Theory of leptonic flavor mixing

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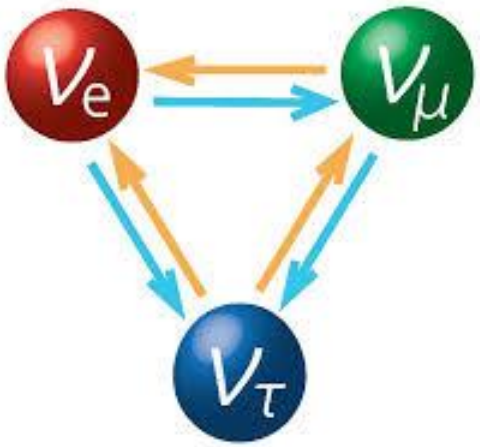
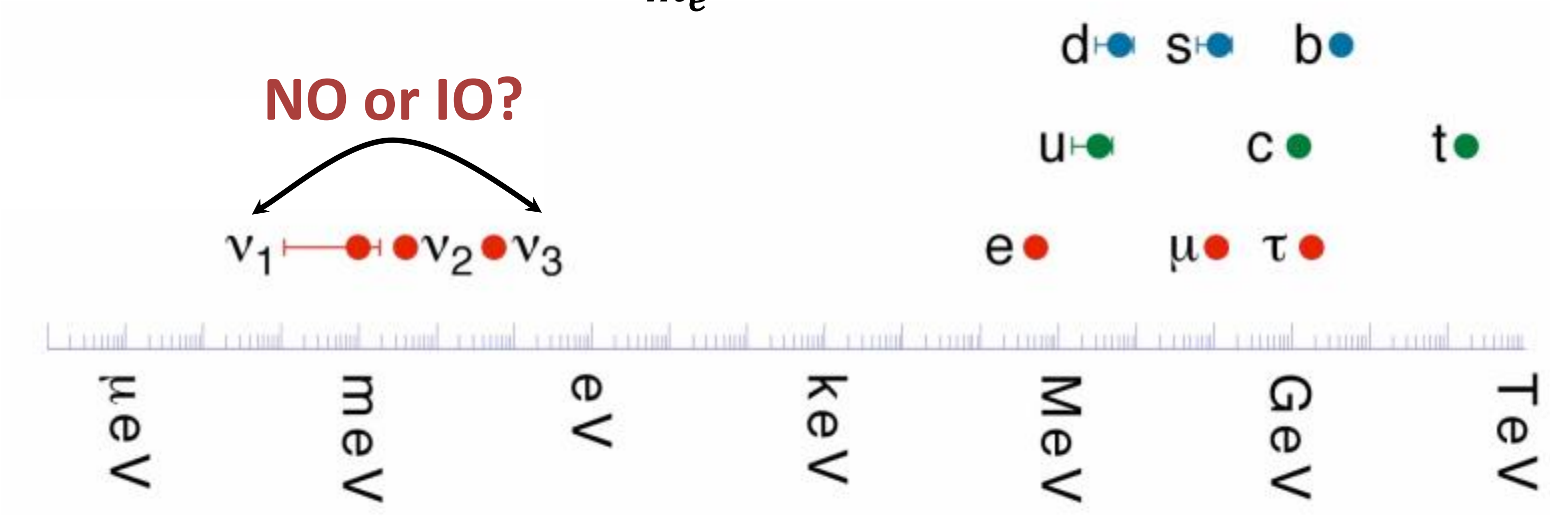


NEUTRINO 2024

Flavor problems of SM

- **Hierarchical masses, e.g.** $\frac{m_t}{m_e} = \mathcal{O}(10^5)$



- **Quark mixing vs lepton mixing**

Quark mixings are small

$$|V_{CKM}| \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

PDG(2024)

Lepton mixings are large

$$|U_{PMNS}| \approx \begin{pmatrix} 0.823 & 0.548 & 0.149 \\ 0.365 & 0.651 & 0.665 \\ 0.436 & 0.525 & 0.731 \end{pmatrix}$$

NuFIT5.3(2024)

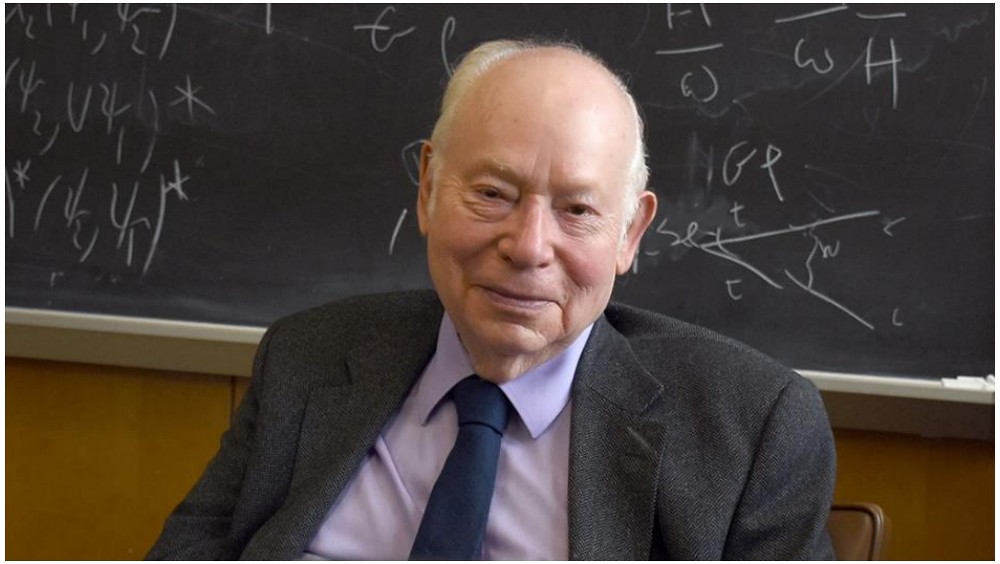
talks by Art McDonald and Mariam Tortola

Quark and lepton mixing matrices have distinctive structures!

Isidor Issac Rabi asked in 1936:

Who ordered that?

Steven Weinberg: ``Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesn’t have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses. …’’

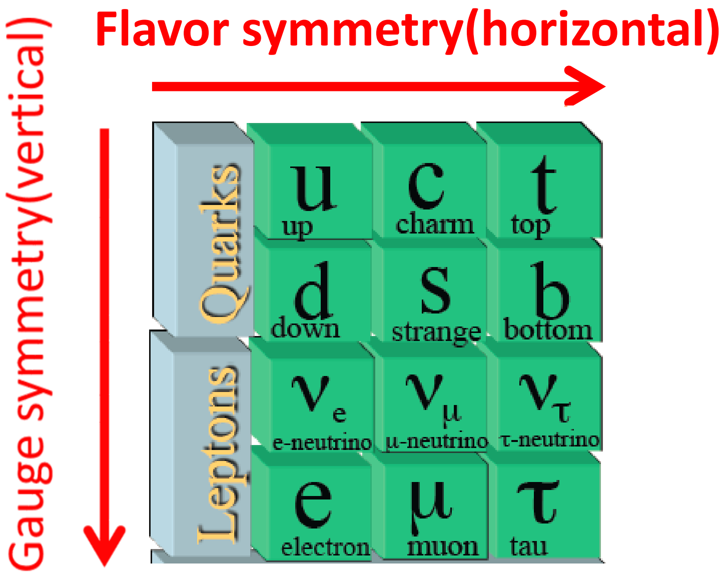


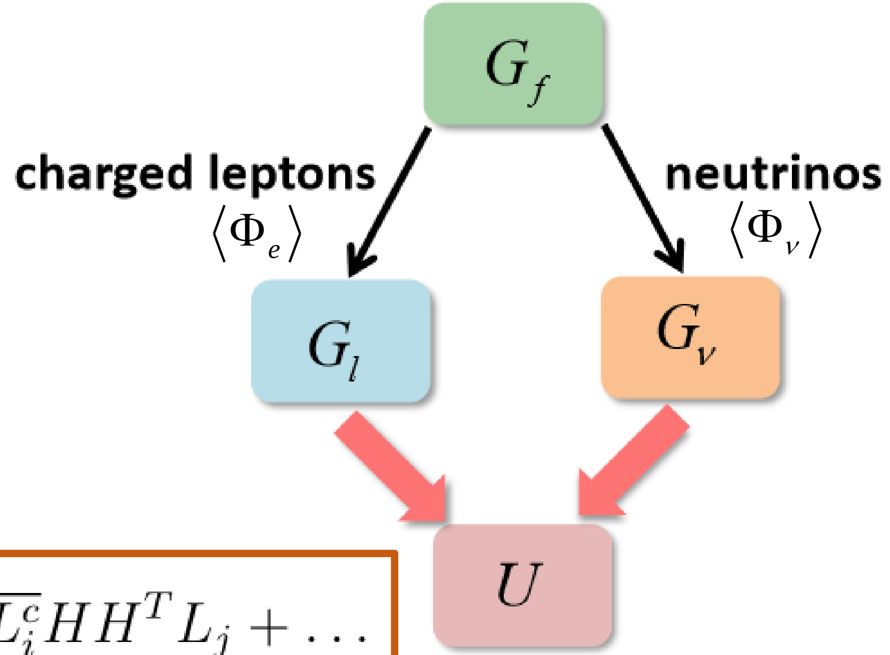
Flavor symmetry approach to flavor problem

The fundamental principle of fermion masses and flavor mixing structure is unkonwn so far.

Symmetry can help to reduce the number of free parameters in the Yukawa coupling.

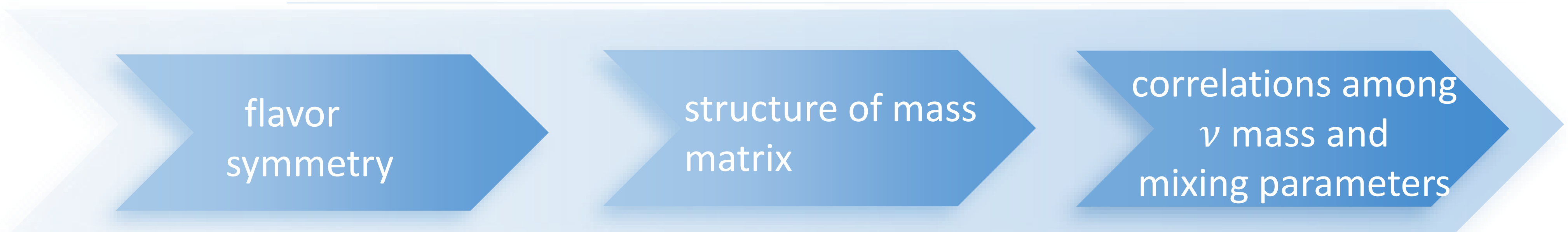
- **Flavor symmetry:** relating three families e-family ⟺ muon-family ⟺ tau-family





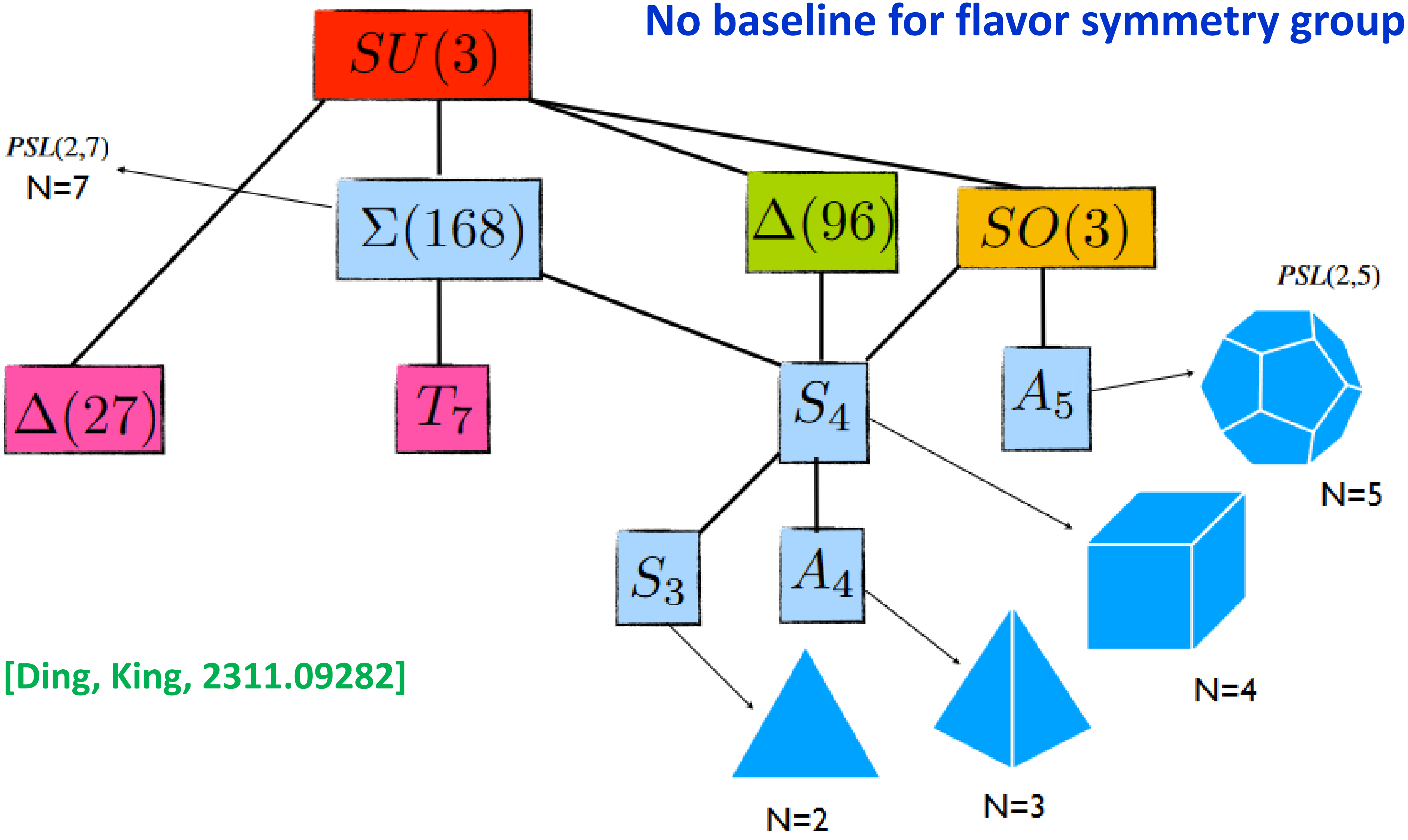
[King,1701.04413; Petcov, 1711.10806; Xing,1909.09610; Feruglio,Romanino, 1912.06028; Ding, King, 2311.09282; Ding,Valle, 2402.16963]

$$\mathcal{L}_m = -Y^e_{ij}(\langle\Phi_e\rangle)\overline{L_i}He_{Rj} - \frac{1}{2}Y^\nu_{ij}(\langle\Phi_\nu\rangle)\overline{L^c_i}HH^TL_j + \ldots$$



Non-Abelian Discrete flavor symmetry

No baseline for flavor symmetry group!



[Ding, King, 2311.09282]

Discrete flavor symmetry approach to lepton mixing

- ①Lepton mixing is **fully** determined by flavor symmetry G_f, **i.e.** $G_l > Z_2$ **&** $G_\nu > Z_2$

$$U = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2}\cos\vartheta & 1 & -\sqrt{2}\sin\vartheta \\ -\sqrt{2}\cos(\vartheta-\pi/3) & 1 & \sqrt{2}\sin(\vartheta-\pi/3) \\ -\sqrt{2}\cos(\vartheta+\pi/3) & 1 & \sqrt{2}\sin(\vartheta+\pi/3) \end{pmatrix}$$

[Lindner et al., 1212.2411; King, Neder, Stuart, 1305.3200; Fonseca, Grimus, 1405.3678;Yao, Ding, 1505.03798; Ding,Valle,2402.16963 ....]

ϑ: discrete, fixed by groups G_f, G_l, G_ν

close to 3σ upper bounds

- Lepton mixing angles:

$$\sin^2\theta_{12} = \sec^2\theta_{13}/3 \simeq 0.341,$$

$$\sin^2\theta_{23} = \frac{1}{2} \pm \frac{1}{2}\tan\theta_{13}\sqrt{2-\tan^2\theta_{13}} = 0.605 \text{ or } 0.395$$

- Dirac CP phase δ_{CP} is **conserved:** $\sin\delta_{CP}=0$

Testable at JUNO, DUNE, Hyper-K

- **Larger** groups required, for example **$|G_f|=648$** for Majorana neutrinos

➢ ②Lepton mixing is **partially** determined by flavor symmetry G_f, i.e. $G_l > Z_2$ & $G_\nu = Z_2$

TM1: $U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \times & \times \\ -\frac{1}{\sqrt{6}} & \times & \times \\ -\frac{1}{\sqrt{6}} & \times & \times \end{pmatrix}$

TM2: $U = \begin{pmatrix} \times & \frac{1}{\sqrt{3}} & \times \\ \times & \frac{1}{\sqrt{3}} & \times \\ \times & \frac{1}{\sqrt{3}} & \times \end{pmatrix}$

GR2: $U = \begin{pmatrix} \times & s^\nu_{12} & \times \\ \times & \frac{c^\nu_{12}}{\sqrt{2}} & \times \\ \times & \frac{c^\nu_{12}}{\sqrt{2}} & \times \end{pmatrix}$

$\tan\theta^\nu_{12} = 2/(1+\sqrt{5}) = 1/\phi$

Sum rules:

$$\begin{cases} 3\cos^2\theta_{12}\cos^2\theta_{13} = 2, \\ \cos\delta = \dfrac{(5\sin^2\theta_{13}-1)\cot 2\theta_{23}}{2\sin\theta_{13}\sqrt{2-6\sin^2\theta_{13}}} \end{cases}$$

$$\begin{cases} 3\sin^2\theta_{12}\cos^2\theta_{13} = 1, \\ \cos\delta = \dfrac{\cos 2\theta_{13}\cot 2\theta_{23}}{\sin\theta_{13}\sqrt{2-3\sin^2\theta_{13}}} \end{cases}$$

$$\begin{cases} \sqrt{5}\,\phi\sin^2\theta_{12}\cos^2\theta_{13} = 1\,, \\ \cos\delta = \dfrac{(\phi^2\cot^2\theta_{13}-2)\cot 2\theta_{23}}{2\sqrt{\phi^2\cot^2\theta_{13}-1}} \end{cases}$$

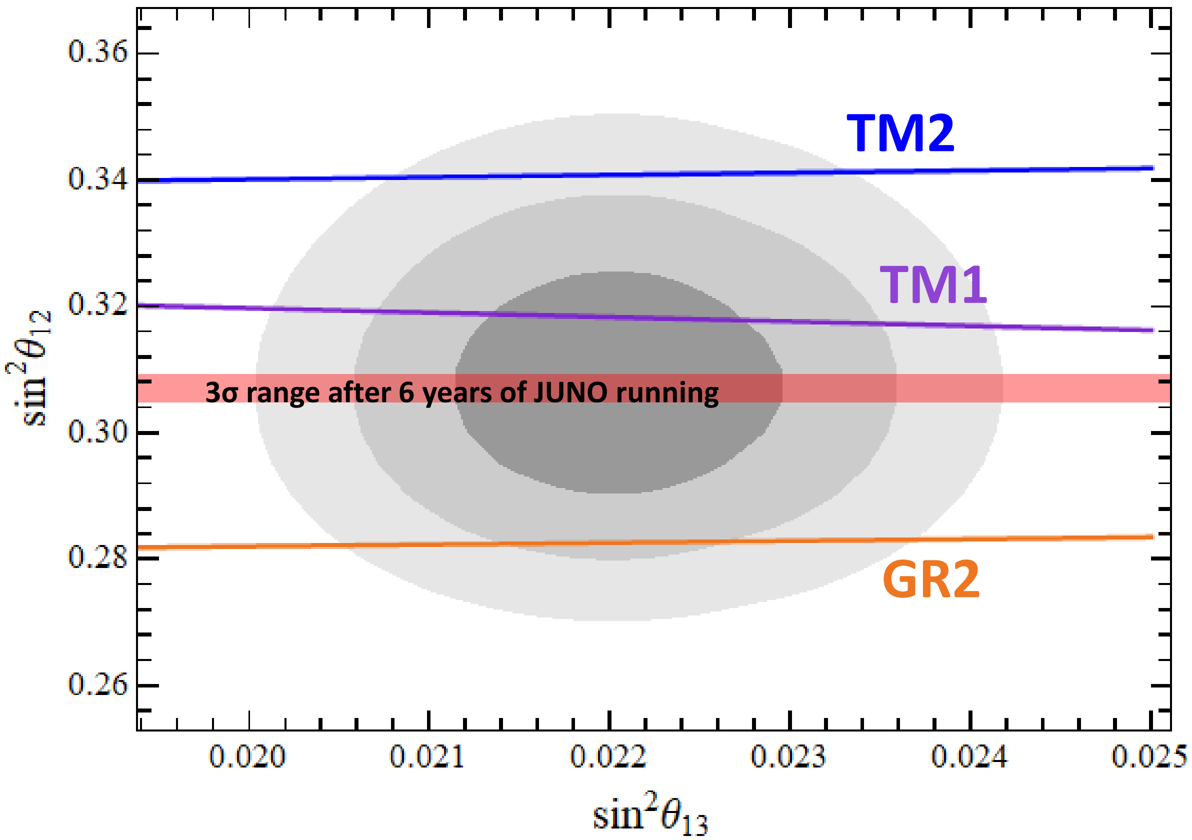
[Albright, Rodejohann, 0812.0436; Albright, Dueck,Rodejohann, 1004.2798; Costa, King, 2307.13895; Ding,Valle,2402.16963 ….]

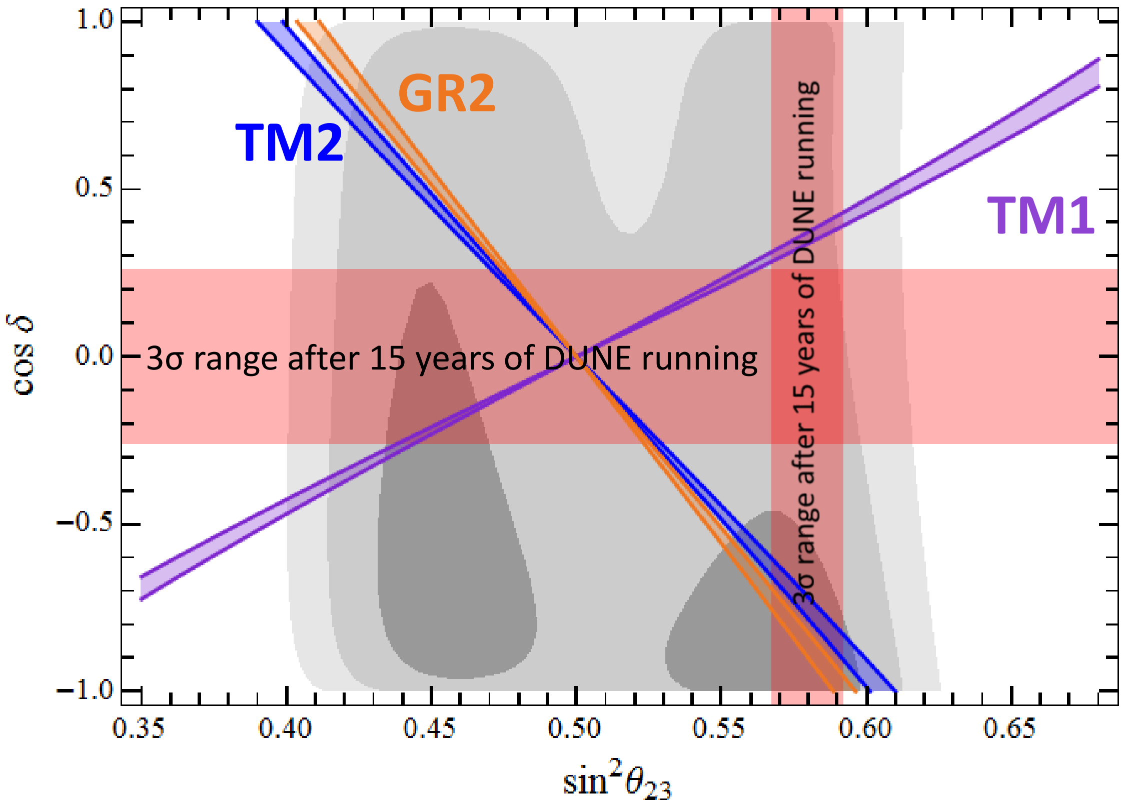
➢ ②Lepton mixing is **partially** determined by flavor symmetry G_f, i.e. $G_l > Z_2$ & $G_\nu = Z_2$

TM1: $U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \times & \times \\ -\frac{1}{\sqrt{6}} & \times & \times \\ -\frac{1}{\sqrt{6}} & \times & \times \end{pmatrix}$ **TM2:** $U = \begin{pmatrix} \times & \frac{1}{\sqrt{3}} & \times \\ \times & \frac{1}{\sqrt{3}} & \times \\ \times & \frac{1}{\sqrt{3}} & \times \end{pmatrix}$ **GR2:** $U = \begin{pmatrix} \times & s^\nu_{12} & \times \\ \times & \frac{c^\nu_{12}}{\sqrt{2}} & \times \\ \times & \frac{c^\nu_{12}}{\sqrt{2}} & \times \end{pmatrix}$

[Costa, King, 2307.13895]

$\tan\theta^\nu_{12} = 2/(1+\sqrt{5}) = 1/\phi$

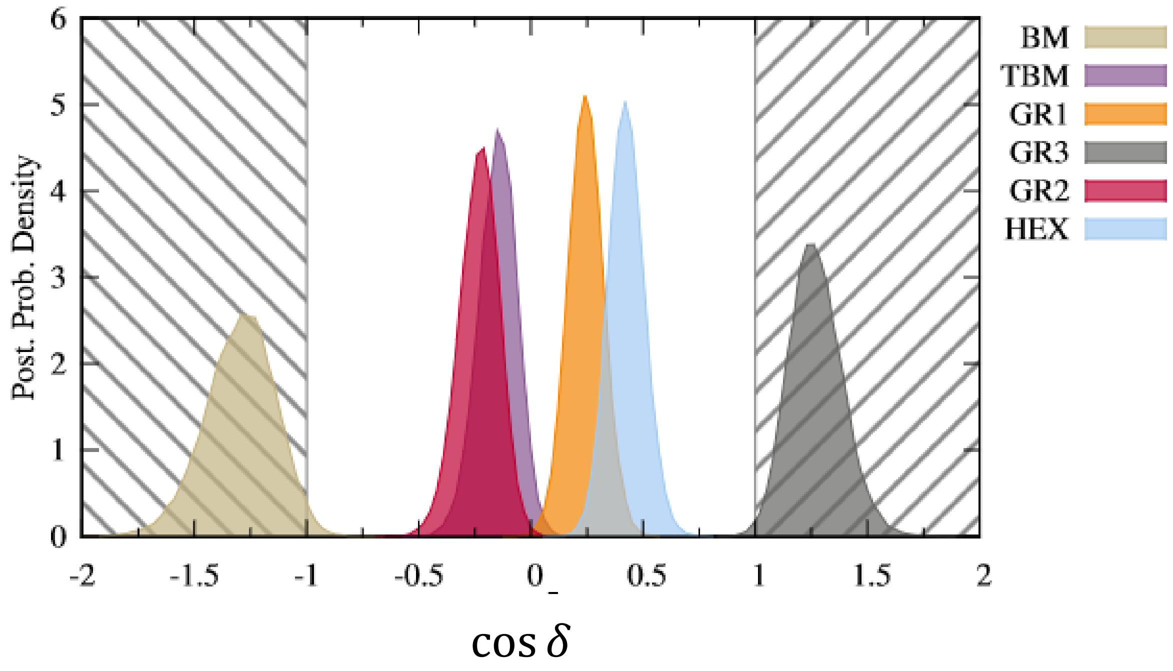


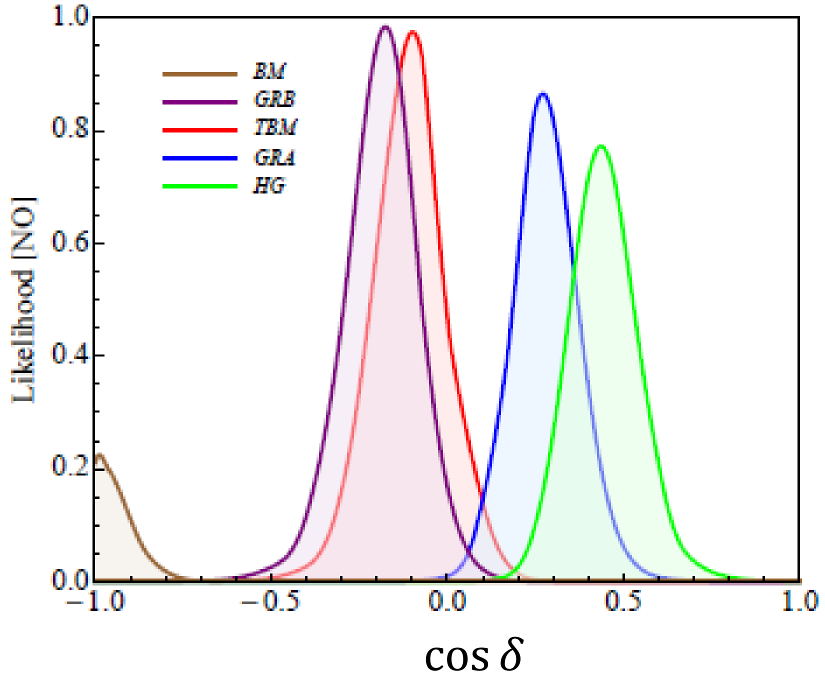


➢ ③Lepton mixing is partially determined by flavor symmetry G_f, i.e. $\boldsymbol{G_l \leq Z_2}$ **&** $\boldsymbol{G_\nu > Z_2}$

$$U = U_{12}^{e\dagger} U_{23}^{e\dagger} R_{23}^{\nu} R_{12}^{\nu}$$

$$\cos\delta = \frac{\tan^2\theta_{23}\sin^2\theta_{12} + \sin^2\theta_{13}\cos^2\theta_{12} - \sin^2\theta_{12}^{\nu}(\tan^2\theta_{23} + \sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}\tan\theta_{23}}$$

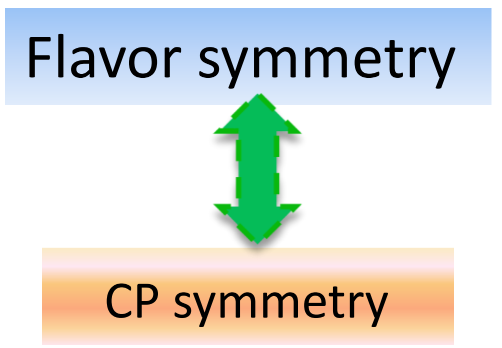




[Petcov, 1405.6006; Ballett, King, et al, 1410.7573; Girardi,Petcov,Titov,1410.8056; Ding,Valle,2402.16963 ….]

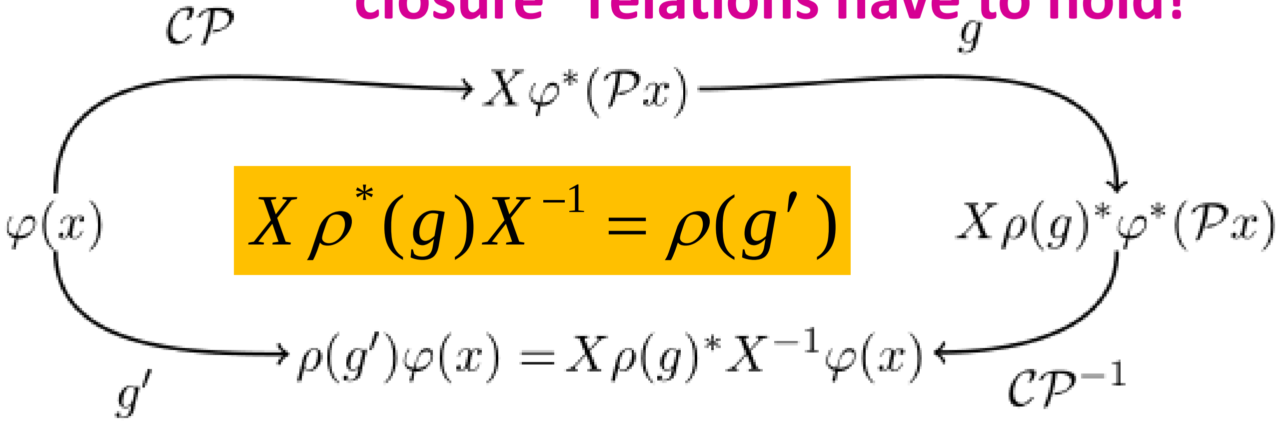
Symmetry origin of CP violation

- **Flavor symmetry → flavor+CP symmetries**



Mixing angles &CP phases

"closure" relations have to hold!



$$X\rho^*(g)X^{-1} = \rho(g')$$

[Grimus,Rebelo,hep-ph/9506272; Feruglio, Hagedorn, Ziegler, 1211.5560; Holthausen, Lindner, Schmidt,1211.6953; Chen, Fallbacher, et al., 1402.0507]

- Simplest example: μτ reflection =μτ exchange+canonical CP

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e^c \\ \nu_\tau^c \\ \nu_\mu^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix}$$

$|U| =$ 

The last two rows have equal magnitudes

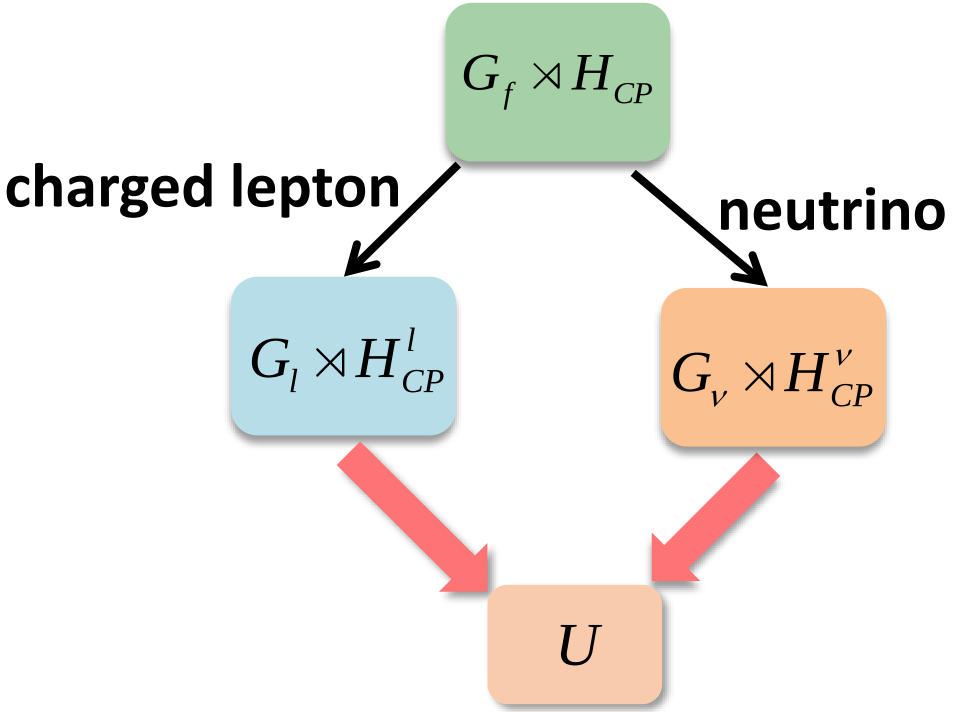
$$\theta_{23} = \frac{\pi}{4}, \quad \delta = \pm\frac{\pi}{2}$$

[Harrison, Scott, hep-ph/0210197; Grimus, Lavoura,hep-ph/0305309; Xing, Zhao, 1512.04207]

Flavor and CP symmetry to lepton mixing

- Flavor + CP symmetries have rich symmetry breaking patterns, and the resulting lepton mixing matrix is determined up to few continuous free parameters.

$G_l \rtimes H^l_{CP}$	$G_\nu \rtimes H^\nu_{CP}$	U	# parameters
Z_n	$K_4 \times CP$	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	**0**
Z_n	$Z_2 \times CP$	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$	**1**
$Z_2 \times CP$	$K_4 \times CP'$	$Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	
$Z_2 \times CP$	$Z_2 \times CP'$	$Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$	**2**
Z_2	$K_4 \times CP$	$Q_l^\dagger P_l^T U_{23}^\dagger(\theta_l, \delta_l) \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	
Z_n	CP	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu O_3(\theta_1, \theta_2, \theta_3) Q_\nu$	**3**
CP	$K_4 \times CP'$	$Q_l^\dagger O_3^T(\theta_1, \theta_2, \theta_3) \Sigma_l^\dagger \Sigma_\nu Q_\nu$	
Z_2	$Z_2 \times CP$	$Q_l^\dagger P_l^T U_{23}^\dagger(\theta_l, \delta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$	



[Ding,Valle,2402.16963]

- The lepton mixing angles as well as Dirac and Majorana CP phases can be predicted by residual symmetry, neutrino masses are not constrained except in concrete models.

Universal flavor symmetry for quark and lepton mixing

$$G_f \rtimes H_{CP}$$

[Lu, Ding, 1610.05682, 1806.02301; Li, Lu, Ding, 1706.04576; Lu, Ding, 1901.07414]

charged lepton

$$Z_2^{g_l} \times X_l$$

$$X_l = \Sigma_l \Sigma_l^T, \quad \Sigma_l^\dagger \rho_3(g_l) \Sigma_l = \pm \mathrm{diag}(1,-1,-1)$$

$$U_l = \Sigma_l R_{23}(\theta_l) P_l Q_l$$

neutrino

$$Z_2^{g_\nu} \times X_\nu$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T, \quad \Sigma_\nu^\dagger \rho_3(g_\nu) \Sigma_\nu = \pm \mathrm{diag}(1,-1,-1)$$

$$U_\nu = \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

$$U(\theta_l, \theta_\nu) = Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

- All mixing angles and CP phases are expressed in terms of two free angles $\theta_{l,\nu} \in [0,\pi)$
- This scheme can be extended to quark sector, and the quark and lepton mixing can be described simultaneously in terms of totally four free angles.

Quark and lepton mixing from Dihedral group D_n and CP

Quark sector:

[Lu, Ding, 1901.07414; Ding,Valle,2402.16963]

Assignment: 2+1 $\begin{pmatrix}\begin{pmatrix}u_L\\d_L\end{pmatrix}\\\begin{pmatrix}c_L\\s_L\end{pmatrix}\end{pmatrix}\sim \mathbf{2}_1,\quad \begin{pmatrix}t_L\\b_L\end{pmatrix}\sim\mathbf{1}_1$

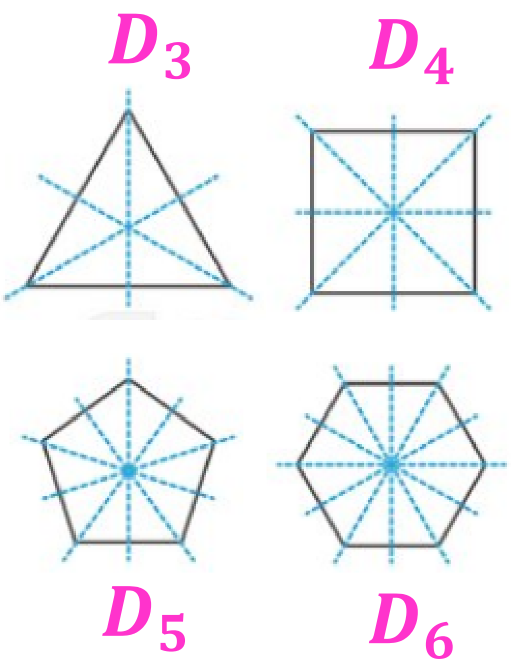
The 3rd generation is much heavier!

Residual symmetry: $Z_2^{g_u}=Z_2^{SR^x},\ X_u=SR^{n/2}, Z_2^{g_d}=Z_2^{SR^y}, X_d=S,\ x,y=0,\ldots,n-1$

➢The CKM matrix is determined as $c_{u,d}\equiv\cos\theta_{u,d},\ s_{u,d}\equiv\sin\theta_{u,d}$

$$V_{CKM}=\begin{pmatrix}-c_d\sin\varphi_1 & \cos\varphi_1 & s_d\sin\varphi_1\\ c_uc_d\cos\varphi_1+is_us_d & c_u\sin\varphi_1 & -c_us_d\cos\varphi_1+ic_ds_u\\ -c_ds_u\cos\varphi_1+ic_us_d & -s_u\sin\varphi_1 & s_us_d\cos\varphi_1+ic_uc_d\end{pmatrix}$$

with $\varphi_1=\frac{y-x}{n}\pi$ ⟵ **depends on choice of residual symmetry**



- Cabibbo angle from group: $\cos^2\theta_{13}^q\sin^2\theta_{12}^q=\cos^2\varphi_1$

- CP phase from mixing angles: $J_{CP}^q\approx\frac{1}{2}\sin2\varphi_1\sin\varphi_2\sin\theta_{13}^q\sin\theta_{23}^q$

- Viable CKM matrix for $\varphi_1 = 3\pi/7$ which can be achieved in D_{14} group

	θ_u^{bf}/π	θ_d^{bf}/π	$\sin\theta_{12}^q$	$\sin\theta_{23}^q$	$\sin\theta_{13}^q$	J_{CP}^q
Our	0.01326	0.00117	0.22252	0.04166	0.00357	3.223×10^{-5}
Data	—	—	0.22500 ± 0.00100	0.04200 ± 0.00059	0.003675 ± 0.000095	$(3.120\pm0.090)\times10^{-5}$

Hierarchical quark mixing angles and irregular CP phase can be accommodated.

- Lepton sector : $\varphi_1 = 2\pi/7$

$$U_{PMNS} = R_{12}(\theta_l)\begin{pmatrix} 0 & \cos\frac{2\pi}{7} & \sin\frac{2\pi}{7} \\ i & 0 & 0 \\ 0 & -\sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{pmatrix} R_{13}(\theta_\nu)Q_\nu \Rightarrow \cos^2\theta_{23}\cos^2\theta_{13} = \cos^2\frac{2\pi}{7}$$

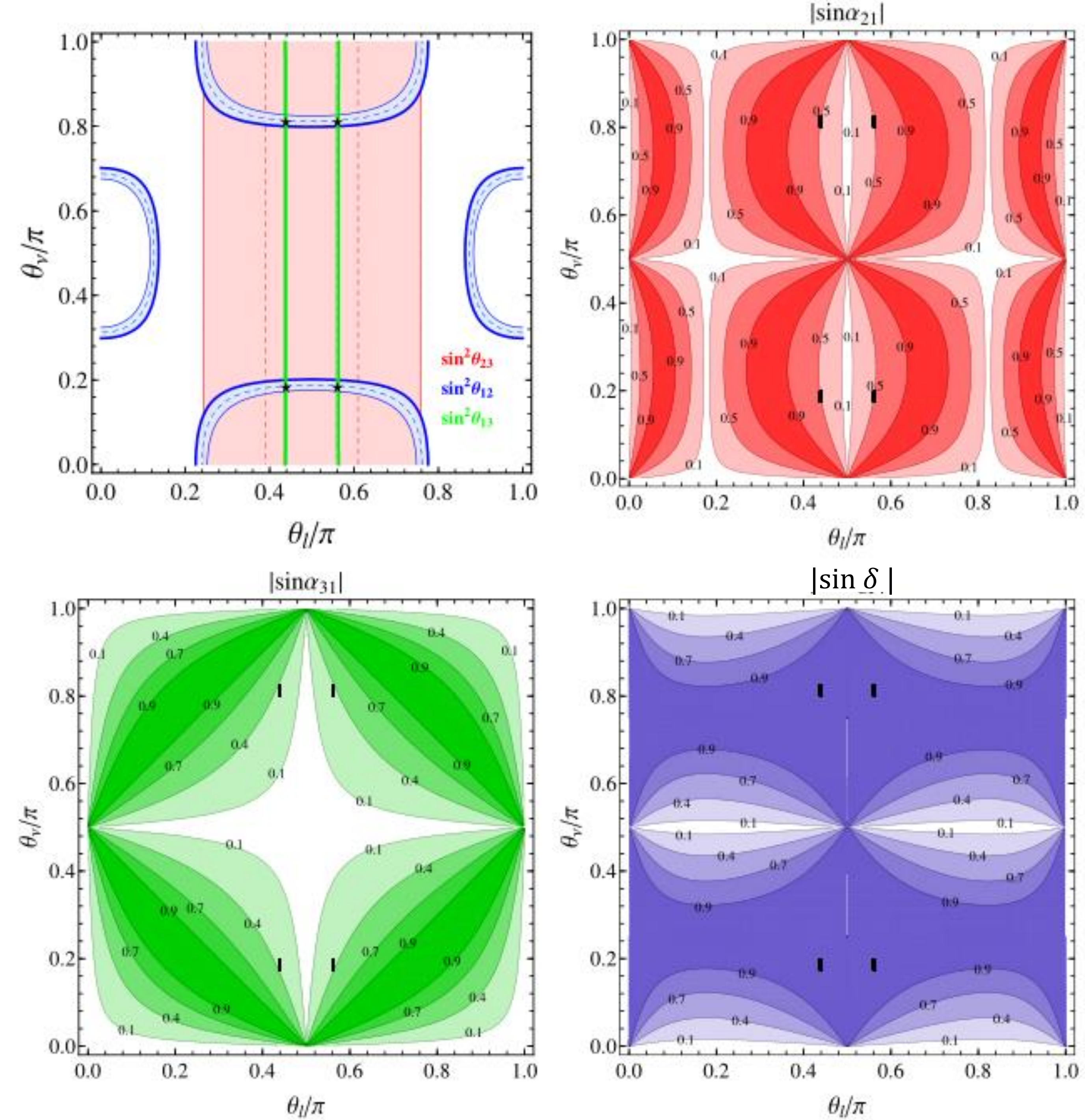
- Numerical benchmark

$$\theta_l = 0.439\pi, \quad \theta_\nu = 0.811\pi, \quad \chi^2_{\min} = 4.147,$$

$$\sin^2\theta_{13} = 0.0220, \quad \sin^2\theta_{12} = 0.318, \quad \sin^2\theta_{23} = 0.603,$$

$$\delta = 1.530\pi, \quad \alpha_{21}/\pi = 0.164\ (\mathrm{mod}\ 1), \quad \alpha_{31}/\pi = 0.112\ (\mathrm{mod}\ 1)$$

Atmospheric angle $\theta_{23} > 45°$ and nearly maximal CP violation $\delta \approx 3\pi/2$

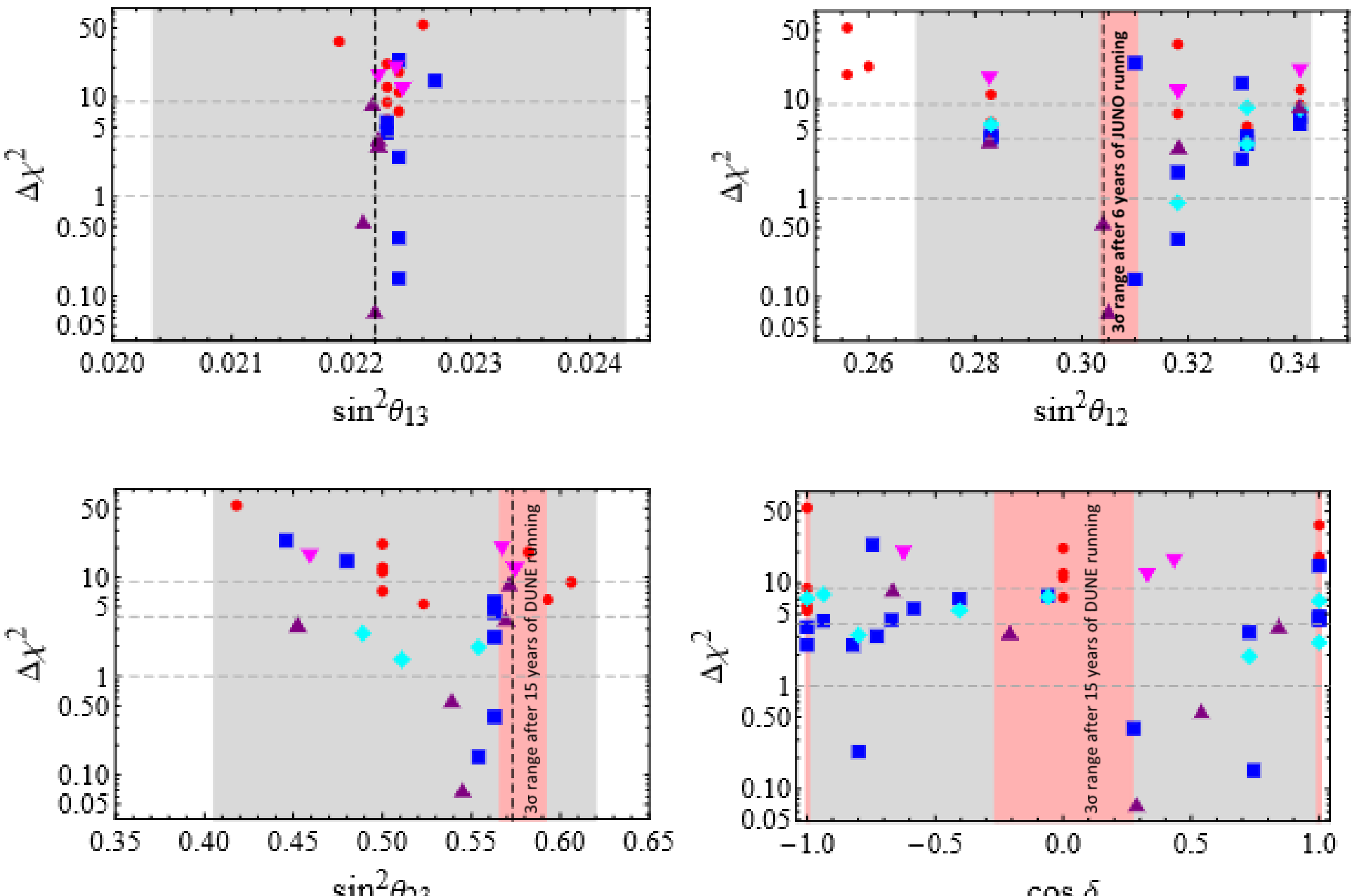


quite predictive!

Testing flavor & CP symmetries

Flavor symmetry models can be ruled out by measuring symmetry protected correlations

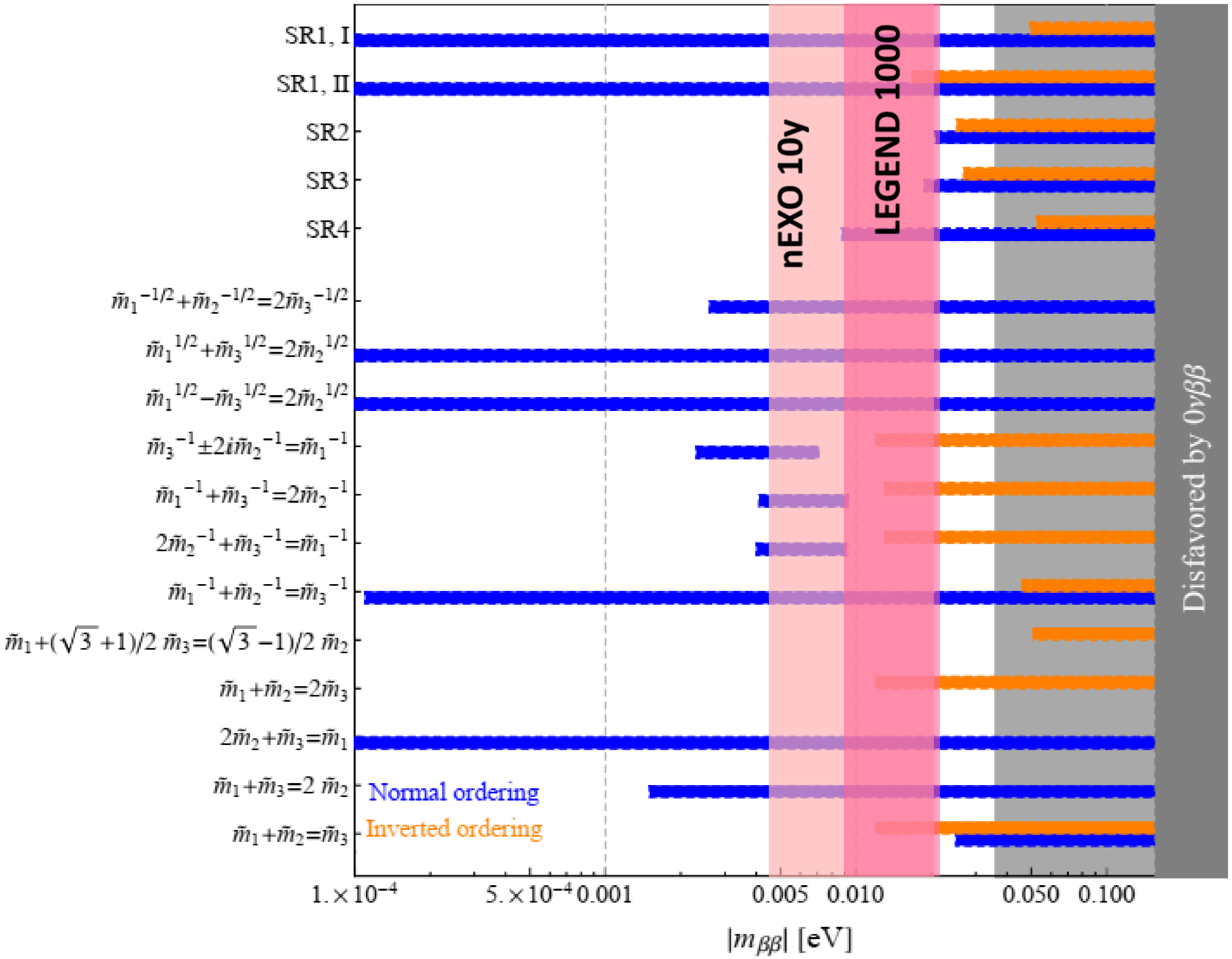
- Precise measurements of mixing angles and CP phase



[Gehrlein, Petcov, Spinrath, Titov, 2203.06219]

- discrete symmetries w/ CP
- discrete symmetries w/o CP (NO)
- discrete symmetries w/o CP (IO)
- modular symmetries (NO)
- modular symmetries (IO)

➢ Test neutrino mass sum rules of flavor symmetry at 0νββ decay



PMNS

neutrino masses

Majorana phases

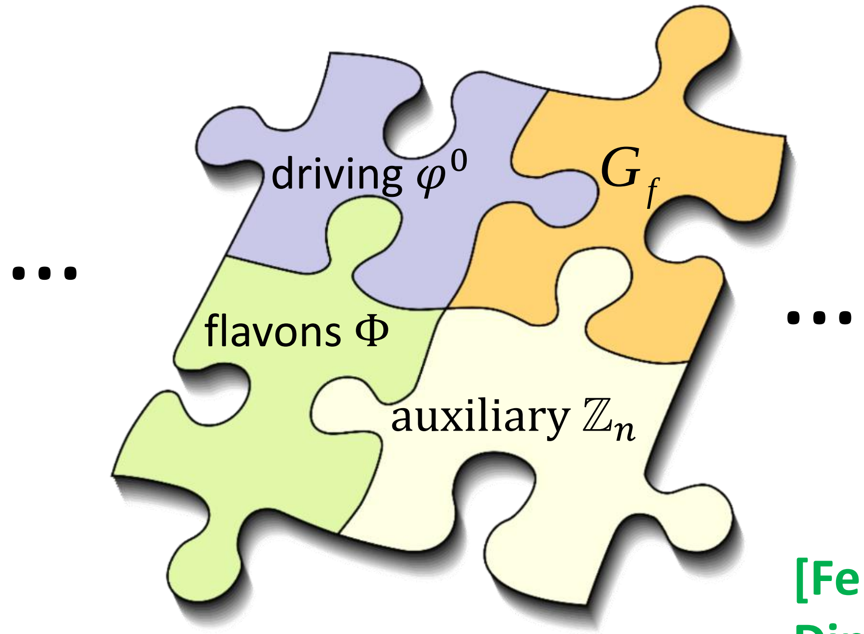
$$|m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$$
$$= \left|c_{12}^2\, c_{13}^2\, m_1 + s_{12}^2\, c_{13}^2\, e^{i\alpha_{21}}\, m_2 + s_{13}^2\, e^{i\alpha_{31}}\, m_3\right|$$

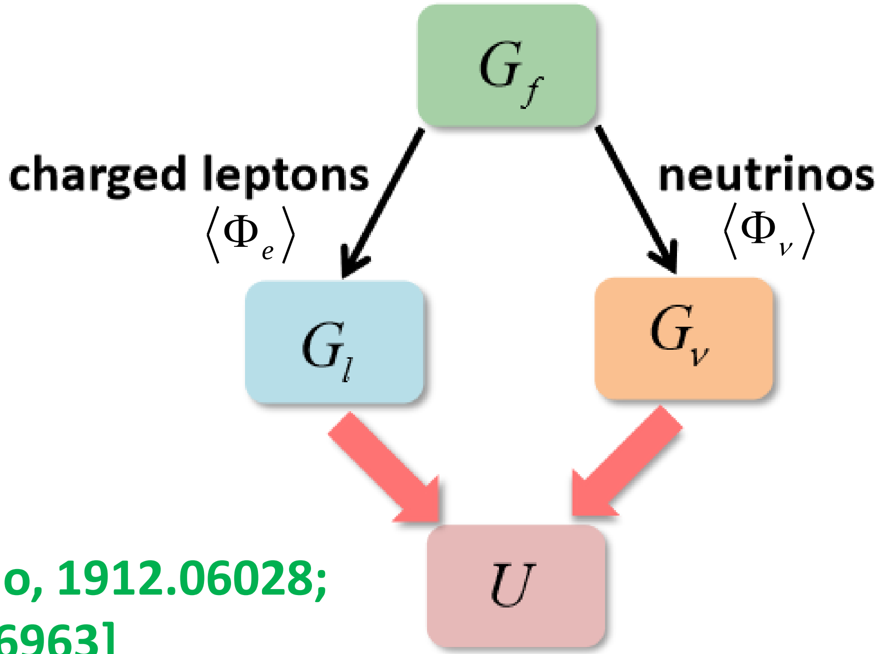
cosines and sines of the mixing angles

talk by Art McDonald

[Snowmass, 2203.12169]

Obstacles of flavor symmetry model building





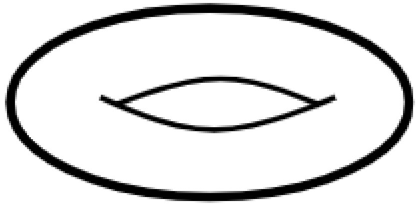
[Feruglio,Romanino, 1912.06028; Ding,Valle, 2402.16963]

- Realistic description of fermion masses and mixing angles requires flavor symmetry G_f be broken by Higgs-like fields "flavons" Φ_e, Φ_ν etc
  - A large number of free parameters in the scalar potential
  - large shaping symmetries and many auxiliary fields

Flavor symmetry models are complicated by the symmetry breaking sector!

Modular symmetry motivated by string compactification

a single complex flavon τ parametrizing shape of torus



$SL(2,Z)$ on torus T^2

- Modular action

[Feruglio, 1706.08749]

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d} \qquad SL(2,Z) = \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a, b, c, d \in \mathbb{Z},\ ad - bc = 1 \right\}$$

- Field **Non-linear** transformation

$$\psi \to (c\tau + d)^{-k} \rho(\gamma) \psi$$

weight $k \in \mathbb{Z}$ ρ is a unitary representation of Γ_N or Γ'_N

finite modular groups as G_f

$$G_f = \begin{cases} \Gamma_N \equiv SL(2,Z) / \pm\Gamma(N) \\ \Gamma'_N \equiv SL(2,Z) / \Gamma(N) \end{cases}$$

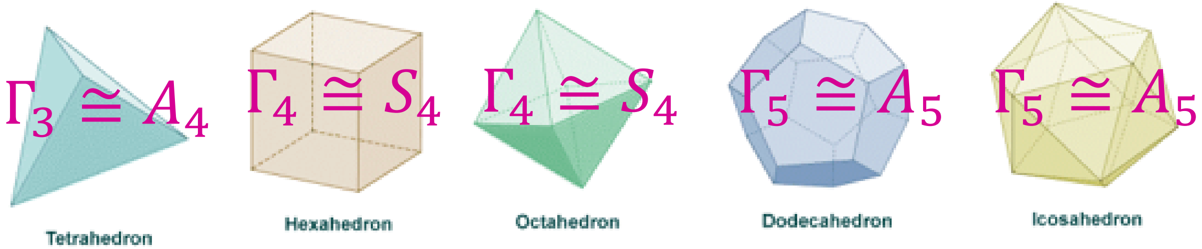
- Superpotential

$$\mathcal{W} = \sum Y_{I_1 I_2 \ldots I_n}(\tau)\, \psi_{I_1} \psi_{I_2} \ldots \psi_{I_n}$$

Principal congruence subgroup of level N

$$\Gamma(N) = \{ \gamma \in SL(2,Z) \mid \gamma = 1_2 \text{ mod } N \}$$

Yukawa coupling $Y_{I_1 I_2 \ldots I_n}$ only depends on τ, and it is strongly constrained by modular invariance



Modular invariant flavor models

Are neutrino masses modular forms?

Ferruccio Feruglio (INFN, Padua and Padua U.) (Jun 27, 2017)

e-Print: 1706.08749 [hep-ph]

pdf   DOI   cite   claim   reference search   254 citations

- Bottom-up models for lepton and quark **[reviews: Kobayashi, Tanimoto, 2307.03384; Ding, King, arXiv:2311.09282]**

	Γ_N/Γ'_N	leptons alone	leptons & quarks	$SU(5)$	$SO(10)$
$N=2$	S_3	Kobayashi et al, 1803.10391...	—	Kobayashi et al, 1906.10341...	—
$N=3$	A_4	Feruglio, 1706.08749,1807.01125; Kobayashi, Tanaka, et al, 1803.10391; Kobayashi, Omoto, et al, 1808.03012...	Okada,Tanimoto,1905.13421; King, King, 2002.00969; Yao, Lu, Ding, 2012.13390...	Anda, King,Perdomo, 1812.05620; Chen, Ding, King,2101.12724...	Ding, King,Lu,2108.09655
	T'	Liu, Ding,1907.01488...	Lu, Liu, Ding,1912.07573...	—	—
$N=4$	S_4	Penedo,Petcov,1806.11040 ; Novichkov, Penedo et al,1811.04933...	Qu, Liu et al,2106.11659	Zhao, Zhang,2101.02266 ; Ding, King, Yao,2103.16311...	—
	S'_4	Novichkov,Penedo,Petcov,2006.03058...	Liu, Yao, Ding, 2006.10722...	—	—
$N=5$	A_5	Novichkov, Penedo et al,1812.02158; Ding, King, Liu, 1903.12588...	—	—	—
	A'_5	Wang, Yu, Zhou, 2010.10159 ...	Yao, Liu, Ding,2011.03501	—	—
$N=6$	Γ_6	—	—	Abe,Higaki et al, 2307.01419	—
	Γ'_6	Li,Liu,Ding,2108.02181	—	—	—
$N=7$	Γ_7	Ding, King et al, 2004.12662	—	—	—
	Γ'_7	—	—	—	—

Minimal modular lepton model

Modular symmetry allows to construct quite predictive lepton models. The modular flavor symmetry is modular binary octahedral group $2O$ which is the Shur double cover of S_4

	L	$E_D^c=(e^c,\mu^c)$	τ^c	N^c	$H_{u,d}$
$2O$	$\mathbf{3}$	$\widehat{\mathbf{2}}'$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{1}$
k_I	-1	6	5	1	0

[Ding, Liu, Yao, 2211.04546; Ding, Liu, Lu, Weng, 2307.14926]

- **Charged leptons:** $\mathcal{W}_E=\alpha\left(E_D^c L Y^{(5)}_{\widehat{\mathbf{2}}'}\right)_{\mathbf{1}} H_d+\beta\left(E_D^c L Y^{(5)}_{\widehat{\mathbf{4}}}\right)_{\mathbf{1}} H_d+\gamma\left(\tau^c L Y^{(4)}_{\mathbf{3}'}\right)_{\mathbf{1}} H_d$

$$M_E=\begin{pmatrix} -\alpha Y^{(5)}_{\widehat{\mathbf{2}}',2}(\tau)-\sqrt{2}\beta Y^{(5)}_{\widehat{\mathbf{4}},3}(\tau) & \sqrt{3}\beta Y^{(5)}_{\widehat{\mathbf{4}},1}(\tau) & \sqrt{2}\alpha Y^{(5)}_{\widehat{\mathbf{2}}',1}(\tau)+\beta Y^{(5)}_{\widehat{\mathbf{4}},4}(\tau) \\ -\alpha Y^{(5)}_{\widehat{\mathbf{2}}',1}(\tau)+\sqrt{2}\beta Y^{(5)}_{\widehat{\mathbf{4}},4}(\tau) & -\sqrt{2}\alpha Y^{(5)}_{\widehat{\mathbf{2}}',2}(\tau)+\beta Y^{(5)}_{\widehat{\mathbf{4}},3}(\tau) & -\sqrt{3}\beta Y^{(5)}_{\widehat{\mathbf{4}},2}(\tau) \\ \gamma Y^{(4)}_{\mathbf{3}',1}(\tau) & \gamma Y^{(4)}_{\mathbf{3}',3}(\tau) & \gamma Y^{(4)}_{\mathbf{3}',2}(\tau) \end{pmatrix} v_d$$

- **Neutrino mass (seesaw mechanism):** $\mathcal{W}_\nu=gH_u(N^cL)_{\mathbf{1}}+\Lambda\left(N^cN^cY^{(2)}_{\mathbf{2}}\right)_{\mathbf{1}}$

$$M_D=g\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}v_u\,,\quad M_N=\begin{pmatrix}-2Y^{(2)}_{\mathbf{2},1}(\tau)&0&0\\0&\sqrt{3}Y^{(2)}_{\mathbf{2},2}(\tau)&Y^{(2)}_{\mathbf{2},1}(\tau)\\0&Y^{(2)}_{\mathbf{2},1}(\tau)&\sqrt{3}Y^{(2)}_{\mathbf{2},2}(\tau)\end{pmatrix}\Lambda$$

Minimal #p: $\alpha, \beta, \gamma, g^2/\Lambda$

Minimal: only 4 real couplings plus modulus τ can explain 12 observables

$$\langle\tau\rangle = -0.1921+1.0854i,\ \ \beta/\alpha = 0.7159,\ \ \gamma/\alpha = 87.4471,$$

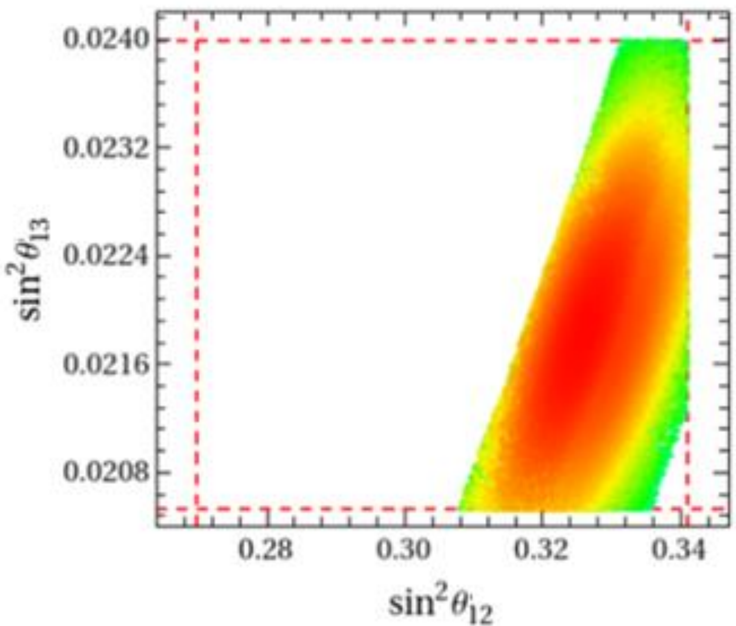
$$\alpha v_d = 0.02881\,\mathrm{MeV},\ \ g^2 v_u^2/\Lambda = 71.8888\,\mathrm{meV}$$

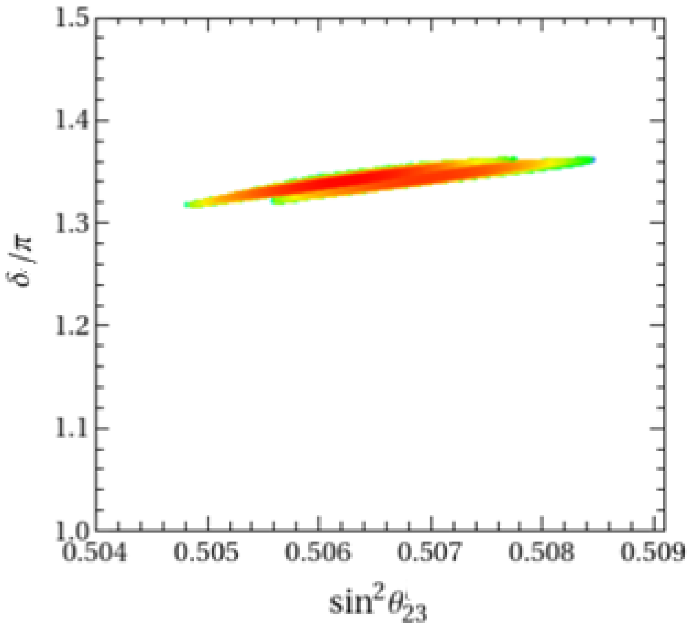
τ is the unique source breaking both modular and CP symmetries. All observables are within the 3σ regions, and neutrino mass spectrum is normal ordering

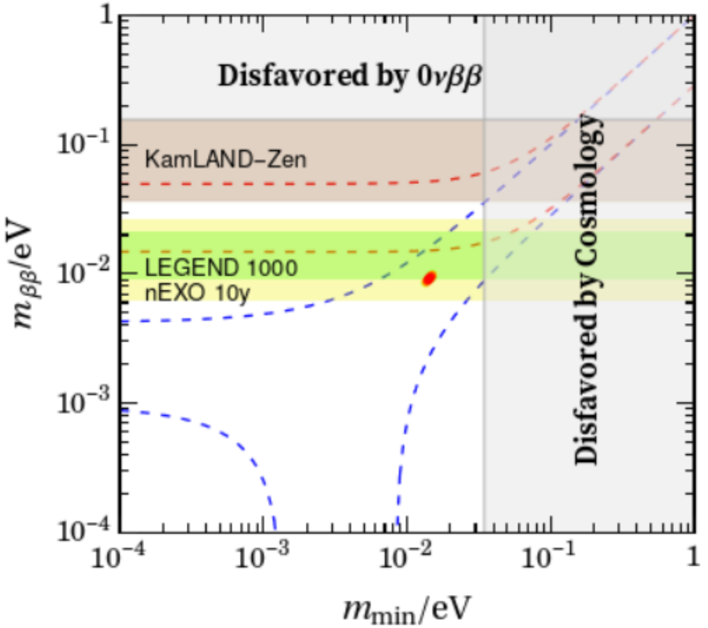
$$\sin^2\theta_{12} = 0.3261,\ \ \sin^2\theta_{13} = 0.02182,\ \ \sin^2\theta_{23} = 0.5063,\ \ \delta = 1.34\pi,$$

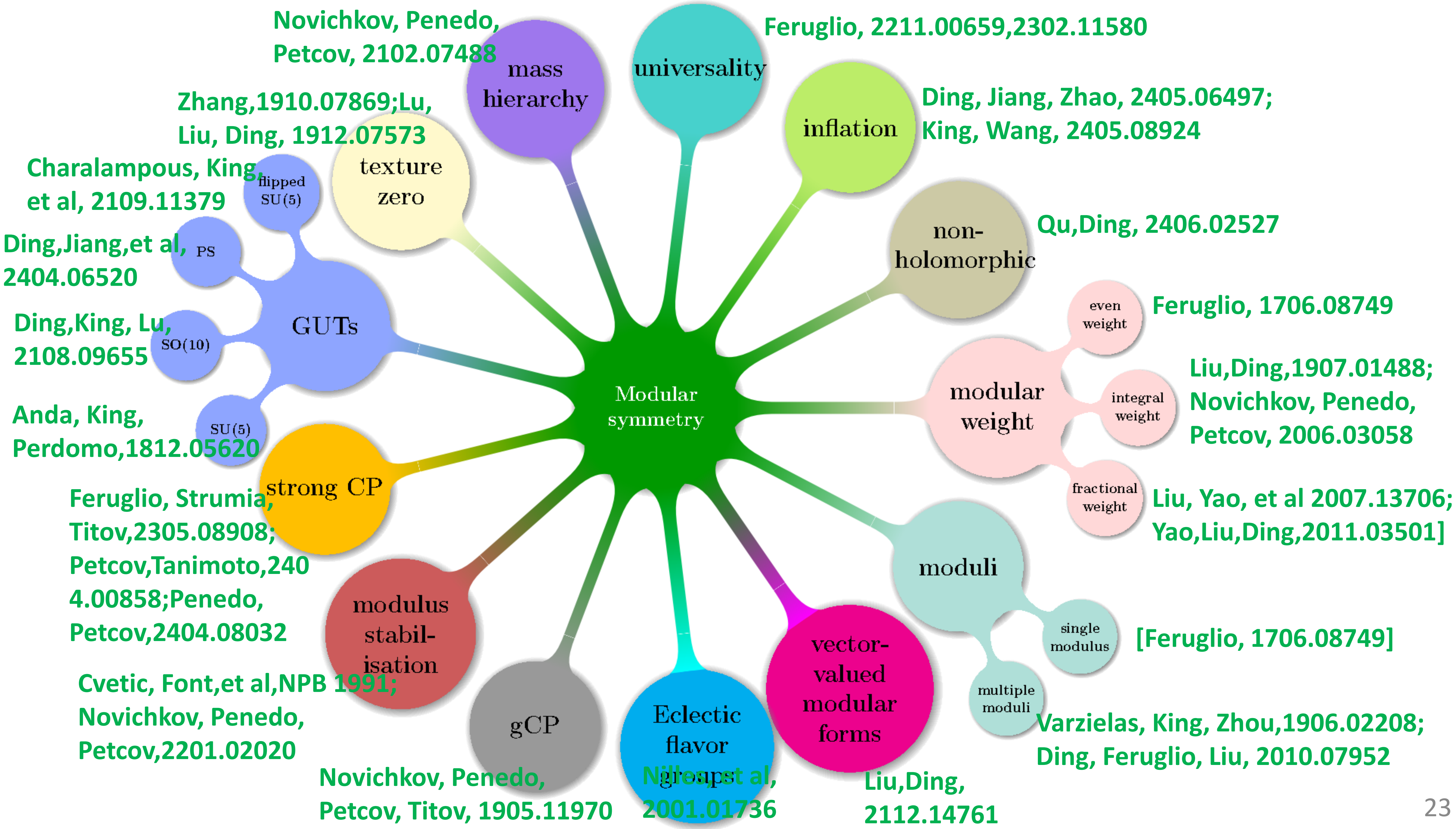
$$\alpha_{21} = 1.3268\pi,\ \ \alpha_{31} = 0.5401\pi,\ \ m_e/m_\mu = 0.004737,\ \ m_\mu/m_\tau = 0.05876,$$

$$m_1 = 14.27\,\mathrm{meV},\ \ m_2 = 16.67\,\mathrm{meV},\ \ m_3 = 51.64\,\mathrm{meV}$$








Modular symmetry
mass hierarchy
Novichkov, Penedo, Petcov, 2102.07488
universality
Feruglio, 2211.00659,2302.11580
inflation
Ding, Jiang, Zhao, 2405.06497; King, Wang, 2405.08924
non-holomorphic
Qu,Ding, 2406.02527
modular weight
even weight
Feruglio, 1706.08749
integral weight
Liu,Ding,1907.01488; Novichkov, Penedo, Petcov, 2006.03058
fractional weight
Liu, Yao, et al 2007.13706; Yao,Liu,Ding,2011.03501]
moduli
single modulus
[Feruglio, 1706.08749]
multiple moduli
Varzielas, King, Zhou,1906.02208; Ding, Feruglio, Liu, 2010.07952
vector-valued modular forms
Liu,Ding, 2112.14761
Eclectic flavor groups
Nilles, et al, 2001.01736
gCP
Novichkov, Penedo, Petcov, Titov, 1905.11970
modulus stabil-isation
Cvetic, Font,et al,NPB 1991; Novichkov, Penedo, Petcov,2201.02020
strong CP
Feruglio, Strumia, Titov,2305.08908; Petcov,Tanimoto,2404.00858;Penedo, Petcov,2404.08032
GUTs
flipped SU(5)
Charalampous, King, et al, 2109.11379
PS
Ding,Jiang,et al, 2404.06520
SO(10)
Ding,King, Lu, 2108.09655
SU(5)
Anda, King, Perdomo,1812.05620
texture zero
Zhang,1910.07869;Lu, Liu, Ding, 1912.07573

Summary

- The fundamental origin of the fermion mass and mixing patterns is still elusive, neutrino provides a unique window to understand the flavor problem and explore BSM physics.

- We have learned a lot from the symmetry consideration, some illuminating and testable examples

  - Flavor and CP symmetries are powerful in constraining lepton and quark mixing parameters, in particular CP violation phases

  - Modular symmetry: enhanced predicitivity and possible connection with string theory

- Future neutrino facilities precisely measuring lepton mixing angles, CP phase δ and $0\nu\beta\beta$ decay can exclude many models, will help to pin down the organizing principle of flavor sector.

Thank you for your attention!

Backup

Anarchy: alternative to flavor symmetry

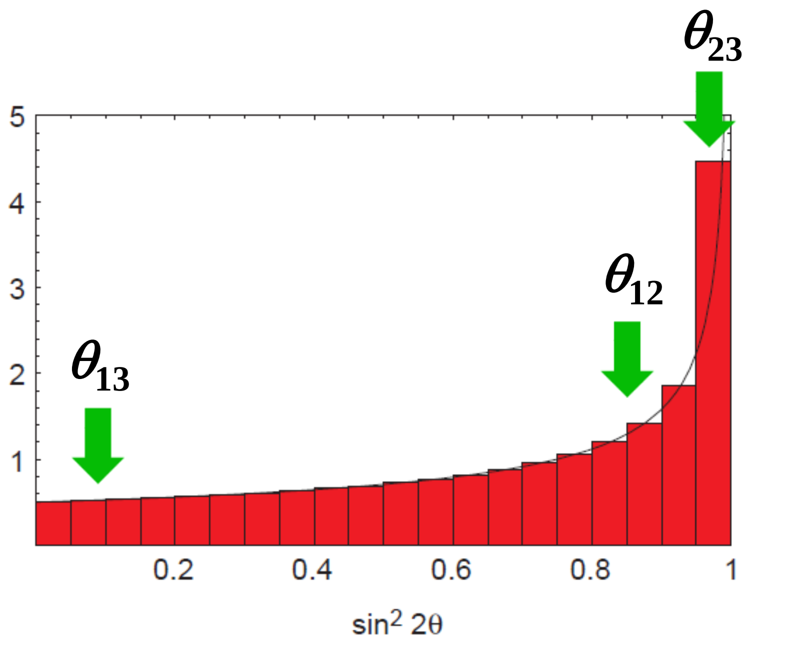
Anarchy: No particular structure in neutrino mass matrix

[Hall, Murayama, Weiner, hep-ph/9911341 ; Gouvea,Murayama, 1204.1249]

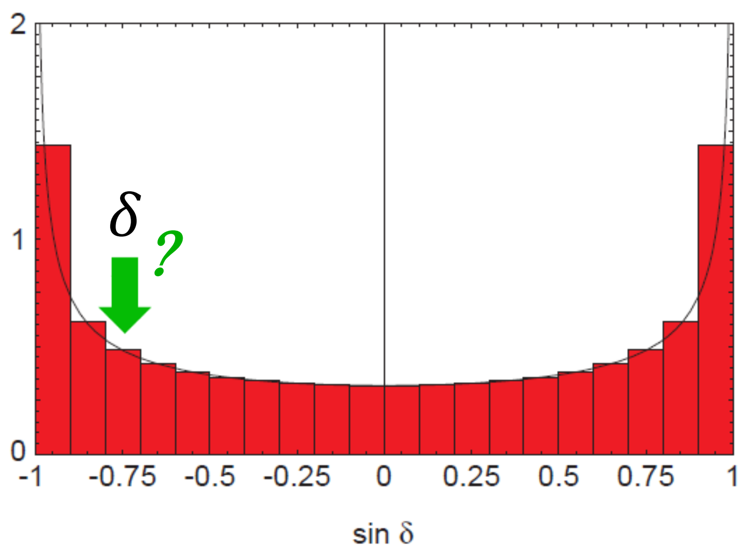
$$m_\nu \propto \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

each matrix element is random

The mixing angles and CP violation phases distributions are given by U(3) Harr distribution



All three mixing angles large



Large CP violation

- large number of O(1) free parameters→ only statistical tests

- Anarchy is not applicable to charged lepton and quark Yukawa couplings

Extending the minimal modular model to quark sector

	$Q_D=(Q_1,Q_2)$	Q_3	$U_D^c=(u^c,c^c)$	t^c	$D_D^c=(d^c,s^c)$	b^c
$2O$	$\mathbf{2}$	$\mathbf{1}'$	$\hat{\mathbf{2}}'$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{1}'$
k_I	k_{Q_D}	k_{Q_D}	$3-k_{Q_D}$	$6-k_{Q_D}$	$6-k_{Q_D}$	$-k_{Q_D}$

$$M_u = \begin{pmatrix} \alpha_u Y^{(3)}_{\hat{\mathbf{4}},3} & -\alpha_u Y^{(3)}_{\hat{\mathbf{4}},2} & 0 \\ \alpha_u Y^{(3)}_{\hat{\mathbf{4}},4} & \alpha_u Y^{(3)}_{\hat{\mathbf{4}},1} & 0 \\ -\beta_u Y^{(6)}_{\mathbf{2},2} & \beta_u Y^{(6)}_{\mathbf{2},1} & \gamma_u Y^{(6)}_{\mathbf{1}} \end{pmatrix} v_u \,,$$

$$M_d = \begin{pmatrix} \alpha_d Y^{(6)}_{\mathbf{1}} - \gamma_d Y^{(6)}_{\mathbf{2},1} & \beta_d Y^{(6)}_{\mathbf{1}'} + \gamma_d Y^{(6)}_{\mathbf{2},2} & -\delta_d Y^{(6)}_{\mathbf{2},2} \\ \gamma_d Y^{(6)}_{\mathbf{2},2} - \beta_d Y^{(6)}_{\mathbf{1}'} & \alpha_d Y^{(6)}_{\mathbf{1}} + \gamma_d Y^{(6)}_{\mathbf{2},1} & \delta_d Y^{(6)}_{\mathbf{2},1} \\ 0 & 0 & \epsilon_d \end{pmatrix} v_d$$

The complex modulus τ is common in both quark and lepton sectors, and its value is fixed by the lepton parameters

$$\langle\tau\rangle = -0.1946 + 1.0799i$$

The quark masses and CKM mixing parameters can be well accommodated with $\chi_q^2 = 6.4$:

$$\theta_{12}^q = 0.229,\quad \theta_{13}^q = 0.00393,\quad \theta_{23}^q = 0.0388,\quad \delta_{CP}^q = 61.27°,$$

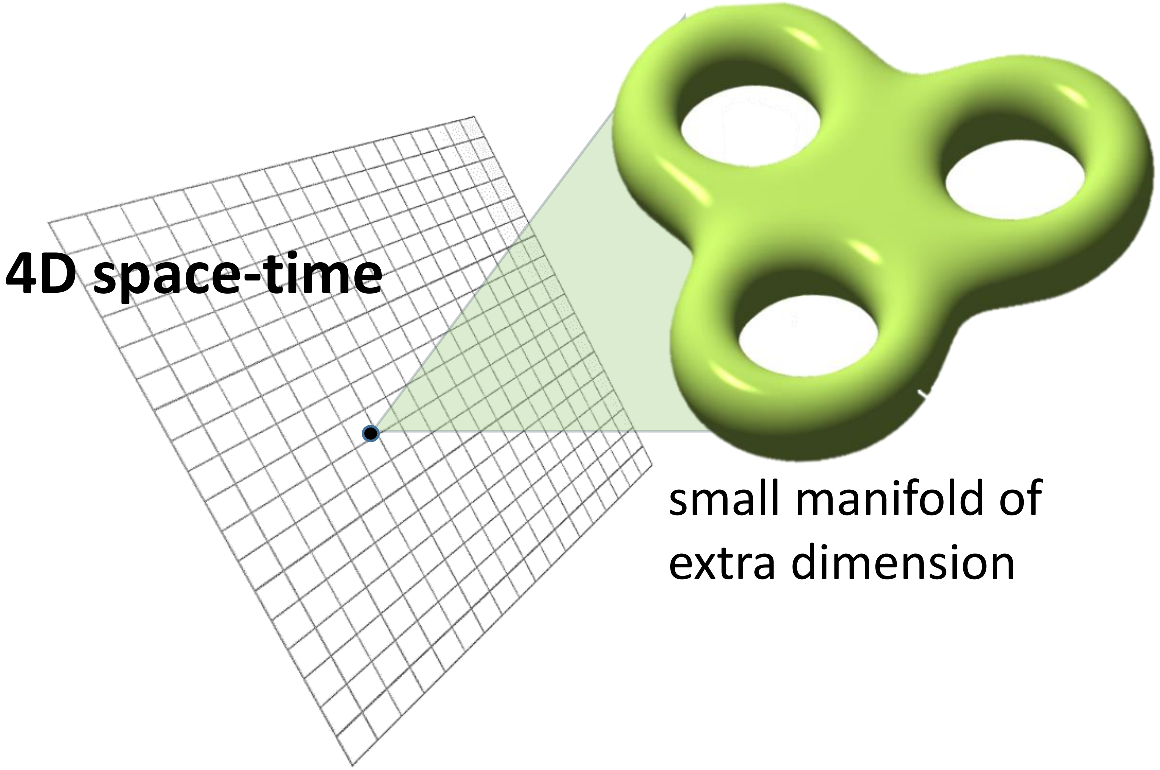
$$m_u/m_c = 0.00243,\quad m_c/m_t = 0.00245,\quad m_d/m_s = 0.0510,\quad m_s/m_b = 0.0234$$

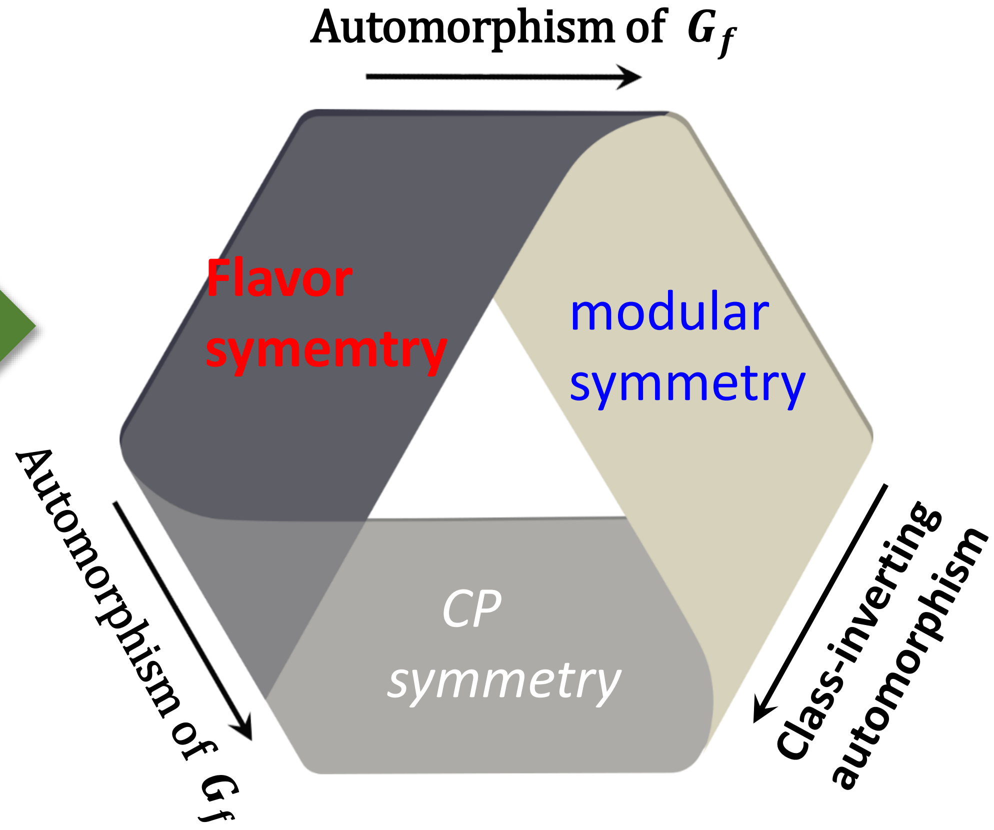
- The model uses **14** parameters to describe the masses and mixing of both quark and lepton sectors: **12** masses+**6** mixing angles+**4** CP phases.

Unification of flavor, CP and modular symmetries

- Top-down approach (orbifold string compactification) gives
  - Normal symmetries of extra dimensions ➜traditional flavor symmetries
  - String duality transformations ➜modular flavor symmetries
  - CP symmetry

[Nilles, Ramos-Sanchez, Vaudrevange, 2001.01736; 2004.05200]





- Top-down and bottom-up approaches do not yet meet, and model building in its infancy

Tests of modulus couplings

Non-standard neutrino interactions [Ding,Feruglio, 2003.13448]

$$\mathcal{L} = i \sum_{f=e,e^c,\nu} \bar{f}\, \bar{\sigma}^\mu \partial_\mu f + \frac{1}{2}\partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha - \frac{1}{2} M^2_\alpha \varphi^2_\alpha - (m_e + \mathcal{Z}^e_\alpha \varphi_\alpha) e^c e - \frac{1}{2}\nu(m_\nu + \mathcal{Z}^\nu_\alpha \varphi_\alpha)\nu + h.c. + \dots.$$

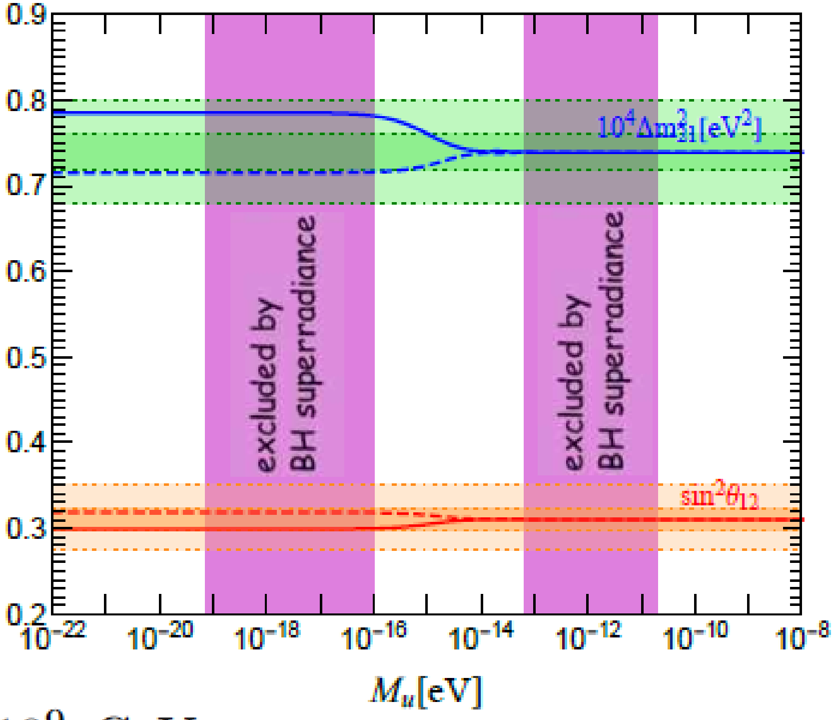
in medium with non-zero electron number density

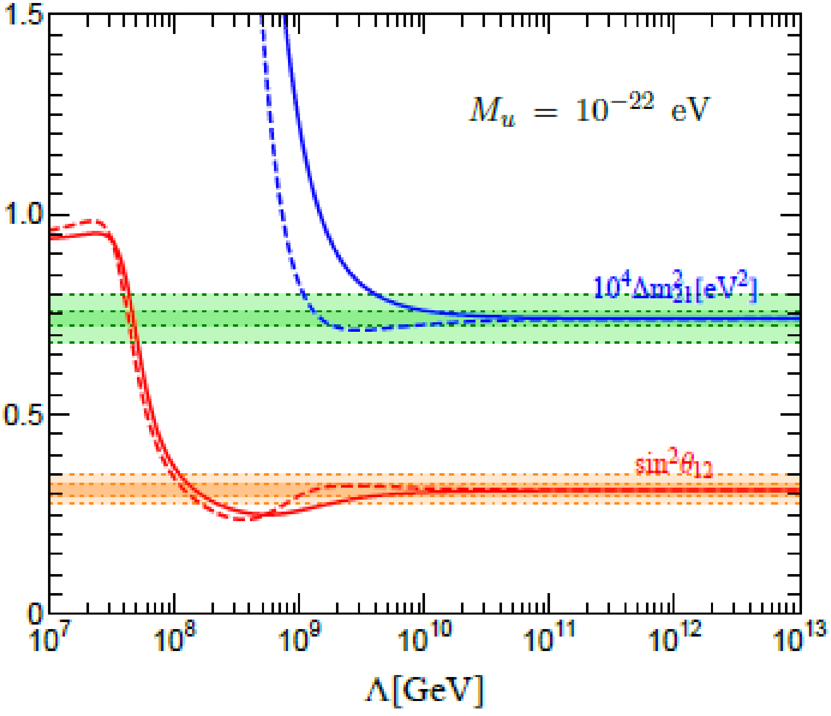
$$\delta m_\nu(0) = -n_e^0 \frac{\mathrm{Re}(\mathcal{Z}^e)\mathcal{Z}^\nu}{M^2(R)}$$

small, unless the modulus is very light

In the sun

$$\tau = <\tau> + \frac{\varphi_u + i\varphi_v}{\sqrt{2}}$$





$$\Lambda = 5 \times 10^9 \ \mathrm{GeV}$$