012, ApJ, 761, 90)





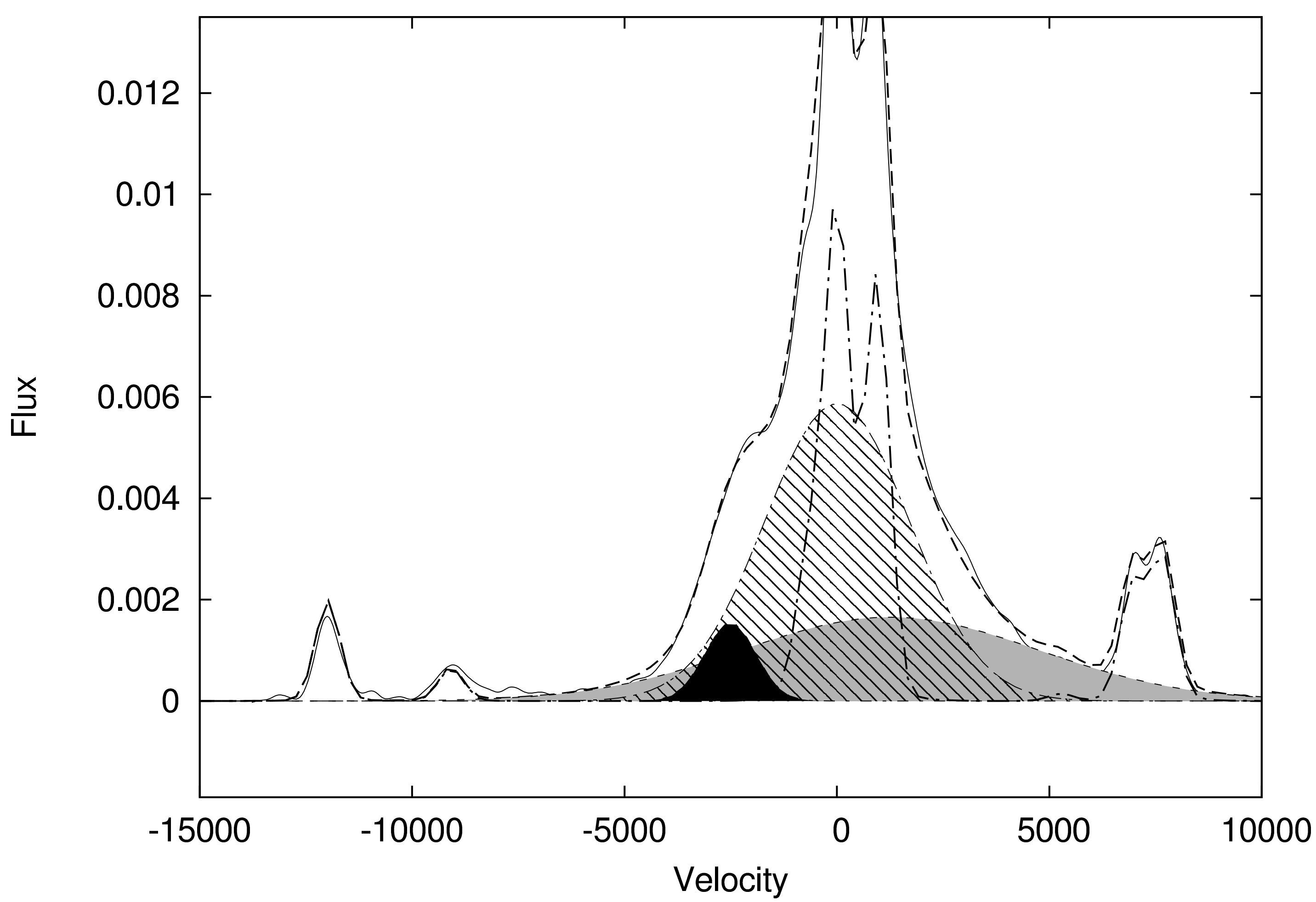
f_λ (10^{-15} erg cm^{-2} s^{-1} Å^{-1})
0
1
2
3
5000
5500
6000
6500
Rest Wavelength (Å)
3C 206
PKS 1451–37


SDSS J105041.36+345631.4
BBH 035
Mg II
u_b
UT 2005/03/02
Hβ
UT 2005/03/02
Hα
f_λ (10^{-17} erg/cm^2/s/Å)
0
15
10
30
20
10
-2×10^4
-10^4
0
10^4
Velocity (km s^{-1})


Centre of Mass
V_2

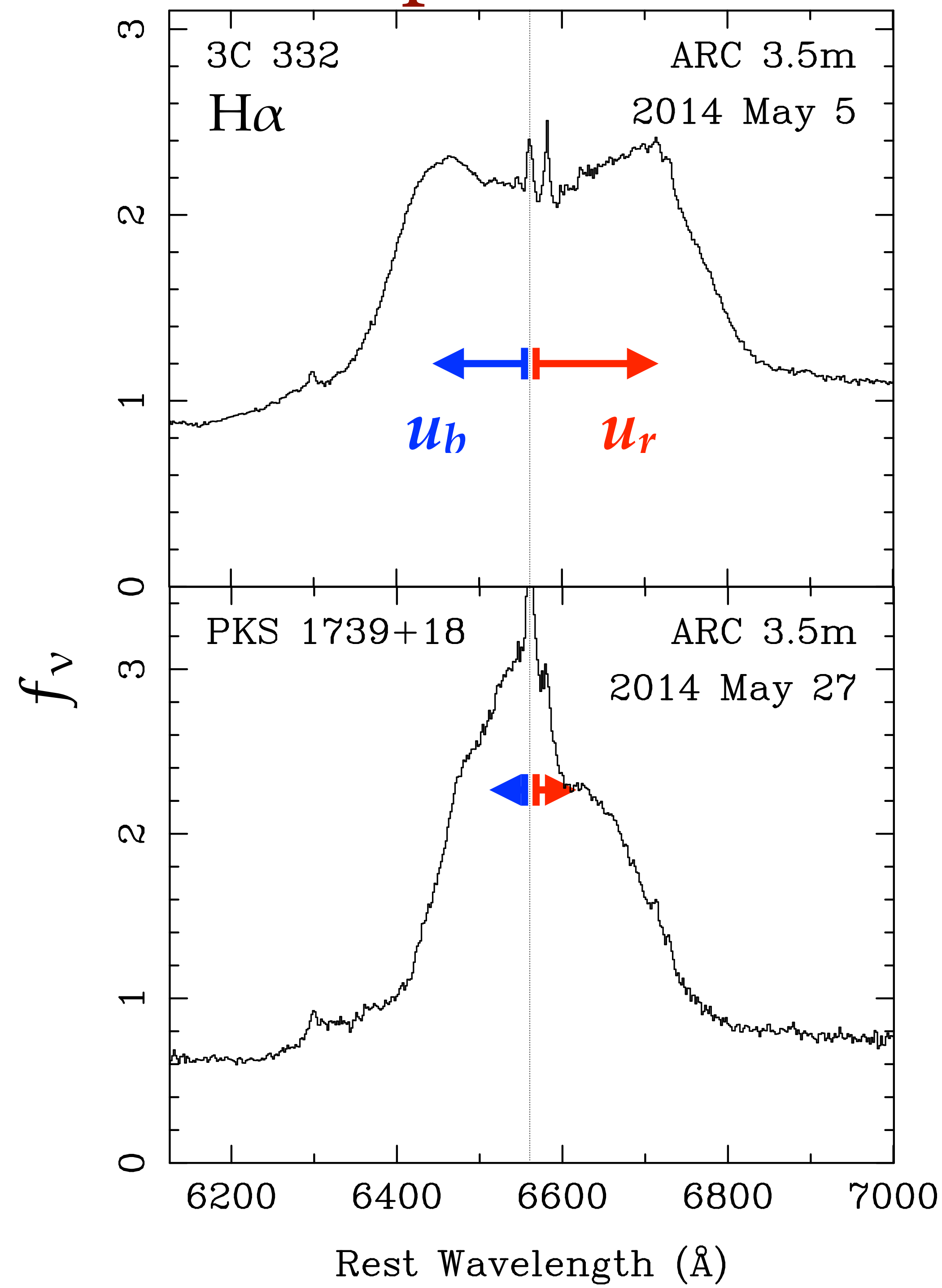
Limitations and next steps

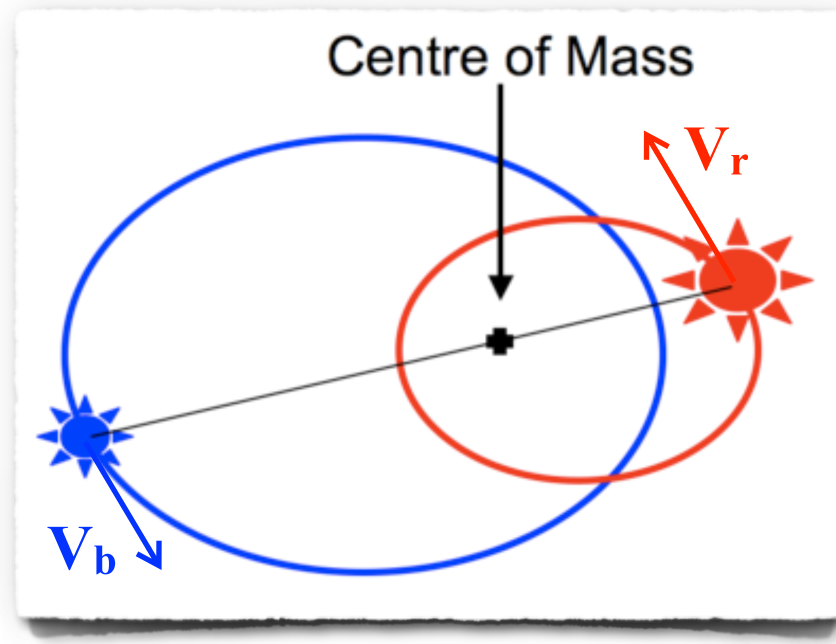
- Continued monitoring (remedy for many problems)
  - record more cycles of photometric modulation
  - establish longer (monotonic?) trend of radial velocity curves
  - Observe bigger samples
- Better empirical characterization of the time variability of "typical" quasars: specifically radial velocity jitter, so that we know what quasars can really do.
- Better theoretical understanding of quasar broad-line regions of single quasars



0.012
0.01
0.008
0.006
0.004
0.002
0
Flux
-15000
-10000
-5000
0
5000
10000
Velocity

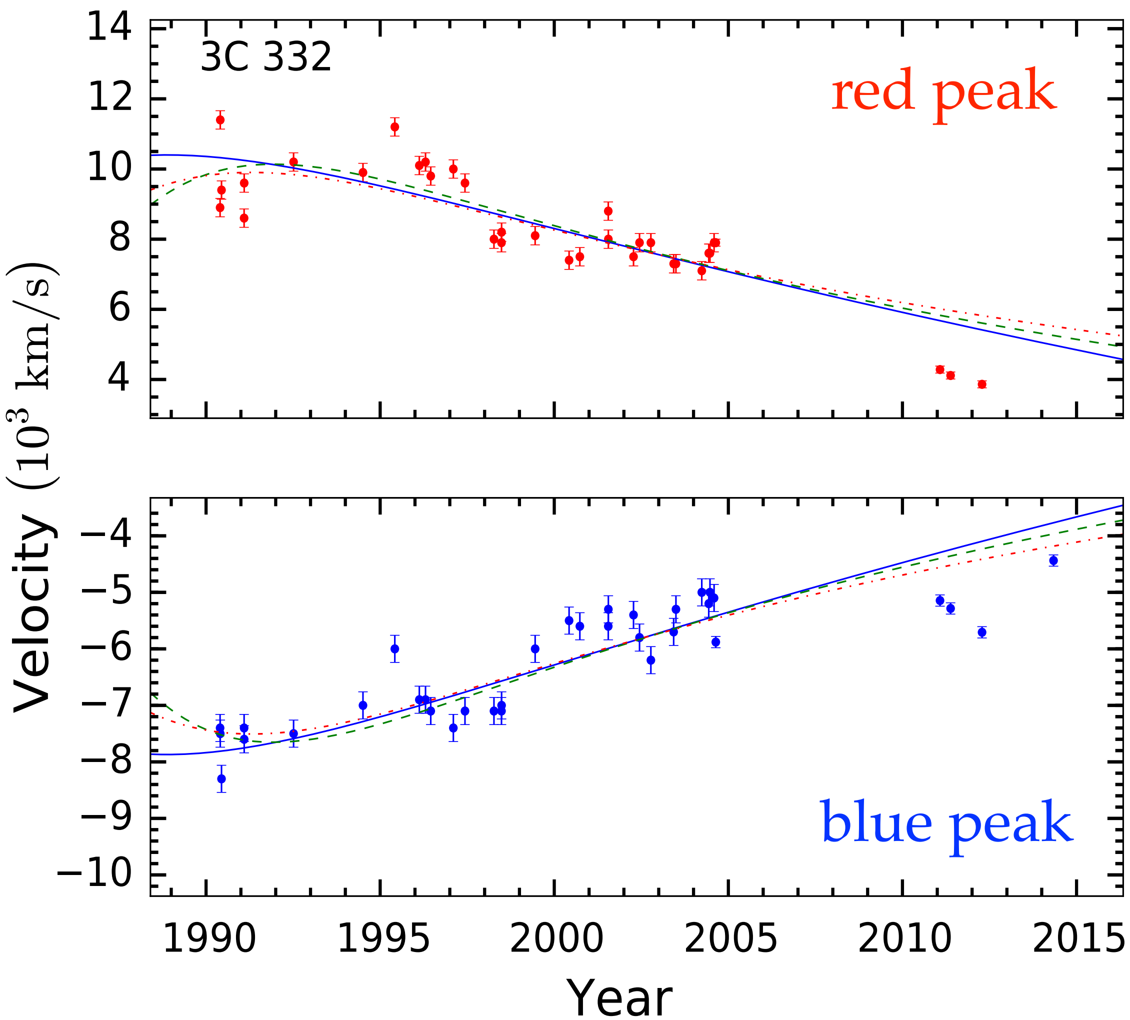
More examples of emission line profiles

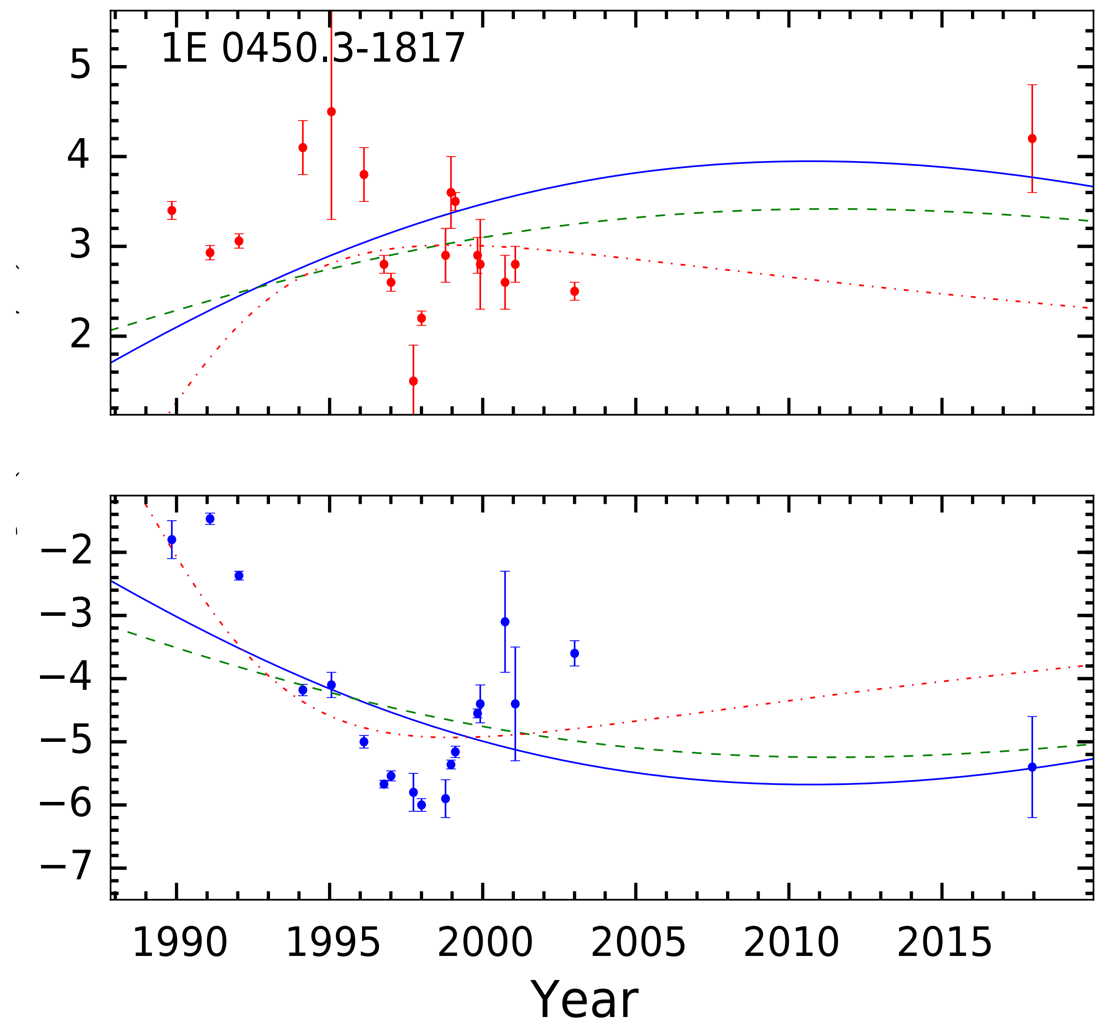




(spectra from Doan+20, MNRAS, 491, 1104)

Fits to radial velocity curves: 14 cases of double-peaked lines are inconsistent with supermassive binaries.



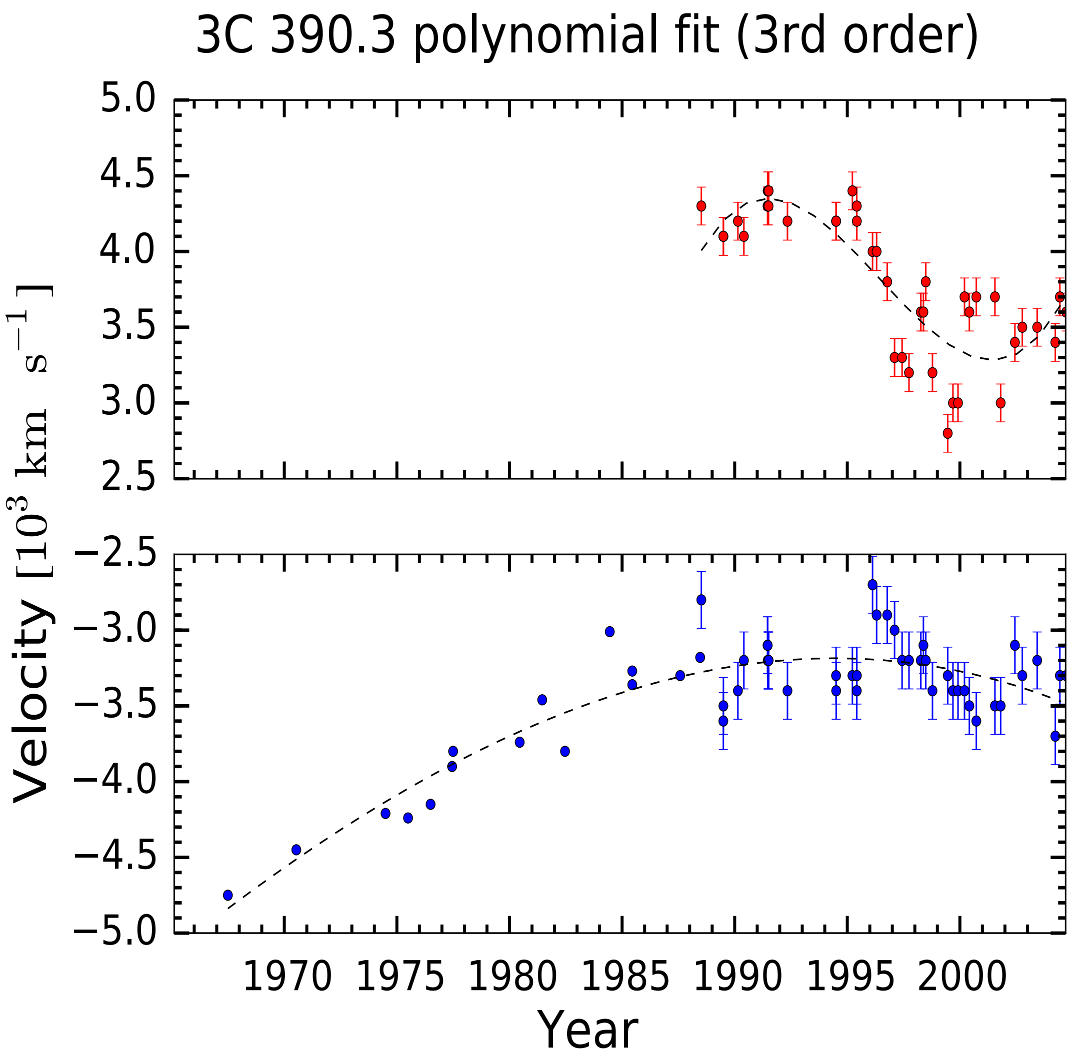


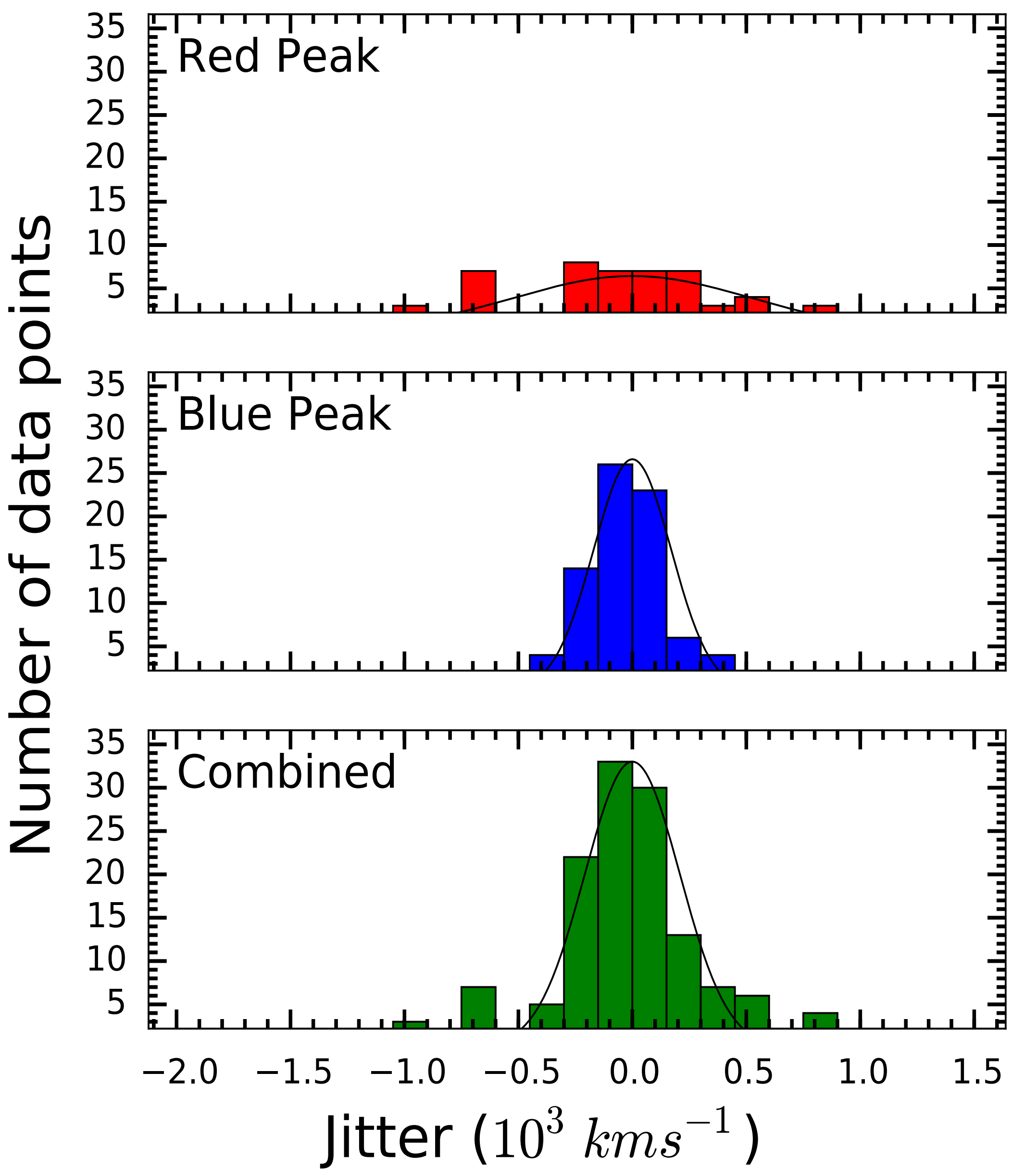
2/14 of cases:
good fits with $M > 10^{10}$ $M_\odot$

1/3 of cases:
very poor fits

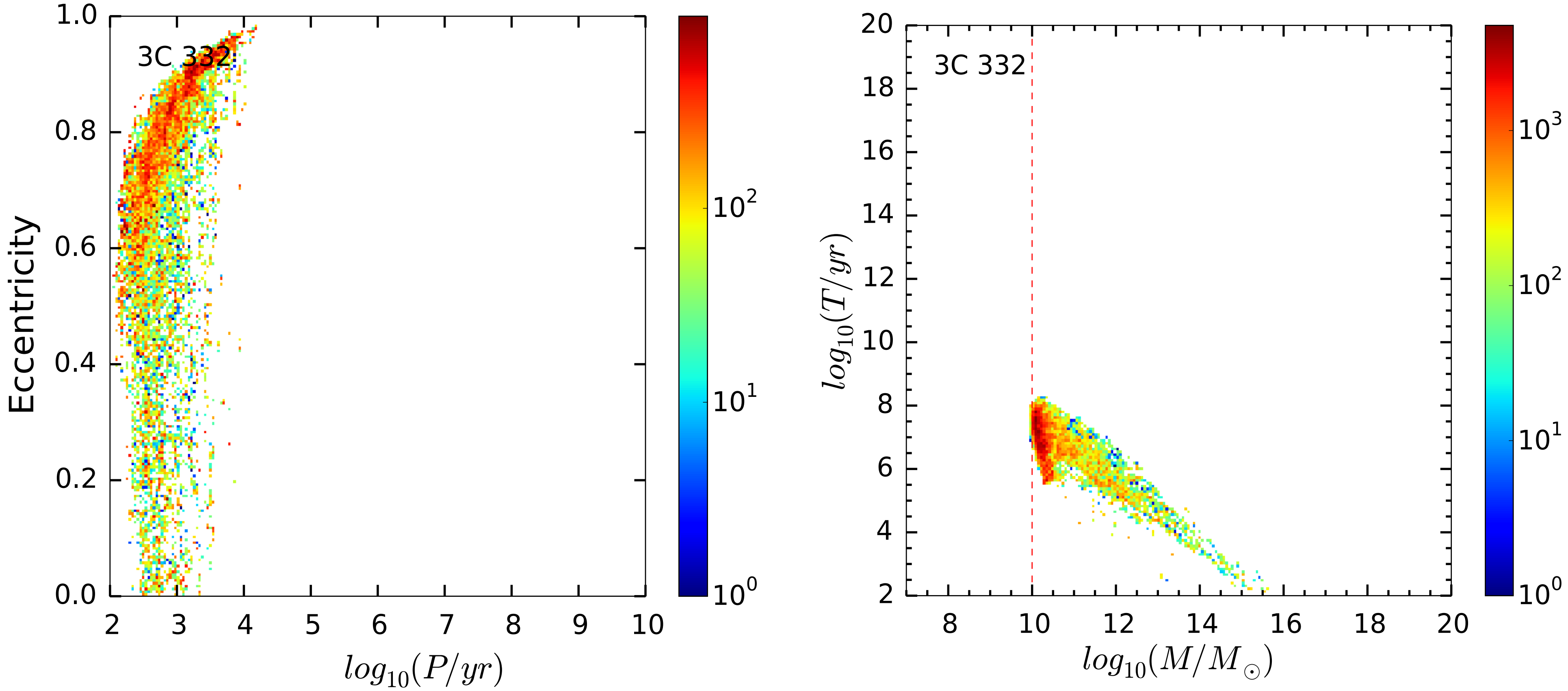
figures from Doan+20, MNRAS, 491, 1104

Radial Velocity Jitter: illustration of the effect

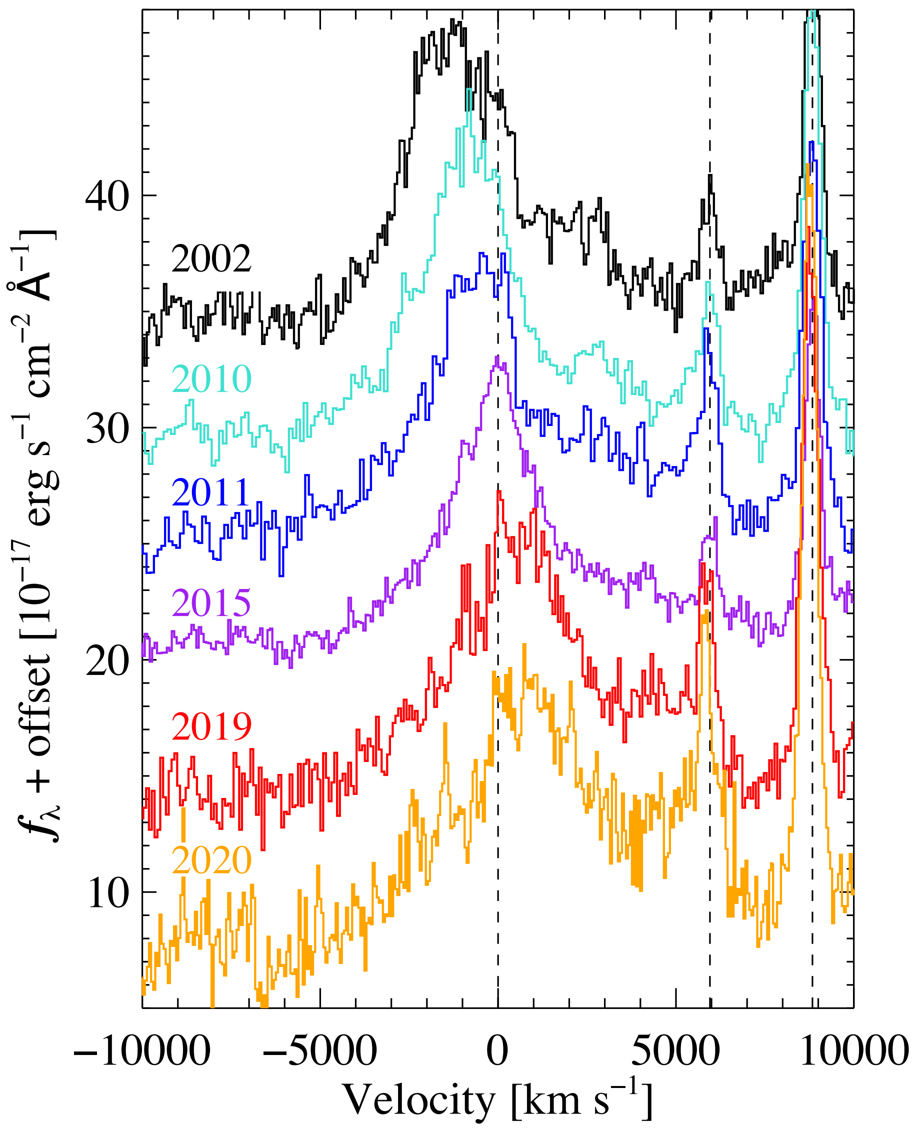


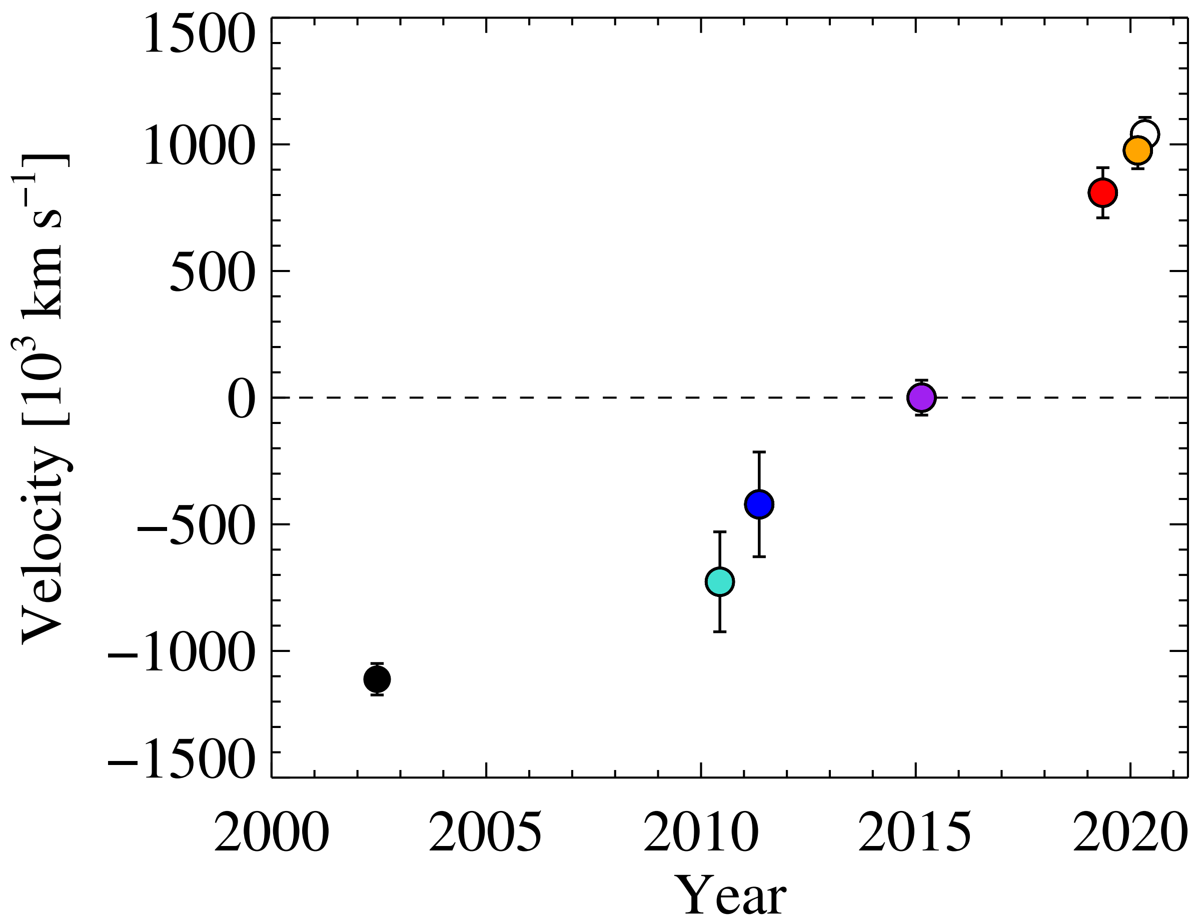


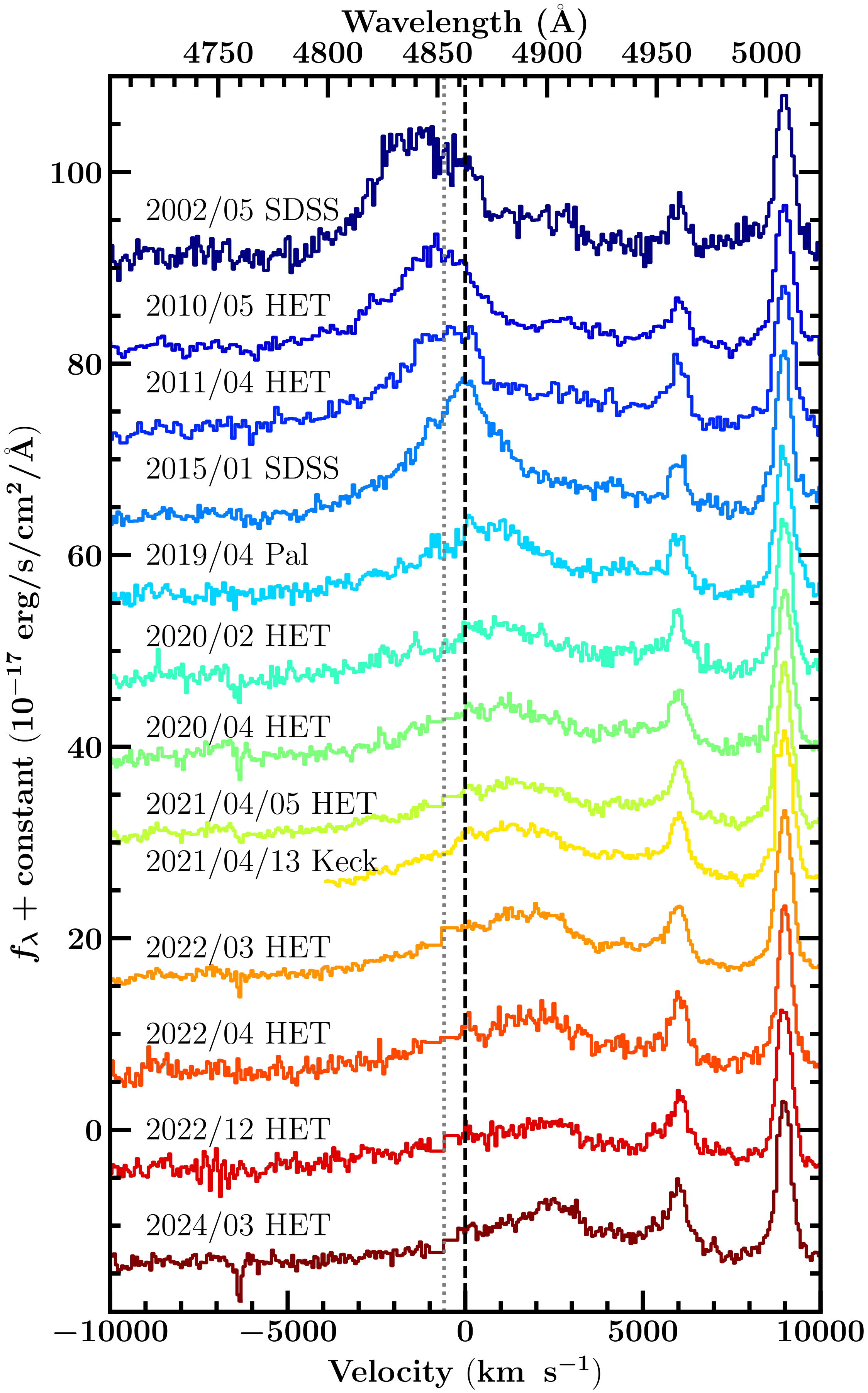
figures from Doan+20, MNRAS, 491, 1104

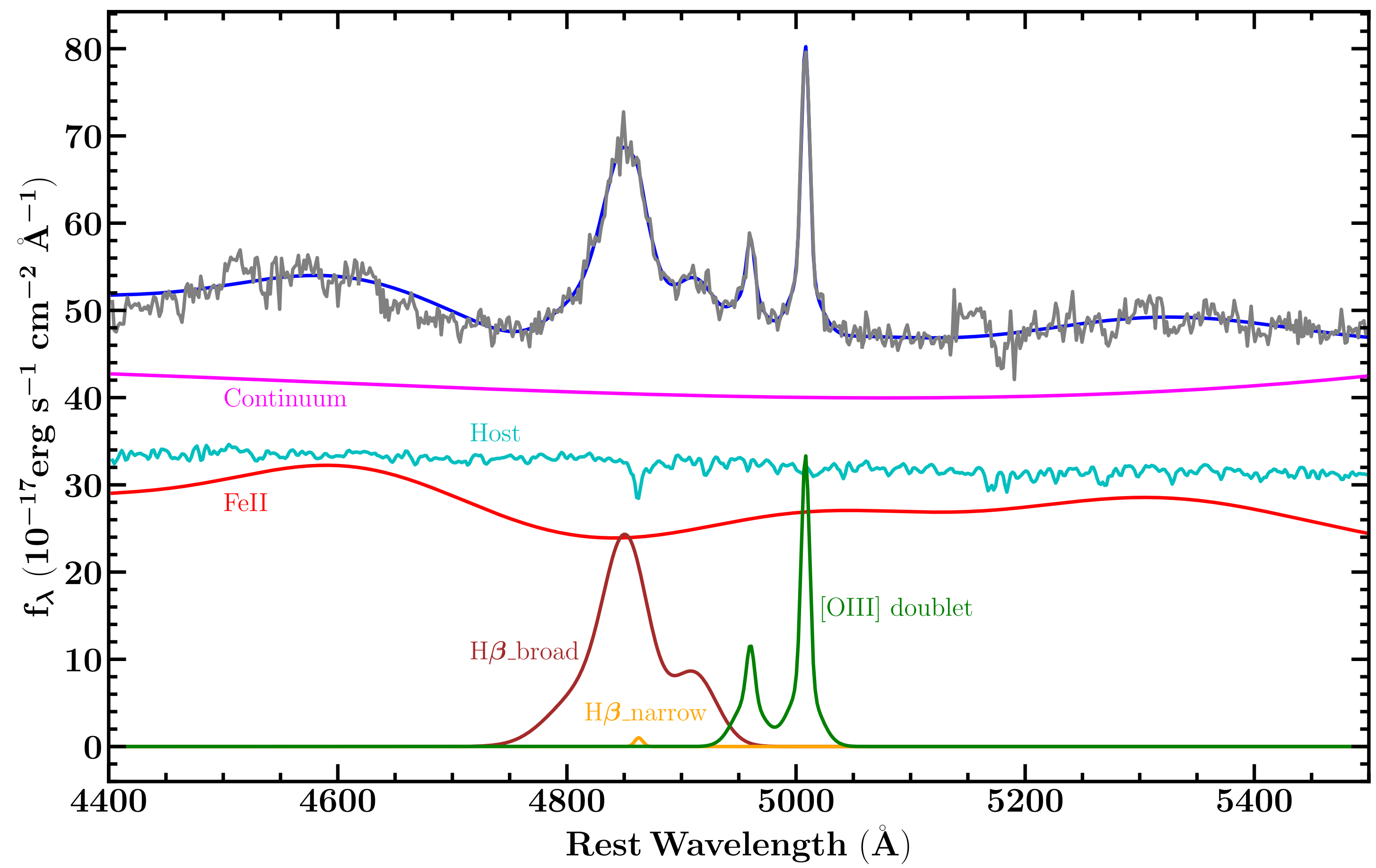

3C 332
Eccentricity
$log_{10}(P/yr)$
1.0
0.8
0.6
0.4
0.2
0.0
2 3 4 5 6 7 8 9 10
10^2
10^1
10^0
3C 332
$log_{10}(T/yr)$
$log_{10}(M/M_\odot)$
20 18 16 14 12 10 8 6 4 2
8 10 12 14 16 18 20
10^3
10^2
10^1
10^0

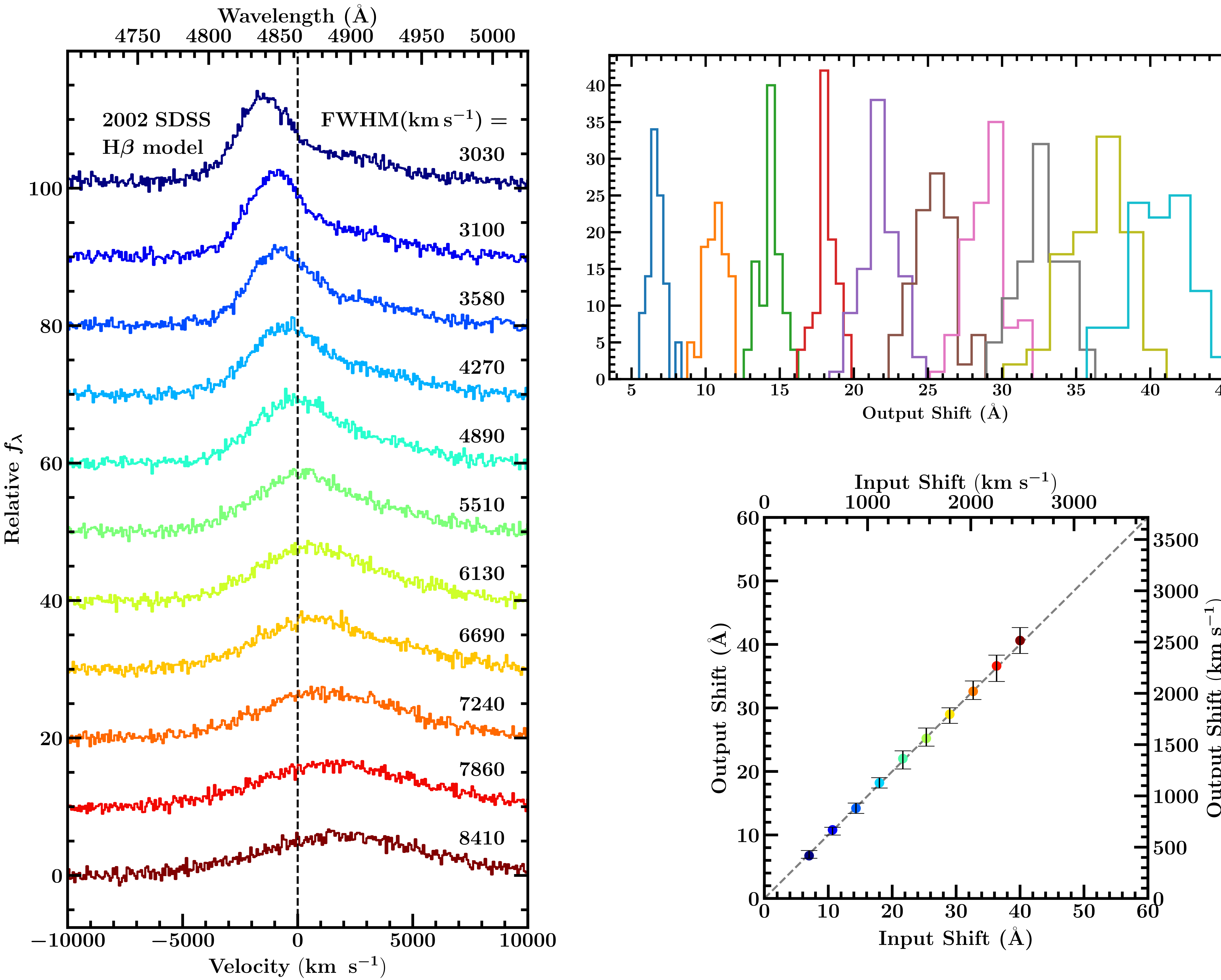
The best candidate so far, P ~ 40 years

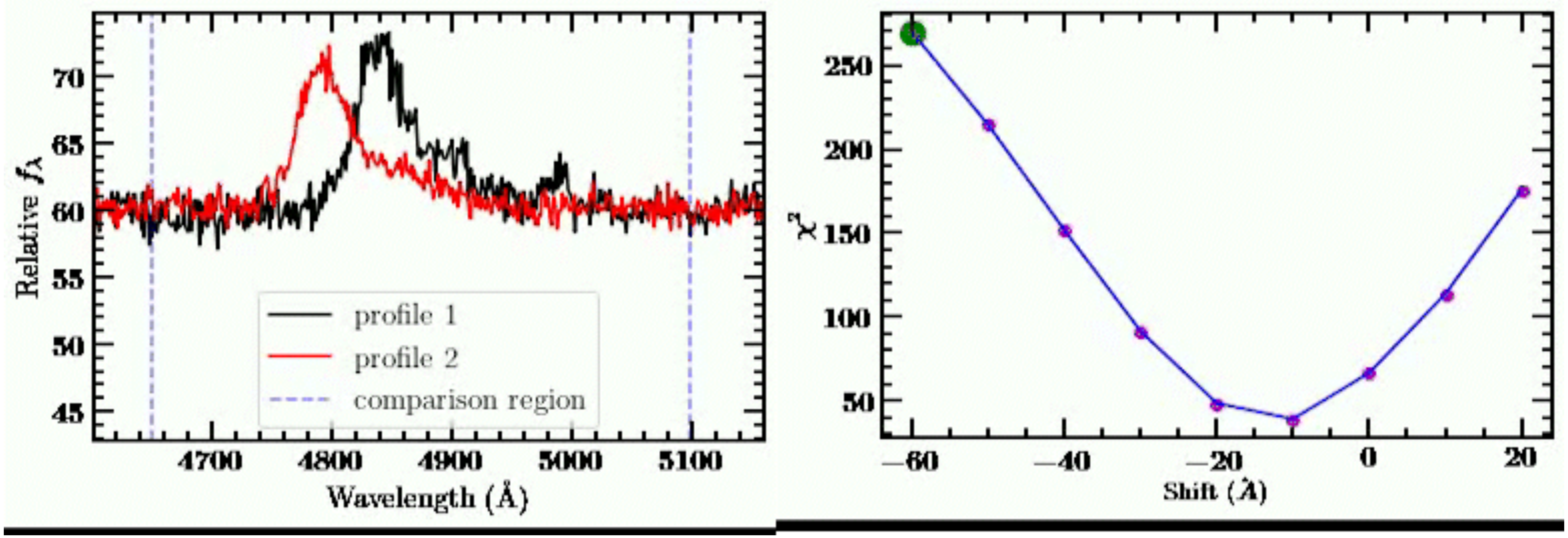





Wavelength (Å)
4750
4800
4850
4900
4950
5000
100
80
60
40
20
0
2002/05 SDSS
2010/05 HET
2011/04 HET
2015/01 SDSS
2019/04 Pal
2020/02 HET
2020/04 HET
2021/04/05 HET
2021/04/13 Keck
2022/03 HET
2022/04 HET
2022/12 HET
2024/03 HET
f_λ + constant (10^{-17} erg/s/cm^2/Å)
−10000
−5000
0
5000
10000
Velocity (km s^{-1})


80
70
60
50
40
30
20
10
0
f$_\lambda$ (10^{-17}erg s^{-1} cm^{-2} Å^{-1})
Continuum
Host
FeII
[OIII] doublet
Hβ_broad
Hβ_narrow
4400
4600
4800
5000
5200
5400
Rest Wavelength (Å)

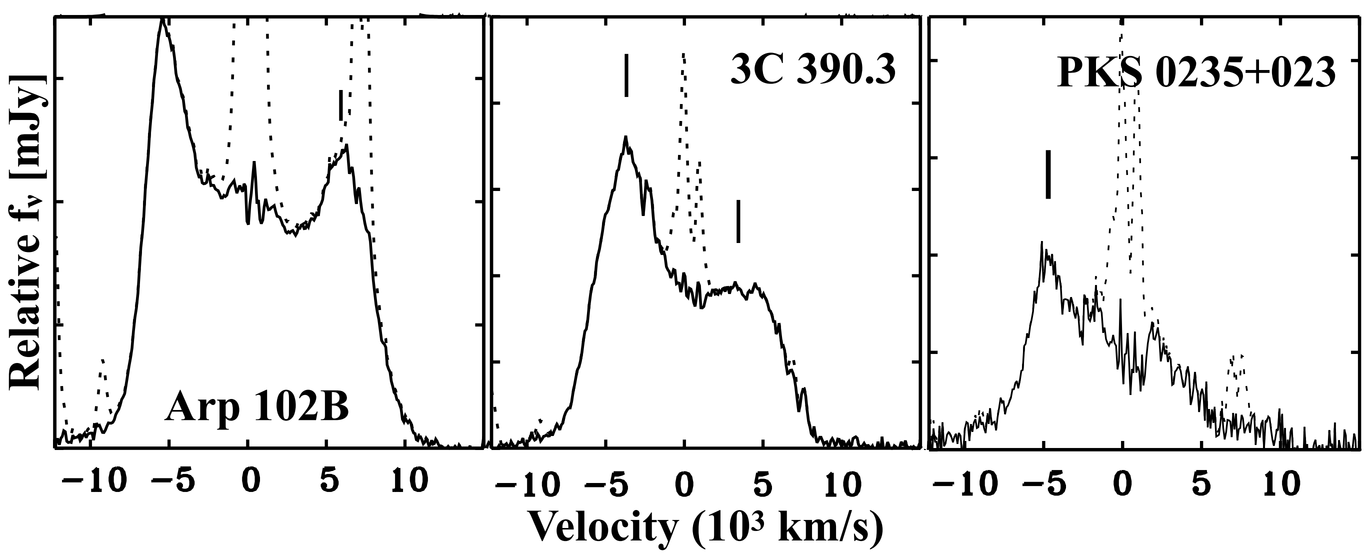

Wavelength (Å)
4750 4800 4850 4900 4950 5000
2002 SDSS
Hβ model
FWHM(km s−1) =
3030
3100
3580
4270
4890
5510
6130
6690
7240
7860
8410
Relative f_λ
Velocity (km s−1)
−10000 −5000 0 5000 10000
Output Shift (Å)
Input Shift (km s−1)
Output Shift (Å)
Output Shift (km s−1)
Input Shift (Å)

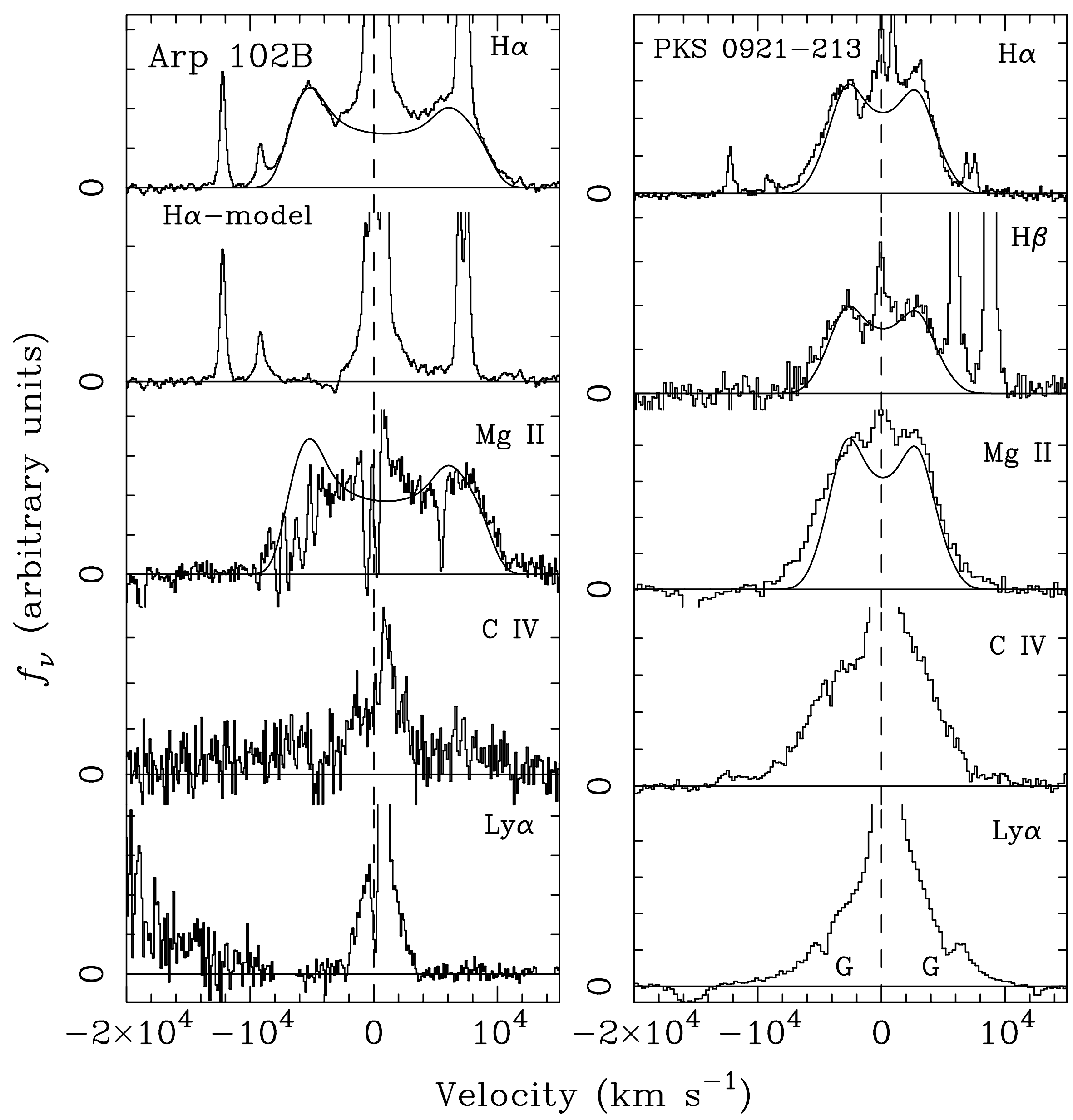

Relative f_λ
70
65
60
55
50
45
4700
4800
4900
5000
5100
Wavelength (Å)
profile 1
profile 2
comparison region
χ^2
250
200
150
100
50
−60
−40
−20
0
20
Shift (Å)

Helpful constraints from the gravitational wave background

- The exercise
  - Take the population of candidates from a given method at face value
  - Extrapolate to short separations and high redshifts and predict grav. wave background
  - Compare to current limits from the PTAs
- Results
  - Shorter-period population found by photometry modulation of light curves in tension with grav. wave background (Sesana+18, ApJ, 856, 42)
  - Longer-period population found by spectroscopy consistent with grav. wave background (Nguyen+20, ApJ, 900, L42)

UV spectroscopic test





Evolution time of the gravitational wave frequency because of orbital decay

For two point masses in a circular orbit, this is the time it takes for the GW frequency to evolve from $f_{\rm i}$ to $f_{\rm f}$. The mass ratio is q (< 1) and the total mass is M_6 (in units of 10^6 $M_\odot$).

$$T \approx 8 \times 10^4 \, \frac{(1+q)^2}{q} \, M_6^{-5/3} \, \left(\frac{f_{\rm i}}{1\,\mu{\rm Hz}}\right)^{-8/3} \left[1 - \left(\frac{f_{\rm i}}{f_{\rm f}}\right)^{8/3}\right] \; {\rm years}$$

The evolution is veeeeery slooooowwww….

Orbital period: $P = \frac{332\ M_8}{(1+q)^3\, u_{2,3}^3} \left(\frac{\sin i}{\sin 45^\circ} \frac{|\sin\phi|}{\sin 45^\circ} \right)^3 \ \mathrm{yr}$

Separation: $a = \frac{0.11\, M_8}{(1+q)^2\, u_{2,3}^2} \left(\frac{\sin i}{\sin 45^\circ} \frac{|\sin\phi|}{\sin 45^\circ} \right)^2 \ \mathrm{pc.}$

Instantaneous acceleration:

$$\left| \frac{du_2}{dt} \right| = 2.4\ \frac{u_{2,3}^4\,(1+q)^3}{M_8\ \sin^3 i} \left| \frac{\cos\phi}{\sin^4\phi} \right| \ \mathrm{km/s/yr}$$

$$= 19\ \frac{u_{2,3}^4\,(1+q)^3}{M_8} \left(\frac{\sin 45^\circ}{\sin i} \right)^3 \frac{|\cos\phi|}{\cos 45^\circ} \left(\frac{\sin 45^\circ}{\sin\phi} \right)^4 \ \mathrm{km/s/yr}$$

Evolution time of the gravitational wave frequency because of orbital decay

For two point masses in a circular orbit, this is the time it takes for the GW frequency to evolve from f_i to f_f. The mass ratio is q (< 1) and the total mass is M_6 (in units of 10^6 M$_\odot$).

(chirp mass) $^{-5/3}$

~1 for $f_\mathrm{i}/f_\mathrm{f} \geq 2$

$$T \approx 8 \times 10^4 \, \frac{(1+q)^2}{q} \, M_6^{-5/3} \left(\frac{f_\mathrm{i}}{1\,\mu\mathrm{Hz}}\right)^{-8/3} \left[1 - \left(\frac{f_\mathrm{i}}{f_\mathrm{f}}\right)^{8/3}\right] \text{years}$$

$= 4$ for $q = 1$

$= q^{-1}$ for $q \ll 1$

~ 10^5 middle of PTA band

~ 10^{-5} "left edge" of LISA band

Types of Sources Detectable by LISA: Binary Supermassive BHs

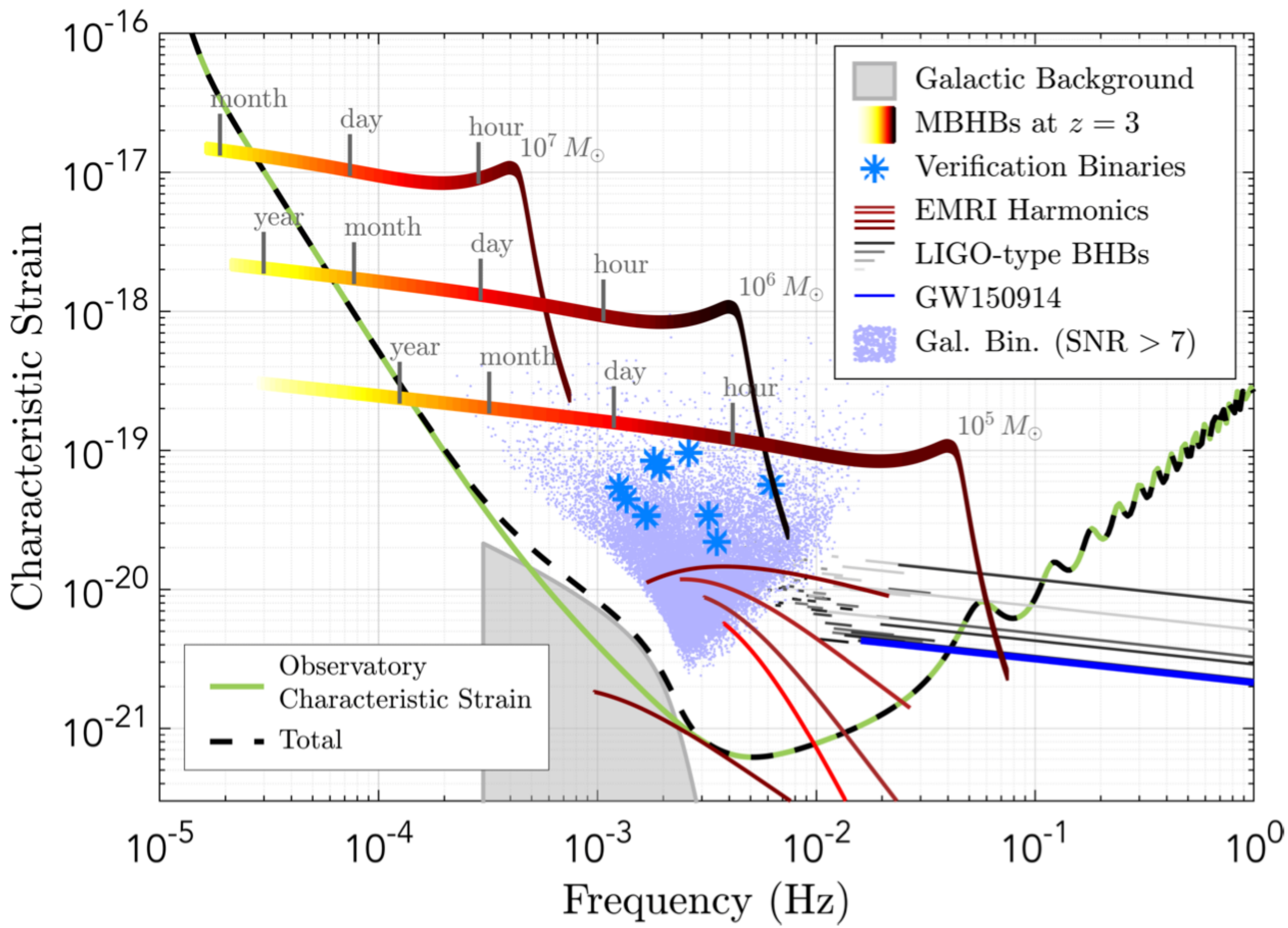


figure from LISA Proposal (K. Danzmann PI, arXiv:1702.00786)

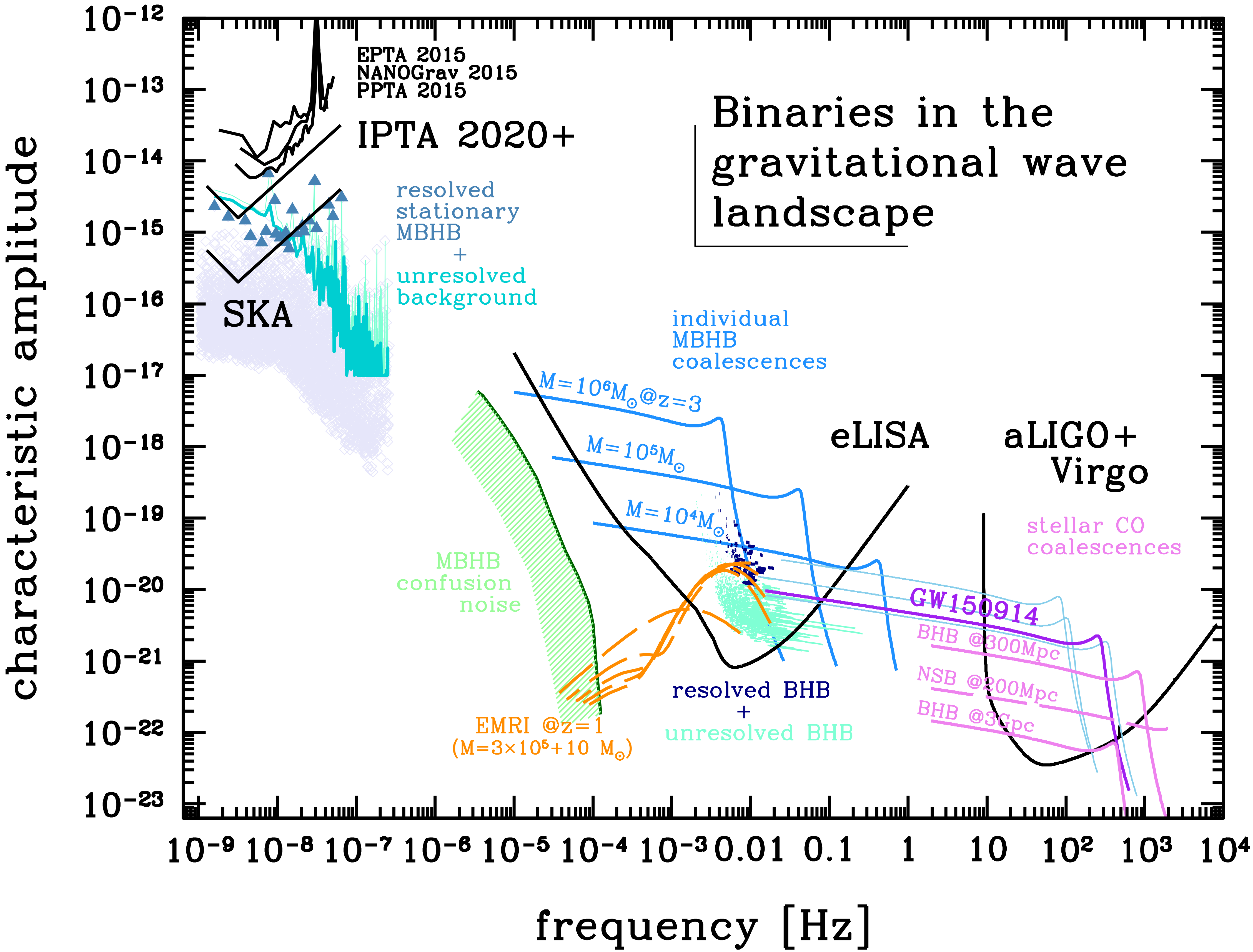


figure 33 from Colpi & Sesana 2017, "An Overview of Gravitational Waves," World Scientific (arXiv:1610.05309)

$$\text{mass ratio}: \quad q \equiv \frac{m_2}{m_1} \leq 1 \quad \text{and} \quad \eta \equiv \frac{q}{(1+q)^2} \leq 0.25$$

$$\text{total mass}: \quad M \equiv m_1 + m_2$$

$$\text{reduced mass}: \quad \mu \equiv \frac{m_1 m_2}{m_1 + m_2} = \left[\frac{q}{(1+q)^2}\right] M = \eta\, M$$

$$\text{chirp mass}: \quad \mathcal{M} \equiv \mu^{3/5}\, M^{2/5} = \left[\frac{q}{(1+q)^2}\right]^{3/5} M = \eta^{3/5}\, M$$

$$\text{GW power}: \quad \frac{dE}{dt} = \frac{32}{5}\,\frac{G^4}{c^5}\,\frac{\eta^2\,\mathcal{M}^5}{a^4}$$

$$\text{GW strain}: \quad h = \left(\frac{32}{5}\right)^{1/2}\,\frac{G^{5/3}}{c^4}\,\frac{\mathcal{M}^{5/3}}{d}(\pi f)^{2/3}$$

$$\text{lifetime} \quad \tau_{GW} = \frac{5}{256}\,\frac{c^5}{G^3}\,\frac{a_0^4}{\eta\,M^3}$$

$$h \equiv \frac{\delta L}{L} \propto \frac{G^2}{c^4}\,\frac{1}{D}\,\frac{M^2}{r}$$

$$t_{\rm chirp} \approx 2\ {\rm hours}\ \frac{(1+q)^{5/3}}{q}\left(\frac{M}{10^6\ {\rm M}_\odot}\right)^{-5/3}\left(\frac{f_{\rm i}}{10^{-3}\ {\rm Hz}}\right)^{-8/3}\left[1-\left(\frac{f_{\rm i}}{f_{\rm f}}\right)^{8/3}\right]$$

$$f_{\rm tidal} \approx 2\,(G\bar{\rho})^{1/2} \sim 0.5\left(\frac{\bar{\rho}}{10^6\ {\rm g\ cm}^{-3}}\right)^{1/2}$$

$$f_{\rm binary} \approx 2\times 10^{-8}\ {\rm Hz}\ \left(\frac{M}{10^6\ {\rm M}_\odot}\right)^{1/2}\left(\frac{a}{10^{-3}\ {\rm pc}}\right)^{-3/2}$$

$$f_{\rm binary} \approx 0.06\ {\rm Hz}\ \left(\frac{M}{10^6\ {\rm M}_\odot}\right)^{-1}\left(\frac{a}{r_{\rm g}}\right)^{-3/2}$$

$$h \approx 2\times 10^{-16}\ \frac{q}{(1+q)^2}\left(\frac{M}{10^6\ {\rm M}_\odot}\right)^{5/3}\left(\frac{f}{10^{-3}\ {\rm Hz}}\right)^{2/3}\left(\frac{D_{\rm L}}{100\ {\rm Mpc}}\right)^{-1}$$