Cavitation and Thermal Dilepton Production in QGP

SREEKANTH V.

Physical Research Laboratory, Ahmedabad

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Instituto Nazionale di Fisica Nucleare - LNF, Rome

Collaborators:

- Dr. Jitesh R. Bhatt
- Dr. Hiranmaya Mishra

- The results are presented in:

  Jitesh R. Bhatt, H. Mishra, and Sreekanth V, JHEP 11 (2010) 106. [arXiv:1011.1969]

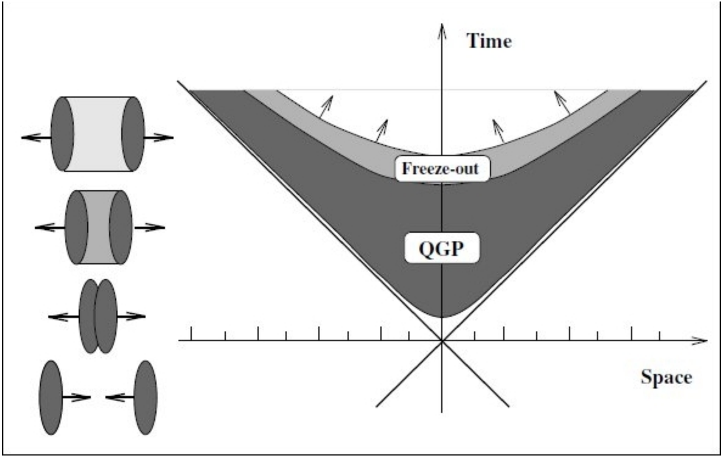
  Jitesh R. Bhatt, H. Mishra, and Sreekanth V, [arXiv:1101.5597]

Plan of the Talk

- Introduction
- 2nd order causal dissipative hydrodynamics (Israel-Stewart)
- Non-ideal effects: EoS and bulk viscosity
- Hydrodyamical evolution and Cavitation
- Thermal photons and dileptons from QGP
- Summary

Introduction

- AIM: To study the role of *non-ideal* effects near T_c arising due to the equation of state (EoS), bulk-viscosity and shear-viscosity (*cavitation*) on the thermal dilepton production from QGP.



Energy momentum tensor of the fluid element in Relativistic dissipative hydrodynamics is defined as

- $$T^{\mu\nu} = \varepsilon\, u^{\mu}\, u^{\nu}\ - P\,\Delta^{\mu\nu} + \Pi^{\mu\nu}$$

  ε, P and u^{μ} are the energy density, pressure and four velocity of the fluid element. $\Delta^{\mu\nu}\ =\ g^{\mu\nu} - u^{\mu}\, u^{\nu}$.
- Viscous contributions to $T^{\mu\nu}$ is represented by

  $$\Pi^{\mu\nu} = \pi^{\mu\nu} -\ \Delta^{\mu\nu}\,\Pi$$
- $\pi^{\mu\nu}$ (traceless) gives the contribution of shear viscosity and Π gives the bulk viscosity contribution.

Formalism: Relativistic Hydrodynamics

Relativistic hydrodyamical equations are

$$\boxed{\begin{array}{rcl} D\varepsilon + (\varepsilon + P)\,\theta - \Pi^{\mu\nu}\nabla_{(\mu}\,u_{\nu)} & = & 0 \\ (\varepsilon + P)\,Du^{\alpha} - \nabla^{\alpha}P + \Delta_{\alpha\nu}\,\partial_{\mu}\Pi^{\mu\nu} & = & 0 \end{array}}$$

$(D \equiv u^{\mu}\partial_{\mu},\ \theta \equiv \partial_{\mu}\,u^{\mu},\ \nabla_{\alpha} = \Delta_{\mu\alpha}\partial^{\mu}$ and $A_{(\mu}\,B_{\nu)} = \frac{1}{2}[A_{\mu}\,B_{\nu} + A_{\nu}\,B_{\mu}])$

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The structure of viscous tensor can be determined with help of the definition of the entropy current s^{μ} and demanding the validity of second law of thermodynamics:

$$\partial_{\mu}s^{\mu} \geq 0 \ (s = \tfrac{\varepsilon+P}{T})$$

Causal dissipative hydrodynamics

- Second order hydrodynamics (Israel-Stewart) is obtained by using
$$s^\mu = su^\mu - \frac{\beta_0}{2T} u^\mu \Pi^2 - \frac{\beta_2}{2T} u^\mu \pi_{\alpha\beta}\pi^{\alpha\beta} + \mathcal{O}(\Pi^3)$$

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- Now $\partial_\mu s^\mu \geq 0$ gives *dynamical evolution equations* for $\pi_{\mu\nu}$ and Π

$$\begin{aligned} \pi_{\alpha\beta} &= \eta\left(\nabla_{<\alpha}u_{\beta>} - \pi_{\alpha\beta}TD\left(\frac{\beta_2}{T}\right) - 2\beta_2 D\pi_{\alpha\beta} - \beta_2\pi_{\alpha\beta}\partial_\mu u^\mu\right) , \\ \Pi &= \zeta\left(\nabla_\alpha u^\alpha - \frac{1}{2}\Pi\ TD\left(\frac{\beta_0}{T}\right) - \beta_0 D\Pi - \frac{1}{2}\beta_0\Pi\partial_\mu u^\mu\right) , \end{aligned}$$

The coefficients β_0 and β_2 are related with the relaxation time by
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\end{aligned}$$

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- Unlike first order (Navier-Stokes) this description is *causal* and no *instabilities* [Hiscock and Lindblom (1985), Baier et. al (2006)]

1D boost invariant flow

Bjorken's prescription to describe the dimensional boost invariant expanding flow:-

- convenient parametrization of the coordinates using the proper time $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity $\eta_s = \frac{1}{2}\, ln[\frac{t+z}{t-z}]$;

$$t = \tau \cosh\eta_s \text{ and } z = \tau \sinh\eta_s$$

- in the local rest frame of the fireball $u^\mu = (\cosh\eta_\mathrm{s}, 0, 0, \sinh\eta_\mathrm{s})$, form of $T^{\mu\nu} = \mathrm{diag.}(\varepsilon, \mathrm{P}_\perp, \mathrm{P}_\perp, \mathrm{P_z})$
- Viscosities can contribute in the effective pressure in the transverse and longitudinal directions

$$\begin{aligned} P_\perp &= P + \Pi + \frac{1}{2}\Phi \\ P_z &= P + \Pi - \Phi \end{aligned}$$

- Φ is the shear ($\pi^{ij} = \mathrm{diag}(\Phi/2, \Phi/2, -\Phi)$) and Π is the bulk viscosity contributions to the equilibrium pressure P.

Equations governing longitudinal expansion

In this 1D formalism;

$$\frac{\partial \varepsilon}{\partial \tau} = -\frac{1}{\tau}(\varepsilon + P + \Pi - \Phi),$$
$$\frac{\partial \Phi}{\partial \tau} = -\frac{\Phi}{\tau_\pi} + \frac{2}{3}\frac{1}{\beta_2 \tau} - \frac{1}{\tau_\pi}\left[\frac{4\tau_\pi}{3\tau}\Phi + \frac{\lambda_1}{2\eta^2}\Phi^2\right],$$
$$\frac{\partial \Pi}{\partial \tau} = -\frac{\Pi}{\tau_\Pi} - \frac{1}{\beta_0 \tau}$$

- $\tau_\pi(T) = \tau_\Pi(T)$ as we don't have any reliable prediction for τ_Π and $\tau_\pi = \frac{2-\ln 2}{2\pi T}$
- EoS is needed to close the system.

Non-ideal effect: EoS and viscosity

- We use lattice QCD result for *non-ideal* ($\varepsilon - 3P \neq 0$) EoS [A. Bazavov *et al*. (2009)], which is more important near T_c.

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- Recent studies show that near the critical temperature T_c effect of bulk viscosity becomes important
- We use the result of Meyer (2008) based upon Lattice QCD calculations which indicate the existence a peak of ζ/s near T_c, however the height and width of this curve are not well understood.
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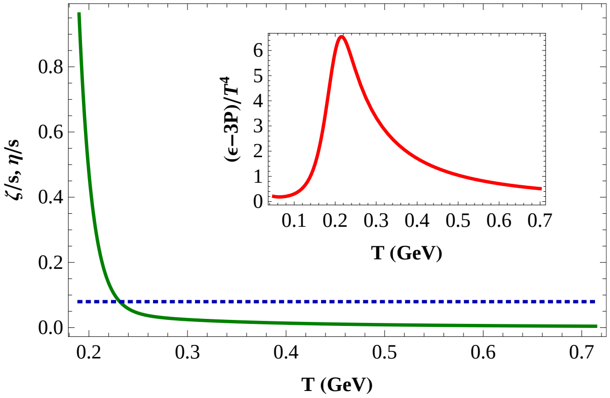
- We use the lower bound of the shear viscosity to entropy density ratio $\eta/s = 1/4\pi$ [KSS (2005)]

Non-ideal Equation of State



- $(\varepsilon - 3P)/T^4, \zeta/s$ (and $\eta/s = 1/4\pi$) as functions of temperature T. Around critical temperature ($T_c = .190$ GeV) $\zeta \gg \eta$ and departure of equation of state from ideal case ($\varepsilon = 3P$) is large.

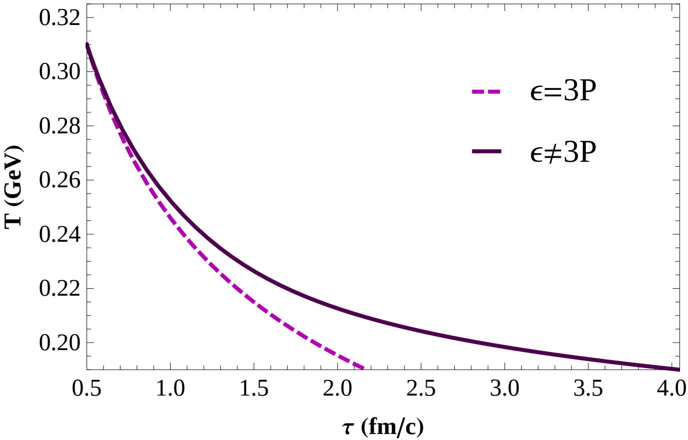
Hydrodyamical Evolution

- In order to understand the temporal evolution of temperature $T(\tau)$, pressure $P(\tau)$ and viscous stresses - $\Phi(\tau) \text{ and } \Pi(\tau)$, we numerically solve the hydrodynamical equations describing the longitudinal expansion of the plasma
- Initial conditions: we use relevant initial condition for RHIC

$$\tau_0 = 0.5 fm/c,\, T_0 = .310 GeV$$

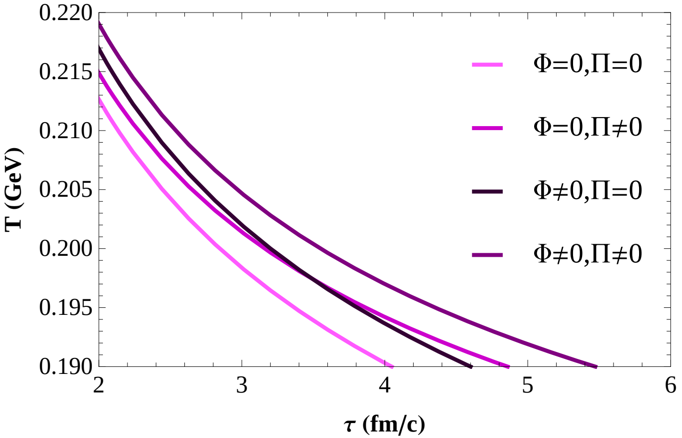
- We will take initial values of viscous contributions as $\Phi(\tau_0) = 0$ and $\Pi(\tau_0) = 0$.

Temperature profile (No viscosity)



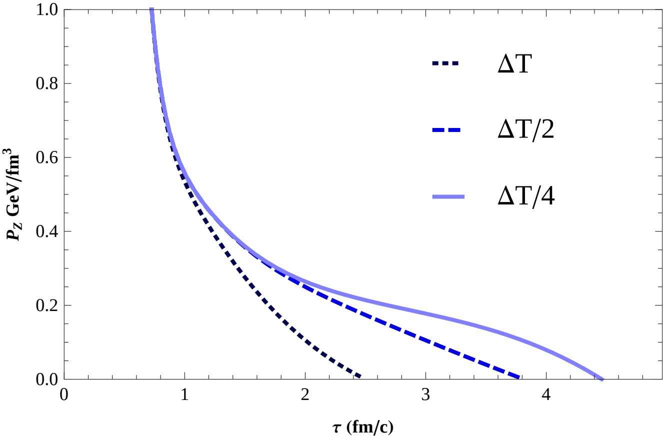
- Temperature profile using massless (*ideal*) and *non-ideal* EoS in RHIC scenario. Viscous effects are neglected in both cases. System evolving with *non-ideal* EoS takes a significantly larger time to reach T_c as compared to *ideal* EoS scenario.

Temperature profile



- Time evolution of temperature with *non-ideal* EoS for different combinations of bulk (Π) and shear (Φ) viscosities.

Longitudinal Pressure

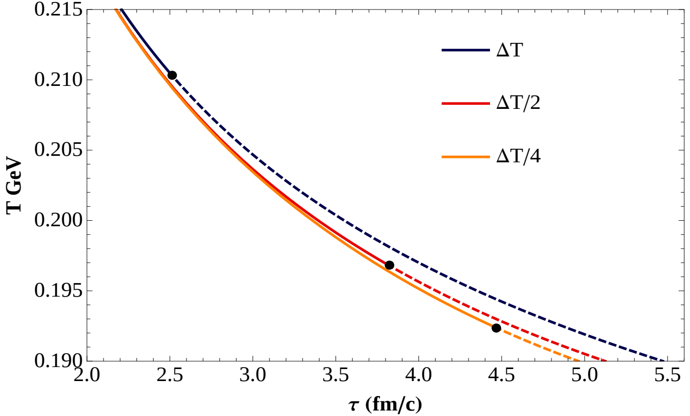


- Longitudinal pressure P_z for various bulk viscosity cases.

- Since $\Pi < 0$, from the definition of longitudinal pressure $P_z = P + \Pi - \Phi$ it is clear that if either Π or Φ is large enough it can drive P_z to negative values.

- $Pz = 0$ defines the condition for the onset of *cavitation*

- At this instant when of P_z becoming zero the expanding fluid will break apart in to fragments and *hydrodynamic treatment looses its validity* [Mishustin et. al. (2008), K. Rajagopal et. al. (2010)]



Temperature is plotted as a function of time. With peak value (a) of ζ/s remains same while width (ΔT) varies. Solid line in the curve ends at the time of cavitation, while the dashed lines shows that how system would continue till T_c if cavitation is ignored. Figure shows that larger the ΔT shorter the cavitation time.

Thermal Dileptons in QGP

- Thermal dilepton emitted from the hot fireball created in relativistic heavy-ion collisions is a promising tool for providing a signature of quark-gluon plasma
- In QGP the dominant mechanism for the production of thermal dileptons is $q\bar{q} \rightarrow \gamma^* \rightarrow l^+ l^-$.
- Spectra of these thermal radiations depend upon the fireball temperature and they can be calculated from the scattering cross-section of the processes.
- Thermal photons can be used as a tool to measure the viscosity of the strongly interacting matter produced in the collisions [J. Bhatt and V. Sreekanth (2010), K Dusling (2010)]

- Rate of dilepton production (number of dileptons produced per unit volume per unit time) for this process is given by

$$\frac{dN}{d^4xd^4p} = \int \frac{d^3\mathbf{p}_1}{(2\pi)^3}\frac{d^3\mathbf{p}_2}{(2\pi)^3} f(E_1,T)f(E_2,T) v_{rel}\, g^2\sigma(M^2)\delta^4(p-p_1-p_2) \quad (1)$$

where $p_{1,2} = (E_{1,2}, \mathbf{p}_{1,2})$ is the four momentum of quark or anti-quark with $E_{1,2} = \sqrt{\mathbf{p}_{1,2}^2 + m_q^2} \simeq |\mathbf{p}_{1,2}|$ neglecting the quark masses. Here $M^2 = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2$ is the invariant mass of the virtual photon $p = (p_0 = E_1 + E_2, \mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2)$ is the four momentum of the dileptons.. The function $f(E,T) = 1/(1+e^{E/T})$ is the quark (anti-quark) distribution function in thermal equilibrium and g is the degeneracy factor. v_{rel} is the relative velocity of the quark-anti-quark pair and $\sigma(M^2)$ is the thermal dilepton production cross section.

Viscous corrections to the distribution functions

- Viscous corrections to the particle distribution functions are determined by

$$f(p) = f_0 + \delta f = f_0 + \delta f_\eta + \delta f_\zeta$$

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 E} p^\mu p^\nu f = T_o^{\mu\nu} + \eta \nabla^{\langle\mu} u^{\nu\rangle} + \zeta \Delta^{\mu\nu}\,\Theta$$

- restricting corrections to f upto quadratic order in momentum,

$$f(p) = f_0 \left(1 + \frac{\eta/s}{2T^3} p^\alpha p^\beta \nabla_{\langle\alpha} u_{\beta\rangle} + \frac{\zeta/s}{2T^3} p^\alpha p^\beta \Delta_{\alpha\beta} \Theta \right)$$

- In 1D Bjorken flow viscous corrections to the distribution function takes the form

$$\begin{aligned} f = f_0 \Bigg(1 \quad &+ \quad \frac{\eta/s}{2T^3} \left[\frac{2}{3\tau} p_T^2 - \frac{4}{3\tau} m_T^2 sinh^2(y - \eta_s) \right] \\ &- \quad \frac{2}{5} \frac{\zeta/s}{2T^3} \left[\frac{p_T^2}{\tau} + \frac{m_T^2}{\tau} sinh^2(y - \eta_s) \right] \Bigg) \end{aligned}$$

Viscous modified thermal dilepton rates

$$
\begin{aligned}
\frac{dN}{d^4x dM^2 d^2p_T dy} &= \frac{1}{2^3}\frac{5\alpha^2}{9\pi^4}\; e^{-p_0/T} \\
&\left[1+\frac{2}{3}\left(\frac{\eta/s}{2T^3}p^\alpha p^\beta \nabla_{\langle\alpha}u_{\beta\rangle}+\frac{2}{5}\frac{\zeta/s}{2T^3}p^\alpha p^\beta \Delta_{\alpha\beta}\Theta\right)-\frac{2}{5}\frac{\zeta/s}{4T^3}M^2\Theta\right].
\end{aligned}
$$

with

$$
\begin{aligned}
p^\alpha p^\beta \nabla_{\langle\alpha}u_{\beta\rangle} &= \frac{2}{3\tau}p_T^2-\frac{4}{3\tau}m_T^2 sinh^2(y-\eta_s), \\
p^\alpha p^\beta \Delta_{\alpha\beta}\Theta &= -\frac{p_T^2}{\tau}-\frac{m_T^2}{\tau}sinh^2(y-\eta_s).
\end{aligned}
$$

Total Particle Production

Once the evolution of temperature is known from the hydrodynamical model, the *total dilepton spectrum* is obtained by integrating the total rate over the space time history of the collision,

$$\left(\frac{dN}{dM^2 d^2p_T dy}\right)_{M,p_T,y} = \pi R_A^2 \int_{\tau_0}^{\tau_1} d\tau\ \tau \int_{-y_{nuc}}^{y_{nuc}} d\eta_s \left(\frac{1}{2}\frac{dN}{d^4x d^4p}\right)$$

- τ_0 and τ_f (with $T(\tau_f) = T_c$) are the initial and final values of time we are interested.

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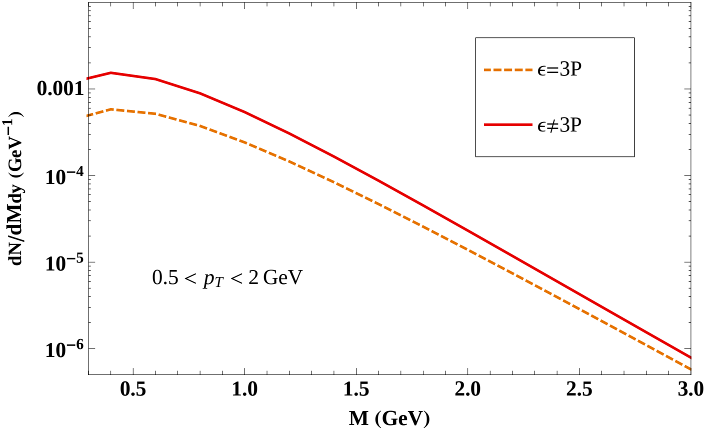
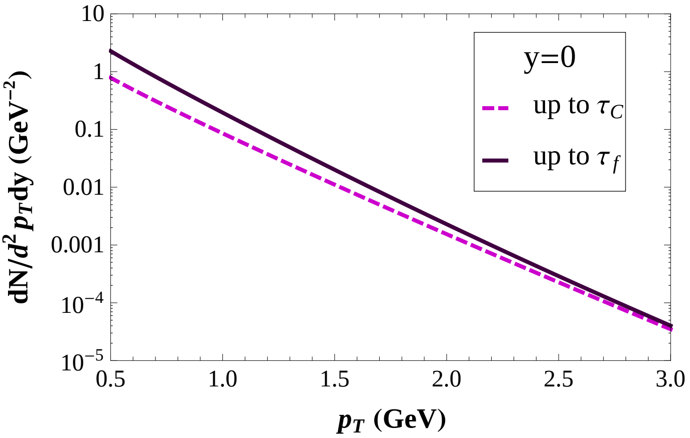
- τ_0 and τ_f (with $T(\tau_f) = T_c$) are the initial and final values of time we are interested.
- If cavitation occurs at τ_c, we have to replace τ_f by τ_c in dilepton yield expression, since hydrodynamics looses its validity after cavitation.

Thermal Dilepton Production (No viscous effects)

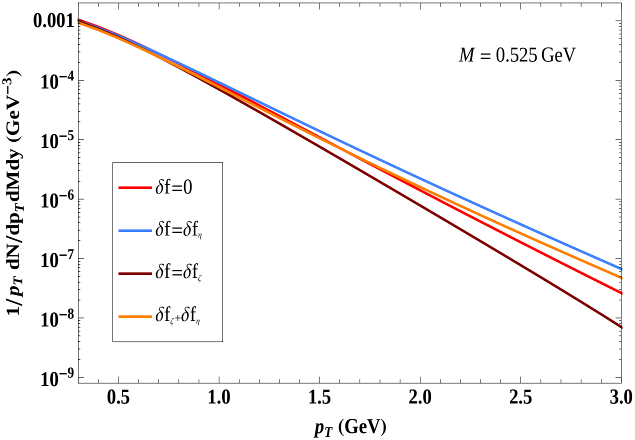


- Dilepton yield $dN/dMdy$ calculated using *ideal* (massless) and *non-ideal* EoS. Effects of viscosity (both in hydrodynamics and distribution function) are ignored. Dilepton flux from the *non-ideal* EoS is about 125% larger than that from the *ideal* EoS case.

Rate over-estimation

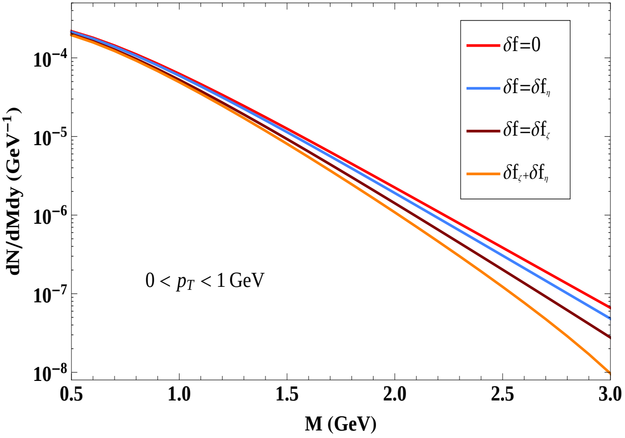


- Photon spectrum obtained by considering the effect of cavitation (dashed line). For a comparison we plot the spectrum without incorporating the effect of cavitation (solid line). (At p=0.5 GeV overestimation of rate is 200% and at p=2 GeV it is 50%).



- Transverse momentum spectra of dileptons from a viscous QGP calculated at $M = 0.525\ GeV$. The solid line shows the dilepton production rate without considering the viscous corrections to the distribution functions. The effect of inclusion of viscous corrections

Thermal Dileptons



- Invariant mass distribution of mid-rapidity thermal dileptons in RHIC scenario calculated at the low p_T regime.

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- Bulk viscosity plays a dual role in heavy-ion collisions: On one hand it enhances the time by which the system attains the critical temperature, while on the other hand it can make the hydrodynamical treatment invalid much before it reaches T_c.
- We have shown that if the phenomenon of cavitation is ignored one can have erroneous estimates of the particle production.

GRAZIE

Ideal Equation of State

In order to understand the effect of *non-ideal* EoS in hydrodynamical evolution and subsequent photon spectra we compare these results with that of an *ideal* EoS ($\varepsilon = 3P$).

- We consider the EoS of a relativistic gas of massless quarks and gluons. The pressure of such a system is given by

$$P = a\ T^4\,;\ a = \left(16 + \frac{21}{2} N_f\right)\ \frac{\pi^2}{90}$$

  where $N_f = 2$ in our calculations.

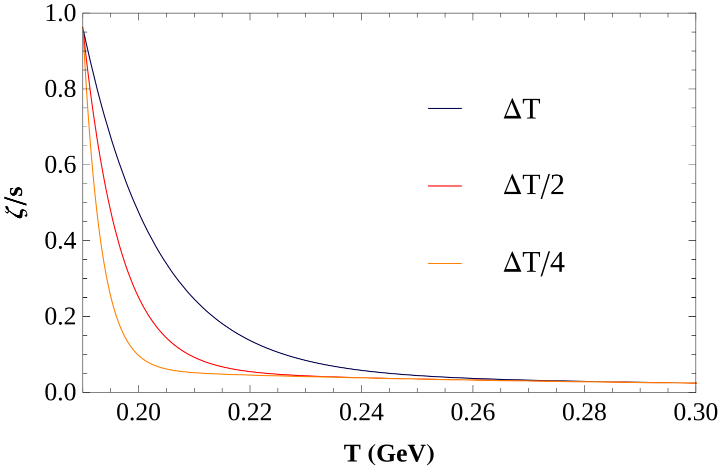
- Hydrodynamical evolution equations of such an EoS within ideal (without viscous effects) Bjorken flow can be solved analytically and the temperature dependence is given by

$$T = T_0\ \left(\frac{\tau_0}{\tau}\right)^{1/3},$$

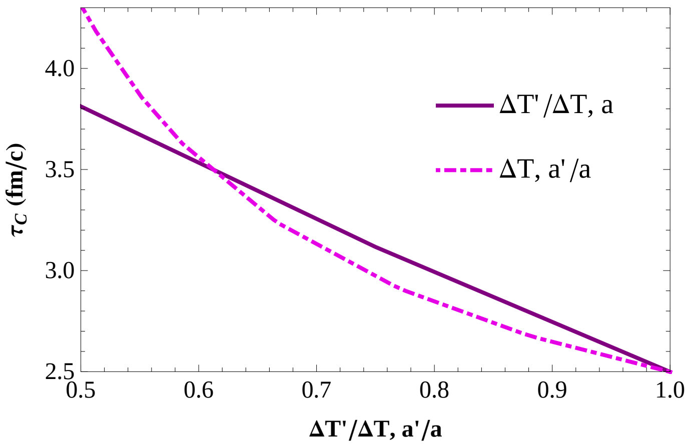
  where τ_0 and T_0 are the initial time and temperature.

- effect of bulk viscosity can be neglected in the relativistic limit when the equation of state $3P = \varepsilon$ is obeyed

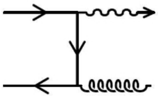
Bulk Viscosity



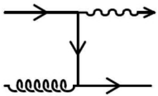
- Various bulk viscosity scenarios by changing the width of the curve through the parameter ΔT.



- Cavitation time τ_c as a function of different values of height (a') and width $(\Delta T')$ of ζ/s curve.



$q\,\bar{q} \to \gamma\, g$



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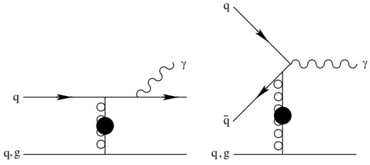

$$E\frac{dN}{d^4x d^3p}\Big|_{cs+ann.} = 0.0281\alpha\alpha_s\ T^2\ e^{-E/T}\ \ln\left(\frac{0.23\mathrm{E}}{\alpha_\mathrm{s}\mathrm{T}}\right)$$

$$E\frac{dN}{d^4x d^3p}\Big|_{brems.} = 0.0219\ \alpha\alpha_s\ T^2\ e^{-E/T}$$

$$E\frac{dN}{d^4x d^3p}\Big|_{aws.} = 0.0105\ \alpha\alpha_s\ ET\ e^{-E/T}$$