# Neutrinoless $\beta\beta$ decay searches: theory of nuclear matrix elements

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# Creation of matter in nuclei: $0\nu\beta\beta$ decay

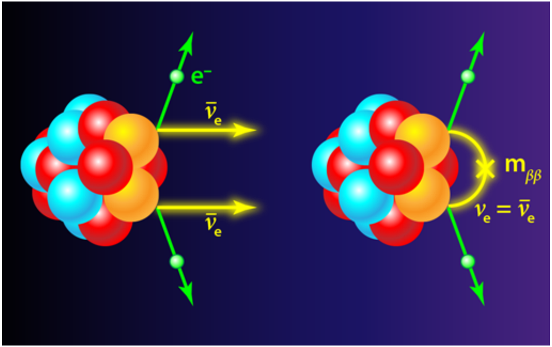
Lepton number conserved
in all processes observed:

single $\beta$ decay,
$\beta\beta$ decay with $\nu$ emission...

Neutral massive particles (Majorana $\nu$'s)
allow lepton number violation:

neutrinoless $\beta\beta$ decay
creates two matter particles (electrons)

Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)



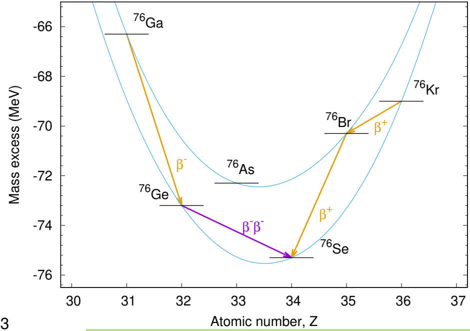
# $\beta\beta$ decay

Second order process in the weak interaction

Only observable in nuclei where (much faster) $\beta$-decay is forbidden energetically due to nuclear pairing interaction

$$BE(A) = -a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + \frac{(A-2Z)^2}{A} + \begin{cases} -\delta_{pairing} & \text{N,Z even} \\ 0 & \text{A odd} \\ \delta_{pairing} & \text{N,Z odd} \end{cases}$$

or where $\beta$-decay is very suppressed by $\Delta J$ angular momentum change



$^{48}$Ca → $^{48}$Ti
$^{76}$Ge → $^{76}$Se
$^{82}$Se → $^{82}$Kr
$^{96}$Zr → $^{96}$Mo
$^{100}$Mo → $^{100}$Ru
$^{110}$Pd → $^{110}$Cd
$^{116}$Cd → $^{116}$Sn
$^{124}$Sn → $^{124}$Te
$^{130}$Te → $^{130}$Xe
$^{136}$Xe → $^{136}$Ba
$^{150}$Nd → $^{150}$Sm

# Scales in new-physics searches using nuclei

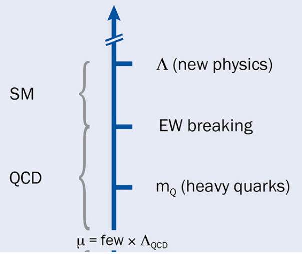
New physics scale: $\Lambda \gg 250$ GeV

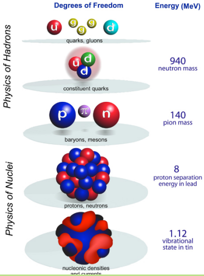
Electroweak scale:

$$v = \left(\sqrt{2}G_F\right)^{-1/2} \sim 250 \text{ GeV}$$

QCD (hadron) scale: $m_N \sim$ GeV

Nuclear scale: $k_F \sim m_\pi \sim 200$ MeV





# Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

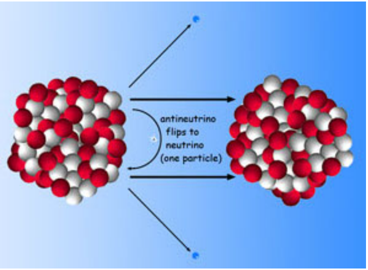
Nuclear structure physics
encoded in nuclear matrix elements
key to plan, fully exploit experiments

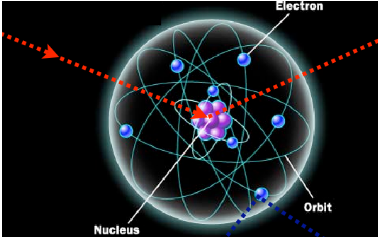
$$0\nu\beta\beta\text{:}\ \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

$$\text{Dark matter:}\ \frac{d\sigma_{\chi\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$$

$$\text{CE}\nu\text{NS:}\ \frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\,\zeta_i\,\mathcal{F}_i\right|^2$$

$M^{0\nu\beta\beta}$: Nuclear matrix element
$\mathcal{F}_i$ : Nuclear structure factor





# $0\nu\beta\beta$ decay half-life

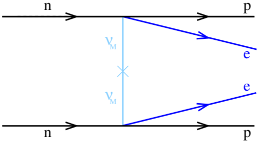
Half-life of $0\nu\beta\beta$ decay sensitive to
$m_{\beta\beta} \sim 1/\Lambda$ (dim-5 operator), new-physics scales $\widetilde{\Lambda}$ (dim-7) or $\widetilde{\Lambda}'$ (dim-9)

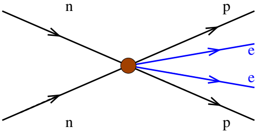
$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M^{0\nu}_{\text{light}}\right)^2 m_{\beta\beta}^2 + m_N^2\, \tilde{G}\, \tilde{g}^4\, \tilde{M}^2 \left(\frac{v}{\widetilde{\Lambda}}\right)^6 + \frac{m_N^4}{v^2}\, \widetilde{G}'\, \widetilde{g}'^4\, \widetilde{M}'^2 \left(\frac{v}{\widetilde{\Lambda}'}\right)^{10}$$

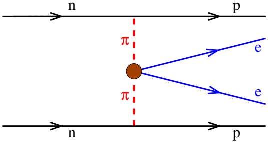
$G_{01}$, $\widetilde{G}$, $\widetilde{G}'$: phase-space factors (electrons), very well known

$g_A$, $g_\nu^{\text{NN}}$, $\widetilde{g}$, $\widetilde{g}'$: coupling to hadron(s), experiment or calculate with QCD

$M^{0\nu}_{\text{long}}$, $M^{0\nu}_{\text{short}}$, $\widetilde{M}$, $\widetilde{M}'$: nuclear matrix elements, many-body challenge



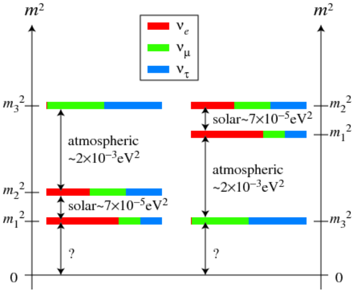


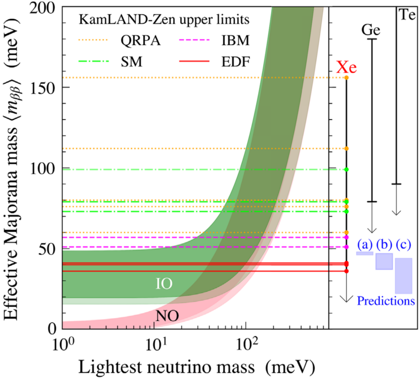


# Next generation experiments: inverted hierarchy

Decay rate sensitive to
neutrino masses, hierarchy
$m_{\beta\beta} = \left| \sum U_{ek}^2 m_k \right|$

$$\left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} = G_{0\nu}\, g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

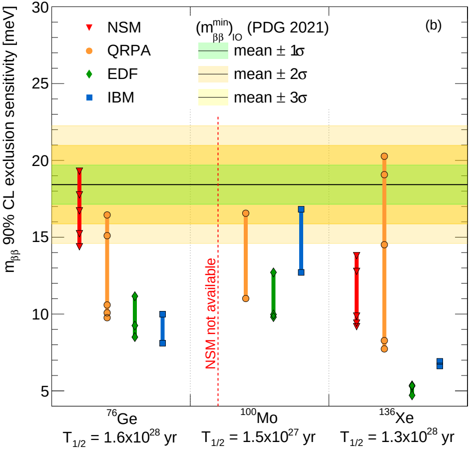




KamLAND-Zen, PRL130 051801(2023)

Matrix elements assess
if next-generation experiments
fully cover "inverted hierarchy"

# Uncertainty in physics reach of $0\nu\beta\beta$ experiments



Agostini, Benato, Detwiler, JM, Vissani

Phys. Rev. C 104 L042501 (2021)

Nuclear matrix element theoretical uncertainty critical to anticipate $m_{\beta\beta}$ sensitivity of future experiments

Current uncertainty in $m_{\beta\beta}$ prevents to foresee if next-generation experiments will fully cover parameter space of “inverted” neutrino mass hierarchy

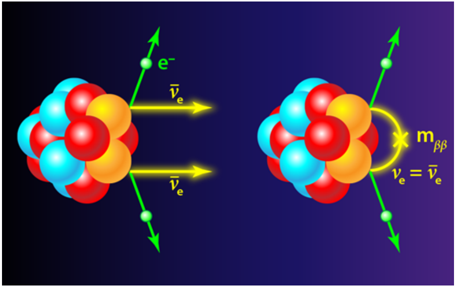
Uncertainty needs to be reduced!

# Nuclear matrix elements

Nuclear matrix elements needed in low-energy new-physics searches

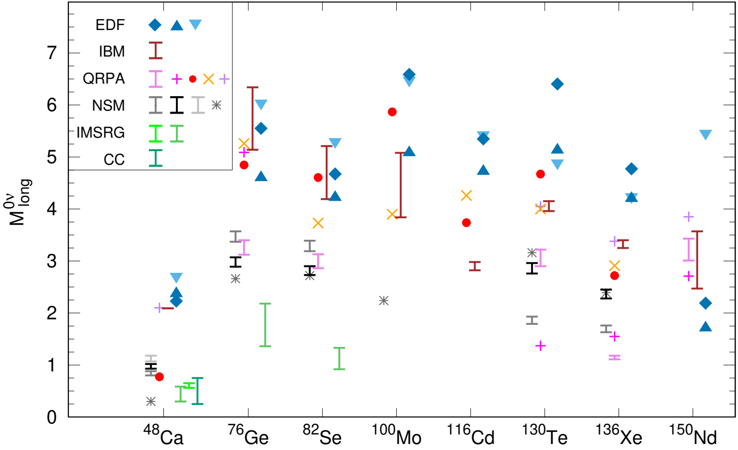
$$\langle\,\text{Final}\,|\mathcal{L}_{\text{leptons}-\text{nucleons}}|\,\text{Initial}\,\rangle = \langle\,\text{Final}\,|\int dx\, j^{\mu}(x) J_{\mu}(x)\,|\,\text{Initial}\,\rangle$$

- Nuclear structure calculation of the initial and final states:
  Shell model, QRPA, IBM,
  Energy-density functional
  Ab initio many-body theory
  QMC, Coupled-cluster, IMSRG...
- Lepton-nucleus interaction:
  Hadronic current in nucleus:
  phenomenological,
  effective theory of QCD



# $0\nu\beta\beta$ decay nuclear matrix elements

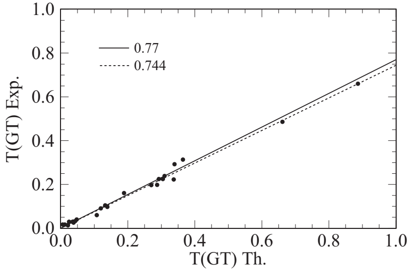
Large difference in nuclear matrix element calculations



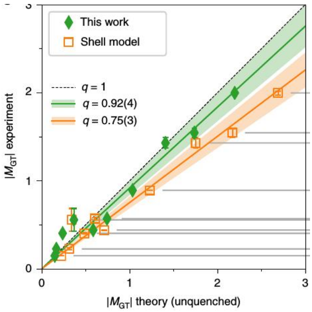
Agostini, Benato, Detwiler, JM, Vissani, Rev. Mod. Phys. 95, 025002 (2023)

# “Quenching”: missing physics in the calculations

## $\beta$ decays ($e^-$ capture): nuclear shell model vs ab initio



Martinez-Pinedo et al. PRC53 2602(1996)



Gysbers et al. Nature Phys. 15 428 (2019)

$$\langle F|\sum_i [g_A\, \boldsymbol{\sigma}_i \tau_i^-]^{\rm eff}\, |I\rangle\,, \quad [\boldsymbol{\sigma}_i\tau]^{\rm eff} \approx 0.7\boldsymbol{\sigma}_i\tau$$

Shell model: $\boldsymbol{\sigma}_i\tau$ “quenching”

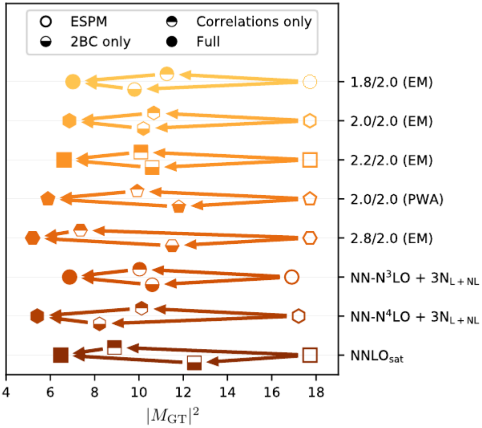
quenching: effects not in model

Ab initio calculations including meson-exchange currents and additional nuclear correlations

do not need “quenching”

# Origin of $\beta$ decay "quenching"

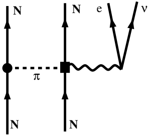
Which are main effects missing in conventional $\beta$-decay calculations?
Test case: GT decay of $^{100}$Sn



Relatively similar and complementary impact of

- nuclear correlations
- meson-exchange currents

Gysbers et al.
Nature Phys. 15 428 (2019)



# 2b currents in $0\nu\beta\beta$ decay

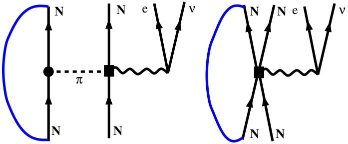
In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

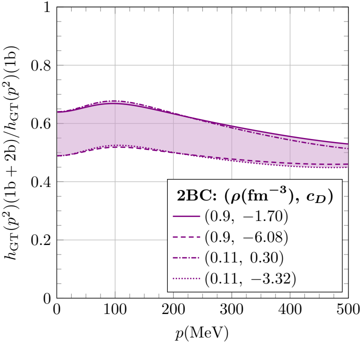
Approximate 2b current as effective 1b current normal ordering with respect to a Fermi gas

JM, Gazit, Schwenk, PRL107 062501(2011)

Normal-odering approximation works remarkably well for $\beta$ decay ($q = 0$)

Gysbers et al. Nature Phys. 15 428 (2019)





Jokiniemi, Romeo, Soriano, JM

PRC 107 044305 (2023)

# Correlations in $0\nu\beta\beta$ decay

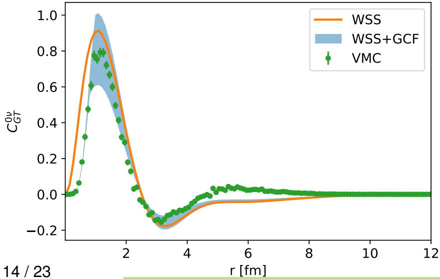
Compare $\beta\beta$ transition densities: shell model vs quantum Monte Carlo agree at long distances, shell model misses short-range correlations

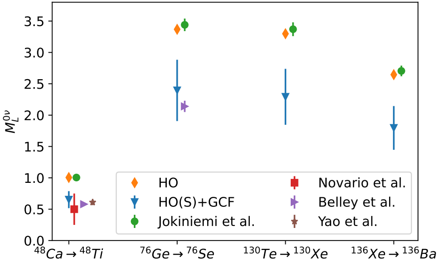
$$4\pi r^2 \rho_{GT}(r) = \langle \Psi_f | \sum_{a<b} \delta(r - r_{ab}) \sigma_{ab} \tau_a^+ \tau_b^+ | \Psi_i \rangle \qquad M_{GT}^{0\nu} = \int_0^\infty dr\, C_{GT}^{0\nu} ,$$

Generalized contact formalism Weiss et al. PRC106 065501 (2022)

Separation of scales: wf, transition density factorize for nearby nucleons

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi^\alpha(\boldsymbol{r}_{ij}) A^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}), \quad \rho_{GT}(r) \xrightarrow[r\to 0]{} -3|\varphi^0(r)|^2 C^0_{pp,nn}(f,i)$$





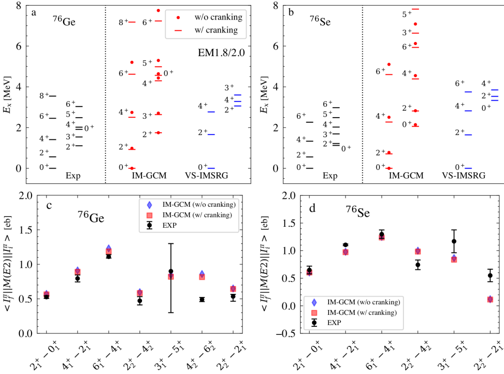
# Ab initio calculations of $^{76}$Ge

Two different
ab initio methods calculate
$0\nu\beta\beta$ decay of $^{76}$Ge:
IM-GCM and VS-IMSRG

IM-GCM better describes spectroscopic properties of $^{76}$Ge, $^{76}$Se

VS-IMSRG less
computationally expensive,
better suited
to study heavier nuclei

The two ab initio $0\nu\beta\beta$
NMEs disagree by $\sim 30\%$



Belley et al. PRL132 182502 (2024)

# Light-neutrino exchange: contact operator

Short-range operator contributes to light-neutrino exchange
for RG invariance of two-nucleon decay amplitude: high-energy $\nu$'s

$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M^{0\nu}_{\text{long}} + M^{0\nu}_{\text{short}}\right)^2 m_{\beta\beta}^2,$$ Cirigliano et al. PRL120 202001(2018)

$$M^{0\nu}_{\text{short}} \equiv \frac{1.2A^{1/3}\,\text{fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \; 1 \,[\frac{2}{\pi} \int j_0(qr)\, 2g_\nu^{\text{NN}}\, g(p/\Lambda)\, p^2 dp]\, |0_i^+\rangle,$$

$$M^{0\nu}_{\text{GT}} \simeq \frac{1.2A^{1/3}\,\text{fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \; \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \,[\frac{2}{\pi} \int j_0(qr)\, \frac{1}{p^2}\, g_A^2\, f^2(p/\Lambda_A)\, p^2 dp]\, |0_i^+\rangle$$

Unknown value (and sign) of the hadronic coupling $g_\nu^{\text{NN}}$!

Lattice QCD calculations can obtain value of $g_\nu^{\text{NN}}$

Davoudi, Kadam, Phys. Rev. Lett. 126, 152003 (2021), PRD105 094502('22)

match $nn \rightarrow pp + ee$ amplitude calculated with dispersion QCD methods

Cirigliano et al. PRL126 172002 (2021), JHEP 05 289 (2021)

charge-independence breaking of nuclear Hamiltonians

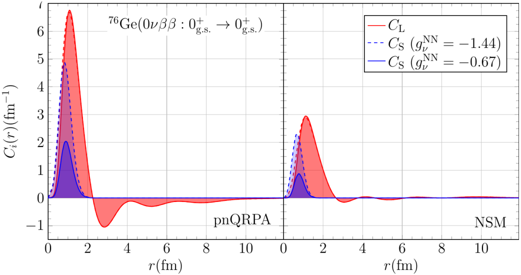
Cirigliano et al. PRC100, 055504 (2019)

# Long and short-range NME in heavy nuclei

Relatively stable contribution of new term $M_S/M_L$:

$20\% - 50\%$ impact of short-range NME in shell model
$30\% - 70\%$ impact of short-range NME in QRPA

consistent with 43% effect in IM-GCM for $^{48}$Ca
using calculated $nn \rightarrow pp + ee$ decay Wirth et al. PRL127 242502 (2021)

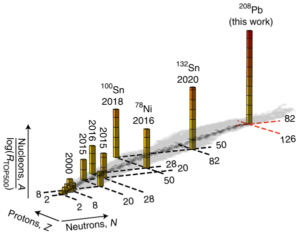


Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

Uncertainty dominated by coupling $g_\nu^{\rm NN}$

# Ab initio predictions for nuclear neutron radius

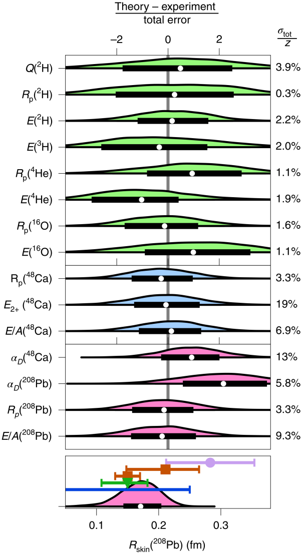
Remarkable progress ab initio calculations of (relatively uncorrelated) heavy nuclei, $^{208}$Pb



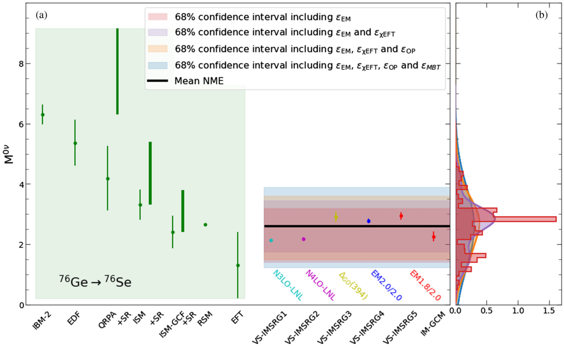
Determine $^{208}$Pb neutron skin using Bayesian approach sampling of $10^9$ (parameters of) nuclear Hamiltonians

Hu, Jiang, Miyagi et al.

Nature Phys. 18 1196 (2022)



# $0\nu\beta\beta$ NMEs: theoretical uncertainty



Belley et al. PRL132 182502 (2024)

Ab initio VS-IMSRG $0\nu\beta\beta$ of $^{76}$Ge provide first robust calculation of NME with theoretical uncertainties

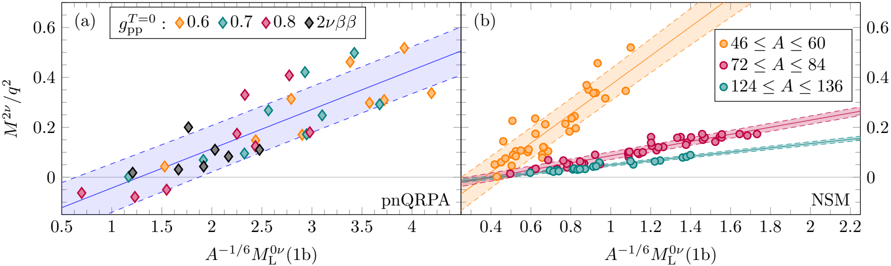
Sampling over $> 8000$ (parameters) of non-implausible nuclear Hamiltonians

Hamiltonians weighted by $^{1}S_0$ phase shift at 50 MeV found to be correlated with $0\nu\beta\beta$ NMEs

# Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between $2\nu$ and $0\nu$ modes of $\beta\beta$ decay
in nuclear shell model (systematic calculations of different nuclei)
and QRPA calculations (decays of $\beta\beta$ emitters with different $g_{pp}$ values)

Similar but not common correlation, depends on mass for shell model
$0\nu\beta\beta - 2\nu\beta\beta$ correlation also observed in $^{48}$Ca, $^{136}$Xe
Horoi et al. PRC106 054302 (2022), PRC107 045501 (2023)



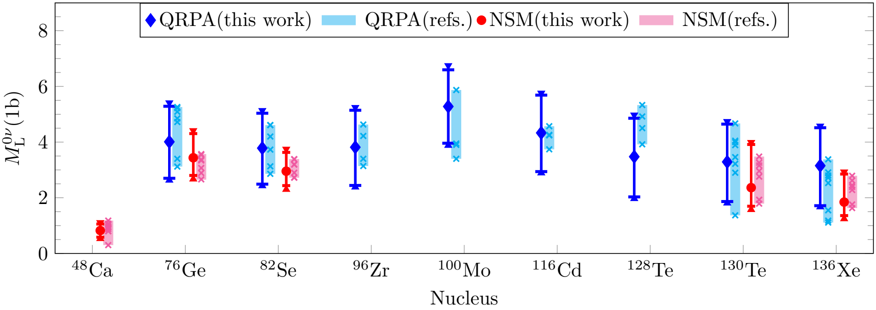
Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

# $0\nu\beta\beta$ NMEs from $2\nu\beta\beta - 0\nu\beta\beta$ correlation

NMEs consistent with previous nuclear shell model, QRPA results

Theoretical uncertainty involves
systematic calculations covering dozens of nuclei and interactions
error of each calculation (eg quenching) and experimental $2\nu\beta\beta$ error

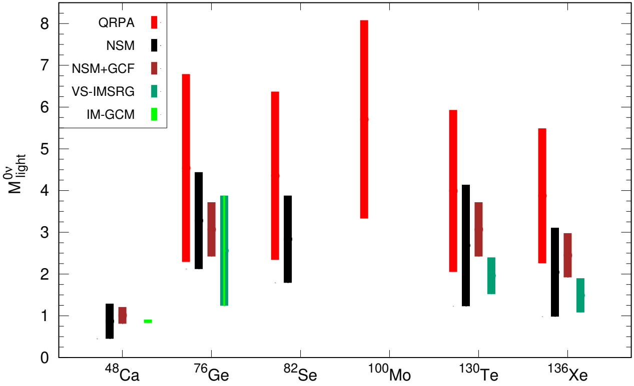
Previous theoretical uncertainty mostly ignored: collection of calculations



Jokiniemi, Romeo, Soriano, JM, PRC 107 044305 (2023)

# $0\nu\beta\beta$ decay total (long- and short-range) NMEs

Not-so-large difference in nuclear matrix element calculations!



Wirth et al.
PRL127 242502 (2021)

Belley et al.
arXiv:2307:15156

Belley et al.
PRL132 182502 (2024)

Jokiniemi et al.
PRC 107 044305 (2023)

Weiss et al.
PRC106 065501 (2022)

Gómez-Cadenas, Martín-Albo, JM, Mezzeto, Monrabal, Sorel, Riv. Nuovo Cim. 46, 619 (2023)

# Summary

Calculations of $0\nu\beta\beta$ NMEs
challenge nuclear many-body methods,
searches demand reliable NMEs

Ab initio results suggest reduced NMEs
due to nuclear correlations
key to understand $\beta$ decay "quenching"

Enhancement by short-range NME:
should be included in all NMEs used to
extract experimental $m_{\beta\beta}$ sensitivities

Theoretical NME uncertainties
by sampling nuclear Hamiltonians
or exploiting $0\nu\beta\beta - 2\nu\beta\beta$ correlation

**Outlook:** 2b currents, key in $\beta$ decays
soon included fully in $0\nu\beta\beta$ calculations!