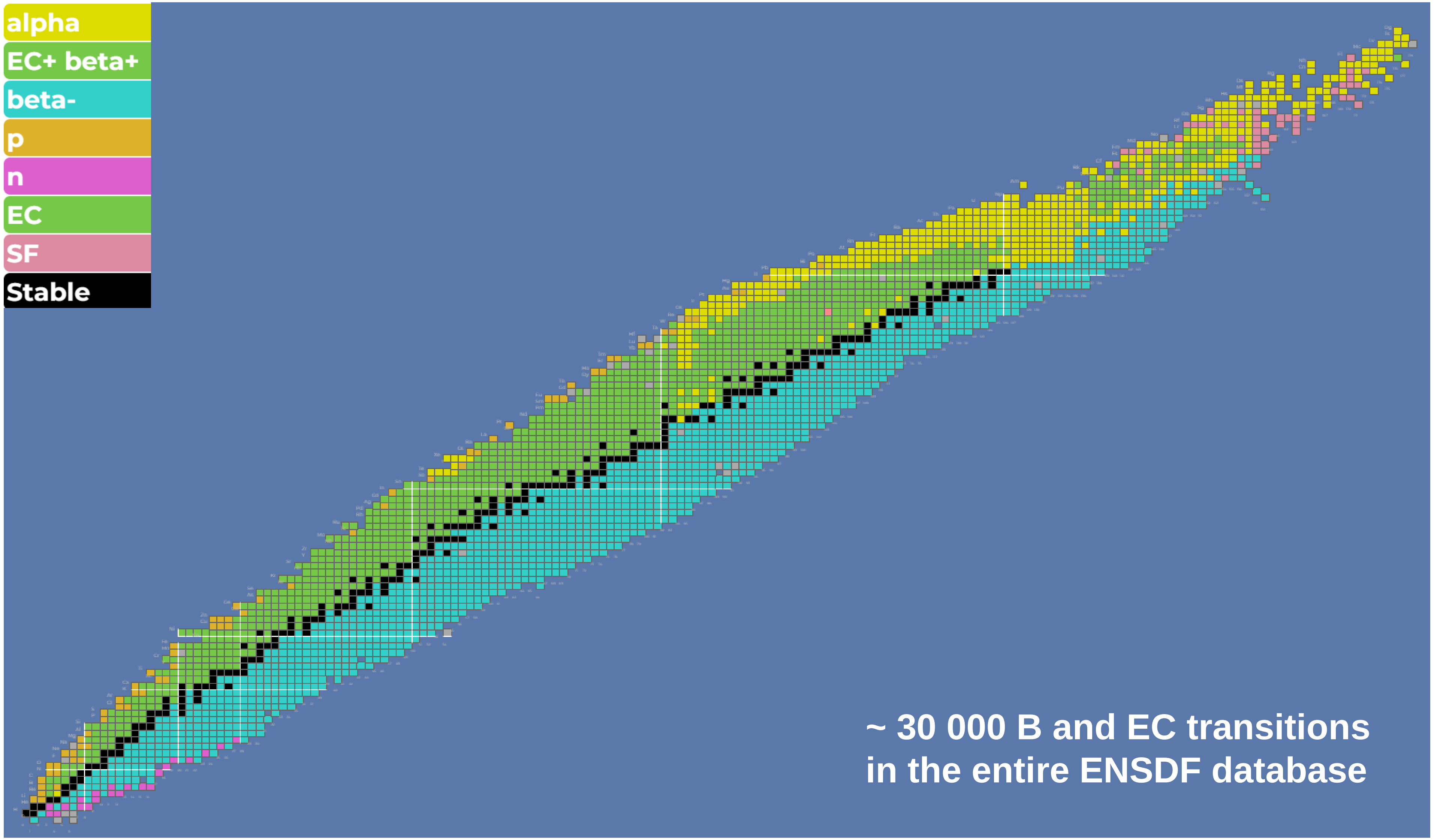
# Single beta spectral shapes and theories

X. Mougeot, CEA-LNHB (France)

Neutrino 2024 Conference

# Context

# The main radioactive decay mode



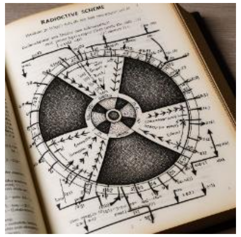
# Importance of beta decays

## Metrology

**Activity measurements**
(Liquid scintillation,
Ionization chambers)

Better knowledge → **Improvement of accuracy and uncertainties**

## Nuclear decay data



**ENSDF**, **DDEP**, **JEFF** databases

**Properties calculated** with the LogFT legacy code (Gove and Martin, 1971)

**log-*ft* for Jπ assignment**, **no beta spectrum**

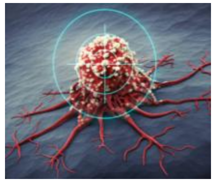
## Fundamental research

Nuclear astrophysics (*r*-process)

**Neutrino physics** (reactor anomaly, reactor monitoring, non-proliferation)

**Standard Model** (CKM matrix unitarity, weak magnetism)

**New physics** (Fierz interference, sterile neutrino, dark matter)

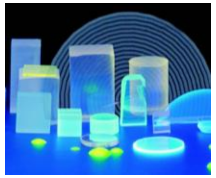


## Medical applications

Micro-dosimetry, internal radiotherapy, contamination

## Nuclear fuel cycle

Decay heat, nuclear waste



## Developments

Beta-voltaic batteries, new detectors (e.g. $LaBr_3$)

# Radionuclide metrology

LNHB (National Laboratory Henri Becquerel) is the French Designated Institute for primary standards in ionizing radiation metrology.

→ **Definition of activity (Bq) and dose (Sv, Gy) units.**

The diversity of radioactive processes makes necessary a certain pre-knowledge to establish primary and secondary standards: decay schemes, atomic and nuclear data.

→ **Evaluation of atomic and nuclear decay data.**

✓ Coordination of the DDEP (Decay Data Evaluation Project) international collaboration.

✓ Decay data officially recommended by the BIPM.

**Extension of SIR to almost pure beta emitters**

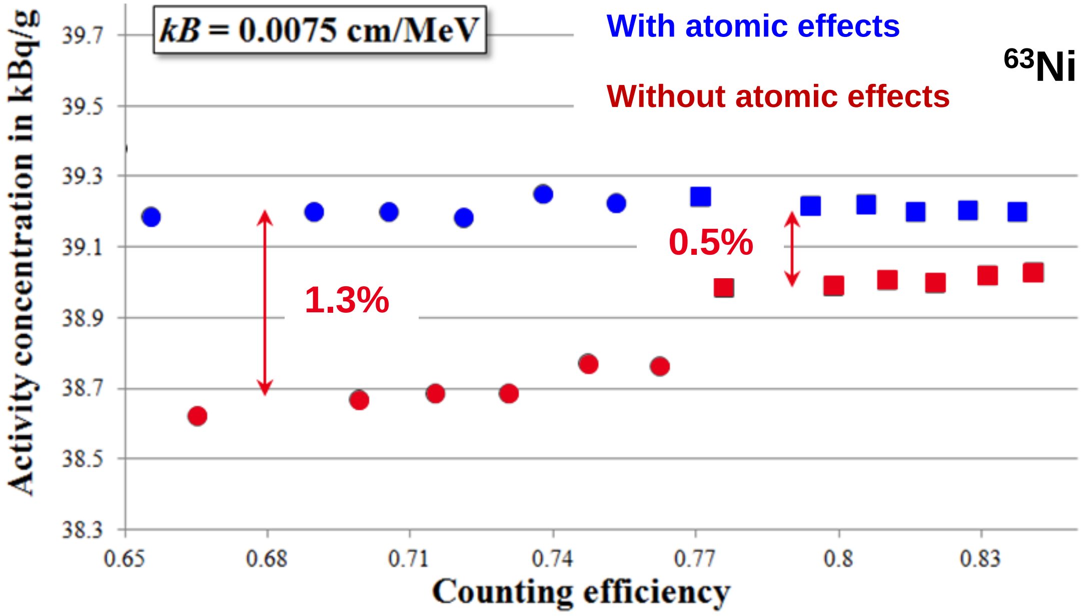
→ Primary activity by liquid scintillation counting.

→ Strong non-linear efficiency at low energy.

→ **Sensitivity to beta spectrum shape**

Estimate of deposited dose in patient's cells

→ **Impact at DNA level**

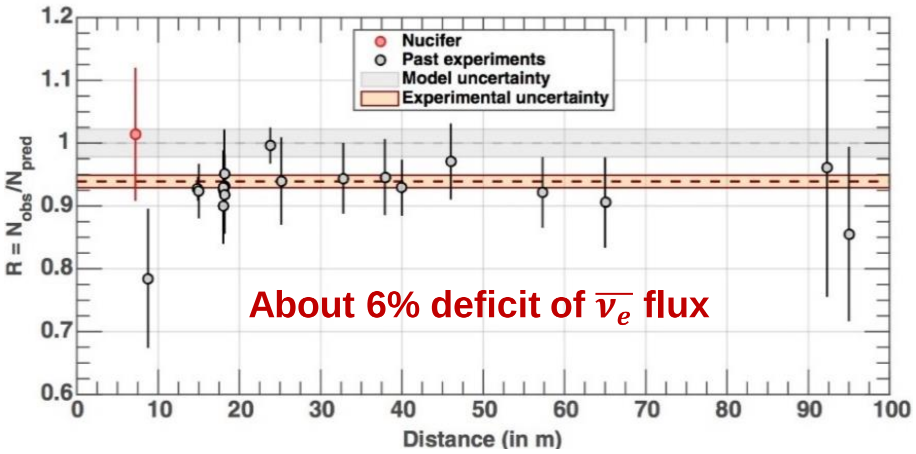


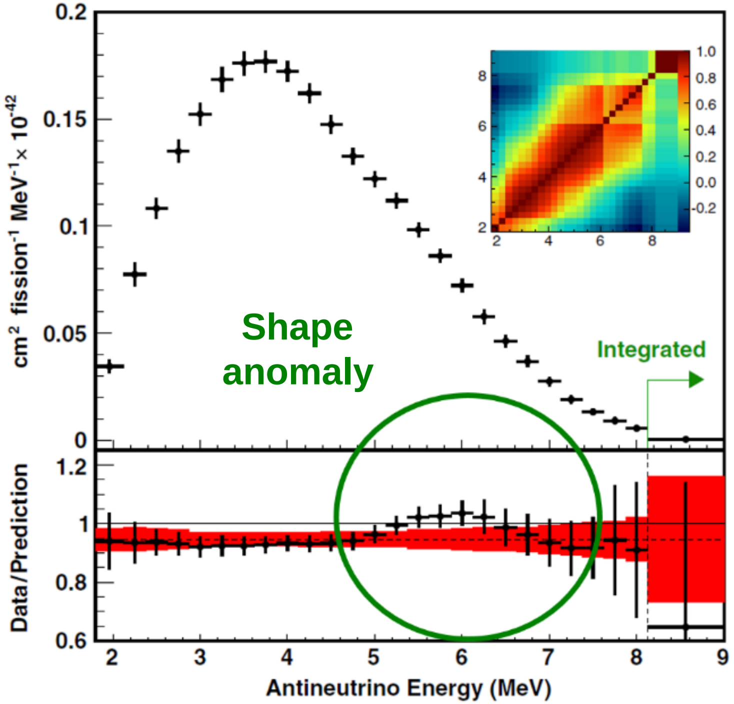

$^{63}$Ni K. Kossert, X. Mougeot, Appl. Radiat. Isot. 101, 40 (2015)

$^{60}$Co K. Kossert et al., Appl. Radiat. Isot. 134, 212 (2018)

$^{90}$Sr/$^{90}$Y K. Kossert, X. Mougeot, Appl. Radiat. Isot. 168, 109478 (2021)

# Sterile neutrino?




- ✓ **First evidence of divergence between measured and predicted antineutrino flux/spectra** from CEA.

  G. Mention et al., Phys. Rev. D 83, 073006 (2011)

- ✓ **Revision of the summation model.**
  - ✓ Nuclear structure included for dominant forbidden non-unique beta transitions.
  - ✓ First complete, robust and detailed uncertainty budget.

  L. Périssé et al., Phys. Rev. C 108, 055501 (2023)

- ✓ **Phenomenological Gamow-Teller decay strength to model missing transitions in databases.**

  → Bias on reference electron spectra.

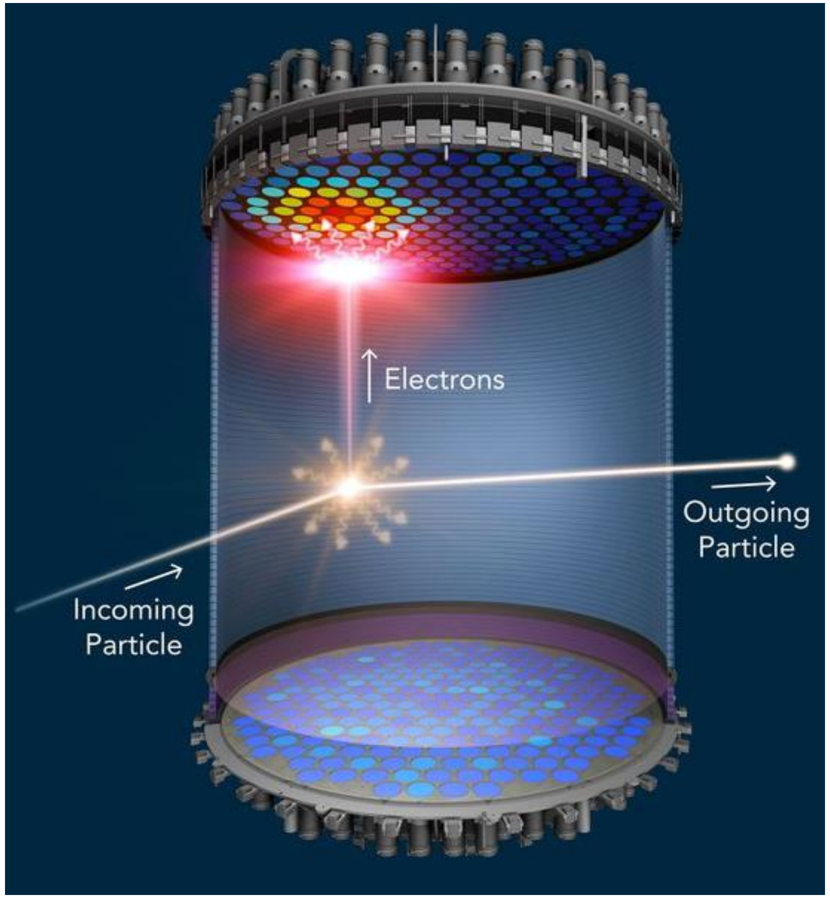
  A. Letourneau et al., Phys. Rev. Lett. 130, 021801 (2023)

# Dark matter

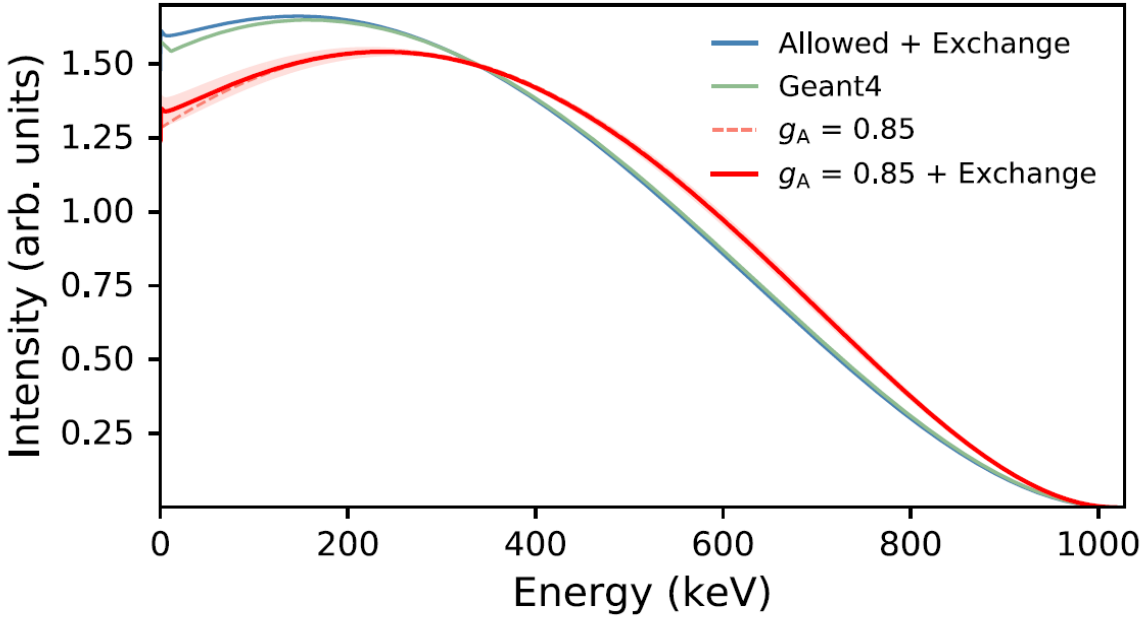
**XENON Collaboration**

Accurate description of low energy radioactive background is essential.

Excess of electron events as a mono-energetic peak at low energy (~ 2 keV).




$^{214}$Pb gs to gs decay: influence of nuclear structure




E. Aprile et al., Phys. Rev. D 102, 072004 (2020)

S.J. Haselschwardt et al., Phys. Rev. C 102, 065501 (2020)

# Testing the Standard Model with induced interactions

$$\frac{\mathrm{d}P}{\mathrm{d}W} \propto pWq^2 \times \left[1 \pm \frac{4}{3}\frac{W}{M} b_{\mathrm{wm}} + \frac{\gamma m_e}{W} \cdot b_{\mathrm{Fierz}}\right]$$

L. Hayen et al., Rev. Mod. Phys. 90, 015018 (2018)

**Weak magnetism**

Point-like nucleons → Finite size nucleons with internal structure.

**Fierz interference**

Additional interactions induced by exotic currents beyond the Standard Model.

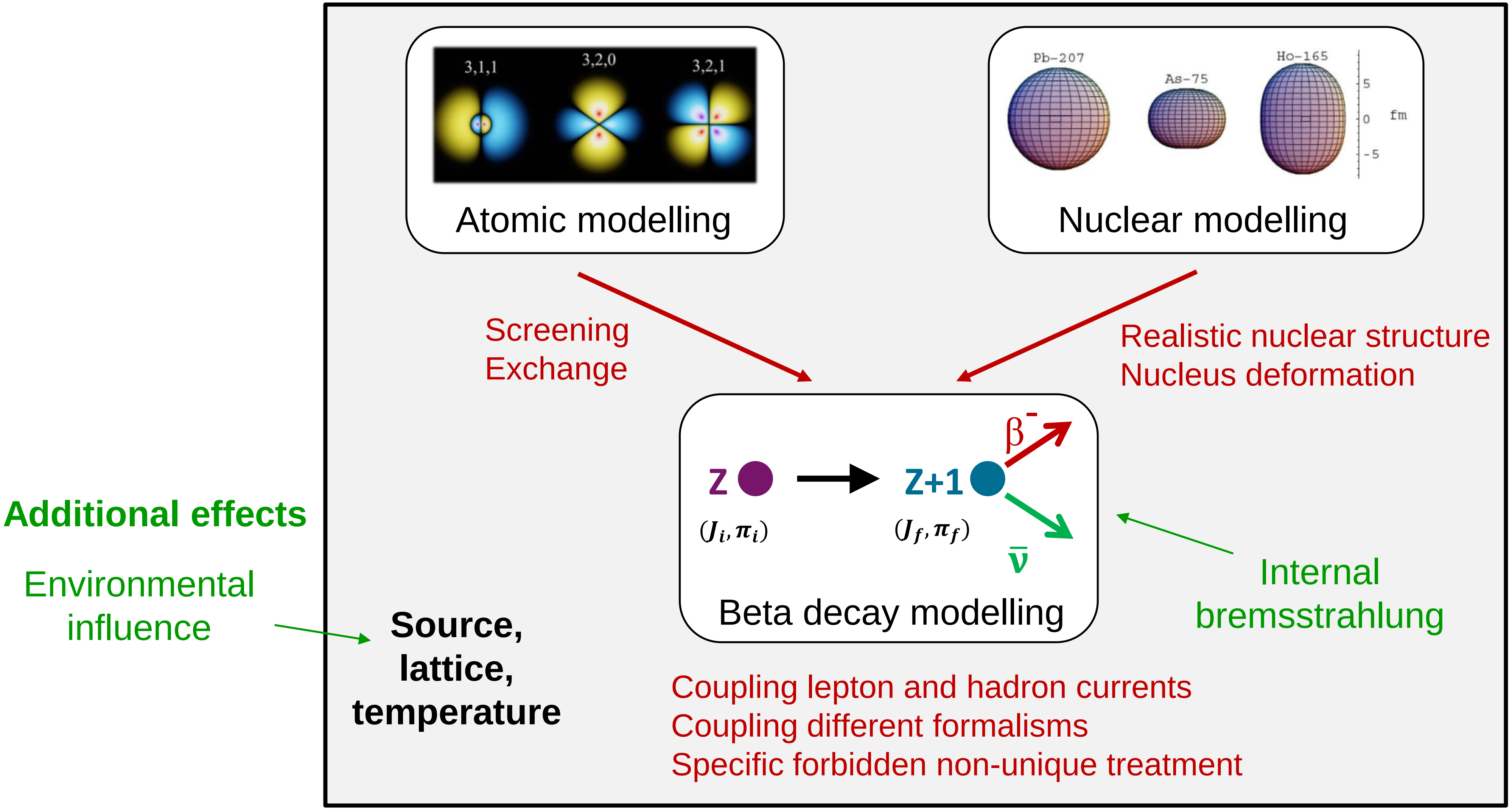
→ Their energy dependency allows an almost independent treatment through the analysis of both low and high energy transitions.

**French project bSTILED**: PI O. Naviliat-Cuncic (LPC Caen).

High-precision measurement of $^6$He beta spectrum at GANIL.

→ Phys. Rev. C 106, 045502 (2022): $T_{1/2} = (807.25 \pm 0.16_{stat} \pm 0.11 sys)$ ms

# Overview


3,1,1
3,2,0
3,2,1
Atomic modelling
Pb-207
As-75
Ho-165
5
0
-5
fm
Nuclear modelling
Screening
Exchange
Realistic nuclear structure
Nucleus deformation
Z
$(J_i, \pi_i)$
Z+1
$(J_f, \pi_f)$
$\beta^-$
$\bar{\nu}$
Beta decay modelling
Additional effects
Environmental influence
Source, lattice, temperature
Internal bremsstrahlung
Coupling lepton and hadron currents
Coupling different formalisms
Specific forbidden non-unique treatment

# Beta decays without nuclear structure

# Current situation in ENSDF database

Weak interaction properties in nuclear decay data are incomplete and come from calculation: mean energies, capture probabilities, log-*ft* values.

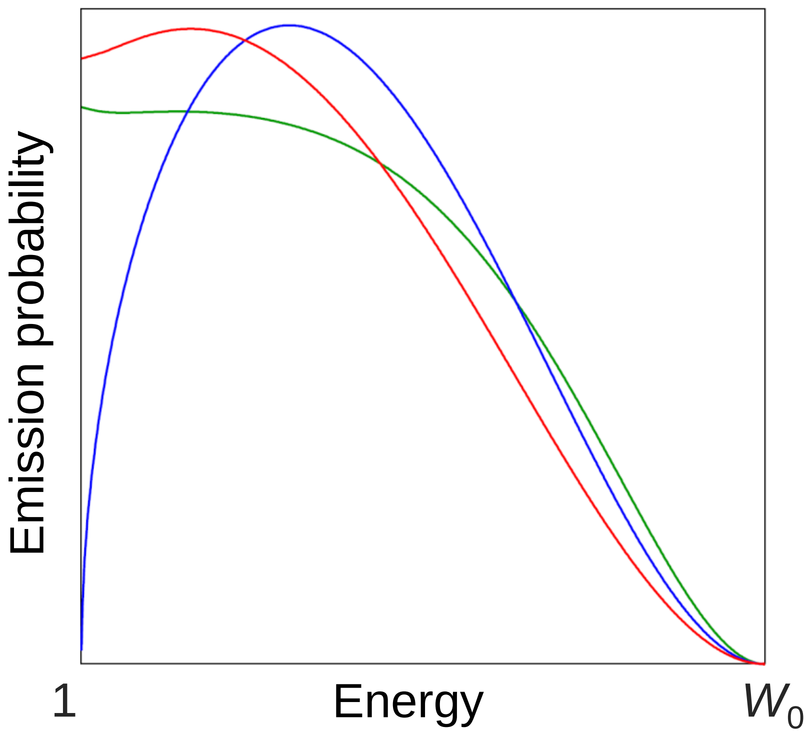
They have been determined for the last 50 years with the LogFT code, developed in the late 1960’s.

Quite simple analytical models of beta transitions and electron captures. Limited in forbiddenness degree.

# Beta spectrum shape

Phase space Fermi function Shape factor

$$\frac{dP}{dW} \propto \boxed{pWq^2} \cdot \boxed{F(Z,W)} \cdot \boxed{C(W)}$$




$W$ electron energy, $W_0$ transition energy

$p$ electron momentum, $q$ neutrino momentum

| | |
|---|---|
| **Allowed** | $C(W) = 1$ |
| **First forbidden unique** | $C(W) = q^2 + \lambda_2 p^2$ |
| **Second forbidden unique** | $C(W) = q^4 + \lambda_2 q^2 p^2 + \lambda_3 p^4$ |
| **Third forbidden unique** | $C(W) = q^6 + \lambda_2 q^4 p^2 + \lambda_3 q^2 p^4 + \lambda_4 p^6$ |
| | Etc. |

- ✓ The BetaShape program (version 2.4) now replaces the LogFT code for the new ENSDF evaluations. Electron captures also treated.

  → Available on IAEA-NSDD GitHub: https://github.com/IAEA-NSDDNetwork

- ✓ $F(Z,W)$ and $\lambda_k$ parameters determined from the relativistic electron wave functions, obtained by numerical solving of the Dirac equation.

- ✓ Included: extended nucleus; atomic exchange, overlap and screening; radiative corrections; database of experimental shape factors.

**For forbidden non-unique transitions, coupling with nuclear structure is necessary.**

→ ξ-approximation possible but accuracy is questionable.

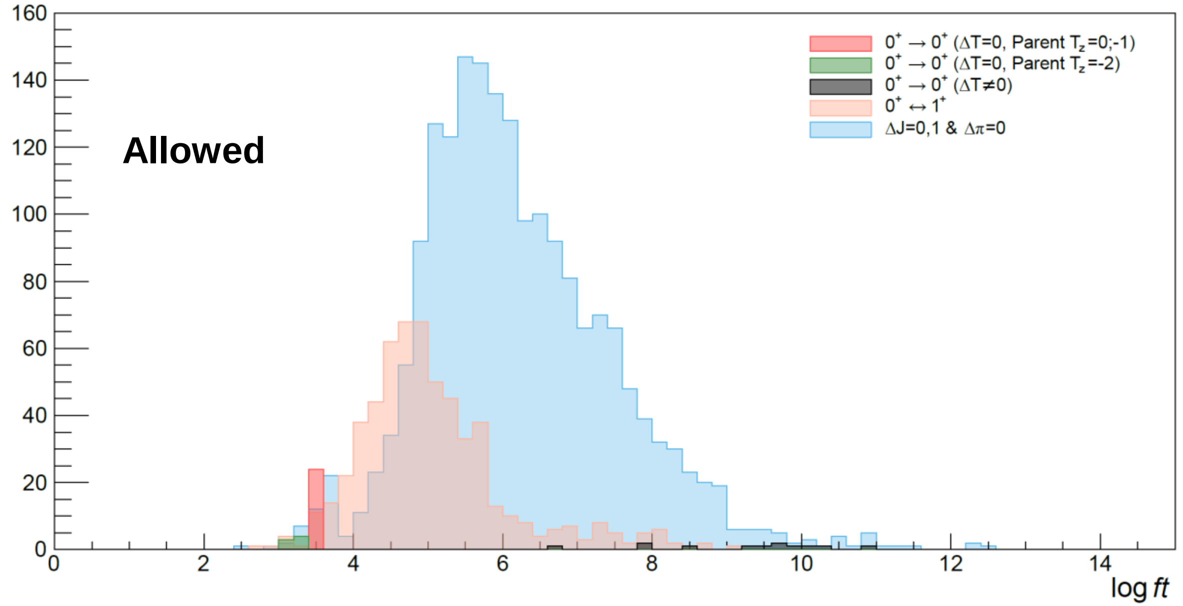
# New review of log-*ft* values

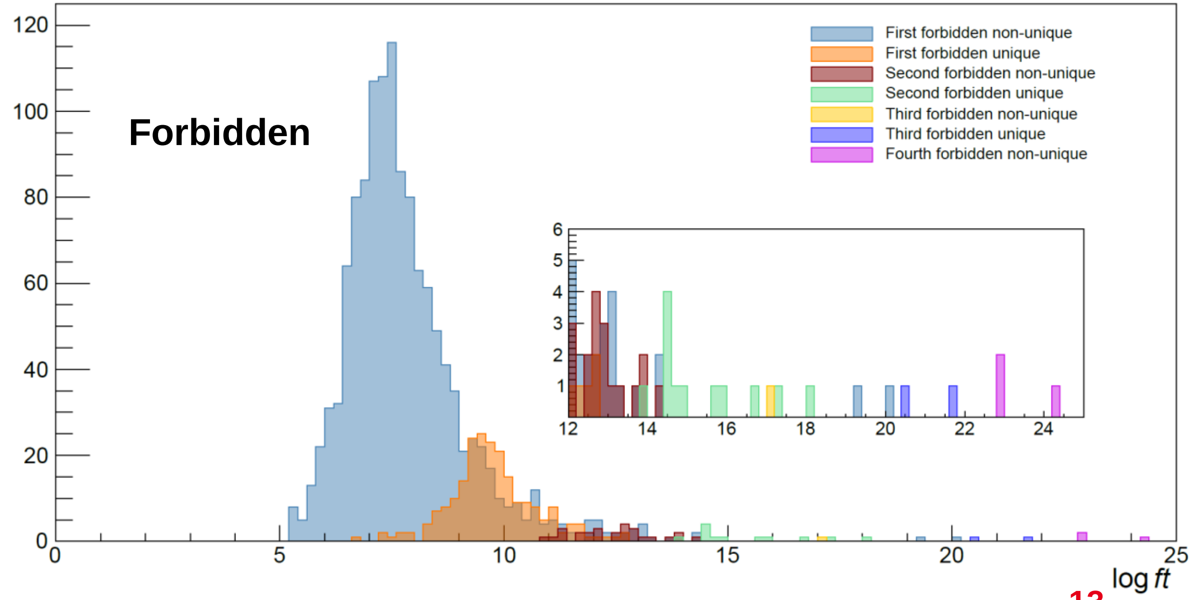
Former review of log-*ft* values, calculated with the LogFT code: *B. Singh et al., Nuclear Data Sheets 84, 487 (1998)*

These values are used in nuclear structure studies, e.g. to assign spin and parity to a level.

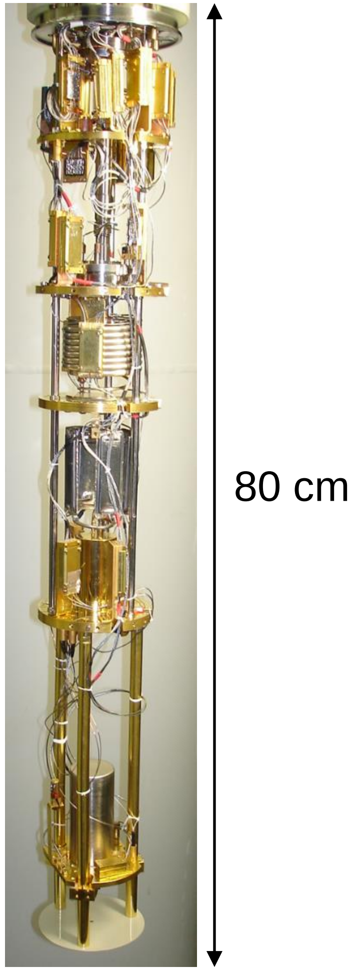
Collaborative work: B. Singh (McMaster University), S. Turkat and K. Zuber (TU Dresden), X. Mougeot (CEA-LNHB)

- ✓ Update of B and EC decays present in ENSDF database (as of mid-April 2023).
- ✓ Use of BetaShape to calculate the log-*ft* values.
- ✓ In total, 26 318 transitions calculated. Selection of well-defined transitions. Possible pandemonium nuclei flagged.
- ✓ 4 038 transitions survived this filtering. All distributions re-established. Specific transitions are discussed.
- ✓ *S. Turkat et al., Atomic Data and Nuclear Data Tables 152, 101584 (2023)*

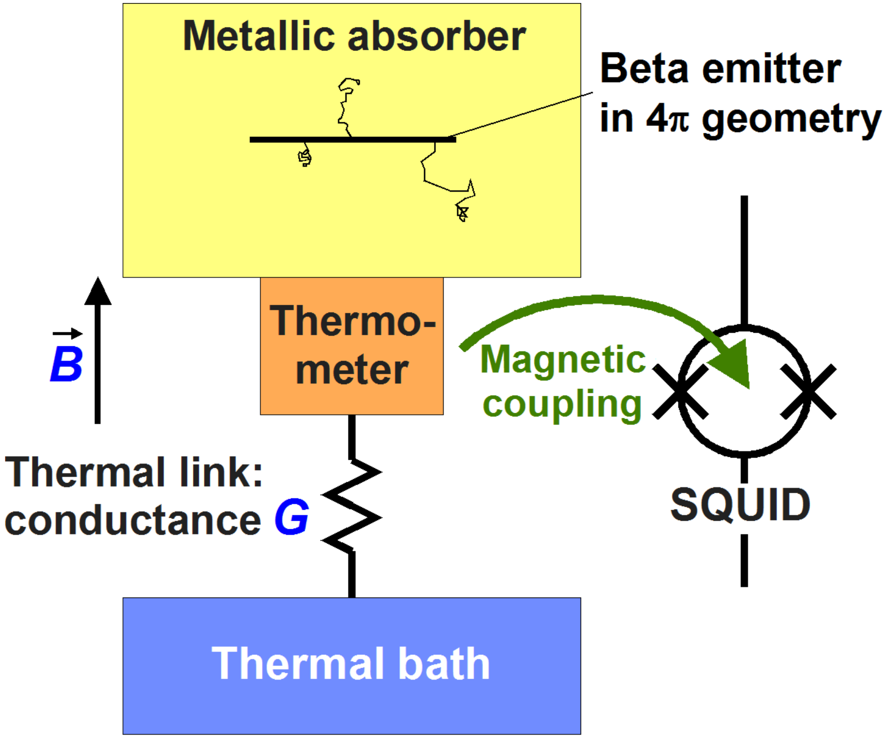


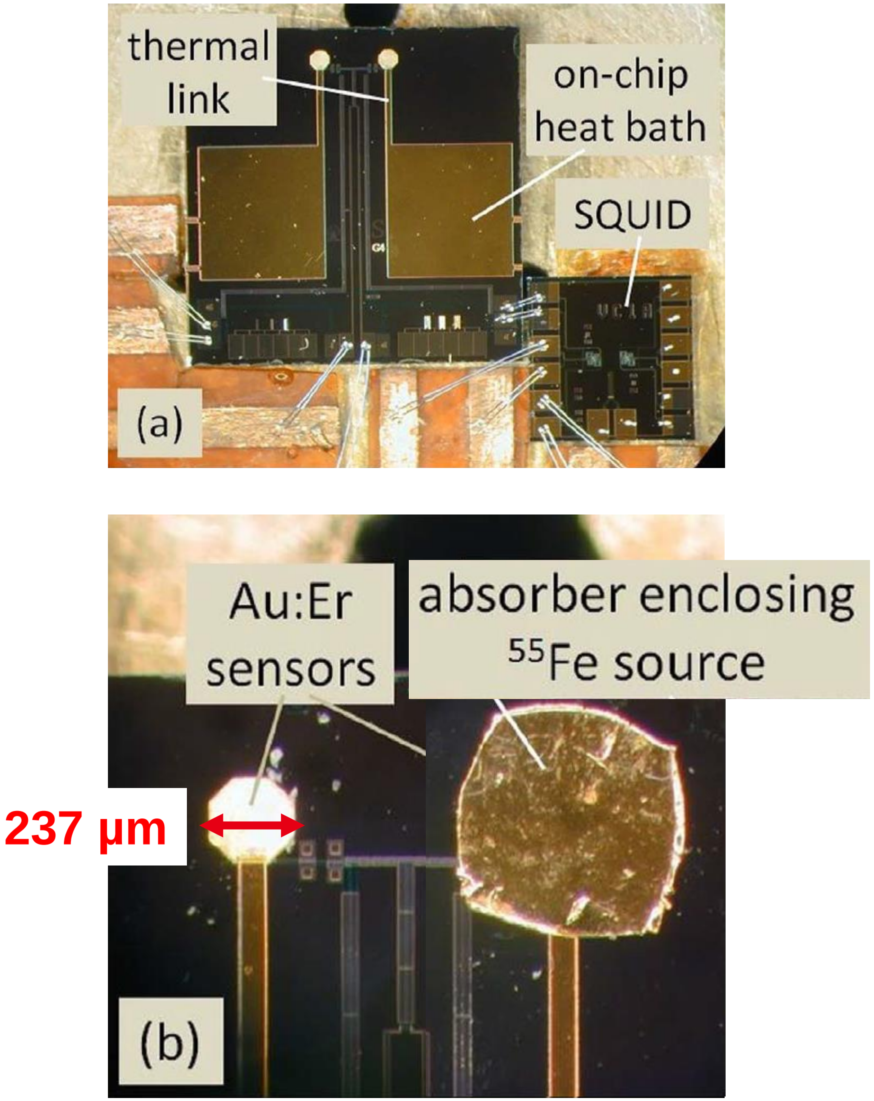
# Metallic magnetic calorimetry (MMC)



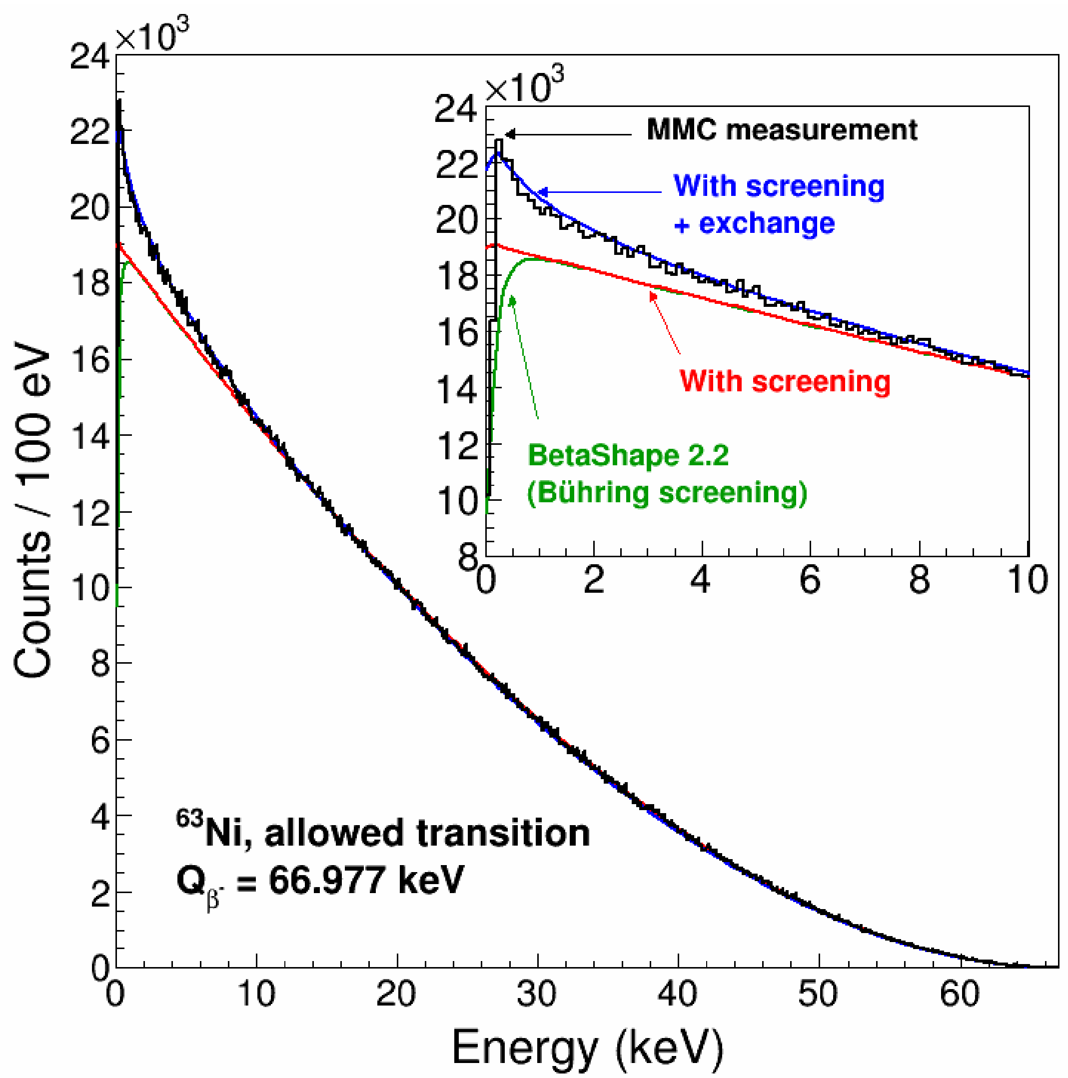

Dilution refrigerator
(10 mK)

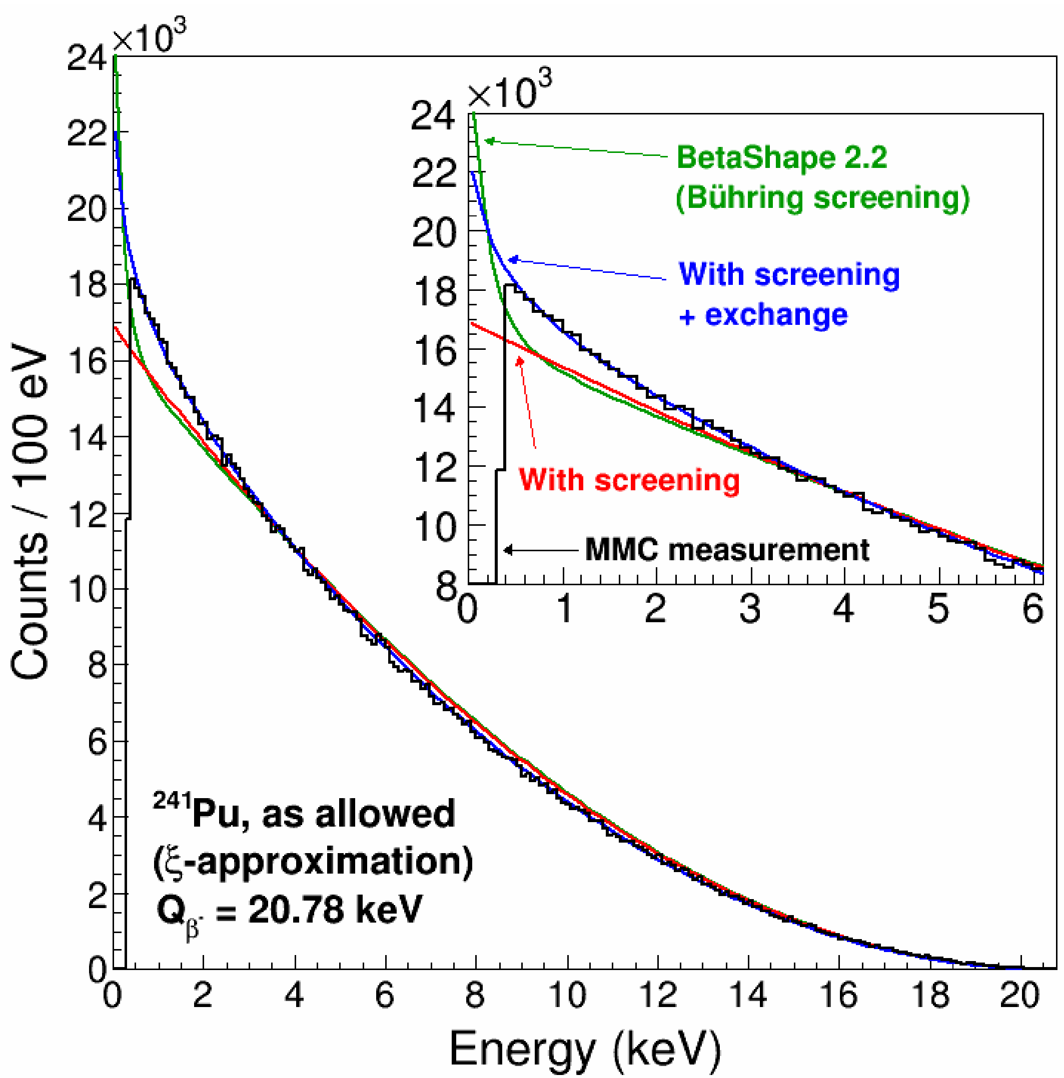



M. Loidl *et al.*, App. Radiat. Isot. 134, 395 (2018)



# Atomic screening and exchange effects



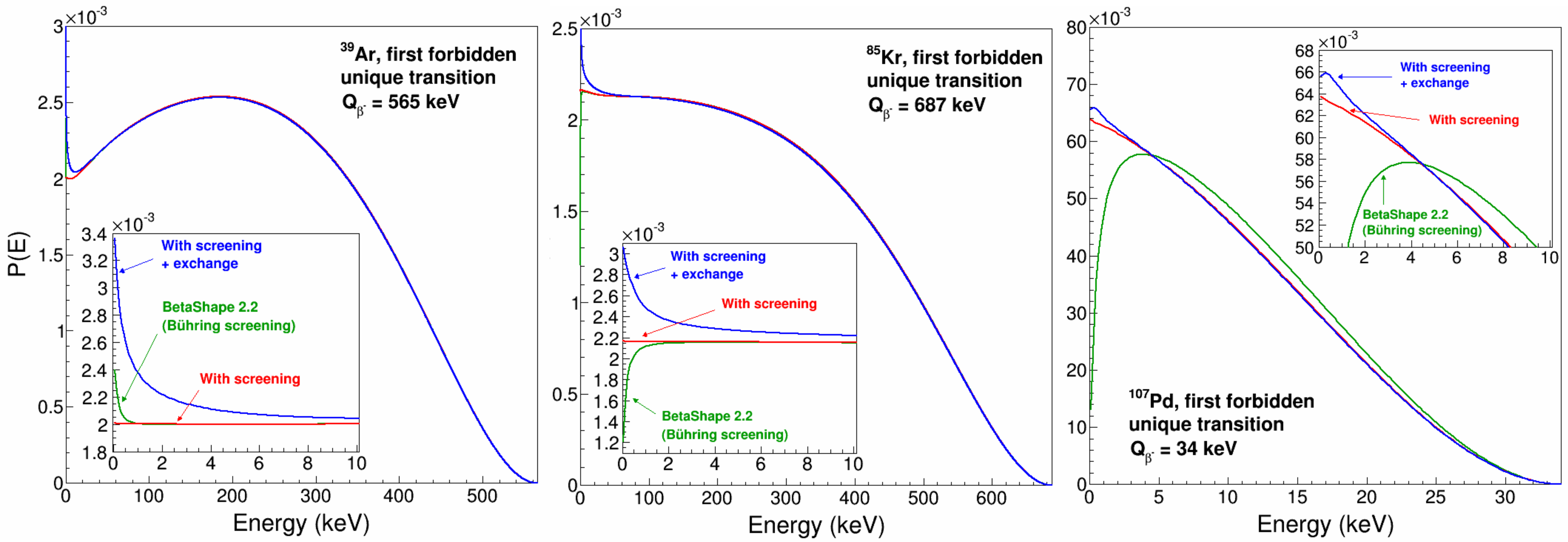
# Extension to forbidden unique transitions

→ **Contribution of additional atomic orbitals**

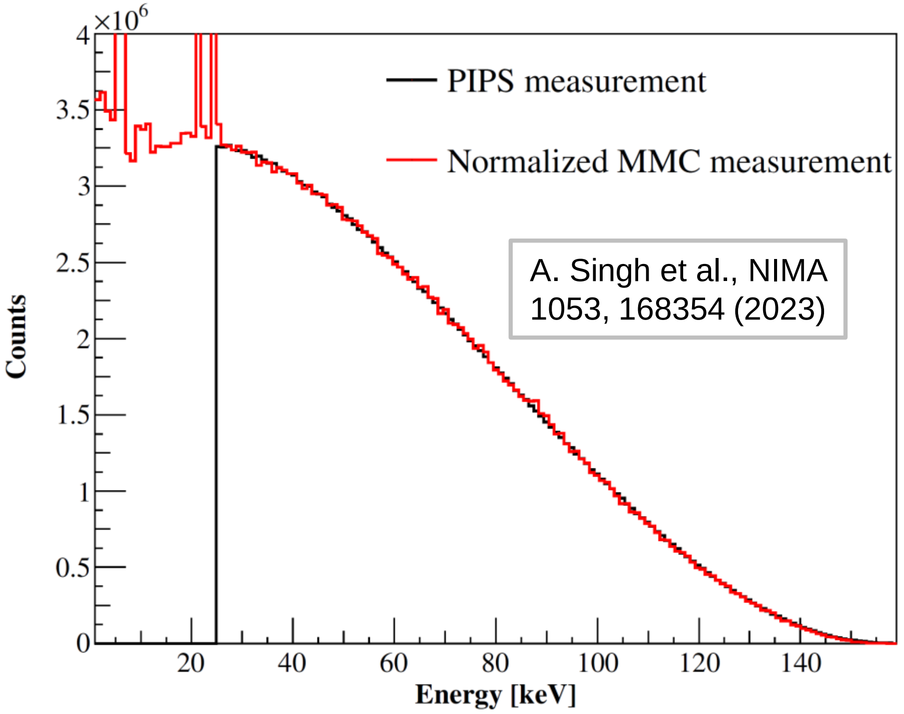
→ **Applied to dark matter (DarkSide-50 collaboration)**

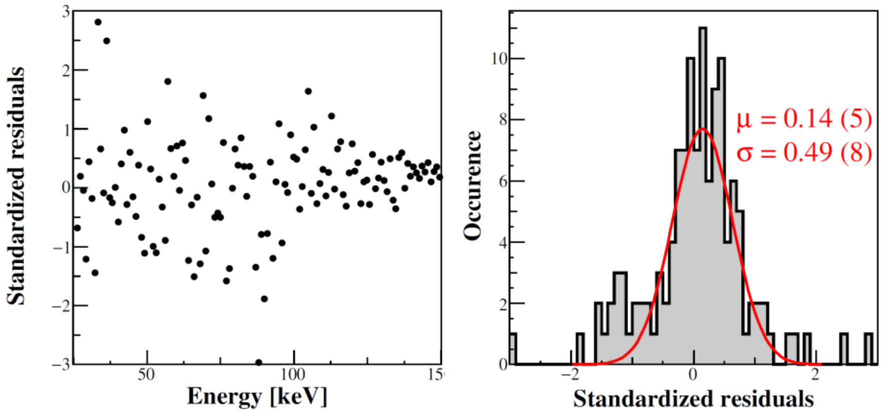
✓ *X. Mougeot, Appl. Radiat. Isot. 201, 111018 (2023)*

✓ *P. Agnes et al., Phys. Rev. D 107, 063001 (2023)*



# $^{14}C$ allowed decay




**Allowed transition with very long half-life → Nuclear structure effect**

- Measurement with silicon detectors and ultra-thin source. Unfolding process based on precise Monte Carlo simulations to correct for residual distortions: particle escape, dead layers, auto absorption.
- Comparison with a high-precision measurement with a Metallic Magnetic Calorimeter (MMC) is possible.
- ✓ Excellent agreement of the spectra in the common energy range.
- ✓ Extracted Q-value = 156.49 (49) keV fully consistent with AME2020 value of 156.476 (4) keV.
- ✓ Controversy on the spectrum shape: weak magnetism term confirmed.

| Study | $a$ in $\mathrm{MeV}^{-1}$ | Comment |
|---|---|---|
| [44] | –0.386 | CVC from exp. not certain |
| [45] | –0.37 (4) | SM, ×2 difference with CVC |
| [36] | –0.43 | SM, consistent with CVC |
| [41] | –0.45 (4) | $^{14}$C-doped Ge detector |
| [8] | –1.038 (28) | Wall-less prop. counter |
| This work | –0.430 (37) | Si detector, with $F_0 L_0$ |

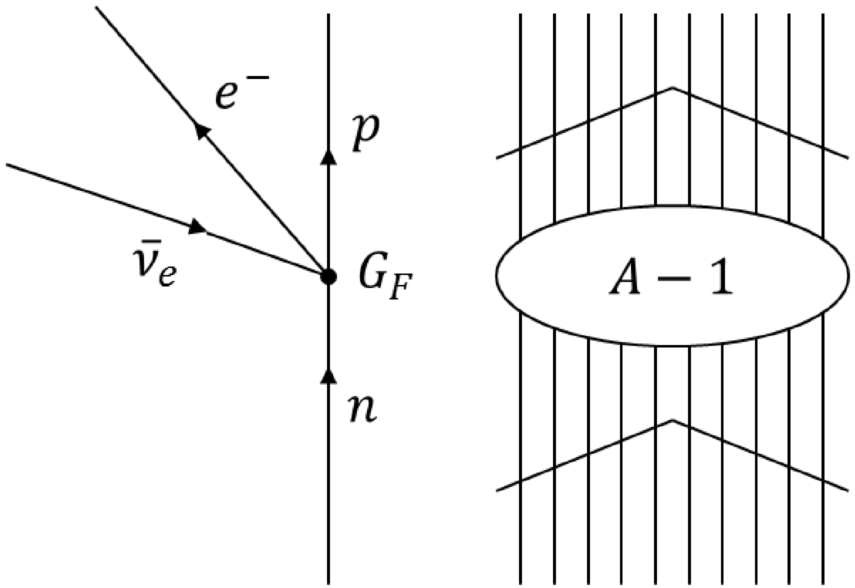
# Including nuclear structure

# Theoretical shape factor

$$C(W_e) = \sum_{K k_e k_\nu} \lambda_{k_e} \left[ M_K^2(k_e, k_\nu) + m_K^2(k_e, k_\nu) - \frac{2\mu_{k_e}\gamma_{k_e}}{k_e W_e} M_K(k_e, k_\nu) m_K(k_e, k_\nu) \right]$$

H. Behrens, W. Bühring, *Electron Radial Wave functions and Nuclear Beta Decay*, Oxford Science Publications (1982)

**Multipole expansion** of lepton and nuclear currents. Calculation of shape factors, half-lives, branching ratios, log-*ft* values.




**Fermi theory**

- Vertex of the weak interaction is assumed to be point-like. No propagation of $W^\pm$ boson.
- Effective coupling constant $G_F$.
- Relativistic lepton and nuclear wave functions.
- Non-relativistic and relativistic vector or axial-vector matrix elements.

**Impulse approximation**

- The nucleon is assumed to feel only the weak interaction.
- Other nucleons are spectators.

# Realistic nuclear structure

Nuclear state described as a **superposition of nucleon states**.

**One-body transition densities** must be given by a nuclear structure model.

$$\langle \xi_f J_f || T_\lambda || \xi_i J_i \rangle = \hat{\lambda}^{-1} \sum_{a,b} \langle a || T_\lambda || b \rangle \left\langle \xi_f J_f \left\| \left[ c_a^\dagger \tilde{c}_b \right]_\lambda \right\| \xi_i J_i \right\rangle$$

transition matrix element

tensor rank

single particle matrix element

one-body transition density (OBTD)

**Transition described as a sum of transformations of single nucleons.**

Example: Nonrelativistic vector matrix element

$${}^{V}\mathcal{M}_{KK0}(q^2) = \frac{\sqrt{2}}{\sqrt{2J_i+1}} \cdot \frac{(2K+1)!!}{(qR)^K} \times \left[ G_{KK0}(\kappa_f, \kappa_i) \int_0^\infty g_f(r,\kappa_f)\, j_K(qr)\, g_i(r,\kappa_i)\, r^2 \mathrm{d}r + S_{\kappa_f} S_{\kappa_i} G_{KK0}(-\kappa_f, -\kappa_i) \int_0^\infty f_f(r,\kappa_f)\, j_K(qr)\, f_i(r,\kappa_i)\, r^2 \mathrm{d}r \right]$$

Components of the relativistic bound wave function of the nucleon

Geometrical coefficients

# Forbidden non-unique transitions

Relativistic matrix element

Non-relativistic matrix elements

$$M_K(k_e,k_\nu) = C_K (pR)^{k_e-1}(qR)^{k_\nu-1}\left\{-\sqrt{\frac{2K+1}{K}}\,{}^VF^{(0)}_{K,K-1,1} - \frac{\alpha Z}{2k_e+1}\,{}^VF^{(0)}_{K,K,0}(k_e,1,1,1)\right.$$

$$\left. -\left[\frac{WR}{2k_e+1}+\frac{qR}{2k_\nu+1}\right]{}^VF^{(0)}_{K,K,0} - \frac{\alpha Z}{2k_e+1}\sqrt{\frac{K+1}{K}}\,{}^AF^{(0)}_{K,K,1}(k_e,1,1,1) - \left[\frac{WR}{2k_e+1}-\frac{qR}{2k_\nu+1}\right]\sqrt{\frac{K+1}{K}}\,{}^AF^{(0)}_{K,K,1}\right\}$$

**Nuclear structure models are non-relativistic, but forbidden non-unique transitions are sensitive to $^{V}F_{K,K-1,1}$**

→ **Conserved Vector Current (CVC) hypothesis**

- Derived from the gauge invariance of the weak interaction.
- Provides relationships between non-relativistic and relativistic vector matrix elements.
- Depends on the Coulomb displacement energy $\Delta E_C$.

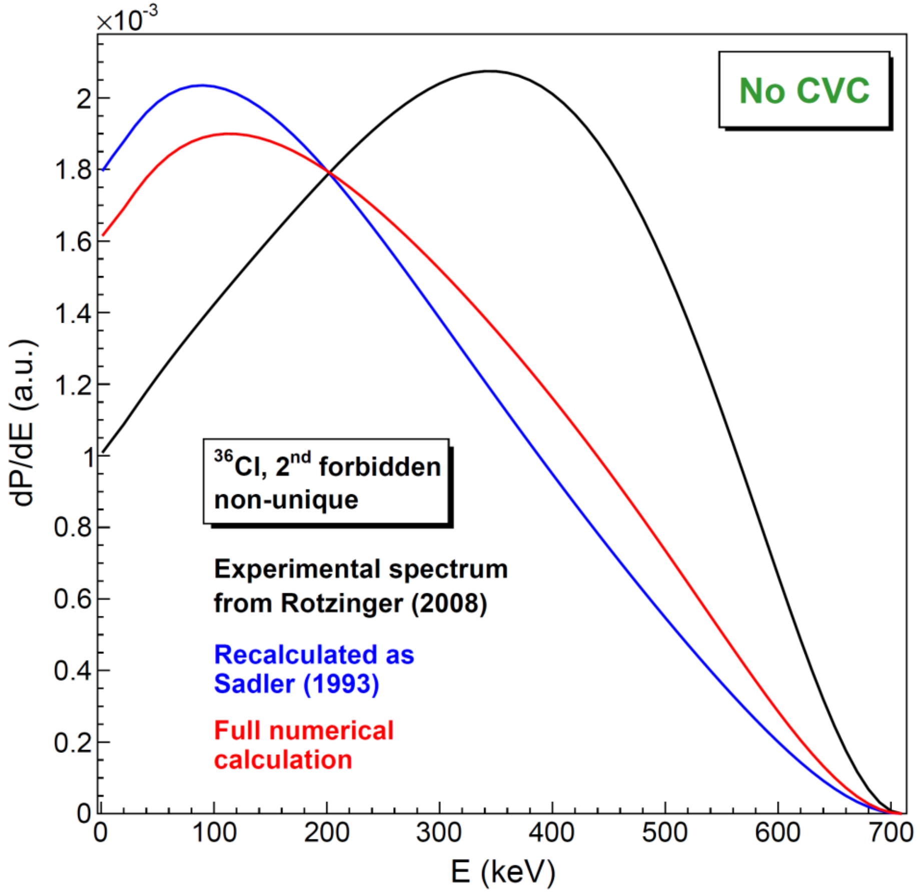
**Influence of lepton current treatment**: usually approximated only to dominant terms (formula here).

→ Full numerical treatment is also possible, avoiding approximations and next-to-leading-order terms.

# $^{36}$Cl 2$^{nd}$ forbidden non-unique decay

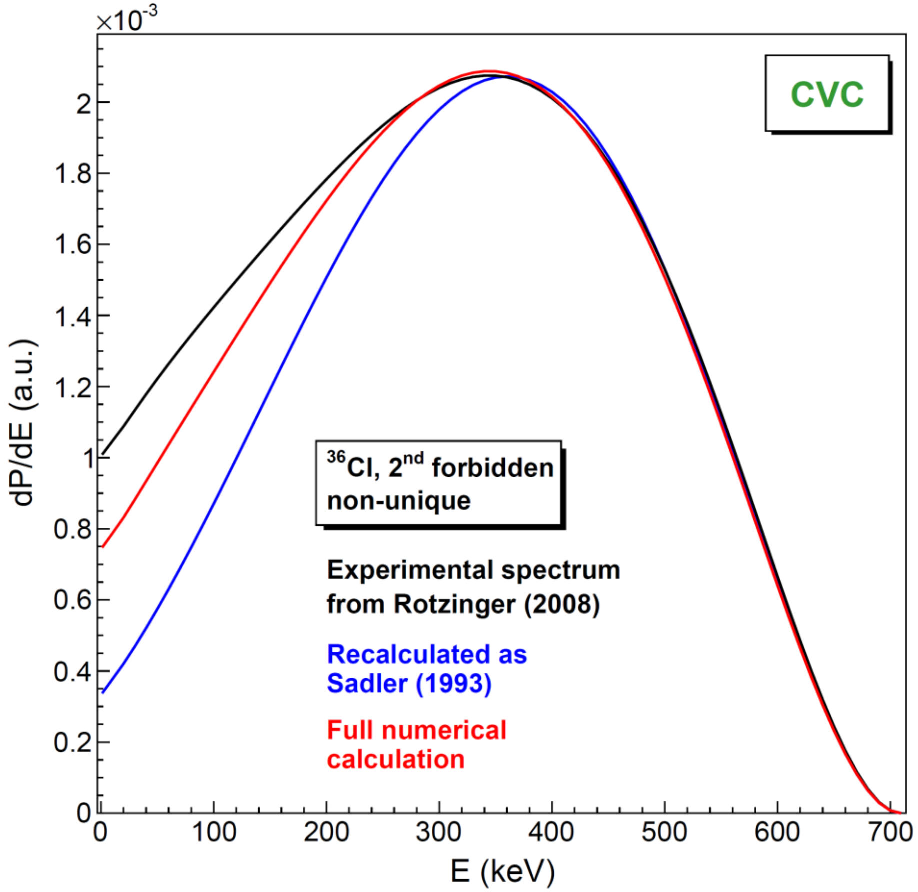
Precise measurement exists

Rotzinger et al., J. Low Temp. Phys. 151, 1087 (2008)



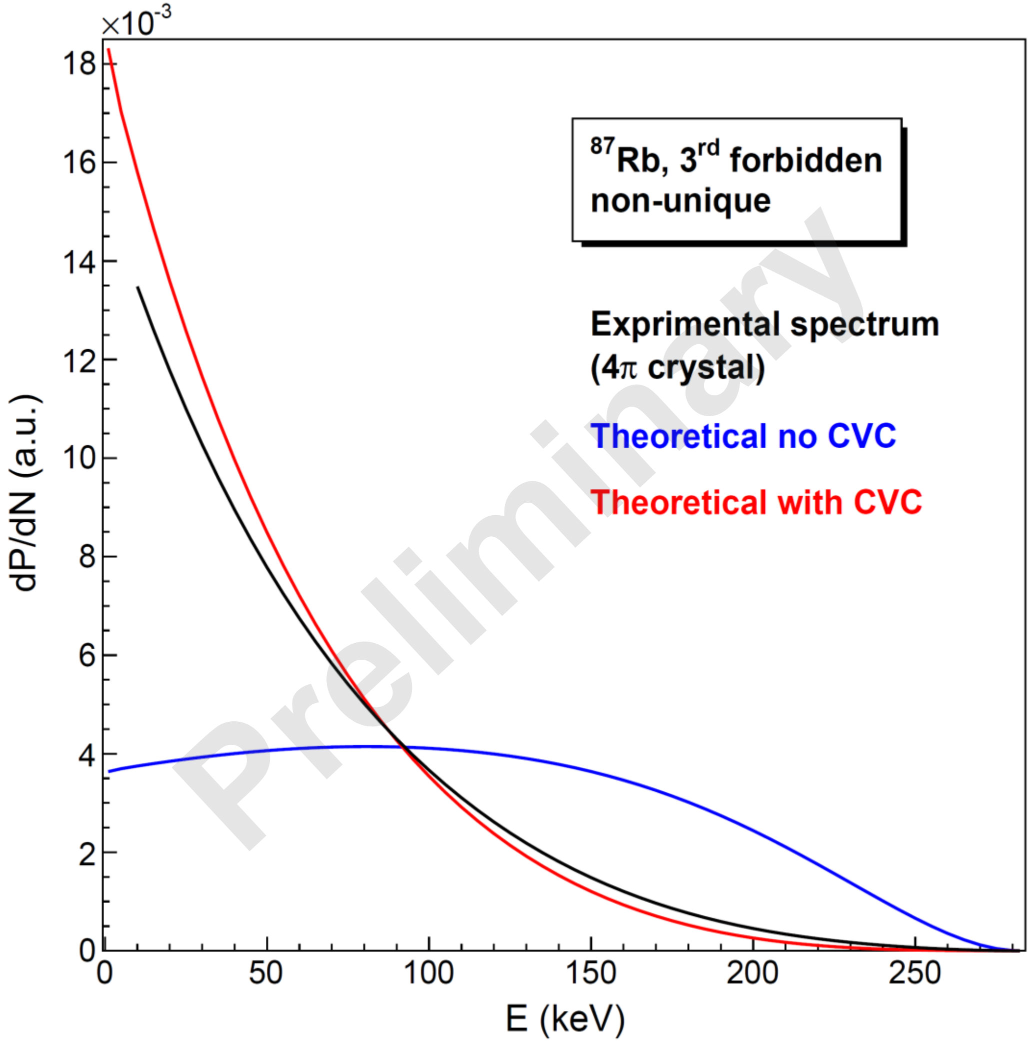

Detailed theoretical study (simplified lepton current)

Sadler, Behrens, Z. Phys. A 346, 25 (1993)




→ **CVC hypothesis mandatory + Influence of lepton current treatment**

# $^{87}$Rb 3$^{rd}$ forbidden non-unique decay

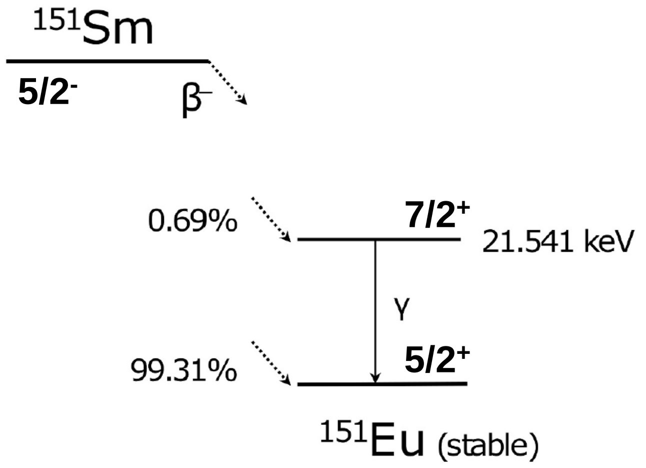



$$^{87}\mathrm{Rb}\ (\mathrm{gs}, 3/2^-) \rightarrow {}^{87}\mathrm{Sr}\ (\mathrm{gs}, 9/2^+)$$

- Third forbidden non-unique transition
- NushellX $^{56}$Ni doubly magic core, jj44 model space, jj44b interaction
- Preliminary measurement from the European MetroBeta project (4π $RbGd_2Br_7$ crystal)

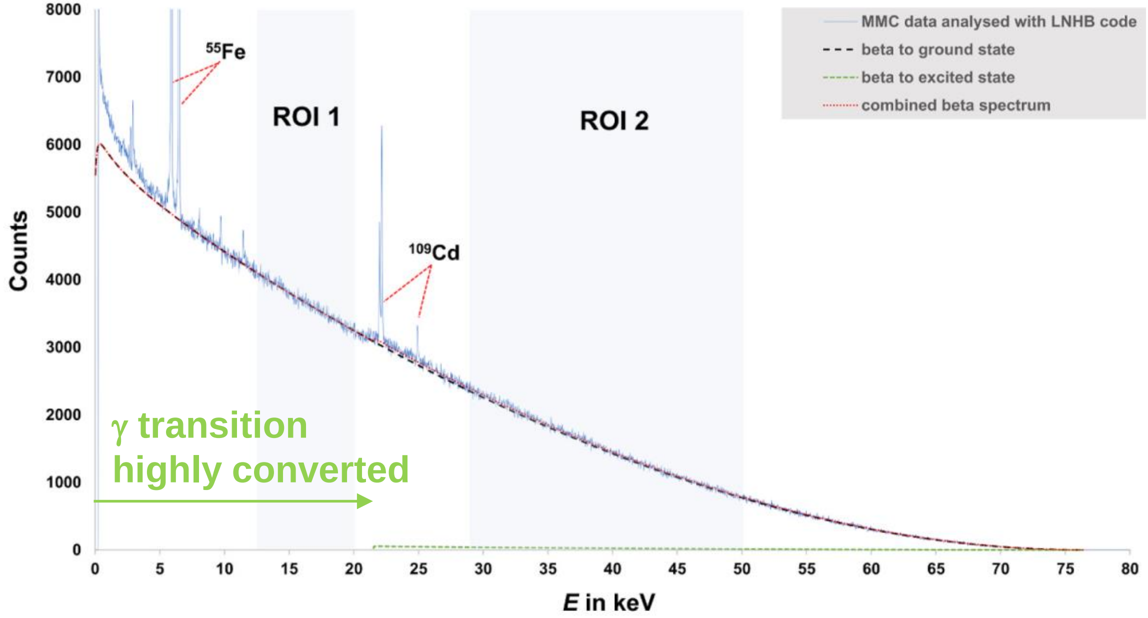
→ **CVC hypothesis mandatory to describe the spectrum correctly**

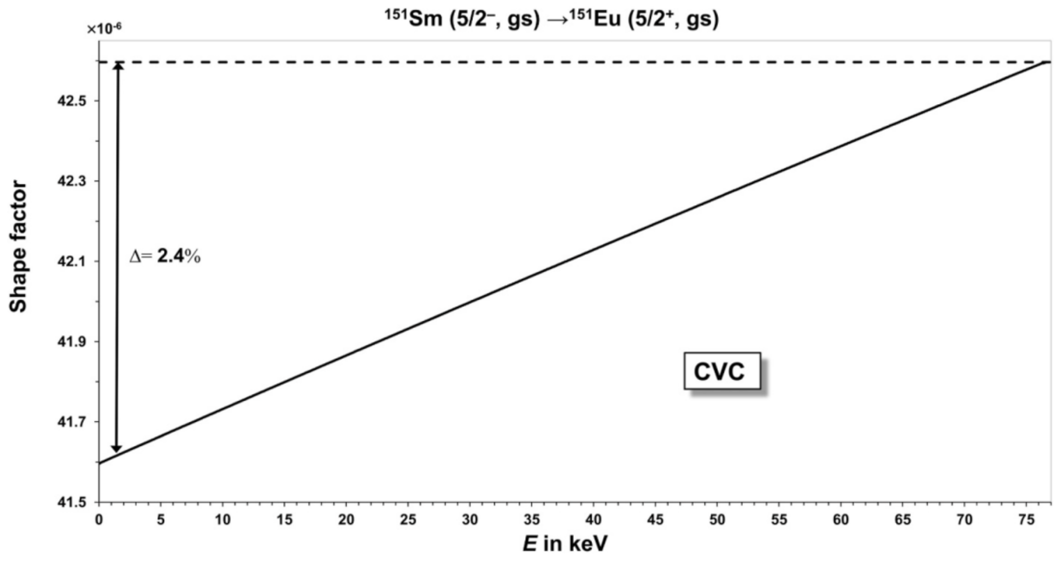
# $^{151}$Sm decay




**First forbidden non-unique transitions**

- ✓ High-precision measurement of $^{151}$Sm spectrum with Metallic Magnetic Calorimeters (MMC) at LNHB.
- ✓ **New Q-value = 76.430 (68) keV** more precise than AME2020 value of 76.5 (5) keV.
- ✓ New determination of branching ratios: 99.31 (11)% and 0.69 (11)%.
- ✓ Kossert et al., Appl. Radiat. Isot. 185, 110237 (2022)

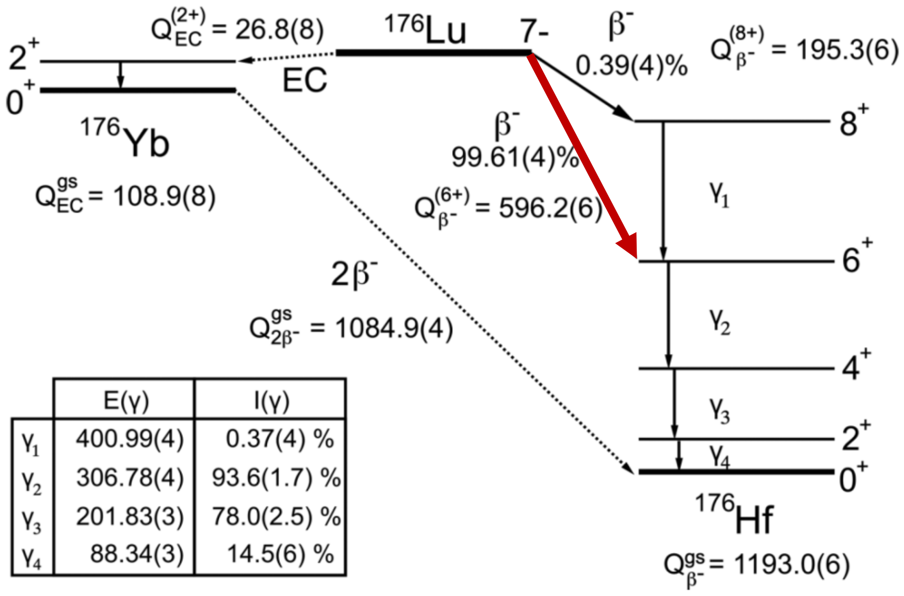


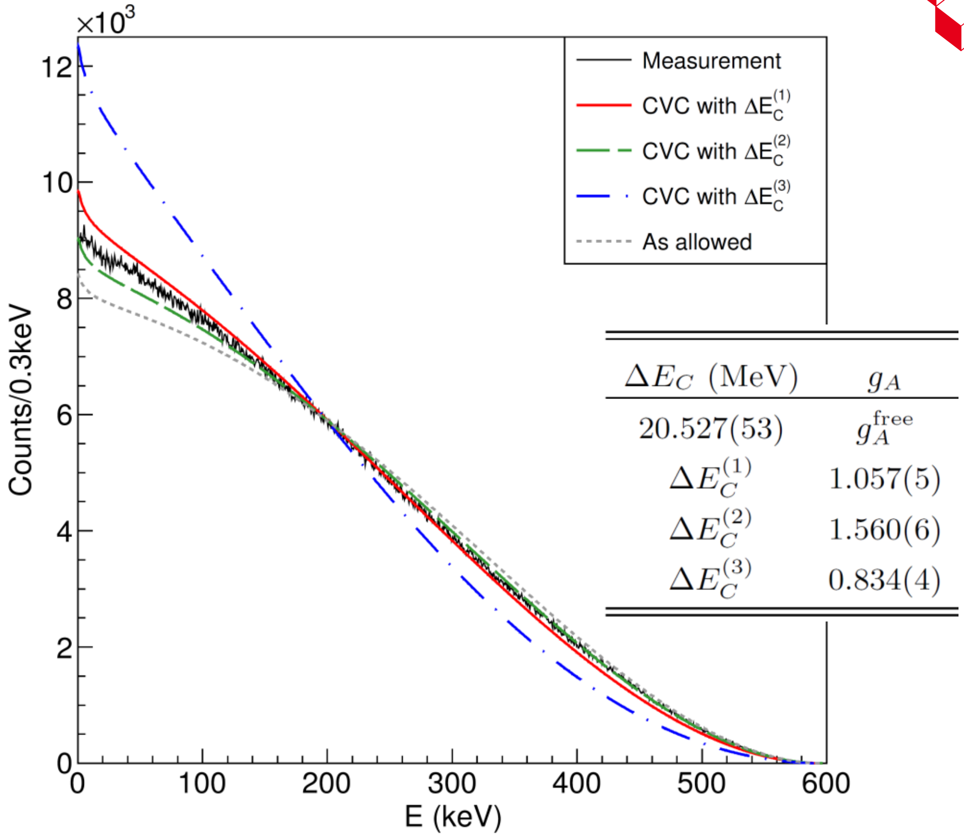

**Detailed calculations with nuclear structure**

→ Testing the accuracy of the ξ-approximation

# $^{176}$Lu decay




| | E(γ) | I(γ) |
|---|---|---|
| $\gamma_1$ | 400.99(4) | 0.37(4) % |
| $\gamma_2$ | 306.78(4) | 93.6(1.7) % |
| $\gamma_3$ | 201.83(3) | 78.0(2.5) % |
| $\gamma_4$ | 88.34(3) | 14.5(6) % |



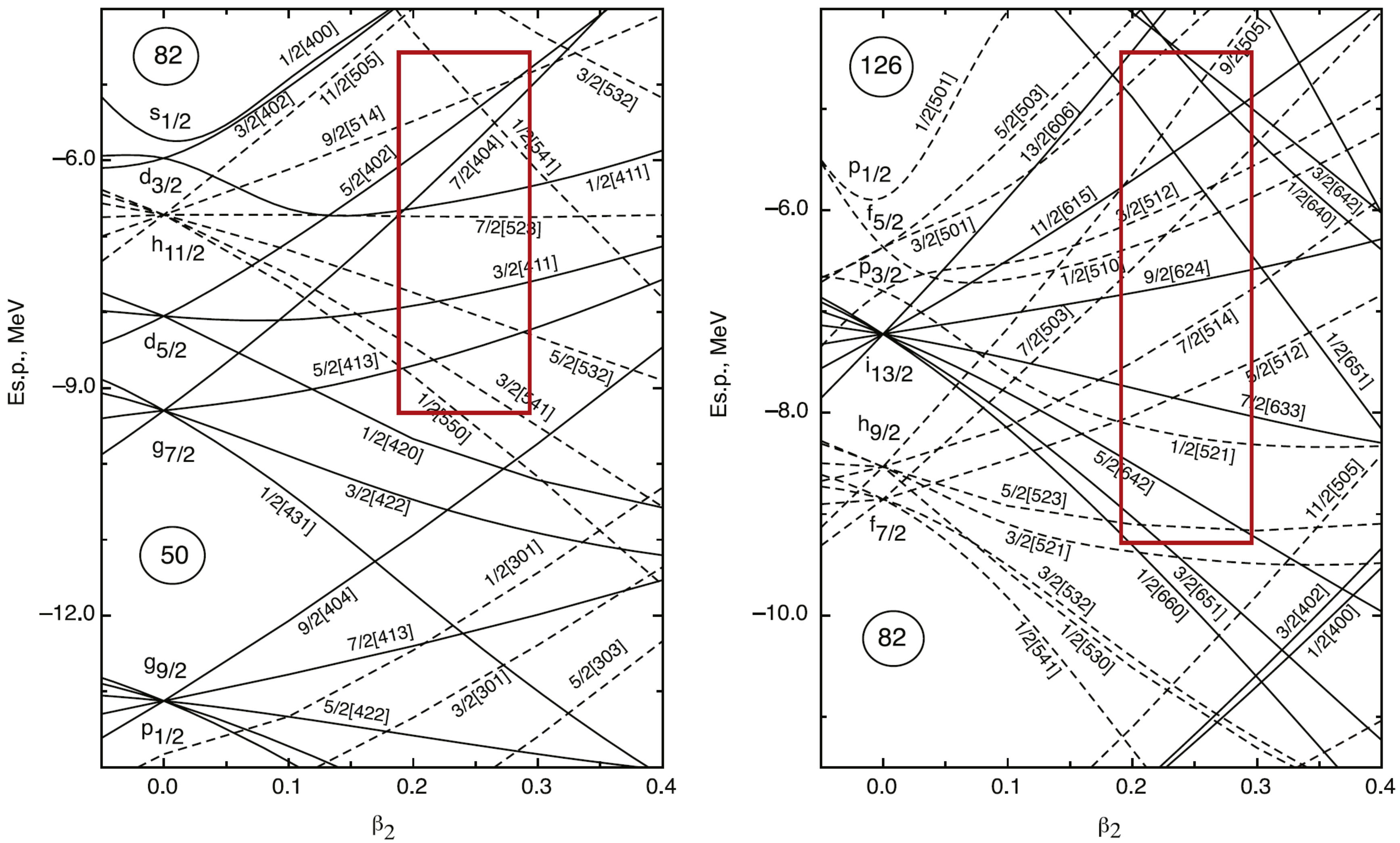

| $\Delta E_C$ (MeV) | $g_A$ |
|---|---|
| 20.527(53) | $g_A^{free}$ |
| $\Delta E_C^{(1)}$ | 1.057(5) |
| $\Delta E_C^{(2)}$ | 1.560(6) |
| $\Delta E_C^{(3)}$ | 0.834(4) |

**First forbidden non-unique transition**

- ✓ First high-precision spectrum measurement from self-scintillation of a LuAG:Pr crystal at TU Delft.
- ✓ New Q-values: $Q_\beta$ = 1193.0 (6) keV and $Q_\varepsilon$ = 108.9 (8) keV. From AME2020: $Q_\beta$ = 1194.1 (9) keV and $Q_\varepsilon$ = 109.0 (12) keV.
- ✓ Spectrum shape retrieved adjusting the Coulomb displacement energy $\Delta E_C$ or the axial-vector coupling constant $g_A$.
- × Calculated half-life shorter by 13 orders of magnitude!

  → **Detailed analysis would require accurate modelling with nuclear deformation: hindered transition ($\Delta K = 7$).**
- ✓ F.G.A. Quarati et al., Phys. Rev. C 107, 024313 (2023).

# K isomers in A ~ 170 – 190 region




Kondev, Dracoulis and Kibédi, ADNDT 103-104, 50-105 (2015)

**Fig. 2.** Nilsson levels for protons (left) and neutrons (right) in the *A* ~ 170–190 region. Boxes indicate the main orbitals of interest.

# Effective coupling constants

- Free-nucleon value $g_V$ = 1 according to CVC
- Free-nucleon value $g_A$ = 1.2754 (13) [PDG 2020]

**Review of J. Suhonen in Front. Phys. 5, 55 (2017)**

→ Coupling constants $g_V$ and $g_A$ of the weak interaction can be affected by:

- Nuclear medium effects

  *The nucleon decays within a finite nucleus. Beyond the impulse approximation.*

- Nuclear many-body effects

  *Simplification of the many-body problem: core excitation, nucleon correlations, etc.*

Unfortunately, it is almost impossible to disentangle between these two categories of effects by analyzing beta decays.

Spectral shapes can help for a better quantification of the effective values → **the Spectrum Shape Method**.

→ **1st forbidden non-unique**

$g_V^{\text{eff}}$ ~ 0.3 – 0.7 and $g_A^{\text{eff}}$ ~ 0.46 – 0.56

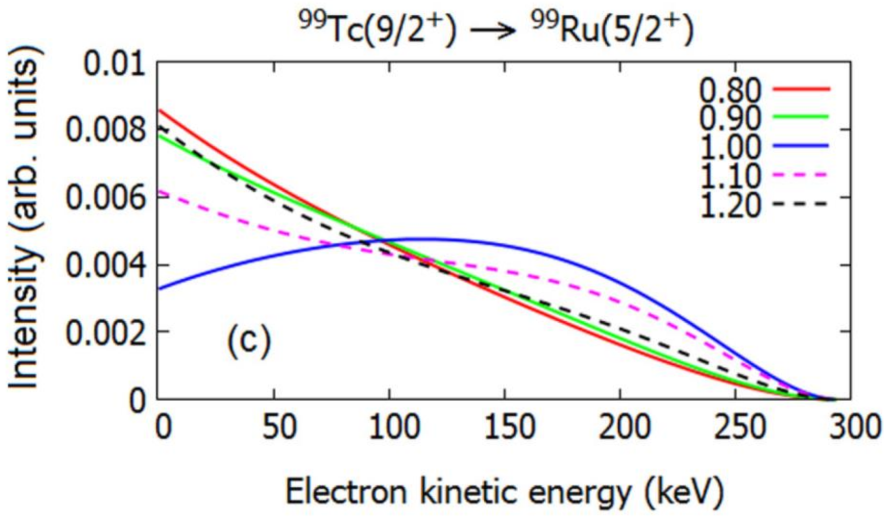
→ **Higher non-unique**

Lack of high-quality measurements

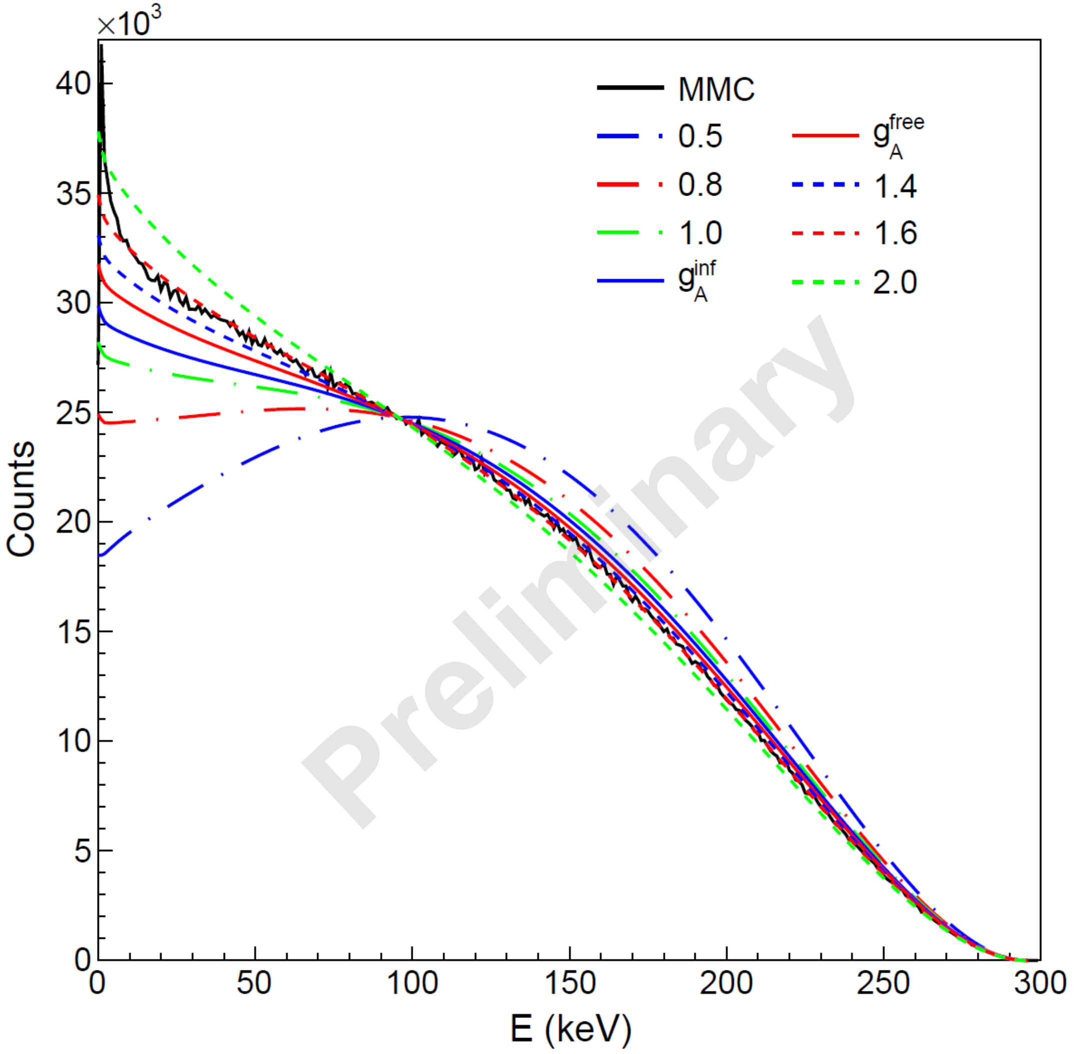
Suggests $g_V$ = 1 and $g_A^{\text{eff}}$ ~ 0.4

$^{99}$Tc beta spectrum, second forbidden non-unique, predicted to be very sensitive to $g_A$.

J. Kostensalo, J. Suhonen, PRC 96, 024317 (2017)



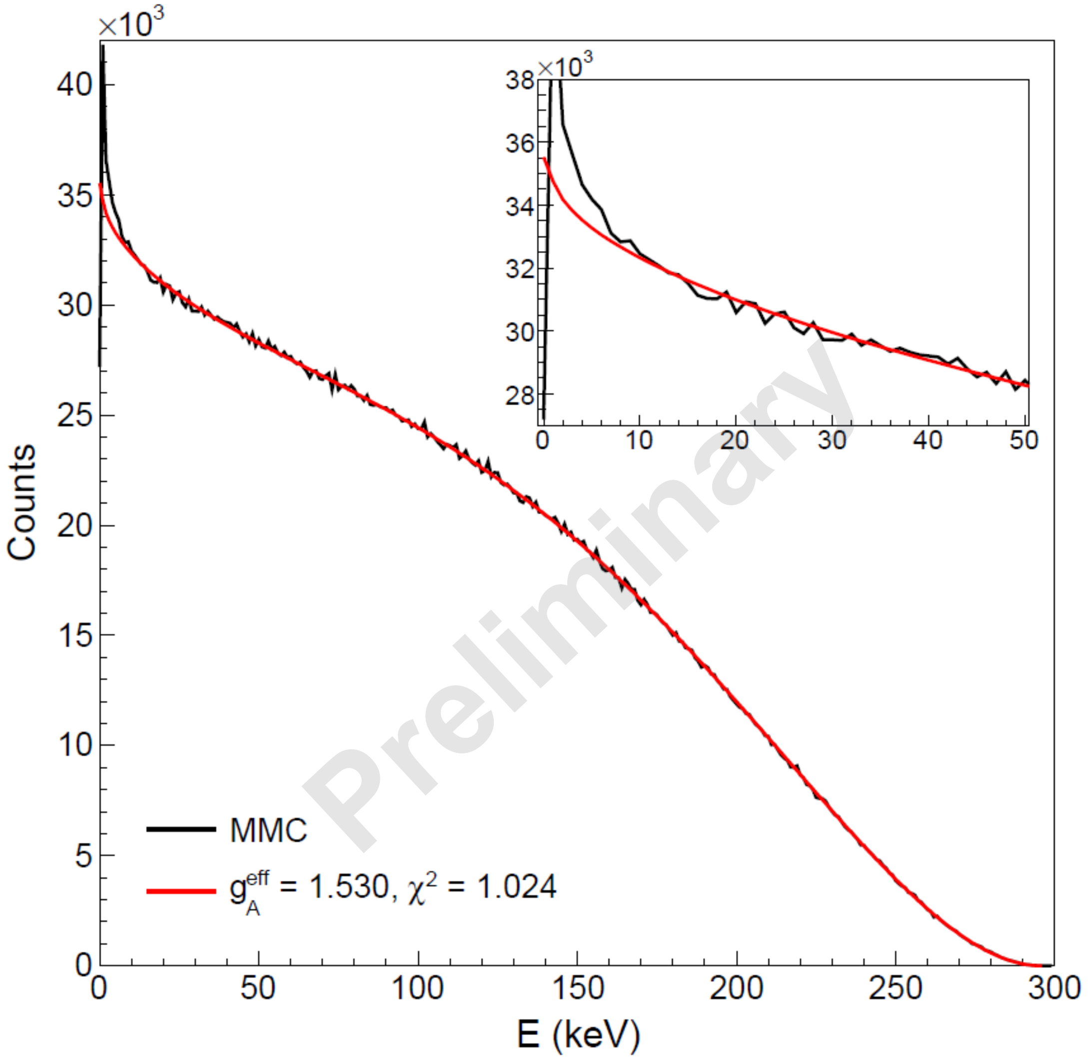
# $^{99}$Tc 2$^{nd}$ forbidden non-unique decay



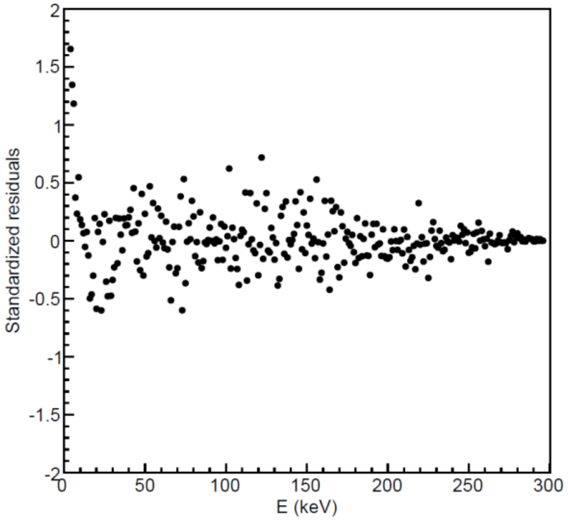

Within the European project PrimA-LTD

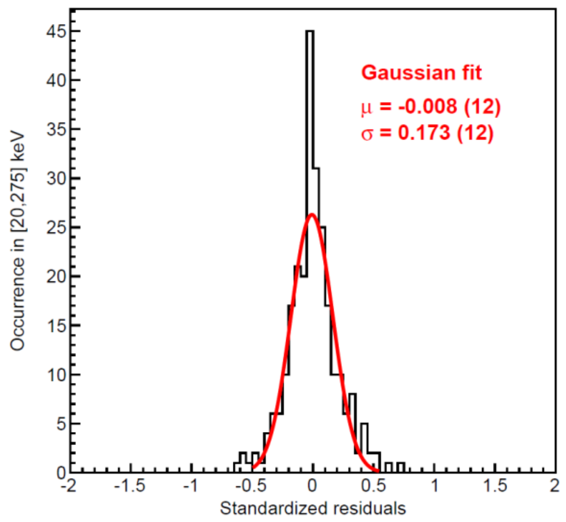
- ✓ High-precision measurements of $^{99}$Tc spectrum with MMC at CEA-LNHB and PTB, and with Silicon detectors at CEA-LNHB.
- ✓ Excellent agreement of all the three spectra.
- ✓ **New Q-value = 295.82 (16) keV** not consistent with AME2020 value of 297.5 (9) keV.
- ✓ **High sensitivity to the effective value of $g_A$ confirmed.**

# $^{99}$Tc 2$^{nd}$ forbidden non-unique decay




- ✓ Three different model spaces used with NushellX (GL, GLEKPN, jj45pn) to quantify the influence of nuclear structure.
- ✓ Theoretical calculations with nuclear structure, CVC and complete lepton current.
- ✓ Best adjustment gives an effective axial-vector coupling constant $g_A^{\mathrm{eff}}$ = 1.53 (8).
- ✓ Excellent residuals, without any trend down to 6 keV.



# $^{99}$Tc 2$^{nd}$ forbidden non-unique decay

‼ Effective $g_A$ value is **enhanced** while should be **quenched**. What happens?

- Calculated half-life is about one order of magnitude too low.
- From DDEP: $T_{1/2}$ = 211,5 (11) · $10^3$ y
  Evaluated value is mainly based on one publication, authors providing a *suggested* value.
- We can use it to renormalize the calculation and obtain a consistent picture: accurate shape, accurate half-life.

First possibility: **renormalization of the OBTD**

Example: GLEKPN

| Multipole | Transition | Original | Corrected |
|---|---|---|---|
| K = 2 | n $1g_{7/2} \rightarrow$ p $1g_{9/2}$ | 0.00994 | 0.00362 |
| | n $2d_{5/2} \rightarrow$ p $1g_{9/2}$ | 0.47752 | 0.17383 |
| K = 3 | n $1g_{7/2} \rightarrow$ p $1g_{9/2}$ | -0.01709 | -0.00622 |
| | n $2d_{5/2} \rightarrow$ p $1g_{9/2}$ | -0.43403 | -0.15800 |
| | n $2d_{3/2} \rightarrow$ p $1g_{9/2}$ | 0.03143 | 0.01144 |

Second possibility: **renormalization of $g_V$ and $g_A$**

$\rightarrow g_V^{\text{eff}} \sim 0.27 – 0.39$

$\rightarrow g_A^{\text{eff}} \sim 0.46 – 0.57$

depending on the calculation hypotheses.

→ **Consistent with 1$^{st}$ forbidden non-unique results**

→ **What about CVC hypothesis?**

- Eventually, one also deduces: log *f* = -0.47660 (22), log *ft* = 12.3478 (23), $\overline{E_\beta}$ = 98.45 (20) keV.

# Conclusion

# Take away

Beta decays are of importance for nuclear decay data, but also for a wide range of scientific topics from fundamental physics to applications.

- **Allowed and forbidden unique transitions can be calculated** without any nuclear structure **with good accuracy**. Except in a few specific cases ("accidental cancellation of nuclear matrix elements").
- If targeted accuracy better than 1-5%, detailed models are necessary.
- ✓ When no nuclear structure is required, the BetaShape code already improves the situation.

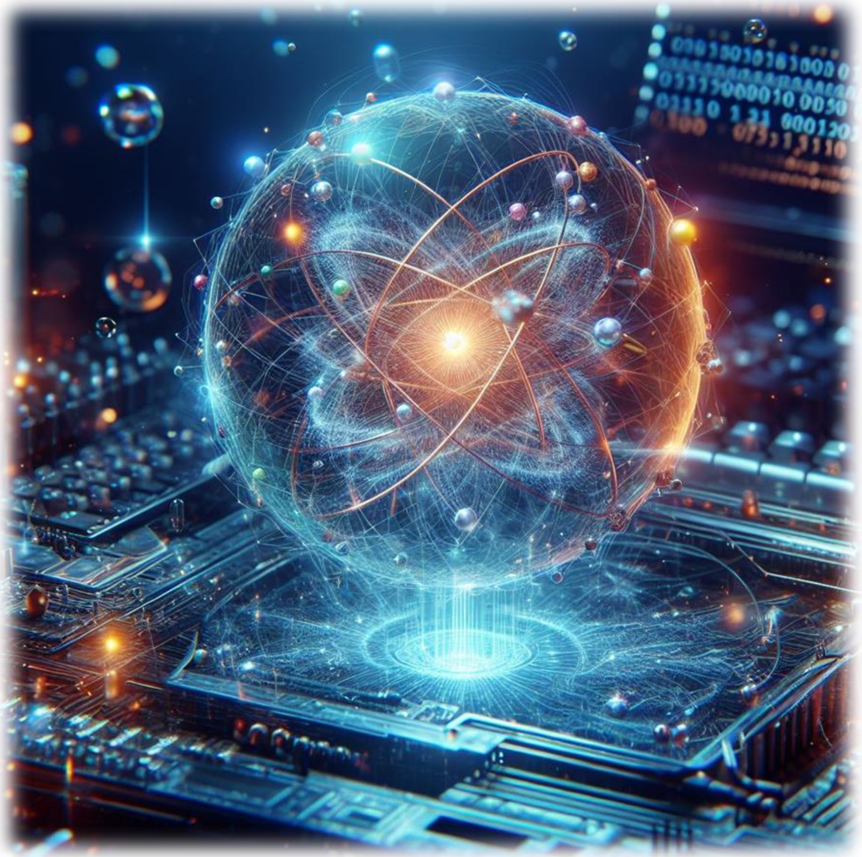
  → Available on IAEA-NSDD GitHub: https://github.com/IAEA-NSDDNetwork

- **Forbidden non-unique transitions are sensitive to nuclear structure**. Some can be calculated accurately as allowed or forbidden unique.

## Ongoing developments and challenges

- **High-precision measurements**, which can be used as a probe of nuclear models.
- **Medium and heavy nuclei, nuclear deformation**

  → Ongoing collaboration in the framework of the European metrology project PrimA-LTD.
- **Effective values of weak interaction coupling constants** is still an open subject.



# Thank you for your attention

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