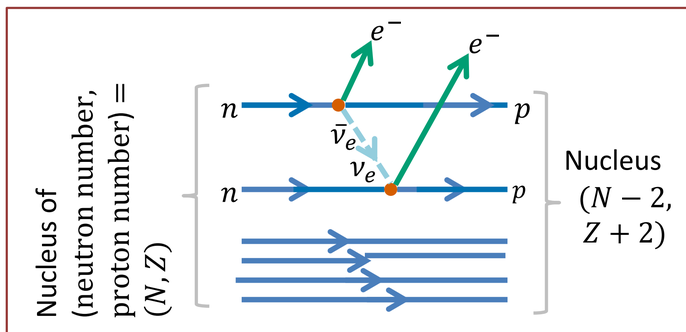
Reliability of Calculated Nuclear Matrix Element of Two-Neutrino Double-β decay

NEUTRINO 2024

ID # 17

J. Terasaki, *Institute of Experimental and Applied Physics, Czech Technical Univ. in Prague*

The goal is to determine the effective mass of the neutrino using the neutrinoless double-β (0νββ) decay of the nuclei. A problem was reported of a significant discrepancy in the running sum for the nuclear matrix element (NME) of the two-neutrino double-β (2νββ) decay between the shell model and QRPA calculations. The reliability of the calculation of the NME of the 2νββ decay is important because of the similarity of the 0νββ and 2νββ decays. This problem is solved in this study.



Possible change of two neutrons to two protons in a nucleus emitting two electrons with neutrino exchange (neutrinoless double-β (0νββ) decay). This decay occurs, if the neutrino (ν) is a Majorana particle ($\nu = \bar{\nu}$), and the effective neutrino mass can be determined, see the equations below. Determination of the effective neutrino mass is one of the most important subjects in modern physics.

Why nuclei?

Because E(final state) < E(initial state) is necessary.

List of nuclei used in the experiments

^{76}Ge→^{76}Se	^{130}Te→^{130}Xe	^{136}Xe→^{136}Ba
^{150}Nd→^{150}Sm	^{48}Ca→^{48}Ti	^{82}Se→^{82}Kr
^{96}Zr→^{96}Mo	^{100}Mo→^{100}Ru	^{110}Pd→^{110}Cd
^{116}Cd→^{116}Sn	^{124}Sn→^{124}Te	and more

Principle to determine effective neutrino mass

$$\langle m_\nu \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

U: Pontecorvo-Maki-Nakagawa-Sakata matrix

m_i: eigen mass (i=1,2,3)

$$1/T_{0\nu}(\text{g.s.} \to \text{g.s.}) = \left|M^{(0\nu)}\right|^2 G_{0\nu} g_A^4 \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Half-life ↑ Experimental measurement

Nuclear matrix element (NME)

Phase-space factor

Effective ν mass

Axial-vector current coupling

Theoretical calculation

m_e: electron mass

Phase-space factor ← Wave functions of emitted electrons

NME ← Nuclear wave functions

Accurate calculation more difficult than the phase-space factor

Approximation is indis-pensable.

NME

$$M^{(0\nu)} = \sum_B \sum_{\substack{pp' \\ nn'}} \langle pp' | V(r_{12}, E_B) | nn' \rangle \langle F | c_{p'}^\dagger c_{n'} | B \rangle \times \langle B | c_p^\dagger c_n | I \rangle$$

Final state, ground state of nucleus(N–2,Z+2)

intermediate state,nucleus (N–1,Z+1)

Initial state, ground state of nucleus (N,Z)

The transition operator used in my calculation is

$$V(r_{12}, E_B) \cong h_+(r_{12}) \{ -\boldsymbol{\sigma}(1) \cdot \boldsymbol{\sigma}(2) + g_V^2/g_A^2 \} \tau^-(1) \tau^-(2)$$

Double-Gamow-Teller + Double-Fermi

Neutrino potential

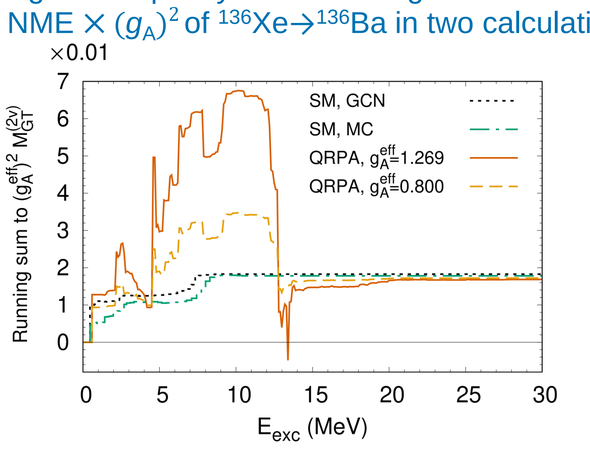
g_V: vector current coupling = 1

Status:

The calculated NMEs by various approximation methods and groups are distributed in a range of the min.-max. ratio of 2–3. The NME cannot be obtained by experiment. Thus, examination and improvement of the calculation are essential.

The problem

The large discrepancy in the running sum for 2νββ NME × $(g_A)^2$ of ^{136}Xe→^{136}Ba in two calculations.



A. Gando et al., PRL **122**, 192501 (2019)

The intermediate nucleus is ^{136}Cs.

In calculation, effective value g_A^{eff} is used for g_A.

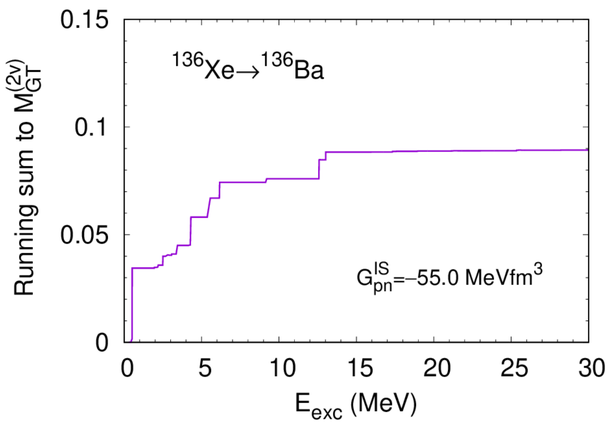
SM: shell model; GCN and MC: interaction names.

QRPA: quasiparticle random-phase approximation.

The calculated $M_{GT}^{(2v)}$ (the right edge) are fitted to the exp. value.

GT: Gamow-Teller; the Fermi NME is negligible for the 2νββ decay.

Variety of calculations



My QRPA calculation

Skyrme + Coulomb + contact isovector (IV) and isoscalar (IS) pairing (*pp*, *nn*, and *pn*) interactions.

p: proton; *n*: neutron

G_{pn}^{IS}: strength of the IS *pn* pairing interaction.

J. T., PRC **108**, 014301 (2023)

	Menéndez	Horoi	Šimkovic	Terasaki
Method	SM		QRPA	
Variation of comp. of $M^{(2v)}$	Small	Large	Large	Small

The calculations of Menéndez et al. (SM) and Šimkovic et al. (QRPA) are those in the above figure (Gando et al.).

The calculation of Horoi et al. is in

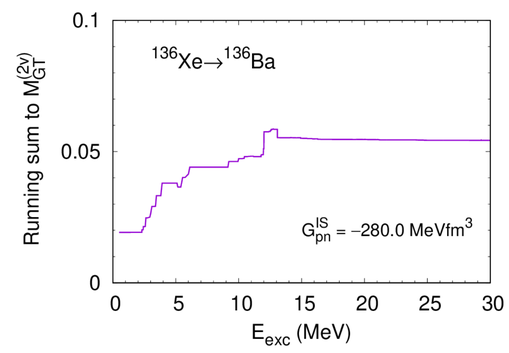
M. Horoi and A. Brown, PRL **110**, 222502 (2013)

The cause of the discrepancy problem is not the theoretical differences between SM and QRPA.

Conclusion 1o3

Investigation of cause

• It turned out that a small decrease is obtained at the energy of the GT⁻ giant resonance (GR⁻) at 12.5 MeV by increasing G_{pn}^{IS}.



• The decrease in the running sum implies that the component of the 2νββ NME changes its sign at that energy. This behavior can be explained by the large strength of the interaction analytically under the separable approximation to the RPA.

2νββ NME

$$M^{(2\nu)} \cong \sum_B \frac{m_e c^2}{E_B - \bar{M}} \langle F | \boldsymbol{\sigma}\tau^- | B \rangle \cdot \langle B | \boldsymbol{\sigma}\tau^- | I \rangle$$

$$|B(\text{GR}^-)\rangle = O_{B_{GR^-}}^{I\dagger} |I\rangle + O_{B_{GR^-}}^{F\dagger} |F\rangle$$

$$O_B^\dagger = \cdots + \frac{N_B C_{\mu i}}{\varepsilon_{\mu i} - E_B} c_\mu^\dagger c_i + \cdots,$$

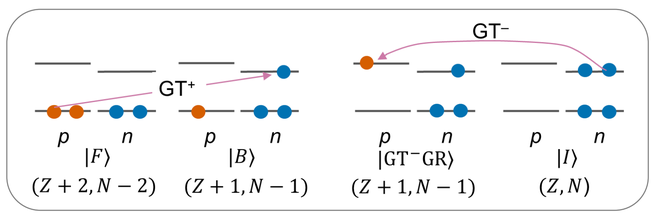
$C_{\mu i}$: matrix element of the one-body operator used for the interaction of the separable approximation. The particle and hole states are denoted by μ and i.

$\varepsilon_{\mu i}$: particle-hole energy of main component of GR⁻.

E_B: energy of intermediate state $|B\rangle$. $\bar{M}$: mean value of initial and final nuclear masses.

N_B: normalization factor > 0.

When the attractive interaction strength is enhanced, all E_B are lowered and the one nearest to the $\varepsilon_{\mu i}$ of the GR⁻ switches from $E_B < \varepsilon_{\mu i}$ to $E_{B'} > \varepsilon_{\mu i}$. This causes the sign change of $\langle B(GR^-) | \boldsymbol{\sigma}\tau^- | I \rangle$. $O_{B_{GR^-}}^{F\dagger} |F\rangle$ does not have this component in the single-Slater approximation; see the figure below. Thus, the possibility of the sign change of $\langle F | \boldsymbol{\sigma}\tau^- | B(GR^-) \rangle$ is low.



GT⁻: GT transition from *n* to *p*.; GT⁺: that from *p* to *n*.

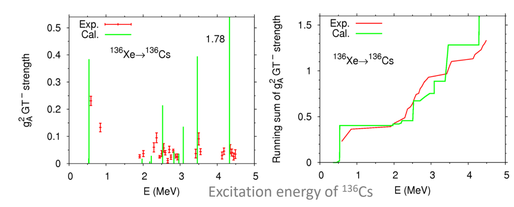
The significant decrease in the running sum at the GT⁻ GR implies that the interaction is stronger than that of calculations with less decrease.

The cause of the problem is the difference in the interaction strength.

Conclusion 2o3

Check of my interaction strength

Exp. data and calculation of g_A^2 x GT⁻ strength



^{136}Xe(^{3}He,*t*)^{136}Cs and *e* capture

Exp. data from D. Frekers et al., Nucl. Phys. A **916**, 219 (2013);

J.T. used g_A^{eff} = 0.49. to reproduce the exp. half-life of the 2νββ decay

Conclusion 3o3

The energy dependence of the strength shows the validity of my interaction.