Probing fundamental constants in primordial nucleosynthesis

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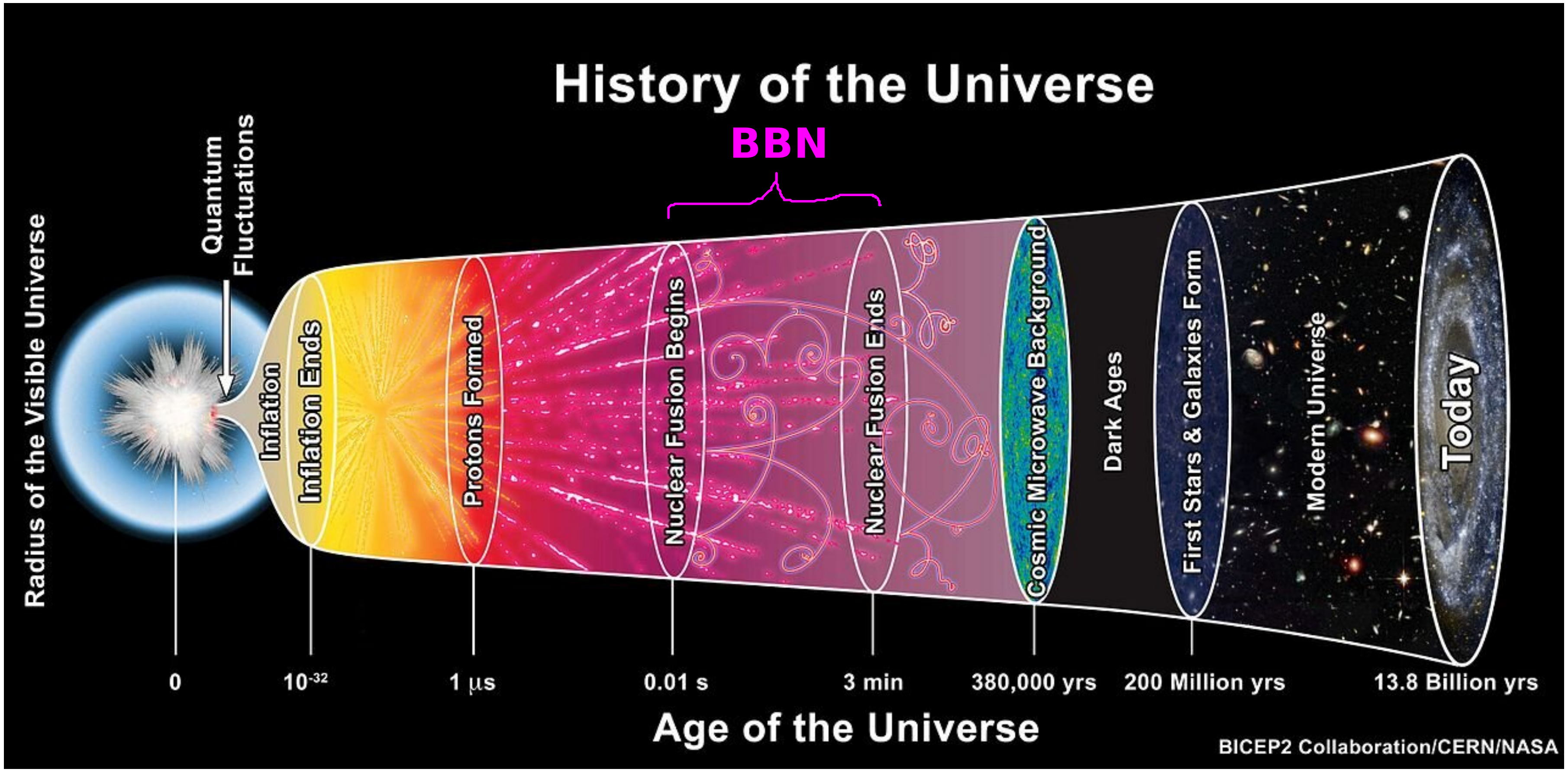
Definition of the physics problem

History of the Universe



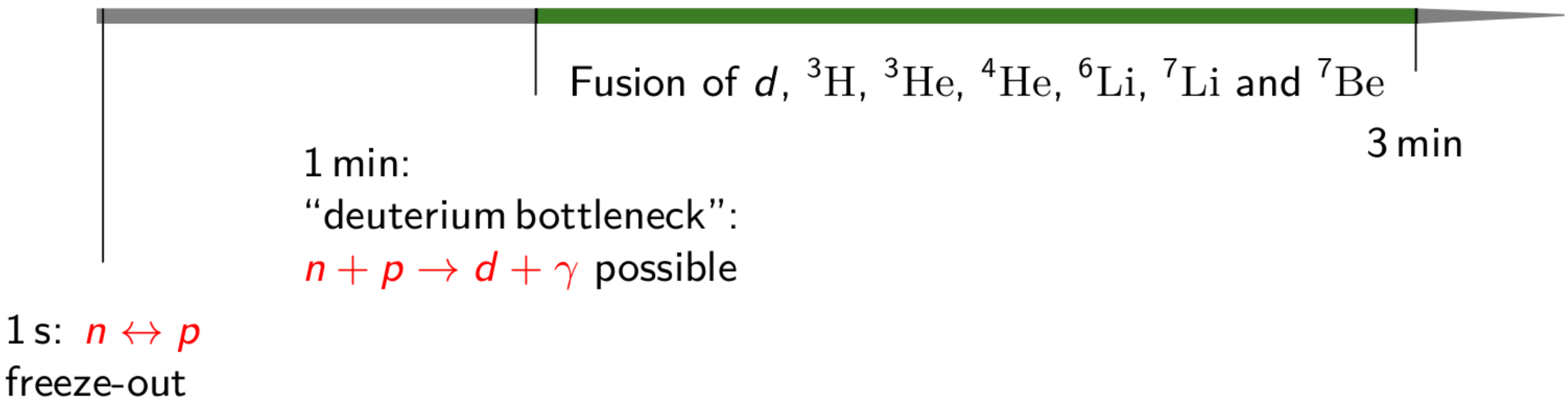
- BBN is a fine probe of our understanding of fundamental physics

Olive et al. (2000), Iocco et al. (2009), Cyburt et al. (2016), Pitrou et al (2018), ...

- Are the fundamental constants really constant? Dirac (1973), and many others

Basics of primordial nucleosynthesis

Timescales



- weak interaction $n \leftrightarrow p$: $\dfrac{n_n}{n_p} = e^{-Q_n/T}$

$$Q_N = m_n - m_p = 1.293\ \mathrm{MeV} \quad \text{[PDG]}$$

- freeze out at $T_f = 14$ MeV $\to$ free neutron decay $\sim \tau_n$

$$\frac{n_n}{n_p} = e^{-Q_n/T}\, e^{(t-t_f)/\tau_n}$$

Evolution of the abundances

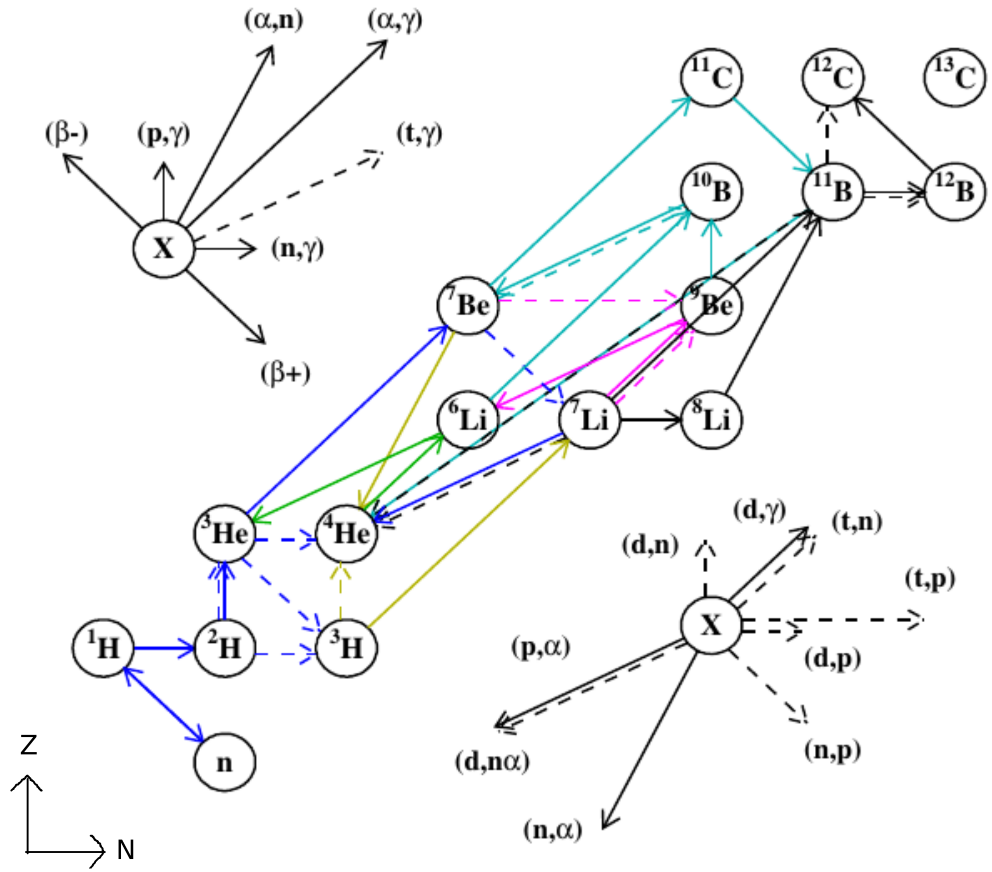
- Abundance defined via $Y_i = \dfrac{n_i}{n_b}$

  n_i = density of species i

  n_b = total baryon density

- Evolution depends on
  - cosmological model: Hubble expansion
  - particle reactions ($\Gamma_{ij\to kl} = n_b \langle \sigma v \rangle_{ij\to kl}$) and decays ($\Gamma_{i\to \ldots}$)

$\Rightarrow$ need to solve the system of rate equations:



Pitrou et al. (2018)

$$\dot{Y}_i \supset -Y_i\Gamma_{i\to\ldots} + Y_j\Gamma_{j\to i+\ldots} + Y_kY_l\Gamma_{lk\to ij} - Y_iY_j\Gamma_{ij\to kl}$$

Evolution of the abundances - results

- 5 different codes:

NUC123 (Kawano-code) [FORTRAN, 88 rate eqs.]

Kawano, FERMILAB-PUB-92-004-A (1992)

PRIMAT [Mathematica, 423 rate eqs.]

Pitrou et al., Phys. Rept. 754 (2018) 1

AlterBBN [C, 100 rate eqs.]

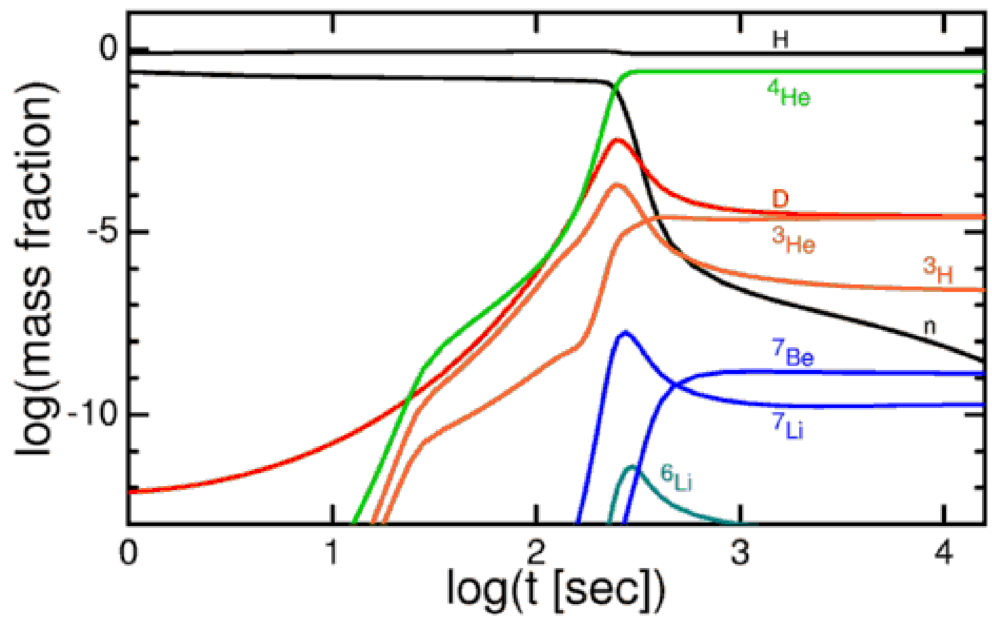
Arbey et al., Comp. Phys. Comm. 248 (2020) 106902

PArthENoPE [FORTRAN, 100 rate eqs.]

Gariazzo et al., Comp. Phys. Comm. 271 (2022) 108205

PRyMordial [Python, 423 rate eqs.]

Burns et al., [arXiv:2307.07061v2]



Burles, Nollett, Turner (2001

- Altho' the results are by and large consistent in spite of the differences (# of rate eqs, QED, ...)

$\hookrightarrow$ must study the dependence of the abundances on these codes & nuclear observables before considering the variation of fundamental parameters such as $\alpha_{\rm EM}$

Probing nuclear observables via primordial nucleosynthesis

UGM, Metsch, Eur. Phys. J. A **58** (2022) 212 [arXiv:2208.12600 [nucl-th]]

Variations of nuclear observables

- Study the nuclear abundances Y_i as a function of nuclear observables:

  $\to$ binding energies, scattering lengths, neutron lifetime, ...

- Use four different BBN codes to study the systematic error

  $\hookrightarrow$ only if this is small / controlled, it makes sense to look at variations of $\alpha_{\rm EM}$ etc.

- Nuclear reactions rates: $\Gamma_{ab\to cd} = n_B\,\gamma_{ab\to cd}$

$$\gamma_{ab\to cd} = N_A \sqrt{\frac{8}{\pi\mu_{ab}(kT)^3}} \int_0^\infty \mathrm{d}E\, E\, \sigma_{ab\to cd}(E)\, \mathrm{e}^{-\frac{E}{kT}}$$

$$\mu_{ab} = m_a m_b/(m_a + m_b)$$

- Inverse reaction (with spin multiplicity g_i):

$$\gamma_{cd\to ab}(T) = \left(\frac{\mu_{ab}}{\mu_{cd}}\right)^{\frac{3}{2}} \frac{g_a\, g_b}{g_c\, g_d}\, \mathrm{e}^{-\frac{Q}{kT}}\, \gamma_{ab\to cd}(T)$$

- Consider now changes in the binding energies by $\pm$ 1 permille $\to$ Q-values change

Variations of the binding energies / Q-values

- Direct reactions $a + b \to c + d$

$$\gamma_{ab\to cd}(\widetilde{Q};T) \approx \gamma_{ab\to cd}(Q_0;T) + \sqrt{\frac{8}{\pi\,\mu_{ab}\,(kT)^3}} \int_0^\infty \mathrm{d}E\,E\,\sigma(Q_0;E)$$
$$\times \left(\frac{\Delta Q}{2\,(Q_0+E)} + \frac{\Delta Q\,\sqrt{E_G}}{2\,(Q_0+E)^{\frac{3}{2}}}\right)\,\mathrm{e}^{-\frac{E}{kT}}$$
$$= \gamma_{ab\to cd}(Q_0;T) + \Delta\gamma(T)_{ab\to cd}$$

$$\widetilde{Q} = Q_0 + \Delta Q$$ change of the Q-value

$$E_G = 2\pi^2\,Z_c^2\,Z_d^2\,\alpha_{\rm EM}^2\,\mu_{cd}\,c^2$$ Gamov energy in the exit channel

- In all earlier investigations, the T-dependence of $\Delta\gamma(T)_{ab\to cd}$ was neglected

- Similar for radiative capture reactions $a + b \to c + \gamma$
  and weak decay rates $a \to b + e^{\pm} + \overset{(-)}{\nu}$

Reactions considered

- Radiative capture reations:

$$n + p \to d + \gamma\,,\ p + d \to {}^3\text{He} + \gamma\,,\ p + {}^3\text{H} \to {}^4\text{He} + \gamma\,,\ {}^3\text{H} + {}^4\text{He} \to {}^7\text{Li} + \gamma\,,$$
$${}^3\text{He} + {}^4\text{He} \to {}^7\text{Be} + \gamma\,,\ \ d + {}^4\text{He} \to {}^6\text{Li} + \gamma\,,\ \ p + {}^7\text{Li} \to {}^4\text{He} + {}^4\text{He} + \gamma\,,$$
$$p + {}^6\mathbf{Li} \to {}^7\mathbf{Be} + \gamma$$

- Neutron-induced reactions:

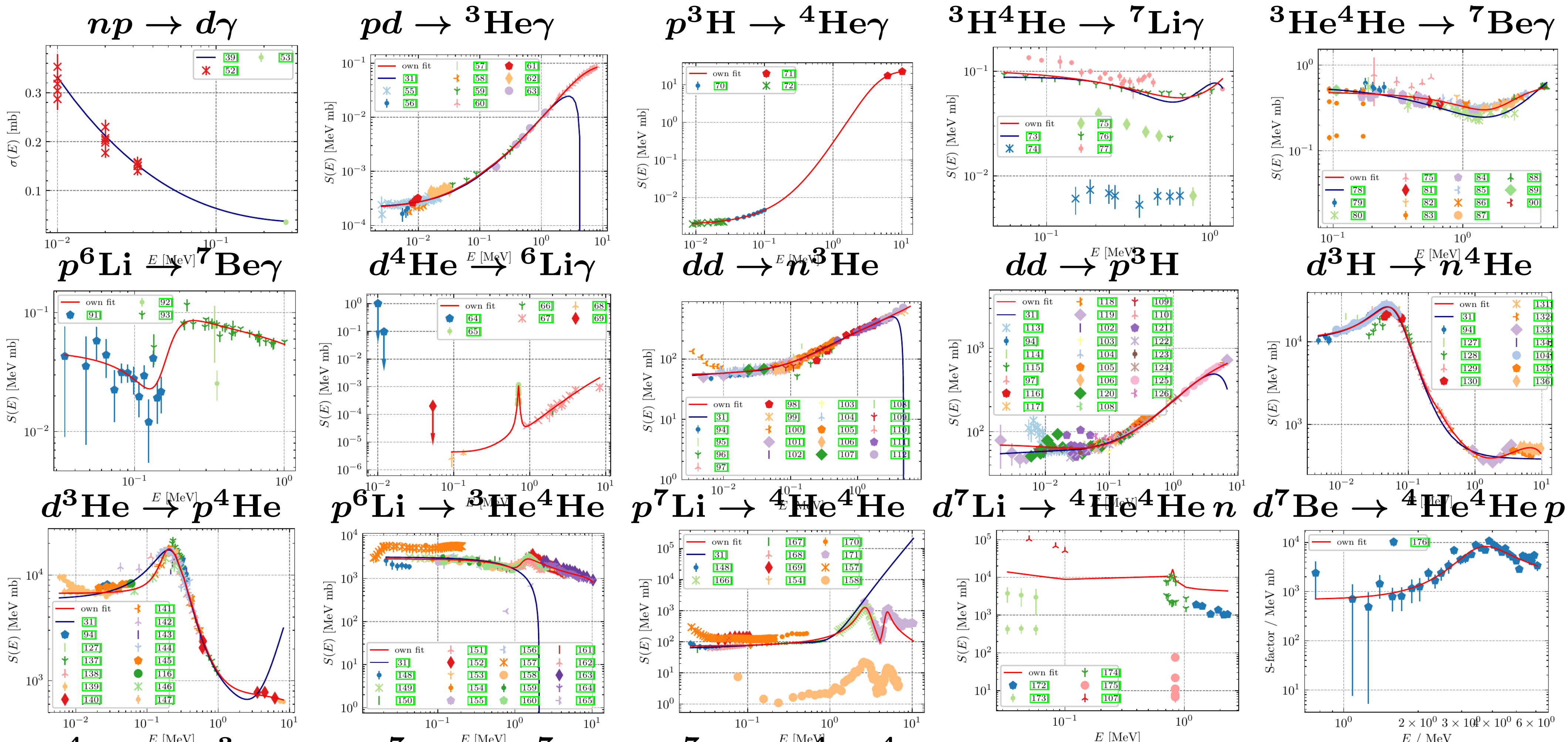
$$n + {}^3\text{He} \to {}^3\text{H} + p\,,\ n + {}^7\text{Be} \to {}^7\text{Li} + p\ \ \ n + {}^7\text{Be} \to {}^4\text{He} + {}^4\text{He}$$

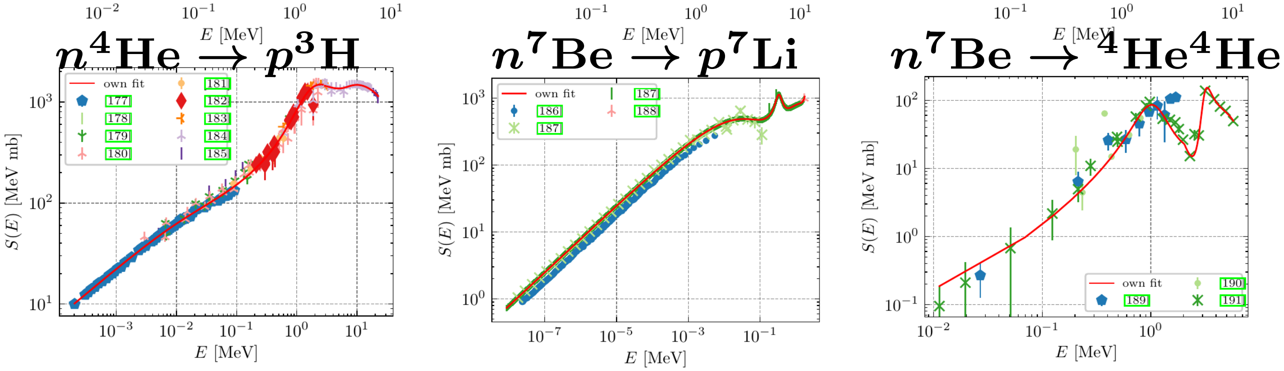
- Direct reactions:

$$d + d \to {}^3\text{H} + p\,,\ d + d \to {}^3\text{He} + n,\ \ d + {}^3\text{H} \to {}^4\text{He} + n\,,\ d + {}^3\text{He} \to {}^4\text{He} + p\,,$$
$$p + {}^7\text{Li} \to {}^4\text{He} + {}^4\text{He}\,,\ \ p + {}^6\text{Li} \to {}^3\text{He} + {}^4\text{He}\,,\ \ d + {}^7\text{Be} \to {}^4\text{He} + {}^4\text{He} + p\,,$$
$$n + {}^7\text{Be} \to {}^4\text{He} + {}^4\text{He}\,,\ \ d + {}^6\text{Li} \to {}^3\text{He} + {}^4\text{He} + n$$

- plus weak decays ....

Reaction parameterizations





$$S(E) = E\sigma(E)\exp(\sqrt{E_G/E})$$
$$= \text{rat. polynom} + \text{BW(s)}$$
$$E_G = 1 \text{ for neutron capture}$$

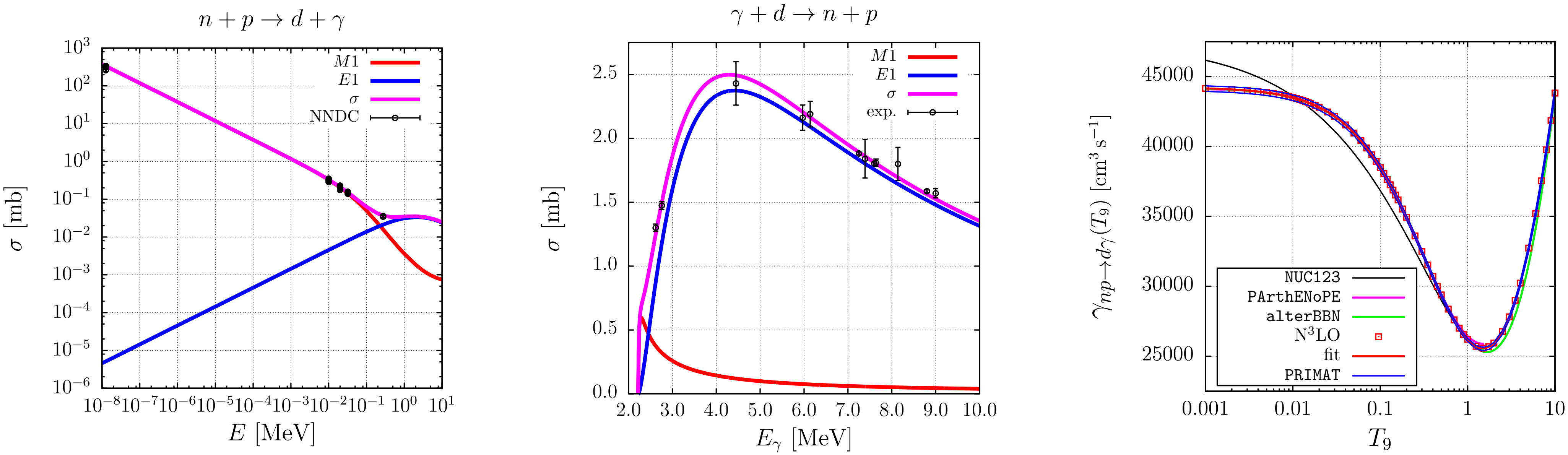
[31] Serpico et al., JCAP **12**, 010 (2004)

The leading reaction $n + p \to d + \gamma$

- Use the pionless EFT description up to N^{3}LO

Chen, Savage (1999), Rupak (2000)

$$\sigma_{np\to d\gamma}(E) = \frac{4\pi\,\alpha_{\rm EM}\,(\gamma^2+p^2)^3}{\gamma^3\,m_N^4\,p}\left[|\chi_{M1}|^2 + |\chi_{E1}|^2\right], \quad \chi_{E1,M1} = f(a_s, B_d)$$



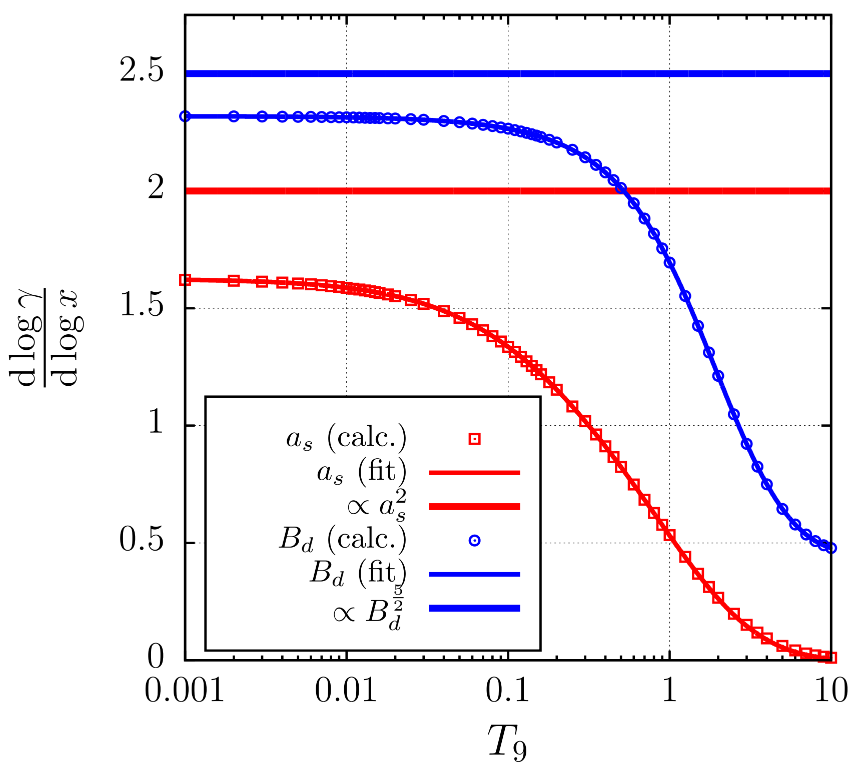
- $M1$ dominance at low energies, all codes agree on the T-dependence

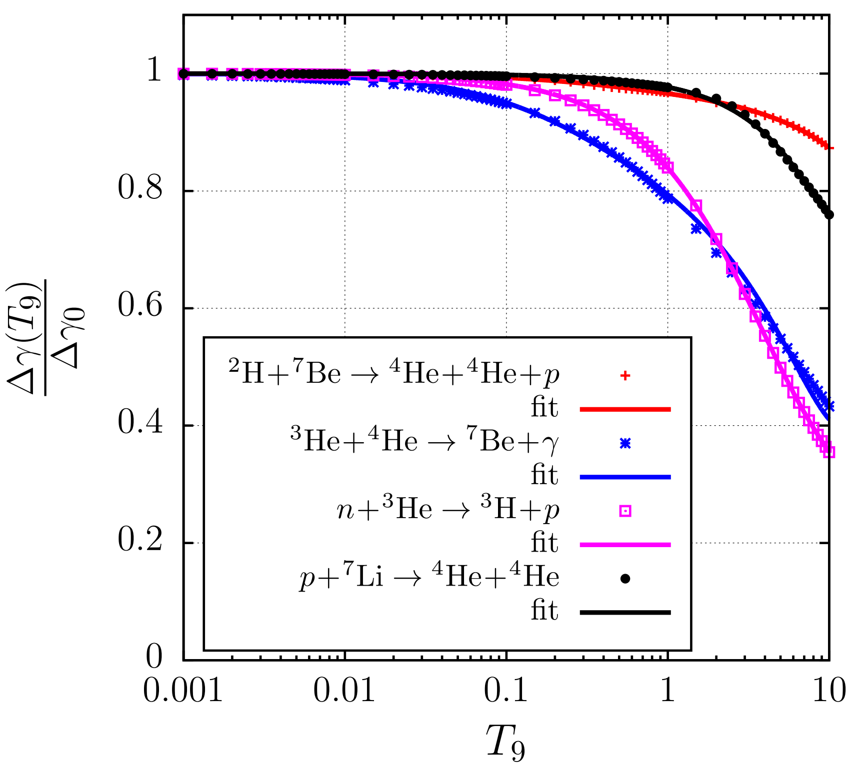
- LO: $\gamma_{M1;np\to d\gamma} \propto B_d^{\frac{5}{2}}\,a_s^2\,, \quad \frac{\partial\log\gamma_{\ldots}}{\partial\log a_s} = 2\,, \quad \frac{\partial\log\gamma_{\ldots}}{\partial\log B_d} = \frac{5}{2}$ appear T-indep.

Temperature-dependence of nuclear reactions

- Consider the leading and the next 17 reactions in the BBN network

  $\rightarrow$ integrate reaction formulae ($\boldsymbol{T_9 = T/[10^9\mathrm{K}]}$):



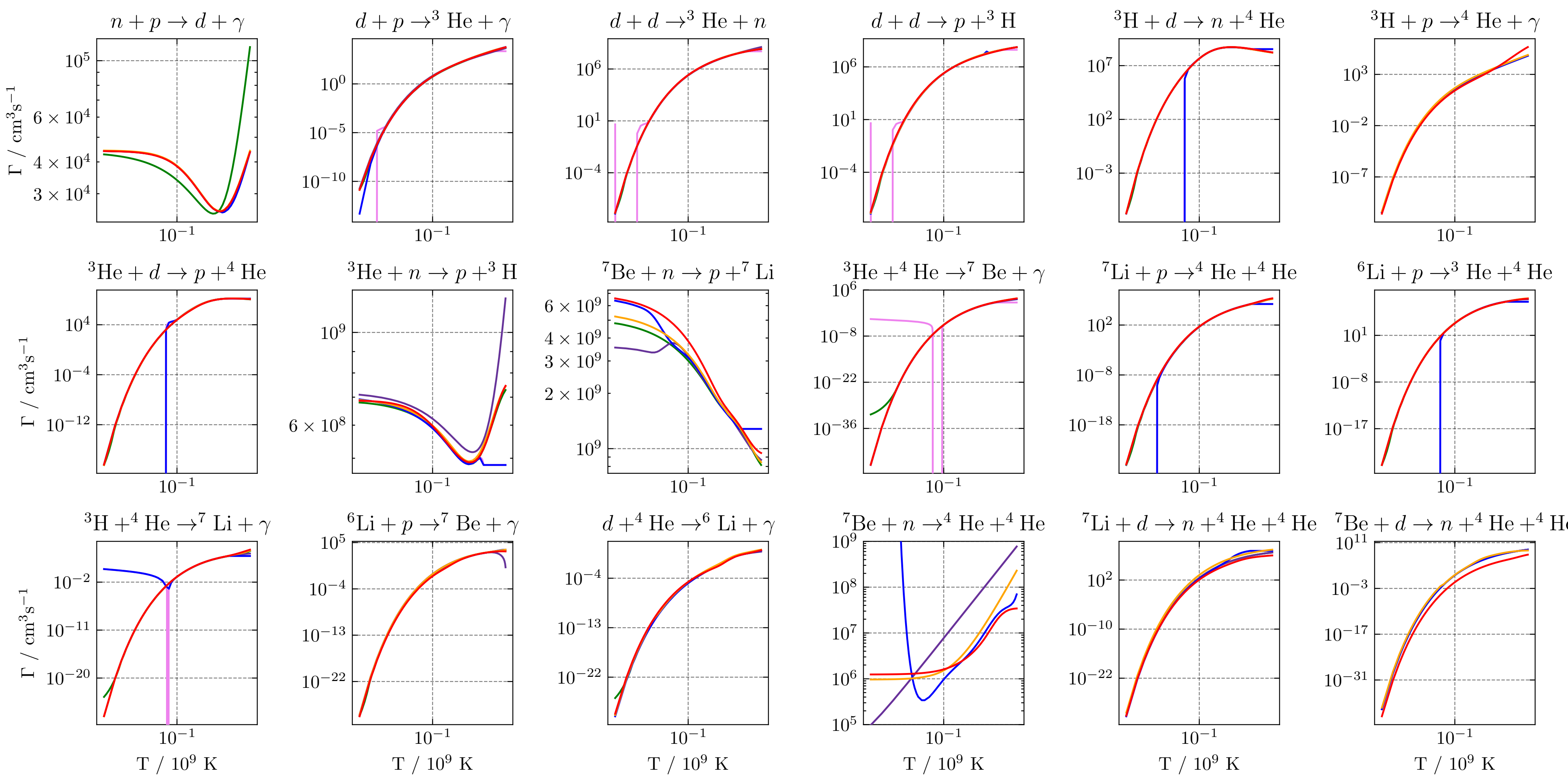


- Strong suppression of the $\boldsymbol{a_s}$ and $\boldsymbol{B_d}$ dependence in the leading reactions due to $\boldsymbol{T}$

- $\boldsymbol{T}$-effect appreciable for most reactions for $\boldsymbol{T_9 \gtrsim 0.1}$

Temperature-dependence of nuclear reactions II

- For all codes, this work and NACRE II [Xu et al., Nucl. Phys. A **918** (2013) 61]



BBN response matrix

- Calculate the linear dependence of the Y_n for small changes in a_S, B_d, τ_n:

$$\partial \log Y_n / \partial \log X_k \,, \qquad n \in \{^2\text{H}\,, ^3\text{H} + {}^3\text{He}\,, ^4\text{He}\,, ^6\text{Li}\,, ^7\text{Li} + {}^7\text{Be}\}$$

- Updating all natural constants, atomic & nuclear masses, and the leading reactions from EFT in all codes ($\eta = 6.14 \cdot 10^{-10}$, $\tau_n = 879.4$ s)

code	^{2}H $\times 10^5$	^{3}H $+ ^3$He $\times 10^5$	Y_p	^{6}Li $\times 10^{14}$	^{7}Li $+ ^7$Be $\times 10^{10}$
NUC123	2.550	1.040	0.247	1.101	4.577
PArthENoPE	2.511	1.032	0.247	1.091	4.672
AlterBBN	2.445	1.031	0.247	1.078	5.425
PRIMAT	2.471	1.044	0.247	1.198	5.413
PDG (2022)	2.547		0.245		1.6
$\pm$	0.025		0.003		0.3

- Four codes are largely consistent
- Lithium problem prevails

Fields, Ann. Rev. Nucl. Part. Sci. **61** (2011) 47

Further results

- T-dependence of the reaction rates on changes in B_A for the first time considered

$\hookrightarrow \dfrac{\partial \log Y_n}{\partial \log a_s}$ reduced by a factor of three

$\hookrightarrow \dfrac{\partial \log Y_n}{\partial \log B_i}$ reduced by about 10 percent

- η-dependence linear and a minor effect for small changes ($\eta = 5.94 - 6.34 \cdot 10^{-10}$)

- Code dependence of $\dfrac{\partial \log Y_n}{\partial X_k}$ is very small

$\Rightarrow$ Now we are in the position to study the dependence of the Y_n on the fundamental parameters, especially on $\alpha_{\rm EM}$

The electromagnetic fine-structure constant in primordial nucleosynthesis revisited

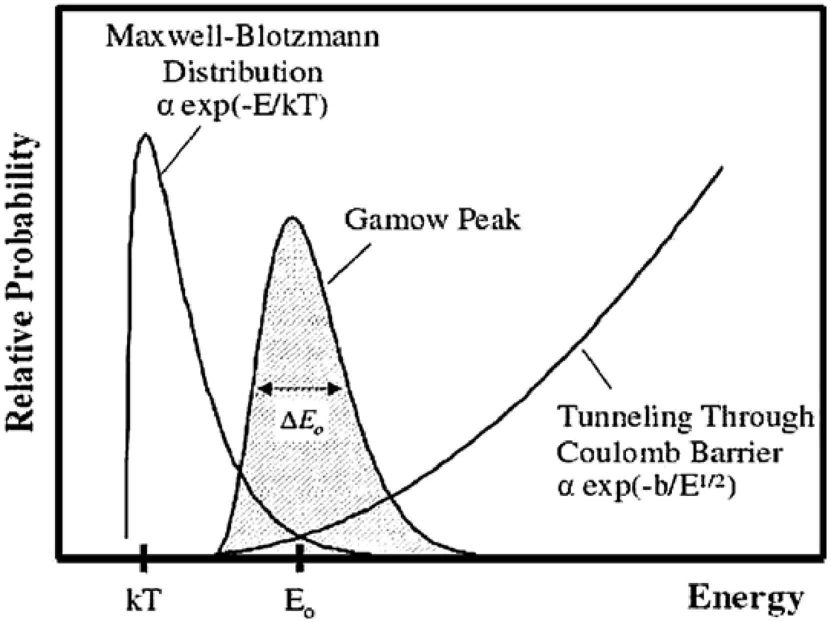
UGM, Metsch, Meyer, Eur. Phys. J. A **59** (2023) 223 [2305.15849 [hep-th]]

The electromagnetic fine-structure constant

- Want to study variations of $\alpha_{\rm EM}$ about its present day value:

$$\alpha_0 = 7.2973525693(11) \times 10^{-3}$$

$\hookrightarrow$ find a bound on $\delta\alpha/\alpha_0$ by comparison with the measured Y_n



from Trache (2010)

- Where does α appear in BBN?
  - Nuclear Rates: Coulomb barrier $\rightarrow$ Gamow factor — Gamow (1928)
  - Weak rates: final-state Coulomb interaction in $n \leftrightarrow p$ rates and β-decays
  - Indirectly: n-p mass difference $Q_n = m_n - m_p$, EM contribution to nuclear binding energies $\rightarrow$ reaction Q-values

Nuclear reaction rates: Coulomb barrier

- Reminder:

$$\boxed{\gamma_{ab\to cd}(T) = \langle\sigma v\rangle \propto \int_0^\infty dE\,\sigma_{ab\to cd}(E)\cdot E\cdot \mathrm{e}^{-\frac{E}{k_BT}}} \qquad E = \frac{1}{2}\mu_{ab}v^2$$

$$\mu_{ab} = \frac{m_a m_b}{m_a+m_b}$$

- Coulomb barrier

↪ The cross section is proportional to the penetration factor (in/out)

$$\sigma \propto v_0 = \frac{2\pi\eta}{\mathrm{e}^{2\pi\eta}-1}$$

with the Sommerfeld parameter

$$\eta = \frac{Z_a Z_b \alpha_{\rm EM} c}{\hbar v} = \frac{1}{2\pi}\sqrt{E_G/E}$$

and the Gamow energy

$$E_G = 2\pi^2\, Z_a^2\, Z_b^2\, \alpha_{\rm EM}^2\, \mu_{ab}\, c^2$$

Nuclear reaction rates: Radiative capture

- Coupling $\propto e \to$ cross section $\sigma \propto e^2 \propto \alpha_{\rm EM}$
- Capture processes are peripheral $\to$ parameterized in $f(\delta\alpha_{\rm EM}) \simeq 1$ Nollett, Lopez (2002)
- Assume dipole dominance
- For the leading $np \to d\gamma$ reactions use again EFT formalism Rupak (2000)

$\Rightarrow$ α-dependence of cross sections ($q_\gamma = 1$ for radiative capture, 0 else)

$$\sigma(\alpha_{\rm EM}, E) \propto \left(\frac{\sqrt{E_G^{\rm in}/E}}{{\rm e}^{\sqrt{E_G^{\rm in}/E}} - 1}\right) \cdot \left(\frac{\sqrt{E_G^{\rm in}/(E+Q)}}{{\rm e}^{\sqrt{E_G^{\rm in}/(E+Q)}} - 1}\right) \cdot \left(\alpha_{\rm EM} f(\delta\alpha_{\rm EM})\right)^{q_\gamma}$$

with $Q = m_a + m_b - m_c - m_d$

$\hookrightarrow$ penetration factors must be modified in there is a neutron in the initial and/or final state

$\hookrightarrow$ for details, see UGM, Metsch, Meyer (2023)

Weak decay rates

- β-decay rate in terms of the Fermi function

Segrè (1964)

$$\lambda = \frac{G_F^2\,|\mathcal{M}_{fi}|^2}{2\pi^3 c^3 \hbar^7} \underbrace{\int_0^{p_{e,\max}} \left(W - \sqrt{m_e^2 c^4 + p_e^2 c^2}\right)^2 F(Z,\alpha,p_e) p_e^2 dp_e}_{=f(\alpha,Q)}$$

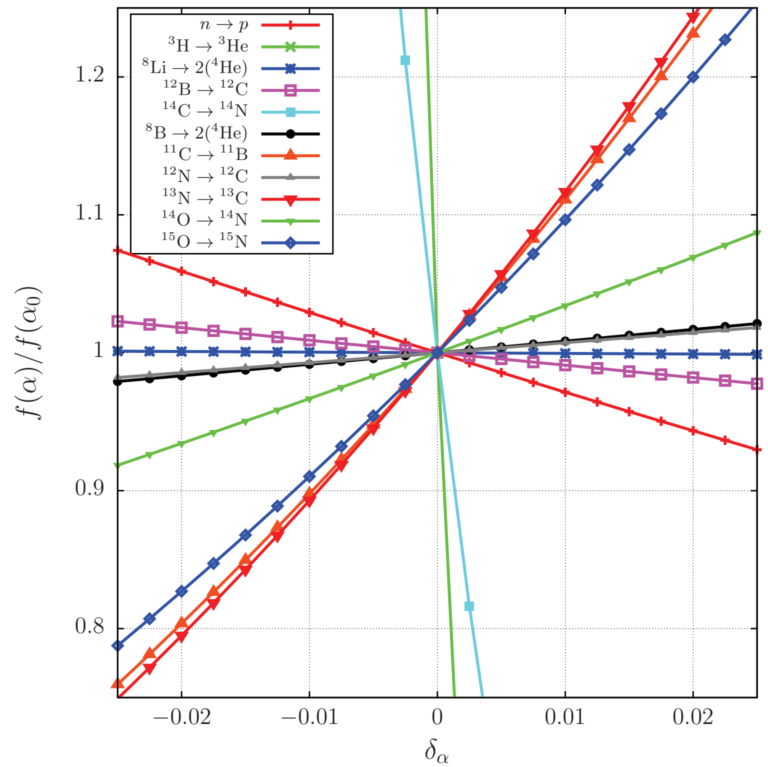
$$p_{e,\max} = \frac{1}{c}\sqrt{W^2 - m_e^2 c^4}\ , \quad W \simeq m_a - m_b = Q$$

- Fermi function (for $Z\alpha \ll 1$):

$$F(\pm Z,\alpha,\epsilon_e) \simeq \frac{\pm 2\pi\nu}{1-\exp(\mp 2\pi\nu)}\ , \quad \nu = \frac{Z\alpha\epsilon_e}{\sqrt{\epsilon_e^2 - 1}}$$

$\hookrightarrow$ α-dependent rates:

$$\lambda(\alpha) = \lambda(\alpha_0)\frac{f(\alpha,Q)}{f(\alpha_0,Q)}$$



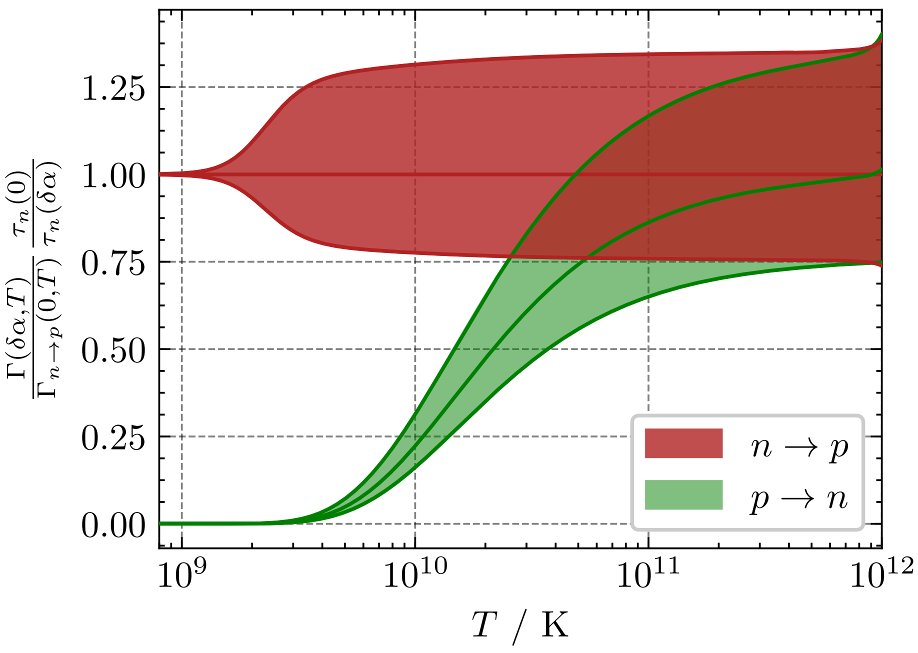
Neutron decay and $n \leftrightarrow p$ rates

- Consider free neutron decay:

$$\tau_n(\alpha) = \tau_n(\alpha_0)\frac{f(\alpha, Q)}{f(\alpha_0, Q)}$$

- But this ignores the Fermi-Dirac distribution of the neutrino and the electron

$\Rightarrow$ strong T-dependence in the α-variation for high T



- $Q_n = m_n - m_p$ has a QED contribution, use the new value:

$$\Delta Q_n = Q_n^{\rm QED} \cdot \delta\alpha = -0.58(16)\ \mathrm{MeV} \cdot \delta\alpha$$

Gasser et al. (2021)

$\hookrightarrow$ affects the weak $n \leftrightarrow p$ rates $\rightarrow$ ^{4}He abundance very sensitive

$\hookrightarrow$ affects the Q-values of the β-decays

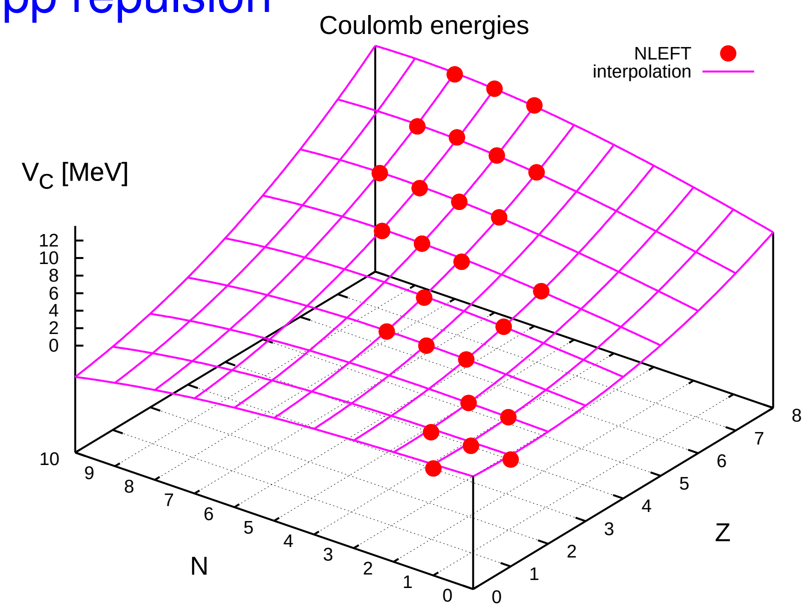
$\hookrightarrow$ α-dependence of $m_N = (m_p + m_n)/2$ in $np \rightarrow d\gamma$ can be neglected

Indirect effects – binding energies

- EM (Coulomb) contributions to nuclear BEs from pp repulsion
- calculated in NLEFT   Elhatisari et al. (2022) ⟶ sildes

↪ change in Q-values

$$\Delta Q = \delta\alpha \left(-\sum_i B_C^i + \sum_j B_C^j \right)$$



⇒ Nuclear reaction cross sections:

$$\sigma(E,\alpha) \propto \underbrace{(E+Q(\alpha))^{p_\gamma}}_{\text{phase space}} \alpha^{q_\gamma} \frac{\sqrt{E_G^{\rm in}/E}}{\exp\left(\sqrt{E_G^{\rm in}/E}\right)-1} \frac{\sqrt{E_G^{\rm out}/(E+Q(\alpha))}}{\exp\left(\sqrt{E_G^{\rm out}/(E+Q(\alpha))}\right)-1}$$

$p_\gamma = 3\,,\quad q_\gamma = 1$ for radiative capture

$p_\gamma = 1/2\,,\quad q_\gamma = 0$ for other reactions

The tool: Nuclear lattice effective field theory

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem

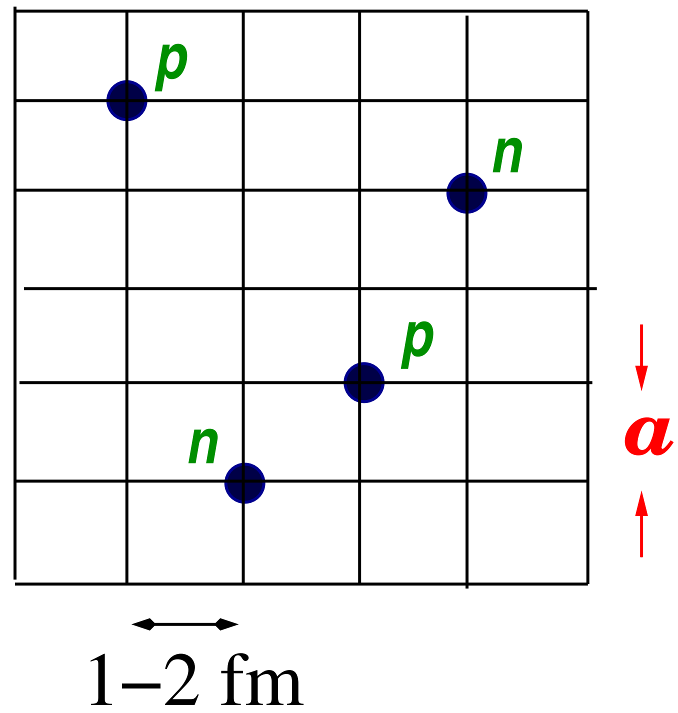
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$: nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges and contact interactions + Coulomb

  → see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters

$$p_{\rm max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

  E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

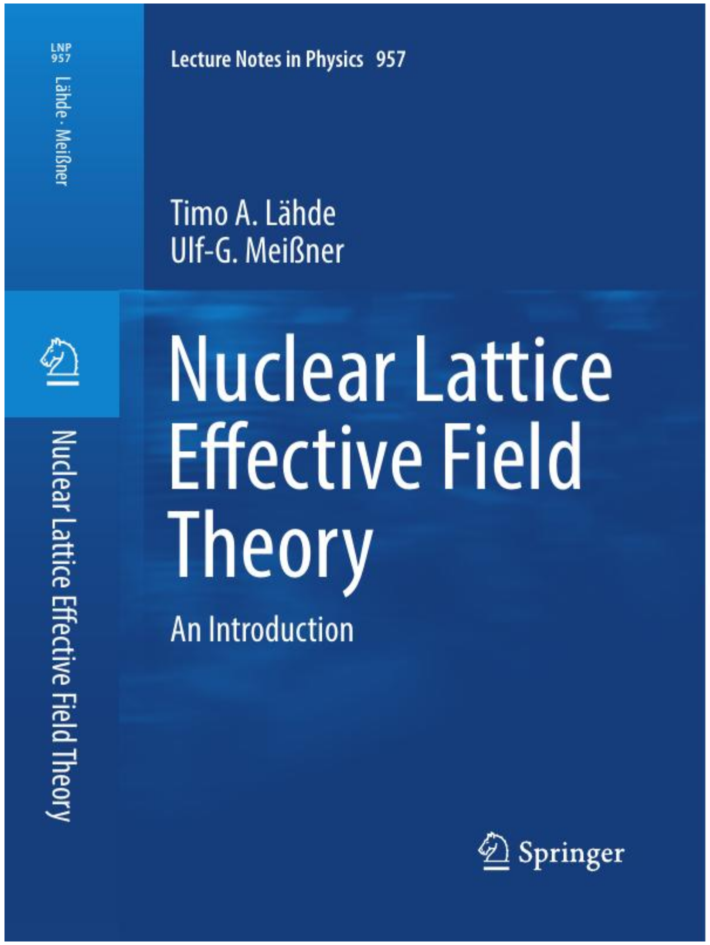
- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

  Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

The tool: Nuclear lattice effective field theory II

- For all details on chiral EFT on a lattice

  T. Lähde & UGM
  Nuclear Lattice Effective Field Theory - An Introduction
  Springer Lecture Notes in Physics **957** (2019) 1 - 396

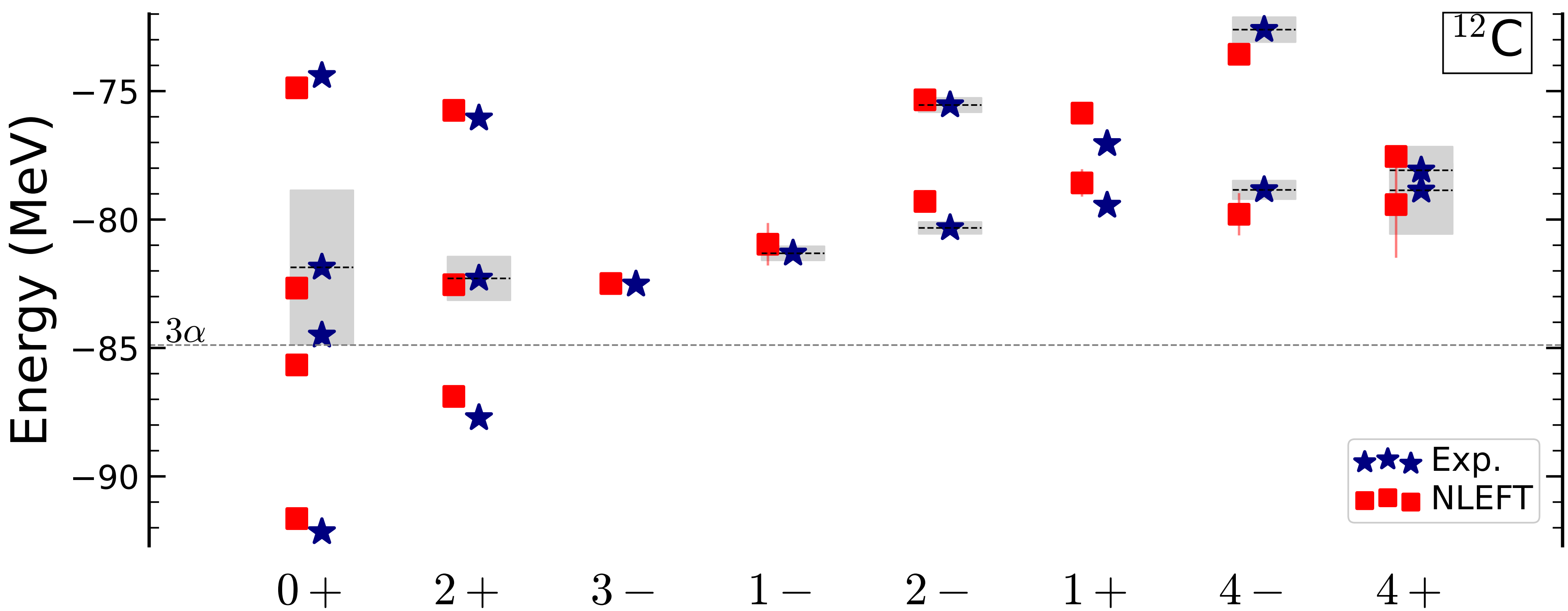


- Computational equipment

NLEFT @ work: The spectrum of carbon-12 A.D. 2023

- with much improved algorithms and methods:

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777



→ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

Results I

- Parameter fit at fixed $\eta = 6.14 \cdot 10^{-10}$ and $\tau_n = 879.4$ s:

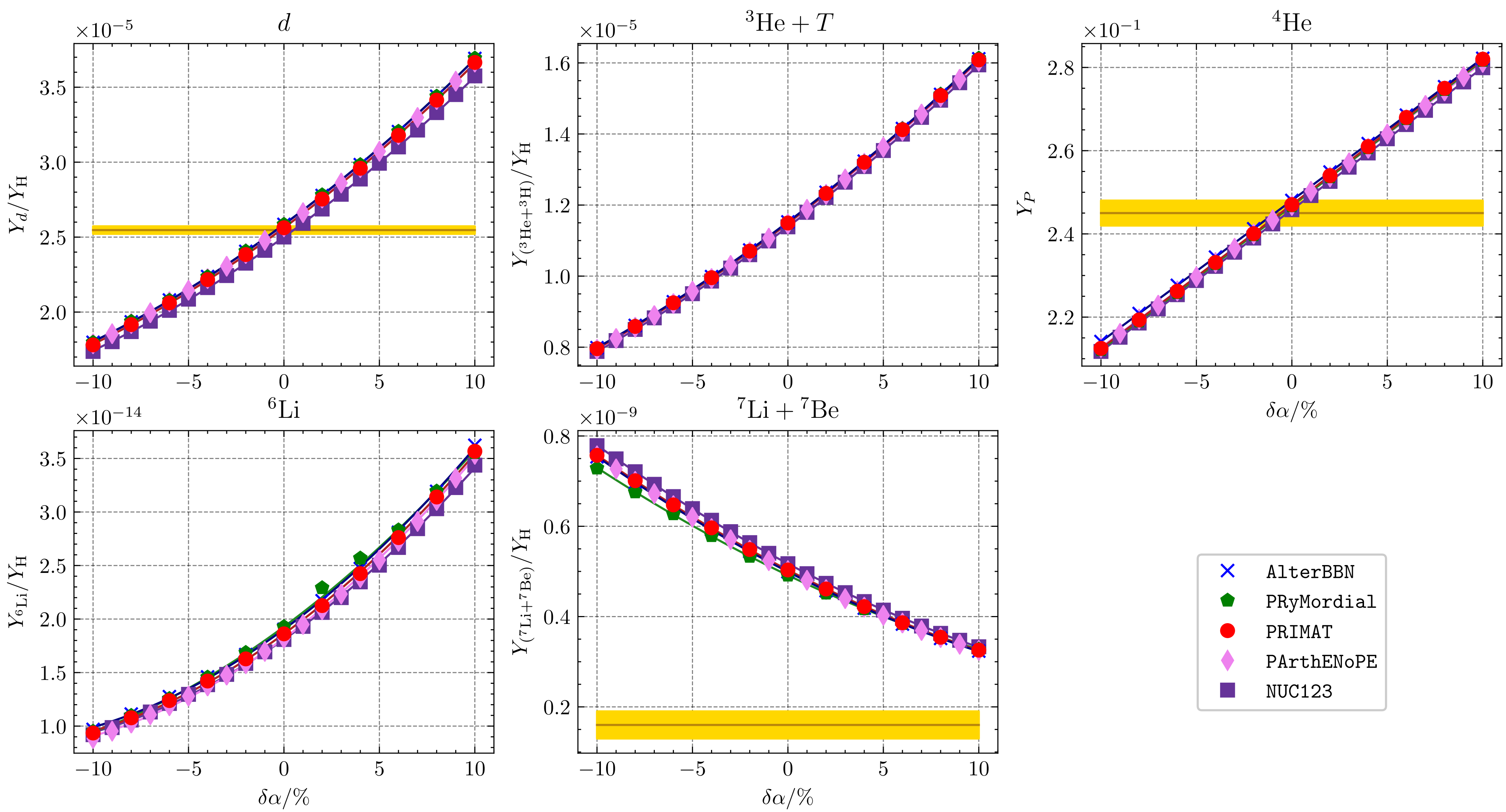
$$\frac{Y(\alpha) - Y(\alpha_0)}{Y(\alpha_0)} = a \cdot \frac{\delta\alpha}{\alpha_0} + b \cdot \left(\frac{\delta\alpha}{\alpha_0}\right)^2$$

- Consider variations in α up to $|\delta\alpha/\alpha_0| \leq 0.1$

- Main results:
  - Temperature-dependence of reaction rates at varying α important
  - For most elements, change in the nuclear reaction rates is the biggest effect
  - ^{4}He abundance indeed very sensitive to ΔQ_n
  - Lithium problem persists

Results II

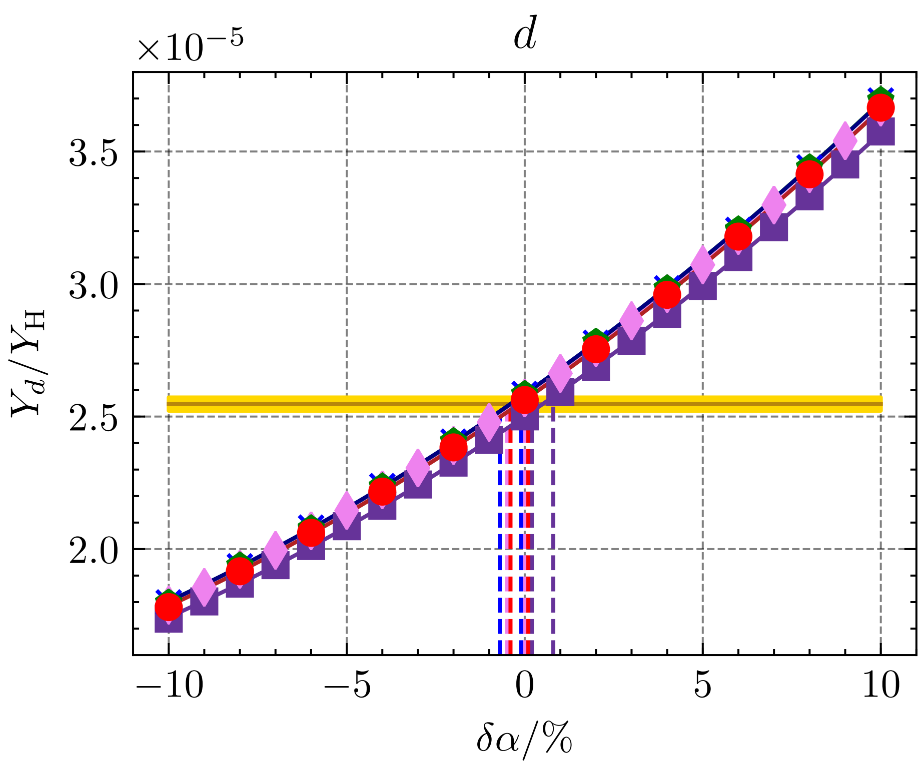
- α-dependence of the abundances:

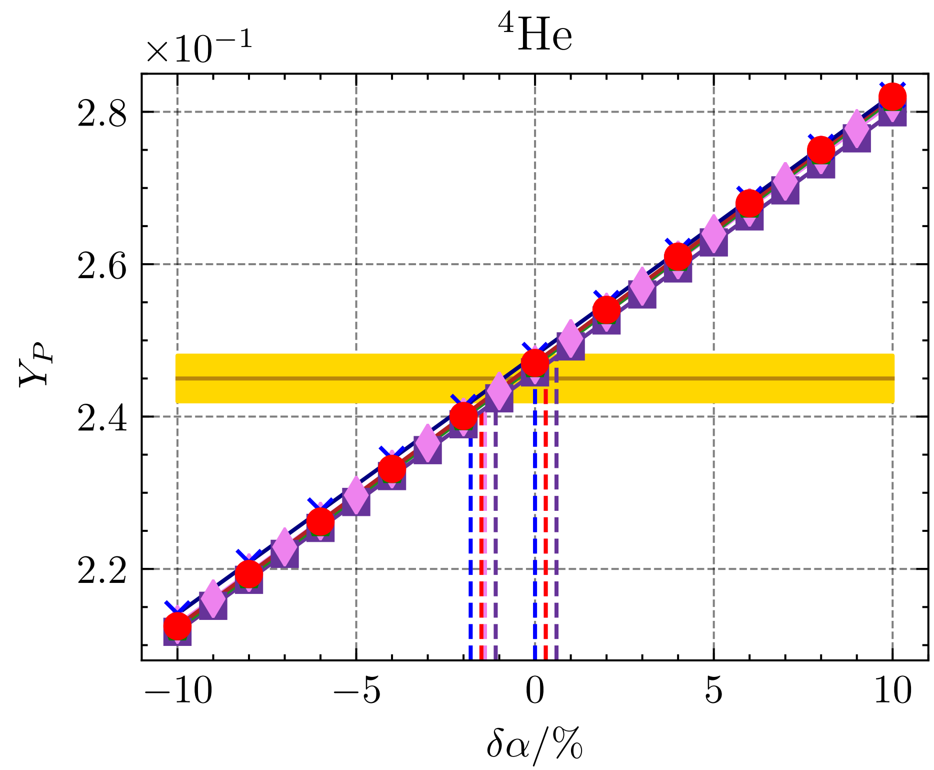


$\hookrightarrow$ largely independent of the codes

Results III

- Extract the allowed α-variation:





- 1σ-bounds on α-variation $\Rightarrow$ from ^{4}He:

$$|\delta\alpha| < 1.8\%$$

Comparison to earlier works

- Compare the coefficient a for various codes and earlier works

Code/work	d	^{3}H+^{3}He	^{4}He	^{6}Li	^{7}Li+^{7}Be
PRIMAT	3.658	3.534	1.408	6.953	-4.302
AlterBBN	3.644	3.526	1.373	6.856	-4.322
Dent et al. (2007)	3.612	0.948	1.898	6.681	-11.307
Nollett et al. (2002)	3.993	1.033	–	–	-9.296
Bergstroem et al. (2002)	5.129	0.778	1.956	–	-13.619

- largest differences in ^{3}H+^{3}He (unmeasured) and ^{7}Li+^{7}Be (Li problem)

- Our bounds are stronger due to:

  $\hookrightarrow$ updated experimental values for masses, constants, ..., smaller $Q_n^{\rm QED}$

  $\hookrightarrow$ different reaction rates due to cross section parametrizations

  $\hookrightarrow$ calculating the corrections exactly or using T-dependent approximations

Primordial nucleosynthesis at varying quark masses

Berengut, Epelbaum, Flambaum, Hanhart, UGM, Nebreda, Pelaez,
Phys. Rev. D **87** (2013) 085018

General remarks

- Fundamental parameters of the strong interactions: the quark masses m_u, m_d

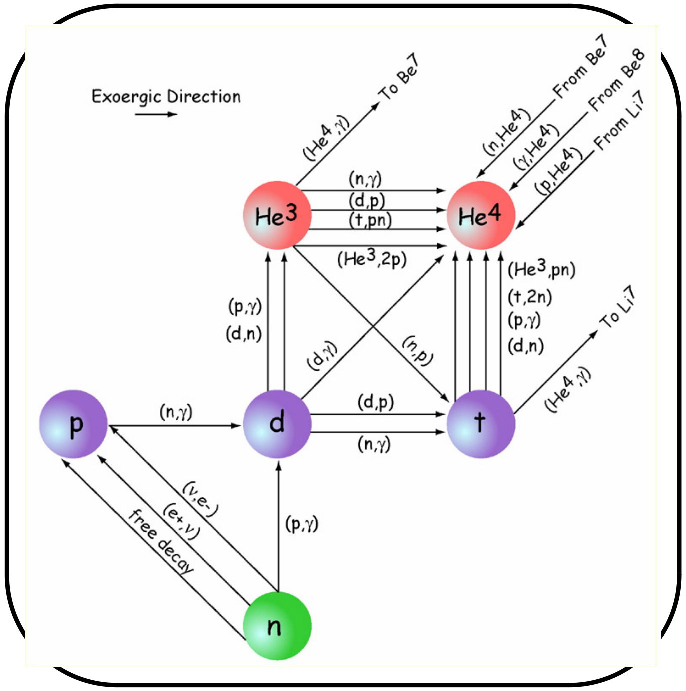
- In almost all nuclear reactions, strong isospin violation $m_d/m_u \simeq 2$ can be neglected because:

$$\boxed{\frac{m_u - m_d}{\Lambda_{\rm QCD}} \simeq \frac{1}{100}}$$

- Perform a first calculation with a simplified network by considering quark mass variations of $m_q = (m_u + m_d)/2$ for nulcei up to ^{4}He in the BBN network

- use the KAWANO code [NUC123]

Kawano, FERMILAB-PUB-92-004-A



from Cococubed.com

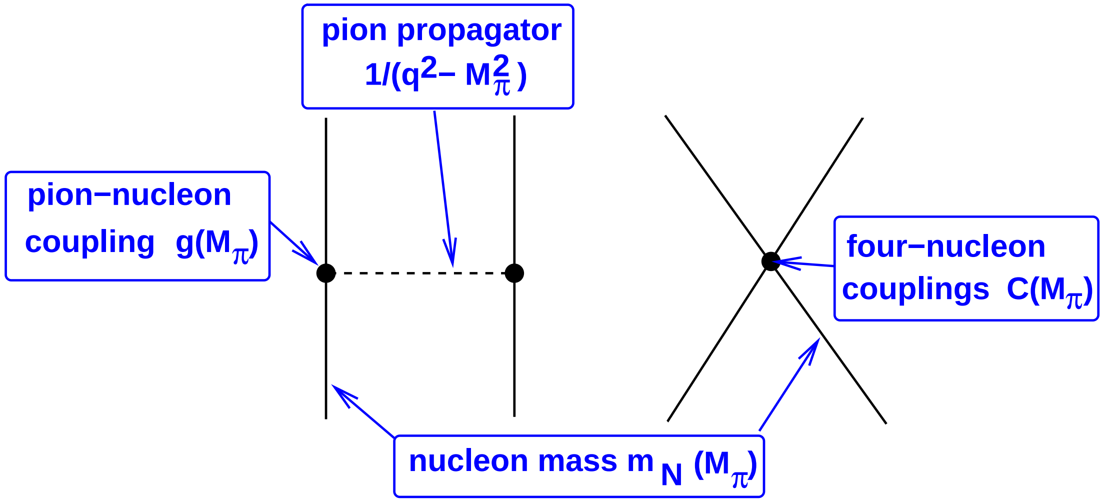
Ingredients

- Nuclear forces are given by chiral EFT based on Weinberg's power counting

Weinberg 1991

$\Rightarrow$ Pion-exchange contributions and short-distance multi-N operators

- graphical representation of the quark mass dependence of the LO potential



- always use the Gell-Mann–Oakes–Renner relation: $M^2_{\pi^\pm} \sim (m_u + m_d)$
- fulfilled to better than 94% in QCD

Colangelo, Gasser, Leutwyler 2001

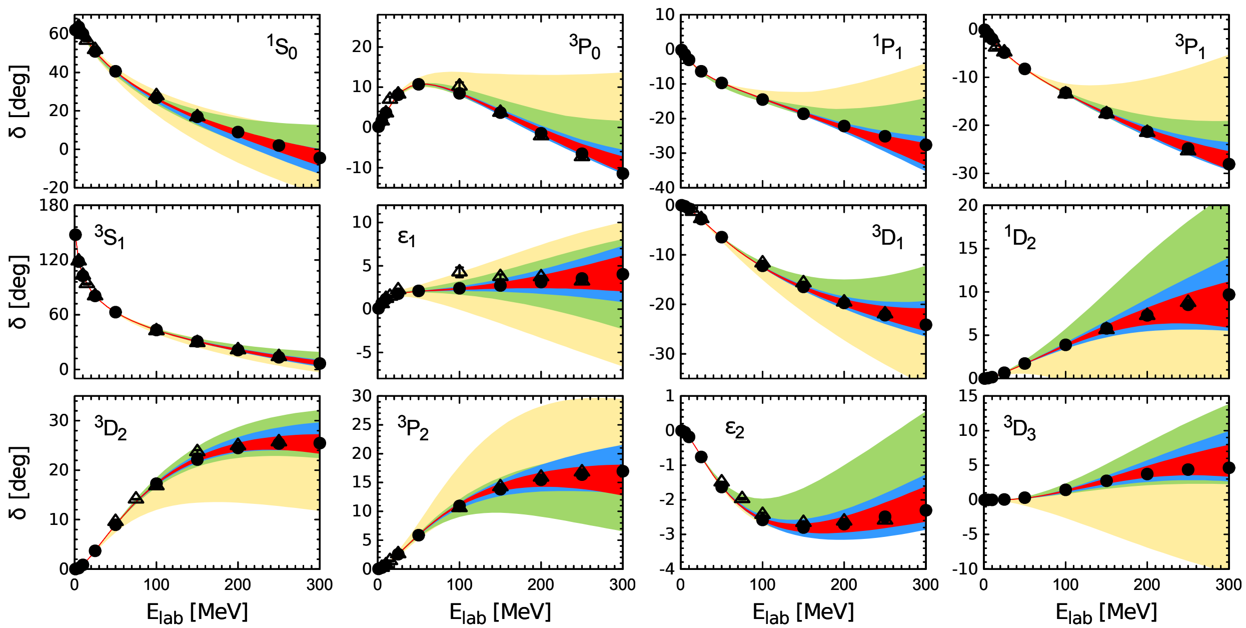
- Strong isospin violation and electromagnetic effects can also be included

Chiral nuclear EFT: Results

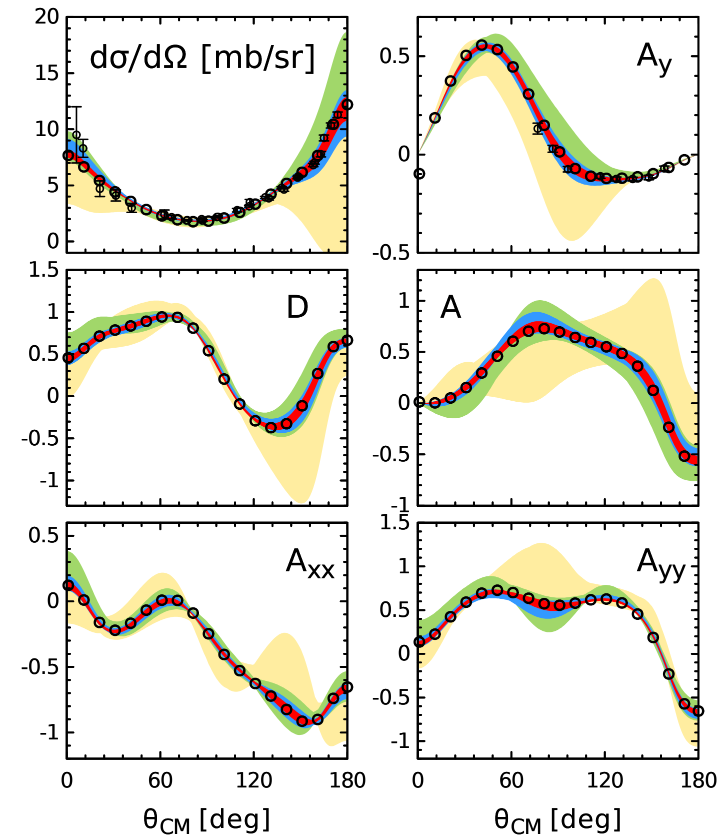
- Expansion to fifth order in the chiral expansion [Weinberg's power counting]

Epelbaum, Krebs, UGM, Phys.Rev.Lett. **115** (2015) 122301; Eur. Phys. J. A **51** (2015) 53

- phase shifts



- np scattering at 200 MeV



NLO N2LO N3LO N4LO

⇒ now a precision tool in nuclear physics

see e.g. Epelbaum, Krebs, Reinert, Front. in Phys. **8** (2020) 98

Quark mass dependence of hadron masses etc

- Quark mass dependence of hadron properties:

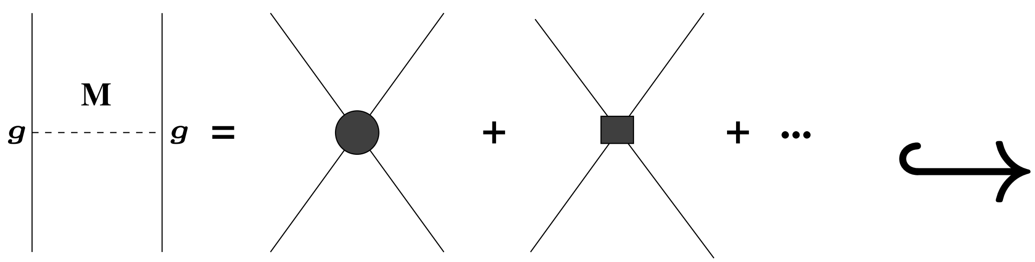
$$\frac{\delta O_H}{\delta m_f} \equiv K_H^f \frac{O_H}{m_f}\ , \quad f = u, d, s$$

- Pion and nucleon properties from lattice QCD combined with CHPT

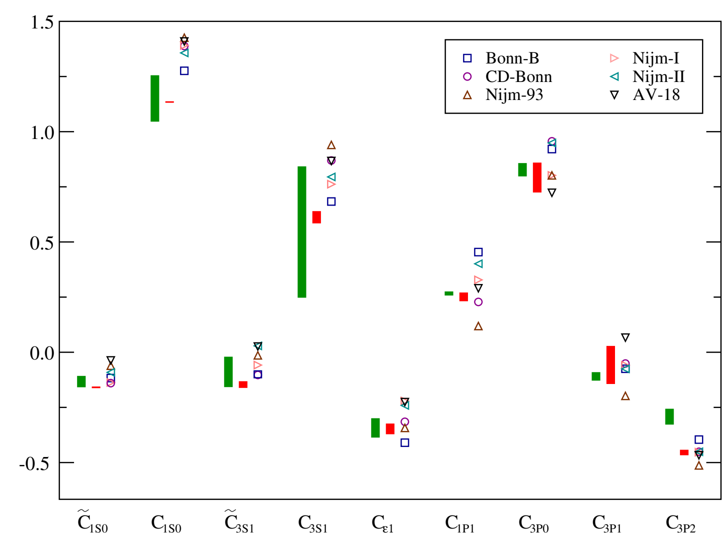
- Contact interactions modeled by heavy meson exchanges + unitarized CHPT

Epelbaum, UGM, Glöckle, Elster (2002)

Hanhart, Pelaez, Rios (2008)



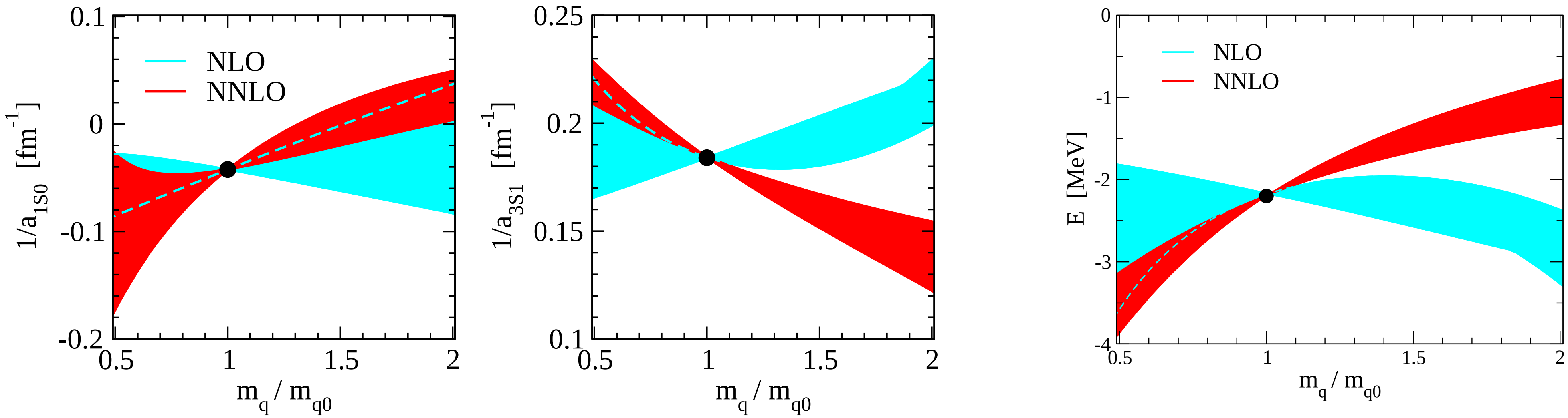
$$\frac{g^2}{t-M^2} = -\frac{g^2}{M^2} - \frac{g^2\,t}{M^4} + \ldots$$



Results for the NN system

- Putting pieces together for the two-nucleon system:

$$K^q_{a,1S0} = 2.3^{+1.9}_{-1.8}, \quad K^q_{a,3S1} = 0.32^{+0.17}_{-0.18}, \quad K^q_{\mathrm{B(deut)}} = -0.86^{+0.45}_{-0.50}$$



- Extends and improves earlier work based on EFTs and models

Beane, Savage (2003), Epelbaum, UGM, Glöckle (2003), Mondejar, Soto (2007), Flambaum, Wiringa (2007), Bedaque, Luu, Platter (2011) [BLP], . . .

- connection to lattice QCD results for NN → later

Quark mass variations of heavier nuclei

- In BBN, we also need the variation of ^{3}He and ^{4}He. All other BEs are kept fixed.

- use the method of BLP: Bedaque, Luu, Platter, PRC 83 (2011) 045803

$$K^q_{^A\mathrm{He}} = K^q_{a,\,1S0} K^{a,\,1S0}_{^A\mathrm{He}} + K^q_{\mathrm{deut}} K^{\mathrm{deut}}_{^A\mathrm{He}}\ , \quad A = 3,4$$

with

$$K^{a,\,1S0}_{^3\mathrm{He}} = 0.12 \pm 0.01\ , \quad K^{\mathrm{deut}}_{^3\mathrm{He}} = 1.41 \pm 0.01$$

$$K^{a,\,1S0}_{^4\mathrm{He}} = 0.037 \pm 0.011\ , \quad K^{\mathrm{deut}}_{^4\mathrm{He}} = 0.74 \pm 0.22$$

so that

$$\Rightarrow \boxed{K^q_{^3\mathrm{He}} = -0.94 \pm 0.75, \quad K^q_{^4\mathrm{He}} = -0.55 \pm 0.42}$$

$\Rightarrow$ calculate BBN response matrix of primordial abundances Y_a
at fixed baryon/photon ratio [first in the isospin limit]

Limits for the quark mass variation

- Average of ^{2}H and ^{4}He:

$$\frac{\delta m_q}{m_q} = 0.02 \pm 0.04$$

- in contrast to earlier studies, we provide reliable error estimates (EFT)
- but: BLP find a stronger constraint due to the neutron life time (affects $Y(^4\mathrm{He})$)
- re-evaluate this under the model-independent assumption that *all* quark & lepton masses vary with the **Higgs VEV** v (CHPT w/ virtual photons)

$\longrightarrow$ slide

$\Rightarrow$ results are dominated by the ^{4}He abundance:

$$\left|\frac{\delta v}{v}\right| = \left|\frac{\delta m_q}{m_q}\right| \leq 0.9\%$$

- Presently updated: larger networks, 5 BBN codes and improved m_q variations

UGM, Metsch, Meyer, on-going

Effects of the neutron lifetime

- The neutron width $\Gamma_n \sim 1/\tau_n$ is given by:

$$\Gamma_n = \frac{(G_F \cos\theta_C)^2}{2\pi^3} m_e^5 (1 + 3g_A^2)\; f\left(\frac{(m_n - m_p)^{\rm QED}}{m_e}\right)$$

- BLP assumed that m_u/m_d stays constant when m_q changes

  $\hookrightarrow$ induces a large dependence of the function f on variation in m_q

  $\hookrightarrow$ is model-dependent, as all other parameters are supposed to be unaffected

- A more natural scenario for $m_u/m_d = \text{constant}$ is that the Higgs VEV changes, while all Yukawa and gauge coupling stay constant

  $\hookrightarrow$ this reduces the dependence of Γ_n on variations of m_q by a factor of 2

  $\hookrightarrow$ the sensitivity to τ_n entirely denotes the ^{4}He sensitivity

Using lattice QCD

Lähde, UGM, Epelbaum, Eur. Phys. J **A 56** (2020) 89

- Use lattice data to determine $\bar{A}_{s,t} = \partial a_{s,t}^{-1}/\partial M_\pi|_{M_\pi^{\rm phys}}$

$$\bar{A}_s = 0.54(24)\ ,\quad \bar{A}_t = 0.33(16)$$

$\hookrightarrow$ $\bar{A}_s$ is consistent w/ earlier determination ($0.29^{+0.25}_{-0.23}$)

$\hookrightarrow$ $\bar{A}_t$ changes sign compared to earlier det. ($-0.18^{+0.10}_{-0.10}$)

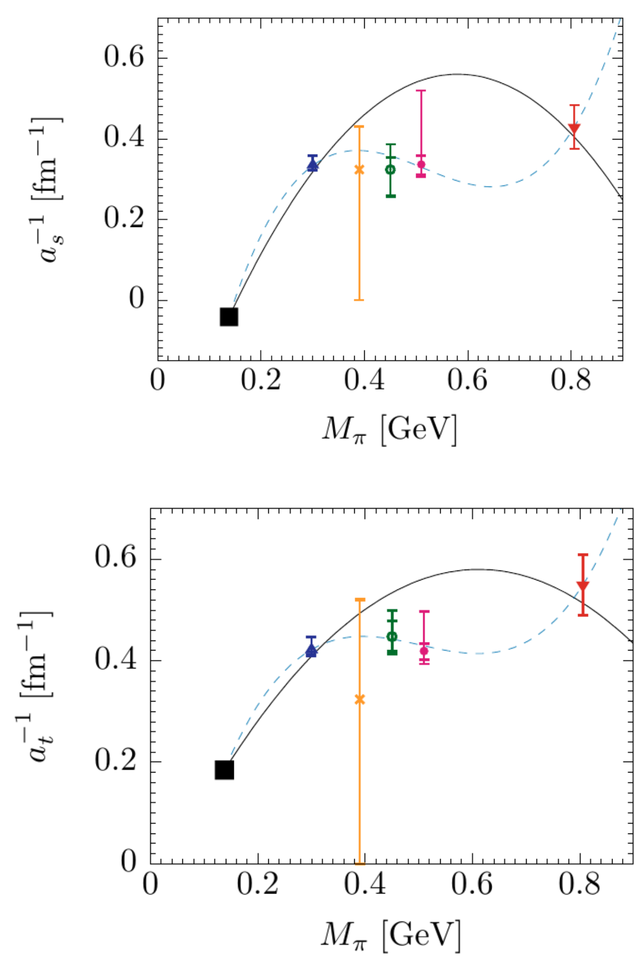
- update $x_1 = \partial m_N/\partial M_\pi$ and $x_2 = \partial g_{\pi N}/\partial M_\pi$ using better LQCD data:

$$x_1 = 0.84(7)\ ,\quad x_2 = -0.053(16)$$

$\hookrightarrow$ x_1 and x_2 more precise

$\hookrightarrow$ x_2 now has a definite sign

$\Rightarrow$ must be updated and included in the BBN network



Beane et al. (2012)
Yamazaki et al. (2015)
Orginos et al. (2015)
Beane et al. (2013)
Yamazaki et al. (2012)

Quark mass dependence of alpha-alpha scattering

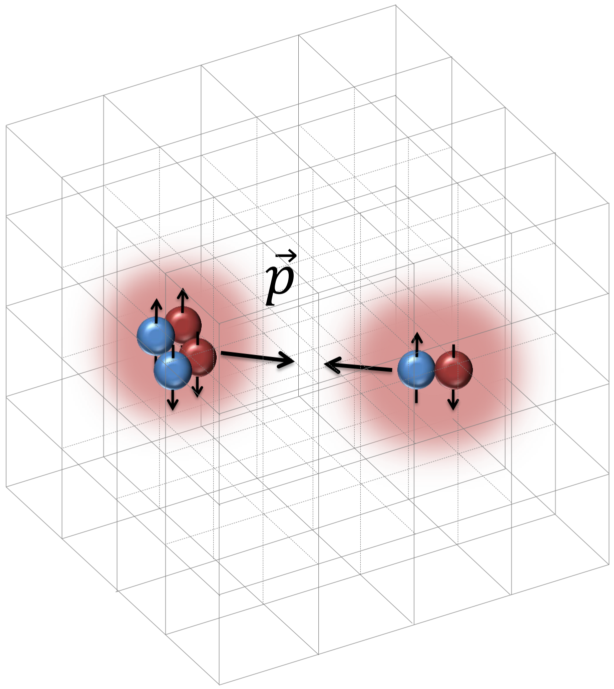
Elhatisari, Lähde, Lee, UGM, Vonk, JHEP **02** (2022) 001

Nucleus-nucleus scattering on the lattice

- Processes involving α-particles and α-type nuclei comprise a major part of stellar nucleosynthesis, and control the production of certain elements in stars

- Ab initio calculations of scattering and reactions using continuum methods suffer from very unfavorable computational scaling with the number of nucleons A in the clusters (either factorial or exponential in A)

- This is very different in NLEFT:



Lattice EFT computational scaling $\Rightarrow (A_1 + A_2)^2$

Rupak, Lee, Phys. Rev. Lett. **111** (2013) 032502
Pine, Lee, Rupak, Eur. Phys. J. A **49** (2013) 151
Elhatisari, Lee, Phys. Rev. C **90** (2014) 064001
Elhatisari et al., Phys.Rev. C **92** (2015) 054612
Elhatisari, Lee, UGM, Rupak, Eur. Phys. J. A **52** (2016) 174

Ab initio alpha-alpha scattering

Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, UGM, Nature **528** (2015) 111

- Construct the so-called adiabatic Hamiltonian

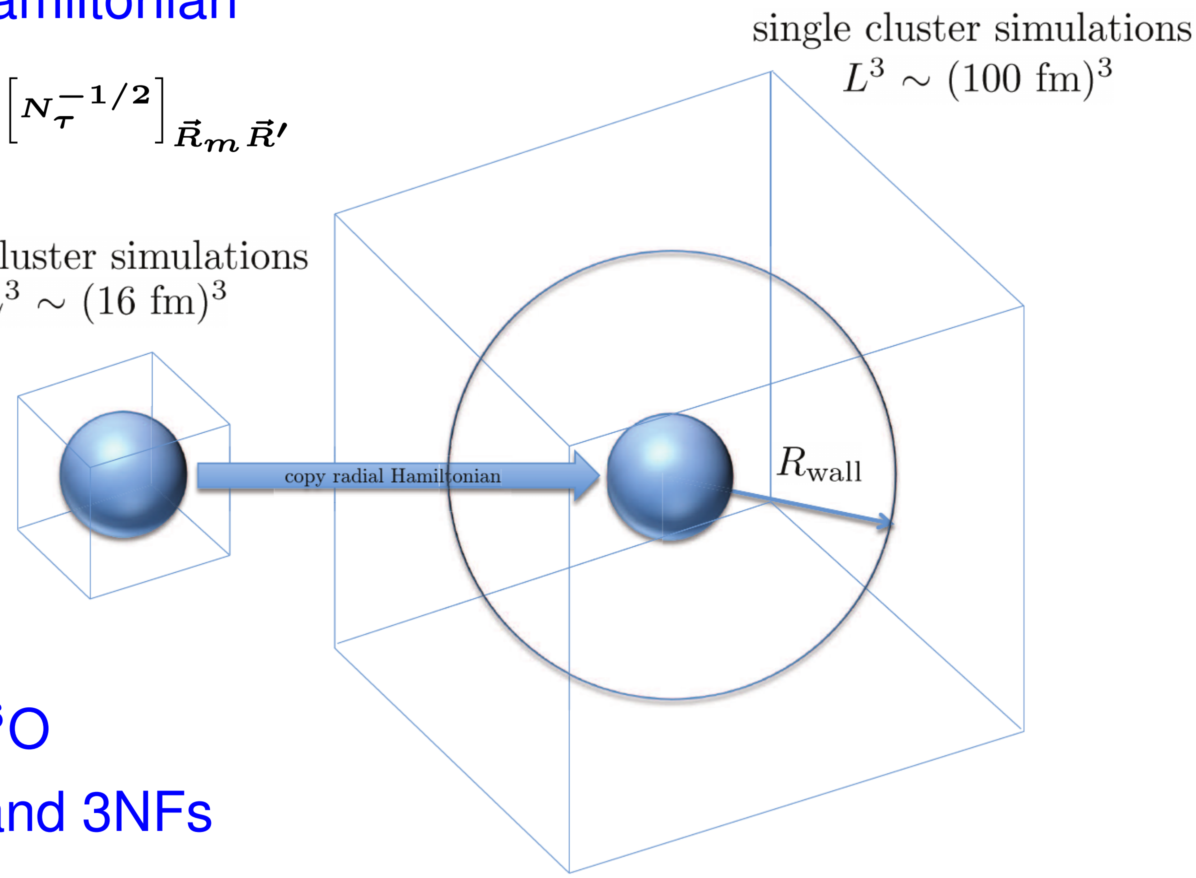
$$[H^a_\tau]_{\vec{R}\vec{R}'} = \sum_{\vec{R}_n \vec{R}_m} \left[N_\tau^{-1/2}\right]_{\vec{R}\vec{R}_n} \left[H_\tau\right]_{\vec{R}_n\vec{R}_m} \left[N_\tau^{-1/2}\right]_{\vec{R}_m\vec{R}'}$$

↪ two-cluster simulations

- Long-range Coulomb via spherical wall method (huge box)

Lu, Lähde, Lee, UGM, Phys. Lett. B **760** (2016) 309

↪ single cluster simulations



- Same action as used for ^{12}C and ^{16}O chiral N2LO Lagrangian w/ 2NFs and 3NFs

↪ all LECs determined before in NN and NNN systems

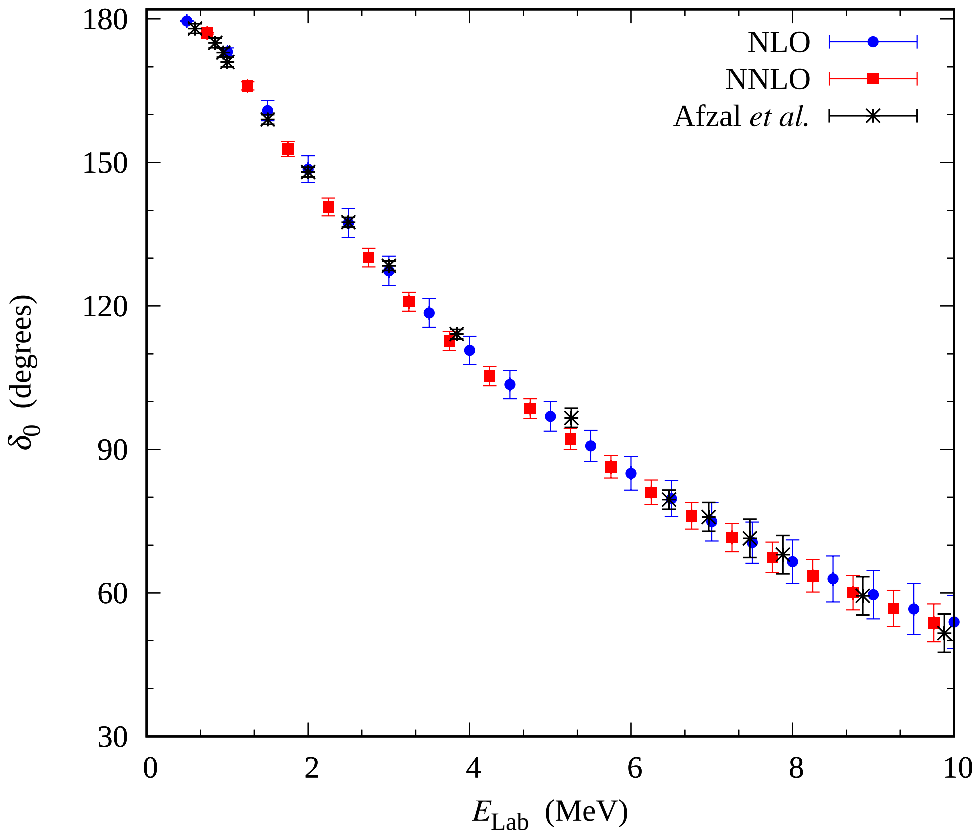
↪ parameter-free predictions

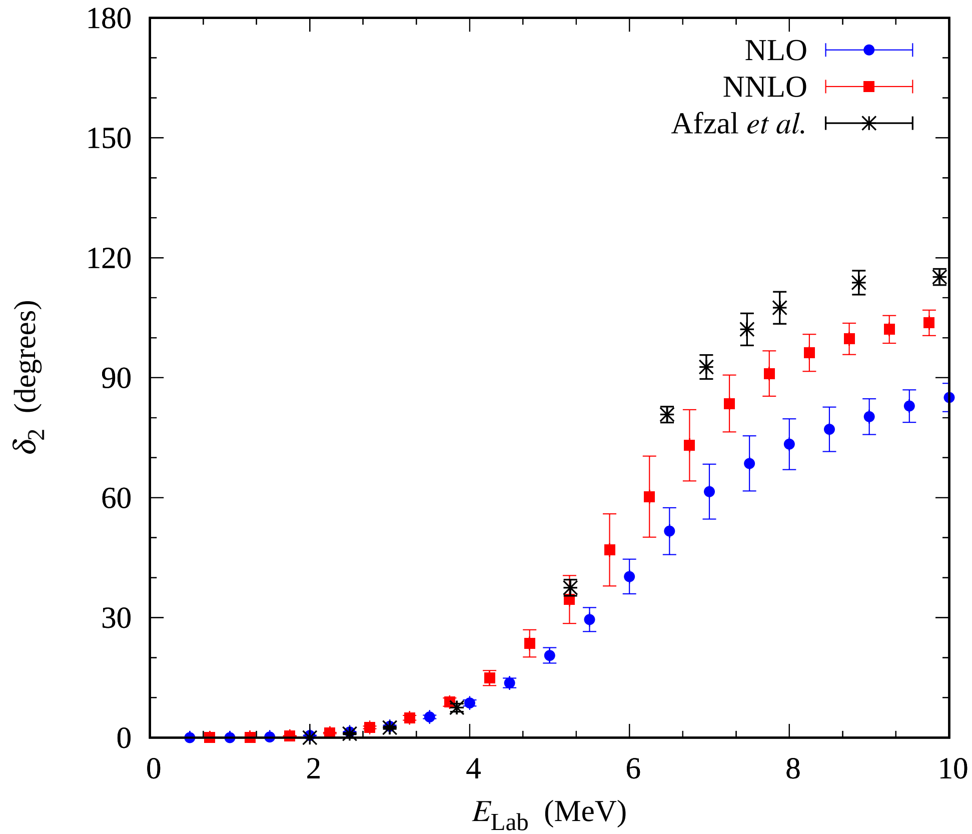
↪ first ever *ab initio* calculation of α-α scattering

Phase shifts of alpha-alpha scattering

- S-wave and D-wave phase shifts, updated in 2022

Elhatisari, Lähde, Lee, UGM, Vonk, JHEP **02** (2022) 001





$$\boldsymbol{E_R^{\text{NNLO}} = -0.11(1)}\ \text{MeV}\quad [\boldsymbol{+0.09}\ \text{MeV}]$$

$$\boldsymbol{E_R^{\text{NNLO}} = 2.93(5)}\ \text{MeV}\quad [\boldsymbol{2.92(18)}\ \text{MeV}]$$

$$\boldsymbol{\Gamma_R^{\text{NNLO}} = 2.00(16)}\ \text{MeV}\quad [\boldsymbol{1.35(50)}\ \text{MeV}]$$

Afzal et al., Rev. Mod. Phys. **41** (1969) 247 [data]

Alpha-alpha scattering in the multiverse

Elhatisari, Lähde, Lee, UGM, Vonk, JHEP **02** (2022) 001

- Now vary the light quark mass m_q and the fine-structure constant $\alpha_{\rm EM}$

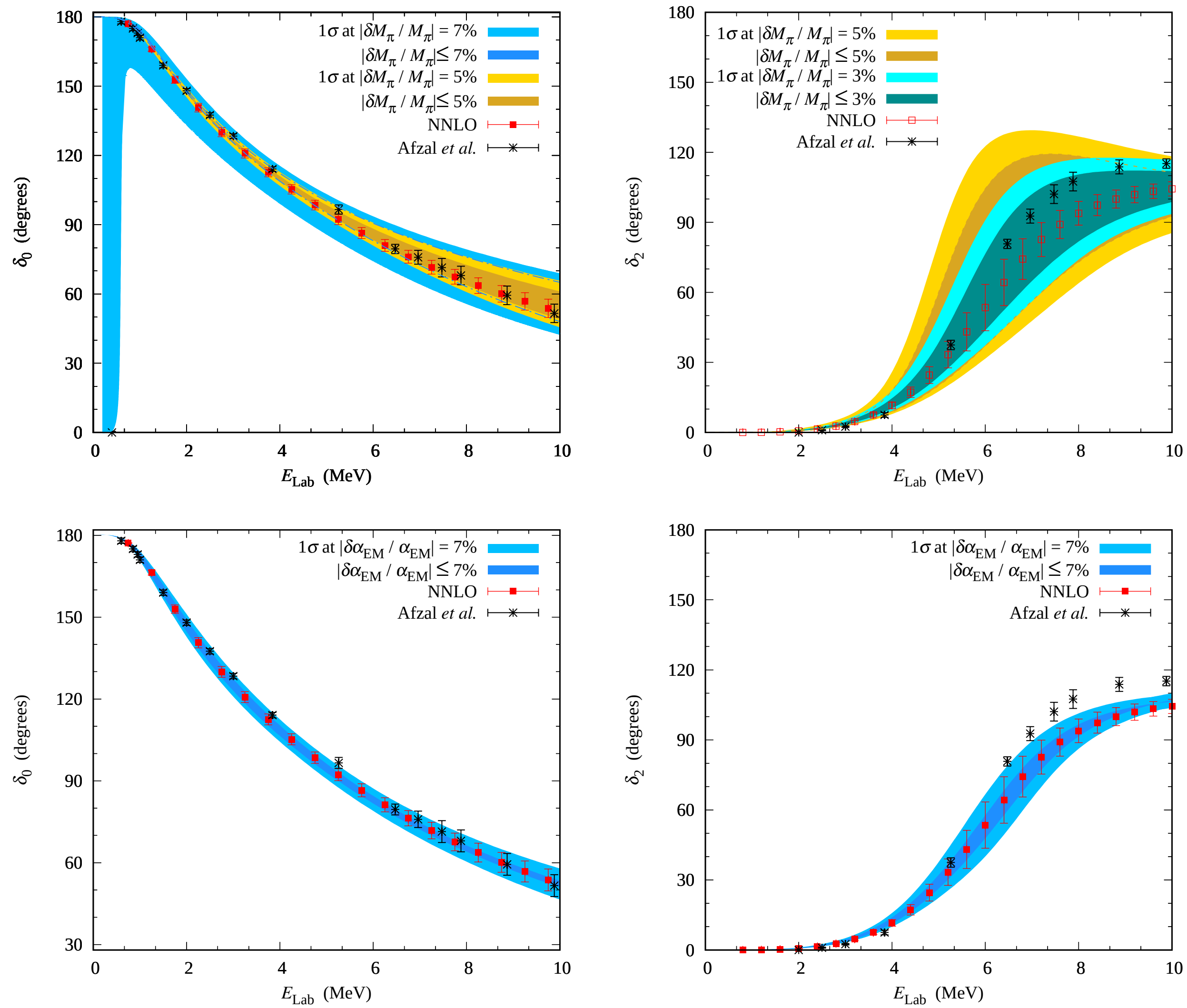
$\Rightarrow$ Dramatic effect in the S-wave
for $\delta M_\pi/M_\pi \simeq 7\%$

$\Rightarrow$ D-wave resonance requires
$\delta M_\pi/M_\pi \lesssim 3\%$

$\Rightarrow$ S- and D-wave phase shifts
tolerate $\delta\alpha_{\rm EM}/\alpha_{\rm EM} \lesssim 7\%$

$\Rightarrow$ weaker bounds as given by the
position of the Hoyle state
but independet of stellar modelling!

- in a next step, consider $\alpha+{}^8$Be$\to{}^{12}$C
as function of m_q and $\alpha_{\rm EM}$



Summary & outlook

- Chiral nuclear EFT: best approach to nuclear forces and few-body systems
- Nuclear lattice simulations as a new quantum many-body approach
  $\longrightarrow$ allow to vary the parameters of QCD+QED
  $\longrightarrow$ investigate changes in nuclear properties + scattering can also be done
- Study of the nuclear force as a function of the quark masses & $\alpha_{\rm EM}$
  $\longrightarrow$ pion-exchanges straightforward, contact interactions require modeling / LQCD
- Impact on BBN:
  $\hookrightarrow$ $|\delta m_q/m_q| \leq 0.9\%$ $\hookrightarrow$ requires update
  $\hookrightarrow$ Variations of $\alpha_{\rm EM}$: many sources, new input $\rightarrow$ $|\delta\alpha_{\rm EM}/\alpha_{\rm EM}| \leq 1.8\%$
- Sensitivity of α-α scattering to m_q and $\alpha_{\rm EM}$ worked out
  $\longrightarrow$ towards study of the triple-alpha process and the “holy grail” of nuclear astrophysics for varying m_q and $\alpha_{\rm EM}$

SPARES

QUARK MASS DEP. of the SHORT-DISTANCE TERMS

- Consider a typical OBEP with $M = \sigma, \rho, \omega, \delta, \eta$

- Quark mass dependence of the sigma and rho from unitarized CHPT

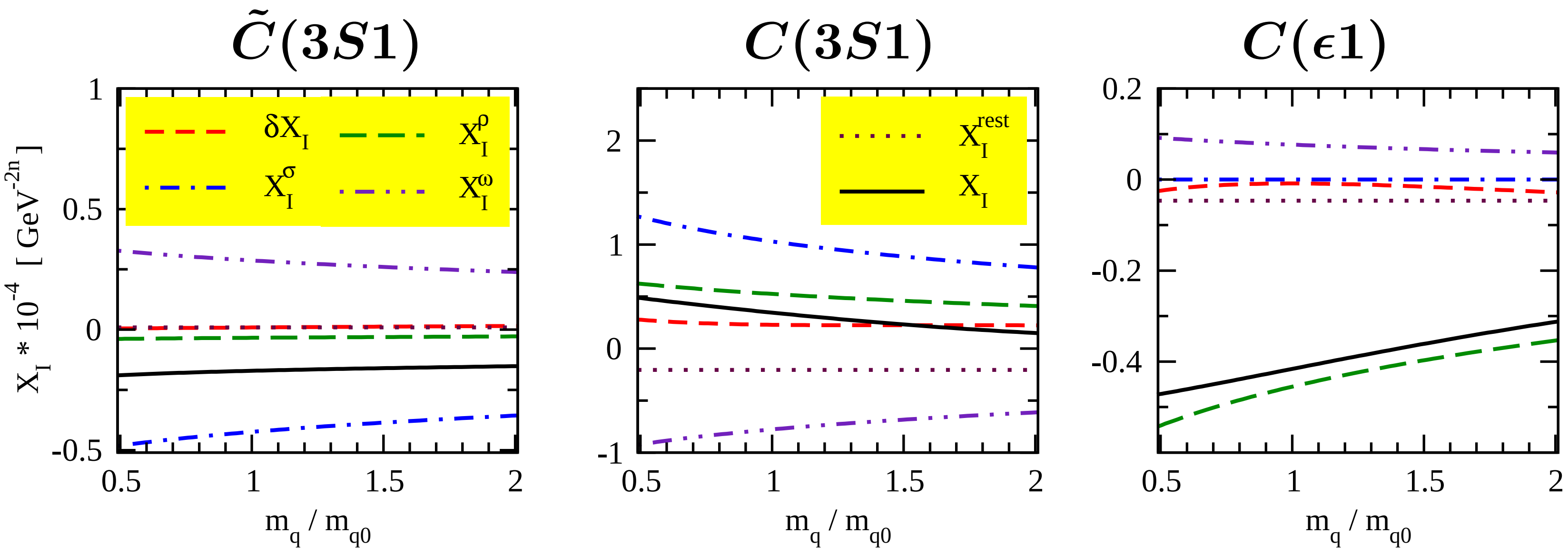
  Hanhart, Pelaez, Rios (2008)

$\Rightarrow$ $K^q_{M_\sigma} = 0.081 \pm 0.007, \quad K^q_{M_\rho} = 0.058 \pm 0.002$

$\Rightarrow$ couplings appear quark mass independent (requires refinement in the future)

- assume a) that $K^q_\omega = K^q_\rho$ and b) neglect dep. of δ, η

$\Rightarrow$



A SHORT HISTORY of the HOYLE STATE

- Heavy element generation in massive stars: triple-α process

  Bethe 1938, Öpik 1952, Salpeter 1952, Hoyle 1954, ...

$$^4\text{He} + {}^4\text{He} \rightleftharpoons {}^8\text{Be}$$
$$^8\text{Be} + {}^4\text{He} \rightleftharpoons {}^{12}\text{C}^* \rightarrow {}^{12}\text{C} + \gamma$$
$$^{12}\text{C} + {}^4\text{He} \rightleftharpoons {}^{16}\text{O} + \gamma$$

- Hoyle's contribution: calculation of relative abundances of ^{4}He, ^{12}C and ^{16}0
  $\Rightarrow$ need a resonance close to the ^{8}Be + ^{4}He threshold at $E_R = 0.35$ MeV
  $\Rightarrow$ this corresponds to a $J^P = 0^+$ excited state 7.7 MeV above the g.s.
- a corresponding state was experimentally confirmed at Caltech at
  $E - E(\text{g.s.}) = 7.653 \pm 0.008$ MeV   Dunbar et al. 1953, Cook et al. 1957
- still on-going experimental activity, e.g. EM transitions at SDALINAC

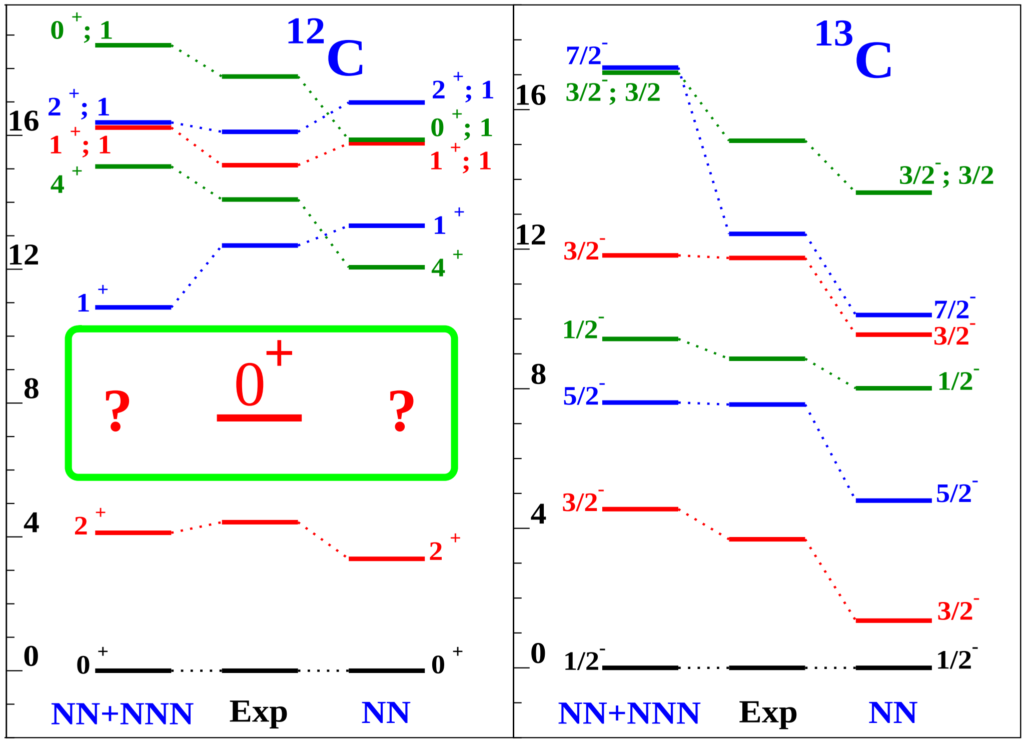
  M. Chernykh et al., Phys. Rev. Lett. 98 (2007) 032501
- and how about theory ? $\rightarrow$ this talk
- side remark: NOT driven by anthropic considerations

  H. Kragh, Arch. Hist. Exact Sci. 64 (2010) 721

AN ENIGMA for NUCLEAR THEORY

- Ab initio calculation in the no-core shell model: $\approx 10^7$ CPU hrs on JAGUAR

P. Navratil et al., Phys. Rev. Lett. **99** (2007) 042501; R. Roth et al., Phys. Rev. Lett. **107** (2011) 072501



$\Rightarrow$ excellent description, but no trace of the Hoyle state

RESULTS for HEAVIER NUCLEI

- calculate BBN response matrix of primordial abundances Y_a at fixed baryon/photon ratio :

$$\frac{\delta \ln Y_a}{\delta \ln m_q} = \sum_{X_i} \frac{\partial \ln Y_a}{\partial \ln X_i} K^q_{X_i}$$

$\Rightarrow$

X	d	^{3}He	^{4}He	^{6}Li	^{7}Li
a_s	−0.39	0.17	0.01	−0.38	2.64
$B_{\rm deut}$	−2.91	−2.08	0.67	−6.57	9.44
$B_{\rm trit}$	−0.27	−2.36	0.01	−0.26	−3.84
$B_{^3{\rm He}}$	−2.38	3.85	0.01	−5.72	−8.27
$B_{^4{\rm He}}$	−0.03	−0.84	0.00	−69.8	−57.4
$B_{^6{\rm Li}}$	0.00	0.00	0.00	78.9	0.00
$B_{^7{\rm Li}}$	0.03	0.01	0.00	0.02	−25.1
$B_{^7{\rm Be}}$	0.00	0.00	0.00	0.00	99.1
τ	0.41	0.14	0.72	1.36	0.43

updated Kawano code

Kawano, FERMILAB-Pub-92/04-A

RESULTS

- putting pieces together:

$$\left.\frac{\partial \Delta E_h}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.455(35)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.744(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.056(10)$$

$$\left.\frac{\partial \Delta E_b}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.117(34)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.189(24)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.012(9)$$

- x_1 and x_2 only affect the small constant terms
- also calculated the shifts of the individual energies (not shown here)

INTERPRETATION

- $(\partial\Delta E_h/\partial M_\pi)/(\partial\Delta E_b/\partial M_\pi) \simeq 4$
  $\Rightarrow$ ΔE_h and ΔE_b cannot be independently fine-tuned

- Within error bars, $\partial\Delta E_h/\partial M_\pi$ & $\partial\Delta E_b/\partial M_\pi$ appear unaffected
  by the choice of x_1 and x_2 $\to$ indication for α-clustering

- the triple alpha process is controlled by :

$$\Delta E_{h+b} \equiv \Delta E_h + \Delta E_b = E_{12}^\star - 3E_4$$

$$\boxed{\left.\frac{\partial\Delta E_{h+b}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = -0.571(14)\left.\frac{\partial a_s^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} - 0.934(11)\left.\frac{\partial a_t^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} + 0.069(6)}$$

$\Rightarrow$ quark mass dependence of the scattering lengths discussed earlier

The fate of carbon-based life as a function of the quark mass

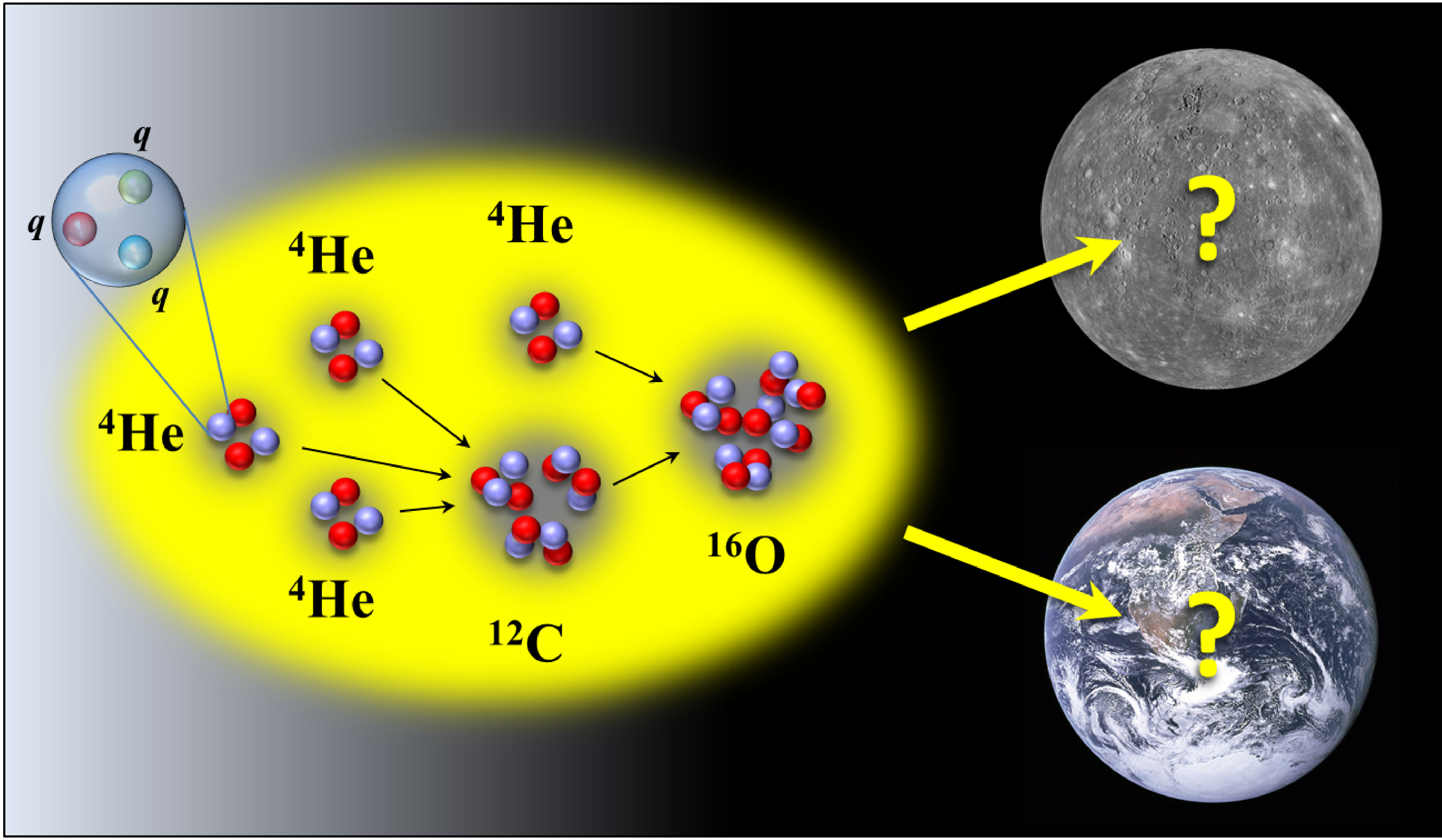
Epelbaum, Krebs, Lähde, Lee, UGM
Phys. Rev. Lett. **110** (2013) 112502; Eur. Phys. J. **A 48** 82 (2013)

update: Lähde, UGM, Epelbaum, Eur. Phys. J. **A 56** (2020) 89

review: UGM, Sci. Bull. **60** (2015) 43

Fine-tuning of the fundamental parameters

Fig. courtesy Dean Lee



The tool: Nuclear lattice effective field theory

Frank, Brockmann (1992), Koonin, Müller, Seki, van Kolck (2000) , Lee, Schäfer (2004), . . .
Borasoy, Krebs, Lee, UGM, Nucl. Phys. **A768** (2006) 179; Borasoy, Epelbaum, Krebs, Lee, UGM, Eur. Phys. J. **A31** (2007) 105

- *new method* to tackle the nuclear many-body problem

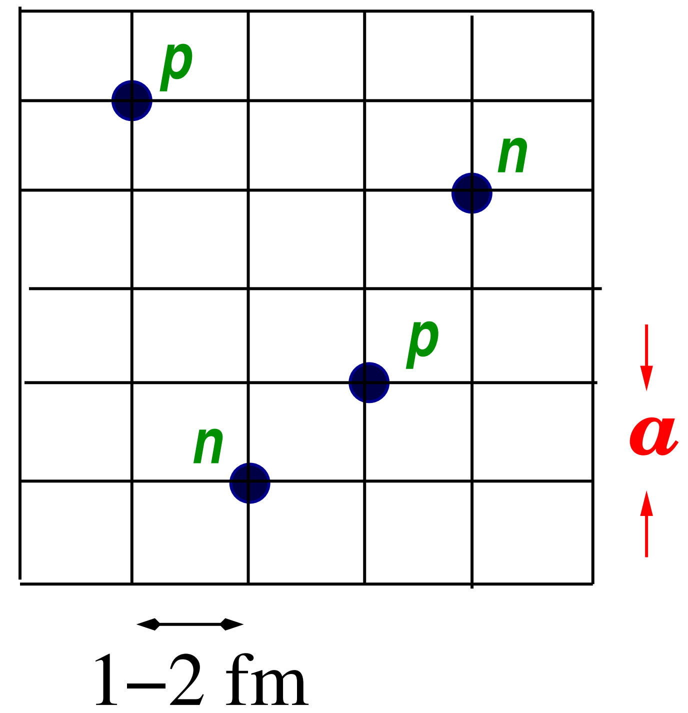
- discretize space-time $V = L_s \times L_s \times L_s \times L_t$:
  nucleons are point-like particles on the sites

- discretized chiral potential w/ pion exchanges
  and contact interactions + Coulomb

  $\to$ see Epelbaum, Hammer, UGM, Rev. Mod. Phys. **81** (2009) 1773

- typical lattice parameters

$$p_{\rm max} = \frac{\pi}{a} \simeq 315 - 630 \text{ MeV [UV cutoff]}$$



- strong suppression of sign oscillations due to approximate Wigner SU(4) symmetry

  E. Wigner, Phys. Rev. **51** (1937) 106; T. Mehen et al., Phys. Rev. Lett. **83** (1999) 931; J. W. Chen et al., Phys. Rev. Lett. **93** (2004) 242302

- physics independent of the lattice spacing for $a = 1 \ldots 2$ fm

  Alarcon, Du, Klein, Lähde, Lee, Li, Lu, Luu, UGM, EPJA **53** (2017) 83; Klein, Elhatisari, Lähde, Lee, UGM, EPJA **54** (2018) 121

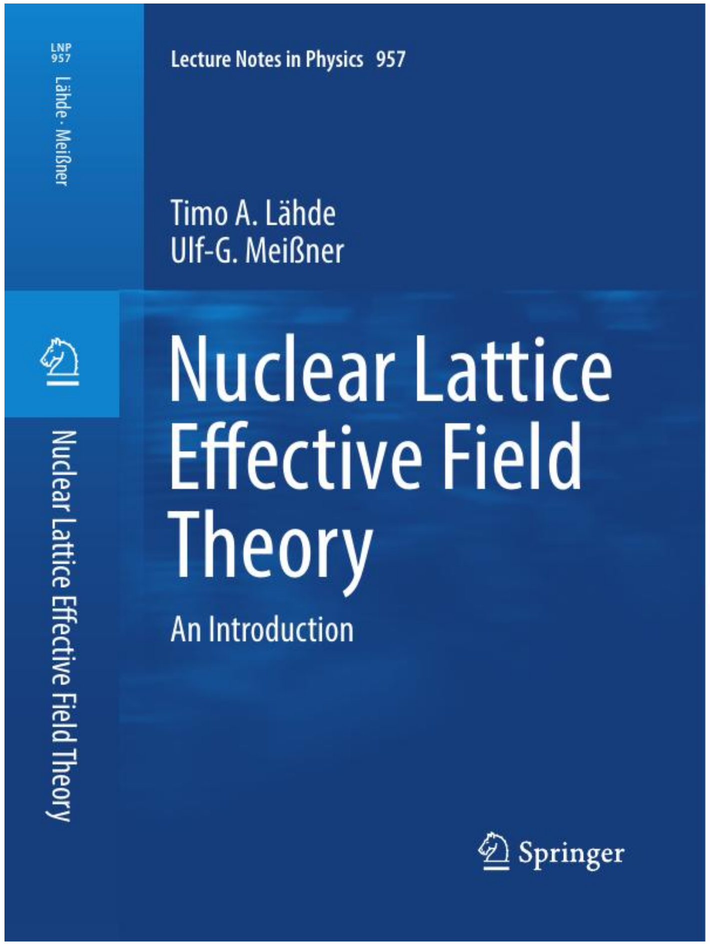
The tool: Nuclear lattice effective field theory II

- For all details on chiral EFT on a lattice

  T. Lähde & UGM

  Nuclear Lattice Effective Field Theory - An Introduction

  Springer Lecture Notes in Physics **957** (2019) 1 - 396



- Computational equipment

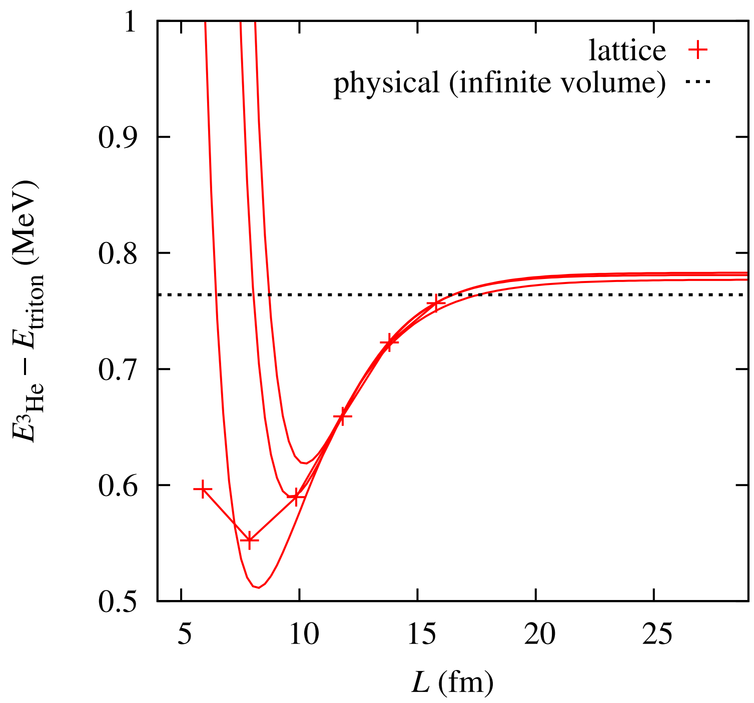
Some early results: Validation of the method

Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **104** (2010) 142501; Eur. Phys. J. A **45** (2010) 335

Lähde, Epelbaum, Krebs, Lee, UGM, Rupak, Phys. Lett. B **732** (2014) 110; Phys. Rev. Lett. **112** (2014) 102501

- Some groundstate energies and differences

E [MeV]	NLEFT	Exp.
^{3}He -^{3}H	0.78(5)	0.76
^{4}He	−28.3(6)	−28.3
^{8}Be	−55(2)	−56.5
^{12}C	−92(3)	−92.2
^{16}O	−131(1)	−127.6
^{20}Ne	−166(1)	−160.6
^{24}Mg	−198(2)	−198.3
^{28}Si	−234(3)	−236.5

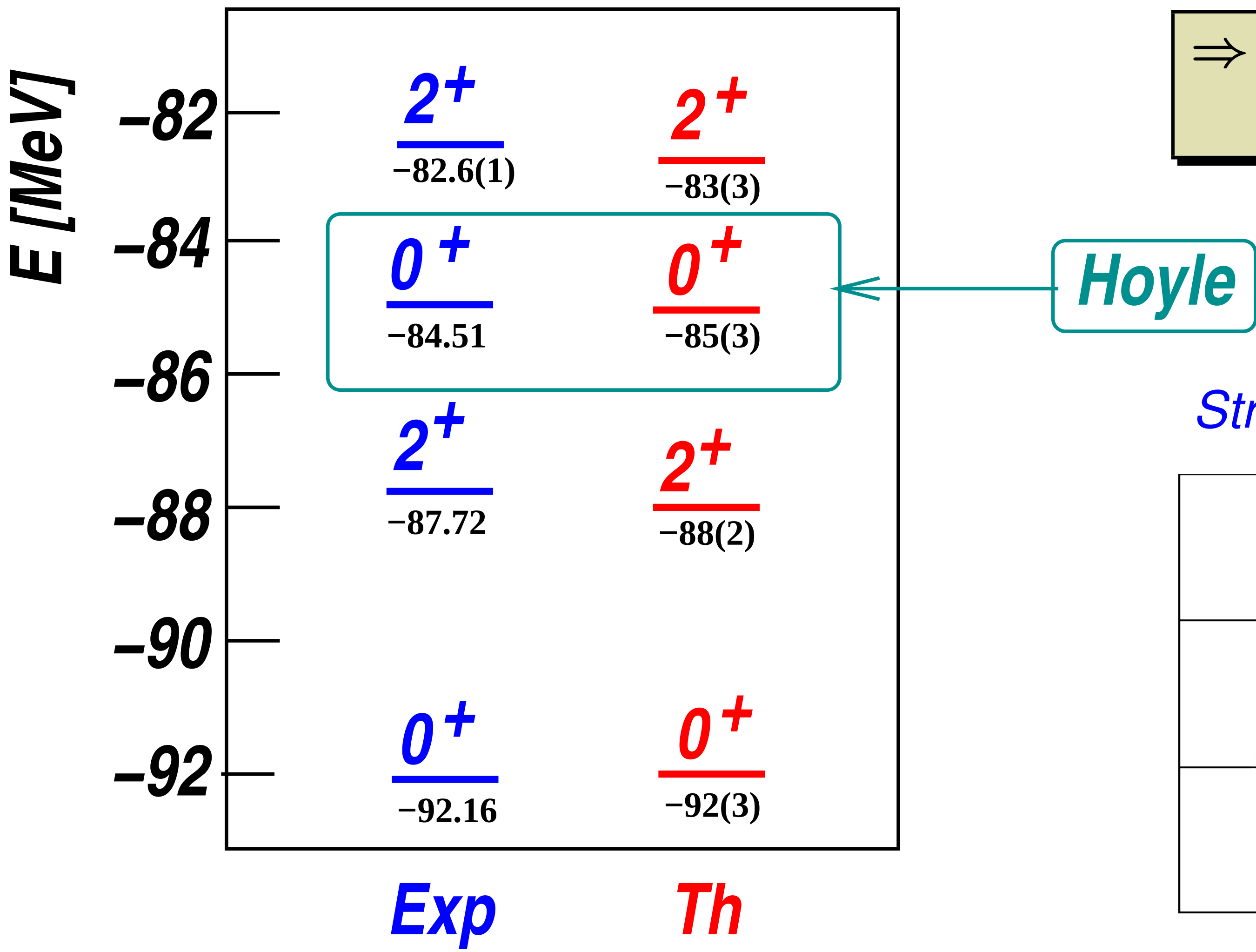


- promising results [much improved by now]

- excited states more difficult, but also doable

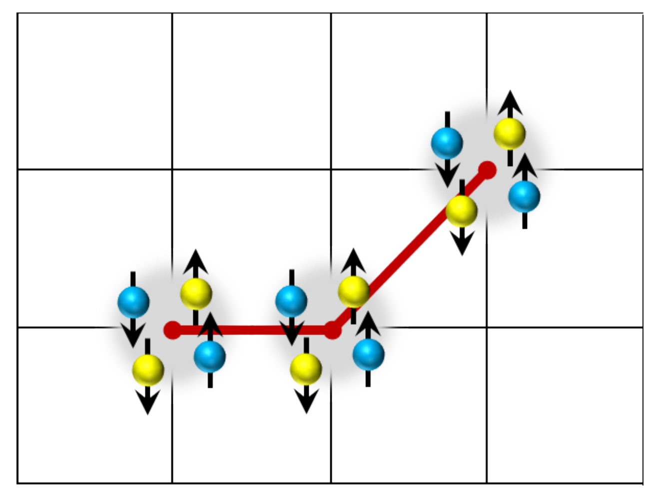
The spectrum of carbon-12 A.D. 2011

- After $8 \cdot 10^6$ hrs JUGENE/JUQUEEN (and “some” human work)



⇒ First ab initio calculation of the Hoyle state ✓

Structure of the Hoyle state:



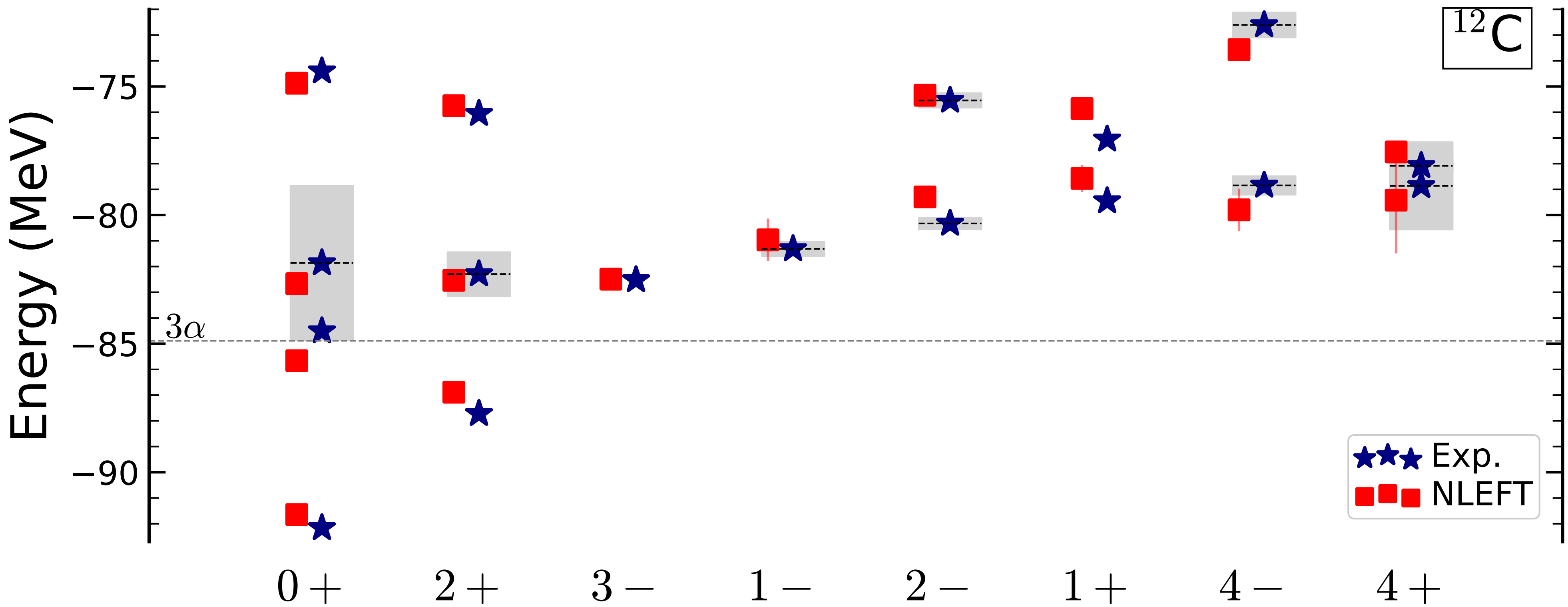
Epelbaum, Krebs, Lee, UGM, Phys. Rev. Lett. **106** (2011) 192501

Epelbaum, Krebs, Lähde, Lee, UGM, Phys. Rev. Lett. **109** (2012) 252501

The spectrum of carbon-12 A.D. 2023

- with much improved algorithms and methods:

Shen, Lähde, Lee, UGM, Nature Commun. **14** (2023) 2777



$\rightarrow$ solidifies earlier NLEFT statements about the structure of the $\mathbf{0_2^+}$ and $\mathbf{2_2^+}$ states

Pion mass dependence from MC simulations

- Consider pion mass changes as *small perturbations* for an energy (difference) E_i

$$\left.\frac{\partial E_i}{\partial M_\pi}\right|_{M_\pi^{\rm phys}} = \left.\frac{\partial E_i}{\partial M_\pi^{\rm OPE}}\right|_{M_\pi^{\rm phys}} + x_1 \left.\frac{\partial E_i}{\partial m_N}\right|_{m_N^{\rm phys}} + x_2 \left.\frac{\partial E_i}{\partial g_{\pi N}}\right|_{g_{\pi N}^{\rm phys}} + x_3 \left.\frac{\partial E_i}{\partial C_0}\right|_{C_0^{\rm phys}} + x_4 \left.\frac{\partial E_i}{\partial C_I}\right|_{C_I^{\rm phys}}$$

with

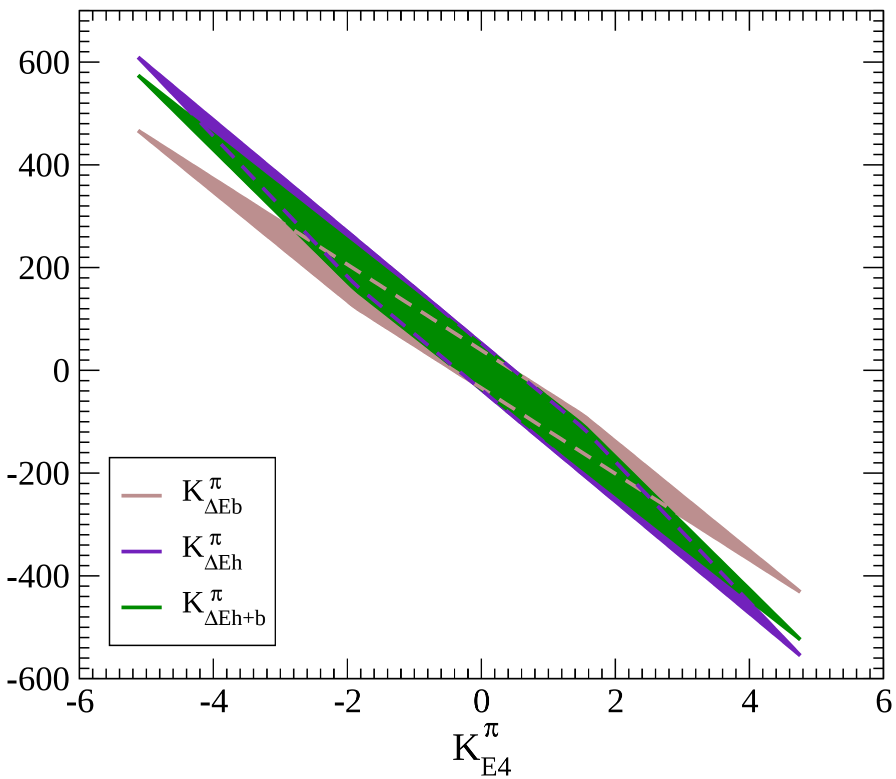
$$x_1 \equiv \left.\frac{\partial m_N}{\partial M_\pi}\right|_{M_\pi^{\rm phys}},\; x_2 \equiv \left.\frac{\partial g_{\pi N}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}},\; x_3 \equiv \left.\frac{\partial C_0}{\partial M_\pi}\right|_{M_\pi^{\rm phys}},\; x_4 \equiv \left.\frac{\partial C_I}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

$\Rightarrow$ problem reduces to the calculation of the various derivatives using AFQMC and the determination of the x_i

- x_1 and x_2 can be obtained from LQCD plus CHPT
- x_3 and x_4 can be obtained from NN scattering and its M_π-dependence $\to \bar{A}_{s,t}$

Correlations

- vary the quark mass derivatives of $\bar{A}_{s,t} = \partial a_{s,t}^{-1}/\partial M_\pi|_{M_\pi^{\rm phys}}$ within $-1,\ldots,+1$:



$$\Delta E_b = E(^8\mathrm{Be}) - 2E(^4\mathrm{He})$$

$$\Delta E_h = E(^{12}\mathrm{C}^*) - E(^8\mathrm{Be}) - E(^4\mathrm{He})$$

$$\Delta E_{h+b} = E(^{12}\mathrm{C}^*) - 3E(^4\mathrm{He})$$

$$\boxed{\frac{\partial O_H}{\partial M_\pi} = K_H^\pi \frac{O_H}{M_\pi}}$$

- clear correlations: the two fine-tunings are not independent

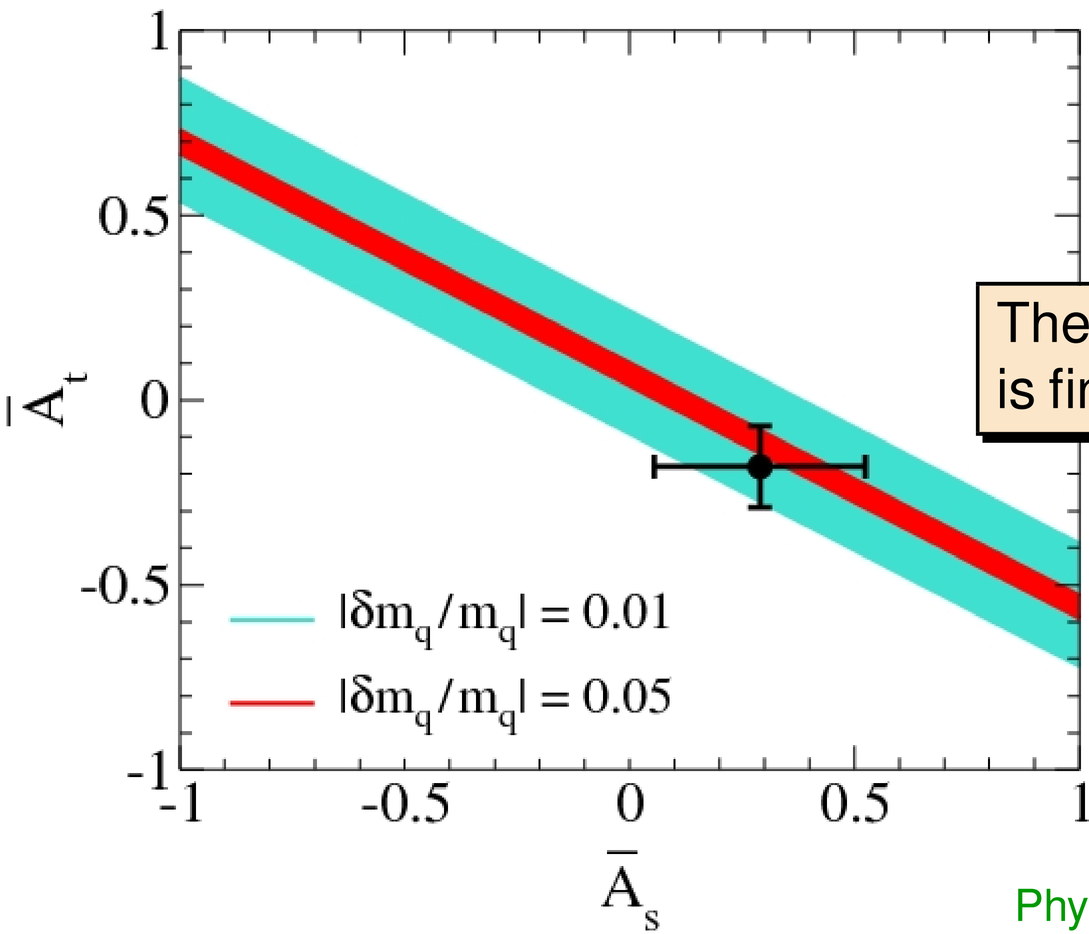
$\Rightarrow$ has been speculated before but could not be calculated Weinberg (2001)

The end-of-the-world plot I

- $|\delta(\Delta E_{h+b})| < 100$ keV

Oberhummer et al., Science (2000)

$$\rightarrow \boxed{\left|\left(0.571(14)\bar{A}_s + 0.934(11)\bar{A}_t - 0.069(6)\right)\frac{\delta m_q}{m_q}\right| < 0.0015}$$



$$\bar{A}_{s,t} \equiv \left.\frac{\partial a_{s,t}^{-1}}{\partial M_\pi}\right|_{M_\pi^{\rm phys}}$$

The light quark mass is fine-tuned to $\simeq 2-3\,\%$

Similarly: $\alpha_{\rm EM}$ is fine-tuned to $\simeq 2.5\%$

Berengut et al., Phys. Rev. **D 87** (2013) 085018

An update on fine-tunings in the triple-alpha process

Lähde, UGM, Epelbaum, Eur. Phys. J **A 56** (2020) 89

- Use lattice data to determine $\bar{A}_s$ and $\bar{A}_t$:

$$\bar{A}_s = 0.54(24)\,, \quad \bar{A}_t = 0.33(16)$$

$\hookrightarrow$ $\bar{A}_s$ is consistent w/ earlier determination

$\hookrightarrow$ $\bar{A}_t$ changes sign compared to earlier determination

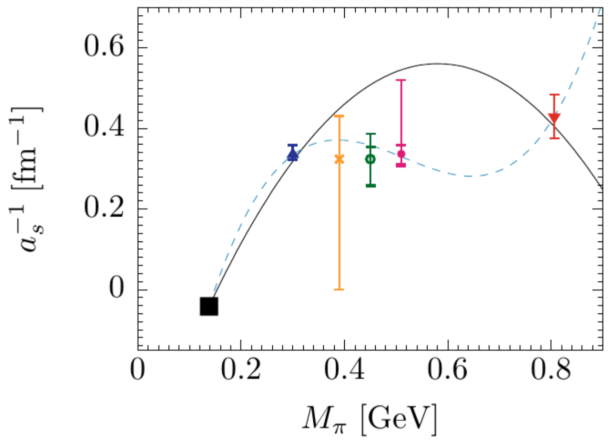
- update x_1 and x_2 using better LQCD data:

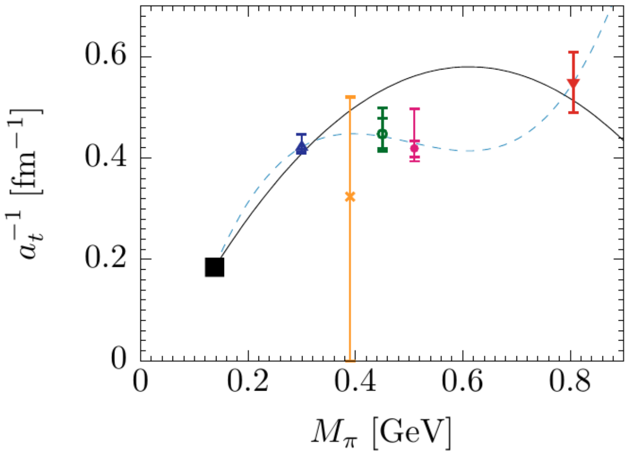
$$x_1 = 0.84(7)\,, \quad x_2 = -0.053(16)$$

$\hookrightarrow$ x_1 and x_2 more precise

$\hookrightarrow$ x_2 now has a definite sign

$\Rightarrow$ update end-of-the-world plot





Beane et al. (2012)
Yamazaki et al. (2015)
Orginos et al. (2015)
Beane et al. (2013)
Yamazaki et al. (2012)

New end-of-the-world plots

Lähde, UGM, Epelbaum, Eur. Phys. J **A 56** (2020) 89

- Constraints now depend on Z, the nucleus and the sign of δm_q

- lattice values for $\bar{A}_{s,t}$:

  > The light quark mass is fine-tuned to $\simeq 0.5\,\%$

- chiral EFT values for $\bar{A}_{s,t}$:

  > The light quark mass is fine-tuned to $\simeq 5\,\%$

- Bound on $\alpha_{\rm EM}$ softened ($\sim 7.5\,\%$)

$\Rightarrow$ need better determinations of $\bar{A}_{s,t}$ from lattice QCD with pion masses closer to the physical point!