A Very Light Dilaton

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Outline

What is a Dilaton?

A Nambu-Goldstone boson of a spont. broken scale (dilatation) symmetry.
If scale symmetry is only approximate, we get a pseudo NGB instead, also called dilaton.

Why would one be interested in light dilaton?

- It can serve as a scalar analog of the graviton. (Sundrum '03)
- If SM is embedded in a CFT, a dilaton could have similar properties as the Higgs. (Goldberger, Grinstein, Skiba '08)
- It can serve as a force mediator between dark matter and normal matter. (Bai, Carena, Lykken '09)
- If the dilaton is sufficiently decoupled, can serve as a dark matter candidate. (Choi, Hong, Matsuzaki '11)

Review of Scale Symmetry

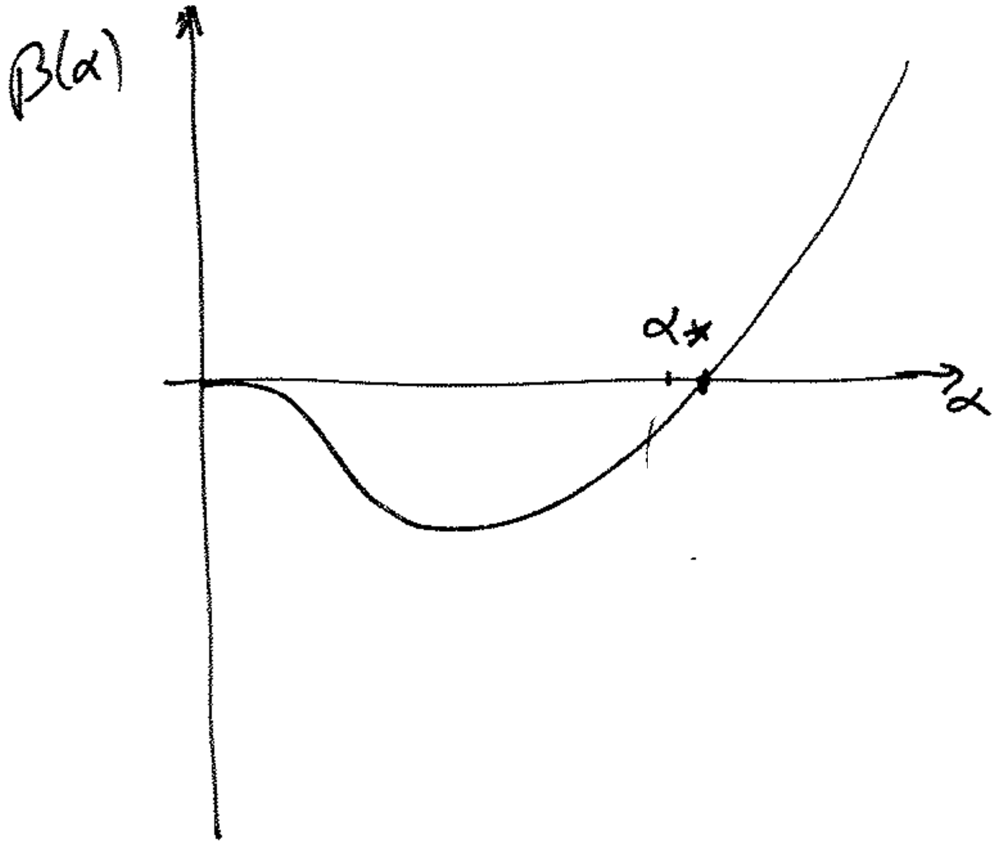
Scale transformation takes

$$x \to e^{\alpha} x$$
$$\phi(x) \to e^{\alpha d_\phi} \phi(e^{\alpha} x)$$

Classical: theory without dimensional parameters is scale invariant.
Quantum: scale invariance is broken by renormalization effect.

Scale invariance is recovered if there is a (non-trivial) fixed point.

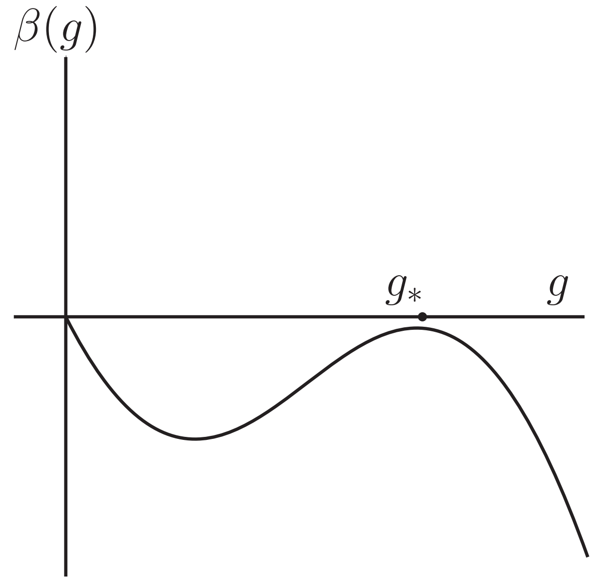
Along RG trajectory close to the fixed point, one usually has approximate scale invariance.



Coupling runs from UV fixed point (origin) to IR fixed point
(which it reaches in exponential RG-time).

(Approximate) Scale Invariant Theory

A schematic β function of the theory. The coupling flow toward the "would be" IR fixed-point, g_*. Close to the fixed-point, the flow is slow and the theory posses approximate scale symmetry.



If the RG-trajectory reaches g_*, scale invariance becomes exact.

However, some degree of freedom in the theory can get a vev. If this happens close to the fixed-point, scale invariance is spontaneously broken and one expect a dilaton in the spectrum.

A field theory with this behavior is not common!

(Approximate) Scale Invariant Theory (cont.)

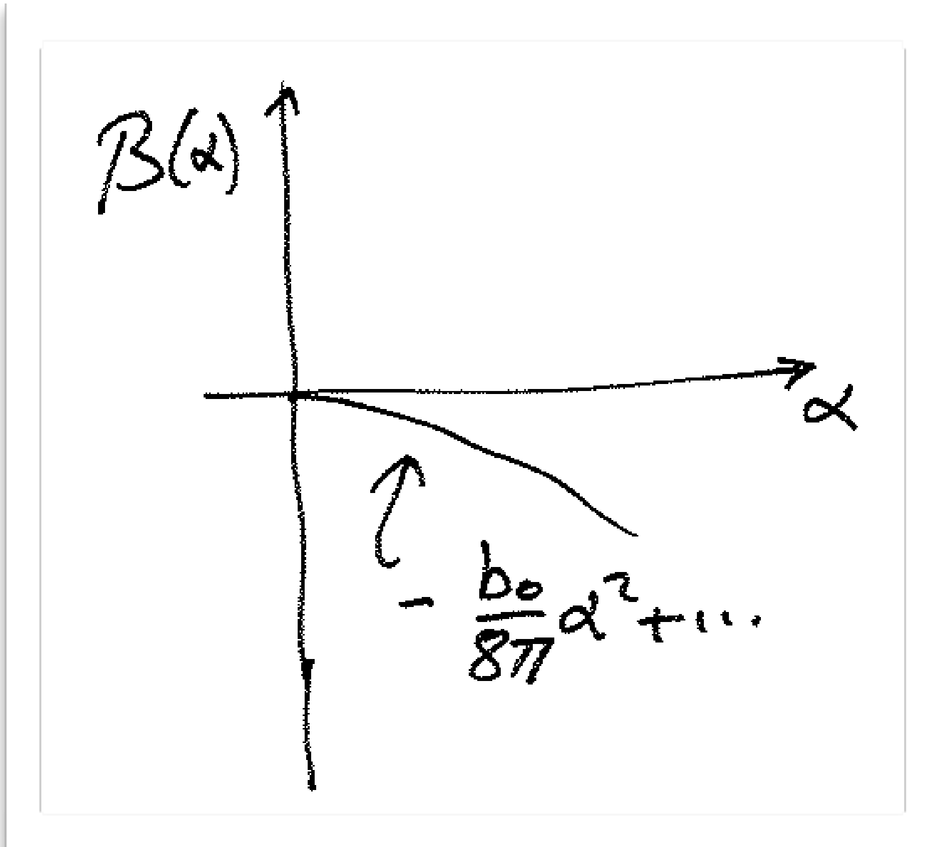
Walking Technicolor has been one of the widely studied theories of this type.
In this framework, the fixed-point is strongly interacting. As a result, fermion condensate is formed and scale invariance is broken.
But a strong interacting nature of the model makes it difficult to analyze analytically.
In particular, the existence of a light dilaton in WTC is not clear and is a subject of a recent debate:

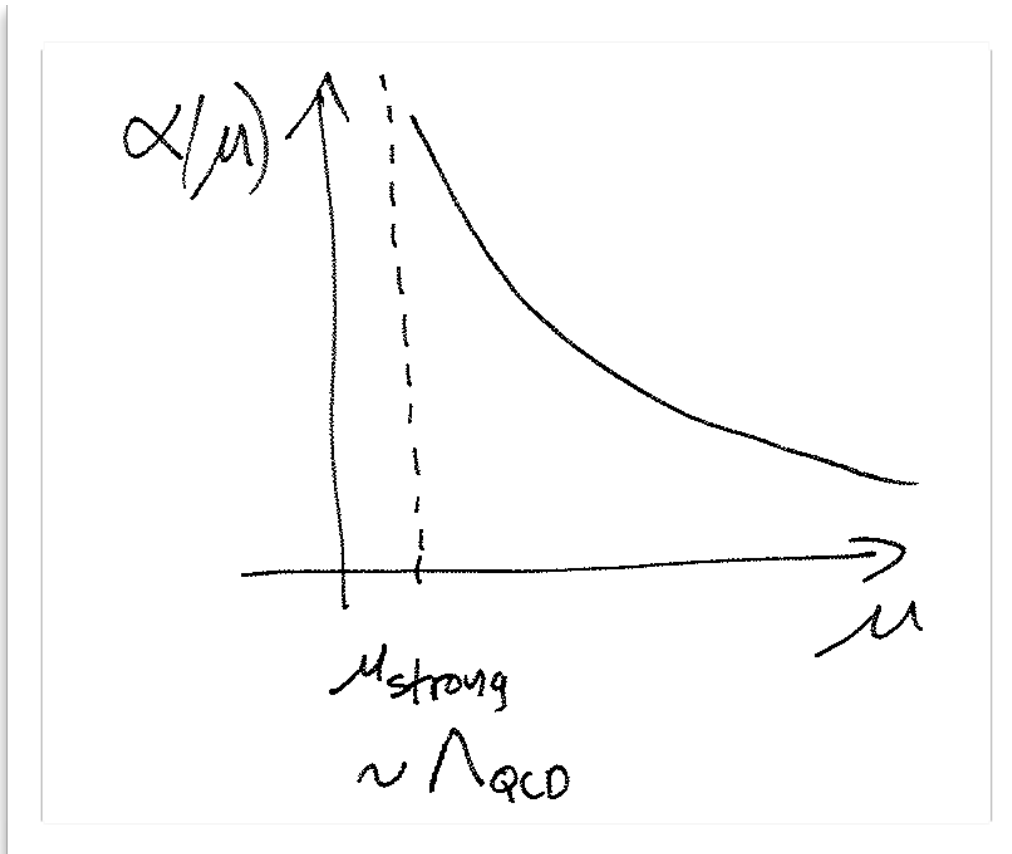
- Yes (Appelquist, Bai '10)
- No (Hashimoto, Yamawaki '10; Vecchi '10)

Having a perturbative toy model with the above properties – an interacting fixed-point and an approximate scale invariance which is broken dynamically, would help us study such a question.

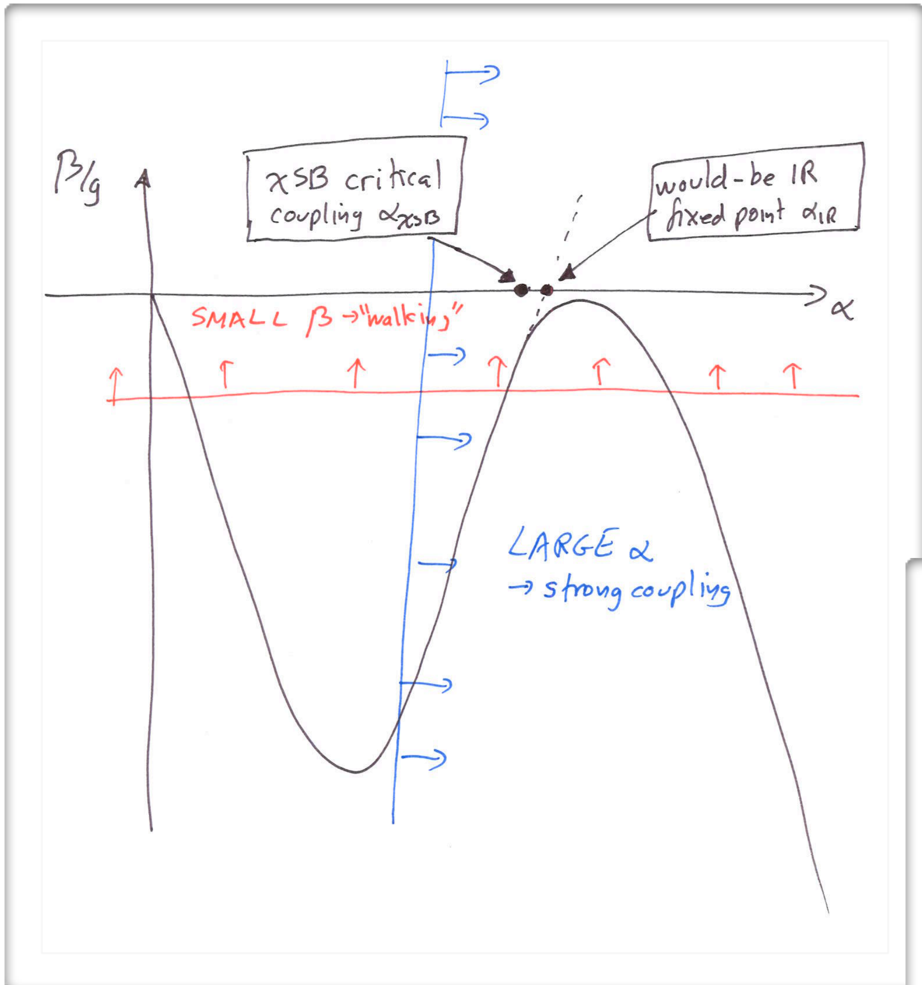
Walking Technicolor in Pictures

First, review QCD:

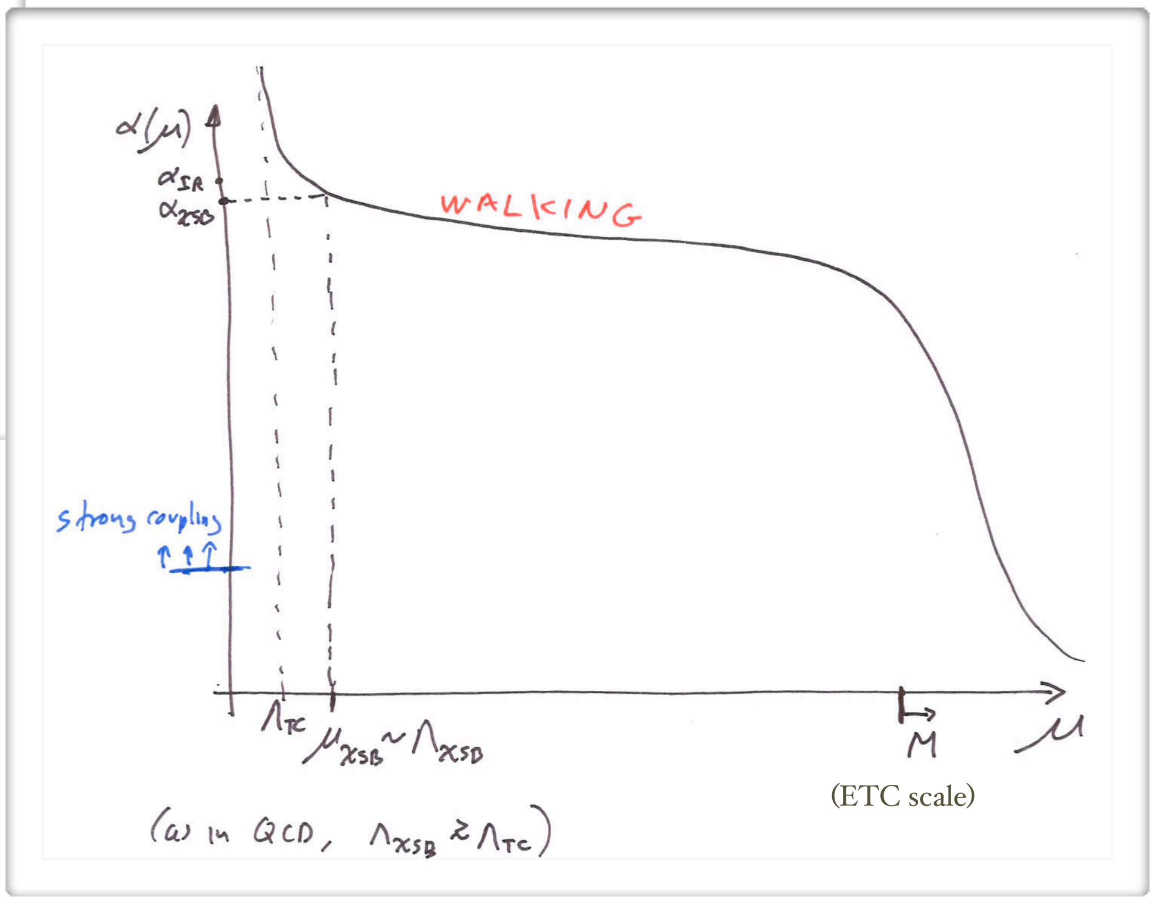




WTC:



β/g
χSB critical coupling α_χSB
would-be IR fixed point α_IR
α
SMALL β → "walking"
LARGE α
→ strong coupling



α(μ)
α_IR
α_χSB
WALKING
strong coupling
Λ_TC
μ_χSB ~ Λ_χSB
M
μ
(ETC scale)
(as in QCD, Λ_χSB ≳ Λ_TC)

The quark mass:

o(1) — hidden dependence on Λ/M — o(Λ^3)

$$\mathcal{L}_{\rm mass} = \frac{C(\mu)}{M^2}\langle\bar{Q}Q\rangle(\mu)\ \bar{q}q = \frac{C(M)}{M^2}\langle\bar{Q}Q\rangle(M)\ \bar{q}q = \frac{C(\Lambda)}{M^2}\langle\bar{Q}Q\rangle(\Lambda)\ \bar{q}q$$

Renormalization Group: $C(\Lambda) = \left(\frac{M}{\Lambda}\right)^{\gamma_*}$

The central observation: large coupling gives large anomalous dimension
It is argued that $\gamma_* \cong 1$

Hence

$$\mathcal{L}_{\rm mass} = \frac{C(M)}{M\Lambda}\langle\bar{Q}Q\rangle(\Lambda)\ \bar{q}q \qquad \Rightarrow \qquad m_q \sim C(M)\frac{\Lambda^2}{M}$$

Bonus: in non-minimal TC this also raises the mass of pseudo-goldstone bosons:

If $\gamma_{(\bar{Q}Q)(\bar{Q}Q)} \approx 2\gamma_{(\bar{Q}Q)} \approx 2$ then $(\bar{Q}Q)(\bar{Q}Q)$ has dimension 4 (marginal operator).

(Approximate) Scale Invariant Theory (cont.)

Walking Technicolor has been one of the widely studied theories of this type.
In this framework, the fixed-point is strongly interacting. As a result, fermion condensate is formed and scale invariance is broken.
But a strong interacting nature of the model makes it difficult to analyze analytically.
In particular, the existence of a light dilaton in WTC is not clear and is a subject of a recent debate:

- Yes (Appelquist, Bai '10) Small beta-function
- No (Hashimoto, Yamawaki '10; Vecchi '10) Anomaly: like eta'

Having a perturbative toy model with the above properties – an interacting fixed-point and an approximate scale invariance which is broken dynamically, would help us study such a question.

Why is it non-straightforward?

- SSB in CFT? How? Take, e.g., $\mathcal{N}$=4 SUSY SU(N)
  Flat directions. Expand about point away from origin.
  SU(N) -> SU(N-k) x SU(k) x U(1). But $\mathcal{N}$=4 SUSY unbroken.
  "Dilaton" is exactly massless and corresponds to radial direction along flat directions. There already, not a real dilaton (does not appear as dilaton in low energy effective theory).

- Better try: Coleman-Weinberg abelian-higgs model.
  Fine tune mass to zero, scale invariance classically.
  Effective potential develops a minimum away from origin.
  Gauge and scale symmetries spontaneously broken.
  Gauge field acquires mass.
  But would-be-dilaton acquires mass too: trace anomaly spoils scaling symmetry.

  M(dilaton)/M(vector) $\sim e^2/16\pi^2$

- Want: arbitrarily light dilaton without turning off interactions

Outline

The Model

SU(N) gauge theory with $n_\psi = n_\chi$ fundamental fermions ψ and χ and two scalar singlets ϕ_1 and ϕ_2.

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu} + \sum_{j=1}^{n_\chi}\left(\bar{\psi}^j i\not{D}\psi_j + \bar{\chi}^j i\not{D}\chi_j\right) + \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2$$

$$- y_1\left(\bar{\psi}\psi + \bar{\chi}\chi\right)\phi_1 - y_2(\bar{\psi}\chi + \bar{\chi}\psi)\phi_2$$

$$- \frac{1}{24}\lambda_1\phi_1^4 - \frac{1}{24}\lambda_2\phi_2^4 - \frac{1}{4}\lambda_3\phi_1^2\phi_2^2$$

This theory is invariant under discrete $\mathbb{Z}_2$ as well as SU(n_χ) symmetry

$$\begin{aligned}\phi_1, \psi &\to \phi_1, \psi \\ \phi_2, \chi &\to -\phi_2, -\chi\end{aligned} \quad \text{and} \quad \begin{aligned}\psi &\to U\psi \\ \chi &\to U\chi\end{aligned}$$

Notate bene:

Masses set to zero (I am not solving the hierarchy problem).
This is precisely as with Coleman and Weinberg.
You can set them to zero and use dimensional regularization.

Theory has Landau pole. This is a UV issue. We study
the IT properties of the model. We can take it to be a cut-off theory.

This is not the theory of everything.
It is a Toy Model that displays some behavior that mimics WTC
and may answer some questions.

$\overline{\mathrm{MS}}$ β Functions

For large N with $n_\chi = 11N/4\,(1-\delta/11)$, the leading terms are

$$(16\pi^2)\frac{\partial g}{\partial t} = -\frac{\delta N}{3} g^3 + \frac{25N^2}{2}\frac{g^5}{16\pi^2}$$

$$(16\pi^2)\frac{\partial y_1}{\partial t} = 4y_1 y_2^2 + 11N^2 y_1^3 - 3Ng^2 y_1$$

$$(16\pi^2)\frac{\partial y_2}{\partial t} = 3y_1^2 y_2 + 11N^2 y_2^3 - 3Ng^2 y_2$$

$$(16\pi^2)\frac{\partial \lambda_1}{\partial t} = 3\lambda_1^2 + 3\lambda_3^2 + 44N^2\lambda_1 y_1^2 - 264N^2 y_1^4$$

$$(16\pi^2)\frac{\partial \lambda_2}{\partial t} = 3\lambda_2^2 + 3\lambda_3^2 + 44N^2\lambda_2 y_2^2 - 264N^2 y_2^4$$

$$(16\pi^2)\frac{\partial \lambda_3}{\partial t} = \lambda_1\lambda_3 + \lambda_2\lambda_3 + 4\lambda_3^2 + 22N^2\lambda_3 y_1^2 + 22N^2\lambda_3 y_2^2 - 264N^2 y_1^2 y_2^2$$

Fixed-point

To get a fixed-point for the gauge coupling, need to balance a 1-loop against a 2-loop.

This is possible because for large N, δ can be made small by a carefully chosen n_χ.

The fixed-point to leading order in $1/N$ is

$$g_*^2 = 16\pi^2 \frac{2}{75}\frac{\delta}{N}$$

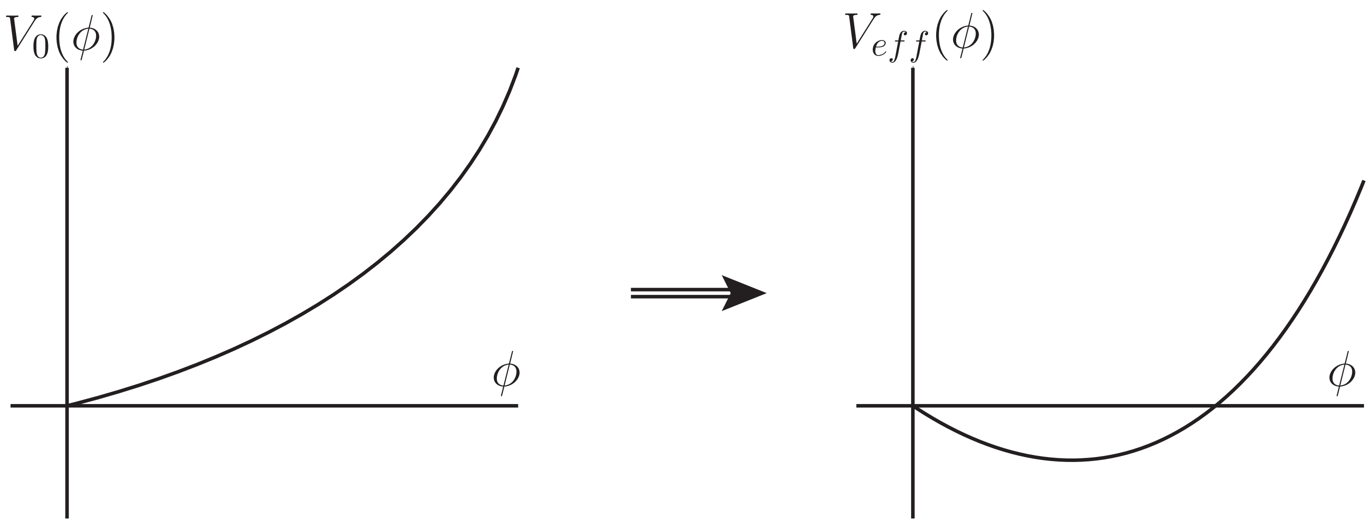
$$y_{1*}^2 = y_{2*}^2 = \frac{3}{11}\frac{g_*^2}{N}$$

$$\lambda_{1*} = \lambda_{2*} = \lambda_{3*} = \frac{18}{11}\frac{g_*^2}{N} = 6y_{1*}^2$$

Effective Potential

At tree-level, $\langle \phi_i \rangle = 0$ and all the particle are massless. The theory flows to the IR fixed-point.

However, quantum effect could drastically change the structure of the vacuum (Coleman, Weinberg '73)



The non-trivial vev gives mass to both fermions and scalars and alters the RG trajectory.

Effective Potential (cont.)

The effective potential in $\overline{\text{MS}}$ is

$$V_{\text{eff}} = -\frac{1}{24}\lambda_1\phi_1^4 - \frac{1}{24}\lambda_2\phi_2^4 - \frac{1}{4}\lambda_3\phi_1^2\phi_2^2 - \frac{11N^2M_{f+}^4}{(64\pi^2)}\left(\ln\frac{M_{f+}^2}{2\mu^2} - \frac{3}{2}\right) - \frac{11N^2M_{f-}^4}{(64\pi^2)}\left(\ln\frac{M_{f-}^2}{2\mu^2} - \frac{3}{2}\right) + \frac{M_{s+}^4}{(64\pi^2)}\left(\ln\frac{M_{s+}^2}{\mu^2} - \frac{3}{2}\right) + \frac{M_{s-}^4}{(64\pi^2)}\left(\ln\frac{M_{s-}^2}{2\mu^2} - \frac{3}{2}\right)$$

$$M_{f\pm} = y_1\phi_1 \pm y_2\phi_2,$$

$$M_{s\pm}^2 = \frac{(\lambda_1+\lambda_3)\phi_1^2 + (\lambda_2+\lambda_3)\phi_2^2}{4} \pm \frac{\sqrt{(\lambda_1-\lambda_3)^2\phi_1^4 + (\lambda_2-\lambda_3)^2\phi_2^4 - 2(\lambda_1\lambda_2 - \lambda_1\lambda_3 - \lambda_2\lambda_3 - 7\lambda_3^2)\phi_1^2\phi_2^2}}{4}$$

Effective Potential (cont.)

Minimizing the potential analytically is difficult. But easy to identify some local minima. Focus on minimum which preserves discrete $\mathbb{Z}_2$ (i.e. $\langle\phi_2\rangle = 0$).

The potential reduces to

$$V_{\text{eff}} = \frac{\lambda_1}{24}\phi_1^4 + \frac{(\lambda_1\phi_1^2)^2}{256\pi^2}\left(\ln\frac{\lambda_1\phi_1^2}{2\mu^2} - \frac{3}{2}\right) + \frac{(\lambda_3\phi_1^2)^2}{256\pi^2}\left(\ln\frac{\lambda_3\phi_1^2}{2\mu^2} - \frac{3}{2}\right) - \frac{22N^2y_1^4\phi_1^4}{64\pi^2}\left(\ln\frac{y_1^2\phi_1^2}{\mu^2} - \frac{3}{2}\right)$$

The extremum, $\partial/\partial\phi_1 V_{\text{eff}}(\langle\phi_1\rangle) = 0$, is at

$$-\frac{\lambda_1}{6} = \frac{\lambda_1^2}{64\pi^2}\left(\ln\frac{\lambda_1\langle\phi_1\rangle^2}{2\mu^2} - 1\right) + \frac{\lambda_3^2}{64\pi^2}\left(\ln\frac{\lambda_3\langle\phi_1\rangle^2}{2\mu^2} - 1\right) - \frac{88N^2y_1^4}{64\pi^2}\left(\ln\frac{y_1^2\langle\phi_1\rangle^2}{\mu^2} - 1\right)$$

Vacuum Expectation

λ_1 can be traded with $\langle\phi_1\rangle$ as a free parameter. For consistency, $\frac{\lambda_1}{16\pi^2}\ln\frac{\langle\phi_1\rangle^2}{\mu^2} \ll 1$. Stability of the vev is determined from the eigenvalues of second derivative matrix

$$\frac{\partial^2}{\partial\phi_1^2}V_{\text{eff}}(\langle\phi_1\rangle, 0) = \frac{\lambda_3^2 - 88N^2y_1^4}{32\pi^2}\langle\phi_1\rangle^2$$
$$\frac{\partial^2}{\partial\phi_2^2}V_{\text{eff}}(\langle\phi_1\rangle, 0) = \frac{\lambda_3}{2}\langle\phi_1\rangle^2 + \mathcal{O}(\text{1-loop})$$

Evaluate V_{eff} at $\langle\phi_1\rangle$ yields

$$V_{\text{eff}}(\langle\phi_1\rangle) = -\frac{\lambda_3^2 - 88N^2y_1^4}{512\pi^2}\langle\phi_1\rangle^4$$

Thus when $\varepsilon \equiv \lambda_3^2 - 88N^2y_1^4 \geq 0$, there is a non-trivial minimum.

Role of ϕ_2

Note that ϕ_2 never entered into any calculation.
Moreover, one can get an attractive IR fixed-point with just one scalar singlet. This raise the question, what is the purpose of the second singlet?

Role of ϕ_2

Note that ϕ_2 never entered into any calculation.
Moreover, one can get an attractive IR fixed-point with just one scalar singlet. This raise the question, what is the purpose of the second singlet?

- It allows us to introduce more couplings, in particular the cross-coupling λ_3.
- Without the second singlet, the extremum found by perturbative analysis would have been the maximum.
  - The scalar potential appears to be unbounded from below.
  - Possible to have non-trivial minimum at higher scale which is inaccessible to perturbative analysis.

Pole mass in Broken Phase

The explicit 1-loop pole masses are

$$
\begin{aligned}
M_\psi(\mu) = M_\chi(\mu) &= y_1 v\left[1-\frac{g^2}{16\pi^2}\frac{N}{2}\left(3\ln\frac{y_1^2v^2}{\mu^2}-4\right)\right] \\
M_{\phi_1}^2 &= \frac{\lambda_1 v^2}{2}+\frac{3\lambda_1^2v^2}{64\pi^2}\left(\ln\frac{\lambda_1 v^2}{2\mu^2}-\frac{5}{3}+\frac{2\pi}{3\sqrt{3}}\right)+\frac{3\lambda_3^2v^2}{64\pi^2}\left(\ln\frac{\lambda_3 v^2}{2\mu^2}-\frac{1}{3}-\frac{2\lambda_1}{3\lambda_3}\right) \\
&\quad+\frac{22N^2y_1^2}{16\pi^2}\left[y_1^2v^2-\frac{\lambda_1 v^2}{12}-3\left(y_1^2v^2-\frac{\lambda_1 v^2}{12}\right)\left(\ln\frac{y_1^2v^2}{\mu^2}\right)\right. \\
&\quad\left.-3\int_0^1 dx\left(y_1^2v^2-\frac{x(1-x)}{2}\lambda_1 v^2\right)\ln\left(1-x(1-x)\frac{\lambda_1}{2y_1^2}\right)\right] \\
&\simeq \frac{\lambda_3^2-88N^2y_1^4}{32\pi^2}v^2=\frac{\varepsilon}{32\pi^2}v^2
\end{aligned}
$$

Since $v=\langle\phi_1\rangle$, it has the same anomalous dimension as ϕ_1.
Using the anomalous dimension and the β functions, one can verify that the masses are RG invariant at 1-loop.

Outline

Dilatation Current

The dilatation current, $\mathcal{D}^\mu$, is constructed from the improved energy-moementum tensor, $\Theta^{\mu\nu}$, of Callan, Coleman and Jackiw.

$$\mathcal{D}^\mu = x_\nu \Theta^{\mu\nu}$$

$$\Theta^{\mu\nu} = -F^{a\mu\lambda}F^{a\nu}_{\lambda} + \frac{1}{2}\bar{\chi} i(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\chi + \frac{1}{2}\bar{\psi} i(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi$$
$$+ \partial^\mu \phi_i \partial^\nu \phi_i - g^{\mu\nu}\mathcal{L} - \frac{1}{2}\kappa(\partial^\mu \partial^\nu - g^{\mu\nu}\partial^2)\phi_i^2$$

κ is the improvement term. It is a total derivative.
The CCJ improved tensor is the one with $\kappa = 1/3$.

- The improvement term does not change the charges constructed from $\Theta^{\mu\nu}$.
- The matrix elements of $\Theta^{\mu\nu}$ is finite, thus it doesn't get renormalized.

Trace Anomaly

The divergence of the dilatation current is the trace of the improved energy-momentum tensor.
Classically Θ^μ_μ vanishes for theory without any dimensional couplings.
Quantum effects make Θ^μ_μ non-zero, this is known as trace anomaly.
For the theory under consideration

$$\Theta^\mu_\mu = \gamma_{\phi_1}\phi_1\partial^2\phi_1 + (4\gamma_{\phi_1}\lambda_1 - \beta_{\lambda_1})\frac{\phi_1^4}{24} + \ldots$$

Terms involving other fields are omitted.

Terms proportional to γ_{ϕ_1} are usually omitted.
They cancel when EOM is applied but can contribute to off-shell matrix element and Green functions.
Also these terms are needed to make the trace RG-invariant.

Dilaton State

The dilaton state, $|\sigma\rangle$, in the theory is identify by the following criteria

- spinless state
- couple strongly to the energy-momentum tensor
- lightest state

Clearly ϕ_1 satisfy all of the above.

- It is the only state whose mass start at 1-loop.
- It is the only state which couples linearly to the energy-momentum tensor when expanded about $\langle\phi_1\rangle = v$, $\langle\phi_2\rangle = 0$

Thus $|\sigma\rangle$ can be identify with a state created by ϕ_1.

Decay Constant

Define the decay constant f_σ by

$$\langle 0|\Theta^{\mu\nu}(x)|\sigma\rangle = \frac{f_\sigma}{3}\left(p^\mu p^\nu - g^{\mu\nu}p^2\right)e^{ip\cdot x}$$

where p is the momentum state $|\sigma\rangle$. The form of the right hand side is constrained by conservation of $\Theta^{\mu\nu}$. The factor $1/3$ comes from

$$\langle 0|\partial_\mu \mathcal{D}^\mu|\sigma\rangle = \langle 0|\Theta^\mu_\mu|\sigma\rangle = -f_\sigma M_\sigma^2 e^{ip\cdot x}$$

Note that $\Theta^{\mu\nu} = -1/3v\partial^\mu\partial^\nu\phi_1 + \cdots$.
Thus to lowest order $f_\sigma = v + \cdots$.
The RG invariant expression is easy to guess

$$f_\sigma = vZ_{\phi_1}^{-1/2}$$

where Z_{ϕ_1} is the wavefunction renormalization factor.

Dilaton Mass

Having determined the decay constant f_σ, the mass of the dilaton can be obtained from the trace anomaly.
To lowest order, the mass is

$$M_\sigma^2 = \frac{\lambda_1^2 + \lambda_3^2 - 88N^2 y_1^4}{32\pi^2} v^2$$
$$= \frac{\varepsilon}{32\pi^2} v^2 = M_{\phi_1}^2$$

where λ_1^2 term is dropped for consistency.
RG invariant of M_σ can be inferred from M_{ϕ_1}.

- Given the vev, we can tune ε to make the dilaton light compares to the vev.

Outline

Broken Phase

Recall the theory admits a non-trivial minimum provided

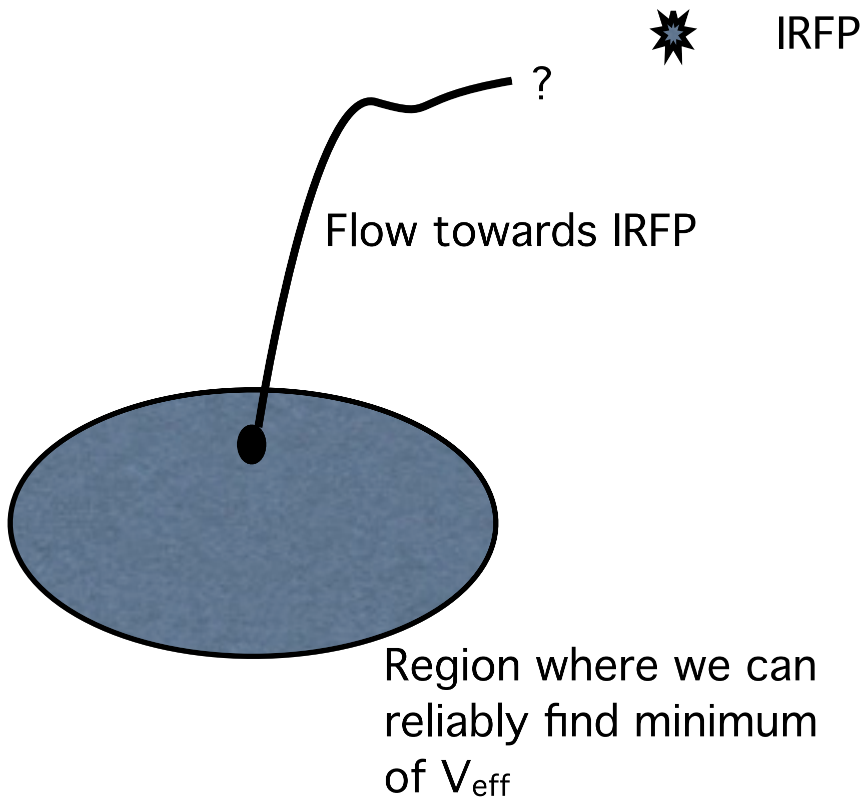
- λ_1 is much smaller than other couplings,
- $\varepsilon \equiv \lambda_3^2 - 88N^2 y_1^4 \geq 0$.

We want to study symmetry breaking close to the IR fixed-point. However, near the fixed-point these conditions are not satisfied.

Use RGE to trace back the RG trajectory to large RG time where perturbative analysis of effective potential yields a non-trivial minimum.

Similary can define the theory at scale μ_0 where perturbative analysis yields a non-trivial minimum. Moreover, if the vev is well below μ_0, RG flows will get closed to the fixed-point before the massive particles decouple.

Theory parameter space: couplings at fixed μ_0



Symmetric Phase

For a point in parameter space where $\varepsilon < 0$

- $V_{eff}(\langle\phi_1\rangle)$ becomes positive and the non-trivial minimum disappears,
- the effective potential seems to be unbounded from below along ϕ_1 direction for large ϕ_1.

The second point threatens the validity of the model.

However, at large ϕ_1 perturbative analysis breaks down.
Can extend the range of perturbativity using the improved effective potential which effectively re-sum large logarithms.

$$V_{eff}^{imp} = \frac{1}{24}\bar{\lambda}_1(t)e^{-4\int_0^{t'}\gamma_{\phi_1}dt'}\phi_1^4$$

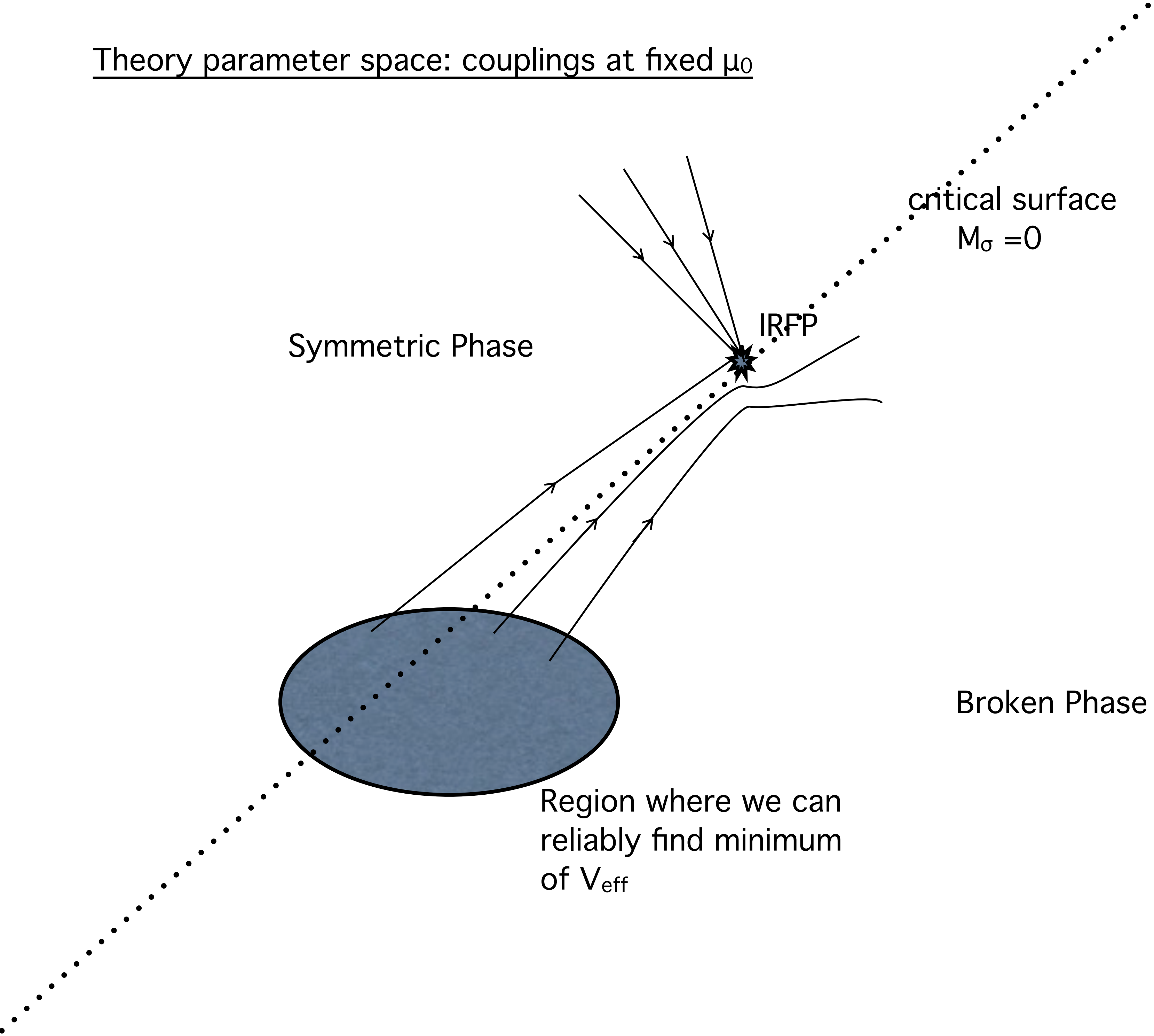
Here $t = \ln\phi_1/\mu_0$. This form is valid as long as $\bar{\lambda}_1(t)$ is perturbative.

Symmetric Phase (cont.)

For points in parameter space closed to the IR fixed-point, gauge coupling drives the Yukawa coupling to 0 in the UV.
Thus in the far UV, the β-function for λ_1 has a Landau pole.

- The effective potential is bounded from below because $\bar{\lambda}_1(t) > 0$ for large ϕ_1.
- The theory need a UV cutoff.
    - One can view the model as being a low energy effective theory of some UV completed models.
    - Since the cutoff will be many order of magnitude above the scale of symmetry breaking, one can (safely) ignore it.

Theory parameter space: couplings at fixed μ_0



Numerical Value

$N = 20$, $n_f = 11/2\,N$, $\delta = 0.2$,
$g(\mu_0) = \frac{4}{9} g_*$, $y_1(\mu_0) = 0.32 y_{1*}$, $y_2(\mu_0) = \frac{1}{5} y_{2*}$,
$\lambda_1(\mu_0) = \frac{1}{30}\lambda_{2*}$, $\lambda_2(\mu_0) = 3\lambda_{2*}$, $\lambda_3(\mu_0) = 5.2\lambda_{3*}$.
These condition corresponds to $\varepsilon \gtrsim 0$.
The vev is at

$$\ln \frac{\langle \phi_1 \rangle}{\mu_0} \simeq -29$$

and the spectrum are

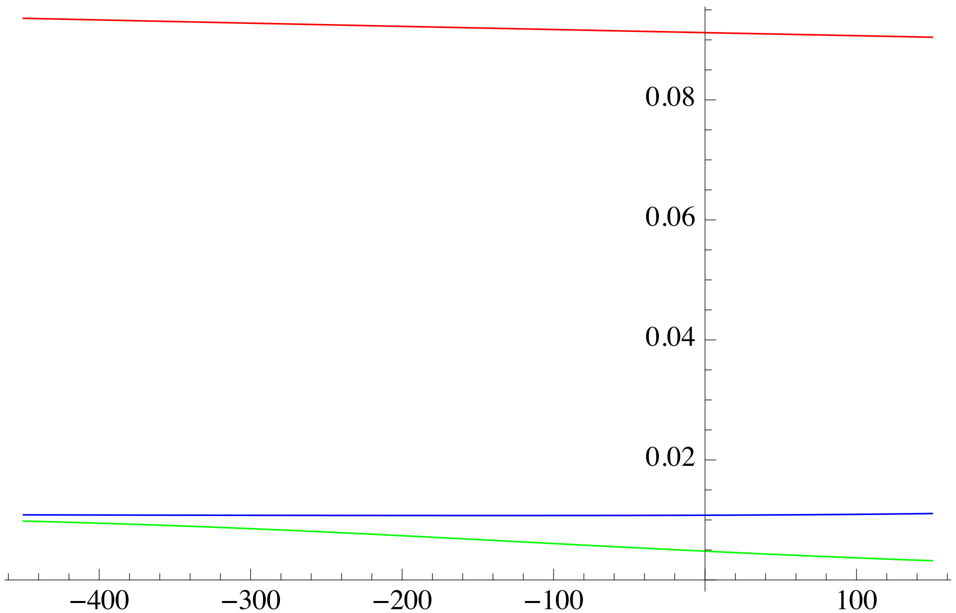
$$\frac{M_{\psi,\chi}}{v} \simeq 8.5 \times 10^{-3}, \quad \frac{M_{\phi_1}}{v} \simeq 7.9 \times 10^{-4}, \quad \frac{M_{\phi_2}}{v} \simeq 9.5 \times 10^{-2}.$$

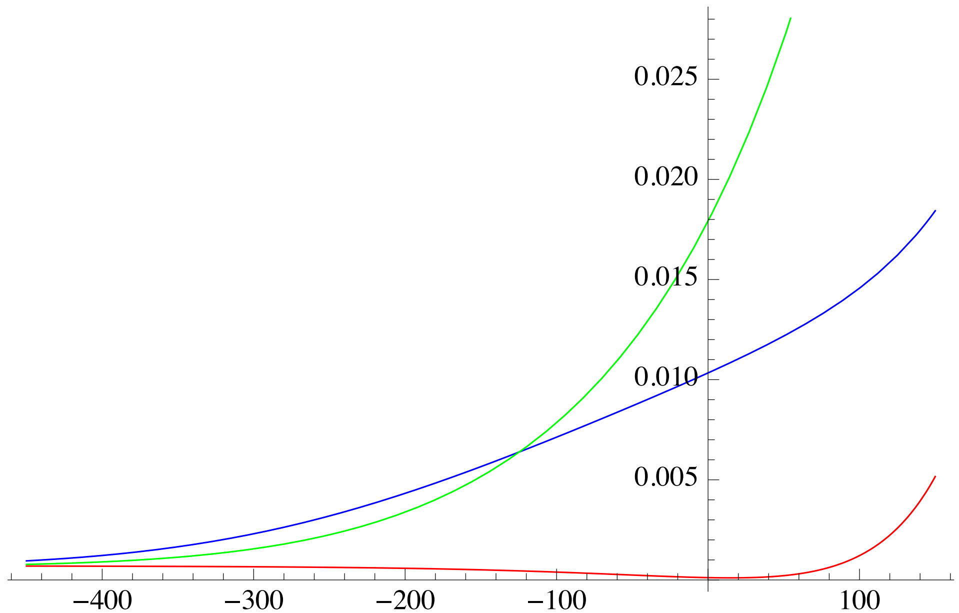
Fractional correction to the effective potential from higher order terms are approximated to be

$$\left| \frac{N g_*^2}{16\pi^2} \ln \left(\frac{y_1^2 v^2}{\mu^2} \right) \right| \simeq 0.2.$$

Numerical: Couplings Evolution

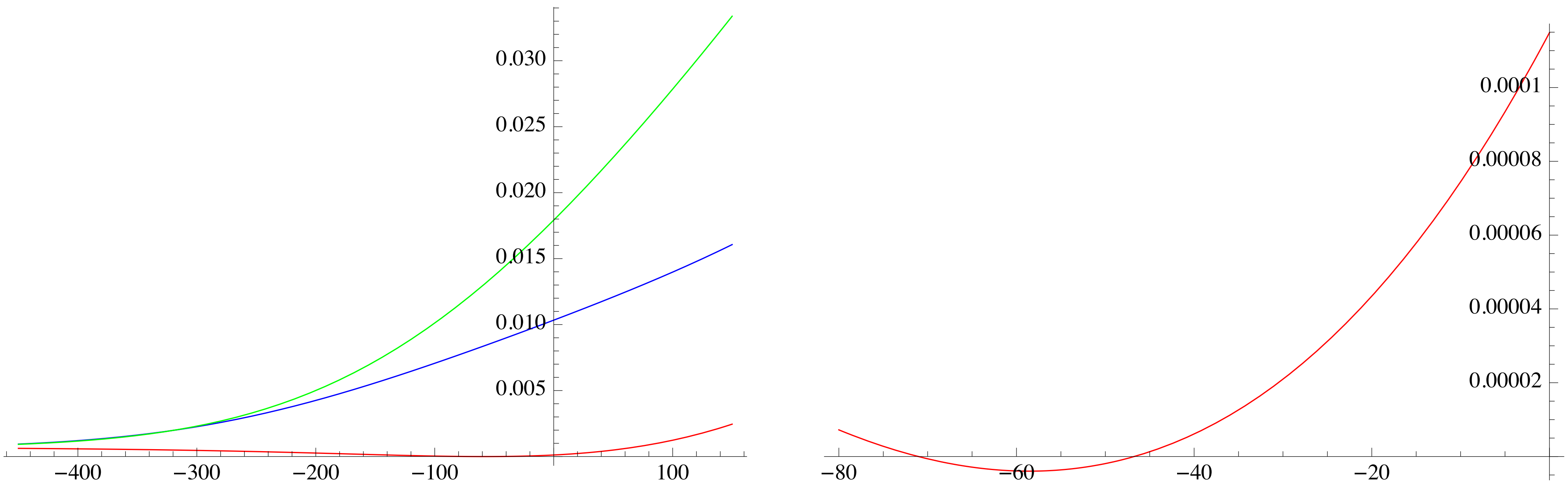
$N = 20$, $n_f = 11/2\,N$, $\delta = 0.2$,
$g(\mu_0) = \frac{4}{9}g_*$, $y_1(\mu_0) = 0.45 y_{1*}$, $y_2(\mu_0) = \frac{1}{5}y_{2*}$,
$\lambda_1(\mu_0) = \frac{1}{30}\lambda_{1*}$, $\lambda_2(\mu_0) = 3\lambda_{2*}$, $\lambda_3(\mu_0) = 5.2\lambda_{3*}$. These condition corresponds to $\varepsilon < 0$.





Numerical: Broken Phase

$y_1(\mu_0) = 0.32 y_{1*}$. This corresponds to a positive ε.



The coupling λ_1 becomes negative during the flow.
This agrees with our expectation from the improved effective potential.

Outline

Applications

We can use this model to verify various results in the literature. For a specific example we will verify the dilaton potential in nearly conformal theory (Goldberger, BG, Skiba) . Taking $\mathcal{L} = \mathcal{L}_{\mathsf{CFT}} + \sum_n \lambda_n \mathcal{O}_n$, GGS arrives, via indirect argument, at the dilaton potential

$$V_{\mathrm{eff}}(\chi) = \frac{M_\sigma^2}{4 f_\sigma^2} \chi^4 \left[\ln\left(\frac{\chi}{f_\sigma} \right) - \frac{1}{4} \right] + \mathcal{O}(\gamma^2).$$

If $$\mathcal{L} = \mathcal{L}_{\rm CFT} + \sum_n \lambda_n \mathcal{O}_n \,.$$ and

$|\gamma_n| \ll 1$ then $$V_{\rm eff}(\chi) = \frac{M_\sigma^2}{4 f_\sigma^2} \chi^4 \left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4} \right] + \mathcal{O}(\gamma^2) \,.$$

or

$|\lambda_n| \ll 1$ $$V_{\rm eff}(\chi) = \frac{M_\sigma^2}{f_\sigma^2 \gamma} \chi^4 \left[\frac{1}{4+\gamma} \left(\frac{\chi}{f_\sigma}\right)^{\gamma} - \frac{1}{4} \right] + \mathcal{O}(\lambda^2) \,,$$

$$V_{\rm eff}(\chi) = \frac{M_\sigma^2}{f_\sigma^2} \chi^4 \sum_n \left\{ x_n \left[\frac{1}{4+\gamma_n} \left(\frac{\chi}{f_\sigma}\right)^{\gamma_n} - \frac{1}{4} \right] \right\} + \mathcal{O}(\lambda^2) \,,$$ $\sum_n \gamma_n x_n = 1.$

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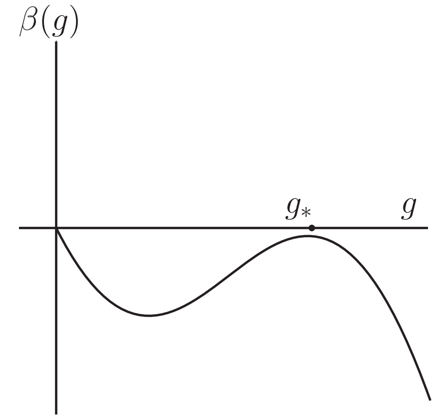
To compare our model with GGS, we view our model as $\mathcal{L}(g) = \mathcal{L}(g_*) + (\mathcal{L}(g) - \mathcal{L}(g_*))$. In our model the dilaton field is identify with ϕ_1 and the anomalous dimensions are small.
Our effective potential for ϕ_1 turns out to be exactly the same as GGS.



Dilaton in WTC?

AB say:

$$M_\sigma^2 \simeq \frac{s(\alpha_* - \alpha_c)}{\alpha_c}\Lambda^2 \simeq \frac{N_f^c - N_f}{N_f^c}\Lambda^2,$$

First equation: In our model the critical coupling is a critical surface, the IRFP is on critical surface, 0=0, correct but not interesting and not what is intended

Second equation: $(N^c - N) / N$ plays role of ε, measures distance to critical surface, and equation is qualitatively correct!

Summary

- We construct a perturbative model which a non-trivial IR fixed point.
- There are two distinct phases in our model:
  - Symmetric phase: the flow reaches the fixed point (in infinite RG time.)
  - Broken phase: the vev is dynamically generated. this phase somewhat mimics the behavior of walking technicolor.
- The broken phase can be used to study the dilaton:
  - The mass of the dilaton can be made arbitrary small compared to the vev by tuning the parameter $\varepsilon = \lambda_3^2 - 88N^2 y_1^4$.
- The model can be used to verify results/conjectures in literature which are obtained via indirect argument.