# A minimalist flavour symmetry for neutrinos: modular $S_3$

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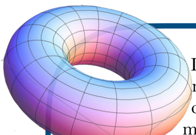
## Motivation

In the recent past, substantial effort has been devoted to exploring flavour symmetries to solve the flavour puzzle. In the lepton sector, the goal is to understand the mass hierarchies of the charged leptons and the neutrinos mass scale and mixing pattern.
However, traditional flavour symmetry models proved to be quite unsatisfactory: the symmetry-breaking sector spoils the predictivity given the large number of free parameters, a to account for the $\theta_{13} \neq 0$ experimental result [1]. In 2017, a new 'bottom-up' approach based on modular invariance was suggested [2]. This represents a nice opportunity in model building since it may allow to increase the predictivity and shed some light on the most important aspects of next-gen neutrino oscillations experiments, e.g. the mass ordering and CP-violation. Up until 2023, no minimalistic construction for leptons using the smallest finite modular group $S_3$ was available. In this work, we found suitable choices for the irreducible representations of the fields that provide a predictive model for neutrino masses and mixing with modular $S_3$.

## Modular Invariance



In the modular framework, the Standard Model Yukawa couplings become modular forms, i.e. special holomorphic functions of a complex variable $\tau$ called "modulus". Modular forms $Y(\tau)$ are defined to transform under the modular group SL(2,ℤ) as

$$Y(\gamma(\tau)) = (c\tau + d)^{k_Y}\, Y(\tau) \qquad \tau \to \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1$$

$\gamma \in \mathrm{SL}(2,\mathbb{Z})$ — Yukawa couplings — $k_Y$ weight $\in \mathbb{Z}^+$

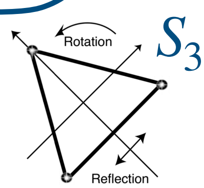
$$Y_i(\tau) \to (c\tau + d)^{k_Y}\, \rho_{ij} Y_j(\tau)$$

$$\varphi_i \to (c\tau + d)^{-k_\varphi}\, \rho_{ij}\, \varphi_j$$

$\rho_{ij}$ unitary irreps of $\Gamma_N = \Gamma/\Gamma(N)$

$$\mathbf{1} \subset \rho_Y \otimes \rho_{\varphi_1} \otimes \rho_{\varphi_2} \otimes \rho_{\varphi_3}: \quad \overbrace{\underbrace{Y(\tau)\, \varphi_1 \varphi_2 \varphi_3}_{k_Y = k_{\varphi_1} + k_{\varphi_2} + k_{\varphi_3}}}$$

| | $E_1^c$ | $E_2^c$ | $E_3^c$ | $D_\ell$ | $\ell_3$ | $H_{d,u}$ |
|---|---|---|---|---|---|---|
| $SU(2)_L \times U(1)_Y$ | $(\mathbf{1},+1)$ | $(\mathbf{1},+1)$ | $(\mathbf{1},+1)$ | $(\mathbf{2},-1/2)$ | $(\mathbf{2},-1/2)$ | $(\mathbf{2},\mp 1/2)$ |
| $\Gamma_2 \cong S_3$ | $\mathbf{1}$ | $\mathbf{1'}$ | $\mathbf{1'}$ | $\mathbf{2}$ | $\mathbf{1'}$ | $\mathbf{1}$ |
| $k_I$ | 4 | 0 | −2 | 2 | 2 | 0 |




## Charged-leptons

$$M_\ell = \begin{pmatrix} \alpha(Y_{\mathbf{2}}^{(3)})_1 & \alpha(Y_{\mathbf{2}}^{(3)})_2 & \alpha_D Y_{\mathbf{1'}}^{(3)} \\ \beta Y_2 & -\beta Y_1 & 0 \\ 0 & 0 & \gamma \end{pmatrix}_{RL} v_d\,,$$

Symmetry-Zero

$$\frac{\beta}{\alpha} \approx \frac{\alpha_D}{\alpha} \approx \frac{\gamma}{\alpha} \sim 1$$

$$m_\ell = m_\tau(1, |Y_1|, |Y_1|^3)$$

## Neutrinos: Weinberg operators

$$m_\nu = \frac{2 g v_u^2}{\Lambda}\left[\begin{pmatrix} -(Y_2^2 - Y_1^2) & 2Y_1Y_2 & \frac{g'}{2g} 2Y_1Y_2 \\ 2Y_1Y_2 & (Y_2^2 - Y_1^2) & -\frac{g'}{2g}(Y_2^2 - Y_1^2) \\ \frac{g'}{2g} 2Y_1Y_2 & -\frac{g'}{2g}(Y_2^2 - Y_1^2) & 0 \end{pmatrix} + \begin{pmatrix} \frac{g''}{g}(Y_1^2 + Y_2^2) & 0 & 0 \\ 0 & \frac{g''}{g}(Y_1^2 + Y_2^2) & 0 \\ 0 & 0 & \frac{g_p}{g}(Y_1^2 + Y_2^2) \end{pmatrix}\right]$$

$$\frac{g''}{g}, \frac{g'}{g}, \frac{g_p}{g} \in \mathbb{R}$$

$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}} = \begin{pmatrix} \frac{7}{100} + \frac{42}{25}q + \frac{42}{25}q^2 + \frac{168}{25}q^3 + \ldots \\ \frac{14\sqrt{3}}{25} q^{1/2}(1 + 4q + 6q^2 + \ldots) \end{pmatrix},$$

$$q = \exp(2\pi i \tau) \qquad \tau = \mathrm{Re}\,\tau + i\,\mathrm{Im}\tau \qquad [\mathrm{Im}\,\tau > 0]$$

CP-violation [3]

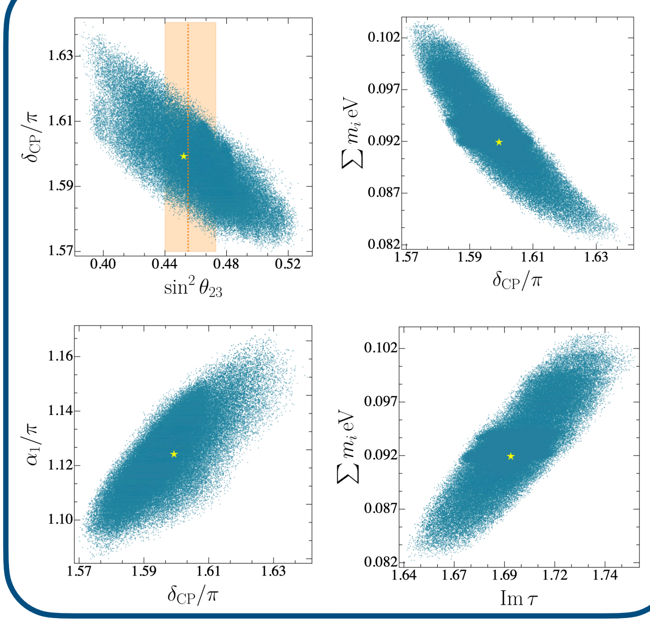
**Fit:** $\{\theta_{12}, \theta_{13}, \theta_{23}, \Delta m^2_{sol}/\Delta m^2_{atm}\}$

**Predict:** $\{\delta_{cp}, \alpha_1, \alpha_2, m_{\text{lightest}}\}$

**Majorana phases**

## Numerical Analysis

All points with $\sqrt{\Delta\chi^2} < 5$




## Results

**Mass ordering: N.O ($\chi^2_{min}$)** $\qquad \delta_{cp} \sim 1.6\,\pi$

$\sum_i m_i \sim 0.09$ eV < recent bounds [4] $\qquad |m_{\beta\beta}| \sim m_\beta \sim 20$ meV

## Conclusions

For the first time in modular flavour literature, a model based on the smallest modular finite group successfully describes the lepton sector data using a number of free parameters < 12 which is the number of observables we wish to reproduce or predict in the Majorana neutrino scenario. A number of predictions are provided, as well as interesting correlation plots between the observables. The CP violation is entirely provided by the VEV of the complex modulus $\tau$ and is predicted to be near maximal violation.

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## References

[1] F. P. An *et al.*, "Observation of electron-antineutrino disappearance at daya bay," *Phys. Rev. Lett.* **108** (Apr, 2012)
[2] F. Feruglio, "Are neutrino masses modular forms?," in *From My Vast Repertoire ...: Guido Altarelli's Legacy*, pp. 227–266. World Scientific, (2018). arXiv:1706.08749
[3] P.P. Novichkov, J.T. Penedo, S.T. Petcov and A.V. Titov, Generalised CP Symmetry in Modular-Invariant Models of Flavour, JHEP **07** (2019) 165 [arXiv:1905.11970]
[4] eBOSS collaboration, Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory, Phys. Rev. D **103** (2021) 083533