Measurement-induced phase transitions in quantum circuits

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Genova
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This talk

LP, Y. Li, R. Vasseur, A. Nahum, PRB **107**, 224303 (2023)

M. Fava, LP, T. Swann, D. Bernard, A. Nahum, arXiv:2302.12820 [Accepted PRX]

Main interests

Many-body physics out of equilibrium

Quantum information & quantum simulation

Tensor networks

Quantum circuits & cellular automata

Low-dimensional quantum field theory (Integrability, CFT)

Cold atoms

Quantum chaos & random-Matrix theory

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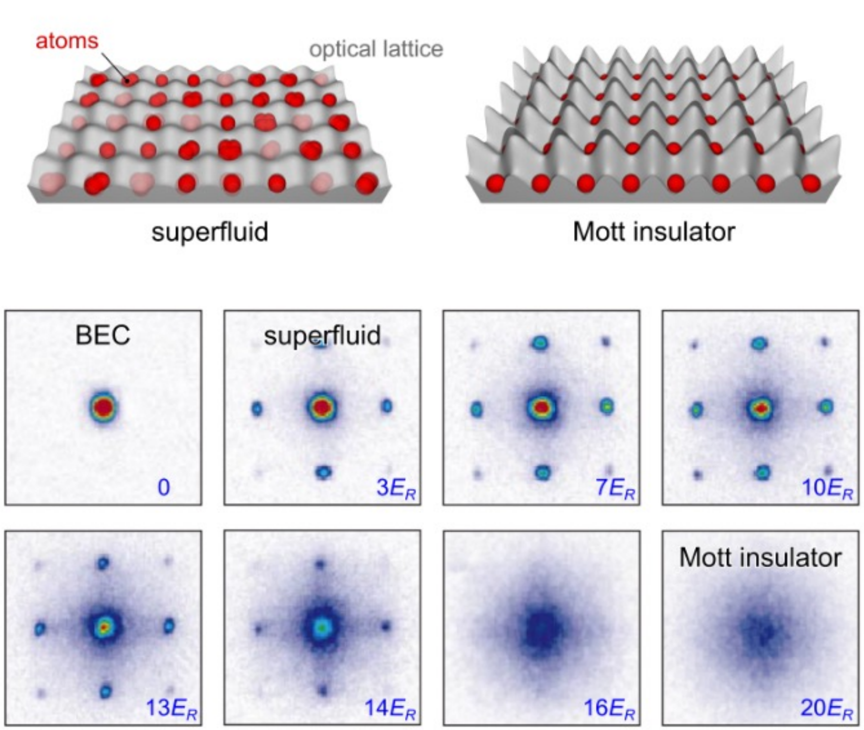
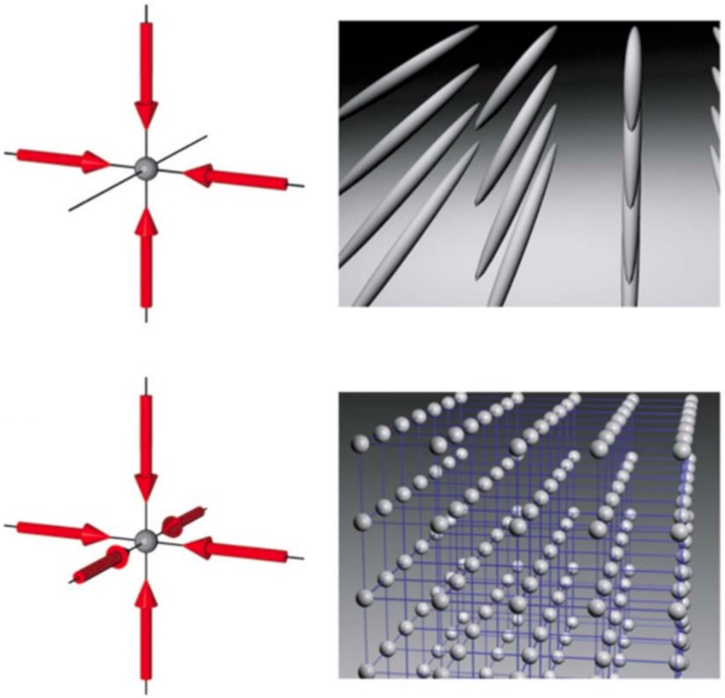
- Renewed interest from physics of ultra-cold atoms:
  - large-number of atoms confined
    in arbitrary geometries and effective dimensions
  - ideal isolation and control of the system parameters
  - real-time dynamics accessible to high precision





Bloch, et al. Rev. Mod. Phys. (2008)

- Non-perturbative problems & hard to study numerically

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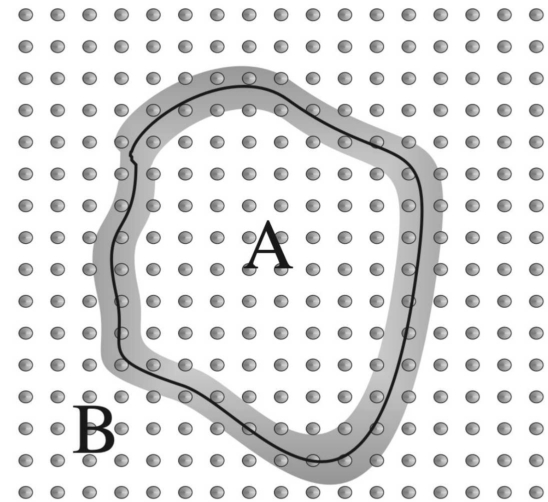
- Entanglement emerged as organizing principle



$$|\psi\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

- Define reduced state $\rho_A = \mathrm{tr}_B\left[|\psi\rangle\langle\psi|\right]$
- Entanglement entropies

$$S_A^{(n)}[\rho] = \frac{1}{1-n}\log[\mathrm{tr}(\rho^n)]$$

$$S_A[\rho] = -\mathrm{tr}\left(\rho\log[\rho]\right)$$

- They quantify quantum correlations, and answer the question:

"how far is $|\psi\rangle$ from a (classical) product state"?

$$|\psi_0\rangle = |\phi_0\rangle \otimes \cdots \otimes |\phi_0\rangle$$

- Entanglement key ingredient for exotic quantum effects
- It makes quantum dynamics hard to simulate classically

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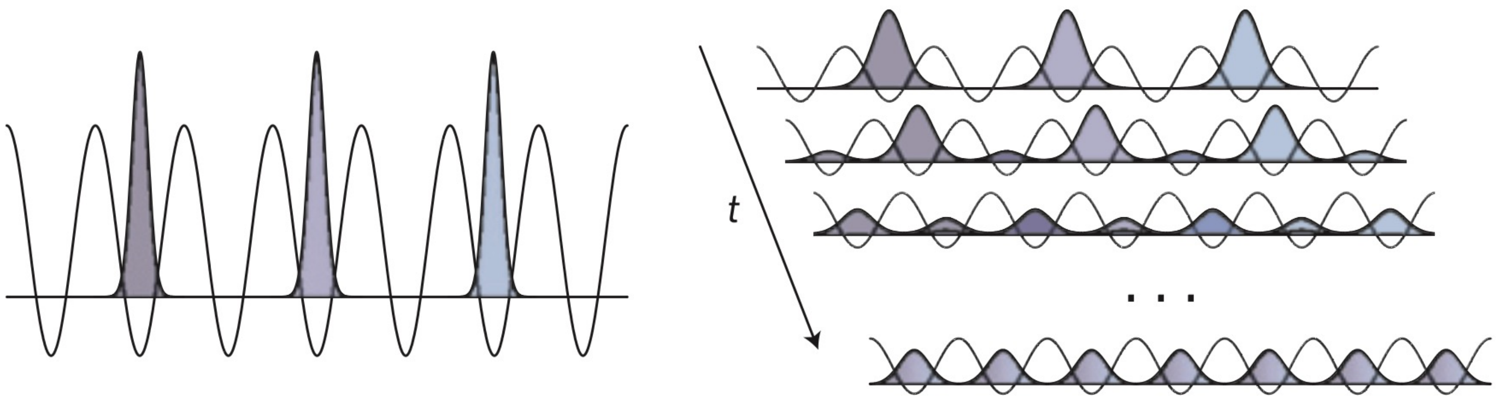
- Ex: “quantum quench”

$$\hat{H} = \sum_j \left[-J \left(\hat{a}_j^\dagger \hat{a}_{j+1} + \text{ h.c. } \right) + \frac{U}{2} \hat{n}_j \left(\hat{n}_j - 1 \right) + \frac{K}{2} \hat{n}_j j^2 \right]$$



(1) Preparation

(2) Evolution

$$|\psi(t=0)\rangle = |\ldots 10101 \ldots\rangle$$

Trotzky et al. Nature Physics (2012)

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- It makes quantum dynamics hard to simulate classically

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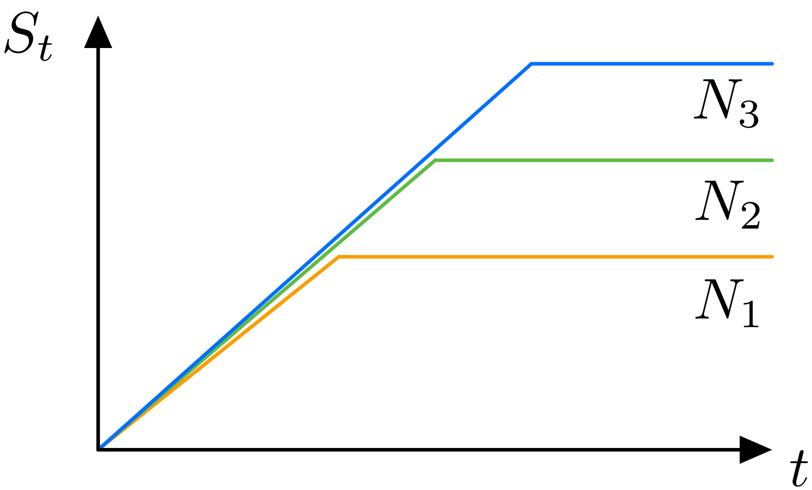
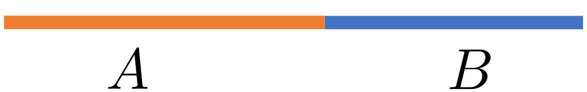


$$|\Psi_0\rangle \xrightarrow{e^{-iHt}} |\Psi_t\rangle$$

$$S(|\Psi\rangle) = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$

$$\rho_A = \mathrm{Tr}_B\left[|\Psi\rangle\langle\Psi|\right]$$

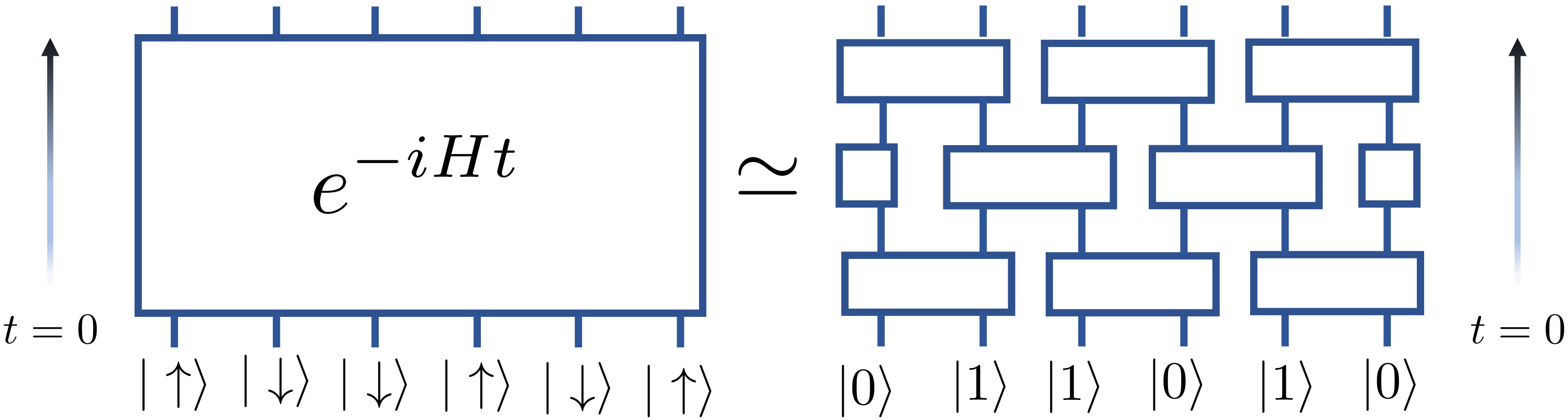


- Large-scale dynamics simulable for low entanglement via tensor-network methods

- Bottleneck: “Entanglement barrier”

Calabrese & Cardy, PRL 96 (2006)

- What if we had a quantum computer?
- Direct access to quantum dynamics via Trotter-Suzuki decomposition



Lloyd (1996)

$$H = \sum_j h_{j,j+1} \quad \Rightarrow \quad e^{-iHt} \simeq \left[e^{-iH_e\delta} e^{-iH_o\delta}\right]^{t/\delta}$$

- Implementation of Digital Quantum Simulation

Feynman, 1982

- Remarkable recent progress in platforms for quantum simulation: cold & Rydberg atoms, trapped ions, superconducting circuits

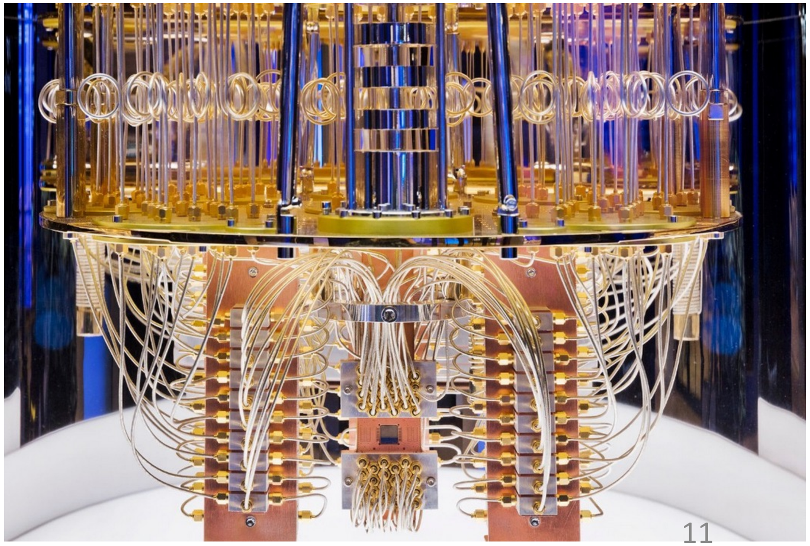
- Most advanced experiments in analog quantum simulation

J. Daley *et al.*, Nature (2022)

- Digital quantum simulation $\Longrightarrow$ greater control and versatility, but experimental research is at an early stage

- Noisy-Intermediate-Scale-Quantum (NISQ) era

J. Preskill, Quantum 2 (2018)



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1. Discrete dynamics with new features compared to Hamiltonian evolution:
   What new phases and universal behavior in NISQ platforms?

2. New paradigm for many-body physics out of equilibrium

- Minimally-structured models to study hard questions in many-body physics
- Implementation feasible in NISQ devices

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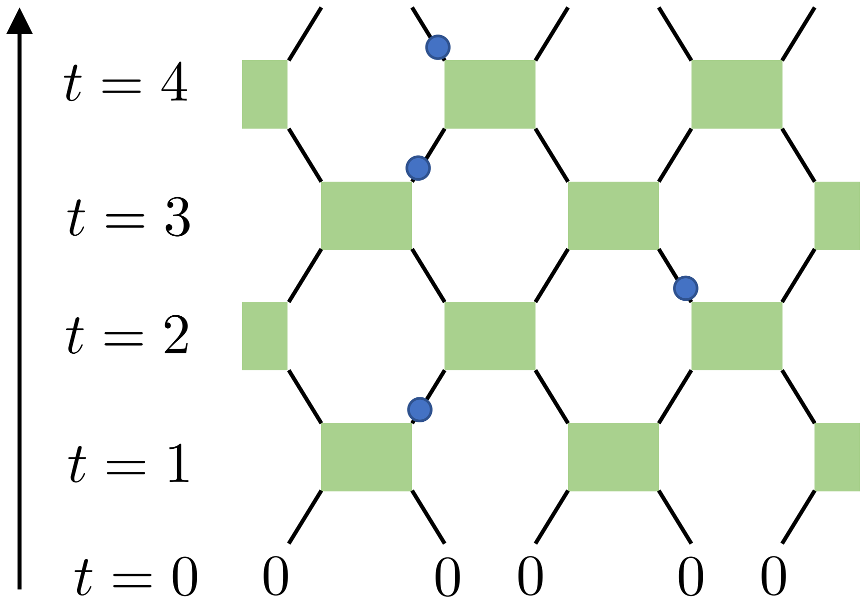
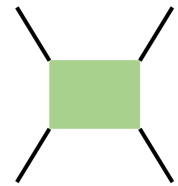
$\Longrightarrow$ This talk: focus on class of stat.-mech. problems arising in this context

Measurement-induced phase transitions





: unitary gate

$U_{j,j+1} = e^{-ih_{j,j+1}\tau}$

: projective measurement

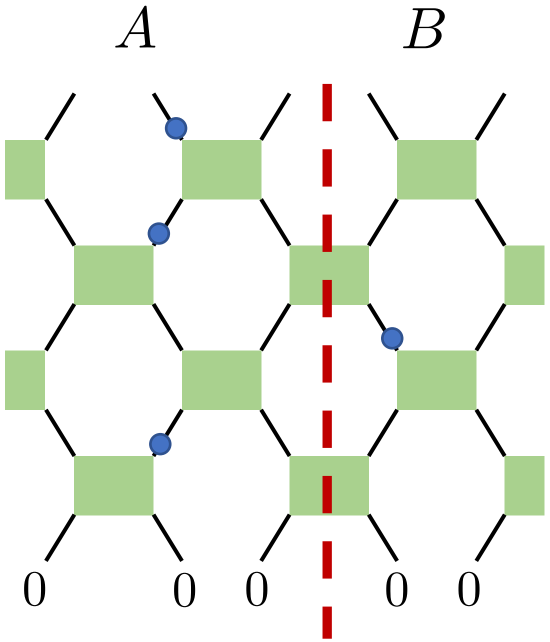
$P_\alpha = |\alpha\rangle\langle\alpha|$

- $\mathcal{H} = \mathbb{C}^q \otimes \cdots \otimes \mathbb{C}^q$

  $\mathbb{C}^{q+1} = \mathrm{span}(\{|0\rangle, |1\rangle, \ldots |q\rangle\})$

- Born's rule $\mathrm{prob}(\alpha) = \langle\psi(t)|(|\alpha\rangle\langle\alpha|_j)|\psi(t)\rangle$

Each qubit measured randomly, with a rate (probability) p

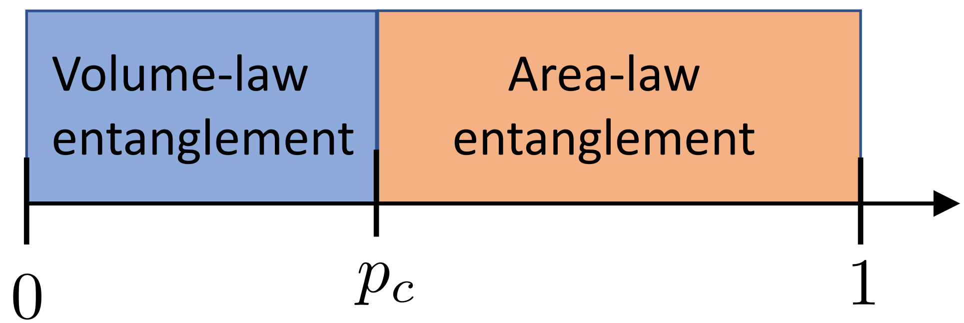


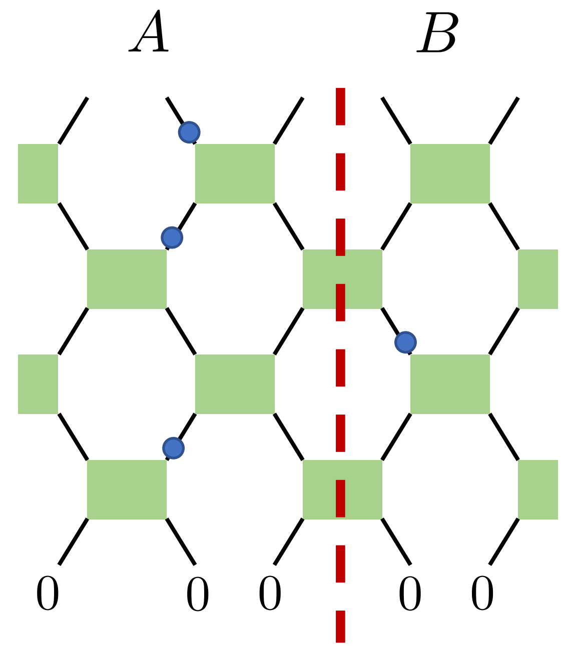
Skinner, Ruhman, Nahum, PRX (2019)
Li, Chen, Fisher, PRB 98, (2018)

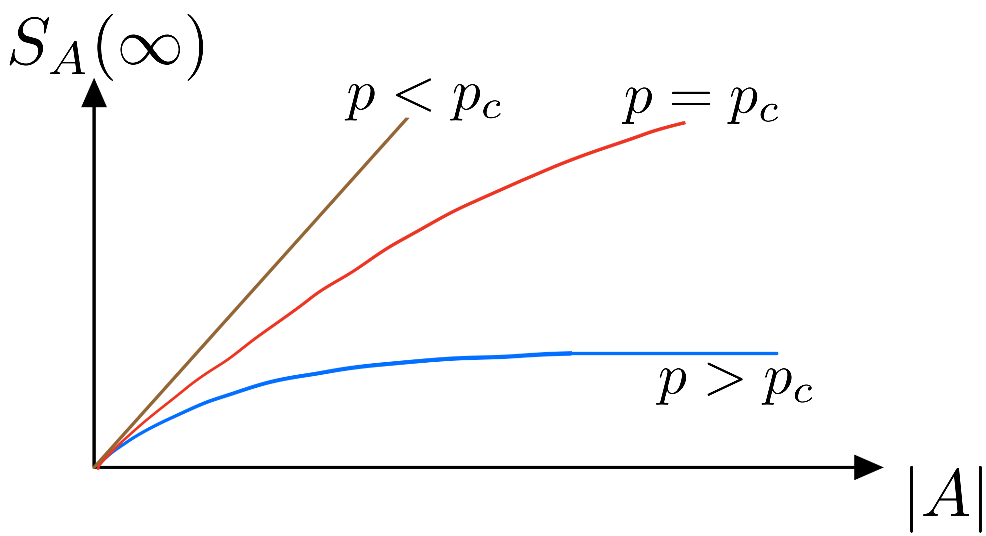
Each qubit measured randomly, with a rate (probability) p

Unitary limit

Zeno limit







$$S(t) = -\mathrm{Tr}\left\{\rho_A(t)\log\left[\rho_A(t)\right]\right\}$$

$$\rho_A(t) = \mathrm{Tr}_B\left[|\Psi(t)\rangle\langle\Psi(t)|\right]$$

Skinner, Ruhman, Nahum, PRX (2019)
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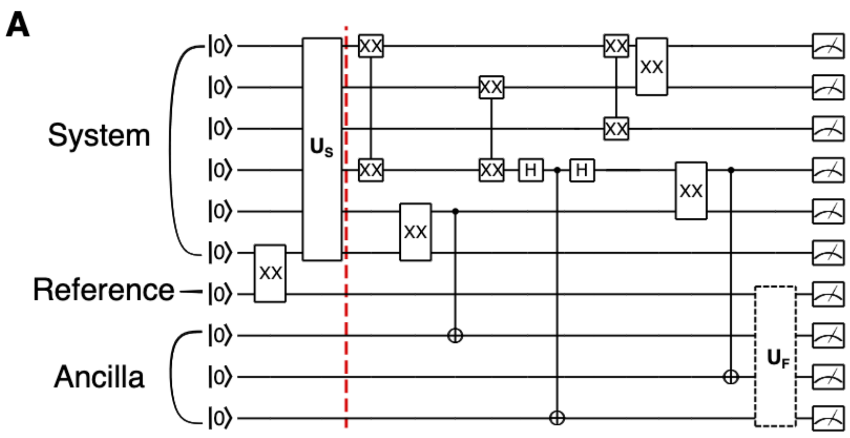
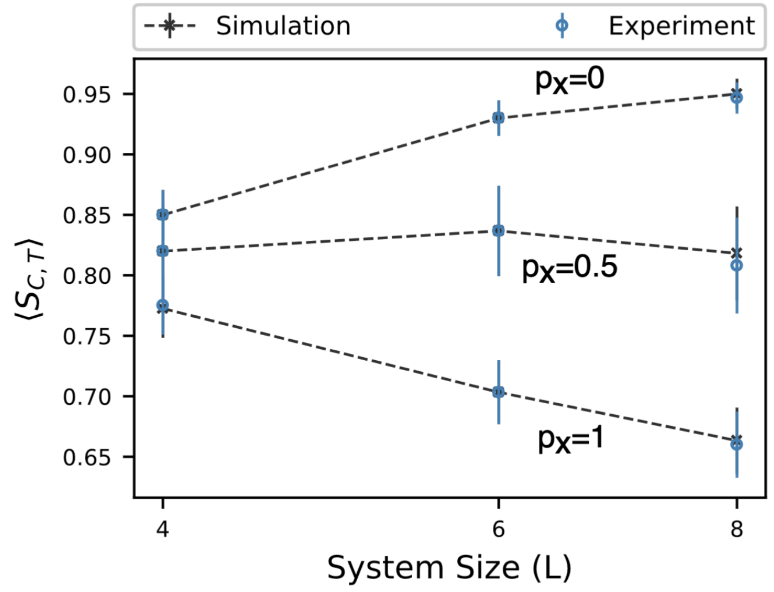
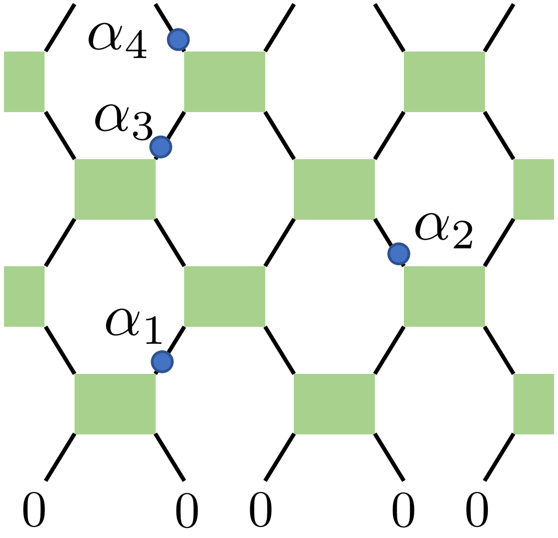
Experimentally probed in point-of-principle experiments





C. Noel et al., Nature Phys. 18, 760 (2022)
J. Ming Koh et *al.* arXiv (2022)



- Measurement histories define quantum trajectories

$$\mathcal{E} = \{|\psi(t, \{\alpha_j\}_j)\rangle, \mathrm{prob}(\{\alpha_j\}_j)\}$$

- Dynamics of averaged density matrix is dephasing

$$\rho \mapsto \mathcal{E}_D^{(j)}[\rho] = \sum_\alpha {}_j\langle\alpha|\rho|\alpha\rangle_j \otimes |\alpha\rangle\langle\alpha|_j$$

- Drives locally system towards infinite-temperature
$\Longrightarrow$ no competition with unitary dynamics

$\Longrightarrow$ transition not detected by local observables

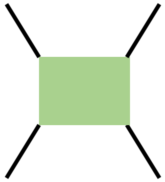
- Example of new universal behavior beyond conventional dynamical phase transitions

- Recent work aimed at characterizing phenomenology of the transition (critical exponents, order parameters, etc.)

- This talk:
  - analytic predictions for simplest type of transition
    [Fava, LP, Swann, Bernard, Nahum, PRX (2023)]

  - enriched phenomenology via adaptive measurements
    [LP, Li, Vasseur, Nahum, PRB (2023)]

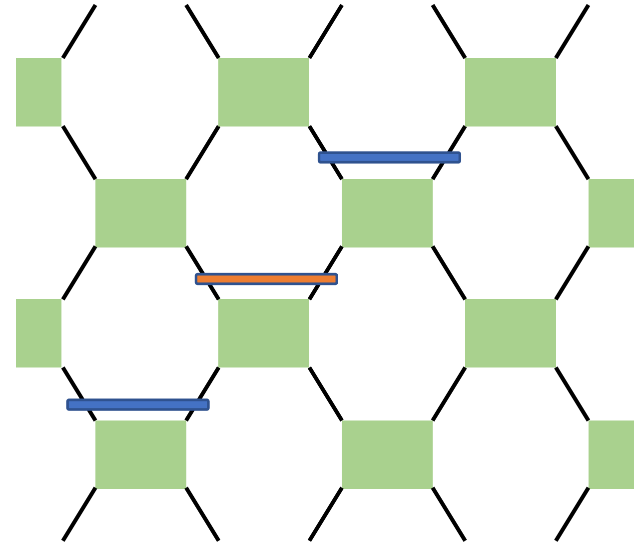
- Non-interacting dynamics



$$\propto \exp\left(\sum_{ij} J_{ij}(t)\hat{\gamma}_i\hat{\gamma}_j\right)$$

$$\{\hat{\gamma}_i, \hat{\gamma}_j\} = \delta_{ij}$$

$$\mathbb{E}\left[J_{i,j}(t)J_{k,l}(t')\right] = J^2\delta(t-t')\delta_{ik}\delta_{jl}$$



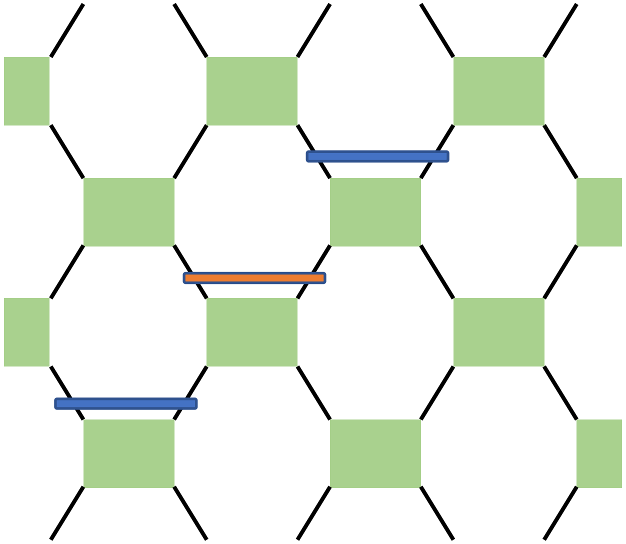
- Measure fermionic local parity $\hat{\gamma}_{2j}\hat{\gamma}_{2j+1}$, $\hat{\gamma}_{2j+1}\hat{\gamma}_{2j+2}$ with rates $\Gamma_1, \Gamma_2 = \Gamma(1 \pm \Delta)$

- Simple model, but monitored dynamics non-trivial

- Numerical evidence suggests entanglement transition from area-law phase to non-trivial phase

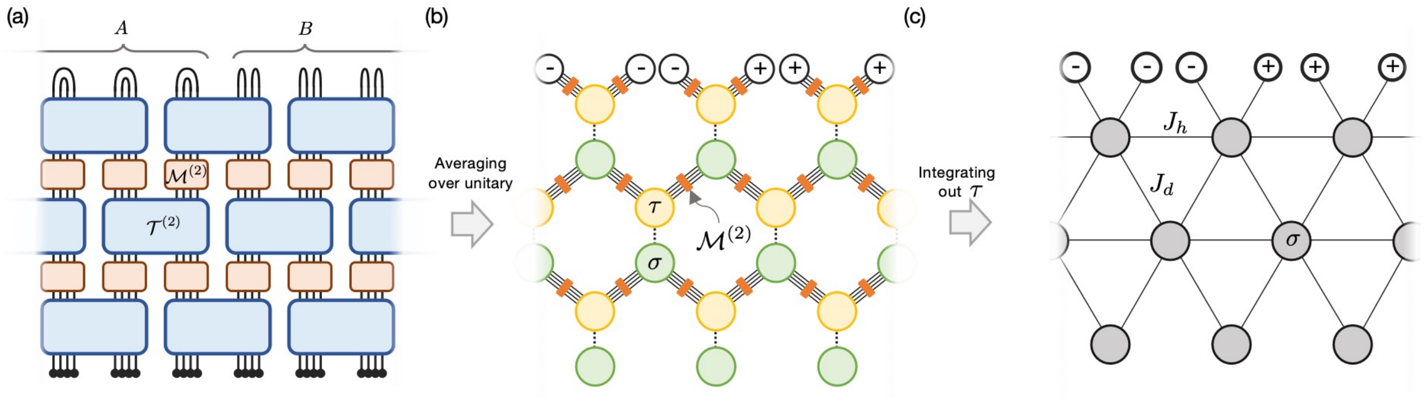
Cao, et al., SciPost Phys. (2019)
Alberton et al. PRL (2021)

- We provide analytic predictions based on mapping to stat. mech. problem [Bao, et al. Ann. Phys. (2021); Jian, et al. arXiv:2302.09094]



- Basic idea:
  - map space-time pattern to 2D static disordered fermions
  - the problem can be tackled using replica trick developed for Anderson localization problem
  - for large space-time scales, develop a field theoretical description in terms of non-linear sigma model

- We provide analytic predictions based on mapping to stat. mech. problem [Bao, et al. Ann. Phys. (2021); Jian, et al. arXiv:2302.09094]



- Replica limit arising due to

$$S_{n,A}[|\psi\rangle] = \frac{1}{1-n} \log \mathrm{Tr}_A \left(\rho_A^n\right)$$

$$\log x = \lim_{m\to 0} \left(x^m - 1\right)/m$$

- Bipartite entanglement entropy mapped to partition function for a matrix field $Q \in SO(N)$

$$\mathcal{S}[Q] = \frac{1}{2g_B} \int \mathrm{d}x \ \mathrm{d}t \, \mathrm{Tr} \left[\frac{1}{v} \partial_t Q^T \partial_t Q + v \partial_x Q^T \partial_x Q \right]$$

N: number of replicas.

- Same description as Anderson transition

- Physical predictions obtained in replica limit $N \to 1$ because of Born probabilities

- We obtain asymptotic exact prediction based on RG analysis

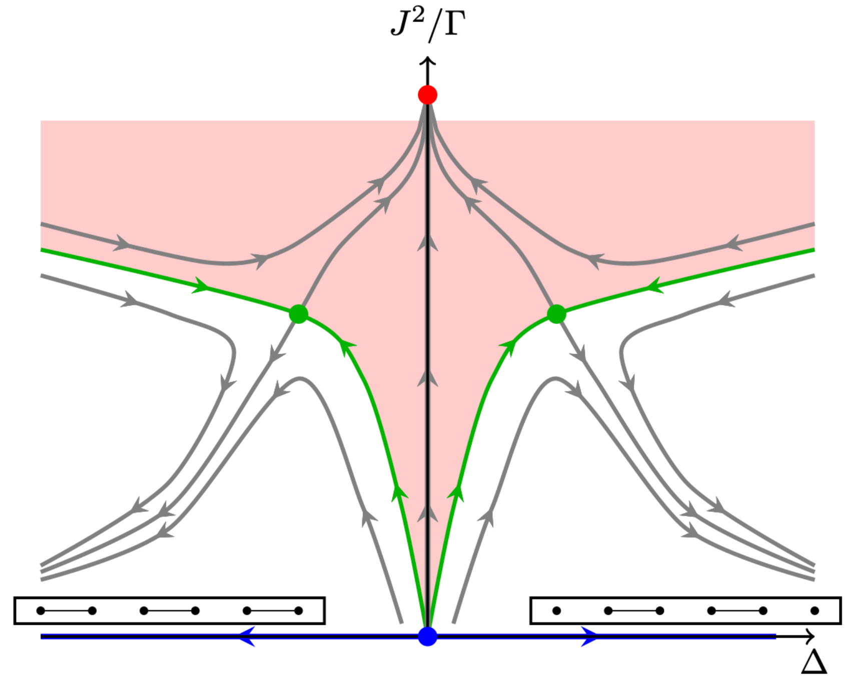
$$\frac{\mathrm{d}g}{\mathrm{d}\ln L} = \frac{1}{8\pi}(N-2)g^2 + O\left(g^3\right)$$

- Non-trivial phase:

$$S_L(\infty) \sim \frac{1}{48}(\ln L)^2$$

- Trivial phase:
  entanglement area law

$$S_L(\infty) \sim O(1)$$



$$\Gamma_1, \Gamma_2 = \Gamma(1 \pm \Delta)$$

$$\mathbb{E}\left[J_{i,j}(t) J_{k,l}(t')\right] = J^2 \delta(t - t') \delta_{ik} \delta_{jl}$$

- Transition out of the non-trivial phase driven by proliferation of vortices, but different from Kosterlitz–Thouless

$\Longrightarrow$ new universality class!

- Simplest example of universal behavior made possible by quantum-circuit setting

- Wide phenomenology, depending on unitary operations and measurement process

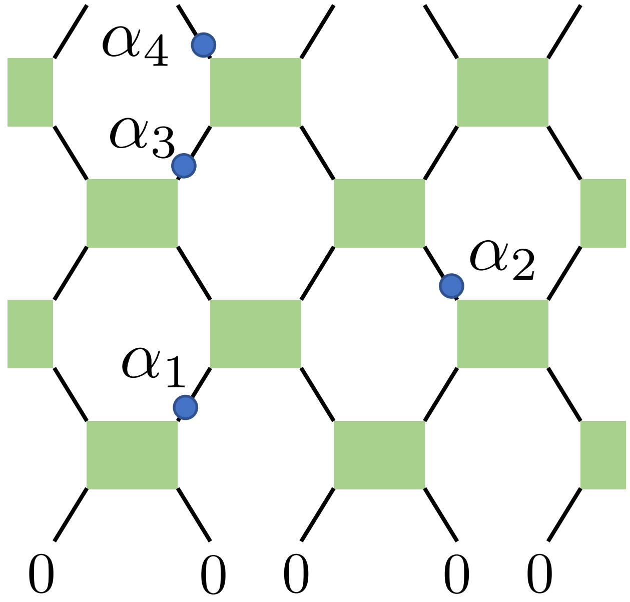
- Our work: generalize measurements via feedback operations, depending on measurement outcome

- Simplest example: local resetting measurements

  - measure qubit k
  - if outcome $\alpha = 0$, do nothing
  - if outcome $\alpha \neq 0$, flip the spin

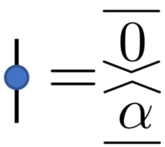
LP, Y. Li, R. Vasseur, A. Nahum PRB 2023

- Resetting described by Kraus operators $M_\alpha = |0\rangle\langle\alpha|$



$$\Rightarrow |\psi\rangle \mapsto \frac{M_\alpha|\psi\rangle}{\sqrt{\text{prob}(\alpha)}}$$

$$\text{prob}(\alpha) = \langle\psi|M_\alpha^\dagger M_\alpha|\psi\rangle$$



- Choose block-diagonal gates

$$U_{j,j+1} = \begin{pmatrix} 1 & 0 \\ 0 & W \end{pmatrix} \Rightarrow U_{j,j+1}|00\rangle_{j,j+1} = |00\rangle_{j,j+1}$$

- The dynamics features absorbing-state

$$|\mathbf{0}\rangle = |0\rangle \otimes |0\rangle \otimes \cdots \otimes |0\rangle$$

fixed by gates and measurements

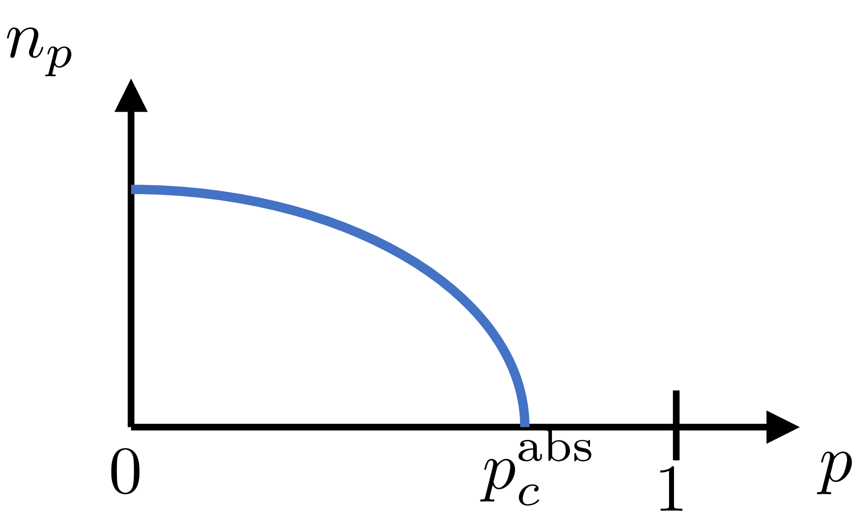
- We expect absorbing-state transition detected by order-parameter

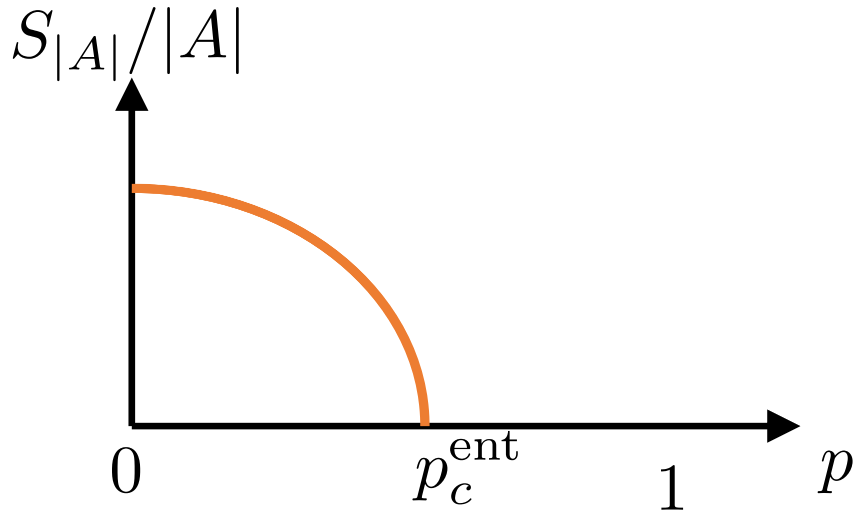
$$n(t) = \frac{1}{L} \sum_{j=1}^{L} \mathrm{Tr}\left\{ \mathbb{E}[\rho(t)] \mathcal{P}_j \right\}$$

$$\mathcal{P}_j = 1 - |0\rangle\langle 0|_j$$

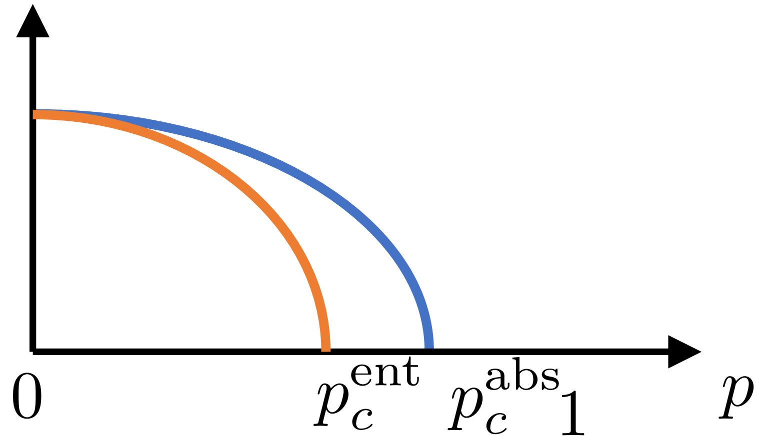
$p < p_c$: active phase $n_p \equiv \lim_{t\to\infty} \lim_{L\to\infty} n(t) > 0$

$p > p_c$: inactive phase $n_p = 0$





- Clearly $p_c^{\mathrm{ent}} \leq p_c^{\mathrm{abs}}$; can the transitions coincide?
- Does feedback modify the universality class of entanglement MIPT?

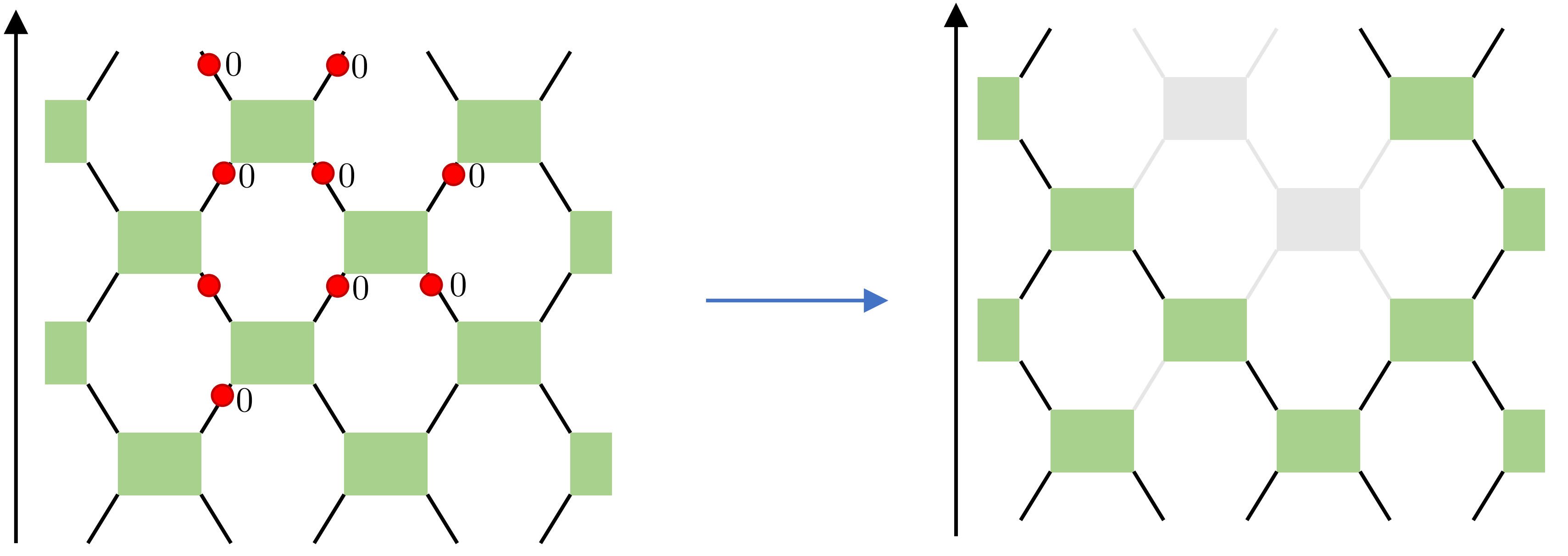


- Clearly $p_c^{\mathrm{ent}} \leq p_c^{\mathrm{abs}}$; can the transitions coincide?

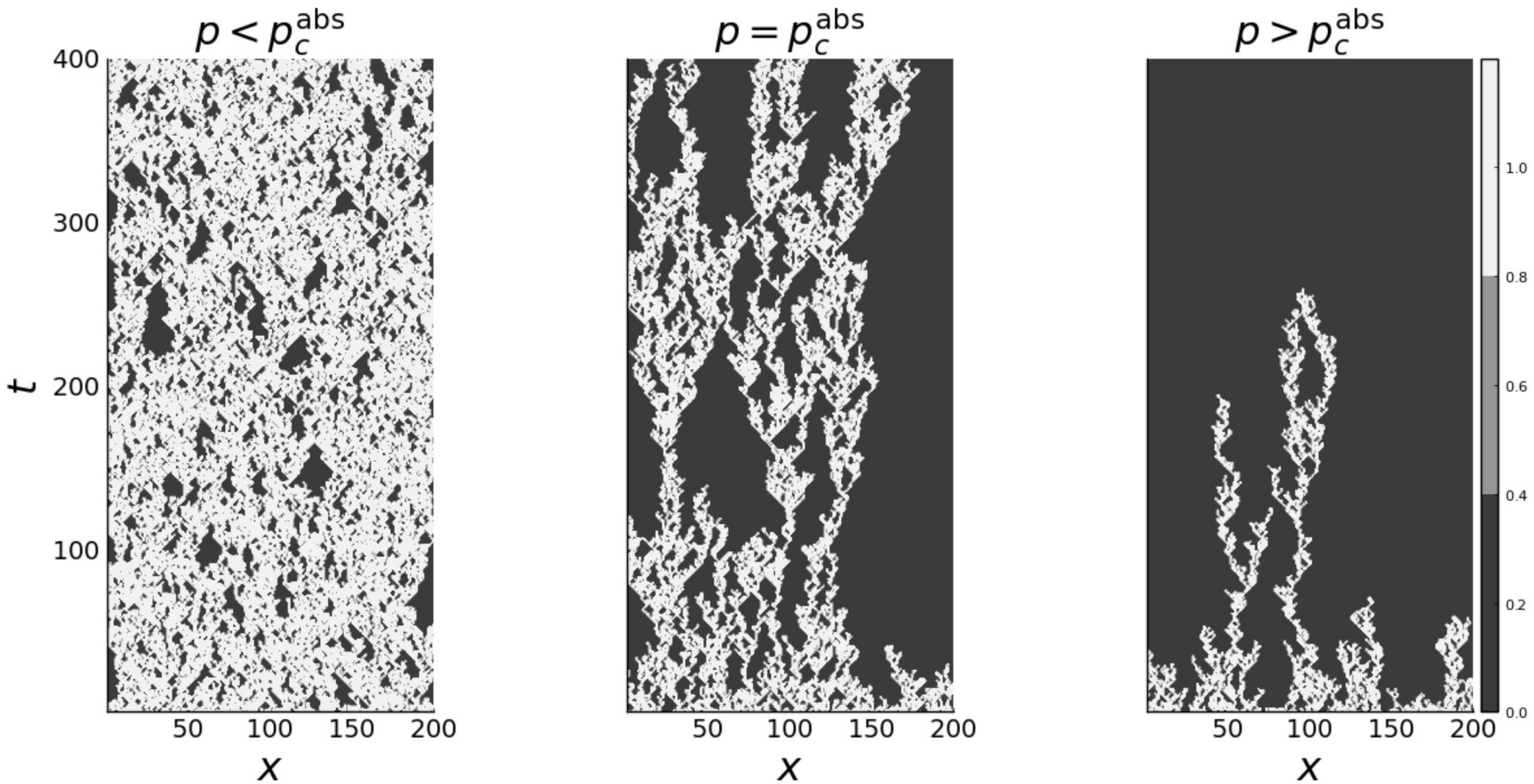
- Does feedback modify the universality class of entanglement MIPT?

- Our work:
  - transitions generically distinct and unrelated
  - they only coincide in ``semiclassical limit’’

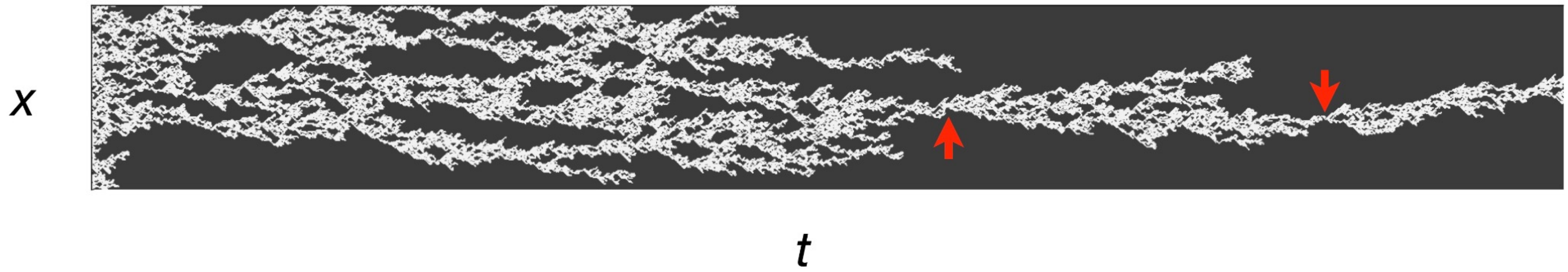
- Basic idea: resetting measurements act as effective cuts in 2D space-time network



- Absorbing-state transition coincides with connectivity transition



- Entanglement ~ number of links connecting the cluster
- Can be studied exploiting standard results in stat. mech.



- Define red bonds as link that disconnect a percolating cluster

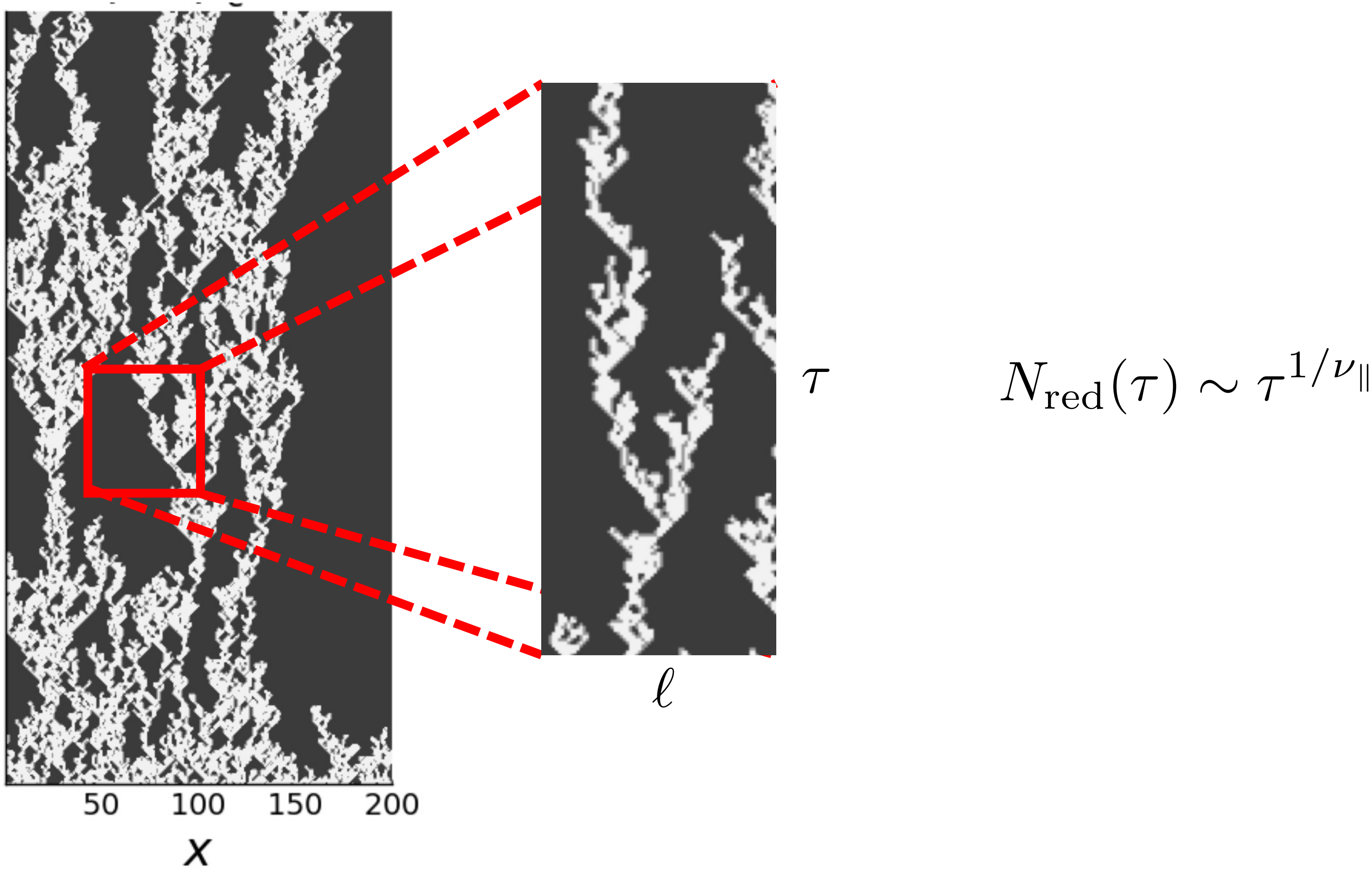
- At criticality

$$N_{\mathrm{red}}(\tau) \sim \tau^{1/\nu_{\parallel}}$$

Huber, Jensen, Sneppen, PRE (1995)

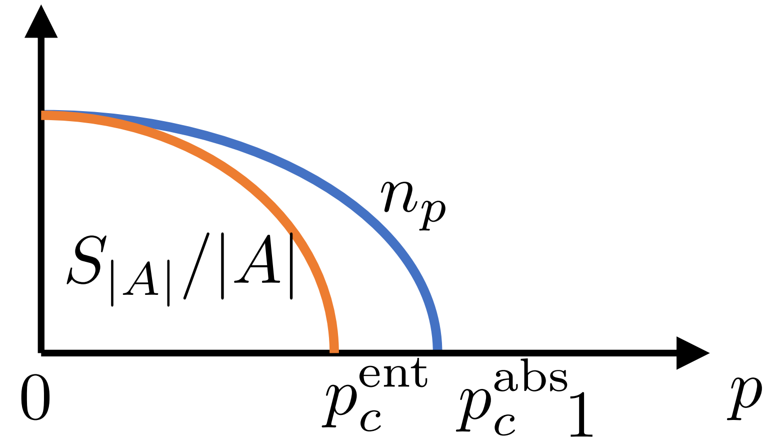
$$\xi_{\perp} \sim |p - p_c^{\mathrm{abs}}|^{-\nu_{\perp}} \quad \nu_{\perp} \simeq 1.097$$

$$\xi_{\parallel} \sim |p - p_c^{\mathrm{abs}}|^{-\nu_{\parallel}} \quad \nu_{\parallel} \simeq 1.73$$



- Near criticality, we can apply this result for a space-time region of dimensions

$$\ell = \xi_\perp \sim |p - p_c^{\mathrm{abs}}|^{-\nu_\perp} \qquad \tau = \xi_\parallel \sim |p - p_c^{\mathrm{abs}}|^{-\nu_\parallel}$$



- Using simple estimates: if

$$|p - p_c^{\rm abs}| \ll 1/q\,,$$

the system is in the area-law phase

⇒ close to DP transition, quantum trajectories are disentangled

- We recover that two transitions only coincide in the limit $q \to \infty$

- Results supported by extensive numerics in simple models

Outlook

- Entanglement phase transitions as new instances of universality out of equilibrium
- Many qualitative and quantitative questions remain open:
  - Characterization of the transition in interacting models
  - Can we efficiently probe the transition?
- More generally, many fundamentally interesting questions are emerging from ``digital quantum physics''
- Important new synergies between groups in quantum-information theory, stat.-mech. and many-body physics

My project QUANTHEM awarded an ERC starting grant!

European Research Council

Established by the European Commission

Research areas:

- *Quantum information theory*
- *Quantum simulation*
- *Many-body physics out of equilibrium*

New postdoc positions at the University of Bologna starting 2024!

QUANTHEM hosted by and interacting with Quantum group in Bologna: E. Ercolessi, P. Pieri, T. Calarco

Thank you for your attention!