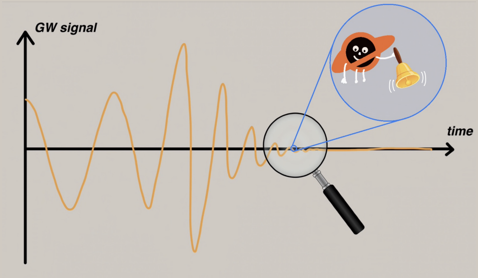
Black holes as ringing bells in and beyond general relativity



PhD Seminars

6th December, 2023

Marco Melis
marco.melis@uniroma1.it

SAPIENZA
UNIVERSITÀ DI ROMA

Table of contents

Long story short: gravitational waves

Gravitational waves are perturbation of the gravitational field propagating at the **speed of light**.

They emerge directly from the theory of General Relativity, as proved by Einstein in 1916.

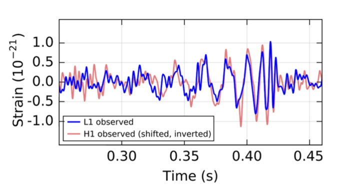
What are the sources of gravitational waves?

- **Compact binaries** $\rightarrow$ binary systems composed of compact objects (e.g. **BHs, NSs**).
- **Supernova explosion** $\rightarrow$ gravitational radiation is produced by the star content blown away.
- **Spinning neutron stars** $\rightarrow$ **slightly deformed** (non-spherical or non-axisymmetric) rotating neutron stars.
- **Stochastic gravitational waves background** $\rightarrow$ first detection June 2023.

Gravitational waves of a BBH merger

GW150914

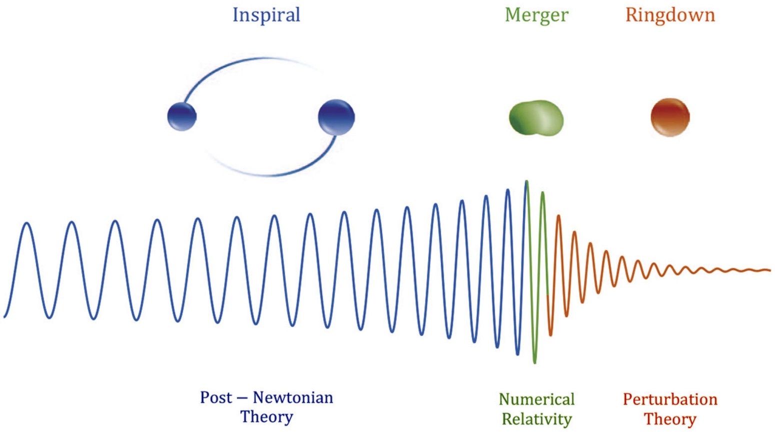
September 14th, 2015:
first direct observation of gravitational waves from the ***coalescence* of two black holes**.
100 years after Einstein's paper!



The GWs from a BBH coalescence can be divided into three phases:

1. **Inspiral** $\rightarrow$ large separation of the objects (Post-Newtonian approximation).
2. **Merger** $\rightarrow$ the two objects coalesce (Numerical Relativity).
3. **Ringdown** $\rightarrow$ the end-product relaxes to its equilibrium configuration (perturbation theory).

Gravitational waves of a BBH merger



N.B.: This is not a strict divide!

The Ringdown

The final object of the BBH coalescence is a *distorted black hole*.

Its GW is described by a set of damped sinusoids called '**quasi-normal modes**'(**QNMs**):

$$h(t) = \sum_i A^{(i)} \sin\left(\omega_R^{(i)} t + \Phi^{(i)} \right) e^{-t/\tau^{(i)}},$$

the ***complex*** frequency is $\omega^{(i)} = \omega_R^{(i)} + i\omega_I^{(i)}$, with $\omega^{(i)} = -1/\tau^{(i)}$.

- The QNMs frequencies depend only on the mass and angular momentum of the BH (*no-hair theorem*).

Fundamental ($i = 0$) mode of a BH of mass $M = nM_{\odot}$:

$$\nu_0 \sim \frac{12}{n} kHz, \qquad \tau_0 \sim n\, 5.5 \times 10^{-5} s.$$

- The ringdown allows us to do **spectroscopy of black holes**!

QNMs from perturbation theory

To compute the quasi-normal modes we need **perturbation theory.**

Toy model: *scalar field* on the Schwarzschild background

Consider a 'test'scalar field.
We wish to solve (to the first order) the *Klein-Gordon equation* on the Schwarzschild background:

$$\Box\psi \equiv \nabla_\mu \nabla^\mu \psi = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g}\partial^\mu \psi) = 0 \; .$$

Perform a *spherical harmonic decomposition*:

$$\psi(t, r, \theta, \varphi) = \sum_{lm} \frac{\psi_{lm}(t, r)}{r} Y^{lm}(\theta, \varphi).$$

QNMs from perturbation theory

Toy model: *scalar field* on the Schwarzschild background

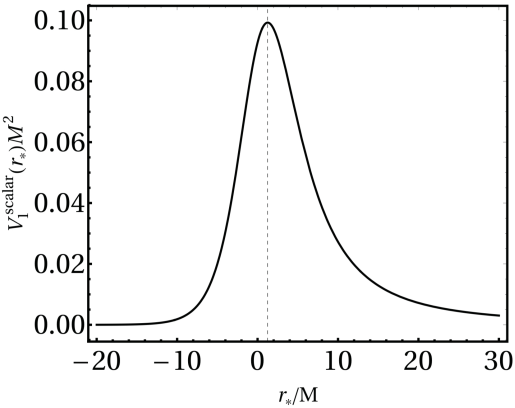
After performing a Fourier transform we obtain the equation in the **frequency domain**, called '**master equation**'

$$\frac{\partial^2 \psi_{lm}(\omega, r)}{\partial r_*^2} + [\omega^2 - V_l^{\text{scalar}}(r)]\psi_{lm}(\omega, r) = 0 \ ,$$

with ***effective potential***

$$V_l^{\text{scalar}}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right).$$

QNMs from perturbation theory



- The **master equation** is identical to the **time-independent Schrödinger equation** in one-dimension!
- Under proper boundary conditions ω represent the **QNMs frequencies.**

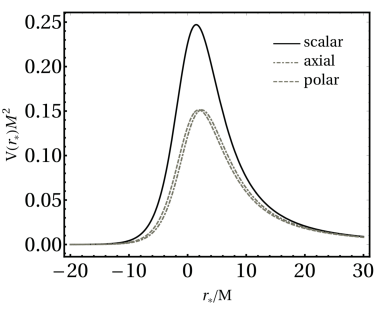
QNMs from perturbation theory

What about gravitational perturbation?

Expand the metric (gravitational field) into *background* + *small perturbation*:

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}.$$

Perform an harmonic decomposition and insert in the Einstein's equations (**up to linear order**).
We get two **master equations**



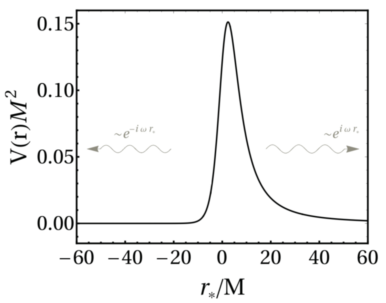
$$\frac{d^2 Q_{lm}}{dr_*^2} + (\omega^2 - V_l^{\text{axial}})Q_{lm} = 0\ ,$$ **Regge-Wheeler (axial)**

$$\frac{d^2 Z_{lm}}{dr_*^2} + (\omega^2 - V_l^{\text{polar}})Z_{lm} = 0\ ,$$ **Zerilli (polar)**

How to extract the QNMs?

- We need to solve an **eigenvalue problem**.
- What are the proper boundary conditions?

1. no wave coming from infinity (**pure outgoing wave**),
2. nothing can escape from BH horizon (**pure ingoing wave**).

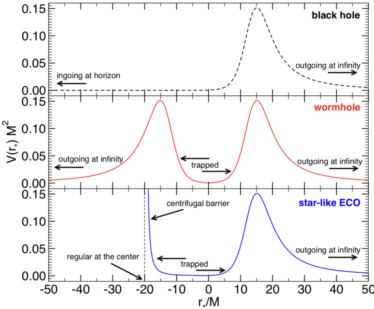


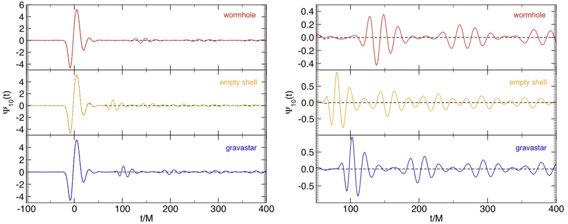
- discrete set of QNMs frequencies:
  $\omega^{(n)} = \omega_R^{(n)} + i\omega_I^{(n)} \qquad n = 1, 2, 3, \ldots$
- **isospectrality**: polar and axial QNMs have the same frequency (in general relativity!).


TO GENERAL RELATIVITY...
... AND BEYOND!
calm dow
calm dow

QNMs beyond general relativity

- In GR the QNMs spectrum only depend on the ***mass*** and ***angular momentum*** of the BH $\rightarrow$ detect few modes from the ringdown signal and infer these parameters.
- **Echoes** from **exotic compact objects** (e.g. fuzzballs, gravastars etc.) $\rightarrow$ able to trap some modes that leak out at late time.





Cardoso, Hopper, Macedo, Palenzuela, Pani (PRD, 2016)

Einstein-Maxwell-scalar theory: a look into my PhD project

Some modifications to general relativity are subject to '**spontaneous scalarization**':

$$S = \int d^4x \sqrt{-g}[R + \text{extra terms}(f(\Phi))],$$

solutions of this modified gravity:

- **GR solutions** (Schwarzschild, RN, Kerr),
- **scalarized black holes** (black holes with scalar field).

Why the black hole acquires a scalar field?

To reach ***stability***!

Einstein-Maxwell-scalar theory

Einstein-Maxwell-scalar theory is a model which provides spontaneous scalarization:

$$S_{\text{EMS}} = \int d^4x\sqrt{-g}[R + (\nabla\Phi)^2 + f(\Phi)F_{\mu\nu}F^{\mu\nu}],$$

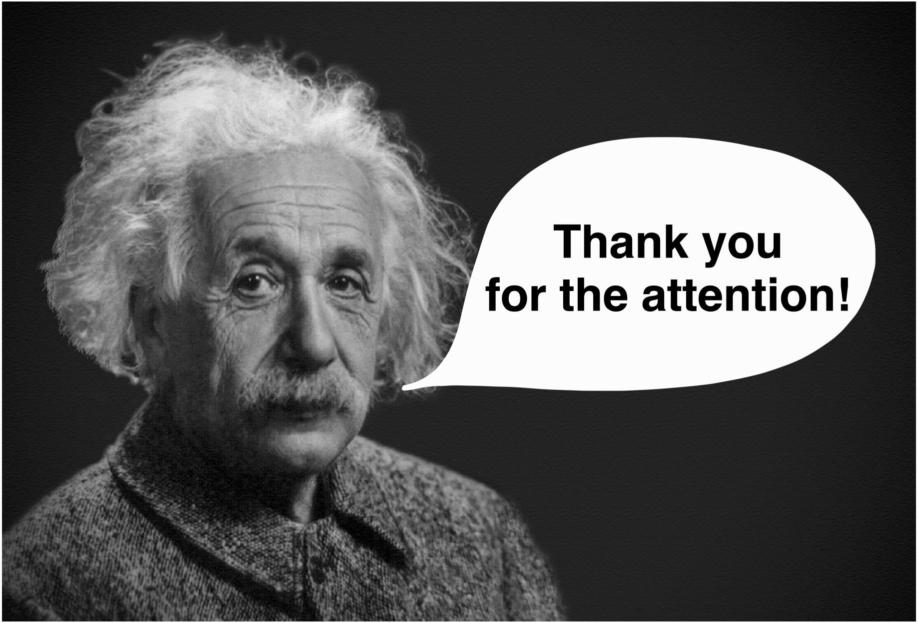
solutions of EMS:

- **RN black hole** (electrically charged BH),
- **scalarized charged black holes** (BH with an electric charge and scalar field).

1. Compute the QNMs spectrum of charged scalarized black holes.
2. These black holes might be prone to trap modes that leak out at late time!
3. QNMs of solutions in other modified gravity theories (5-dim EMS theory,
   Heidmann, Speeney, Berti, Bah PRD 2023).

Conclusions and key takeaway

- **Ringdown**: final part of the GW signal from a binary black hole merger.
- It is *almost* fully described by the "**quasi-normal modes**" (computed in perturbation theory).
- In general relativity the QNMs (axial and polar) only depend on the BH parameters (**mass** and **angular momentum**).
- Some *exotic compact objects* trap the modes in a *gravitational cavity* and release them at late time as "**echoes**".
- In modified gravity or beyond GR the QNMs can be a signature of modifications! ("echoes might emerge: EMS in 4 or more dimensions").
- Is the linear order enough?


Thank you
for the attention!