



CP violation due to a Majorana phase in two flavor neutrino oscillations with decays

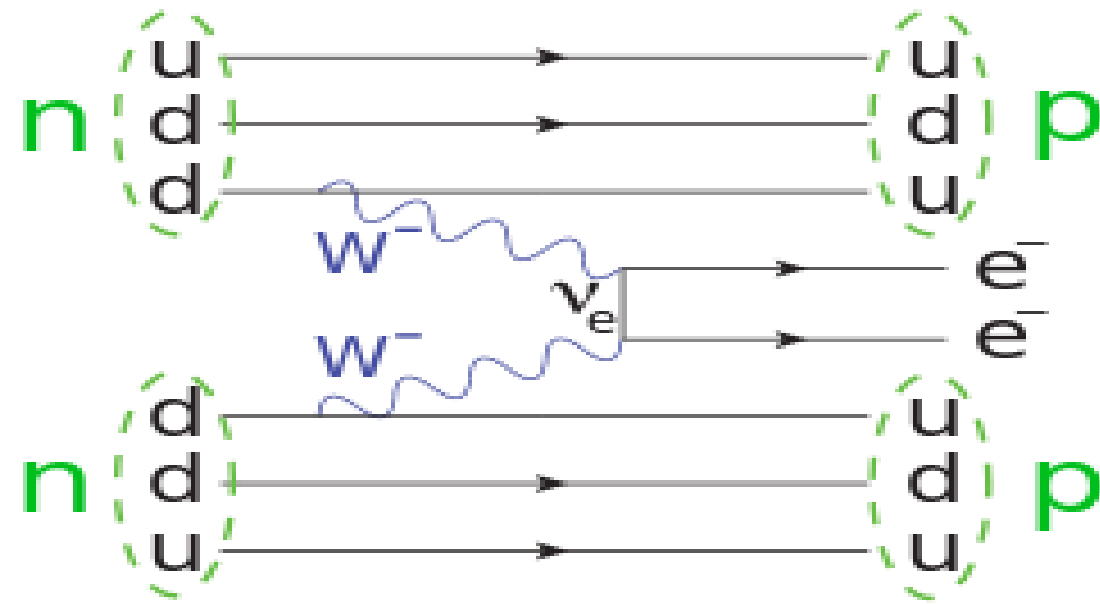
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Introduction

- The fundamental nature of most intriguing particle neutrinos, whether they are Dirac or Majorana fermions, is still an open question.
- To probe **Majorana nature**, many experiments looking for signals of neutrinoless double beta decay.



Objective of our work

- Well established fact: Vacuum oscillation probabilities do not depend on the Majorana phases.
- But oscillation probabilities depend on Majorana phases, with an **off-diagonal decoherence** term and also these probabilities are **CP-violating**.
[F.Benatti et al.Phys.Rev.D 64,085015](#)
- The question we ask: “**what are the other possibilities under which the Majorana phases appear in neutrino oscillation probabilities and lead to CP-violation?**”.

Oscillations with Decoherence

- Neutrino mass eigenstates ν_i mix via a unitary matrix with flavour states ν_α as

$$\nu_\alpha = U \nu_i = O U_{ph} \nu_i$$

where, $\nu_\alpha = (\nu_e \ \nu_\mu)^T$, $\nu_i = (\nu_1 \ \nu_2)^T$ and

$$O = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad U_{ph} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

- The evolution of the neutrino considered as an **open system**, can be expressed by the Lidblad–Kossakowski master equation

$$\frac{d\rho}{dt} = -i[H_{eff}, \rho(t)] + D[\rho(t)]$$

$$\begin{pmatrix} \dot{\rho}_1(t) \\ \dot{\rho}_2(t) \\ \dot{\rho}_3(t) \end{pmatrix} = \begin{pmatrix} -\gamma & \Delta - \alpha & 0 \\ \Delta - \alpha & -\gamma & 0 \\ 0 & 0 & -\gamma_3 \end{pmatrix} \begin{pmatrix} \rho_1(t) \\ \rho_2(t) \\ \rho_3(t) \end{pmatrix}$$

$$P_{\mu e} = \frac{1}{2} \left(1 - e^{-\gamma t} \cos^2(2\theta) - e^{-\gamma t} \sin^2(2\theta) \left[\cosh(\Omega_\alpha t) + \frac{\alpha \sin(2\phi) \sinh(\Omega_\alpha t)}{\Omega_\alpha} \right] \right)$$

where, α is the off-diagonal decoherence term, $\Delta = \frac{\Delta m^2}{2E}$ and $\Omega_\alpha = \sqrt{\alpha^2 - \Delta^2}$.

⇒ The Majorana phase appears in probability expression due to decoherence as long as the **off-diagonal term** $\alpha \neq 0$.

$$\Delta_{CP}(t) = P_{\mu e}(t) - P_{\bar{\mu} \bar{e}}(t) = -e^{-\gamma t} \sin^2(2\theta) \frac{\alpha \sin(2\phi) \sinh(\Omega_\alpha t)}{\Omega_\alpha}$$

Oscillations with decay-Hamiltonian

- The general decay-Hamiltonian

$$\mathcal{H} = M - i\Gamma/2,$$

$$M = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \quad \Gamma/2 = \begin{pmatrix} b_1 & \frac{1}{2}\eta e^{i\xi} \\ \frac{1}{2}\eta e^{-i\xi} & b_2 \end{pmatrix}$$

System of two particles which can oscillate into each other, these matrices can have off-diagonal terms, as in the case of neutral meson system.

- The matrix Γ needs to be positive semi-definite, *i.e.*, **non negative** ⇒ $b_1, b_2 \geq 0$ and $\eta^2 \leq 4b_1b_2$.
- The mass eigenstates are **not** decay eigenstates ($\eta \neq 0$) and evolution equation:

$$i \frac{d}{dt} \nu_\alpha(t) = \left[\frac{(a_1 + a_2)}{2} \sigma_0 - \frac{(a_2 - a_1)}{2} O \sigma_z O^T - \frac{i}{2} (b_1 + b_2) \sigma_0 - \frac{i}{2} O U_{ph} (\vec{\sigma} \cdot \vec{\Gamma}) U_{ph}^\dagger O^T \right] \nu_\alpha(t)$$

where $\vec{\Gamma} = [\eta \cos \xi, -\eta \sin \xi, -(b_2 - b_1)]$.

- Since σ_x and σ_y do not commute with U_{ph} matrix, the Majorana phase ϕ **remains** in the evolution equation.

Oscillation Probability

- Time evolution operator in the mass eigenbasis is $\mathcal{U} = e^{-i\mathcal{H}t}$, can be expanded in the basis spanned by σ_0 and Pauli matrices.

$$n_\mu \equiv (n_0, \vec{n}), \quad n_\mu = Tr[(-i\mathcal{H}t) \cdot \sigma_\mu] / 2.$$

$$\mathcal{U} = e^{n_0} \left[\cosh n \sigma_0 + \frac{\vec{n} \cdot \vec{\sigma}}{n} \sinh n \right],$$

$$n = \sqrt{n_x^2 + n_y^2 + n_z^2}$$

- Oscillation probabilities can be obtained as $P_{\alpha\beta} = |(\mathcal{U}_f)_{\alpha\beta}|^2$, $\mathcal{U}_f = \mathcal{U} \mathcal{U}^{-1}$
- In the limit $b_1 = b = b_2$ and $\eta \ll |a_2 - a_1|$,

$$P_{ee} = e^{-2bt} (P_{ee}^{\text{vac}} - \eta \cos(\xi - \phi) \mathcal{A})$$

$$P_{\mu\mu} = e^{-2bt} (P_{\mu\mu}^{\text{vac}} + \eta \cos(\xi - \phi) \mathcal{A})$$

$$P_{e\mu} = e^{-2bt} (P_{e\mu}^{\text{vac}} + 2\eta \sin(\xi - \phi) \mathcal{B})$$

$$P_{\mu e} = e^{-2bt} (P_{\mu e}^{\text{vac}} - 2\eta \sin(\xi - \phi) \mathcal{B}).$$

where, $\mathcal{A} = \frac{\sin(2\theta) \sin[(a_2 - a_1)t]}{(a_2 - a_1)}$,
 $\mathcal{B} = \frac{\sin(2\theta) \sin^2[\frac{1}{2}t(a_2 - a_1)]}{(a_2 - a_1)}$

CP-violation

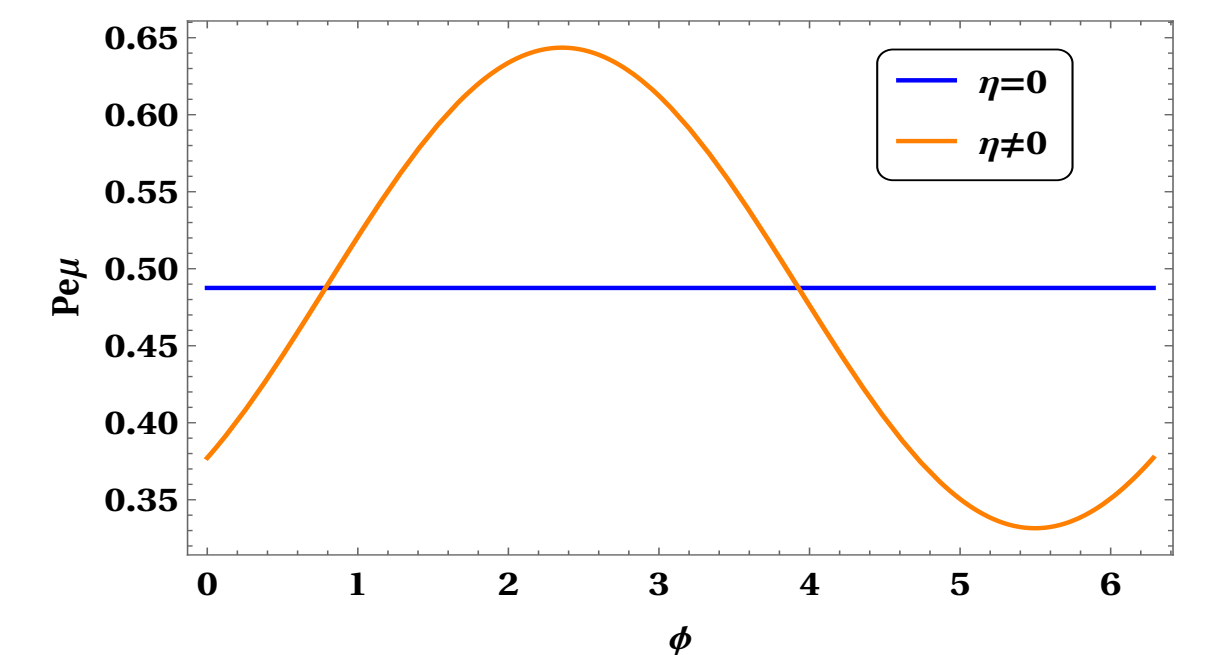
- We assume **CPT**-conservation which implies $\vec{M} = M$ and $\vec{\Gamma} = \Gamma^*$.
- For antineutrino probabilities, substitute $\phi \rightarrow -\phi$ and $\xi \rightarrow -\xi$.

$$P_{\bar{e}\bar{\mu}} \neq P_{e\mu} \Rightarrow \text{CP-violation}$$

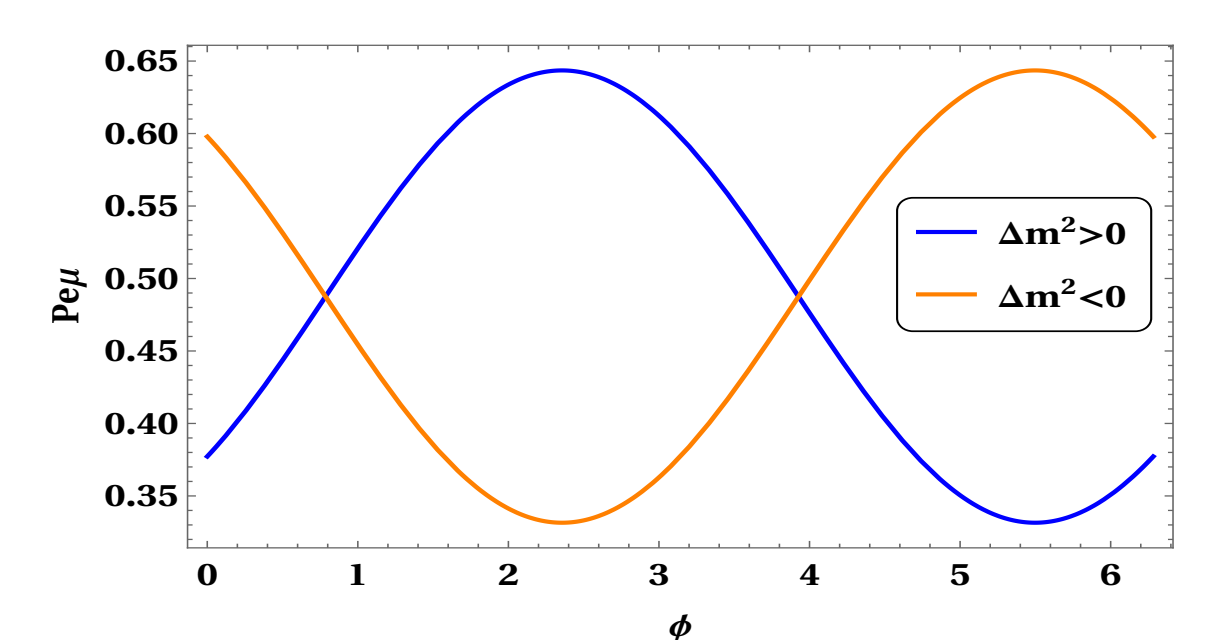
$$P_{e\mu} \neq P_{\mu e} \Rightarrow \text{T-violation}$$

Results and Discussions

- When decay eigenstates **are not aligned** with the mass eigenstates (off-diagonal term in Γ), probability expressions are sensitive to Majorana phase ϕ .



- Presence of η **violates** the equalities $P_{\mu\mu} = P_{ee}$ and $P_{\mu e} = P_{e\mu}$ that we see in the case of two flavour vacuum oscillations.
- The terms with \mathcal{B} , present in oscillation probabilities, have opposite signs for the two cases: $a_2 > a_1$ ($m_2 > m_1$) and $a_2 < a_1$ ($m_2 < m_1$).
 ⇒ Oscillation probability (Not the survival probability) is sensitive to **mass hierarchy**.



Different types of CP-violation are possible:

- due to the Majorana phase ϕ which we call **CP-violation in mass**.
- due to the phase ξ of Γ_{12} which we call **CP-violation in decay**.
- most general possibility is $\eta \neq 0$, $\xi \neq 0$ and $\phi \neq 0$. In this case, we have **CP-violation** due to both mass and decay provided $\phi \neq \xi$.

- In two special cases, when $\phi = \xi$ or when $\phi = \xi = 0$, there is **no CP-violation** even if $\eta \neq 0$. In such a situation, the flavour conversion probabilities are insensitive to off-diagonal elements of Γ but the flavour survival probabilities do depend on them.

Observational effects

- Supernova 1987A, $\tau_\nu \geq 5.7 \times 10^5$ (m_ν/eV)
 ⇒ $\Gamma_\nu \equiv b \approx 10^{-21}$ eV for $m_\nu \sim 1$ eV
[J.A.Frieman et al. Phys.Lett.B 200\(1988\)](#)
- The new effects considered in this work are of order $\eta/(a_2 - a_1) = \eta E / \Delta m^2$. These effects are of order 10% for $\Delta m^2 \approx 10^{-4}$ eV² if $E \approx 10^7$ GeV.
- Ultra high energy neutrinos from **astrophysical sources** provide a platform to study the effect.

Acknowledgements

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Reference

A.K. Pradhan et al. Phys. Rev. D 107, 013002