3D IMAGING in GRAIN via Multiple View Geometry: Track Reconstruction

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Italia

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GENERAL PLAN

MULTIPLE VIEW PROJECTIVE GEOMETRY applied to

Previously

LIGHT POINTS RECONSTRUCTION ALGORITHM

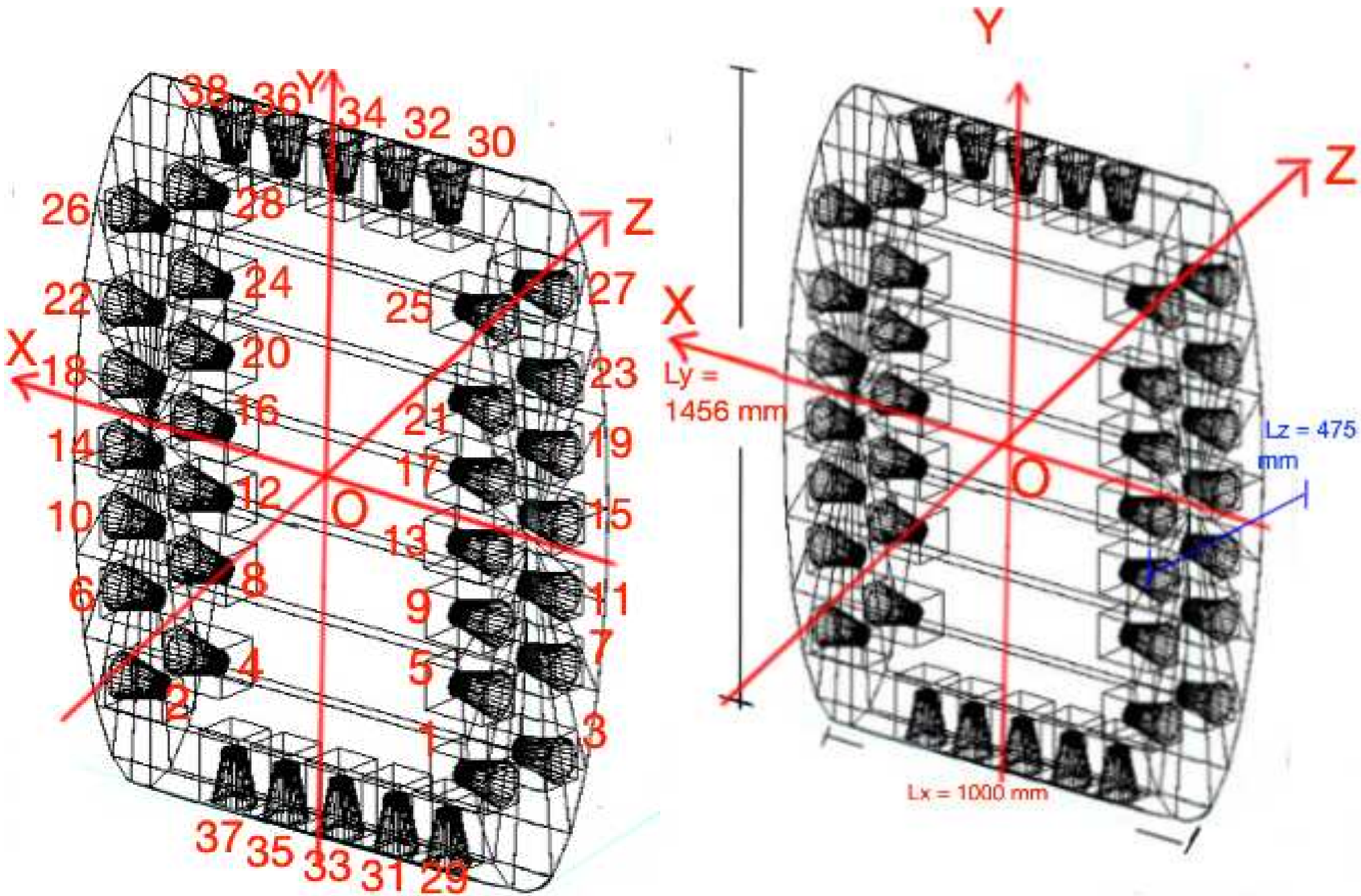
Now

TRACK RECONSTRUCTION ALGORITHM

Then

MULTIPLE EVENTS ALGORITHM

GRAIN as a multiple lens system

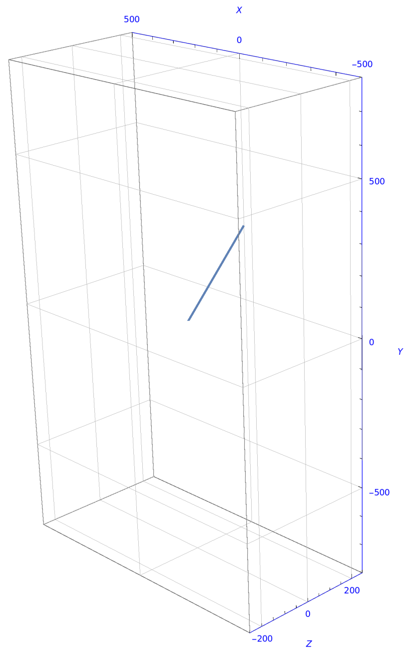


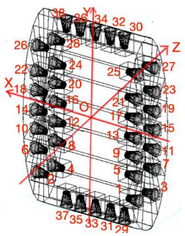
Event 1: Single Track in Grain

Simulated Track: a muon from the center of GRAIN

We consider a semiline starting from the **origin** of Grain and directed as

$$\theta_X = 90^\circ, \qquad \theta_Y = 45^\circ, \qquad \theta_Z = 45^\circ$$

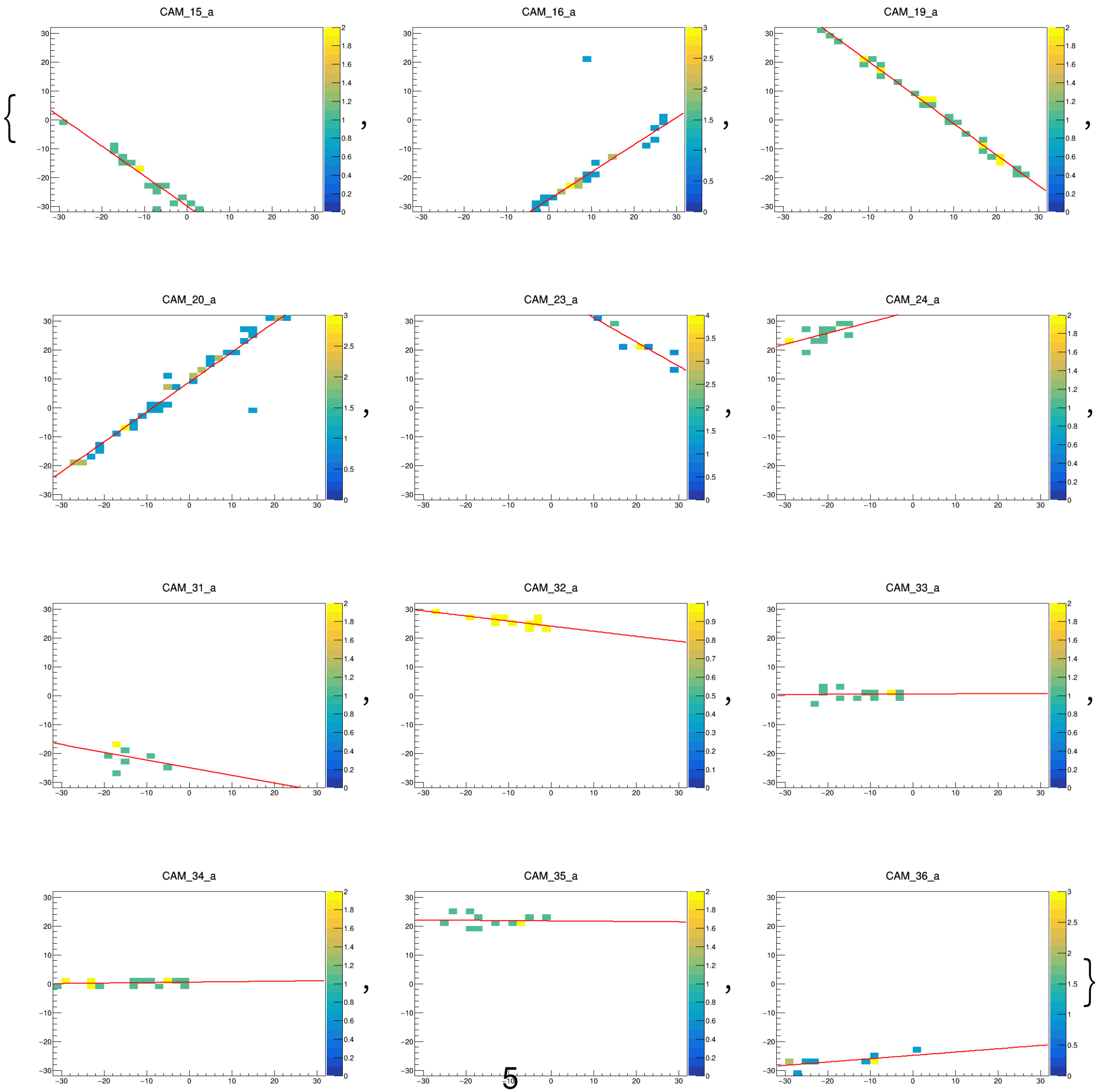




Cameras involved: 15,16,19,20,23,24, 31-36

2D Reconstructions

Out[ ]=



Event 2: two Tracks and Vertex

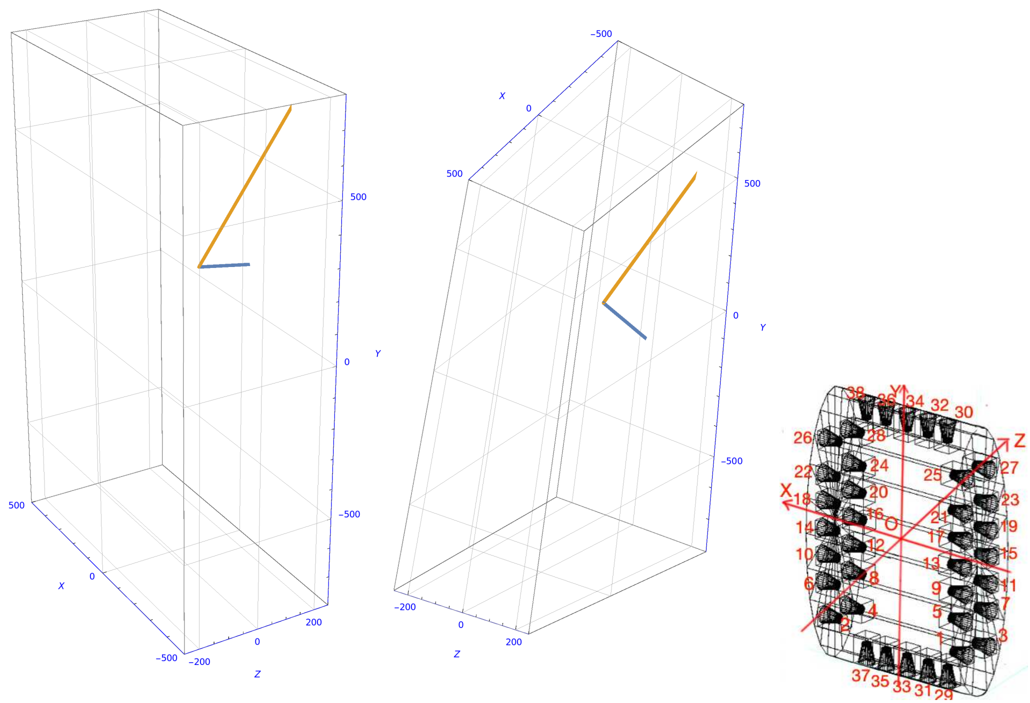
Simulation Details

track 0, $P_x, P_y, P_z \rightarrow (0.338, -0.512, 2.241)$ muon

track 1, $P_x, P_y, P_z \rightarrow (-0.042, 0.120, 0.046)$ proton

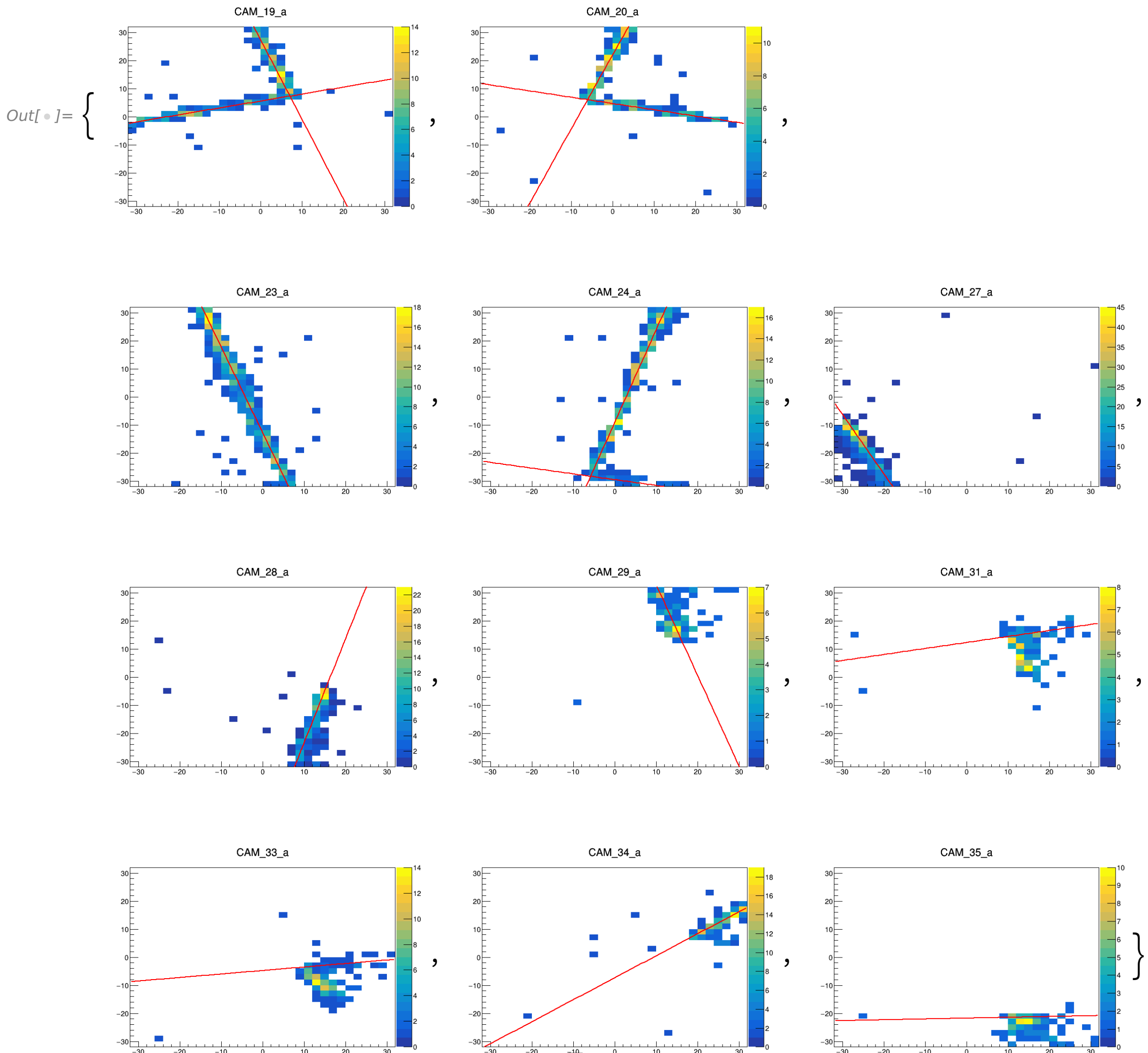
Vertex (-30, 170, 79) (units: mm)

3D Event Representation



Camera Images from MC Simulations: 2D reconstruction

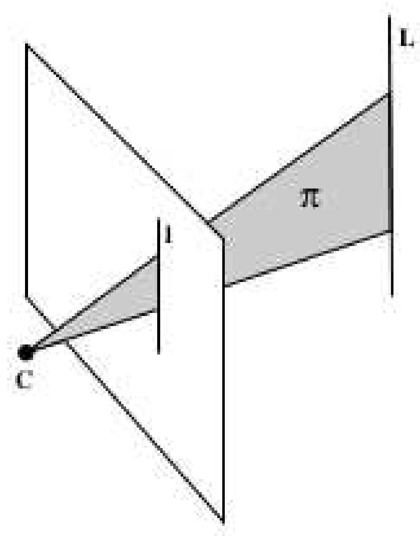
Cameras involved: 19, 20, 23, 24, 27, 28, 29, 31, 33, 34, 35



FROM POINTS TO TRACKS

Tracks reconstruction: theoretical preliminaries

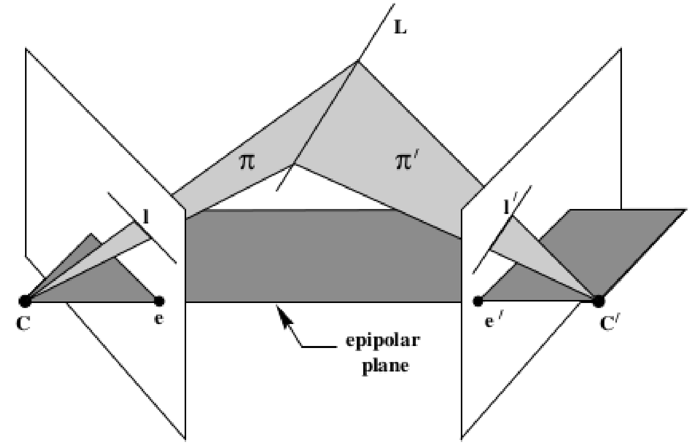
Back-projection of lines



$$\pi = \mathbf{P}^T \mathbf{l}$$

π vector of plane parameters in 3D space, $\mathbf{P}$ camera matrix, $\mathbf{l}$ vector of line parameters on the sensor, $\mathbf{L}$ infinite line in 3D space to be reconstructed

Line Reconstruction



$$\mathbf{L} = \begin{pmatrix} \mathbf{l}^T\mathbf{P} \\ \mathbf{l}'^T\mathbf{P}' \end{pmatrix}$$

$\mathbf{l}^T\mathbf{P}$ vector of plane π parameters in 3D space, $\mathbf{l}'^T\mathbf{P}'$ vector of plane π' parameters in 3D space, $\mathbf{P}, \mathbf{P}'$ camera matrices, $\mathbf{l}, \mathbf{l}'$ vectors of lines l, l' parameters on the sensor, $\mathbf{L}$ infinite line in 3D space (to be reconstructed)

Reconstruction Formula

$$\mathbf{L}\mathbf{X} = \mathbf{0} \tag{1}$$

$\mathbf{X}$: generic point on the 3D line

$\mathbf{L}$: 2×4 matrix of plane parameters

$$\mathbf{L} = \begin{pmatrix} \mathbf{l}^T\mathbf{P} \\ \mathbf{l}'^T\mathbf{P}' \end{pmatrix}$$

Event 1: Single Track in Grain

Simulated Track: a muon from the center of GRAIN

We consider a semiline starting from the origin of Grain and directed as

$$\theta_X = 90^\circ, \qquad \theta_Y = 45^\circ, \qquad \theta_Z = 45^\circ \tag{2}$$

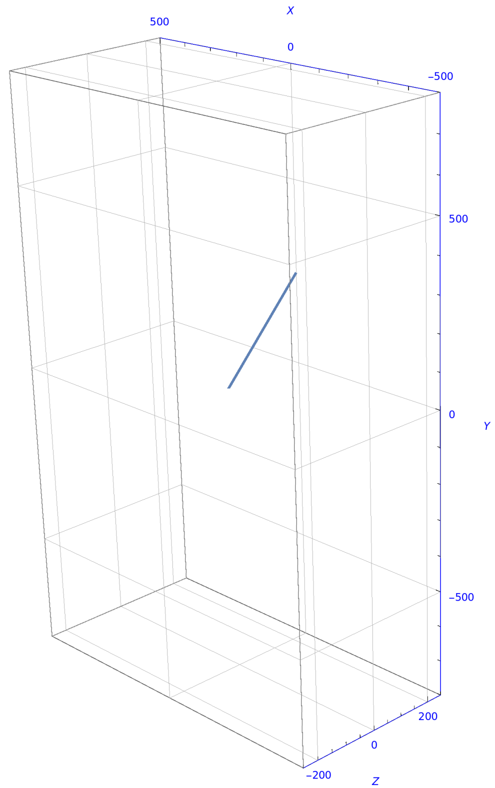
Director cosines

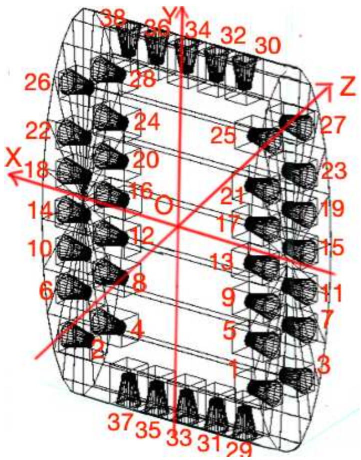
$$l = \cos\theta_X = 0, \qquad m = \cos\theta_Y = n = \cos\theta_Z = \frac{\sqrt{2}}{2} \tag{3}$$

Starting point: $(0, 0, 0)$

$$X = 0, \qquad Y = Z$$

Track in Grain

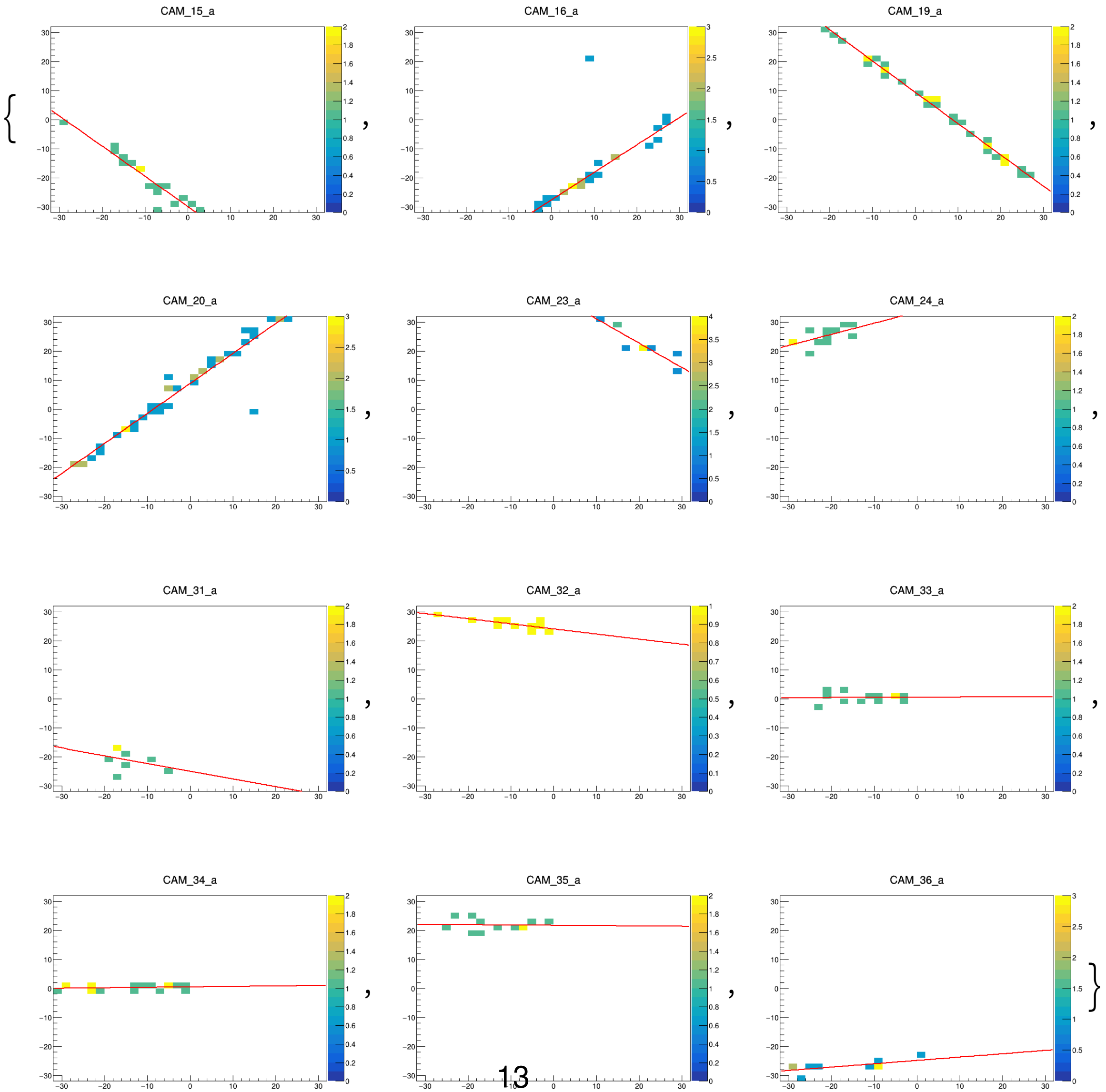




2D Reconstructions

Cameras involved: 15,16,19,20,23,24, 31-36

Out[∘]=



Global Multiple View Reconstruction of a Track

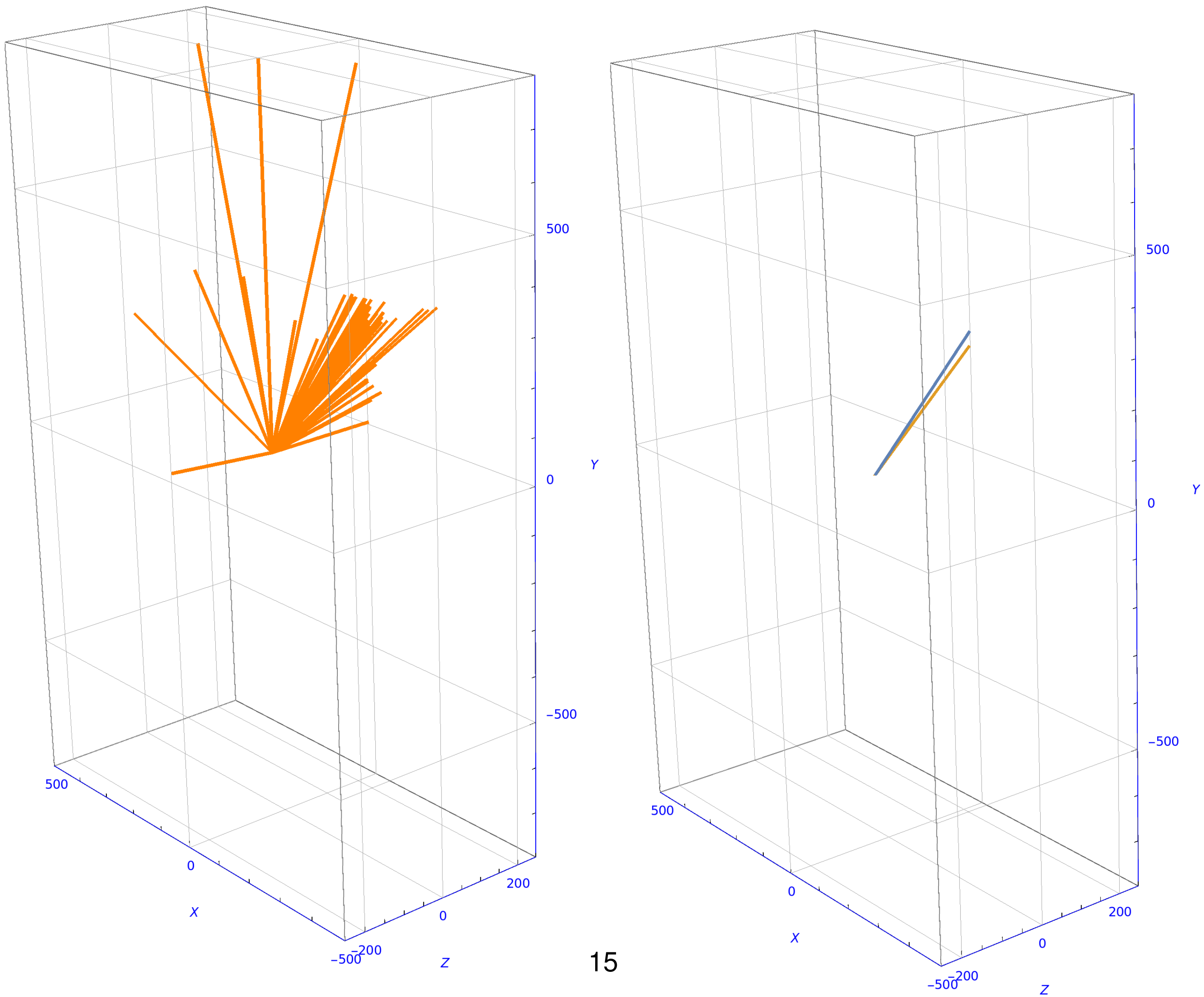
- The track is detected/seen by N cameras
- There are $M = \frac{N!}{2!(N-2)!}$ possible double-view reconstructions for the track
- We perform M reconstructions
- We take the mean value of the M possible reconstructions for each line parameter (director cosines (l, m, n))

$$\boxed{l = \frac{\sum_{i<j}^{N} l_{ij}}{M}} \boxed{m = \frac{\sum_{i<j}^{N} m_{ij}}{M}} \boxed{n = \frac{\sum_{i<j}^{N} n_{ij}}{M}} \quad (4)$$

i, j camera indices

- Analysis and averaging of intercepts and lines projected onto GRAIN coordinate planes XY, XZ, YZ of the M reconstructions

3D Global Reconstruction of the track



Track Reconstruction

- Theoretical

$$X = 0 \quad Y = Z \tag{5}$$

$$\theta_X = 90^\circ \quad \theta_Y = 45^\circ, \quad \theta_Z = 45^\circ$$

$$l = \cos\theta_X = 0, \qquad m = \cos\theta_Y = \frac{\sqrt{2}}{2}, \quad n = \cos\theta_Z = \frac{\sqrt{2}}{2} \tag{6}$$

Vertex $(0, 0, 0)$

- Reconstructed

$$X = 0 \quad Y = -3 + 1.05Z \tag{7}$$

$$\theta_X = 89.2^\circ \quad \theta_Y = 50^\circ, \quad \theta_Z = 45^\circ$$

Director cosines

$$l = \cos\theta_X = 0.006, \qquad m = \cos\theta_Y = 0.6, \quad n = \cos\theta_Z = 0.7 \tag{8}$$

Vertex $(0, 3, 6)$ units: mm

Event 2: two Tracks and Vertex

Simulation Details

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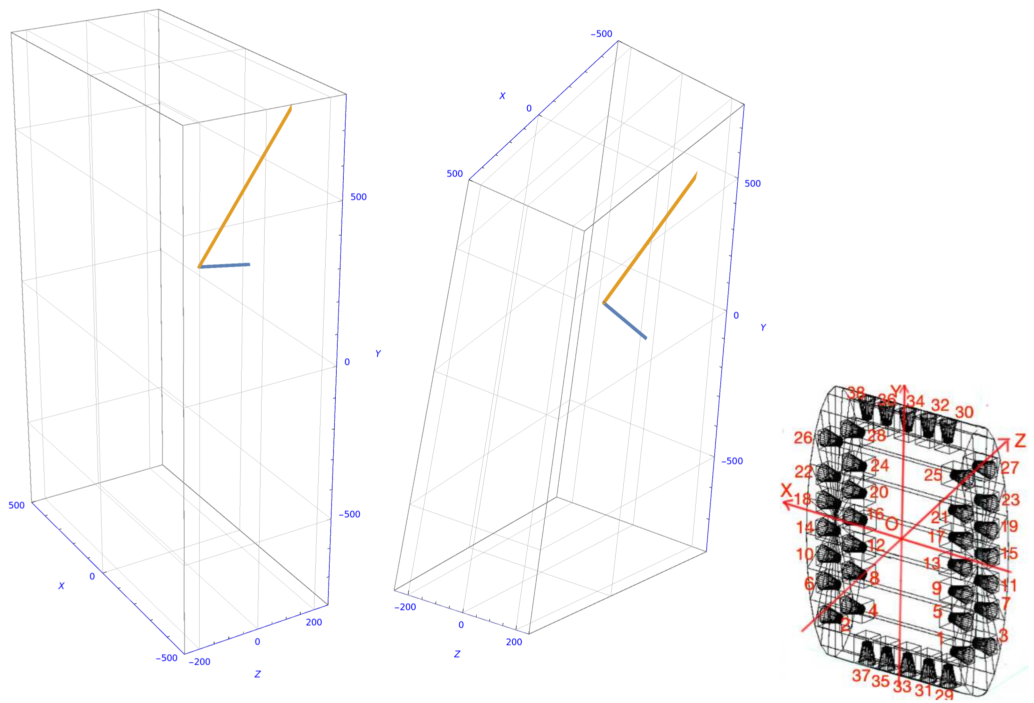
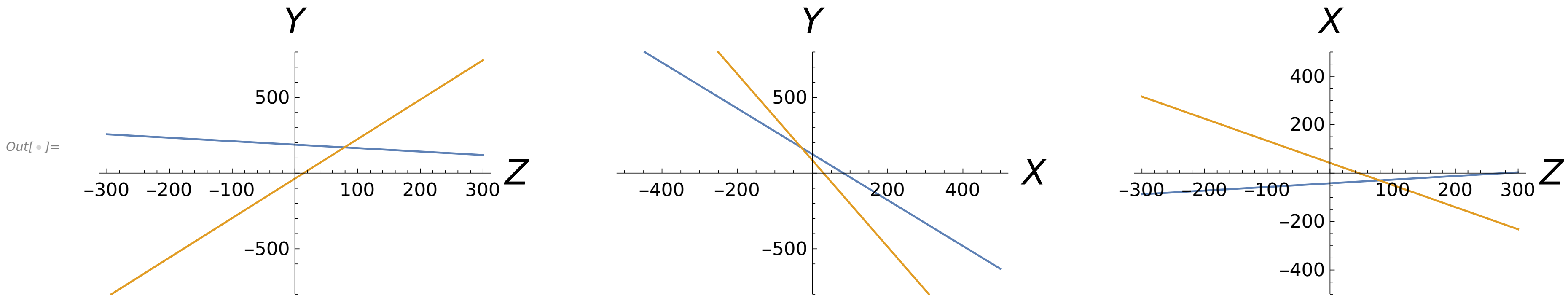
Vertex (-30, 170, 79) (units: mm)

3D Event Representation



Orthogonal Projections onto Grain coordinate planes



Track 0

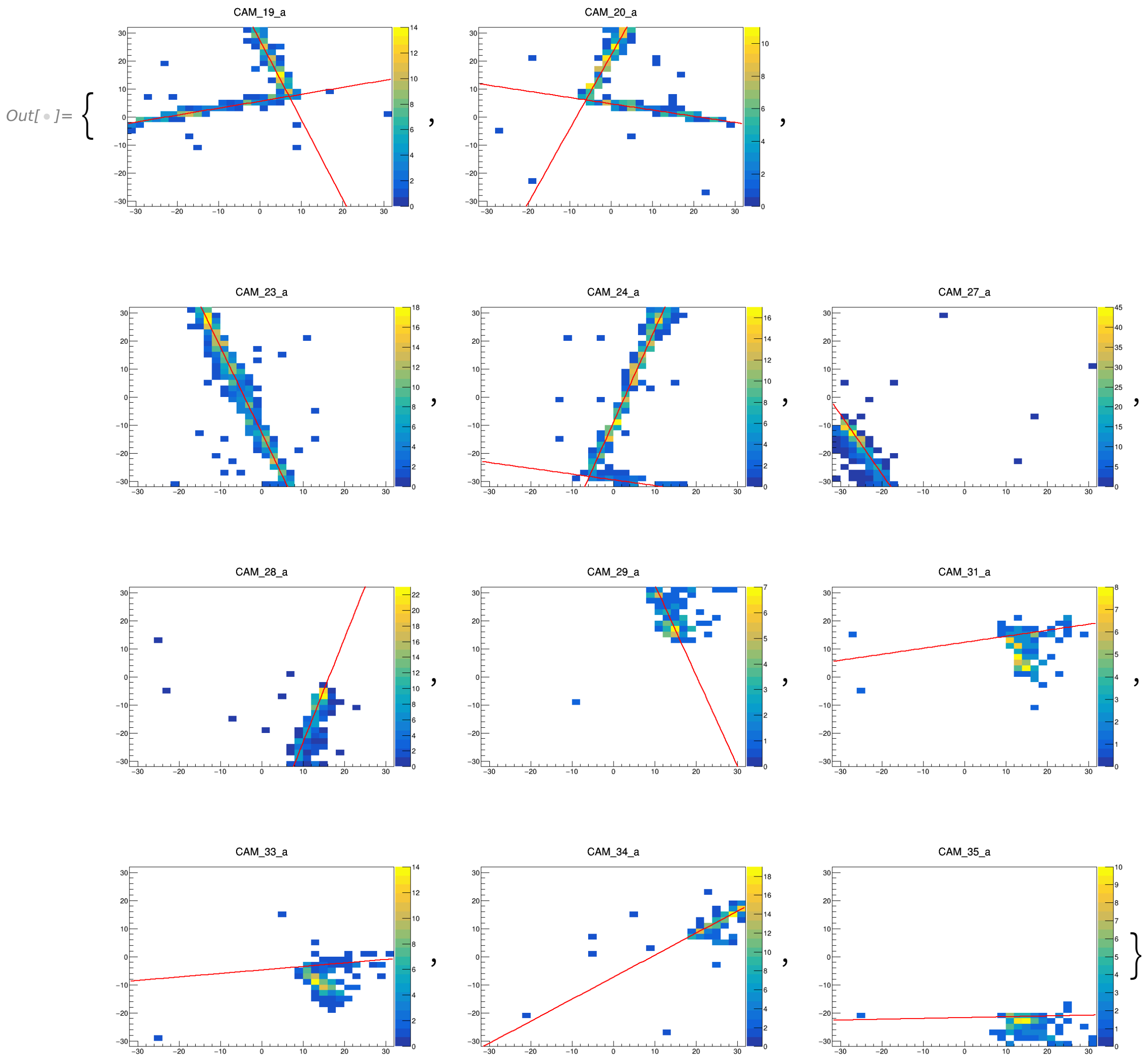
$$Y = 188.049 - 0.228Z, \quad Y = 124.556 - 1.515X, \quad X = -41.915 + 0.151Z$$

Track 1

$$Y = -36.087 + 2.609Z, \quad Y = 84.286 - 2.857X, \quad X = 42.130 - 0.913Z$$

Camera Images from MC Simulations: Fit

Cameras involved: 19, 20, 23, 24, 27, 28, 29, 31, 33, 34, 35



Reconstruction Algorithm

Global Multiple View Reconstruction of two Tracks with Vertex

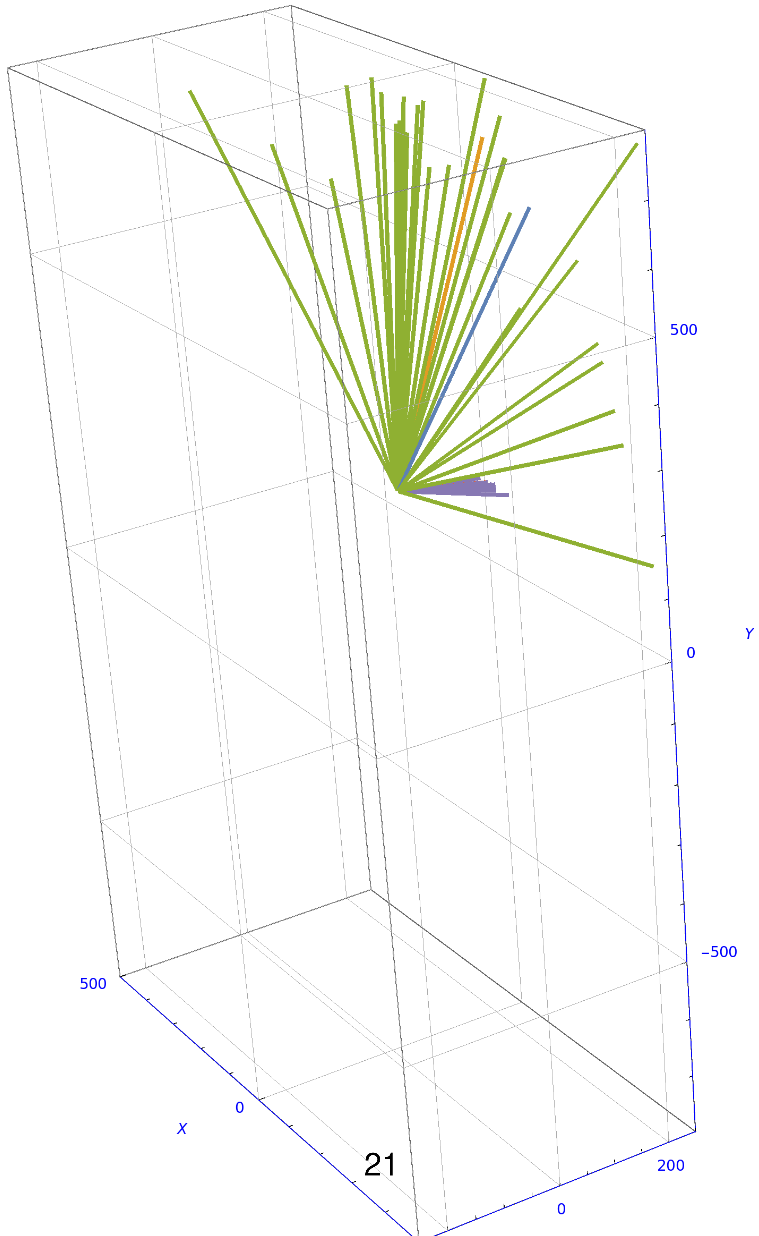
- The track is detected/seen by N cameras
- There are $M = \frac{N!}{2!(N-2)!}$ possible double-view reconstructions for the track
- We perform M reconstructions
- We take the mean value of the M possible reconstructions for each line parameter (director cosines (l, m, n))

$$\boxed{l = \frac{\sum_{i<j}^{N} l_{ij}}{M}} \boxed{m = \frac{\sum_{i<j}^{N} m_{ij}}{M}} \boxed{n = \frac{\sum_{i<j}^{N} n_{ij}}{M}} \quad (9)$$

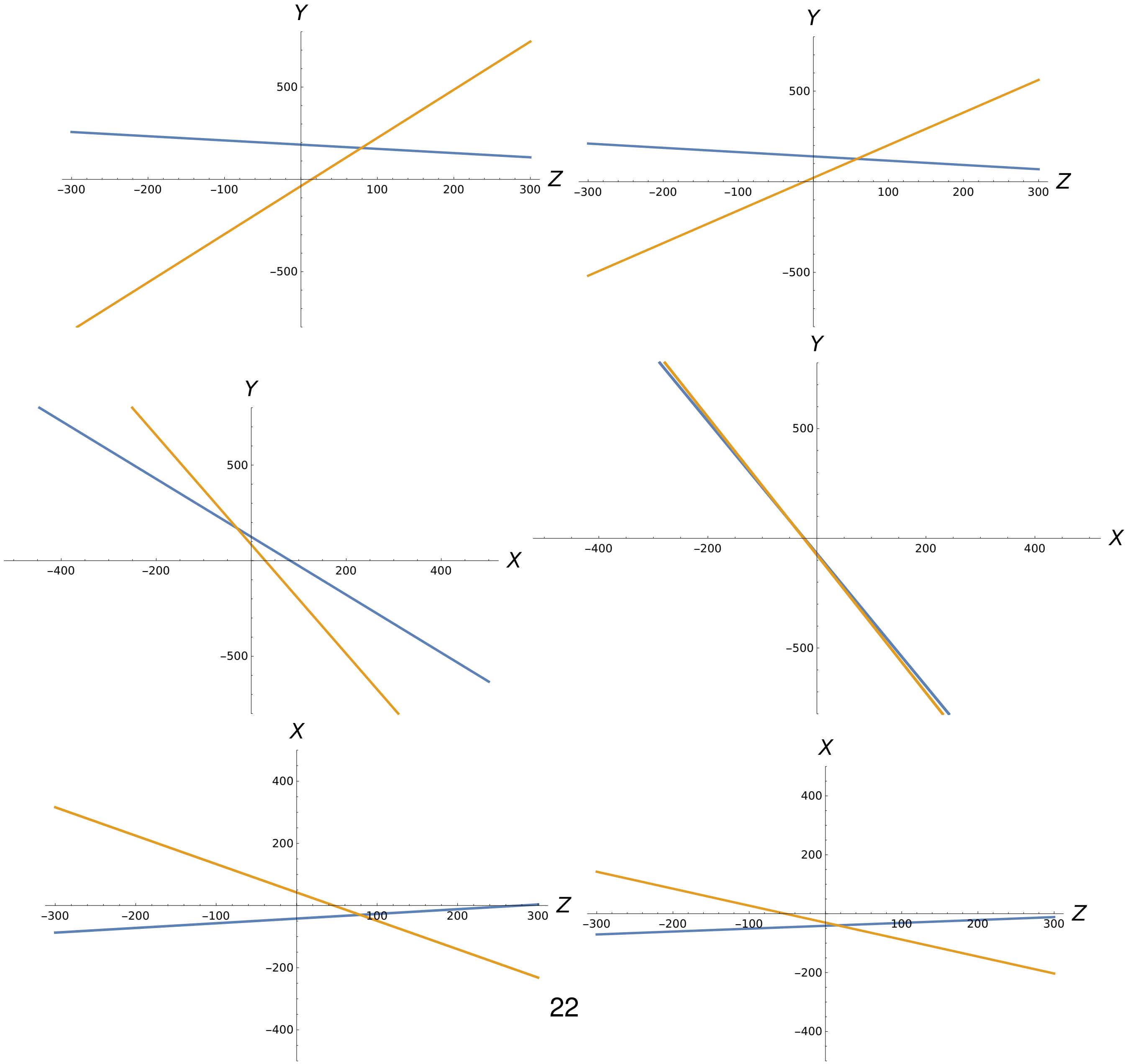
i, j camera indices

- Analysis and averaging of intercepts and lines projected onto GRAIN coordinate planes XY, XZ, YZ of the M reconstructions

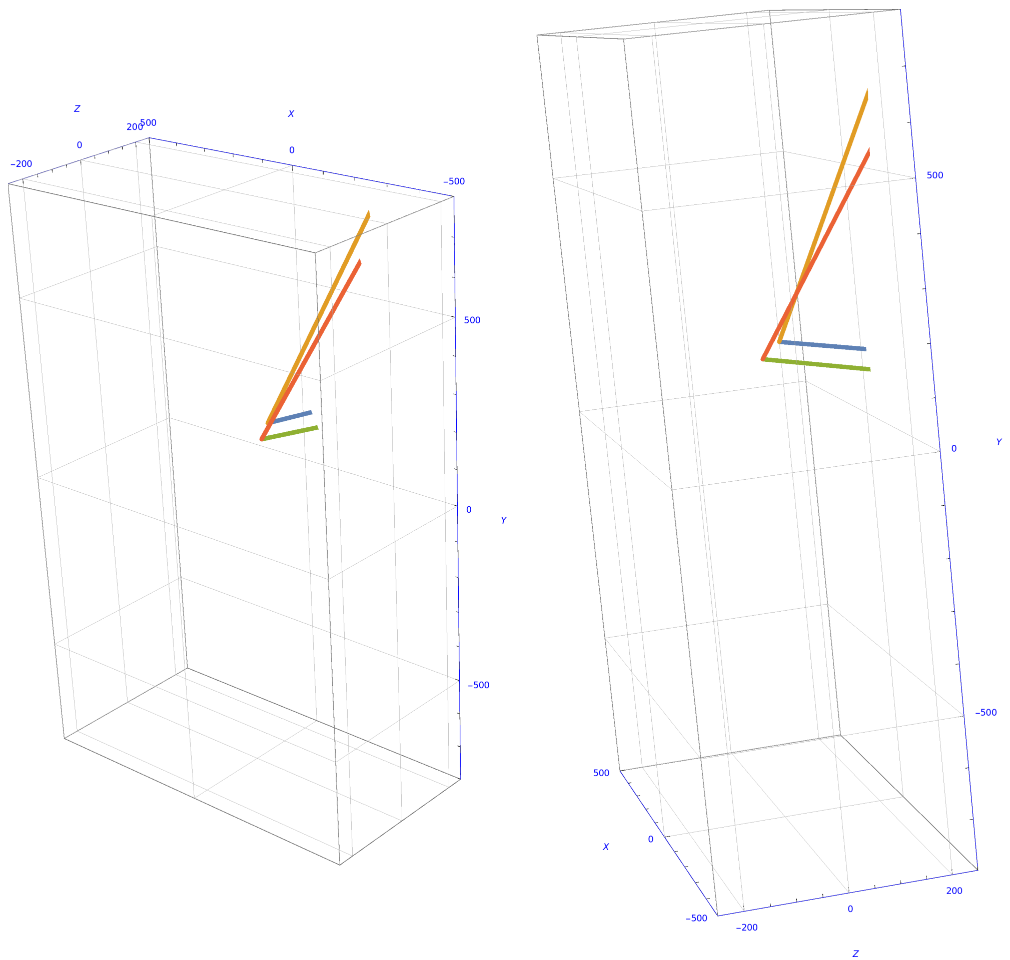
Center Back Projection in 3D



Mean Track Projections on Grain Coordinate Planes for Vertex Reconstruction



Reconstruction in GRAIN



Vertex and Line reconstruction: numerical results

MC truth: Vertex$(-30, 170, 79)$

direction parameters:

$$(0.145, -0.220, 0.965) \quad \theta_X = 81^\circ, \theta_Y = 103^\circ, \theta_Z = 15^\circ \quad \text{Track 0}$$

$$(-0.311, 0.888, 0.340) \quad \theta_X = 108^\circ, \theta_Y = 27^\circ, \theta_Z = 70^\circ \quad \text{Track 1}$$

Reconstruction values: Vertex$(-42, 142, 40)$

direction parameters

$$(0.095, -0.228, 0.961) \quad \theta_X = 85^\circ, \theta_Y = 103^\circ, \theta_Z = 16^\circ \quad \text{Track 0}$$

$$(-0.273, 0.835, 0.472) \quad \theta_X = 106^\circ, \theta_Y = 33^\circ, \theta_Z = 62^\circ \quad \text{Track 1}$$

Conclusions

So far

- **Reconstruction of Light Points in GRAIN via Multiple View Projective Geometry**
- We started the **Reconstruction of Tracks in GRAIN via Multiple View Projective Geometry**

TO BE DONE

- **Improve the method by points and directions correspondence**
- **Triple view geometry for IMAGE TRANSFER: Trifocal Tensor**
- **Extension to events with multiple vertices and multiple tracks**
- **Software**

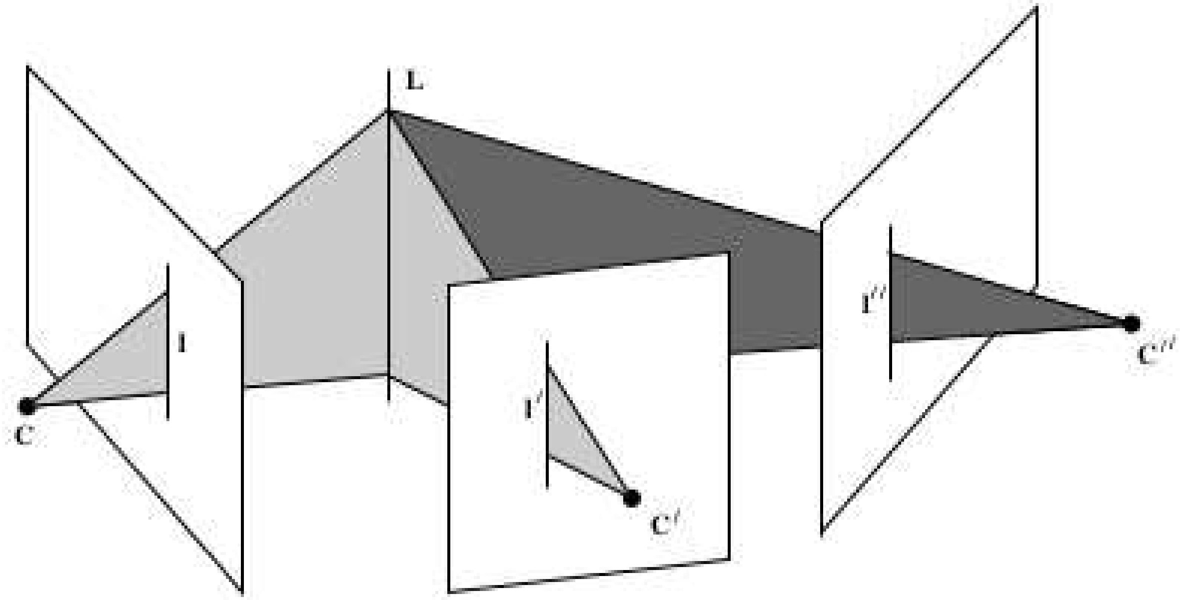
and ...

THANK YOU

For Your Attention!

BACK-UP

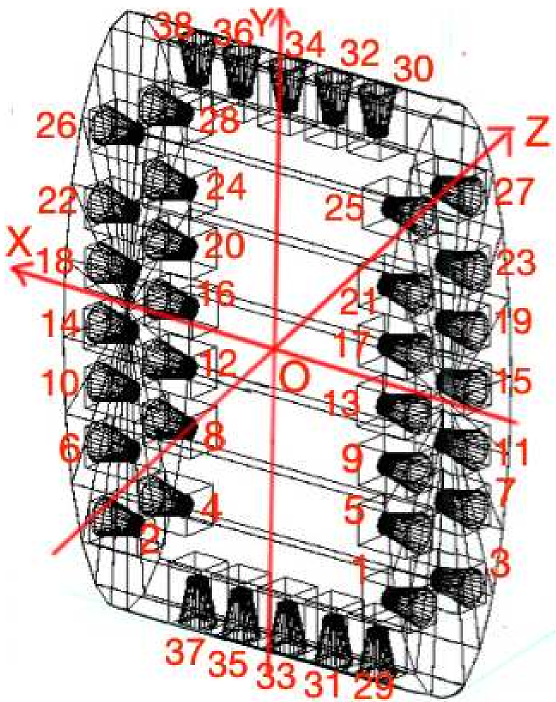
Triple-View Geometry and image correspondences: the Trifocal Tensor



From Lenses to P-matrices

A $\mathbf{P}$-matrix is associated to each of the 38 camera-lenses of GRAIN

$$\text{GRAIN} \Longleftrightarrow \{\mathbf{P}_j\}_{j=1,\ldots,38} \tag{10}$$



3D reconstruction of Points and Tracks

$$i, j = 1, ...38$$
$$\pi_i = \mathbf{P}_i^T \mathbf{l}_i$$

π_i vector of plane parameters in 3D space, $\mathbf{P}_i$ camera matrix i, $\mathbf{l}_i$ vector of line parameters on the sensor i,

$$\mathbf{L}_{ij} = \begin{pmatrix} \mathbf{l}_i^T \mathbf{P}_i \\ \mathbf{l}_j^T \mathbf{P}_j \end{pmatrix}$$

$\mathbf{L}_{ij}$ infinite line in 3D space to be reconstructed $\mathbf{l}_i^T \mathbf{P}$ vector of plane π_i parameters in 3D space,$\mathbf{l}_j^T \mathbf{P}_j$ vector of plane π_j parameters in 3D space, $\mathbf{P}_i, \mathbf{P}_j$ camera matrices, $\mathbf{l}_i, \mathbf{l}_j$ vectors of lines l_i, l_j parameters on the sensor, $\mathbf{L}_{ij}$ infinite line in 3D space (to be reconstructed)

$$\mathbf{L}_{ij} \mathbf{X} = \mathbf{0} \tag{11}$$

$\mathbf{X}$: generic point on the 3D line

$\mathbf{L}_{ij}$: 2×4 matrix of plane parameters

$$\mathbf{L}_{ij} = \begin{pmatrix} \mathbf{l}_i^T \mathbf{P}_i \\ \mathbf{l}_i^T \mathbf{P}_j \end{pmatrix}$$

As for points

$$\mathbf{0} = \mathbf{x}_i \times \mathbf{x}_i = \mathbf{x}_i \times \mathbf{P}_i \mathbf{X}_{ij} \tag{12}$$

$$\mathbf{0} = \mathbf{x}_j \times \mathbf{x}_j = \mathbf{x}_j \times \mathbf{P}_j \mathbf{X}_{ij} \tag{13}$$

$$\boxed{\mathbf{X}_{ij} = \mathbf{P}_i^+ \mathbf{x}_i + \left[\frac{(\mathbf{P}_j \mathbf{P}_i^+ \mathbf{x}_i \times \mathbf{x}_j) \cdot (\mathbf{x}_j \times \mathbf{P}_j C_i)}{(\mathbf{x}_j \times \mathbf{P}_j C_i) \cdot (\mathbf{x}_j \times \mathbf{P}_j C_i)} \right] C_i} \tag{14}$$

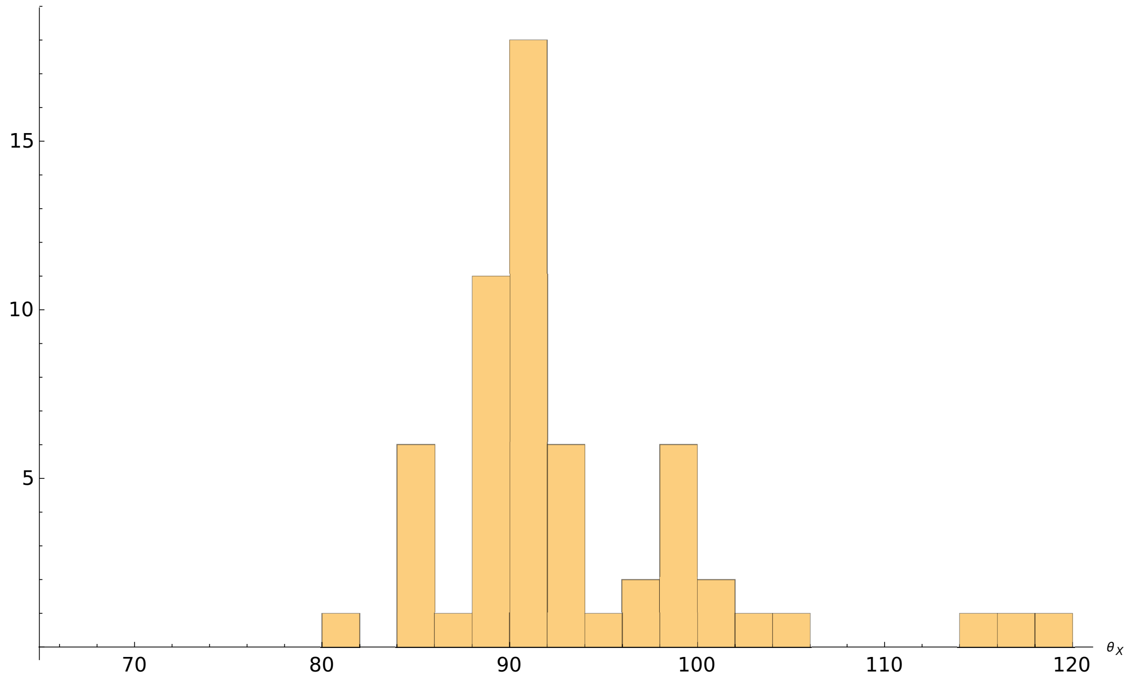
$$\mathbf{P}_i^+ = \mathbf{P}_i^T \left(\mathbf{P}_i \mathbf{P}_i^T \right)^{-1} \tag{15}$$

$\mathbf{P}_i$ camera matrix i

$\mathbf{X}_{ij}$ reconstructed 3D point using cameras i and j, $i \neq j$.

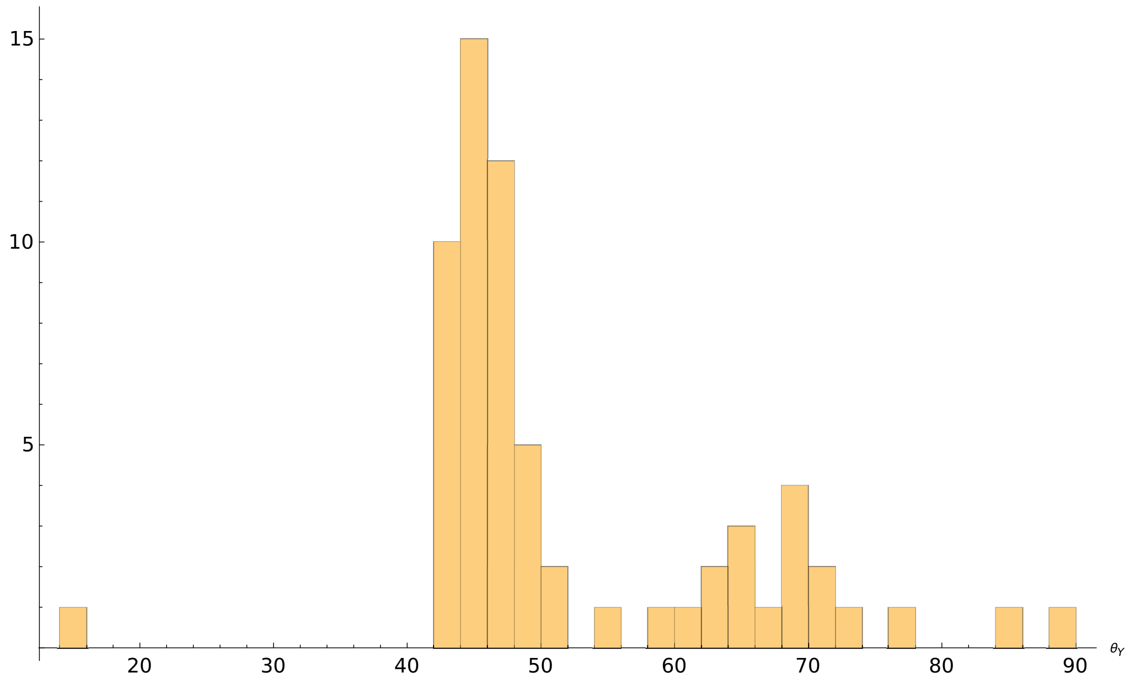
$\mathbf{x}_i$ image point on camera i, C_i camera center i

θ_X Distribution



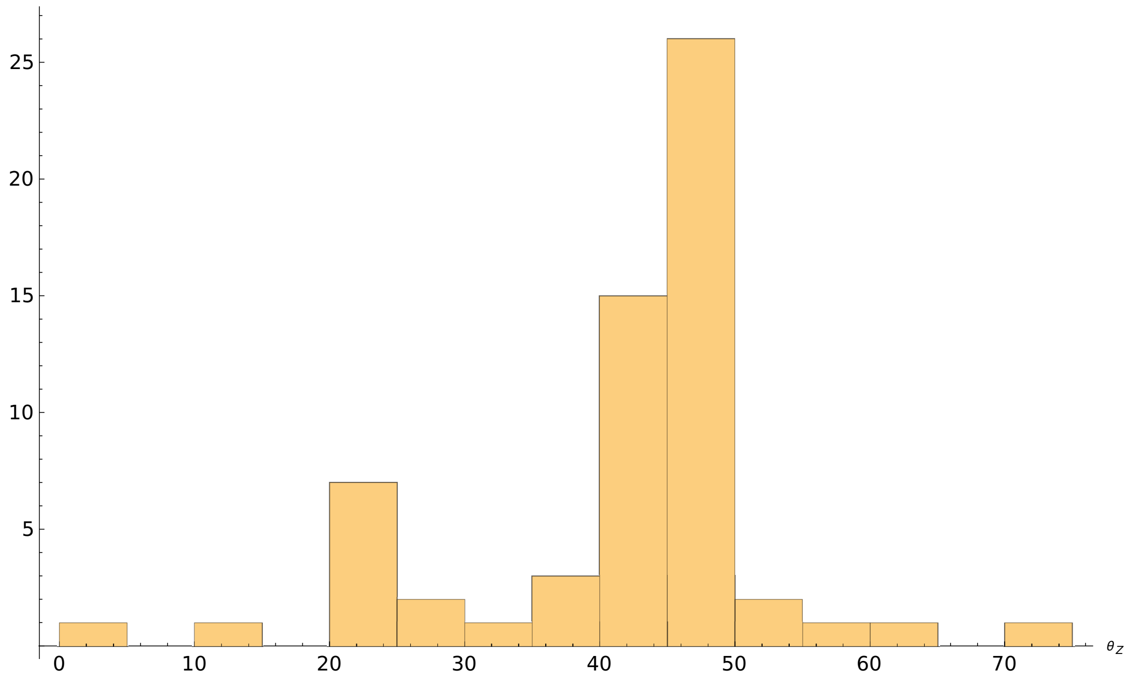
Mean$\approx 89.2^\circ$, RMS$\approx 16^\circ$

θ_Y Distribution



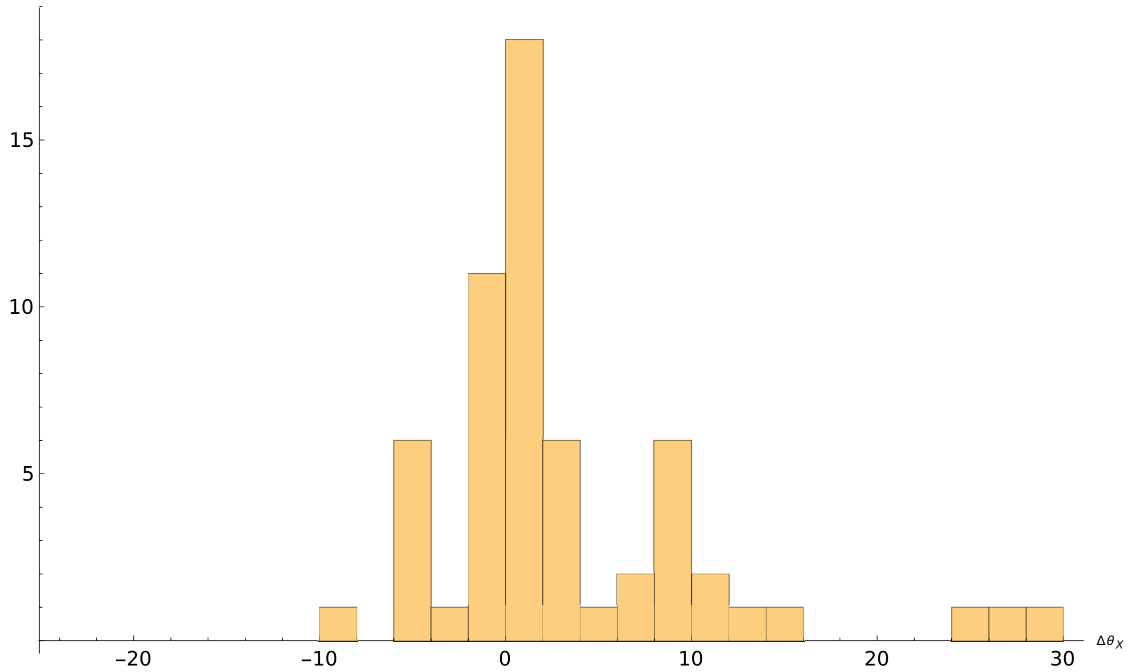
Mean$\approx 50^\circ$, RMS$\approx 12^\circ$

θ_Z Distribution



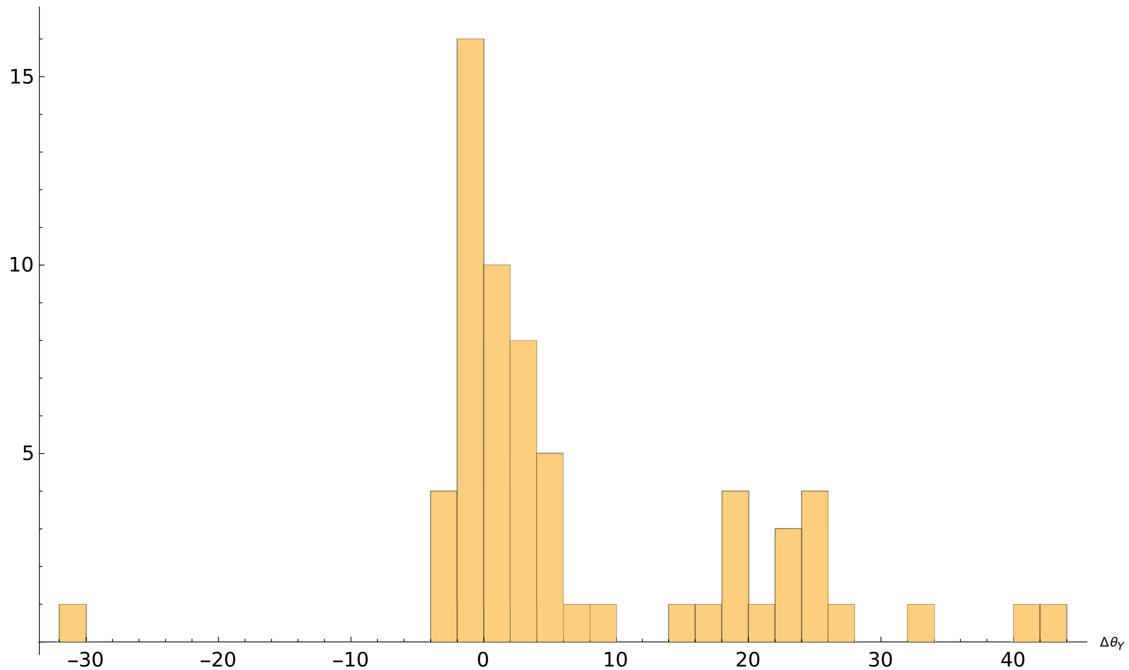
Mean$\approx 45^\circ$, RMS$\approx 20^\circ$

$\Delta\theta_X$ Distribution



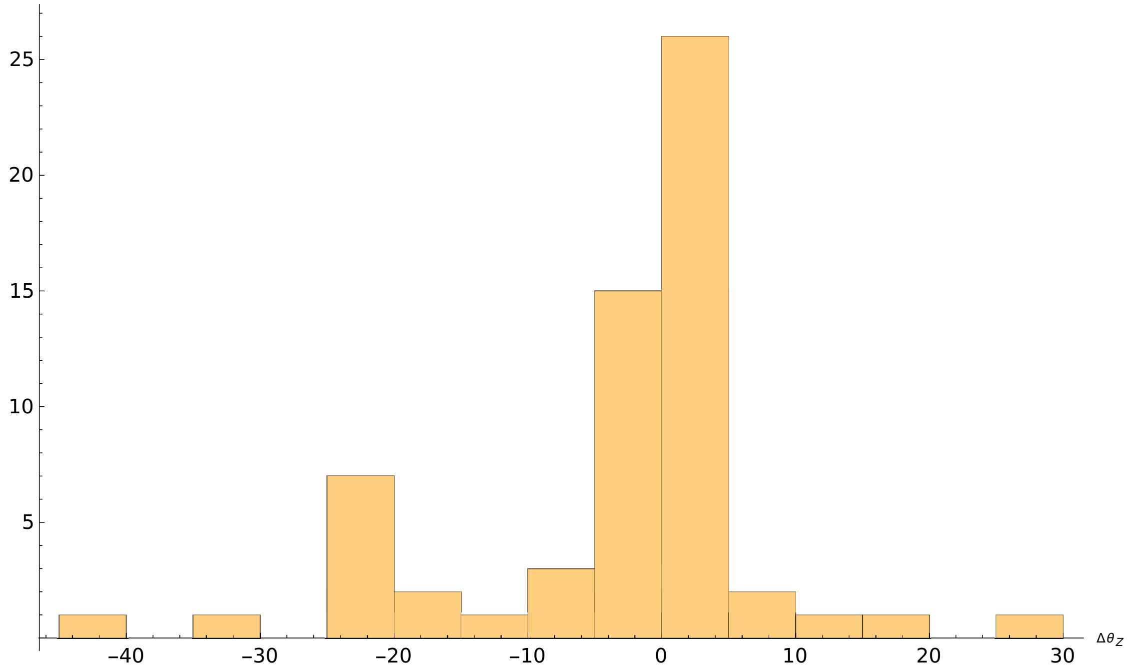
Mean$\approx -0.8^\circ$, RMS$\approx 16^\circ$

$\Delta\theta_Y$ Distribution



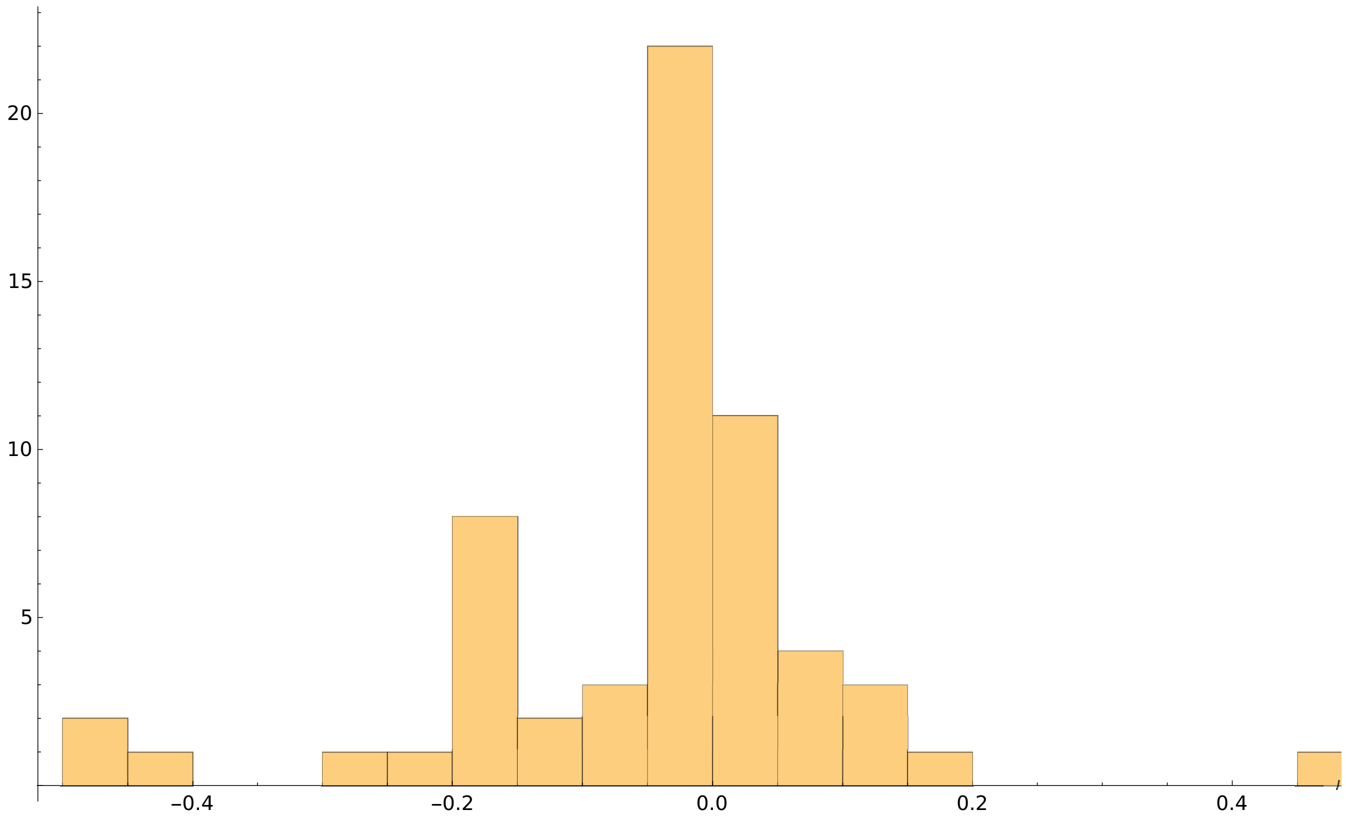
Mean$\approx 7^\circ$, RMS$\approx 12^\circ$

$\Delta\theta_Z$ Distribution



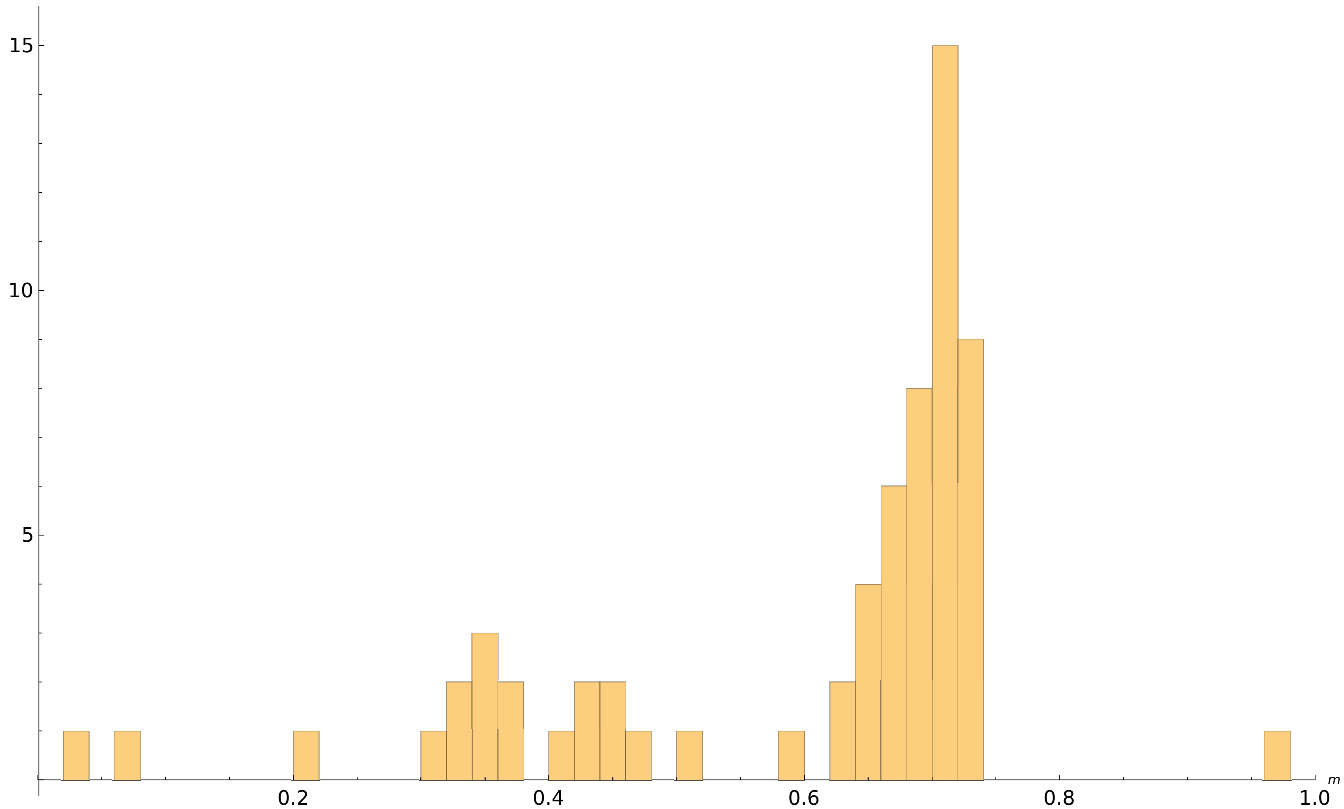
Mean$\approx -0.4^\circ$, RMS$\approx 20^\circ$

$l = \cos\theta_X$ Distribution



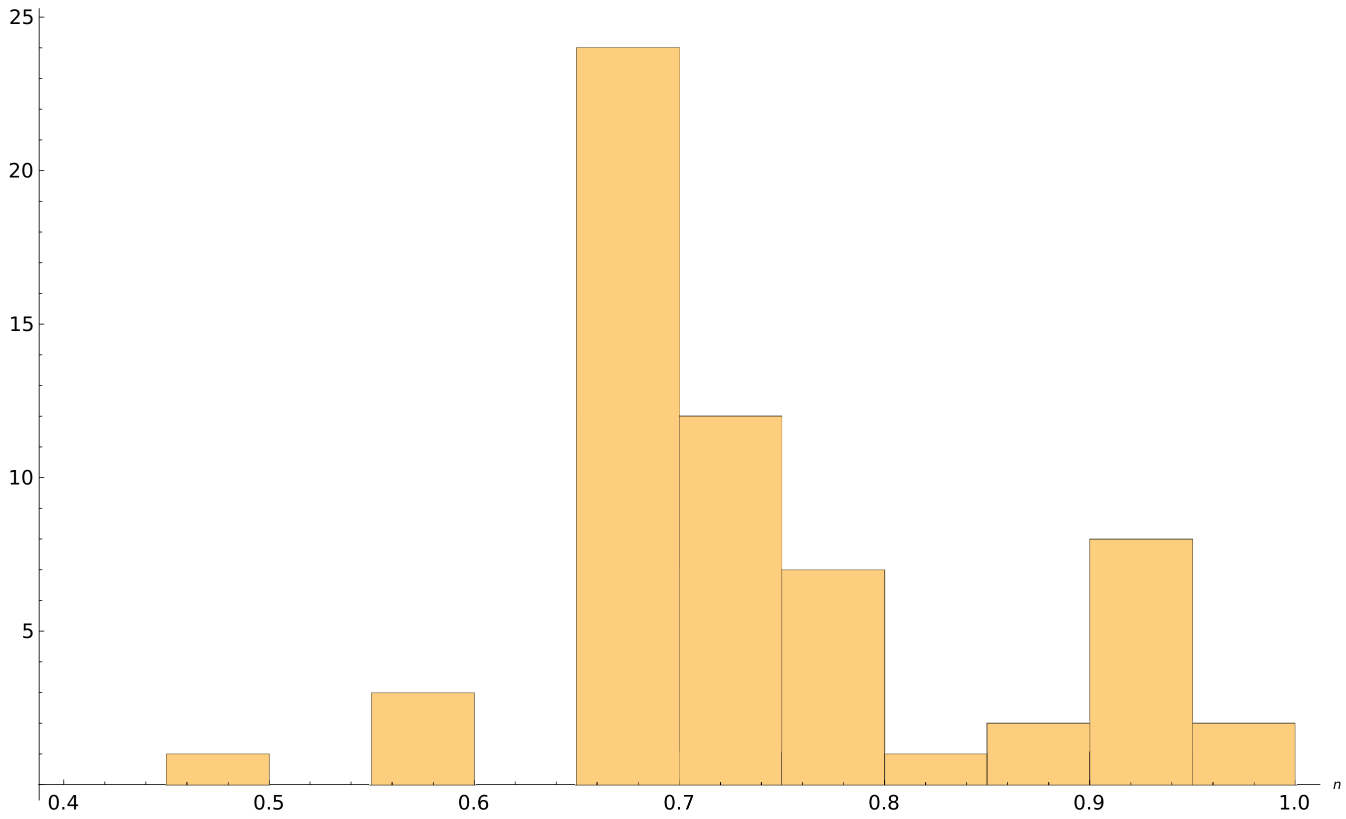
Mean≈ 0.006,   RMS≈ 0.25

$m = \cos\theta_Y$ Distribution



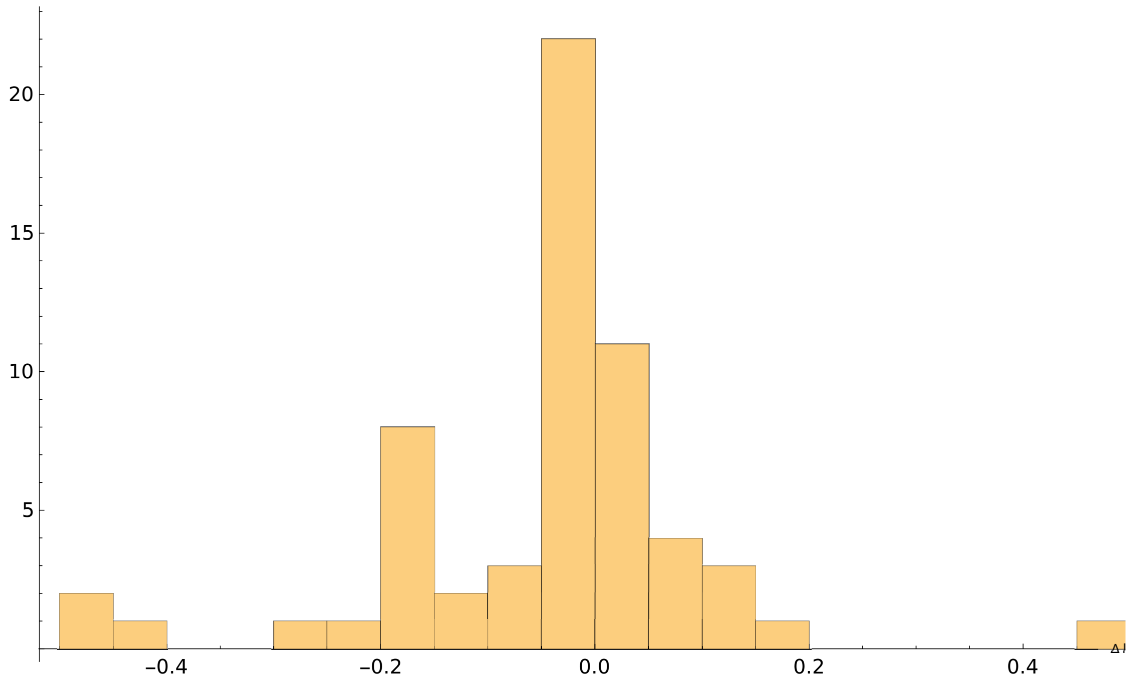
Mean≈ 0.6,   RMS≈ 0.18

$n = \cos\theta_Z$ Distribution



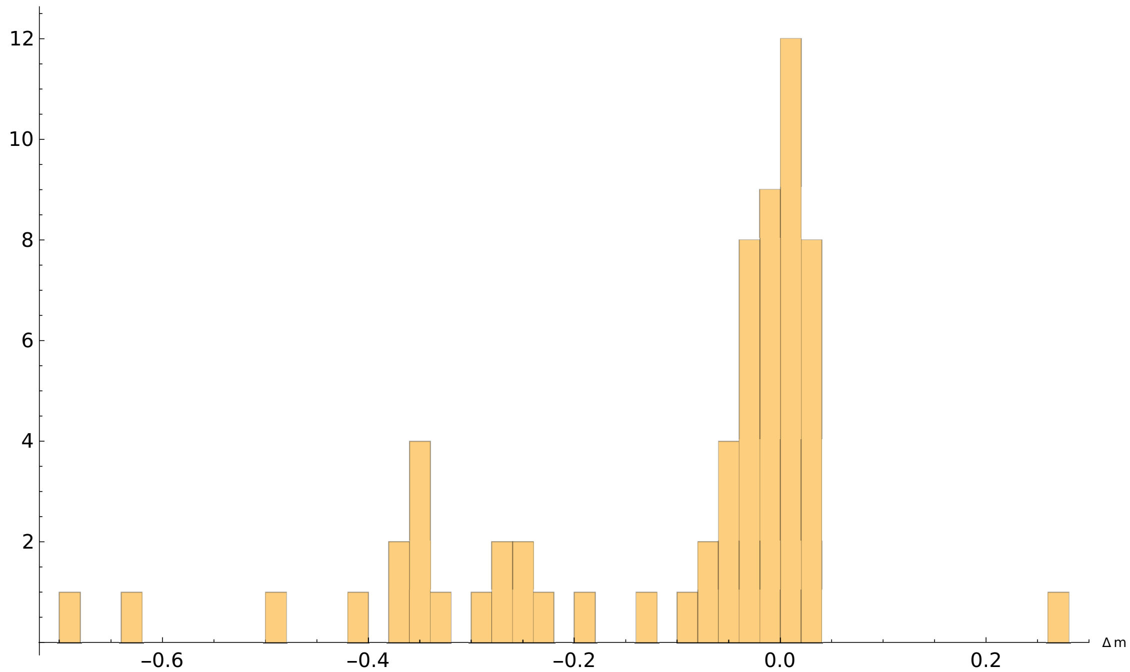
Mean ≈ 0.7,   RMS ≈ 0.27

$\Delta l = \Delta \cos\theta_X$ Distribution



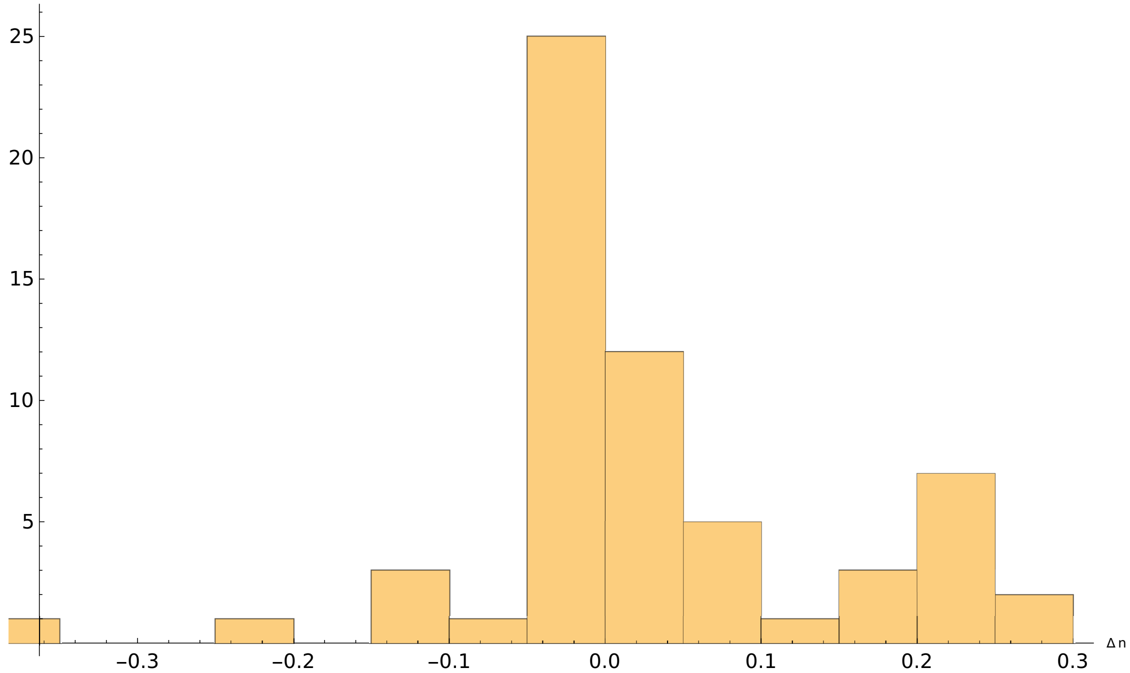
Mean$\approx 0.006,$ RMS≈ 0.25

$\Delta \cos \theta_Y$ Distribution



Mean$\approx -0.1,$ RMS≈ 0.18

$\Delta \cos\theta_Z$ Distribution



Mean≈ -0.02,   RMS≈ 0.27

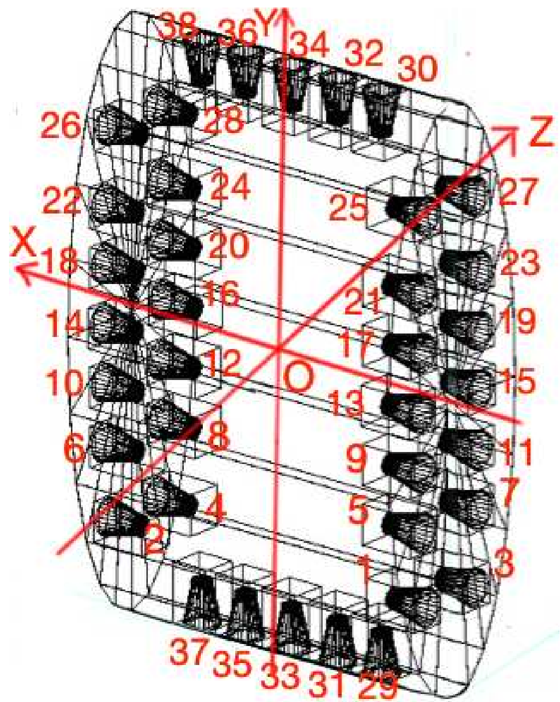
Details on Simulations

- 1000 light point sources
- 4×10^6 photons per point
- light-yield 4×10^4 photons/MeV
- energy release: 100 MeV per point
- processes: scattering, absorption, refractive indices depending on the wavelength

From Lenses to P-matrices

A $\mathbf{P}$-matrix is associated to each of the 38 camera-lenses of GRAIN

$$\text{GRAIN} \Longleftrightarrow \{\mathbf{P}_j\}_{j=1,\ldots,38} \tag{16}$$



Cameras on the 4 sides

$$\text{elliptic side 1, even numbers} \quad \mathbf{P}_{2j}, \quad j = 1, ..., 14 \tag{17}$$

$$\text{elliptic side 2, odd numbers} \quad \mathbf{P}_{2j+1}, \quad j = 0, ..., 13 \tag{18}$$

$$\text{top side} \quad \mathbf{P}_{2j}, \quad j = 15, ..., 19 \tag{19}$$

$$\text{bottom side} \quad \mathbf{P}_{2j+1}, \quad j = 14, ..., 18 \tag{20}$$

Elliptic side 1, even numbers

$$\mathbf{P}_{2j} = \mathbf{K}\left[\mathbf{R}| - \mathbf{R}C_{2j}\right], \quad j = 1, ..., 14 \tag{21}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \tag{22}$$

$$C_{2j}, \quad j = 1, ...14 \qquad \text{lens centers in GRAIN} \tag{23}$$

parameters: $c = 399, f_x = f_y = f = 100, p_z = 110, q_z = 105,$

$r_z = 90, p_y = 145, q_y = 290, r_y = 475, s = x_0 = y_0 = 0$

units: mm

Further parameterization of P-matrices for general Calibration

In fact, each of the 38 $\mathbf{P}$ matrices can be further parameterized by including **radial distorsion**

$$\mathbf{P}_{2j} = \mathbf{L}_{2j}\mathbf{K}\left[\mathbf{R}| - \mathbf{R}C_{2j}\right], \quad j = 1, ..., 14 \tag{24}$$

$$\mathbf{L}_{2j} = \begin{pmatrix} L_{2j}(r) & 0 & x_c \\ 0 & L_{2j}(r) & y_c \\ 0 & 0 & 1 \end{pmatrix} \tag{25}$$

$$L_{2j}(r) = 1 + k_1^{2j} r + k_2^{2j} r^2 + k_3^{2j} r^3 + ..., \qquad r = \sqrt{(x - x_c)^2 + (y - y_c)^2} \tag{26}$$

(x, y) image coordinates on the sensor coordinate local frame, (x_c, y_c) distortion center on he sensor coordinate local frame

Elliptic side 2, even numbers

$$\mathbf{P}_{2j+1} = \mathbf{K}\left[\mathbf{R}_1 | - \mathbf{R}_1 C_{2j+1}\right], \quad j = 0, ..., 13 \tag{27}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R}_1 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{28}$$

$$C_{2j+1}, \quad j = 0, ...13 \qquad \text{lens centers in GRAIN} \tag{29}$$

parameters: $c = 399, f = 100, p_z = 110, q_z = 105,$

$r_z = 90, p_y = 145, q_y = 290, r_y = 475, s = x_0 = y_0 = 0$

units: mm

Top side, even numbers

$$\mathbf{P}_{2j} = \mathbf{K}\left[\mathbf{R}_2 | -\mathbf{R}_2 C_{2j}\right], \quad j = 15, ..., 19 \tag{30}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R}_2 = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \tag{31}$$

$$C_{2j}, \quad j = 15, ...19 \qquad \text{lens centers in GRAIN} \tag{32}$$

parameters: $d = 609, f = 100, w_x = 145, v_x = 280, s = x_0 = y_0 = 0$

units: mm

Bottom side, odd numbers

$$\mathbf{P}_{2j+1} = \mathbf{K}\left[\mathbf{R}_3 | -\mathbf{R}_3 C_{2j+1}\right], \quad j = 14, ..., 18 \tag{33}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R}_3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{34}$$

$$C_{2j+1}, \quad j = 14, ...18 \qquad \text{lens centers in GRAIN} \tag{35}$$

parameters: $d = 609, f = 100, w_x = 145, v_x = 280, s = x_0 = y_0 = 0$

units: mm

Further parameterization of P-matrices for general Calibration

In fact, each of the 38 $\mathbf{P}$ matrices can be further parameterized by including **radial distorsion**

$$\mathbf{P}_j = \mathbf{L}_j \mathbf{K} \left[\mathbf{R}_j | - \mathbf{R}_j C_j \right], \quad j = 1, ..., 38 \tag{36}$$

$$\mathbf{L}_j = \begin{pmatrix} L_j(r) & 0 & x_c \\ 0 & L_j(r) & y_c \\ 0 & 0 & 1 \end{pmatrix} \tag{37}$$

$$L_j(r) = 1 + k_1^j r + k_2^j r^2 + k_3^j r^3 + ..., \qquad r = \sqrt{(x - x_c)^2 + (y - y_c)^2} \tag{38}$$

(x, y) image coordinates on the sensor local coordinate frame, (x_c, y_c) distortion center on the sensor local coordinate frame